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## **A Decision Support Tool for Vehicle Relocation Operations in Carsharing Systems**

Alvina G. H. Kek

Asia Pacific Product Scheduling, ExxonMobil Asia Pacific Pte Ltd

18 Pioneer Road, PE02-235, Singapore 628498

Telephone: +65-6660-6486

Email: Alvina.gh.kek@exxonmobil.com

Ruey Long Cheu<sup>#</sup>

Department of Civil Engineering, The University of Texas at El Paso

500 West University Avenue, El Paso, TX 79968-0516

Telephone: 915-747-6107

Email: rcheu@utep.edu

Qiang Meng

Department of Civil Engineering, National University of Singapore

1 Engineering Drive 2, Singapore 117576

Telephone: +65-6516-5494

Email: cvemq@nus.edu.sg

Chau Ha Fung

Department of Civil Engineering, National University of Singapore

1 Engineering Drive 2, Singapore 117576

Telephone: +65-6516-5035

Email: cvefchj@nus.edu.sg

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<sup>#</sup> Corresponding author

**Abstract:** This paper presents a novel three-phase Optimization-Trend-Simulation (OTS) decision support tool for carsharing operators with flexible return time and stations. The tool assists the operators to find a set of near-optimal manpower and relocation parameters for their vehicle relocation systems. Phase one of the OTS is a mixed integer linear programming model, which minimizes the total generalized cost of vehicle movements, staff time, penalty of unfulfilled customer demand and returns. This model takes input from historical trip patterns, fleet size, station capacity and etc to solve the optimal vehicle relocation problem without the specification of any relocation threshold at any station. Phase two of the OTS tool is a trend filter which takes the output from the optimization model to heuristically deduce the manpower needs, staff shift hour, relocation logic and vehicle inventory thresholds values at each station that will trigger a relocation move. The last phase of the OTS tool employs a time-stepping simulation model, using a set of historical customer usage data, combined with the operating parameters inferred from the trend filter, to evaluate the Level of Service (LOS) provided by the carsharing operator to its customers. Two LOS indicators, namely Full-Port Time (FPT) and Zero-Vehicle Time (ZVT) are used. Tested on a set of commercially operational data from a carsharing company in Singapore, the simulation results suggest that the parameters recommended by the OTS tool leads to a reduction in staff cost of 50%, reduction in ZVT ranging between 4.6% and 13.0%, a maintenance of the already low FPT and a reduction in Number of Relocations (NR) ranging between 37.1% and 41.1%.

## BACKGROUND AND MOTIVATION

Over the last decade, carsharing has emerged as an alternative to owning a vehicle. Most of this form of transportation has been taking place in Europe, North America, Japan and Singapore, with a total of 40 programs deployed in North America (1), 18 in Japan and 4 in Singapore (2). A range of market demand studies, conducted principally in Europe, has estimated a market potential of anything from 3%-25% of the population (3), and in North America, the market potential in major metropolitan regions is estimated at 10% of individuals over the age of 21 (1).

In cities with high population densities, car sharing exhibit great promise in improving mobility, lowering emissions and congestion problems (4). It is shown to also reduce vehicle ownership (3), making carsharing potentially viable as a parking management strategy (5). Simulation-based research carried out to investigate the viability of carsharing suggests that it has the potential to become economically profitable (6). This finding is later supported by the rapid growth of commercial carsharing companies such as Zipcar (7) and Flexcar (8) in the U.S. and Canada since 1999. Another mobility scenario conducted for the Sacramento region in California indicates a modest reduction in vehicle travel and emissions and a significant net economic benefit for home-based work trips (9). To ensure that these positive effects of carsharing remain sustainable, the number of carsharing customers must grow (10). It hence becomes vital for there to be supportive regulatory legislations, financial support and easily adoptable operational tools, to boost the growth of the carsharing industry.

Conventional carsharing systems usually requires users to pickup and return vehicles at the same stations. Stiff competitions from public transportation systems and competing carsharing companies have prompted some operators to provide users with the flexibility in return stations. Carshrining operators who have adopted such strategy include the Honda ICVS (hereafter referred to as ICVS) (11), Praxitele in France (12) and IntelliShare in California (13, 14). Taking it a step further, ICVS also provides users with flexibility in return time. A key issue that arises from such systems (having flexible return time and stations) is the dynamically disproportionate distribution of vehicles across stations, with no pre-emptive knowledge. As a result, periodic relocation becomes necessary to ensure an even distribution of vehicles to serve customer demands.

The IntelliShare research team at the University of California, Riverside proposed and experimented with two user-based relocation methods, namely trip-splitting and trip joining, which successfully reduced the NR required (15). These user-based relocation techniques cleverly shift the burden of relocating vehicles to the users through a price incentive mechanism. These techniques may not be viable in cities where commuters value privacy and convenience over minor cost savings in transportation. In pursuit of offering privacy, simplicity and convenience to commuters, ICVS make use of in-house staff to relocate vehicles (11).

A clear need thus arises for a set of operational tools to support operator-based relocation, for carsharing systems with flexible return time and stations. Related research in this area includes forecasting of vehicle trip demands by Cheu *et al.* (16), and vehicle relocation simulation by Kek *et al.* (17). The motivation behind these studies arises from a fundamental desire to enhance both operational efficiency and service levels at the lowest possible cost. This paper is thus driven to focus on how to bring about an overall optimization of the relocation system. A three-phase

OTS decision support tool is developed to find a near-optimal set of parameters for vehicle relocation operations. The OTS tool is tested and validated on real data from ICVS.

## OVERVIEW OF HONDA ICVS

The ICVS is a commercial multiple-station carsharing system in Singapore which began operation in March 2002 (11). Its competitors are three other carsharing companies. The key differences between ICVS and its competitors is that ICVS allows its users the flexibility of pickup vehicles without having to commit to a return time. The returning station specified by a user prior to a trip may also be changed en-route. In addition, no advance reservation is necessary for a user to pick up a vehicle at a station. The ICVS is thus constantly challenged to maintain a distribution of vehicles at all stations to meet customer demand, with as efficient a relocation system as possible. A delicate balance needs to be struck between having available vehicles for pickup and having enough empty parking stalls for customers to return vehicles to.

The ICVS has experienced continuous growth in terms of number of stations, fleet size and membership. At the time of writing, it has 15 stations, mostly located in the center part of Singapore with 10 of them concentrated in the CBD. A map of existing station locations can be found in the company's website (11). Fifty three environment-friendly Honda Civic Hybrid vehicles are available for more than 2000 members to use. More than 70 trips are being made in an average day. The system makes use of technologies such as GPS and RFID vehicle location systems, contactless smartcard access, internet or phone-based reservations and touchscreen display units. Communicating with the backend computer system periodically, information from these vehicles prompts the system manager to relocate vehicles between the stations when needed. Essentially, the relocation algorithm periodically checks the number of available vehicles at each station against a set of predefined, station specific lower and upper threshold values (termed relocation thresholds), and recommends the origin and destination of a relocation trip, if necessary. More details of the vehicle relocation operations can be found in (17).

ICVS trip data has been collected and stored in a database. This information has proven to be particularly useful in helping to understand user behavior, forecast future trip data (16) and develop simulation models to test the effectiveness of various relocation techniques (17).

## INTEGRATED HEURISTIC APPROACH

The vehicle relocation simulation model developed by (17) provides a means of evaluating the impact of different relocation techniques and operating parameters. Using it to perform system optimization, however, would mean an impractical number of iterative runs to enumerate all the possible permutations of the parameters. A novel three-phase OTS decision support tool is thus developed to overcome this problem. A schematic of this approach is presented in Figure 1.

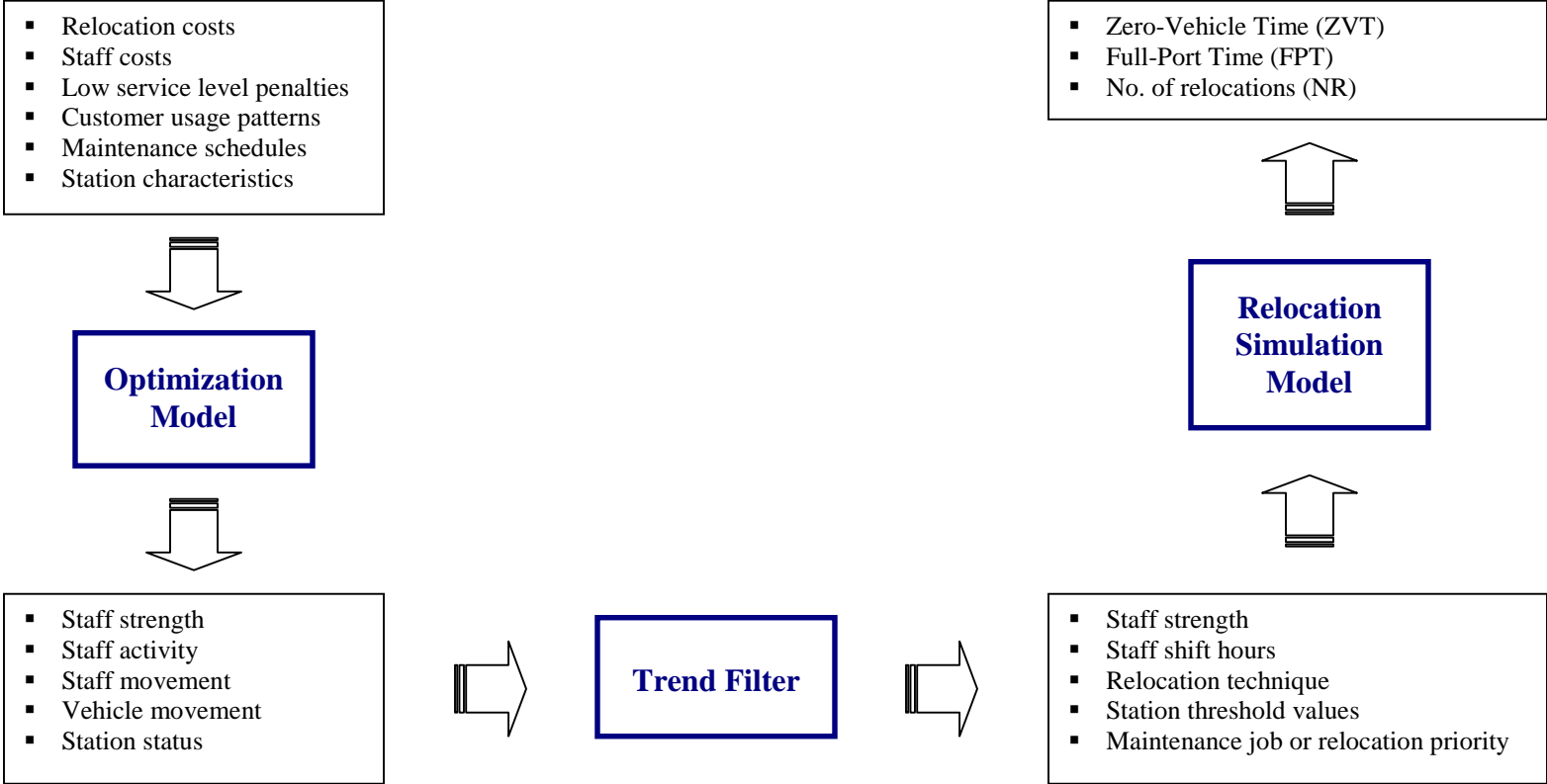
Phase one of the OTS is an *Optimization Model*, which receives inputs from the carsharing system on its inter-station vehicle relocation costs, staff cost, penalties for low LOS for customers, typical or historical customer usage patterns, vehicle maintenance schedules and stations' characteristics such as the number of parking stalls. The low LOS is reflected by the duration of vehicle shortage (i.e., ZVT) and parking stall unavailability (i.e., FPT). The model then proceeds to determine the lowest cost resource allocation, giving the optimized staff

strength, staff activities, relocations and the resulting station status (number of vehicles at the stations at each time step). The availability of staff (in terms of number and their shift hours) who perform vehicle relocation and maintenance activities can be arbitrarily set in phase one and later revised through a series of heuristics in phase two. For example, one can start by assuming that all the staff are available for 24 hours in a day. The shift hours and number of staff may be reduced based on the results of the optimization. The vehicle relocation thresholds, which are station specific, are not required at this phase as the optimization model will move staff and vehicles as needed to achieve the minimum cost. The relocation thresholds will be set in the next phase by examining the optimized relocation activities.

In phase two, the *Trend Filter* receives the optimized outputs from phase one and ‘filters’ them through a series of heuristics. A set of recommended operating parameters (staff strength and shift hours, relocation technique, relocation thresholds and whether priority should be given to maintenance jobs or relocations) is thus obtained.

Entering the set of operating parameters obtained in phase two into the vehicle relocation simulation model developed in (17), phase three of the OTS evaluates the effectiveness of this set of parameters using three performance indicators, namely ZVT, FPT and NR. When ZVT occurs, the station is without an available vehicle and customers requesting for vehicles at that station (either by making advance reservation or walk-in) will be rejected. Conversely, when FPT occurs, the station has no empty parking stall and users wanting to return vehicles to that station will also be rejected. Both ZVT and FPT reduce the attractiveness of the carsharing system to users. From the operator’s point of view, ZVT implies a possible lost in revenue. From the user’s point of view, ZVT forces him/her to use other stations, or turn to other modes of travel, while FPT forces the user to return the vehicle later or to another station, incurring additional usage cost. A greater value of NR means a higher the cost of vehicle relocation operations. Thus, for optimal performance of the system, the values for ZVT, FPT and NR should all be zero. The development of the *Optimization Model* and *Trend Filter* are described in greater detail over the next two sections in this paper. Readers can refer to (17) for more details of the vehicle relocation simulation model which is used in phase three of the OTS.

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**FIGURE 1 Schematic of the Three-Phase OTS Integrated Heuristic Approach**

## OPTIMIZATION MODEL

### Problem Definition

Given a set of geographically scattered stations, with each station having a capacity (number of parking stalls) and customer usage patterns, plus the maintenance schedule for the vehicle fleet, the objective of the optimization problem is to allocate staff resource so as to minimize the operating cost associated with vehicle relocation activities. The operating cost here includes staff cost, vehicle relocation cost and penalty cost of low LOS. The staff resource here refers to the number of staff and their shift hours. This problem may be viewed as somewhat similar to the typical pickup and delivery problem, in which staff are assigned to routes that traverse between different stations while engaged in various activities, e.g., picking up and dropping off vehicles. The problem definition is as described below:

- (i) A staff route can start at any one station.
- (ii) At any time, each staff is engaged in exactly one of the four types of activities. The four types of activities are namely *waiting* (wait at a station for the next activity), *maintenance* (perform a maintenance activity at a station or drive a vehicle to a workshop for maintenance), *movement* (travel between two stations without driving a vehicle) or *relocation* (drive a vehicle from one station to another).
- (iii) A staff route can end at any one station or in the midst of any one of three activities, namely maintenance, movement or relocation.
- (iv) At every time step, there are three non-negative numbers to be monitored at each station. They are the number of available vehicles, the number of unavailable vehicles, and the number of empty parking stalls. The available vehicles refer to those ready for customer use. The unavailable vehicles refer to those already reserved by customers or are awaiting maintenance. The sum of available and unavailable vehicles and the number of empty parking stalls equal to the station capacity.
- (v) The number of available vehicles at each station varies with each time step. It is affected by vehicles relocated into and out of the station, vehicles returning to the station after maintenance, vehicles requiring maintenance at the station and vehicles picked up and returned by customers.
- (vi) The number of unavailable vehicles at each station varies with each time step according to vehicles taken out for maintenance from the station, vehicles requiring maintenance at the station, and vehicles reserved by customers.
- (vii) The movement of a vehicle into and out of a station for maintenance or relocation is accompanied by a movement of staff (the driver) into and out of the same station, engaged in the maintenance or relocation activity respectively.

### Network Representation

The problem is best visualized by constructing a two-dimensional time-space network as shown in Figure 2, with the x-axis representing time and the y-axis representing space.

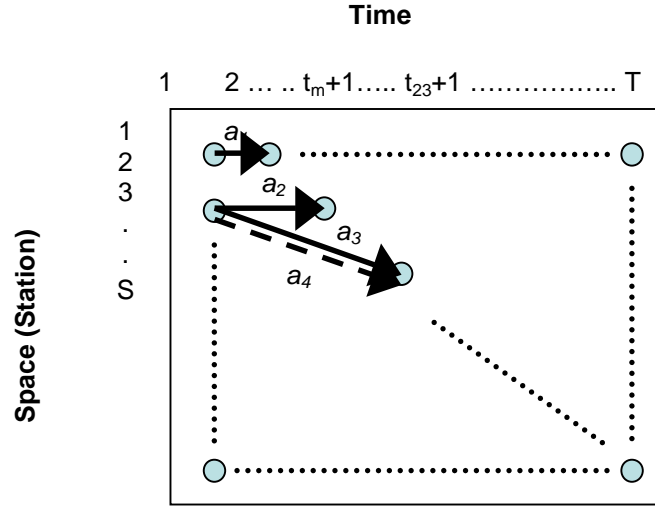


FIGURE 2 Time-Space Network

The time-space network has  $S \times T$  nodes arranged in a rectangular grid, where  $S$  is the number of stations in the carsharing system, and  $T$  is the number of discrete time steps within the period of operations to be optimized. The nodes are positioned in the x-axis at constant time intervals (e.g., 15 minutes) and along the y-axis at fixed positions. Nodes in each row are for a particular station ( $i = 1, 2, \dots, S$ ). Nodes in each column represent all the stations at a time step ( $t = 1, 2, \dots, T$ ). Note that  $t$  denotes the start of each time step. Let  $N = \{1, \dots, i, \dots, S\}$  be the set of stations. For each  $i \in N$ , create  $T$  nodes representing the station  $i$  at time step  $t$ , for  $t = 1, 2, \dots, T$ , and use  $i_t$  to represent node  $i$  at time step  $t$ . Repeat this process for  $i = 1, 2, \dots, S$ . Denote all the  $S \times T$  nodes by a row vector  $V = \{1_1, \dots, i_{t-1}, i_t, i_{t+1}, \dots, S_T\}$ .

Next, define arcs between the nodes in the time-space network to represent staff activities such as waiting, maintenance, movement and relocation. There are four sets of arcs, with each set representing one of the four types of activities. For each node  $i_t \in V$ , create an arc that represents waiting activity at the same station  $i$  from time step  $t$  to time step  $t+1$ . Denote this set of *waiting* arcs as  $A_1 = \{\dots, a_1(i_t, i_{t+1}), \dots\}$ . For each node  $i_t \in V$ , create an arc that represents maintenance activity at station  $i$  from time step  $t$  to time step  $t+t_m$ , where  $t_m$  is the fixed number of time steps required for all types of maintenance activities (refueling, inspection, cleaning, etc). Denote this set of *maintenance* arcs as  $A_2 = \{\dots, a_2(i_t, i_{t+t_m}), \dots\}$ . During maintenance, a vehicle may be taken out of station  $i$  by the staff and returned to the same station  $t_m$  time steps later. For each node  $i_t \in V$ , create a set of  $S-1$  arcs that represent movements of staff from stations  $i$  to  $j$ ,  $\forall j \in N, j \neq i$ , from time step  $t$  to time step  $t+t_{ij}$ , where  $t_{ij}$  is the travel time (in number of time steps) from stations  $i$  to  $j$ . Denote this set of *movement* arcs as  $A_3 = \{\dots, a_3(i_t, i_{t+t_{ij}}), \dots\}$ . For each node  $i_t \in V$ , create a set of  $S-1$  arcs that represent vehicle relocation activities from stations  $i$  to  $j$ ,  $\forall j \in N, j \neq i$ , from time steps  $t$  to  $t+t_{ij}$ , where  $t_{ij}$  is the driving time from stations  $i$  to  $j$ ,  $\forall i, j \in N, i \neq j$ . Denote this set of *relocation* arcs as  $A_4 = \{\dots, a_4(i_t, i_{t+t_{ij}}), \dots\}$ . Note that, the driving time to



relocate a vehicle and the movement time (without driving a vehicle) between stations  $i$  and  $j$  are assumed to take the same number of time steps  $t_{ij}$ .

A sample of each arc can be seen in Figure 2. Arc  $a_1$  represents a waiting activity at station 1 from time steps 1 to 2; arc  $a_2$  represents a maintenance activity at station 2 from time steps 1 to  $1+t_m$ ; arc  $a_3$  represents a movement activity from stations 2 to 3 from time steps 1 to  $1+t_{23}$  ( $t_{23}$  is the travel time from stations 2 to 3); arc  $a_4$  represents a relocation activity from stations 2 to 3 from time steps 1 to  $1+t_{23}$ .

Finally, define a set of staff available to carry out these activities,  $L=\{1, \dots, k, \dots, W\}$ . Here,  $W$  is the maximum number of staff available.

### Mixed Integer Programming Formulation

The mixed integer programming formulation for this problem involves seven types of decision variables:

- $x^k$ : Binary variable, taking the value 1 if staff  $k$  is ever used from  $t=1, \dots, T$ , and 0 otherwise,  $\forall k \in L$ .
- $y_{i,i_{t+1}}^k$ : Binary variable associated with  $A_1$ , taking the value 1 if staff  $k$  waits at station  $i$  from time steps  $t$  to  $t+1$ , and 0 otherwise,  $\forall (i, i_{t+1}) \in A_1, k \in L$ .
- $z_{i,i_{t+t_m}}^k$ : Binary variable associated with  $A_2$ , taking the value 1 if staff  $k$  maintains a vehicle at station  $i$  from time steps  $t$  to  $t+t_m$ , and 0 otherwise,  $\forall (i, i_{t+t_m}) \in A_2, k \in L$ .
- $u_{i,j_{t+t_{ij}}}^k$ : Binary variable associated with  $A_3$ , taking the value 1 if staff  $k$  moves from station  $i$  at time step  $t$  to station  $j$  at time step  $t+t_{ij}$ , and 0 otherwise,  $\forall (i, j_{t+t_{ij}}) \in A_3, k \in L$ .
- $v_{i,j_{t+t_{ij}}}^k$ : Binary variable associated with  $A_4$ , taking the value 1 if staff  $k$  relocates a vehicle from station  $i$  at time step  $t$  to station  $j$  at time step  $t+t_{ij}$ , and 0 otherwise,  $\forall (i, j_{t+t_{ij}}) \in A_4, k \in L$ .
- $d_i^r$ : Integer variable representing the number of rejected customer demand for vehicles at station  $i$  from time steps  $t-1$  to  $t$ ,  $\forall i \in V$ .
- $s_i^r$ : Integer variable, representing rejected customer return of vehicles at station  $i$  from time steps  $t-1$  to  $t$ ,  $\forall i \in V$ .

The four binary variables,  $y_{i,i_{t+1}}^k$ ,  $z_{i,i_{t+t_m}}^k$ ,  $u_{i,j_{t+t_{ij}}}^k$  and  $v_{i,j_{t+t_{ij}}}^k$  are associated with the sets of arcs,  $A_1$ ,  $A_2$ ,  $A_3$  and  $A_4$  respectively. In each of the four variables, there is a  $k$  variable (that denotes staff  $k$ ) associated with each arc. This means that there can be a maximum of  $W$  staff activities being carried out on any arc.

The known constants are:

- $c_{ij}$ : Fixed cost of a movement or relocation trip from stations  $i$  to  $j$ ,  $\forall i, j \in N, i \neq j$ .
- $c_x$ : Fixed cost of utilizing one staff.

- $c_d$ : Fixed cost of rejecting the demand of one customer-vehicle trip.  
 $c_s$ : Fixed cost of rejecting the return of one vehicle by a customer.  
 $r_{i_0}$ : Number of available vehicles at station  $i$  at time step  $t=0$ ,  $\forall i \in N$ .  
 $\bar{r}_{i_0}$ : Number of unavailable vehicles at station  $i$  at time step  $t=0$ ,  $\forall i \in N$ .  
 $d_{i_t}$ : Demand for vehicles at station  $i$  from time steps  $t-1$  to  $t$ ,  $\forall i_t \in V$ .  
 $s_{i_t}$ : Number of vehicles returned by customers at station  $i$  from time steps  $t-1$  to  $t$ ,  $\forall i_t \in V$ .  
 $m_{i_t}$ : Number of returned vehicles becoming in need of maintenance at station  $i$  from time steps  $t-1$  to  $t$ , where  $m_{i_t} \leq s_{i_t}$ ,  $\forall i_t \in V$ .  
 $p_i$ : Number of parking stalls at station  $i$ ,  $\forall i \in N$ .

Two additional variables are:

- $r_{i_t}$ : Number of available vehicles at station  $i$  at time step  $t$ ,  $\forall i_t \in V$ .  
 $\bar{r}_{i_t}$ : Number of unavailable vehicles at station  $i$  at time step  $t$ ,  $\forall i_t \in V$ .

The mixed integer linear programming formulation for the problem is:

$$\text{Min } Z = c_{ij} \left( \sum_{(i_t, j_{t+ij}) \in A_3} \sum_{k \in K} u_{i_t j_{t+ij}}^k + \sum_{(i_t, j_{t+ij}) \in A_4} \sum_{k \in K} v_{i_t j_{t+ij}}^k \right) + c_x \sum_{k \in L} x^k + c_d \sum_{i_t \in V} d_{i_t}^r + c_s \sum_{i_t \in V} s_{i_t}^r \quad (1)$$

subject to

$$\sum_{i \in N} y_{i_1 i_2}^k + \sum_{i \in N} z_{i_1 i_1 + m}^k + \sum_{\substack{i, j \in N \\ i \neq j}} u_{i_1 j_1 + ij}^k + \sum_{\substack{i, j \in N \\ i \neq j}} v_{i_1 j_1 + ij}^k = x^k \quad \forall k \in L \quad (2)$$

$$\begin{aligned} & y_{i_{t-1} i_t}^k + z_{i_{t-1} i_t}^k + \sum_{(i_{t-1} i_t) \in A_2} u_{j_{t-1} j_t}^k + \sum_{(j_{t-1} j_t) \in A_4} v_{j_{t-1} j_t}^k \\ & - y_{i_t i_{t+1}}^k - z_{i_t i_{t+1}}^k - \sum_{(i_t, j_{t+ij}) \in A_3} u_{i_t j_{t+ij}}^k - \sum_{(i_t, j_{t+ij}) \in A_4} v_{i_t j_{t+ij}}^k = 0 \end{aligned} \quad \forall i_t \in V, k \in L, t > 1 \quad (3)$$

$$\begin{aligned} r_{i_t} &= r_{i_{t-1}} + \sum_{(j_{t-1} j_t) \in A_4} \sum_{k \in L} v_{j_{t-1} j_t}^k - \sum_{(i_t, j_{t+ij}) \in A_4} \sum_{k \in L} v_{i_t j_{t+ij}}^k \\ &+ \sum_{\substack{(i_{t-1} i_t) \in A_2 \\ k \in L}} z_{i_{t-1} i_t}^k + (s_{i_t} - s_{i_t}^r) - (d_{i_t} - d_{i_t}^r) - m_{i_t} \end{aligned} \quad \forall i_t \in V \quad (4)$$

$$\bar{r}_{i_t} = \bar{r}_{i_{t-1}} - \sum_{k \in L} z_{i_{t-1} i_t}^k + m_{i_t} \quad \forall i_t \in V \quad (5)$$

$$r_{i_t} + \bar{r}_{i_t} \leq p_i \quad \forall i_t \in V \quad (6)$$

$$d_{i_t}^r \leq d_{i_t} \quad \forall i_t \in V \quad (7)$$

$$s_{i_t}^r \leq s_{i_t} \quad \forall i_t \in V \quad (8)$$

$$x^k = (0, 1) \quad \forall k \in L \quad (9)$$

$$y_{i_t i_{t+1}}^k = (0, 1) \quad \forall (i_t, i_{t+1}) \in A_1, k \in L \quad (10)$$

$$z_{i_t i_{t+m}}^k = (0, 1) \quad \forall (i_t, i_{t+m}) \in A_2, k \in L \quad (11)$$

$$u_{i_t i_{t+ij}}^k = (0, 1) \quad \forall (i_t, i_{t+ij}) \in A_3, k \in L \quad (12)$$

$$v_{i_t i_{t+ij}}^k = (0, 1) \quad \forall (i_t, i_{t+ij}) \in A_4, k \in L \quad (13)$$

$$d_{i_t}^r \geq 0 \quad \forall i_t \in V \quad (14)$$

$$s_{i_t}^r \geq 0 \quad \forall i_t \in V \quad (15)$$

$$r_{i_t} \geq 0 \quad \forall i_t \in V \quad (16)$$

$$\bar{r}_{i_t} \geq 0 \quad \forall i_t \in V \quad (17)$$

The objective function (1) minimizes the total generalized cost, taking into consideration movement and relocation costs, staff cost and penalty costs of rejecting the demand for or return of vehicles from customers. Constraint (2) serves the dual purpose of assigning a non-zero value to  $x^k$  when staff  $k$  is used from time step  $t=1$  and restricting staff  $k$  to only performing one type of activity at  $t=1$ . Constraint (3) ensures the conservation of a staff's activity at each node at  $i_t$ . It restricts each staff to only starting on exactly one new activity after the previous one is completed. Constraints (4) and (5) update the number of available and unavailable vehicles respectively. The number of available vehicles is adjusted by the vehicles relocated into and out of the station, vehicles returning to the station after maintenance, vehicles moving into and out of the station due to customer usage and vehicles requiring maintenance at the station. The number of unavailable vehicles is adjusted by vehicles taken out for maintenance from the station and vehicles requiring maintenance at the station. Constraint (6) ensures that the total number of available and unavailable vehicles does not exceed the station's capacity at any time step. Constraint (7) ensures that the rejected demand for vehicles do not exceed the requested demand for vehicles at a station at time step  $t$ . Constraint (8) ensures that the rejected return of vehicles does not exceed the requested return of vehicles. Constraints (9) to (13) impose binary conditions on the variables  $x^k, y_{i_t i_{t+1}}^k, z_{i_t i_{t+m}}^k, u_{i_t i_{t+ij}}^k$  and  $v_{i_t i_{t+ij}}^k$  respectively. Constraints (14) to (17) impose non-negativity conditions on the variables  $d_{i_t}^r, s_{i_t}^r, r_{i_t}$  and  $\bar{r}_{i_t}$  respectively.

This optimization is thus run from time steps  $t=1, \dots, T$ . As mentioned earlier, the length of staff shift hours can be arbitrarily set in this phase and later revised by the *Trend Filter* in phase two. No relocation thresholds are assumed at this stage. These thresholds will be deduced from the optimal solution. The optimization model developed here is a mixed integer linear programming model and is commonly solved using the branch-and-bound technique (18).

## TREND FILTER

This *Trend Filter* ‘filters’ the optimized results obtained from phase one through a series of heuristics to output a recommended set of operating parameters. The key information extracted from the filter includes staff strength and shift hours, relocation techniques, station threshold values and whether priority should be given to maintenance jobs or relocation trips.

**Selection of staff strength and shift hours.** A set of recommend staff strength and shift hours may be derived by observing the optimized  $x^k$ ,  $z_{i_t i_{t+m}}^k$ ,  $u_{i_t j_t \#ij}^k$  and  $v_{i_t j_t \#ij}^k$  values from phase one. A summation of all the  $x^k$  values provides an initial estimate of the recommended staff strength. This value is then adjusted depending on the  $z_{i_t i_{t+m}}^k$ ,  $u_{i_t j_t \#ij}^k$  and  $v_{i_t j_t \#ij}^k$  values, which depict staff work load.

A sample activity graph for one staff on a typical day is shown below in Figure 3. A non-zero variable value in  $z$ ,  $u$  or  $v$  variables (with a fixed  $k$  value) indicates that the staff is engaged in carrying out a maintenance, movement or relocation activity respectively. In this figure, the shift hours was arbitrarily set at eight hours per shift, i.e., three runs per day on the *Optimization Model* (0000 hrs to 0800 hrs, 0800 hrs to 1600 hrs and 1600 hrs to 0000 hrs). An initial estimate of the staff strength by the *Optimization Model* was thus no staff from 0000 hrs to 0800 hrs and one staff from 0800 hrs to 0000 hrs. It can be observed from Figure 3 however, that the staff is essentially only active from 1200 hrs to 2200 hrs. It can thus be interpreted that a more efficient choice of shift hours would be to have no staff from 0000 hrs to 1200 hrs, one staff from 1200 hrs to 2200 hrs and no staff from 2200 hrs to 0000 hrs.

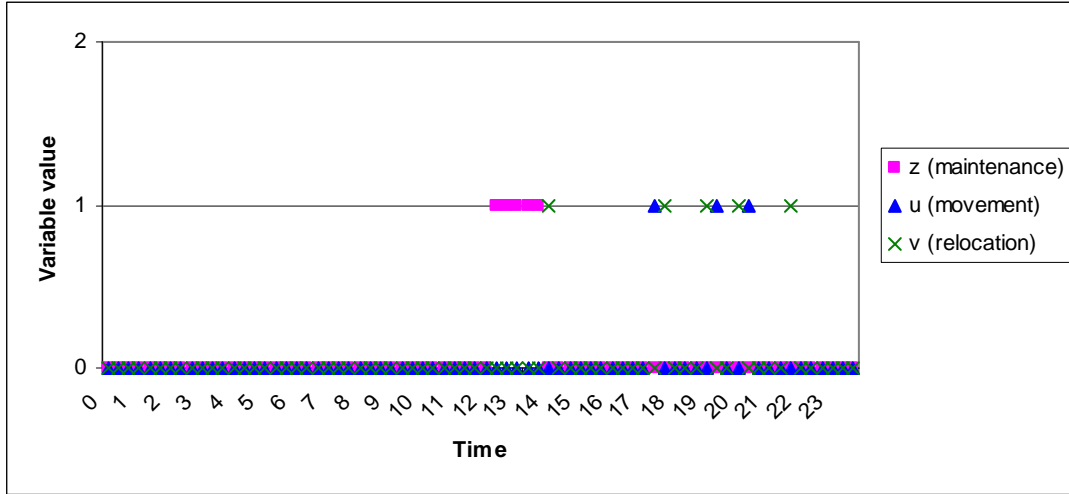


FIGURE 3 Activity Graph for One Staff on a Typical Day

**Selection of relocation technique.** Two techniques of relocating vehicles were proposed in (17), namely ‘shortest time’ and ‘inventory balancing’. Relocating vehicles by *shortest time* means moving vehicles to or from a neighboring station in the shortest possible time (including staff movement time, if necessary). Relocating by *inventory balancing* means filling a station which vehicle shortage with a vehicle from another station which has over supply. The recommended relocation techniques for use during different times of the day may be derived by observing the optimized  $v_{i,j,t+n_j}^k$  values from phase one. A non-zero value in  $v_{i,j,t+n_j}^k$  implies a relocation from

stations  $i$  to  $j$  and may be classified as a *shortest time* relocation or an *inventory balancing* relocation as follows. When a vehicle is relocated from stations  $i$  to  $j$ , it is either because station  $i$  is experiencing an over supply, station  $j$  is experiencing a shortage or both. It is thus defined that if a relocation from stations  $i$  to  $j$  is followed by a reduction in available vehicles in station  $j$  (in the next time step, due to customer pickup or sending for maintenance) and at the same time an increases in available vehicles in station  $i$  (in the next time step, due to customer return or completion of maintenance), the relocation is considered *inventory balancing*. Otherwise, the relocation is considered *shortest time* since the optimization model is cost minimizing, and time is reflected in the various cost components in the objective function. Through an observation of the classified relocation techniques across the time steps, consistent relocation techniques for use during different times of the day are thus recommended.

**Selection of relocation thresholds.** There are a total of four relocation thresholds for each station: two critical thresholds and two buffer thresholds (17). When the number of available vehicles in the station goes above the high critical threshold or below the low critical threshold, a relocation request is generated from and to the station respectively. When a relocation is required in one station, the supporting station will only allow a vehicle to be taken out or brought in if the number of vehicles in the supporting station is at and above the low buffer threshold or at and below the high buffer threshold, respectively. The buffer threshold values are thus naturally bounded by the critical threshold values, i.e., the high critical threshold value is greater than the high buffer threshold value while the low critical threshold value is smaller than the low buffer threshold value. Choosing more extreme critical threshold values (which gives a larger

range between the upper and lower limits) and less extreme (smaller range, and hence more conservative) buffer threshold values would trigger fewer relocation requests and allow fewer relocations to be carried out, thus reducing cost but possibly compromising the LOS. Conversely, more conservative critical thresholds (smaller range) and extreme buffer thresholds (larger range) would trigger more relocation requests and allow more relocations to be carried out, thus maintaining a higher LOS but at the expense of increased cost.

A recommended set of relocation thresholds may be derived by observing the optimized  $r_i$  values (number of available vehicles in station  $i$  at time step  $t$ ) together with the relocation technique identified. Where relocations are identified as *shortest time*, the low buffer threshold is taken to be the minimum  $r_i$  value of the supporting station from which the vehicle is removed while the high buffer threshold is taken to be the maximum  $r_i$  value of the supporting station to which the vehicle is inserted. For critical thresholds however, there is a need to look at the station requesting for relocation in both *inventory balancing* and *shortest time* relocation techniques. The high critical threshold is taken to be the minimum  $r_i$  value of the requesting station from which the vehicle is removed while the low critical threshold is taken to be the maximum  $r_i$  value of the requesting station to which the vehicle is inserted. Given the advantage of perfect knowledge in the optimization model, a conservative allowance of one and two vehicles may be recommended for the buffer and critical thresholds, respectively. A set of four threshold values for each station is thus derived.

**Selection of job priority.** This refers to the decision to give priority to either maintenance jobs or relocation trips when both are required. Depending on the cost structure of the carsharing operation, it may be more cost effective to give priority to either maintenance jobs or relocation trips at different time periods of the day. Once again, a recommended priority may be derived from observing the optimized  $v_{i,j,t+ij}^k$  and  $r_i$  values. A non-zero value of  $v_{i,j,t+ij}^k$  coupled with a positive value of  $r_i$  implies that priority is being given to relocation trips over maintenance jobs. That is, relocations are still being carried out although there are available vehicles at a station. Otherwise, the default priority is given to maintenance jobs. Through an observation of this priority across time, a recommended set of priorities for different time periods of the day is thus derived.

## COMPUTATIONAL RUNS

The new three-phase OTS tool was tested using one plus three months of commercially operational data to evaluate its effectiveness. One typical month of data (prior to the three months) with 1236 customer trips was first passed through the *Optimization Model*, where a branch and bound algorithm was applied with a node selection strategy to branch on the best bound, i.e. branching was always done on the pending node giving the smallest value to the objective function. The problem was coded into ILOG OPL Studio, Version 3.7.1 and solved using the ILOG CPLEX 9.1 Mixed Integer Programming module (19).

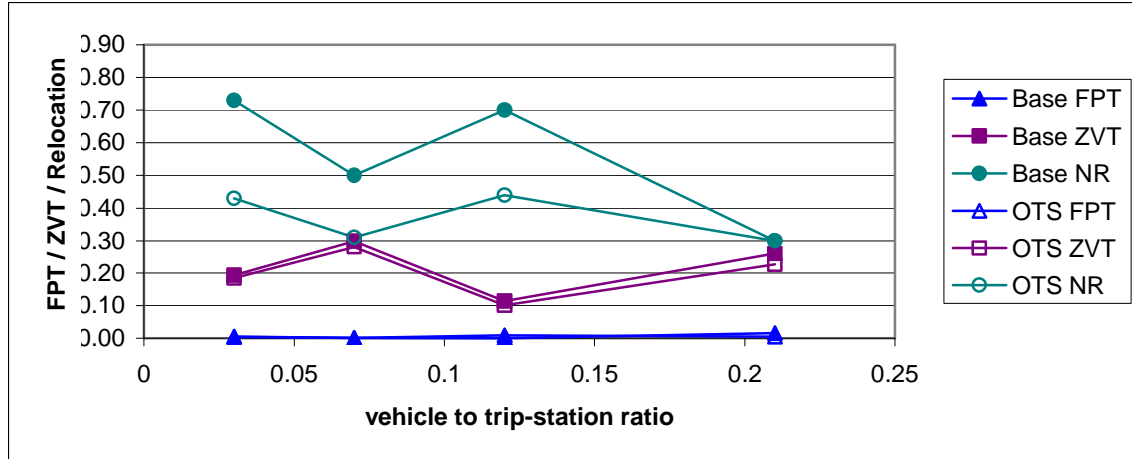
The cost coefficients used are estimated from publicly available data, with  $c_x$  being valued at S\$47 per shift (8 hours multiplied by the minimum average wage rate),  $c_s$  and  $c_d$  equally valued (based on potential commercial losses, calculated from published fare structure) at S\$271.58 on weekdays (Mondays to Fridays, 0800 hrs to 1900 hrs), S\$397.70 on weekends (Saturdays, Sundays and public holidays, 1900 hrs on the eve to 0800 hrs the following day) and S\$833.30 for overnight (1900 hrs to 0800 hrs) and estimated  $c_{ij}$  ranged between S\$1.50 to \$9.00.

The optimized results were then passed through the *Trend Filter* to extract a set of recommended operating parameters (staff strength, staff shift hours, relocation techniques, station threshold values and whether priority should be give to maintenance jobs or relocation trips). The filtered results suggest a 50% reduction in staff strength with minor adjustments to shift hours during weekdays and weekends, use of inventory balancing vehicle relocation technique throughout, less conservative station threshold values and priority to be given to maintenance jobs over relocation trips.

This set of parameters was then entered into the simulation model and evaluated with three months of test data. This data set was selected for its maximum range of vehicle to trip-station ratio, thus enabling a better assessment of the potential benefits from across a wider range of system setup, when new vehicles and/or stations are added to the system. When there are  $x$  vehicles,  $y$  trips a day and  $z$  stations, this ratio is calculated as  $x/(yz)$ . A low ratio implies a high intensity of vehicle usage and/or a wide spread of vehicles across stations. It is important to note that the value of  $y$  simply indicates the trip frequency (which reflect the intensity of vehicle pickups and returns) and not vehicle utilization (vehicle-hour used or vehicle-km traveled). The three performance indicators, namely ZVT, FPT and NR were used to gauge system performance. Due to the confidentiality of the operational data from ICVS, all ZVT and FPT presented have been scaled by the factor  $m$  to dimensionless values. Similarly, NR has also been similarly scaled by a factor  $p$ .

## Results and Analysis

Besides being influenced by the relocation technique adopted and the operating parameters, the performance indicators are also primarily influenced by the vehicle to trip-station ratio. The results generated from the simulation are plotted against their vehicle to trip-station ratios. A comparison of the performance indicators when the system was operating with the OTS-generated parameters and with the current parameters used by ICVS (referred to as the base model) is shown in Figure 4. The percentage improvements in the performance indicators are presented in Table 1.



**FIGURE 4 Comparison of OTS against the Base Model**

**TABLE 1 Percentage Improvement in Performance Indicators of OTS from Base Model**

Vehicle to Trip-Station Ratio	Improvement (%)		
	ZVT	FPT	NR
0.03	4.6	0.0	41.1
0.07	6.0	8.1	38.0
0.12	11.2	0.0	37.1
0.21	13.0	71.3	0.0

It can be observed from Figure 4 that all the three performance indicators are either maintained or reduced. Because all the FPT, ZVT have been scaled by a factor  $m$ , and NR by a factor  $p$ , the improvements from the base model to the one generated by the OTS tool are best expressed in percent. As mentioned earlier, lower ZVT and FPT correspond to a better LOS for customers, while a lower NR means a reduction in operating cost. Figure 4 shows that the ZVT have reduced for the various vehicles to trip-station ratios. From Table 1, it can be seen that ZVT levels show consistent improvements ranging between 4.6% and 13.0% from the base model. It is also obvious from Figure 4 that NR have significantly been reduced by up to 41.1%. The magnitude of the improvements in the FPT is relatively smaller compared to that of the ZVT, as suggested in Figure 4. This is because the FPT in the base model already have small values, due to the conservative setting of the upper critical and buffer threshold values in the base model to favor zero or low FPT. Any slight reduction in the FPT is therefore a significant percentage improvement. The computed percentage improvements in FPT listed in Table 1 range between 0% to 71.3%.

The above performance surpasses the results of the previous simulations conducted by Kek *et al.* (17) which uses iterative methods to select the parameters for the vehicle relocation operations. Although it is an improvement from the base case, the iterative approach is not able to find the optimal or near-optimal combination of the parameters. Although the previous simulation generated a cost savings of up to 12.8%, it resulted in a trade-off relationship between the indicators from the base case (see Figure 4 in (17), where a reduction in ZVT is compromised by



an increase in NR, and vice versa, using a different data set from the ICVS). This new three-phase OTS tool enables all three performance indicators to be consistently maintained or improved, coupled with a 50% reduction in staff cost.

## CONCLUSION

Motivated by a fundamental desire to enhance both operational efficiency and service levels at the lowest possible cost, this paper proposes and studies for the first time, the new problem of finding a set of near-optimal parameters for vehicle relocation operations in a multiple-station carsharing system with flexible return time and stations. A three-phase OTS decision support tool is proposed and developed to solve this problem. Simulation tests, based on a set of commercially operational data, have produced statistics that indicate a better system performance than the existing system. The three-phase OTS tool recommends a set of parameters for vehicle relocation operations, enabling a reduction in staff cost of 50%, an improvement of ZVT from 4.6% to 13.0%, a maintenance of the low FPT level and a reduction in NR from 37.1% to 41.1%. Where previous simulations (17) show only marginal improvements in the performance indicators or a trade-off relationship between the performance indicators, this new three-phase OTS tool enables all three performance indicators to be consistently maintained or improved, coupled with a 50% reduction in staff cost. Other intangible benefits such as increased operational efficiency and potential increase in profit are also present.

Finally, it is important to note the versatility of the OTS tool and their adaptability to a wide variety of multiple-station carsharing systems with flexible return time and stations. Operators can thus easily apply these models to their unique systems to identify a recommended set of operating parameters, effectively removing excesses in their system and bringing about enhanced service levels and increased operational efficiency.

A potentially useful application of these models arises when the operator decides to open up a new station. After starting operations for a month, data may be entered and processed through the OTS tool for a recommended set of operating parameters.

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