VIP - Assignment 1

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3. Derivatives

1. From the chain rule: $f'(x) = -e^{\frac{x^2}{2}} \cdot x$

2. From product rule: $f''(x) = -e^{\frac{x^2}{2}} + -e^{\frac{x^2}{2}} \cdot x = -e^{\frac{x^2}{2}}(1+x)$

3. The plot of the function is shown below:

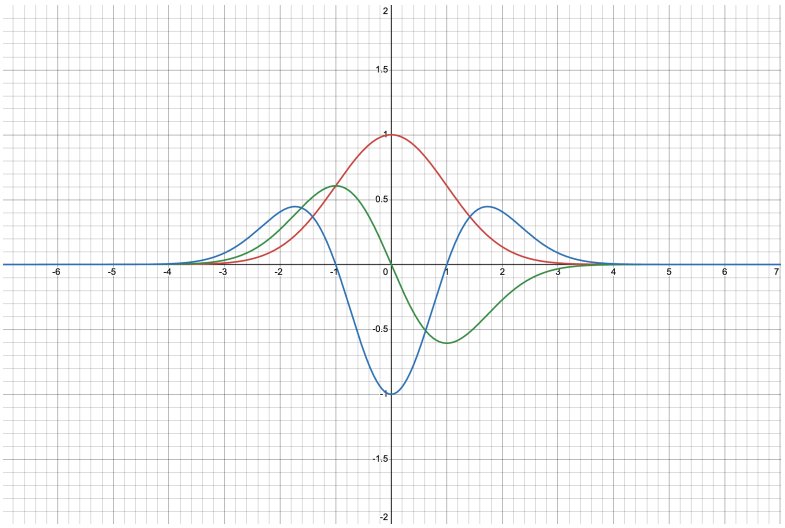

Figure 1: Made in Desmos: `https://www.desmos.com/calculator/vwhzrydfot`

4. From quotient rule: $g'(t) = \frac{-\sin(x)^2 - \cos(x)^2}{\sin(x)^2} = \frac{-1}{\sin(x)^2} = -\csc^2(x)$

5.

6. If t is a constant, we can factor out the beginning fraction and continue on: $\frac{\partial f}{\partial x}(x,t) = \frac{1}{\sqrt{2\pi t}} \cdot -e^{-\frac{x^2}{2t}} \frac{-x}{t} = -\frac{e^{-\frac{x^2}{2t}} x}{t\sqrt{2\pi t}}$

7. When x is a constant:

$$\frac{\partial f}{\partial t}(x,t) = \frac{(x^2-t)e^{\frac{-x^2}{2t}}}{2^{\frac{3}{2}}\sqrt{\pi}\sqrt{t^5}}$$

8. Again, the denominator is a constant in x-world so $\frac{\partial^2 f}{\partial x^2} = \frac{(x^2-t)e^{-\frac{x^2}{2t}}}{\sqrt{2}\sqrt{\pi}\sqrt{t^5}}$

9. $\frac{1}{2}\frac{\partial^2 f}{\partial x^2} = \frac{1}{2} \cdot \frac{(x^2-t)e^{-\frac{x^2}{2t}}}{2^{\frac{1}{2}}\sqrt{\pi}\sqrt{t^5}} = \frac{(x^2-t)e^{-\frac{x^2}{2t}}}{2^{\frac{3}{2}}\sqrt{\pi}\sqrt{t^5}} = \frac{\partial f}{\partial t}(x,t)$

10. - The plot of the functions with t=1 is shown below:

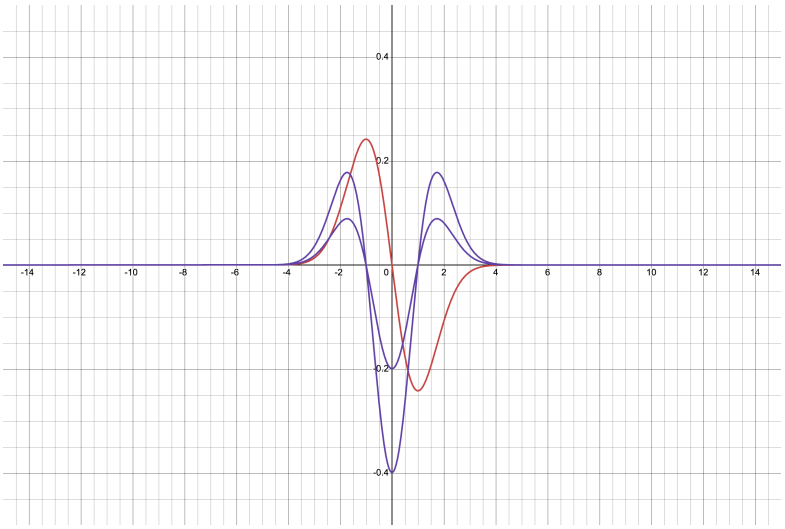

Figure 2: Made in Desmos: `https://www.desmos.com/calculator/xj3dii4bfv`

- The plot of the functions with t=2 is shown below:

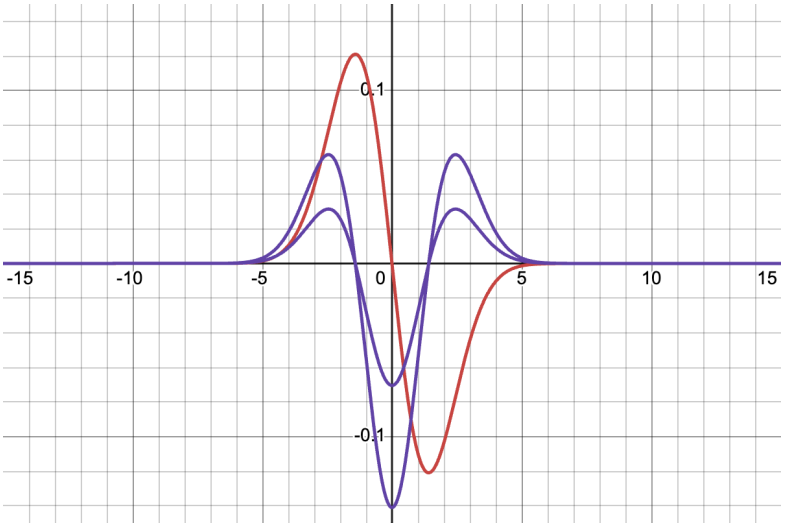

Figure 3: Made in Desmos: `https://www.desmos.com/calculator/1mtqeorgla`

- The plot of the functions with t=4 is shown below:

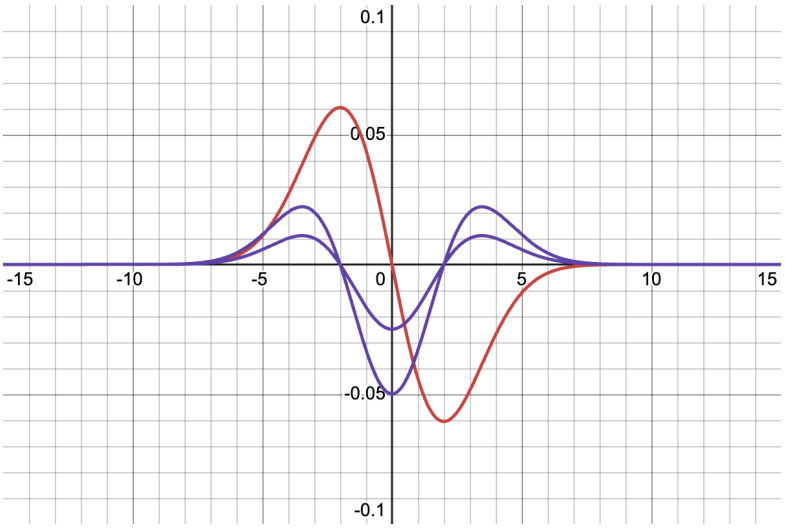

Figure 4: Made in Desmos: `https://www.desmos.com/calculator/t0uyphcscx`