Mostafa ElShamy - Assignment 1 - Vision and Image Processing
VDX832

"Linear Algebra & Differential Calculus"

1)

1. if $v = [-2, b, 7, 2, a]^T$, then $v = \begin{bmatrix} -2 \\ b \\ 7 \\ 2 \\ a \end{bmatrix}$

$$v^T v = [-2, b, 7, 2, a]_{1\times5} \cdot \begin{bmatrix} -2 \\ b \\ 7 \\ 2 \\ a \end{bmatrix}_{5\times1} = \overbrace{(-2\times-2)}^{4} + \overbrace{(b\times b)}^{b^2} + \underbrace{(7\times7)}_{49} + \underbrace{(2\times2)}_{4} + \underbrace{(a\times a)}_{a^2}$$

$$\underline{v^T v = 57 + a^2 + b^2}$$

Furthermore, knowing that for any vectors $x, y \in \mathbb{R}^n$, $x^T y = y^T x$, we can conclude that $vv^T = v^T v = 57 + a^2 + b^2$

Citation: "Linear Algebra Review and Reference"
Zico Kolter, 2015.

I will continue to reference the above citation throughout Section 1 of this assignment.

2.

$$V = \begin{bmatrix} -4 & 8 \\ -5 & -9 \\ 4 & -7 \end{bmatrix}_{3\times2} \cdot \begin{bmatrix} x \\ y \end{bmatrix}_{2\times1} = \begin{bmatrix} -4x+8y \\ -5x-9y \\ 4x-7y \end{bmatrix}_{3\times1}$$

$$W = \begin{bmatrix} -5 & 4 & -6 \\ 3 & 4 & -6 \\ 8 & -6 & 8 \end{bmatrix}_{3\times3} \cdot \begin{bmatrix} -4x+8y \\ -5x-9y \\ 4x-7y \end{bmatrix}_{3\times1} =$$

$$= \begin{bmatrix} -5(-4x+8y)+4(-5x-9y)-6(4x-7y) \\ 3(-4x+8y)+4(-5x-9y)-6(4x-7y) \\ 8(-4x+8y)-6(-5x-9y)+8(4x-7y) \end{bmatrix}$$

$$= \begin{bmatrix} -24x-34y \\ -56x+30y \\ 30x+62y \end{bmatrix}$$

$$C = \begin{bmatrix} (-5\times-4)+(+4\times-5)+(-6\times4), & (-5\times8)+(4\times-9)+(-6\times-7) \\ (3\times-4)+(4\times-5)+(-6\times4), & (3\times8)+(4\times-9)+(-6\times-7) \\ (8\times-4)+(-6\times-5)+(8\times4), & (8\times8)+(-6\times-9)+(8\times-7) \end{bmatrix}_{3\times}$$

$$C = \begin{bmatrix} -24 & -34 \\ -56 & 30 \\ 30 & 62 \end{bmatrix}_{3\times2}, \quad Z = \underset{3\times2}{C} \cdot \begin{bmatrix} x \\ y \end{bmatrix}_{2\times1}$$

$$Z = \begin{bmatrix} -24x-34y \\ -56x+30y \\ 30x+62y \end{bmatrix}.$$

3.

$$A^2 = \begin{bmatrix} 1(1)-8+x-x+7, & -1-x+6-5+x, & -1-x+6-5+x \\ 8-x+8x-x^2-48+6x+x^2-7x-6x+42, & -8-x+x^2-6x-6x+36+5x-x^2-30+6x, & -8-x+x^2-6x-6x+36+5x-x^2-30+6x \\ x-7+40-5x-8x+x^2+5x-35-x^2+7x & -x+7+5x-30-x^2+6x+25-5x-5x+x^2 & -x+7+5x-30-x^2+6x+25-5x-5x+x^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 2 & -2 & -2 \\ -2 & 2 & 2 \end{bmatrix}$$

$$A^3 = A \cdot A \cdot A = A^2 \cdot A = \begin{bmatrix} 0 & 0 & 0 \\ 2 & -2 & -2 \\ -2 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & -1 \\ 8-x & x-6 & x-6 \\ x-7 & 5-x & 5-x \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0, & 0, & 0 \\ 2-16+2x-2x+14, & -2-2x+12-10+2x, & -2-2x+12-10+2x \\ -2+16-2x+2x-14, & 2+2x-12+10-2x, & 2+2x-12+10-2x \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4. Let $f_1 = \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{1z} \end{bmatrix}$, and likewise for f_2, f_3

By the properties of matrix multiplication, we can concatinate the f and e matrices to produce:

$$A \cdot f = e$$

where $f = \{f_1, f_2, f_3\}$ & $e = \{e_1, e_2, e_3\}$

This provides us with the following equation:

$$\overset{A}{\begin{bmatrix} -95 & 72 & -4 \\ -144 & 109 & -6 \\ 76 & -57 & 3 \end{bmatrix}} \overset{\bullet}{\bullet} \overset{f}{\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}} \overset{=}{=} \overset{e}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

Now we can see clearly that e is the identity matrix. Therefore we can conclude that f is actually A^{-1}, as by definition of the inverse matrix, it is the matrix such that $AA^{-1} = \underline{I}$. As such, to compute f_1, f_2 and f_3, we will start by computing A^{-1}.

$$A^{-1} = \frac{\operatorname{adj}(A)}{|A|} \qquad \operatorname{adj}(A) = \begin{bmatrix} \begin{vmatrix} 109 & -6 \\ -57 & 3 \end{vmatrix}, & -\begin{vmatrix} -144 & -6 \\ 76 & 3 \end{vmatrix} & \begin{vmatrix} -144 & 109 \\ 76 & -57 \end{vmatrix} \\ -\begin{vmatrix} 72 & -4 \\ 109 & -6 \end{vmatrix} & \begin{vmatrix} -95 & -4 \\ 76 & 3 \end{vmatrix} & -\begin{vmatrix} -95 & 72 \\ 76 & -57 \end{vmatrix} \\ \begin{vmatrix} 72 & -4 \\ 109 & -6 \end{vmatrix} & \begin{vmatrix} -95 & -4 \\ -144 & -6 \end{vmatrix} & \begin{vmatrix} -95 & 72 \\ -144 & 109 \end{vmatrix} \end{bmatrix}^T$$

$$\operatorname{adj}(A) = \begin{bmatrix} -15 & 12 & 4 \\ -24 & 19 & 6 \\ -76 & 57 & 13 \end{bmatrix}$$

$$|A| : \begin{array}{cccccc} -95 & 72 & -4 & -95 & 72 & -4 \\ -144 & 109 & -6 & -144 & 109 & -6 \\ 76 & -57 & 3 & 76 & -57 & 3 \end{array}$$

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$$|A| = (-95)(109)(3) + (72)(-6)(76) + (-4)(-144)(-57)$$
$$-(-95)(-6)(-57) - (72)(-144)(3) - (-4)(109)(76)$$

$$|A| = 1, \quad A^{-1} = \frac{\operatorname{adj}(A)}{|A|} = \begin{bmatrix} -15 & 12 & 4 \\ -24 & 19 & 6 \\ -76 & 57 & 13 \end{bmatrix}$$

Hence, $P_1 = \begin{bmatrix} -15 \\ -24 \\ -76 \end{bmatrix}$, $P_2 = \begin{bmatrix} 12 \\ 19 \\ 57 \end{bmatrix}$, $P_3 = \begin{bmatrix} 4 \\ 6 \\ 13 \end{bmatrix}$.

The answers to the subsequent questions have been solved through the solution above. B would be equivalent to what I labelled P, and $AB = I$, $BA = I$, hence $B = A^{-1}$.