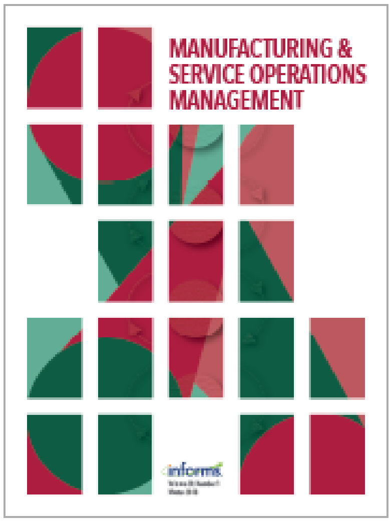

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Optimizing the Profitability and Quality of Service in Carshare Systems Under Demand Uncertainty

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Abstract. Carsharing has been considered as an effective means to increase mobility and reduce personal vehicle usage and related carbon emissions. In this paper, we consider problems of allocating a carshare fleet to service zones under uncertain one-way and round-trip rental demand. We employ a two-stage stochastic integer programming model, in the first stage of which we allocate shared vehicle fleet and purchase parking lots or permits in reservation-based or free-floating systems. In the second stage, we generate a finite set of samples to represent demand uncertainty and construct a spatial–temporal network for each sample to model vehicle movement and the corresponding rental revenue, operating cost, and penalties from unserved demand. We minimize the expected total costs minus profit and develop branch-and-cut algorithms with mixed-integer, rounding-enhanced Benders cuts, which can significantly improve computation efficiency when implemented in parallel computing. We apply our model to a data set of Zipcar in the Boston–Cambridge, Massachusetts, area to demonstrate the efficacy of our approaches and draw insights on carshare management. Our results show that exogenously given one-way demand can increase carshare profitability under given one-way and round-trip price differences and vehicle relocation cost whereas endogenously generated one-way demand as a result of pricing and strategic customer behavior may decrease carshare profitability. Our model can also be applied in a rolling-horizon framework to deliver optimized vehicle relocation decisions and achieve significant improvement over an intuitive fleet-rebalancing policy.

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Keywords: carshare fleet management • demand uncertainty • two-stage stochastic integer programming • Benders decomposition • mixed-integer rounding

1. Introduction

Increasing energy prices and parking fees in large cities have led to increased vehicle ownership costs, resulting in many individuals turning to other means of transportation. Public transportation has traditionally been a common alternative to private vehicle ownership. However, it is far from being a perfect replacement because of limited accessibility, fixed schedules, and the fact that users have to share common space and routes.

In recent years, carsharing has become a popular means of alternative transportation, serving as a middle ground between public transport and private ownership. It provides car rental services to customers on a short-term basis as opposed to long-term rentals provided by typical car rental companies. The popularity of carsharing is attributed to carshare users benefiting from private use of vehicles without having to bear the costs or responsibilities associated with car ownership. Carsharing also provides many benefits through reducing vehicle ownership in cities, such as reduced traffic congestion and reduced fuel consumption and vehicle emissions resulting from the use of clean-fuel vehicles and lowered overall miles traveled (Fan et al. 2008). In North America, studies suggest that carsharing has reduced vehicle mileage by 44% on average per carshare user with each carshare vehicle replacing between 6 and as many as 23 vehicles (Shaheen and Cohen 2007). The benefit of carsharing is further expanded by the use of electric vehicles; advances in electric vehicle charging technology will lower carshare fleet sizes and operational costs and bring greater reduction in CO_2 emissions (He et al. 2017). Hence, it is not surprising that almost 1,000 cities worldwide have adopted carsharing with more than 1.3 million individuals sharing almost 20,000 vehicles through carshare programs in the United States alone as of July 2015 (EcoPlan Association 2015).

Carshare systems can be broadly categorized into two types: reservation-based, in which customers must reserve vehicles prior to using them (e.g., Zipcar), and free-floating, in which customers can pick up any available car for immediate use (e.g., Car2Go). Carshare rentals can be categorized into one-way and round-trip rentals with the former allowing customers to rent and return vehicles in different locations and the latter only allowing returning rental cars to the same location.

From the customers' point of view, the availability of one-way rentals provides two main benefits. First, there can be more flexibility in vehicle use as vehicles do not have to be dropped off at the same location where they were picked up. Second, customers can potentially save on rental fees by splitting a round-trip rental into two one-way rentals.

From a carshare service provider's point of view, although one-way rentals have been requested by customers for a long time, providing such a service has not been a priority because of added management complexities (de Almeida Correia and Antunes 2012, Shaheen et al. 2006). Among these complexities, the most significant one is planning for imbalances in demand, for example, via vehicle relocation, which can be costly. Some of the cost can be offset by pricing one-way rentals at higher hourly rates; for example, Zipcar charges \$7.50–\$8.50 per hour for round-trip rentals and \$12 per hour for one-way rentals (see Zipcar 2015). However, the overall profitability of one-way rentals is questionable even with higher hourly rental prices. To carshare companies, optimally locating their car fleets in response to demand is important to their profitability and quality of service (QoS). As more carshare companies begin to offer one-way rentals in addition to traditional round-trip rentals, such as Zipcar through its ONE > WAY beta program, the problem of optimal fleet allocation and vehicle relocation becomes increasingly complex.

1.1. Problem Description

In this paper, we consider allocating a carshare fleet in a region serviced by a carshare service provider to satisfy uncertain travel demand. The region is discretized into smaller zones with parking costs different from zone to zone. To regulate carshares, city governments issue parking lot contracts and free-floating parking permits for shared cars (Cabanatuan 2014, Lindblom 2014). Parking lot contracts grant carshare exclusivity to lots and are typically taken up by reservation-based carshares, for which the carshare company takes into account parking capacities when accepting vehicle reservations. On the other hand, we consider an alternative way to be purchasing parking permits, often used by free-floating carshares, whose vehicles can be parked at any available city parking lot. We are tasked to allocate a homogeneous fleet of vehicles to zones by determining the number of contracted parking lots (for reservation-based systems) in each zone or the number of parking permits (for free-floating systems) to purchase.

These car fleet allocation decisions are made "here and now" before one-way and round-trip demand is realized at discrete time periods over a finite horizon. In each period, should the demand in a zone exceed the number of vehicles available for use at that zone, any excess demand is immediately lost (i.e., we assume that any excess demand is not carried over to the next period). During the finite horizon, we may relocate vehicles as recourse actions, and customers will be unable to use vehicles that are being relocated. Given each demand realization, we find the optimal vehicle movement to maximize the net profit minus the penalty of undesirable QoS results associated with unserved demand.

Indeed, efficient and economic carshare fleet allocation and reallocation is an emerging problem in practice, faced by almost all carshare service providers as an important step toward optimizing their profitability and QoS. Toyota, for example, provides carshare companies that purchase their vehicles with a fleet management system, which allows the fleet operator to optimize vehicle allocation in the network (Toyota Europe Newsroom 2013). Miveo, an award-winning carshare solutions provider in Europe, launched fleet management services to help customers improve fleet allocation planning (Frost & Sullivan 2016). In addition to fleet allocation, service providers and local governments also need to determine the number of parking spaces or parking permits to acquire or allocate within each region (Cabanatuan 2014, Chesto 2015, Lindblom 2014). This paper considers both car fleet allocation and parking planning. When it comes to carsharing in emerging economies, according to a recent survey (Lane et al. 2015), high capital investment and limited parking spaces are two of the major barriers for carshare companies while low labor cost is recognized as an opportunity. Therefore, in addition to fleet allocation, the problem considered in this paper seeks to overcome the barriers of high capital investment and limited parking spaces by optimizing vehicle and parking lot/permit allocation to service zones and takes advantage of the opportunity of low labor cost by optimizing vehicle relocation.

1.2. Methodology Overview

A carshare company may periodically check the distribution of vehicles in different zones to decide how to relocate them based on predictions of future demand. It follows a dynamic program, which reexamines vehicle locations and demand at each period to decide real-time vehicle reservation and relocation. However, the computation of such a model suffers from the "curse of dimensionality" and usually cannot provide efficient solutions under large-scale uncertainty.

In this paper, we consider a two-stage stochastic integer programming approach to approximate the results of the multistage dynamic model by aggregating recourse decisions and uncertainties in the second-stage problem after the first-stage problem determines car fleet allocation and parking lot/permit acquisition. We employ the sample average approximation (SAA) method (Kleywegt et al. 2002, Shapiro et al. 2009) based on generated i.i.d. samples of the random one-way and round-trip rental demand. For each sample, we model the movement of vehicles from zone to zone as flows on a spatial–temporal network with each node in the network representing the state of a zone in each time period during the finite horizon. Furthermore, we modify the two-stage stochastic programming model and implement it in a rolling-horizon manner to optimize vehicle relocation in each period. This allows carshare service providers to obtain not only the initial parking lot/permit acquisition and fleet allocation decisions, but also dynamic solutions of vehicle relocation over time.

We present three advantages of using spatial–temporal networks to model recourse decisions and outcomes related to the uncertainty. First, since each zone is replicated by the number of time periods, it is straightforward to represent vehicle movement as flows that are conserved between the spatial–temporal nodes and to keep track of the overall status of the vehicles (e.g., whether they are in use or, if available, where they are located). Second, we do not need to consider complex topologies of real-world road networks as they will not affect the construction of spatial–temporal networks. Third, when focusing on the reservation based– or free floating–only systems, the second-stage problem becomes a standard minimum cost-flow problem (Ahuja et al. 1993) on the corresponding spatial–temporal network, which can be solved very efficiently as a linear program.

A spatial–temporal network constructed in this paper could still involve a large number of nodes and arcs as a result of the large number of locations, the large number of periods, or both. To address the computational challenge, we employ the Benders decomposition approach (see e.g., Benders 1962, Van Slyke and Wets 1969) as a common method for optimizing two-stage stochastic programs. Moreover, we generalize a branch-and-cut algorithm with the mixed-integer rounding (MIR) subroutine, first proposed by Bodur and Luedtke (2017) for a call center staffing problem, to derive stronger cuts from Benders cuts.

1.3. Contribution and Main Results

We summarize our contribution and main results as follows.

• We study the strategic planning problem of purchasing parking lots/permits and allocating the initial car fleet in service zones to satisfy uncertain one-way and round-trip carshare demand. We formulate a two-stage, stochastic, integer-programming model to optimize both profitability and QoS by formulating strategic decisions in the first stage and using spatial–temporal networks to capture detailed vehicle movement in the second stage. Via decomposition, we develop a branch-and-cut algorithm with strengthened valid inequalities, which shows promising computational performance in parallel computing.

• Via extensive numerical experiments using augmented real-world data, we show that the impact of increasing one-way demand to carshare systems is complex and depends upon whether one-way demand is exogenously given or endogenously generated. With exogenous demand, our results show that higher one-way proportion can increase a carshare system's profitability. On the other hand, if one-way demand is endogenously determined by pricing and strategic customer behavior, our results show that higher one-way proportion could decrease a carshare system's profitability. Our results also suggest that the effective use of vehicle relocation plays an important role in carshare systems with one-way demand. The number of vehicle relocations will increase as one-way demand increases.

• We further leverage our model to optimize real-time vehicle relocation decisions in a rolling-horizon framework. When applied to the same real-world data, our proposed rolling-horizon method achieves a significant advantage in both profitability and QoS over a benchmark policy that periodically rebalances the fleet according to future demand concentration. Furthermore, we find that increasing the carshare fleet's vehicle relocation capacity can result in substantial profitability and QoS improvement.

1.4. Structure of the Paper

The remainder of this paper is organized as follows. Section 2 reviews the most relevant studies in carshare optimization and solution techniques. Section 3 introduces the construction of the spatial–temporal network and the two-stage stochastic integer program for fleet management and describes how to implement the two-stage model in a rolling-horizon framework to obtain dynamic vehicle relocation solutions. Section 4 develops branch-and-cut algorithms with MIR-enhanced Benders cuts for solving these models. Section 5 provides the insights drawn from applying the models on real-world data and also demonstrates the computational efficiency of the proposed algorithms. Section 6 concludes the paper and discusses future research directions.

2. Literature Review

There is a rapidly growing literature on sharing economies of various forms, including peer-to-peer product rental and service provisioning (Benjaafar et al. 2017b). We focus on carsharing in which a single service provider owns the vehicles. Shaheen et al. (1998) and Shaheen and Cohen (2007) review the history and recent rapid growth, respectively, of the carsharing industry. Katzev (2003) explores the early adoption processes of several carshare systems and evaluates the effects of carsharing on commuter mobility behavior and the environment. Bellos et al. (2017) study the impact of carsharing on an automobile manufacturer's product line design decisions in vehicle driving performance and fuel efficiency. He et al. (2017) optimize service regions for fleets of electric vehicles deployed and shared in urban environments. Jorge et al. (2014) and Kaspi et al. (2014) both consider one-way carshare systems: the former optimizes vehicle relocation via mathematical modeling, and the latter seeks effective parking reservation policies via a Markovian model and simulation. Kaspi et al. (2016) apply mixed-integer linear programming (MILP) models for regulating one-way carshare systems and designing parking reservation policies, which take into account a user behavior model for each policy design. Boyacı et al. (2015) formulate an MILP model for planning one-way carsharing station locations, sizes, and their fleet sizes, subject to vehicle relation and electric vehicle charging requirements. They optimize the model for large-scale problems by aggregating stations into virtual hubs and applying the branch-and-bound algorithm.

After fixing carshare stations and transport networks, how to relocate and redistribute vehicles during operation is a major consideration for satisfying customer demand. Relocating vehicles can be a complex process if dynamic shared fleet and demand distributions both must be taken into consideration. Weikl and Bogenberger (2013) summarize and categorize strategies used by carshare companies to relocate their fleets with real-life examples. Similar to our work, a variety of papers consider two-stage stochastic programming formulations to approximate the multistage dynamic optimization results. For example, Nair and Miller-Hooks (2011) develop a stochastic MILP model for optimizing vehicle relocation plans for shared vehicle systems and use a joint chance constraint to ensure a high rate of satisfying random demand. Benjaafar et al. (2017a) formulate a stochastic dynamic programming model and characterize the structure of the optimal policy for inventory repositioning in a product rental network. Similar planning and operational problems arise in bicycle sharing and have been tackled extensively in the past few years. For example, Shu et al. (2013) formulate a stochastic network flow optimization model to minimize the size of each sharing station and the cost of bicycle redistribution and approximate the optimal solutions by using a deterministic linear program. Moreover, Febbraro et al. (2012), Waserhole et al. (2013), and Pfrommer et al. (2014) all suggest real-time price incentives as a means to shape demand and reduce the need for excessive vehicle relocations. Vehicle allocation and repositioning has also been studied in manufacturing systems for automated guided vehicles (e.g., Ganesharajah et al. 1998; Hall et al. 2001a, b; Asef-Vaziri et al. 2001) and healthcare systems for emergency response vehicles (e.g., Brotcorne et al. 2003, Alanis et al. 2013).

To represent vehicle movement, Kek et al. (2009) use spatial–temporal network remodeling techniques to determine a set of nearly optimal manpower and operations to satisfy given relocation needs. de Almeida Correia and Antunes (2012) and Fan (2014) deploy spatial–temporal networks for optimizing the division of zones in a network and the allocation of vehicles to zones in one-way carshare systems, respectively. Similar network structures have also been used to study other logistics and transportation problems, for example, deployment of containers (Shu and Song 2013). However, the integration of one-way with round-trip rentals in reservation-based and free-floating carshare systems and the difficulty of handling a large spatial–temporal network have not been addressed in the literature mentioned here.

We provide Table 1 to summarize the most recent literature of carsharing or bike sharing, classified based on the decisions, sharing types, and methodologies employed. To our best knowledge, we are the first to integrate both one-way and round-trip rentals and strategic and operations decisions into comprehensive stochastic optimization models that take into account time-varying demand uncertainty, which we model by using spatial–temporal networks. However, we do not consider electric vehicles (EVs) and their charging problems.

Ferrero et al. (2015) provide a comprehensive survey of carsharing literature, classifying the papers according to several taxonomical axes, including (i) rental mode, (ii) vehicle engine for the service, (iii) optimization objective, (iv) time horizon, and (v) methodologies for the research model. According to this taxonomy, our paper targets several areas on the axes at which carsharing literature is lacking. In particular, our paper addresses one-way and round-trip demand in both reservation-based and free-floating modes and can be applied to shared car fleets with both fully thermic and green engines. The objectives of our paper pertain mainly to fleet management with the target time horizon being the strategic design of the system. Finally, our models are based on two-stage stochastic integer programming and network optimization techniques, which can be further implemented in a rolling-horizon framework to make relocation decisions at the operational level.

3. Problem Formulation

In this paper, we consider reservation-based or free-floating carshare systems. We first present a general

Table 1. Classification of the Related Literature

	Carshare or bike sharing	
	One-way only	Round-trip or mix
Decisions		
Planning (location, fleet size, reservation policy)	Boyacı et al. (2015), Fan (2014), de Almeida Correia and Antunes (2012), Kaspi et al. (2014, 2016)	Barrios and Godier (2014), Nair and Miller-Hooks (2014), Nourinejad and Roorda (2014), Chang et al. (2017), Martinez et al. (2012), Shu et al. (2013), He et al. (2017)
Operational (trip selection, fleet relocation, EV charging)	Boyacı et al. (2015), Fan (2014), Febbraro et al. (2012), Jorge et al. (2014), de Almeida Correia and Antunes (2012)	Kek et al. (2009), Weikl and Bogenberger (2013), Fan et al. (2008), Pfrommer et al. (2014), Shu et al. (2013)
Methods		
Deterministic (MILP, game, DP)	Boyacı et al. (2015), Kaspi et al. (2016)	Nair and Miller-Hooks (2011), Chang et al. (2017), Fan et al. (2008), Martinez et al. (2012)
Stochastic (sampling, MILP)	Fan (2014)	Nair and Miller-Hooks (2011), Shu et al. (2013), He et al. (2017)
Simulation, predictive models, decision-support systems	Jorge et al. (2014)	Barrios and Godier (2014), Kek et al. (2009), Nourinejad and Roorda (2014), Weikl and Bogenberger (2013), Pfrommer et al. (2014)
Spacial–temporal network	Kek et al. (2009), de Almeida Correia and Antunes (2012), Fan (2014)	
Involving EVs in the car fleet	Boyacı et al. (2015), He et al. (2017), Chang et al. (2017)	

model for a hybrid system, that is, one with both types of carshare modes, and then treat pure reservation-based and free-floating systems as special cases of the general model.

Consider the problem in which a carshare company needs to allocate a given budget of S vehicles in a set of zones, denoted by I, to maximize its profit and QoS over T time periods, using contracted parking lots (corresponding to a reservation-based system) and purchased parking permits (corresponding to a free-floating system). Decisions include the number of parking lots to purchase in zone i, denoted by w_i; the number of vehicles deployed in zone i that require contracted parking spaces, denoted by x_i^1; and the number of vehicles allocated in zone i with purchased parking permits, denoted by x_i^2, for all $i \in I$. We assume that customers are indifferent to whether a vehicle has a free-floating parking permit or requires a parking space. Let c_i^{lot} be the cost of acquiring one parking space, c_i^{loc} be the cost of allocating a vehicle in zone i for all $i \in I$, and c^{ffp} be the cost of one free-floating parking permit. Note that carsharing is also applicable under certain cases in which parking space may not be a concern, for example, nonmetropolitan areas and theme parks. In those cases, the parking-related cost can be set to zero, and the parking-related decisions will be trivial. In this paper, we focus on carshare systems that are used in areas in which parking space is limited, for example, metropolitan areas, and thus parking cost is significant.

We denote the demand for one-way rentals from zone i to zone j starting at period t and ending at period s by d_{ijts}^{one} and denote the demand for round-trip rentals from zone i starting at period t and ending at period s by d_{its}^{two}. The time taken to travel from zone i to zone j is denoted by l_{ij}, and the data satisfies $s - t \geq l_{ij}$ for any $d_{ijts}^{\text{one}} > 0$. As carshare companies typically use a time-based payment scheme, we assume that cost and revenue parameters for rentals are independent of zones but dependent on usage time. The revenue comes solely from customers using vehicles while costs incurred by the company include the costs of relocating vehicles, maintenance (resulting from wear and tear from car usage), and vehicle idleness (such as opportunity costs and depreciation costs). We denote the revenue per period, per vehicle from one-way rentals by $r^{\text{one}} \geq 0$ and that from round-trip rentals by r^{two}; the relocation cost per period per vehicle is c^{rel}. When a vehicle is in use, whether during one-way rentals, round-trip rentals, or relocation, it incurs a maintenance cost of c^{mnt} per period; when it is not in use, it incurs an idle cost of c^{idle} per period. Note that these costs can be set to zero without loss of generality. We include them for completeness in the paper.

3.1. Construction of the Spatial–Temporal Network

To model zone-to-zone vehicle movement over T periods, we construct a spatial–temporal network $G(N, A)$ with each node $n_{it} \in N$ representing a zone $i \in I$ at period $t \in \{0, 1, \ldots, T\}$. The arcs in this network are directed and represent a spatial–temporal movement of vehicles from one zone to another from an earlier period to a later one. There are four types of arcs in the network:

- One-way arcs $(n_{it}, n_{js}) \in A^{\text{one}}$ for each $d_{ijts}^{\text{one}} > 0$ with capacity d_{ijts}^{one} (for both types of vehicles, i.e., those that require parking spaces and those with free-floating parking permits) and cost $-(r^{\text{one}} - c^{\text{mnt}})(s - t)$ per unit flow. Flows on these arcs represent vehicles being rented one-way from zone i starting from period t and being returned to zone j in period s.

Table 2. Unit Flow Costs and Capacities for Each Arc Type

Type of arc	Cost per unit flow f_a	Capacity u_a
One-way arc (n_{it}, n_{js})	$-(r^{\text{one}} - c^{\text{mnt}})(s-t)$	d^{one}_{ijts}
Round-trip arc (n_{it}, n_{is})	$-(r^{\text{two}} - c^{\text{mnt}})(s-t)$	d^{two}_{its}
Relocation arc $(n_{it}, n_{j,t+l_{ij}})$	$(c^{\text{rel}} + c^{\text{mnt}})l_{ij}$	$+\infty$
Idle arc $(n_{it}, n_{i,t+1})$	c^{idle}	w_i $(m=1)$; $+\infty$ $(m=2)$

- Round-trip arcs $(n_{it}, n_{is}) \in A^{\text{two}}$ for each $d^{\text{two}}_{its} > 0$ with capacity d^{two}_{its} (for both types of vehicles, similar to one-way arcs) and cost $-(r^{\text{two}} - c^{\text{mnt}})(s-t)$ per unit flow. Flows on these arcs represent vehicles being rented round-trip from zone i starting from period t and ending in period s.
- Relocation arcs $(n_{it}, n_{j,t+l_{ij}}) \in A^{\text{rel}}$ for all pairs of zones i and j and periods $0 \le t \le T - l_{ij}$ with infinite capacity and cost $(c^{\text{rel}} + c^{\text{mnt}})l_{ij}$ per unit flow. Flows on these arcs represent vehicles being relocated from zone i in period t and arriving at zone j in period $t + l_{ij}$.
- Idle arcs $(n_{it}, n_{i,t+1}) \in A^{\text{idle}}$ for each zone i and period $0 \le t \le T-1$ with capacity w_i for reservation-based vehicles (i.e., $m=1$) or infinity for free-floating vehicles (i.e., $m=2$) and cost c^{idle} per unit flow. Flows on these arcs represent vehicles being idle in zone i from period t to $t+1$.

The set A is the union of the four types of arcs described, that is, $A = A^{\text{one}} \cup A^{\text{two}} \cup A^{\text{rel}} \cup A^{\text{idle}}$. For convenience, we use arc-based notation subsequently. We denote the unit cost of flow and the capacity of arc a by f_a and u_a, respectively, while $\delta^+(n_{it})$ and $\delta^-(n_{it})$ denote the sets of arcs for which n_{it} are their origin and destination nodes, respectively. The unit flow costs and capacities of each type of arcs are summarized in Table 2. In particular, the capacities of idle arcs for reservation-based vehicles (i.e., $m=1$) depend on the decision w_i of parking lot purchases.

We illustrate the construction of a spatial–temporal network with the following example. Consider two zones labeled B and B′ that require two periods of time to travel between them. Figure 1 shows the corresponding spatial–temporal network over periods $t = 0, \ldots, 3$. Included in the spatial–temporal network is a one-way arc corresponding to a demand of four vehicles to travel from B to B′ starting at period 0 (note that the ending period is automatically two periods after) and a round-trip arc corresponding to a demand of two vehicles picked up at and returned to zone B′, starting in period 1 and ending in period 3. The numbers on the two arcs denote the respective capacities.

3.2. A Two-Stage Stochastic Integer Programming Formulation

We employ a two-stage stochastic program, where $\mathbf{w} \in \mathbb{Z}^{|I|}_+$ denotes the vector of w_i's, and $\mathbf{x}^m \in \mathbb{Z}^{|I|}_+$ denotes the vector of x^m_i's with $m = 1, 2$. Moreover, let $c^1_i = c^{\text{loc}}_i$, $c^2_i = c^{\text{ffp}} + c^{\text{loc}}_i$, for each zone $i \in I$, and $M = \{1, 2\}$. We formulate the carshare fleet allocation problem as

$$\min_{\mathbf{w}, \mathbf{x}^1, \mathbf{x}^2} \left\{ \sum_{i \in I} \left(c^{\text{lot}}_i w_i + \sum_{m \in M} c^m_i x^m_i \right) + Q(\mathbf{w}, \mathbf{x}^1, \mathbf{x}^2) \right\} \tag{1}$$

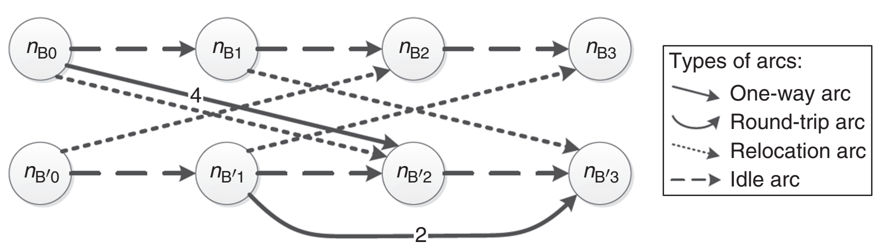
$$\text{s.t. } (\mathbf{w}, \mathbf{x}^1, \mathbf{x}^2) \in X = \left\{ \mathbf{w}, \mathbf{x}^1, \mathbf{x}^2 \in \mathbb{Z}^{|I|}_+ : \sum_{i \in I} \sum_{m \in M} x^m_i \le S, \; x^1_i \le w_i, \; \forall\, i \in I \right\}. \tag{2}$$

The set X requires that the number of vehicles initially deployed to each zone that require parking lots does not exceed the number of parking lots purchased, and the total number of vehicles does not exceed the given budget S. We minimize the total costs of allocating vehicles and purchasing parking lots/permits. The function $Q(\mathbf{w}, \mathbf{x}^1, \mathbf{x}^2)$ returns the optimal cost in the second stage, given first-stage decisions $\mathbf{w}$, $\mathbf{x}^1$, $\mathbf{x}^2$, of which the formulation details are given as follows.

The second-stage problem optimizes flows in the spatial–temporal network given that the supply level at each node n_{i0} is x_i and the capacity on each arc a is u_a determined by the random demand and the number of parking spaces w_i for all $i \in I$. We define recourse decisions y^m_a, $a \in A$, $m \in M$, to represent reservation-based $(m=1)$ and free-floating $(m=2)$ vehicle movements in the spatial–temporal network. Letting $\mathbf{u}$ be the vector of arc capacities u_a, the feasible region of shared vehicle movement is given by

$$Y(\mathbf{w}, \mathbf{x}^1, \mathbf{x}^2, \mathbf{u}) := \left\{ y^m_a \ge 0, \; \forall\, a \in A, \; \forall\, m \in M: \sum_{a \in \delta^+(n_{it})} y^m_a \right.$$

Figure 1. A Spatial–Temporal Network Example for a Two-Zone, Three-Period Instance



$$- \sum_{a \in \delta^-(n_{it})} y_a^m = \begin{cases} x_i^m & \text{if } t = 0 \\ 0 & \text{if } t = 1, \ldots, T-1 \\ -x_i^m & \text{if } t = T, \end{cases} \quad \forall\, i \in I,\ m \in M \tag{3}$$

$$\sum_{m \in M} y_a^m \le u_a, \quad \forall\, a \in A^{\text{one}} \cup A^{\text{two}} \tag{4}$$

$$y_a^1 \le w_i, \quad \forall\, i \in I,\ a = (n_{it}, n_{i,t+1}) \in A^{\text{idle}} \tag{5}$$

$$\left. y_a^m \in \mathbb{Z}_+, \quad \forall\, a \in A \right\}, \tag{6}$$

where (3) is the multi-commodity flow balance constraint, (4)–(5) are the capacity constraints, and (6) is the integrality constraint. The flow balance constraint for the spatial–temporal nodes in the last period requires the final allocation of vehicles being the same as the initial allocation for the purpose of operating the carshare system every T periods with the same initial deployment of vehicles.

Capacities u_a of the one-way and round-trip arcs in Table 2 are random as a result of demand uncertainty. In this paper, we employ Monte Carlo sampling to generate a finite number of demand scenarios from a joint distribution of one-way and round-trip rentals. We index the scenarios by $k \in K$ and denote the vector of capacities of the arcs in scenario k by $\mathbf{u}^k = [u_a^k,\ a \in A^{\text{one}} \cup A^{\text{two}}]^\mathsf{T}$ and the probability of occurrence of scenario k by p^k. For the objective, we minimize the expected cost of scenario-based vehicle movements $\mathbf{y}^k = [y_a^{km},\ m \in M,\ a \in A]^\mathsf{T}$, $k \in K$, plus some random penalty incurred from unserved demands. Thus,

$$Q(\mathbf{w}, \mathbf{x}^1, \mathbf{x}^2) = \min_{\mathbf{y}^1, \ldots, \mathbf{y}^{|K|}} \left\{ \sum_{k \in K} p^k \sum_{a \in A} f_a \sum_{m \in M} y_a^{km} + g(\mathbf{y}^1, \ldots, \mathbf{y}^{|K|}) \right\} \tag{7}$$

$$\text{s.t.} \quad \mathbf{y}^k \in Y(\mathbf{w}, \mathbf{x}^1, \mathbf{x}^2, \mathbf{u}^k) \quad \forall\, k \in K. \tag{8}$$

The first term in the objective function (7) denotes the expected cost of vehicle traveling, idleness, and relocation while the second denotes the penalty incurred in all the scenarios. According to (8), each decision vector $\mathbf{y}^k$ needs to satisfy constraints defined earlier in (3)–(5) with $\mathbf{u} = \mathbf{u}^k$, $\forall\, k \in K$.

3.2.1. A Risk-Neutral Model—Minimizing the Expected Penalty. Note that the number of unserved customers equals unused capacities on the one-way and round-trip arcs. We impose penalty $G_a \ge 0$ for each unit of unused capacities on arcs $a \in A^{\text{one}} \cup A^{\text{two}}$ and propose a risk-neutral model for maintaining a desired QoS level. As a result, we specify

$$g(\mathbf{y}^1, \ldots, \mathbf{y}^{|K|}) = \sum_{k \in K} p^k \sum_{a \in A^{\text{one}} \cup A^{\text{two}}} G_a \left(u_a^k - \sum_{m \in M} y_a^{km} \right).$$

Remark 1. Penalty in the risk-neutral case can be incorporated into regular arc flow cost. We keep the penalty term separate in order to present a general model. Different risk measures can be used in function g to formulate risk-averse optimization models. In Online Appendix A, we describe one that penalizes the conditional value-at-risk (CVaR) of unserved demand. CVaR is a coherent risk measure that is well studied in the stochastic programming literature; moreover, when demand has a finite support with a moderate number of scenarios, we can reformulate a CVaR-based model by using linear constraints and thus keep the same computational tractability of the solution methods proposed in Section 4 for solving the risk-neutral model.

Remark 2. Models (1) and (2) formulate a hybrid carshare system that possibly involves both reservation-based and free-floating vehicle flows with a two-commodity network flow problem in the second stage. In the rest of the paper, we focus on single-commodity carshare systems with either $M = \{1\}$ (reservation-based) or $M = \{2\}$ (free-floating). They can also be viewed as two special cases of the two-commodity model with c^{ffp} or c_i^{lot} set to infinity. These systems are more common in practice with Zipcar and Car2Go as the respective examples. (Zipcar also has free-floating vehicles in selected markets for experimental purposes. Car2Go also has contracted parking garages to encourage round trips. However, the majority of their vehicles are still pure reservation-based or free-floating, respectively.) Moreover, focusing on a single commodity also allows us to solve the second-stage problem efficiently as linear programs.

3.3. Rolling-Horizon Method for Vehicle Relocation

These models provide an initial assignment of vehicles to regions on a day-to-day basis. However, a carshare service provider may also need to make real-time vehicle relocation decisions for each period during the day. Here, we elaborate how to extend our models in a rolling-horizon framework to solve this problem. For ease of exposition, we consider the reservation-based system, that is, $M = \{1\}$. Development for the free-floating system or the general hybrid system will be similar.

Let $s \in \{1, 2, \ldots, T-1\}$ denote the current time period. We assume the following sequence of events in each period. First, vehicles that will finish rental or relocation in the current period are returned and immediately become available for rental. Second, rental demand is realized and fulfilled on a first-come, first-served basis. Any unmet demand is lost. Third, remaining vehicles, if any, after fulfilling demand are repositioned or remain in the same location. Let r_i^s be the number of available vehicles in zone i after demand has been fulfilled in the current period. Let v_{it}^s be the number of vehicles that are currently rented out or being relocated but that will be returned to zone i in period $t > s$. Let R^s be the relocation capacity in period s, for example, the number of employed relocation drivers who are available. Note that R^s is updated periodically. Define decision variable z_{ij}^s as the number of vehicles that start being relocated from zone i to zone j in the current period. Recall that l_{ij} is the time for relocating a vehicle from i to j. Let c_{ij}^{rel} denote the cost of relocating a vehicle from i to j. Let z_{ii}^s denote the number of vehicles that remain in zone i with $l_{ii} = 1$ and $c_{ii}^{\text{rel}} = c^{\text{idle}}$. Let $\mathbf{r}^s$, $\mathbf{v}^s$, and $\mathbf{z}^s$ denote vectors of r_i^s, v_{it}^s, and z_{ij}^s, respectively. The rest of the notation follows those previously defined. We present the rolling-horizon vehicle relocation problem in period s as

$$\min_{\mathbf{z}^s} \left\{ \sum_{i \in I} \sum_{j \in I} c_{ij}^{\text{rel}} z_{ij}^s + \sum_{k \in K} p_k Q_k^{s+1}(\mathbf{z}^s, \mathbf{v}^s) \right\} \tag{9}$$

$$\text{s.t.} \quad \sum_{j \in J} z_{ij}^s = r_i^s, \quad \forall\, i \in I \tag{10}$$

$$z_{ii}^s \le w_i, \quad \forall\, i \in I \tag{11}$$

$$\sum_{i} \sum_{j \ne i} z_{ij}^s \le R^s \tag{12}$$

$$z_{ij}^s = 0, \quad \forall\, i \in I, \forall\, j, l_{ij} > T - s \tag{13}$$

$$z_{ij}^s \in \mathbb{Z}_+, \quad \forall\, i, j \in I \tag{14}$$

The objective in (9) minimizes relocation and idling cost in period s plus expected total cost from all scenarios in set K over periods $s+1$ through T. Constraint (10) guarantees that all available vehicles either remain at the current zone or are relocated to a different zone. Constraint (11) is the parking space constraint. Constraint (12) is the relocation capacity constraint. Constraint (13) guarantees that all relocations must finish by the beginning of period T.

The second-stage problem in this formulation is also a minimum cost network flow problem on a spatial–temporal network, denoted by $G(V_s, A_s)$, which is similar to the one described in Section 3.1 except that it only includes periods $s+1, \ldots, T$ and all the nodes in $G(V_s, A_s)$ may have supplies. Let $J(i,t) = \{j \in I\colon l_{ji} = t\}$, that is, the set of zones from which it takes t periods to relocate a vehicle to zone i. Let $I_{t'}(t) = 1$ if $t = t'$ and $I_{t'}(t) = 0$ if $t \ne t'$. For each $k \in K$, the second-stage cost $Q_k^{s+1}(\mathbf{z}^s, \mathbf{v}^s)$ in (9) is the total cost over periods $s+1$, $\ldots, T$ under scenario k. It can be specified as

$$Q_k^{s+1}(\mathbf{z}^s, \mathbf{v}^s) = \min_{\mathbf{y}} \left\{ \sum_{a \in A_s} f_a y_a + \sum_{a \in A_s^{\text{one}} \cup A_s^{\text{two}}} G_a(u_a^k - y_a) \right\} \tag{15}$$

s.t.

$$\sum_{a \in \delta^+(n_{it})} y_a - \sum_{a \in \delta^-(n_{it})} y_a = v_{it}^s + \sum_{j \in J(i,t-s)} z_{ji}^s - I_{T+1}(t) x_i^1, \quad \forall (i,t) \in V_{s+1} \tag{16}$$

$$0 \le y_a \le u_a^k, \quad \forall\, a \in A_{s+1}. \tag{17}$$

Note that the flow balance constraint (16) has node supplies resulting from rentals and relocations, that is, v_{it}^s and z_{ji}^s. A detailed description of the rolling-horizon approach is included in Online Appendix B.

The rolling-horizon model provides carshare systems with real-time vehicle relocation decisions that can be implemented in practice. In Section 5.5, we implement the rolling-horizon model and show that it could achieve significant improvement over an intuitively appealing benchmark policy that periodically rebalances fleet allocation according to future demand concentration. Note that Models (9)–(14) assume that vehicles have already been allocated to fulfill current period demand. We could also include vehicle allocation decisions in the model to select the optimal subset of rental requests to fulfill and further maximize profit.

4. Solution Approaches

Because of the large number of variables and constraints involved in the MILP models proposed in Section 3, in this section, we develop a branch-and-cut algorithm with cuts enhanced by mixed-integer rounding (MIR), which was first proposed by Bodur and Luedtke (2017) for a stochastic call-center staffing problem. We demonstrate later that this algorithm outperforms the state-of-the-art solver in optimizing our model for instances with diverse sizes generated from real data.

The basic procedure branches on the integer variables and solves individual nodes via Benders decomposition (see Benders 1962, Van Slyke and Wets 1969); the Benders cuts at each node are added to the master problems of subsequent nodes, which, however, could be weak as a result of the relaxed integer constraints in the first stage. Consequently, the branch-and-cut algorithm may branch many times before termination. This motivates us to apply MIR to pairs of previously generated Benders cuts to obtain stronger valid cuts.

We present our solution algorithm in two parts. Section 4.1 decomposes the problem to solve with a branch-and-cut algorithm and Section 4.2 describes the MIR-enhanced algorithm.

4.1. Benders Decomposition

Consider Models (1) and (2) with $M = \{1\}$ and the objective specified as minimizing $\sum_{i \in I}(c_i^{\text{lot}} w_i + c_i^1 x_i^1) + Q(\mathbf{w}, \mathbf{x}^1)$. We decompose the problem into a master problem, consisting of the variables not indexed by k, and $|K|$ subproblems, consisting of the remaining variables separable by index k. Each subproblem corresponds to the spatial–temporal network based on a scenario in K. The Benders approach iteratively generates cuts from each subproblem and adds them to a relaxed version of the master problem. Specifically, the variables in the master problem are the first-stage variables

$\mathbf{w}$, $\mathbf{x}^1$ and auxiliary variables $\mathbf{q} = (q^1, \ldots, q^{|K|})^\mathsf{T}$, denoting the values of the subproblems at optimality. We formulate the master problem as

$$\textbf{MP}: \quad \min_{(\mathbf{w},\mathbf{x}^1)\in\tilde{X},\,\mathbf{q}\in\mathbb{R}^{|K|}} \left\{ \sum_{i\in I}(c_i^{\text{lot}} w_i + c_i^1 x_i^1) + \sum_{k\in K} p^k q^k : \right. \\ \left. L^k(q^k, \mathbf{w}, \mathbf{x}^1) \geq \mathbf{0},\ \forall\, k\in K \right\},$$

where set $\tilde{X}$ includes all the constraints in X except the integrality constraints on variables w_i and x_i^1, $\forall\, i\in I$; $L^k(q^k, w, x) \geq \mathbf{0}$ includes the set of cuts generated from solving the kth subproblem, $\forall\, k\in K$. We formulate the subproblems as the duals of the primal $Q(\mathbf{w},\mathbf{x}^1)$, separated by $k\in K$:

$\mathbf{SP}_k(\mathbf{w},\mathbf{x}^1)$:

$$\max_{\pi,\lambda} \left\{ \sum_{i\in I} x_i^1(\pi_{i0} - \pi_{iT}) + \sum_{a\in A^{\text{one}}\cup A^{\text{two}}} u_a^k \lambda_a \right. \\ \left. + \sum_{i\in I} w_i \left(\sum_{a=(n_{it}, n_{i,t+1})\in A^{\text{idle}}} \lambda_a \right) \right\}$$

$$\begin{aligned} \text{s.t.}\ & \pi_{it} - \pi_{j,t+l_{ij}} + \lambda_a \leq f_a - G_a \quad \forall\, a = (n_{it}, n_{js}) \in A^{\text{one}} \\ & \pi_{it} - \pi_{is} + \lambda_a \leq f_a - G_a \quad \forall\, a = (n_{it}, n_{is}) \in A^{\text{two}} \\ & \pi_{it} - \pi_{j,t+l_{ij}} \leq f_a \quad \forall\, a = (n_{it}, n_{j,t+l_{ij}}) \in A^{\text{rel}} \\ & \pi_{it} - \pi_{i,t+1} + \lambda_a \leq f_a \quad \forall\, a = (n_{it}, n_{i,t+1}) \in A^{\text{idle}} \\ & \lambda_a \leq 0 \quad \forall\, a\in A^{\text{one}}\cup A^{\text{two}}\cup A^{\text{idle}}, \end{aligned}$$

where π_{it} and λ_a are the dual variables associated with the flow balance constraint (3) and the capacity constraints (4) and (5), respectively.

At each iteration of the Benders decomposition algorithm, we optimize a relaxed master problem $\mathbf{MP}$, to obtain optimal solutions $(\hat{\mathbf{w}}, \hat{\mathbf{x}}^1, \hat{\mathbf{q}})$. We pass the solutions to the respective subproblems and optimize $\mathbf{SP}_k(\hat{\mathbf{w}}, \hat{\mathbf{x}}^1)$. If the optimal objective value of the subproblem corresponding to scenario k is greater than the optimal value of $\hat{q}^k$ given by the master problem, an optimality cut is generated to the master problem to remove this solution. For an optimal dual solution $(\hat{\pi}, \hat{\lambda})$ to $\mathbf{SP}_k(\hat{\mathbf{w}}, \hat{\mathbf{x}}^1)$ for scenario k, the Benders optimality cut is of the form

$$q^k - \sum_{i\in I}\left(\sum_{a=(n_{it}, n_{i,t+1})\in A^{\text{idle}}} \hat{\lambda}_a \right) w_i - \sum_{i\in I}(\hat{\pi}_{i0} - \hat{\pi}_{iT}) x_i^1 \\ - \sum_{a\in A^{\text{one}}\cup A^{\text{two}}} \hat{\lambda}_a u_a^k \geq 0. \tag{18}$$

Feasibility cuts are not generated because for any feasible $\mathbf{x}^1$ and $\mathbf{w}$, having the vehicles idle until the last period (i.e., $y_a = x_i^1$ for all arcs $a = (n_{it}, n_{i,t+1}) \in A^{\text{idle}}$ and $y_a = 0$ for all other arcs $a\in A^{\text{one}}\cup A^{\text{two}}\cup A^{\text{rel}}$) is always a feasible solution to the primal problem.

4.2. MIR Procedure

MIR is a procedure used to remove noninteger extreme point solutions from the linear relaxation of a mixed-integer program. We first introduce a generic form of the MIR inequality with a nonnegative real variable and multiple nonnegative integer variables.

Proposition 1 (Wolsey 1998)**.** *Let* $U := \{(\psi, \boldsymbol{\omega}) \in \mathbb{R}_+ \times \mathbb{Z}_+^m : \psi + \sum_{i=1}^m \alpha_i\omega_i - \delta \geq 0\}$ *and let* $\Delta > 0$. *If* $\text{frac}(\Delta\delta) > 0$, *then the cut*

$$\psi + \sum_{i=1}^m \frac{\min\{\lceil \Delta\alpha_i\rceil \text{frac}(\Delta\delta), \text{frac}(\Delta\alpha_i) + \lfloor \Delta\alpha_i\rfloor \text{frac}(\Delta\delta)\}}{\Delta}\omega_i \\ - \frac{\lceil \Delta\delta\rceil \text{frac}(\Delta\delta)}{\Delta} \geq 0$$

is valid for U.

The function $\text{frac}(b)$ is defined as $\text{frac}(b) := b - \lfloor b\rfloor$, that is, the fractional part of a scalar b. Bodur and Luedtke (2017) extend Proposition 1 for a set defined by two inequalities. Following their idea, consequently, one can generate a different valid cut from two valid Benders cuts. We describe this result in Theorem 1 with nomenclature relevant to our model and the corresponding Benders cuts.

Theorem 1. *Let* $k\in K$ *and*

$$q^k - \sum_{i\in I}\left(\sum_{a=(n_{it}, n_{i,t+1})\in A^{\text{idle}}} \hat{\lambda}_a^j \right) w_i - \sum_{i\in I}(\hat{\pi}_{i0}^j - \hat{\pi}_{iT}^j) x_i^1 \\ - \sum_{a\in A^{\text{one}}\cup A^{\text{two}}} \hat{\lambda}_a^j u_a^k \geq 0, \quad j = 1, 2$$

be any pair of Benders cuts (18) *that are valid for the set of cuts* $L^k(q^k, w, x) \geq \mathbf{0}$ *and let* $\Delta > 0$. *Define*

$$\begin{aligned} \alpha_i &:= -\sum_{a=(n_{it}, n_{i,t+1})\in A^{\text{idle}}} (\hat{\lambda}_a^2 - \hat{\lambda}_a^1), \\ \beta_i &:= -((\hat{\pi}_{i0}^2 - \hat{\pi}_{i0}^1) - (\hat{\pi}_{iT}^2 - \hat{\pi}_{iT}^1)), \\ \delta &:= \sum_{a\in A^{\text{one}}\cup A^{\text{two}}} (\hat{\lambda}_a^2 u_a^k - \hat{\lambda}_a^1 u_a^k). \end{aligned}$$

If $\text{frac}(\Delta\delta) > 0$, *then the cut*

$$q^k + \sum_{i\in I}\left(\frac{\min\{\lceil \Delta\alpha_i\rceil \text{frac}(\Delta\delta), \text{frac}(\Delta\alpha_i) + \lfloor \Delta\alpha_i\rfloor \text{frac}(\Delta\delta)\}}{\Delta} \right. \\ \left. - \sum_{a=(n_{it}, n_{i,t+1})\in A^{\text{idle}}} \hat{\lambda}_a^1 \right) w_i \\ + \sum_{i\in I}\left(\frac{\min\{\lceil \Delta\beta_i\rceil \text{frac}(\Delta\delta), \text{frac}(\Delta\beta_i) + \lfloor \Delta\beta_i\rfloor \text{frac}(\Delta\delta)\}}{\Delta} \right. \\ \left. - (\hat{\pi}_{i0}^1 - \hat{\pi}_{iT}^1) \right) x_i^1 \\ - \left(\frac{\lceil \Delta\delta\rceil \text{frac}(\Delta\delta)}{\Delta} + \sum_{a\in A^{\text{one}}\cup A^{\text{two}}} \hat{\lambda}_a^1 u_a^k \right) \geq 0 \tag{19}$$

is valid for $L^k(q^k, \mathbf{w}, \mathbf{x}^1) \geq \mathbf{0}$.

We present a detailed proof in Online Appendix C. In the branch-and-cut algorithm, we directly apply Theorem 1 to pairs of Benders cuts (18) generated by $\mathbf{SP}_k(\mathbf{w}, \mathbf{x}^1)$ to generate new valid cuts. We include in Online Appendix D the implementation details of the branch-and-cut algorithms with MIR-enhanced Benders cuts.

5. Numerical Results

In this section, we test instances generated based on real company data and conduct computational studies to demonstrate the results of our approach. Section 5.1 describes the details of our experimental setup. Section 5.2 analyzes how one-way and round-trip demand proportions affect solution profitability and QoS given fixed prices used in practice. Section 5.3 studies the impact of other important factors, such as the size of the available fleet and the penalty cost of lost demand. Section 5.4 examines the profitability and QoS impact of endogenously generated one-way demand as a result of pricing and strategic customer behavior. Section 5.5 presents results of the rolling-horizon method for optimizing vehicle relocation. Section 5.6 demonstrates the efficacy of MIR-enhanced Benders cuts and parallel computing. Section 5.7 compares the results of our models with the ones of a deterministic benchmark and presents the value of stochastic solutions (VSS). It should be pointed out that these numerical results are obtained based on one data set and specific assumptions. We have tested the robustness of our results by modifying demand distribution and variability as well as important parameters such as relocation cost. Our key observations still hold under these changes. However, as more real-world carshare demand data become publicly available in the future, it is important to test our models with new data and generalize the observations to carshare operations in different situations and areas.

5.1. Numerical Experiment Settings

5.1.1. Data Generation. We use Zipcar's rental data collected from the Boston—Cambridge, Massachusetts, area. There are in total 61 days of data, from October 1 to December 1, 2014, containing the information of the starting–ending time and the zip codes of the origin and destination zones of each rental. Only successful rentals were recorded, that is, demand is right censored with unobservable lost sales. (In practice, uncensored data could be available to the service provider from users' search history.) We divide the Boston–Cambridge area into nine zones, according to significant traveling patterns shown in the data, which is also consistent with Zipcar's current zone partition. Note that these zones should not be considered as individual service locations. Allocating vehicles to individual service locations requires much finer zone division. The rentals are labeled as one-way or round-trip, depending on whether they have different or the same starting and ending zones, respectively. In this data set, one-way rental service is available in all zones, and thus, there is no one-way demand censoring as a result of limited service coverage. (In other situations in which one-way service is only available in selected zones, one must also consider potential one-way demand that is lost because of limited coverage.) Table 3 shows some summary statistics of the data. Since we consider a discrete time horizon with one period equal to one hour, all trip durations are rounded to the nearest integer greater than or equal to one hour. One-way rentals are aggregated by the quadruple (origin, destination, starting hour, ending hour) while round-trip rentals are aggregated by the triple (origin, starting hour, ending hour). There is one observation for each rental demand record (quadruple/triple) every day, resulting in 61 observations for each record.

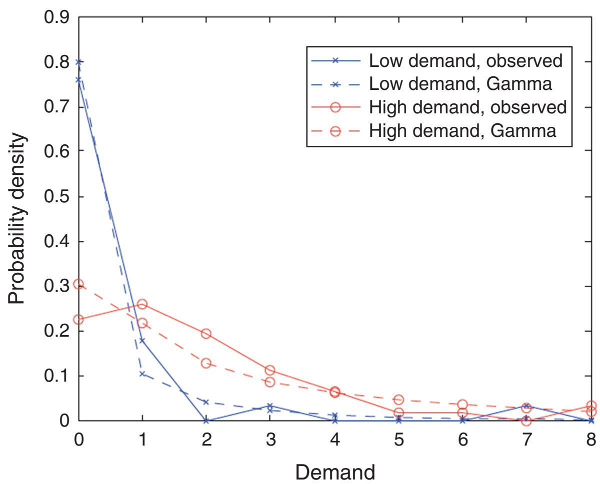
Table 3. Summary Statistics of Demand Data

	One-way	Round-trip
Average number of trips per day	91.39	1,510.90
Proportion of trips (%)	5.70	94.30
Average trip duration	36 min	2 hr 14 min
Standard deviation of trip duration	21 min	1 hr 16 min

To generate more samples for the stochastic programming model and vary the proportion of one-way rentals in our analysis, we generate the number of rentals for each aggregated record using gamma distributions with means and variances equal to the empirical means and variances of the Zipcar data. The simulated gamma random variables are rounded to the nearest integers. We choose the gamma distribution because demand is nonnegative and has high probability of being zero. We have performed Kolmogorov–Smirnov tests and found no statistically significant difference between the distributions of the observed data and the data simulated using gamma distribution. Figure 2 shows two examples with high and low demand volumes, respectively. Furthermore, we have also tested other demand distributions, such as the log-normal distribution, and found that our key observations still hold.

In our experiments, we fix the mean total rental hours over a 24-hour period (one day) to 1,000 vehicle-hours. In order to vary the proportion of one-way demand, the means and variances of the number of hourly rental requests are scaled accordingly. For example, to have approximately 40% one-way rental hours, we divide $1{,}000 \times 40\% = 400$ by the average daily one-way rental hours to obtain a scale factor σ. We first multiply the means and standard deviations of the number of hourly rental requests for each one-way rental triplets by σ and then compute the scale and

Figure 2. (Color online) Observed Distributions Compared with Gamma Distributions with the Same Means and Variances



shape parameters of the gamma distributions for generating one-way demand.

For each mix of one-way and round-trip demand, we use 10 training samples each having 100 scenarios, independently generated following the previously described procedures, for computing optimal first-stage decisions $(\mathbf{w}, \mathbf{x}^1)$ for the reservation-based system and $\mathbf{x}^2$ for the free-floating system. The performance of the first-stage solutions is evaluated using a test sample of 1,000 scenarios with the same mix of one-way and round-trip demand. The models used in this out-of-sample evaluation are (7) and (8) with the penalty term $g(\mathbf{y}^1, \ldots, \mathbf{y}^{|K|}) = 0$ (i.e., the second-stage problem with zero penalty on lost demand). We have compared different choices of the number of training samples and the sizes of training/test samples used for the SAA method. The current settings of 10 training samples each with 100 scenarios and 1,000 scenarios in the evaluation sample result in very small optimality gaps between the estimated upper and lower bounds with all being under 0.5%.

5.1.2. Parameter Settings. In our numerical study, we consider daily operations of a carshare fleet. All reported profitability and QoS results are daily numbers. We use a time granularity of one hour per period, and the problem is solved over $T = 24$ periods. Up to $S = 100$ vehicles are available for initial allocation in all the zones. The parking lot costs are set as $c_i^{\text{lot}} = \$9.6$ per day in zones $i = 1, 2, 5, 6, 9$ and $c_i^{\text{lot}} = \$7.4$ per day in other zones. The parking permit cost is set as $c^{\text{ffp}} = \$9.6$ per day. These are based on annual parking lot reservation costs being \$3,500 per lot in Boston downtown, \$2,700 per lot outside downtown, and annual parking permit costs being \$3,500 per car (Chesto 2015). We use revenue parameters $r^{\text{two}} = \$7.75$ per hour and $r^{\text{one}} = \$12$ per hour based on Zipcar's Boston rental rates for round-trip and one-way rentals in its ONE > WAY program (see Zipcar 2015). Relocation cost is $c^{\text{rel}} = \$10$ per car per hour. Note that vehicle relocation cost is not necessarily higher than the hourly wage of drivers employed to relocate vehicles. Low-cost relocation is possible through innovative methods, such as providing bonuses to attract potential drivers who need to travel along desired routes during desired time. Furthermore, we have tested different relocation costs ranging from \$6 to \$18 per hour and found that our observations regarding the impact of one-way demand on profitability and QoS still hold. The maintenance cost c^{mnt} and idling cost c^{idle} are assumed to be negligible and thus set to zero. Finally, $G_a = -5 f_a$ for all arcs $a \in A^{\text{one}} \cup A^{\text{two}}$, that is, the per unit penalty for lost demand on each arc is five times per unit net revenue.

5.2. Vehicle Allocation, Profitability, and QoS Under Different Demand Mix

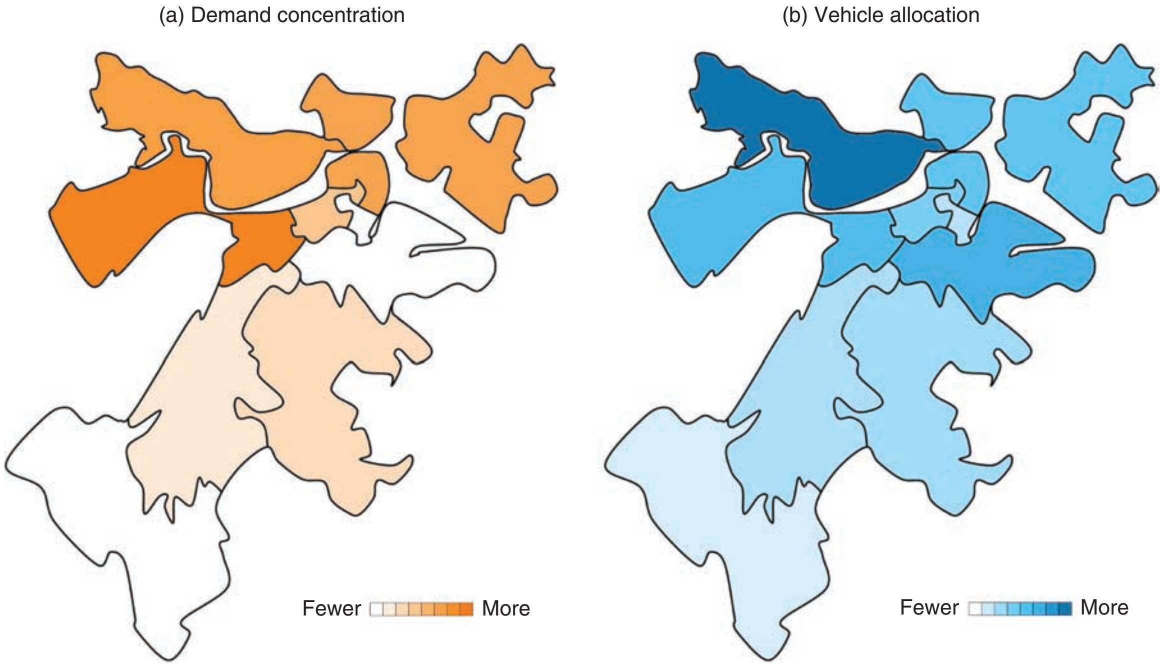
A primary factor affecting profit and QoS of a carshare service is the mix of one-way and round-trip demand. Volatile prices of rentals often negatively impact customer experience, but customer demand, and consequently the ratio of one-way to round-trip rentals, are volatile and change from day to day. Hence, it is important to understand the impact that this ratio has on carshare operations and management. In our first set of experiments, we evaluate the effects of the mix of one-way and round-trip rentals on reservation-based and on free-floating carshare systems.

5.2.1. Demand Concentration versus Vehicle Allocation. First, we compare the average total number of rentals demanded in each zone against the first-stage vehicle allocation obtained through the models. Figure 3 shows the relative concentration of demand starting in each zone and the relative allocation of vehicles obtained by solving `Default` when the proportion of one-way rentals is 40%. A darker colored zone represents a higher demand concentration in Figure 3(a) and more initially allocated vehicles in Figure 3(b).

The data indicates that more demand starts from the zones to the north of the Boston–Cambridge area than from the southern zones. Meanwhile, we observe more vehicles allocated to the northern zones. However, the allocation of vehicles does not match the demand concentrations directly as the optimal allocation of vehicles also depends on ending zones and relocation. This validates the nontriviality of the problem in this paper.

5.2.2. Profitability. The profitability of the carshare system includes the following main components: the cost of purchasing parking lots or free-floating permits, the expected revenue from one-way and round-trip rentals, and the expected cost of relocating vehicles. Table 4 shows these components for all proportions of one-way rentals we tested in the two types of carshare systems.

Figure 3. (Color online) Visual Comparison of Demand Concentration (by Starting Zone) vs. Vehicle Allocation



In Table 4, for both systems, the total profit increases as the proportion of one-way rentals increases. This suggests that one-way rentals are more profitable than round-trip rentals under the current price difference ($12 versus $7.75 per hour) and estimated relocation cost ($10 per hour). Higher proportions of one-way rentals require more vehicle relocation. However, the additional relocation cost is well offset by the higher revenue generated from one-way rentals. Overall, the reservation-based system has higher relocation cost but lower parking cost than the free-floating system. This shows limited parking spaces lead to more vehicle relocation. In the meantime, the effective use of vehicle relocation allows reservation-based systems to strategically locate parking spaces in regions where parking cost is lower.

5.2.3. QoS Performance. We evaluate the QoS of the solutions given by the models using the expected number of unserved rental requests, the expected proportion of unserved rental requests, and the expected number of unserved vehicle hours. All three metrics are given in Table 5 together with the expected total number of vehicle hours spent idle. These measures are widely used in practice and can be obtained from our models. Determining other important carsharing QoS measures through user survey remains as an important research problem. We see that, as one-way demand proportion increases, the QoS measures will improve in general. This is because the one-way rentals in our data set have much shorter durations compared with round-trip rentals, making one-way rentals more flexible and easier to fulfill. Consequently,

Table 4. Profitability of Solutions for Carshare Systems with Different One-Way Proportions

One-way proportion (%)	Carshare system	Parking cost ($)	Revenue from one way ($)	Revenue from round trip ($)	Relocation cost ($)	Total profit ($)
0	Free-floating	960.00	—	6,964.79	840.64	5,164.15
	Reservation-based	918.20	—	6,964.03	867.82	5,178.02
20	Free-floating	960.00	1,921.74	5,514.01	1,117.05	5,358.71
	Reservation-based	911.60	1,922.49	5,511.61	1,202.03	5,320.47
40	Free-floating	960.00	4,302.11	4,041.96	1,327.59	6,056.48
	Reservation-based	910.20	4,302.81	4,040.49	1,415.05	6,018.04
60	Free-floating	960.00	6,832.03	2,567.84	1,541.66	6,898.21
	Reservation-based	899.20	6,832.84	2,564.95	1,620.39	6,878.21
80	Free-floating	960.00	9,407.29	1,169.23	1,782.97	7,833.55
	Reservation-based	897.00	9,407.88	1,166.72	1,833.06	7,844.54
100	Free-floating	960.00	11,950.58	—	2,033.96	8,956.62
	Reservation-based	861.00	11,951.42	—	2,092.89	8,997.53

Table 5. QoS of Solutions for Carshare Systems with Different One-Way Proportions

One-way proportion (%)	Carshare system	Unserved rentals			Idle vehicle-hours
		Requests	Proportion (%)	Vehicle-hours	
0	Free-floating	38.87	8.72	84.17	1,317.25
	Reservation-based	38.97	8.75	84.27	1,314.63
20	Free-floating	40.39	7.42	74.54	1,316.66
	Reservation-based	40.46	7.43	74.79	1,308.41
40	Free-floating	38.13	5.64	68.63	1,287.19
	Reservation-based	38.16	5.64	68.76	1,278.57
60	Free-floating	33.31	4.06	56.03	1,245.16
	Reservation-based	33.37	4.07	56.34	1,237.60
80	Free-floating	26.33	2.72	43.61	1,186.89
	Reservation-based	26.39	2.73	43.89	1,182.16
100	Free-floating	17.39	1.69	17.39	1,100.72
	Reservation-based	17.32	1.69	17.32	1,094.76

the QoS will improve as one-way proportion increases. The reduction in idle vehicle hours as one-way proportion increases is a joint effect of more rental hours being fulfilled and more vehicle relocation required by higher one-way proportion.

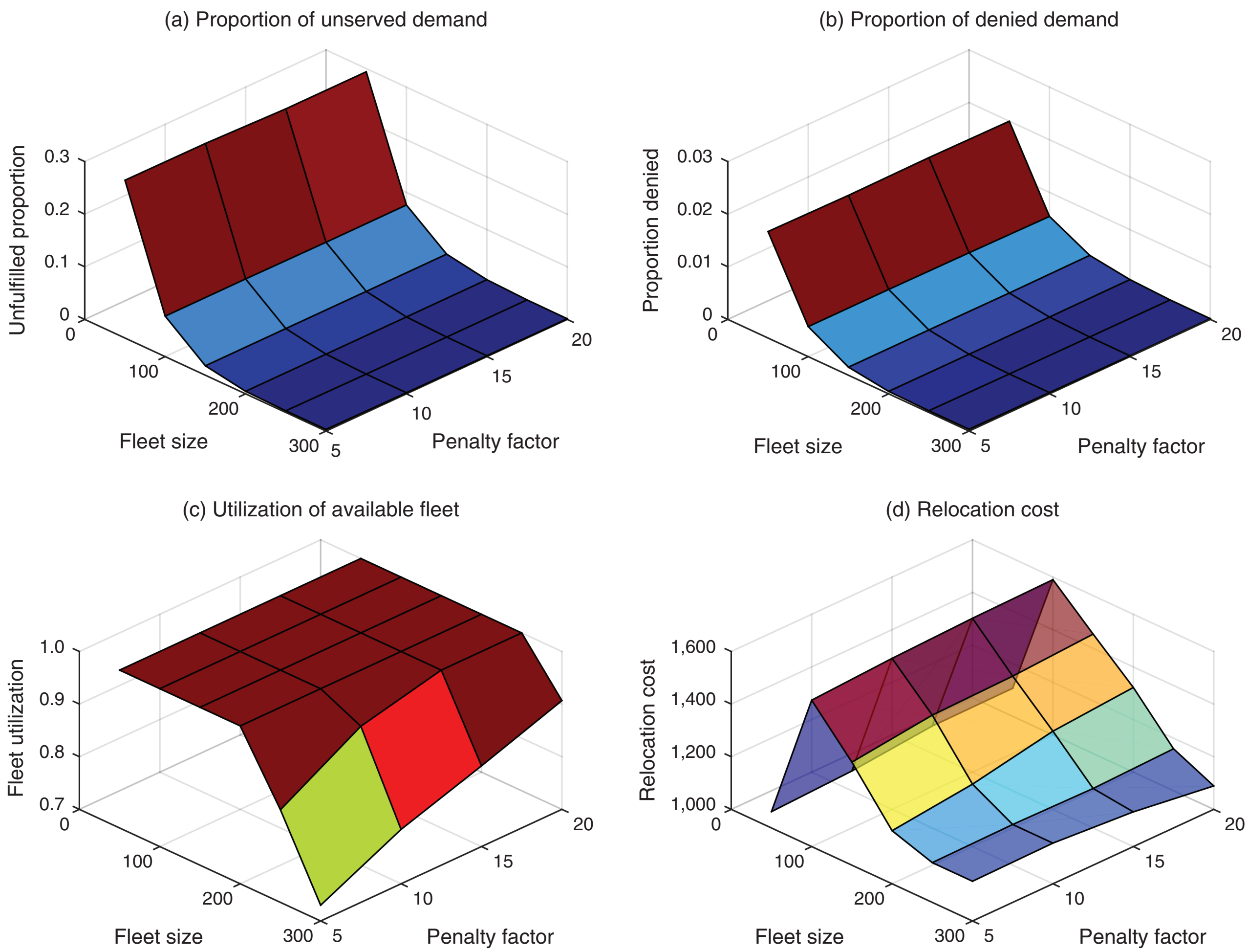
5.2.4. Denied Rentals. A denied rental is defined as a rental request that is unserved even though there is a vehicle available for use in that zone at that time. Such a phenomenon is unlikely to occur in a free-floating system as companies cannot prevent customers from renting available vehicles. Constraints (3)–(5) in our model do not specify that all available vehicles must be rented out if there exists unserved demand. This is mainly a result of computational tractability concerns. Allowing trip denial, we can formulate the subproblem of each scenario as a minimum-cost flow problem that can be solved as a linear program. Adding constraints to prevent trip denial, on the other hand, will make the subproblems nonconvex. Because of the existence of service denial, for free-floating systems, the second-stage problem in our model is less accurate. However, in a reservation-based system, customers could potentially be blocked from renting certain available vehicles. Denied rentals, in this case, provide another measure of QoS.

We present the average proportion of denied rentals and selected percentiles of this proportion in Table 6. The proportion of denied rentals decreases as the one-way proportion increases. This is because one-way rentals are more profitable under the current settings of price and relocation cost and also more flexible and easier to fulfill because of their shorter durations. Also, the proportion of denied rentals is not substantial in most of the cases. With 100% round trip, the proportion of denied rentals is around 1.1%, on average. However, there does exist a heavy tail with which more than 15% of the rentals are denied at the 99% percentile. Note that the results in Table 6 are obtained with fleet size $S = 100$. When more vehicles are available, the

Table 6. Proportions of Denied Rentals for Carshare Systems with Different One-Way Proportions

One-way proportion (%)	Carshare system	Mean	Percentile (%)					
			25	50	75	90	95	99
0	Free-floating	1.12	0.00	0.00	0.60	3.39	6.45	15.04
	Reservation-based	1.12	0.00	0.00	0.59	3.35	6.33	15.61
20	Free-floating	0.60	0.00	0.00	0.25	1.50	3.10	9.52
	Reservation-based	0.57	0.00	0.00	0.24	1.46	2.98	8.89
40	Free-floating	0.36	0.00	0.00	0.19	0.88	1.69	5.48
	Reservation-based	0.33	0.00	0.00	0.19	0.85	1.57	4.59
60	Free-floating	0.25	0.00	0.00	0.15	0.66	1.14	3.05
	Reservation-based	0.23	0.00	0.00	0.16	0.66	1.09	2.55
80	Free-floating	0.18	0.00	0.00	0.13	0.52	0.85	1.88
	Reservation-based	0.23	0.00	0.00	0.22	0.62	0.96	2.34
100	Free-floating	0.18	0.00	0.00	0.24	0.58	0.81	1.35
	Reservation-based	0.18	0.00	0.00	0.27	0.59	0.83	1.32

Figure 4. (Color online) The Impact of Fleet Size and Penalty Factor on Some System Performance Measures



proportion of denied rentals will be much smaller. Thus, we think our model can well approximate the free-floating system under practical choices of fleet size while keeping the problem computationally tractable. Designing efficient optimization algorithms for free-floating systems in which service denial is explicitly prohibited remains as a direction for future research.

5.3. Sensitivity Analysis of Fleet Size and QoS Penalty

In this section, we study the impact of fleet size and penalty factor for unserved demand. Fleet size is defined as the number of vehicles that is available for deployment and is varied from 50 to 300 vehicles in increments of 50. The penalty factor is defined as the ratio between the per-unit penalty and lost revenue of unserved demand and is varied from 5 to 20 in increments of 5. We report the results of the free-floating system with a trip mix of 40% one-way rentals. Results of the reservation-based system are very similar and thus omitted.

In Figure 4, we show several system performance measures under different choices of fleet size and penalty factor. We see from Figures 4(a) and 4(b) that the proportion of unserved or denied trips decreases rapidly as more vehicles are available. On average with 100 vehicles, less than 10% of the demand is unserved, and less than 1% of the demand is denied. In Figure 4(c), we show the utilization of the available fleet defined as the number of vehicles that is actually deployed and the number of vehicles that is available. The available fleet is fully utilized when the fleet size is less than or equal to 200. When the fleet size is larger than 200, it may be optimal not to deploy all available vehicles. Higher penalty cost will lead to more vehicles being deployed. In Figure 4(d), when the fleet size is small (i.e., 50), the use of vehicle relocation is limited since relocating vehicles will further reduce available vehicle hours. When the fleet size increases to 100, relocation cost increases significantly. As the fleet size further grows, the need for vehicle relocation decreases as more vehicles are available. These results suggest that fleet deployment and vehicle relocation can be either complements (i.e., deploying more vehicles will increase relocation) or substitutes (i.e., deploying more vehicles will reduce relocation) to each other.

The impact of penalty factor on vehicle relocation is more complex. When the available fleet is fully utilized (i.e., $S \le 200$), a higher penalty leads to more vehicle relocation to satisfy one-way demand. When fleet size is large (i.e., $S \ge 250$), a higher penalty causes more vehicles to be deployed and, in turn, reduces vehicle relocation. In general, the impact of the penalty factor is not significant in most cases. Thus, we choose a penalty factor of five in our other experiments. For fleet size, we choose $S = 100$ in our other experiments since it leads to satisfactory QoS performance, a fully utilized fleet, and effective use of vehicle relocation.

5.4. Impact of One-Way Rental Pricing

In Section 5.2, we study the carshare system with different exogenously given proportions of one-way rentals. In this section, we consider the case in which this proportion is endogenously determined by the price of one-way rentals. Since the original Zipcar data set only includes one fixed one-way rental price, we must generate new demand realizations under different prices. We hold the price of round-trip rentals fixed at \$7.75 and only vary the rental price of one-way rentals. We assume that a lower one-way rental price does not attract additional one-way rental customers but rather results in some round-trip customers dividing their trips into two segments, *departure* and *return*, each being a one-way rental. We start with the original data set with 100% round-trip rentals (i.e., row 0% in Table 4). For each round-trip, we randomly select a zone different from the departure zone as its "destination." We assume that the customer can divide a round-trip rental into two one-way segments by returning the vehicle at the destination and renting another vehicle for the trip back. The customer then compares the total rental price of the round-trip and its corresponding one-way segments and chooses the option with the lower total price.

Table 7 shows the revenue, cost, and profit under different one-way rental prices. We only show results for price = \$8, \$16, and \$24 per hour because they are the price break points for our data set. The last row, in which one-way price is set to infinity, corresponds to the pure round-trip case in Table 4. We see that when the one-way price is \$24, 15.55% of the customers will have cost savings by splitting their round-trips to two one-way segments. As the one-way price further decreases, more customers will choose one-way (40.67% when price is \$16 and 75.03% when price is \$8). When customers split their trips into one-way segments, the carshare system will lose part of the original rental hours because round-trip customers will pay for the time they spend at the "destination" even though they are not driving the vehicle. This can be seen from the column "Total rental hours." As a result, the total profit decreases significantly as more customers rent one-way. In this case, it is more appropriate to compare the profit generated per rental hour (the last column in Table 7). We see that endogenous one-way demand as a result of pricing has a different impact on the carshare system's profitability as opposed to exogenous one-way demand. When one-way demand is exogenously given, a higher proportion of one-way demand increases the system's profitability. However, when one-way demand is endogenously generated by pricing, a higher proportion of one-way demand reduces the system's profitability. Comparing the free-floating and the reservation-based systems, we see that the reservation-based system suffers from higher losses in profitability as one-way demand increases. This is because limited parking spaces lead to higher relocation costs.

Because of the reduction in total rental hours, the QoS measures will improve as one-way demand increases, and the results are shown in Table 8. Note that, when one-way service is first introduced in a pure round-trip system (i.e., when one-way proportion increases from 0 to 15.55%), the QoS measures improve dramatically. This is because round trips that first get split into one-way segments are those with longer durations, which are also the most difficult and costly to fulfill. By converting these trips into one way, the QoS measures are improved, and the improvement is sufficient to offset the demand imbalance caused by one-way rentals.

Table 7. Profitability with Different One-Way Rental Prices and Endogenous One-Way Demand

One-way price	One-way proportion (%)	Total rental hours	Carshare system	Parking cost (\$)	Revenue from one-way (\$)	Revenue from round-trip (\$)	Relocation cost (\$)	Total profit (\$)	Profit per hour (\$)
8	75.03	381.86	Free-floating	960.00	3,381.13	714.72	1,517.78	1,618.07	4.24
			Reservation	937.60	3,381.19	714.66	1,705.56	1,452.69	3.80
16	40.67	475.15	Free-floating	960.00	2,260.62	2,127.15	1,232.10	2,195.66	4.62
			Reservation	922.00	2,260.51	2,126.99	1,417.89	2,047.61	4.31
24	15.55	657.15	Free-floating	960.00	1,162.17	4,126.51	1,058.47	3,270.21	4.98
			Reservation	907.20	1,162.54	4,125.87	1,198.71	3,182.50	4.84
∞	0	990.69	Free-floating	960.00	0	6,941.61	1,030.48	4,951.12	5.00
			Reservation	916.00	0	6,939.82	1,074.60	4,949.22	5.00

Table 8. QoS Measures with Different One-Way Rental Prices and Endogenous One-Way Demand

One-way price	One-way proportion (%)	Total rental hours	Carshare system	Unserved demand				Idle vehicle hours
				Requests	Proportion (%)	Vehicle hours	Proportion (%)	
8	75.03	381.86	Free-floating	7.88	1.68	7.88	2.06	1,774.24
			Reservation	7.88	1.67	7.88	2.06	1,755.46
16	40.67	475.15	Free-floating	10.67	2.31	12.29	2.59	1,713.93
			Reservation	10.69	2.31	12.32	2.59	1,695.38
24	15.55	657.15	Free-floating	19.18	4.27	27.85	4.24	1,564.85
			Reservation	19.17	4.26	27.91	4.25	1,550.88
∞	0	990.69	Free-floating	44.39	10.32	95.00	9.59	1,301.26
			Reservation	44.49	10.34	95.23	9.61	1,297.08

5.5. Performance of the Rolling-Horizon Method for Vehicle Relocation

In this section, we present results from the rolling-horizon approach for optimizing real-time vehicle relocation discussed in Section 3.3. In this approach, first, Models (1) and (2) are solved to obtain initial vehicle allocation. Then, in each period, demand is realized and fulfilled on a first-come, first-served basis; after demand has been fulfilled, any remaining available vehicles are relocated using Models (9)–(14). We focused on the free-floating system. We used a trip mix with 40% one-way rentals and rental rates of $7.75 per hour round trip and $12 per hour one way. Relocation cost is assumed to be $10 per hour. Fleet size and QoS penalty factor are 100 and 5, respectively. We considered different levels of relocation capacity, which is defined as the maximum number of vehicles that can be in the process of relocation at the same time.

We compared the performance of the rolling-horizon model with that of an intuitively appealing benchmark policy that rebalances the entire fleet according to future demand concentration with a prespecified frequency. For more details of the benchmark policy, please refer to Online Appendix B. Table 9 presents the expected total cost plus QoS penalty minus rental revenue over 24 periods when vehicles are relocated using the proposed rolling-horizon model and the benchmark policy. The numbers shown are averages from 1,000 replications, each of which includes 24 periods of demand realizations. For the benchmark policy, we tested different frequencies with which the entire fleet is rebalanced every 1, 2, 3, 4, 6, 8, 12, or 24 periods. For each relocation capacity, the result using the best relocation frequency is shown in boldface. We can see that the rolling-horizon model performs better than the benchmark policy with the best relocation frequency under all relocation capacities. Moreover, as the relocation capacity increases, the benefit of using the rolling-horizon model increases significantly.

Next, we take a closer look at the performance of the proposed rolling-horizon vehicle relocation model. Table 10 shows detailed profitability and QoS results under different relocation capacities. Increasing relocation capacity can increase revenue, reduce unserved demand and QoS penalty, and improve (i.e., decrease) the objective value. Moreover, these benefits can be achieved to a large extent with relatively low relocation capacity. For example, increasing relocation capacity from five to infinity, the objective value can be improved from $4,829.56 to $3,525.92, that is, a reduction of $1,303.64. However, increasing relocation capacity to 10 leads to a reduction of $1,119.22, which is equal to 86% of the reduction achieved by infinite capacity. Similar patterns are observed for revenue and QoS performance.

Table 9. Expected Objective Value (Total Cost Plus QoS Penalty Minus Revenue) Using the Rolling-Horizon Vehicle Relocation Model and the Benchmark Policy, Denoted by "RBx," Where x Is the Rebalancing Frequency

Relocation capacity	Benchmark policy								Rolling horizon	Benefit (%)	
	RB1	RB2	RB3	RB4	RB6	RB8	RB12	RB24			
5	**5,482.45**	7,240.96	8,188.97	8,724.41	9,523.99	9,630.47	10,171.57	10,911.23	4,829.56	652.90	13.52
7	**5,060.91**	6,478.34	7,487.37	8,057.48	9,056.33	9,167.24	9,857.64	10,886.03	4,105.75	955.16	23.26
10	**5,058.99**	5,830.97	6,750.62	7,309.85	8,512.96	8,561.60	9,454.18	10,822.66	3,710.34	1,348.65	36.35
12	**5,106.14**	5,648.15	6,422.97	6,932.72	8,259.26	8,218.27	9,184.58	10,773.72	3,608.91	1,497.23	41.49
15	**5,300.48**	5,550.45	6,100.06	6,535.17	7,912.36	7,851.97	8,882.07	10,770.84	3,553.93	1,746.55	49.14
17	**5,526.41**	5,591.57	5,999.16	6,380.26	7,728.05	7,646.12	8,673.97	10,759.27	3,537.86	1,988.54	56.21
20	5,800.86	**5,720.08**	5,947.52	6,250.21	7,529.36	7,450.16	8,415.52	10,752.38	3,529.54	2,190.54	62.06
∞	7,841.97	7,024.51	**6,591.04**	6,751.11	7,360.72	7,427.24	7,909.69	10,880.92	3,525.92	3,065.12	86.93

Note. For each relocation capacity, the result using the best relocation frequency is shown in boldface.

Table 10. Summary of Profitability and QoS Measures Using the Rolling-Horizon Vehicle Relocation Model

Relocation capacity	Revenue one way	Revenue round trip	Relocation cost	Relocation utilization (%)	Total profit	Unserved demand: Requests	Unserved demand: Proportion (%)	Unserved demand: Hours	Idle hours	QoS penalty	Objective value
5	4,146.12	2,979.66	884.39	73.70	6,241.39	127.41	20.48	260.00	1,581.57	11,070.95	4,829.56
7	4,240.90	3,035.66	1,065.25	63.41	6,211.31	116.74	18.77	244.88	1,548.37	10,317.06	4,105.75
10	4,303.80	3,069.00	1,247.31	51.97	6,125.49	109.80	17.65	235.33	1,520.60	9,835.83	3,710.34
12	4,320.49	3,080.19	1,313.17	45.60	6,087.51	107.92	17.35	232.50	1,511.18	9,696.42	3,608.91
15	4,331.32	3,086.58	1,361.50	37.82	6,056.40	106.72	17.16	230.77	1,504.62	9,610.33	3,553.93
17	4,333.70	3,089.13	1,375.06	33.70	6,047.78	106.41	17.10	230.24	1,502.74	9,585.64	3,537.86
20	4,335.37	3,090.18	1,383.02	28.81	6,042.53	106.22	17.08	229.97	1,501.67	9,572.07	3,529.54
∞	4,335.52	3,090.92	1,384.73	—	6,041.71	106.17	17.07	229.86	1,501.22	9,567.63	3,525.92

5.6. Computational Efficacy of MIR-Enhanced Benders Cuts

In this section, we evaluate the computational efficiency of the MIR procedure for problems with different sizes. We generate samples with 100, 200, 500, and 1,000 scenarios from the Boston–Cambridge Zipcar data (following the procedures in Section 5.1.1). Five samples are generated for each problem size to obtain the average results. For all the samples, we consider 40% proportion of one-way demand. All the other parameters are set according to Section 5.1.2.

We compare the solution time of three methods, that is, solving the MILP directly (`Default`), solving it via branch-and-cut without MIR cuts (`B&C`), or solving it via branch-and-cut with MIR cuts (`MIR`). When solving the MILP model directly, we use Gurobi 6.0.3 with its Java API and all default settings. For branch-and-cut with or without MIR cuts, we use parallel computing via a master-worker scheme by OpenMPI 1.6 and perform all the computation on Flux HPC cluster with each computing node having 12 2.67-GHz Intel Xeon X5650 processors and 48 GB RAM. We use 20 cores for processing all subproblems in parallel. The same computer settings are also used before for obtaining the results in Sections 5.2–5.4.

Table 11 compares the computation time for different problem sizes (number of scenarios) averaged over five samples for each problem size. All the results in Table 11 are reported in milliseconds. The computation times are decomposed into several components. The column "MP solve time" gives the total solution time in the case of solving directly (no suffix). For `B&C` and `MIR`, this column gives the total time spent solving the master problem (MP) and updating the flow balance constants according to the current first-stage solution. The next column, relevant only to the `B&C` and `MIR` solution methods, gives the average solution time for each subproblem (SP). In other words, this is the total amount of time spent solving the subproblems divided by the number of scenarios. The last column gives the number of iterations required to converge on the solution—also only relevant to `B&C` and `MIR`.

Our results show that `B&C` and `MIR` are generally faster than Gurobi's default MILP solver if the subproblems are computed in parallel. As the number of scenarios increases, the gap in parallel solve time becomes even more significant. Moreover, the method `MIR` results in shorter times for solving the master problem and shorter overall solution times when being implemented in parallel.

Table 11. Computational Time (in Milliseconds) Comparison Between Models for Different Problem Sizes

# scenarios (subproblems)	Method	MP solve time	Avg solve time per SP	Series solve time	Parallel solve time	# iterations
100	`Default`	4,251	—	4,251	4,251	—
	`B&C`	181	78	9,311	1,857	31
	`MIR`	173	75	8,953	1,771	30
200	`Default`	13,436	—	13,436	13,436	—
	`B&C`	602	80	19,703	2,837	32
	`MIR`	659	82	20,338	2,549	34
500	`Default`	65,231	—	65,231	65,231	—
	`B&C`	2,839	76	48,933	5,387	29
	`MIR`	2,284	78	49,251	3,795	28
1,000	`Default`	236,207	—	236,207	236,207	—
	`B&C`	18,529	252	291,389	34,430	39
	`MIR`	16,015	126	160,272	27,175	37

5.7. The Value of Stochastic Solutions (VSS)

Finally, we compute optimal solutions to a benchmark deterministic model that uses empirical mean values of the demand to examine the VSS (Birge and Louveaux 2011). We compare the results of the deterministic model with the ones of `Default` based on the same test instances. While all the approaches result in similar numbers of denied trips, the stochastic approaches have better QoS results than the deterministic model with 5%–7% shorter vehicle idle hours when $c^{\text{rel}} =$ \$10 and almost no unserved demands when $c^{\text{rel}} =$ \$10. The stochastic approaches dominate the deterministic one in metrics of profitability, especially under higher one-way proportions (e.g., when $\geq 60\%$ of the total demand are one-way rentals). The optimal solutions by the stochastic approaches yield 60.9%–153.2% more total revenue while the deterministic model yields 1.75 to 2.24 times more relocation cost. This confirms the importance of using stochastic optimization for carshare service planning under demand uncertainty.

6. Concluding Remarks

In this paper, we develop two-stage stochastic integer programming models and branch-and-cut algorithms for optimizing strategic parking planning and vehicle allocation for carshare systems under uncertain demand with both one-way and round-trip rentals. The models are applied to diverse instances generated based on a real-world data set of Zipcar in the Boston–Cambridge area. Our results indicate that the impact of one-way demand on profitability and QoS is significant. Furthermore, depending on whether one-way demand is exogenously or endogenously generated, a higher proportion of one-way demand may have opposite effects. When exogenously given, for example, from natural market penetration and user adoption, higher one-way demand could increase a carshare system's profitability. On the other hand, when exogenously generated by pricing and strategic customer behavior, higher one-way demand could decrease profitability. Thus, understanding the market dynamics of one-way rentals is of great importance for carshare companies as well as local governments that try to solve their urban transportation problems by promoting carsharing.

We also leverage our model to optimize real-time vehicle relocation operations in a rolling-horizon framework. Comparison with an intuitive benchmark policy shows that our model could lead to significant improvement in both profitability and QoS. With respect to computational performance, our proposed branch-and-cut algorithms perform significantly better than the state-of-the-art commercial mixed-integer program solver. Also, using MIR-strengthened cuts results in more improvement than using only Benders cuts when using parallel computing.

There are several limitations of our models. First, we focus on the reservation-based and free-floating systems, which are two special cases of a general hybrid system with both contracted parking lots and free-floating parking permits. Studying a general system requires new optimization algorithms and is left as a future research direction. Second, we formulate a two-stage stochastic programming model, which provides an approximation of the expected profit from uncertain demand. Robust optimization is also suitable for modeling and optimizing such systems. For future research, we plan to study multistage stochastic programming and multistage robust models for optimizing vehicle relocation in real-time operations. Third, we focus on carshare systems in metropolitan areas where parking is extremely limited. It is also important to study carshare systems where parking is abundant. Finally, our numerical results are obtained using one specific data set and thus may not be easily generalizable. As more data become publicly available, it is necessary to test our models with new data sets to verify current observations and draw more general insights.

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