Convolutional Neural Networks

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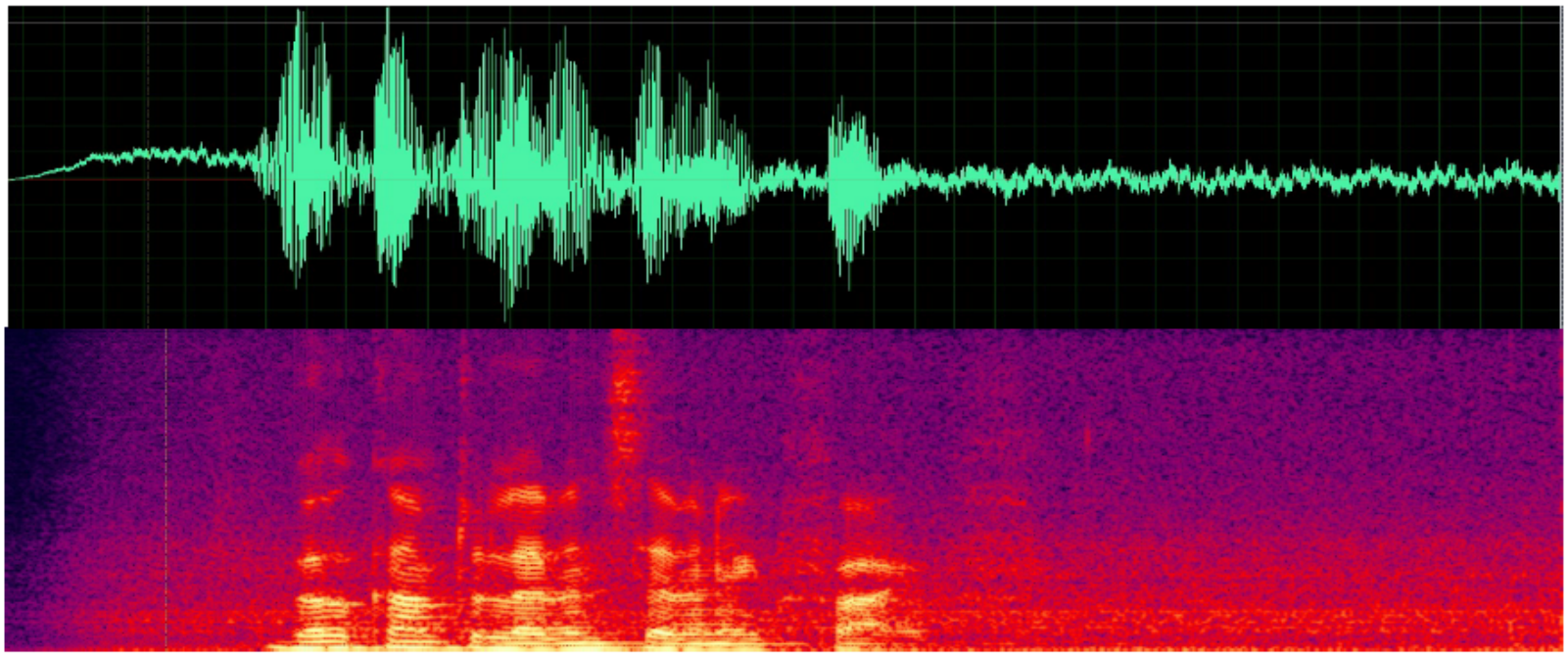
Most slides have been adapted from Bhiksha Raj, 11-785, CMU 2020, Justin Johnson, EECS 498-007, University of Michigan 2020, and Fei Fei Li, cs231n, Stanford 2017

Today's agenda

Recap

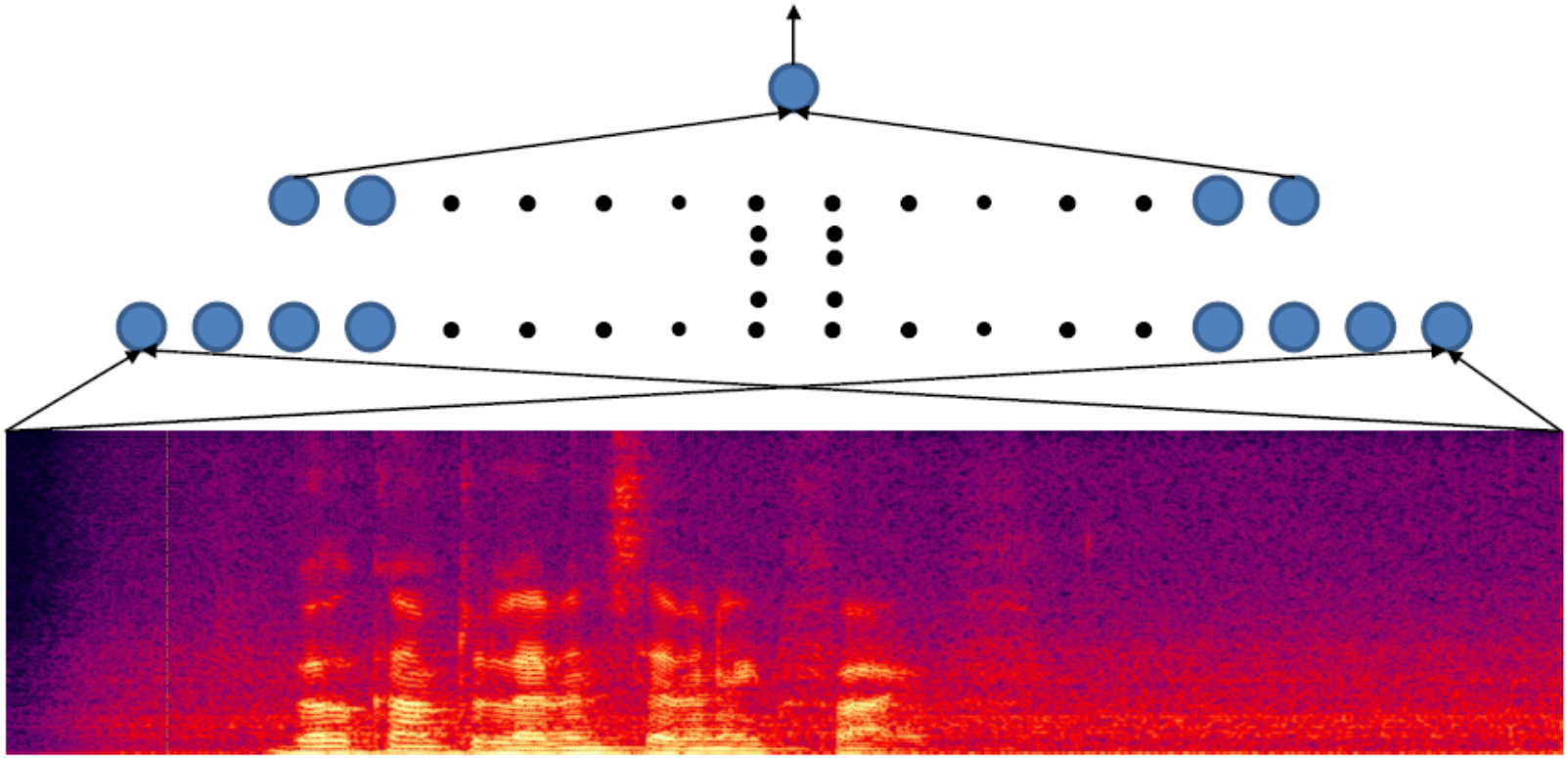
A new problem

- Does this signal contain the word “Welcome”?
- Compose an MLP for this problem.
  - Assuming all recordings are exactly the same length.



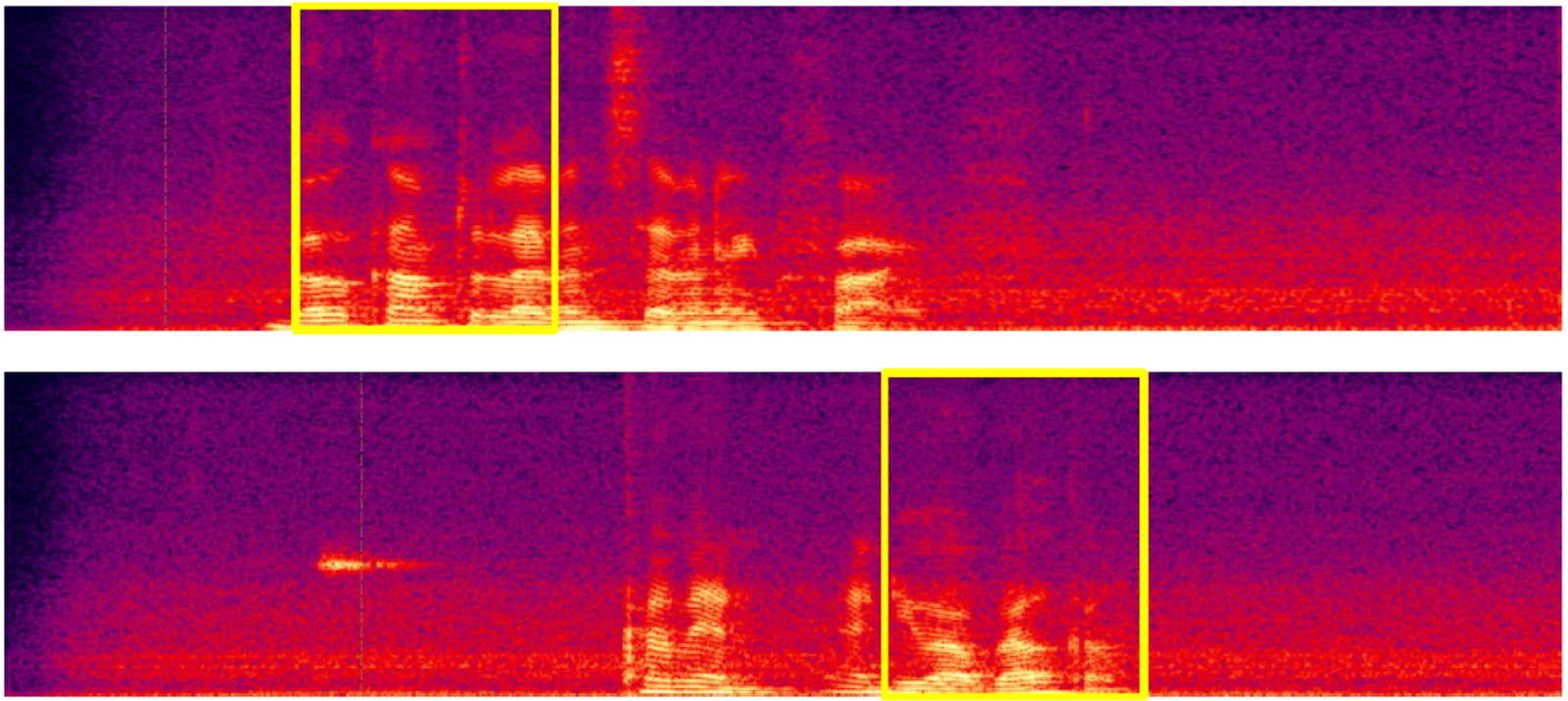
Finding a Welcome

- Trivial solution: Train an MLP for the entire recording



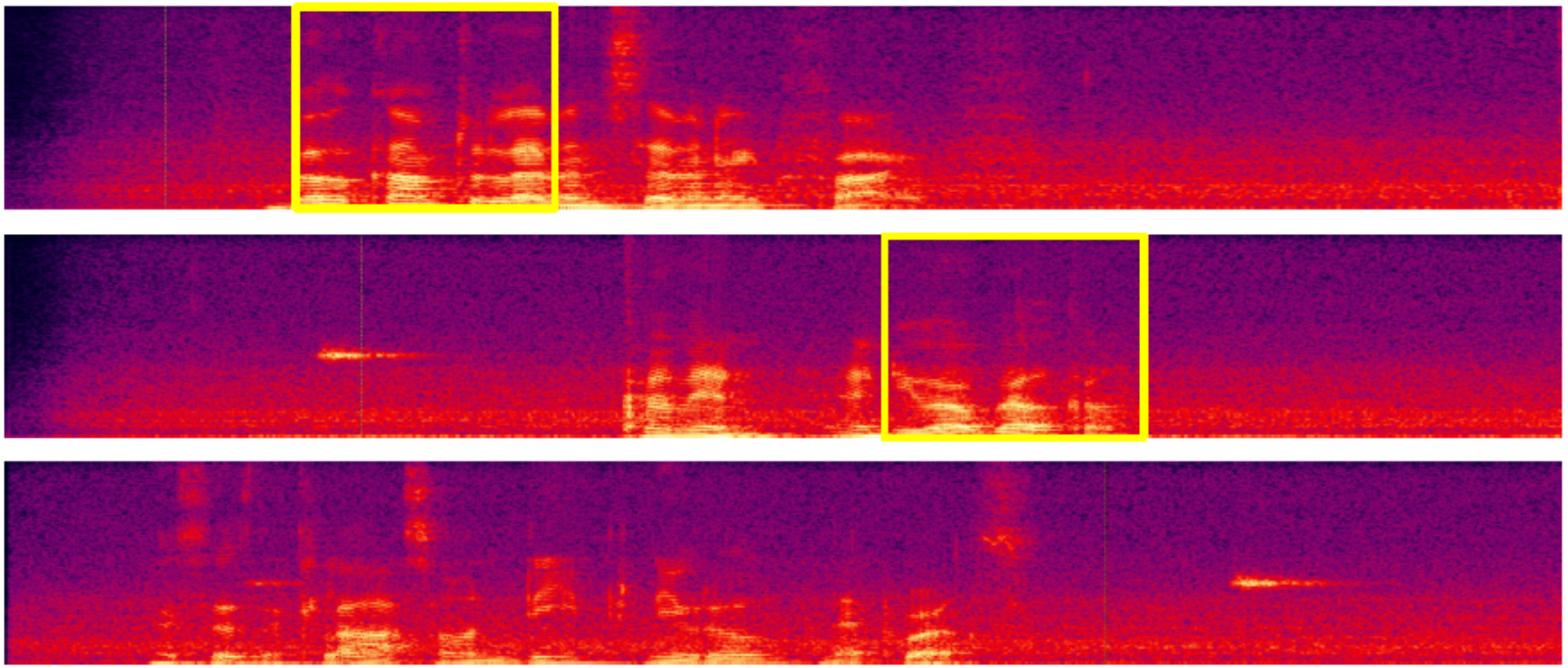
Finding a Welcome

- Problem with trivial solution: Network that finds a “welcome” in the top recording will not find it in the lower one
  - Unless trained with both
  - Will require a very large network and a large amount of training data to cover every case



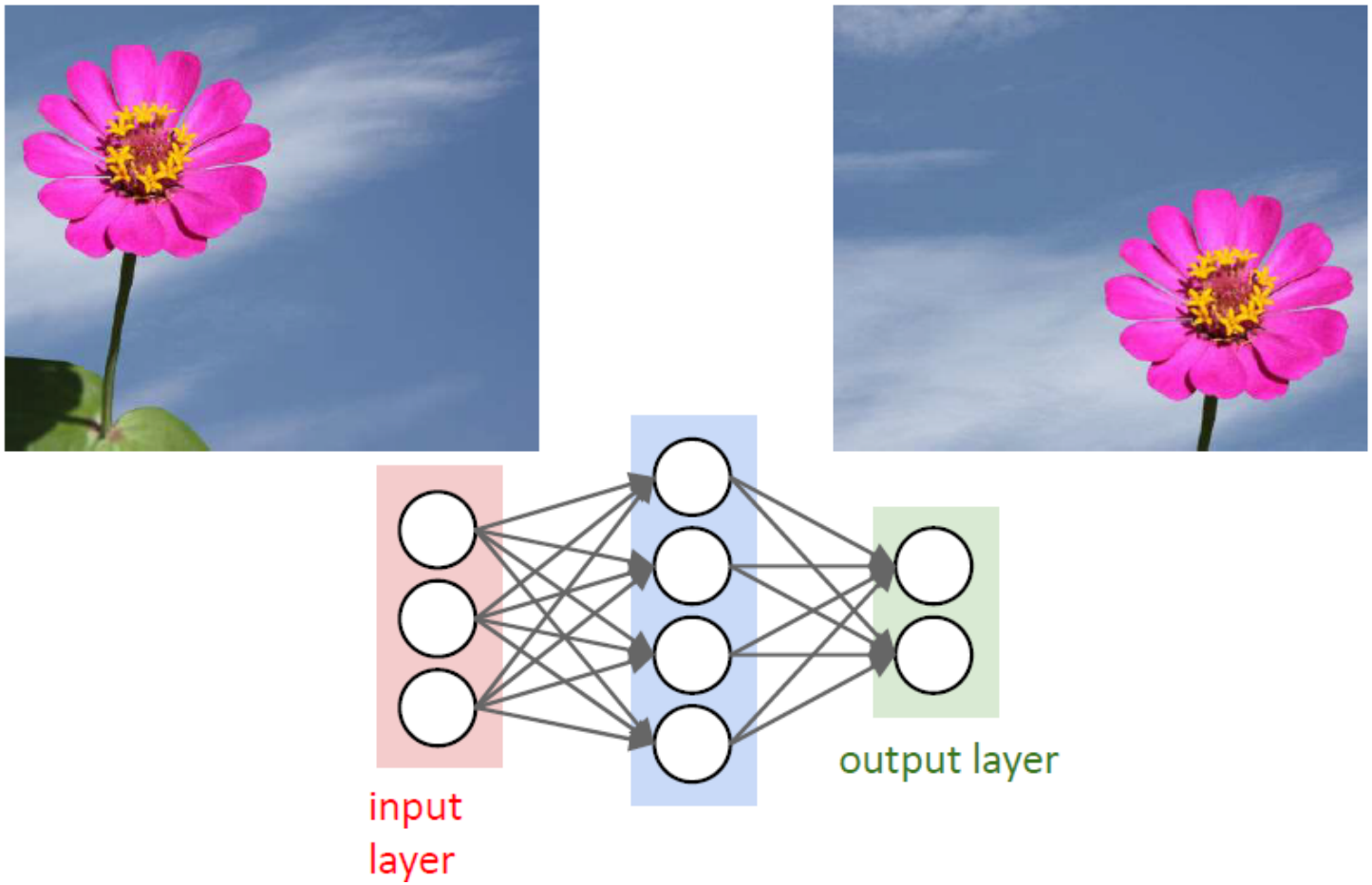
Finding a Welcome

- Need a simple network that will fire regardless of the location of “Welcome”
  - and not fire when there is none



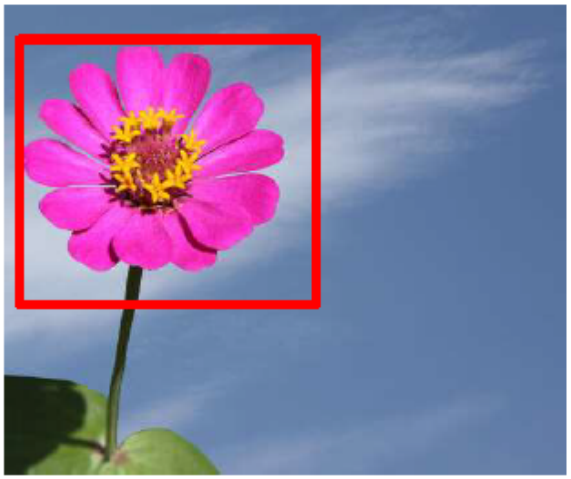
A problem

- Will an MLP that recognizes the left image as a flower also recognize the one on the right as a flower?



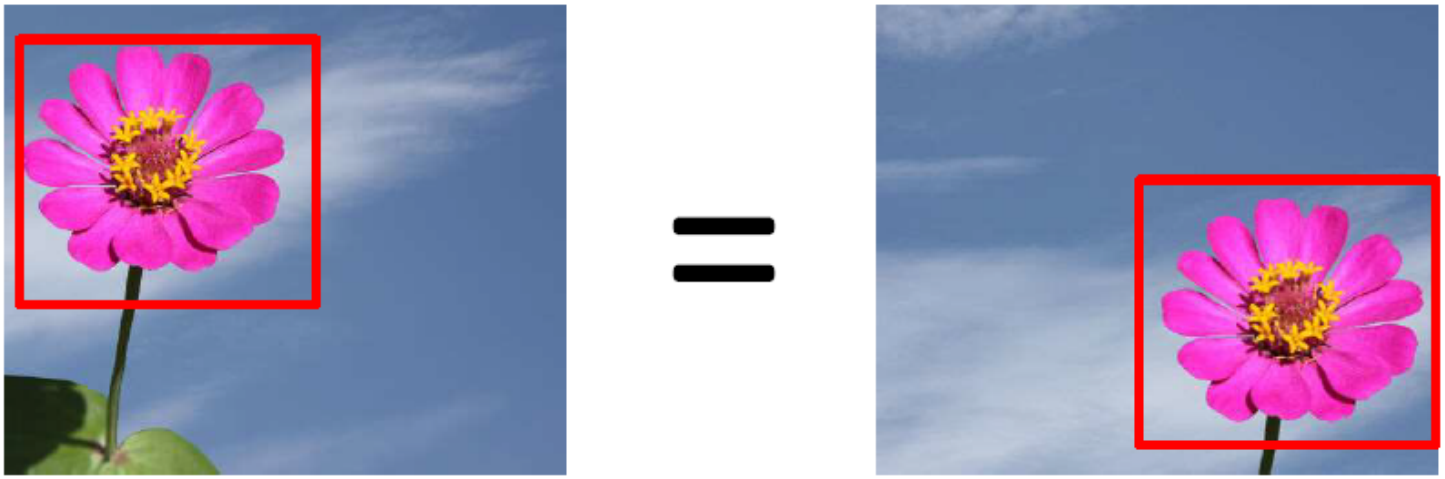
A problem

- Need a network that will “fire” regardless of the precise location of the target object



A problem

- In many problems the location of a pattern is not important
  - Only the presence of the pattern
- Conventional MLPs are sensitive to the location of the pattern
  - Moving it by one component results in an entirely different input that the MLP wont recognize
- Requirement: Network must be **shift invariant**



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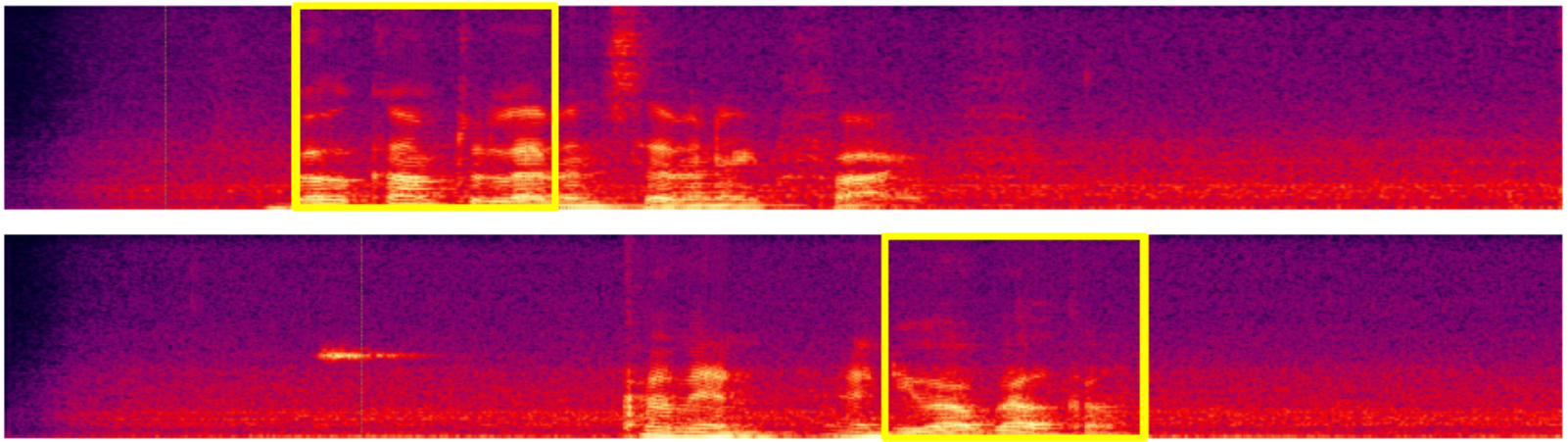
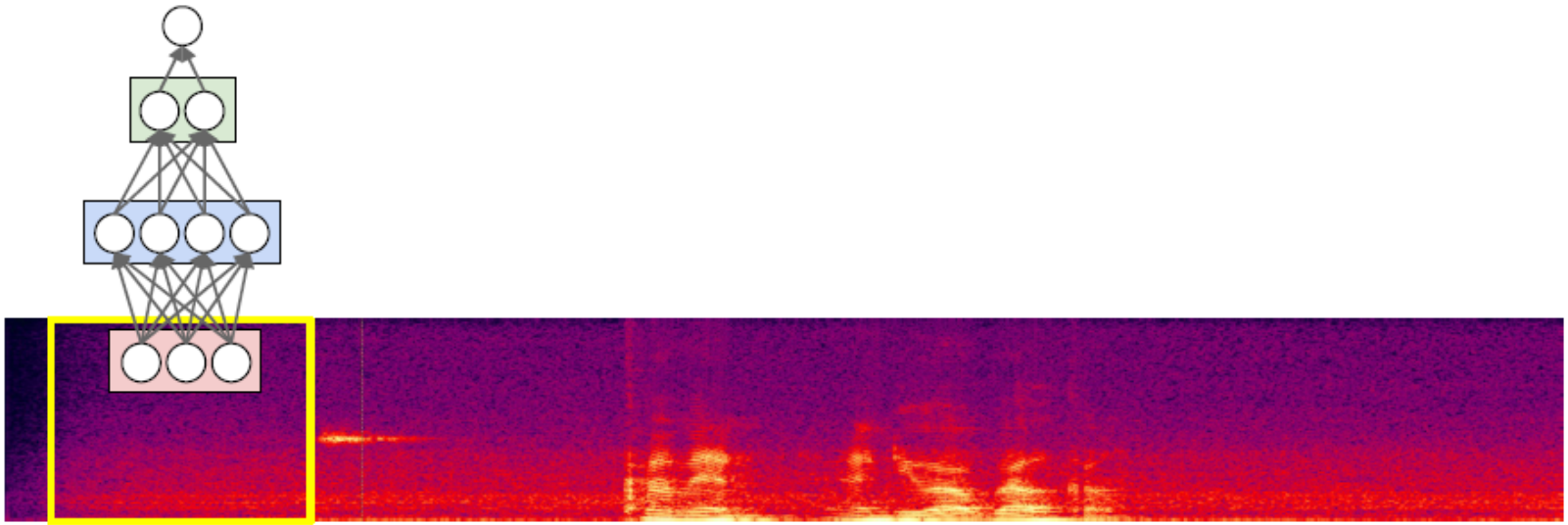


Solution: Scan

- Scan for the target word
  - The spectral time-frequency components in a “window” are input to a “welcome-detector” MLP



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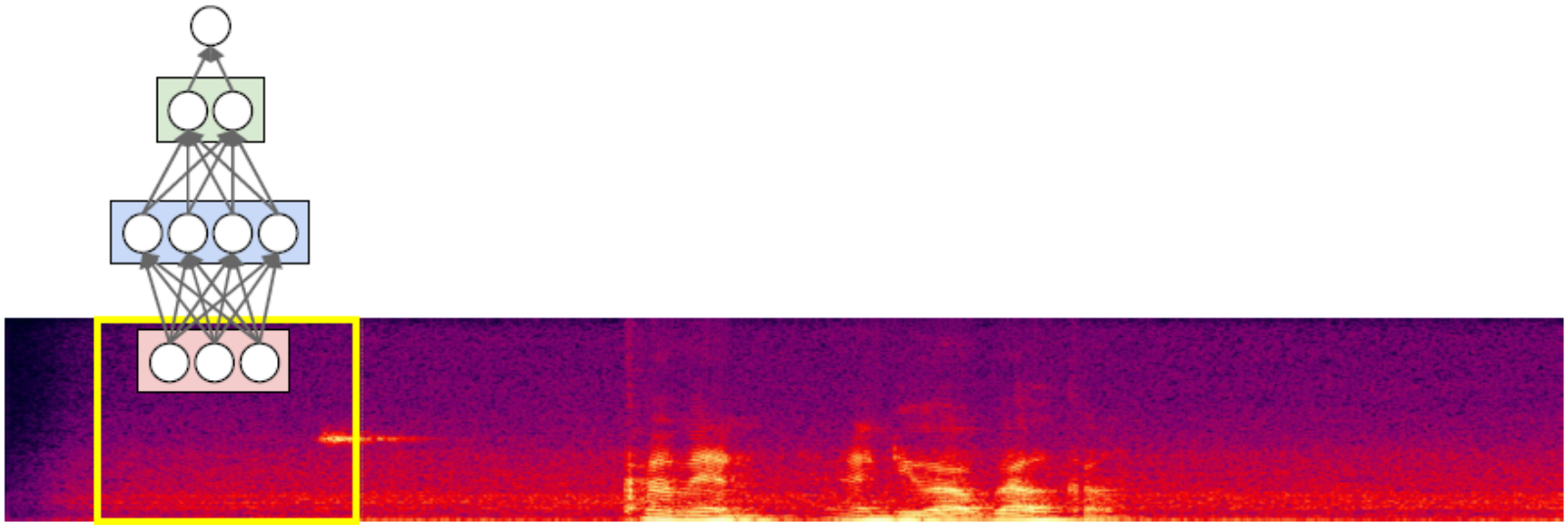

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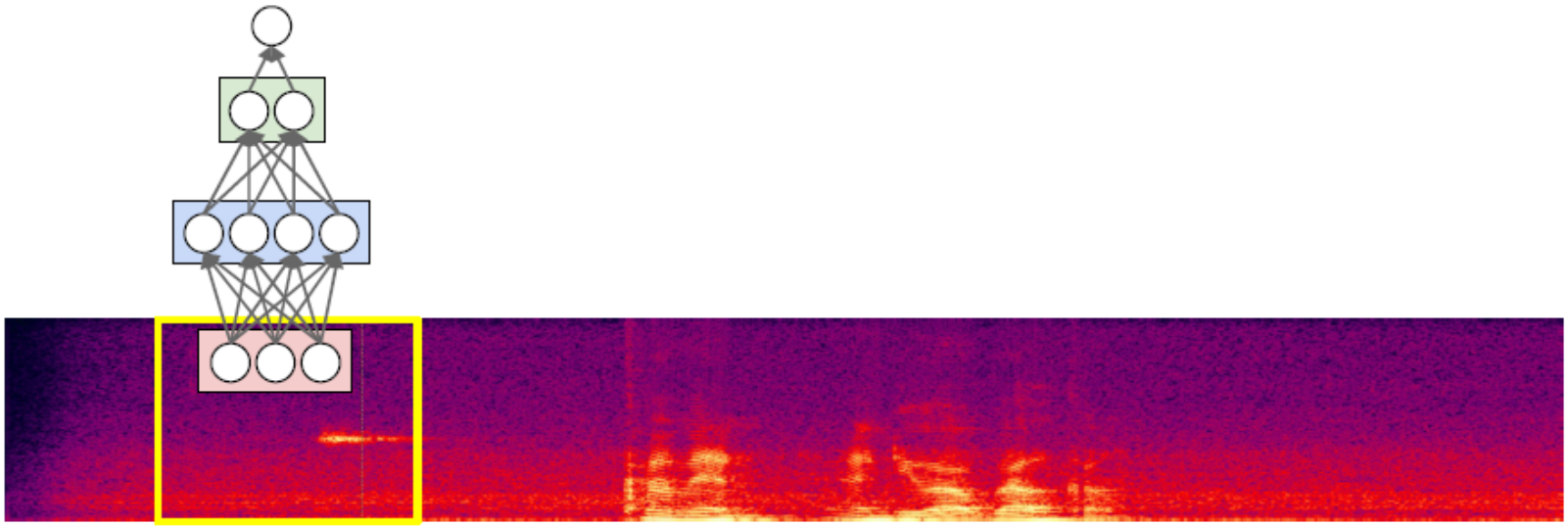

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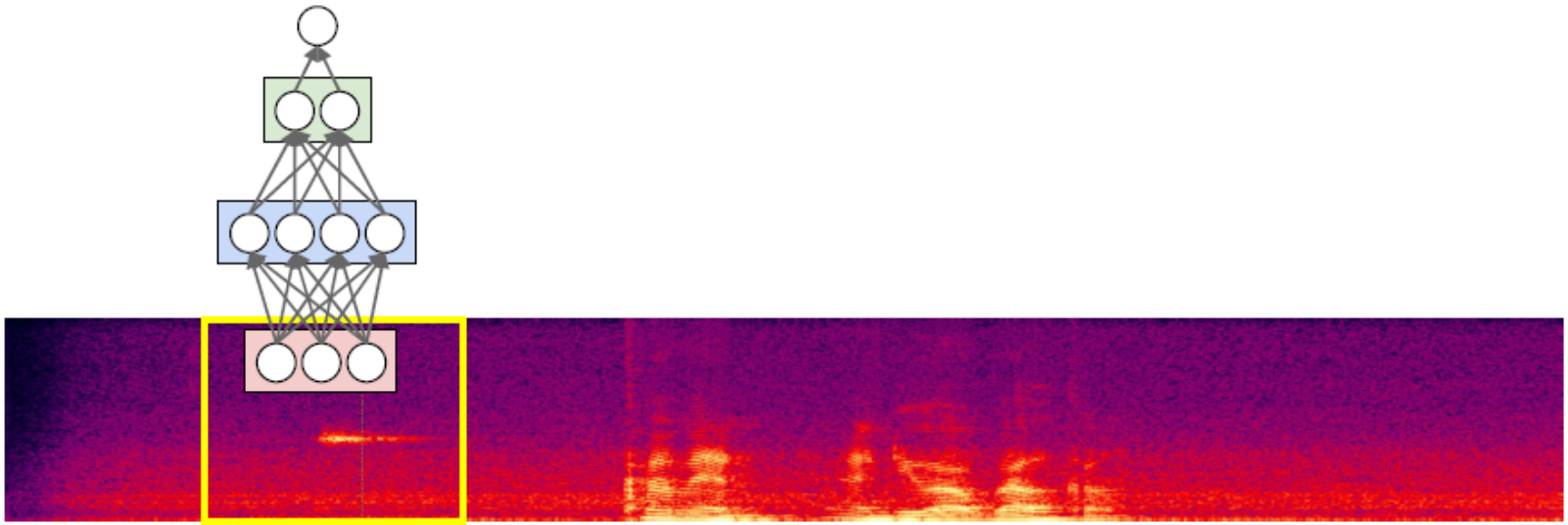

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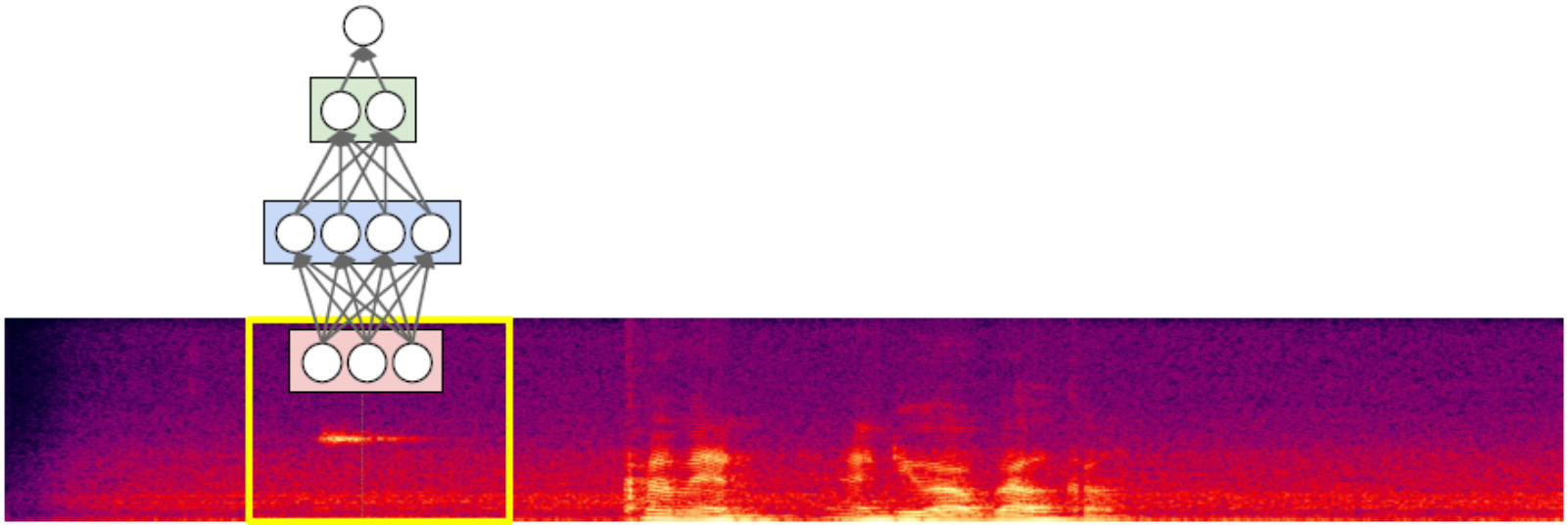

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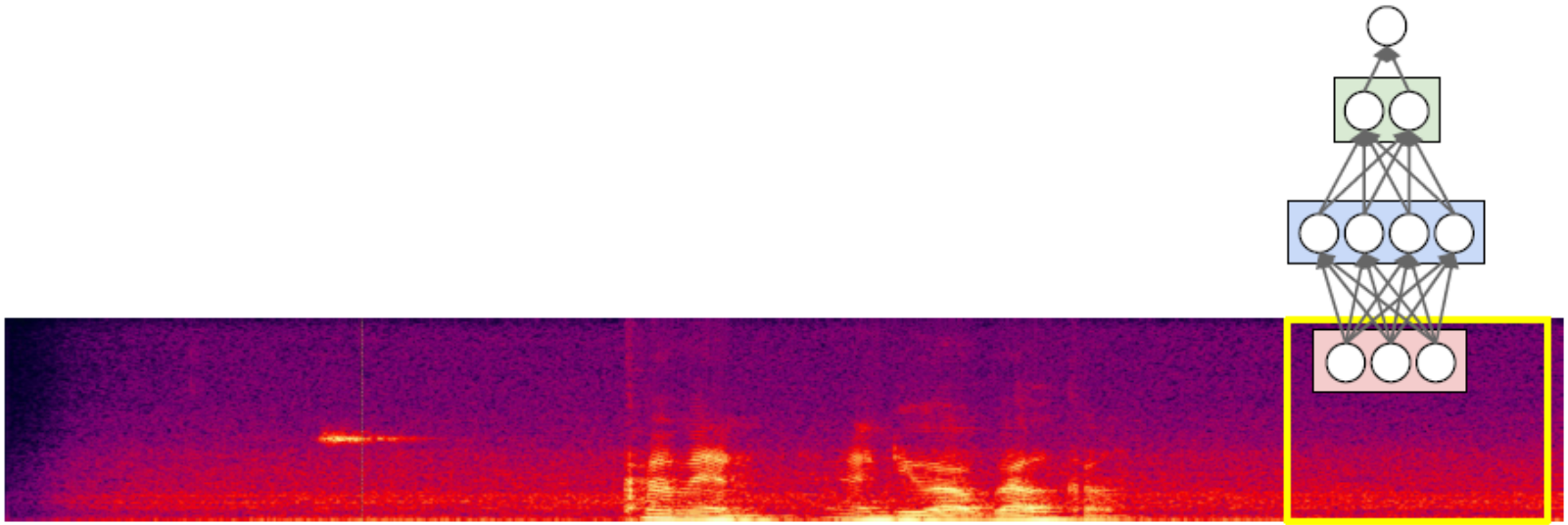
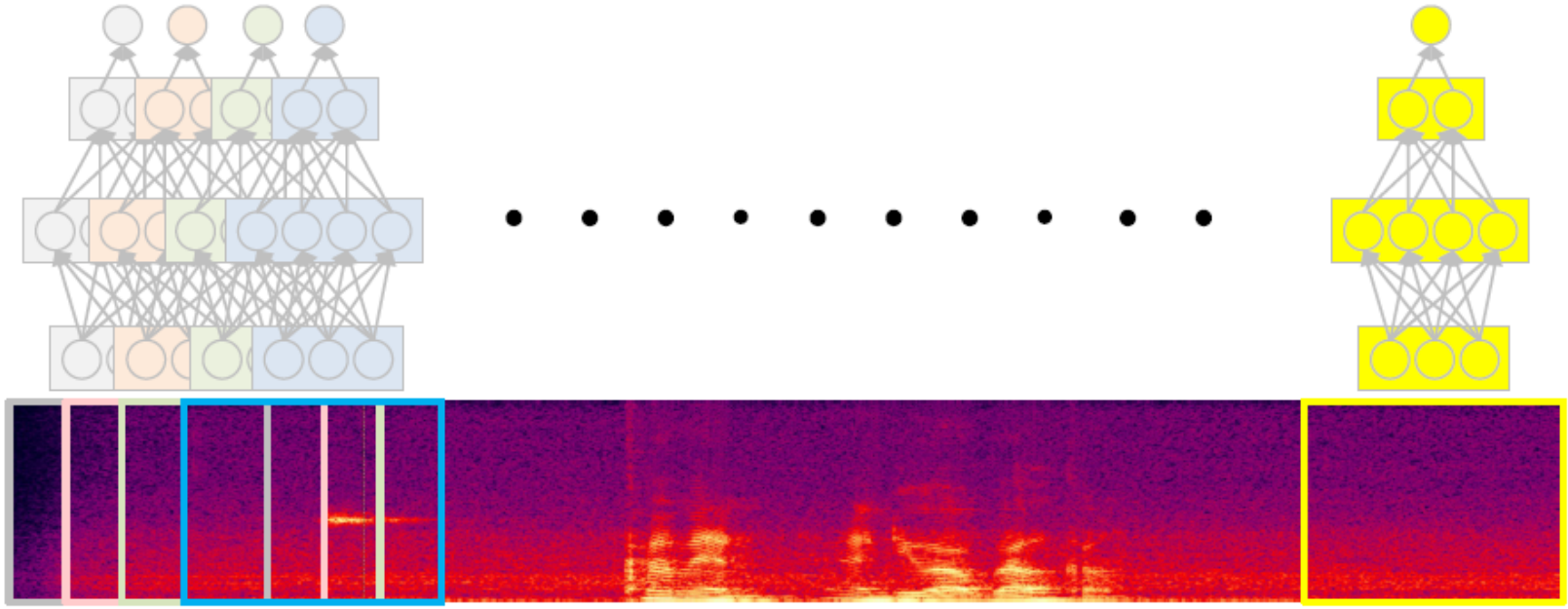


Solution: Scan

- “Does welcome occur in this recording?”
  - We have classified many “windows” individually
  - “Welcome” may have occurred in any of them



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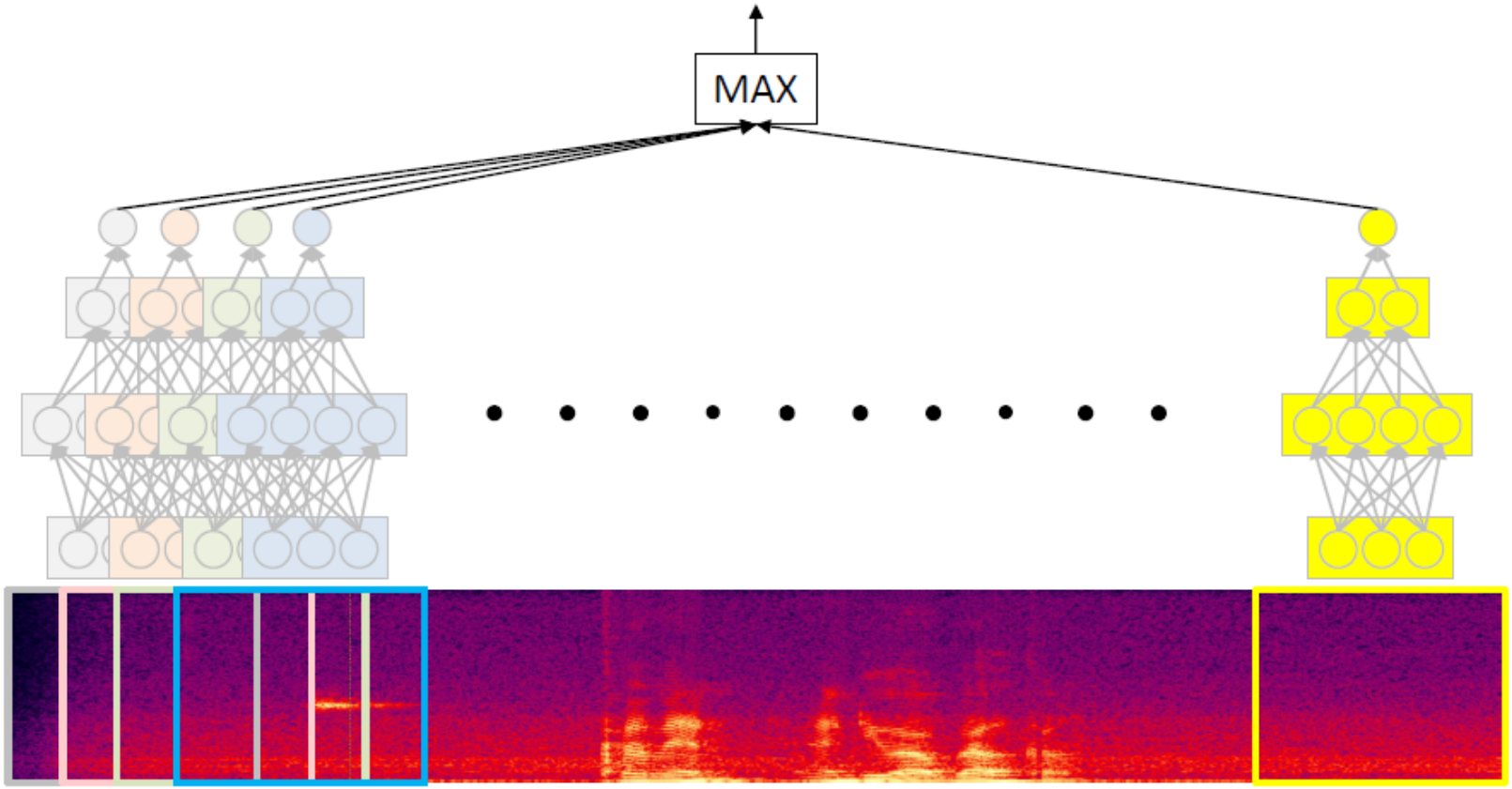
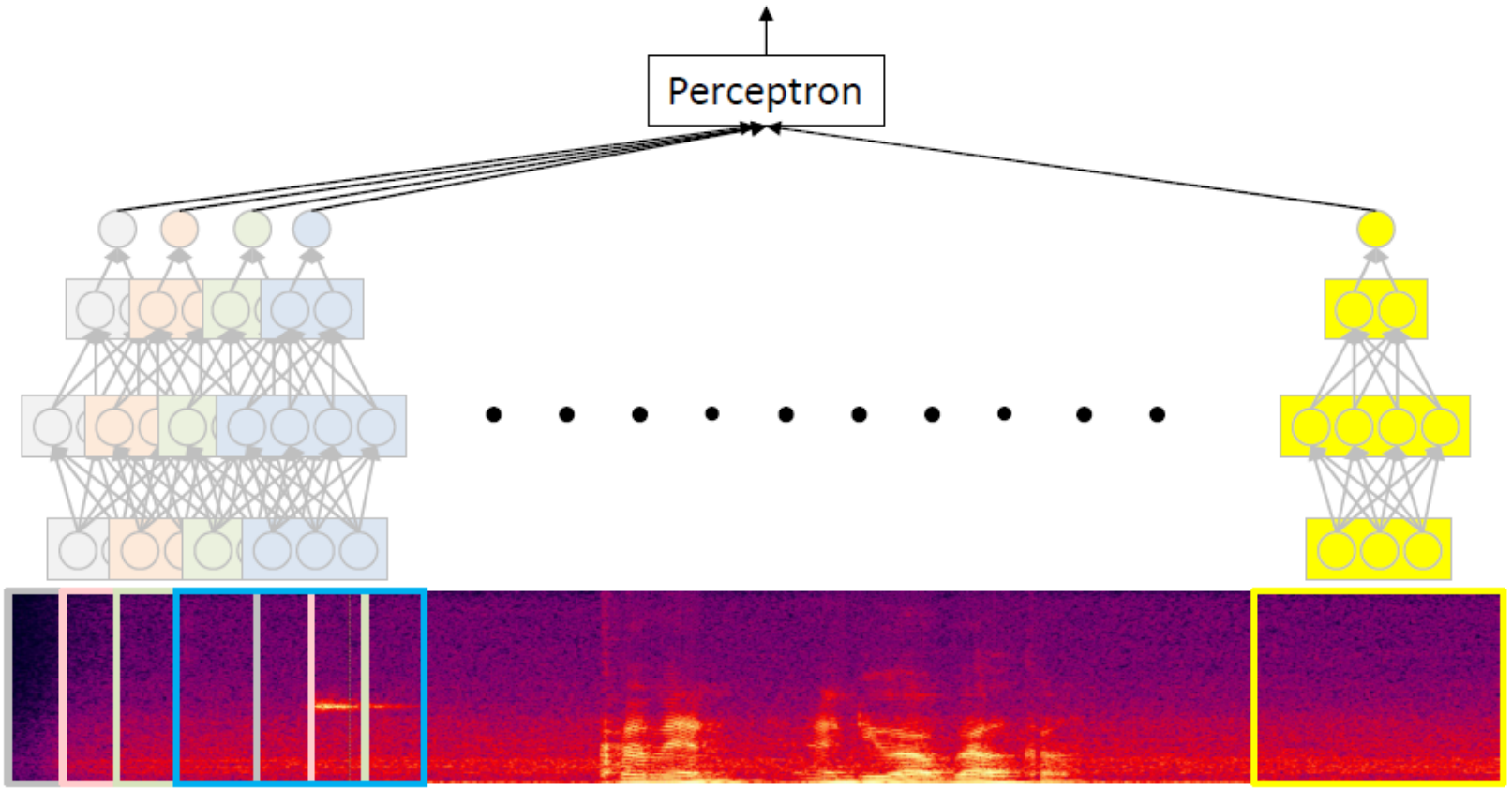
  - Maximum of all the outputs (Equivalent of Boolean OR)



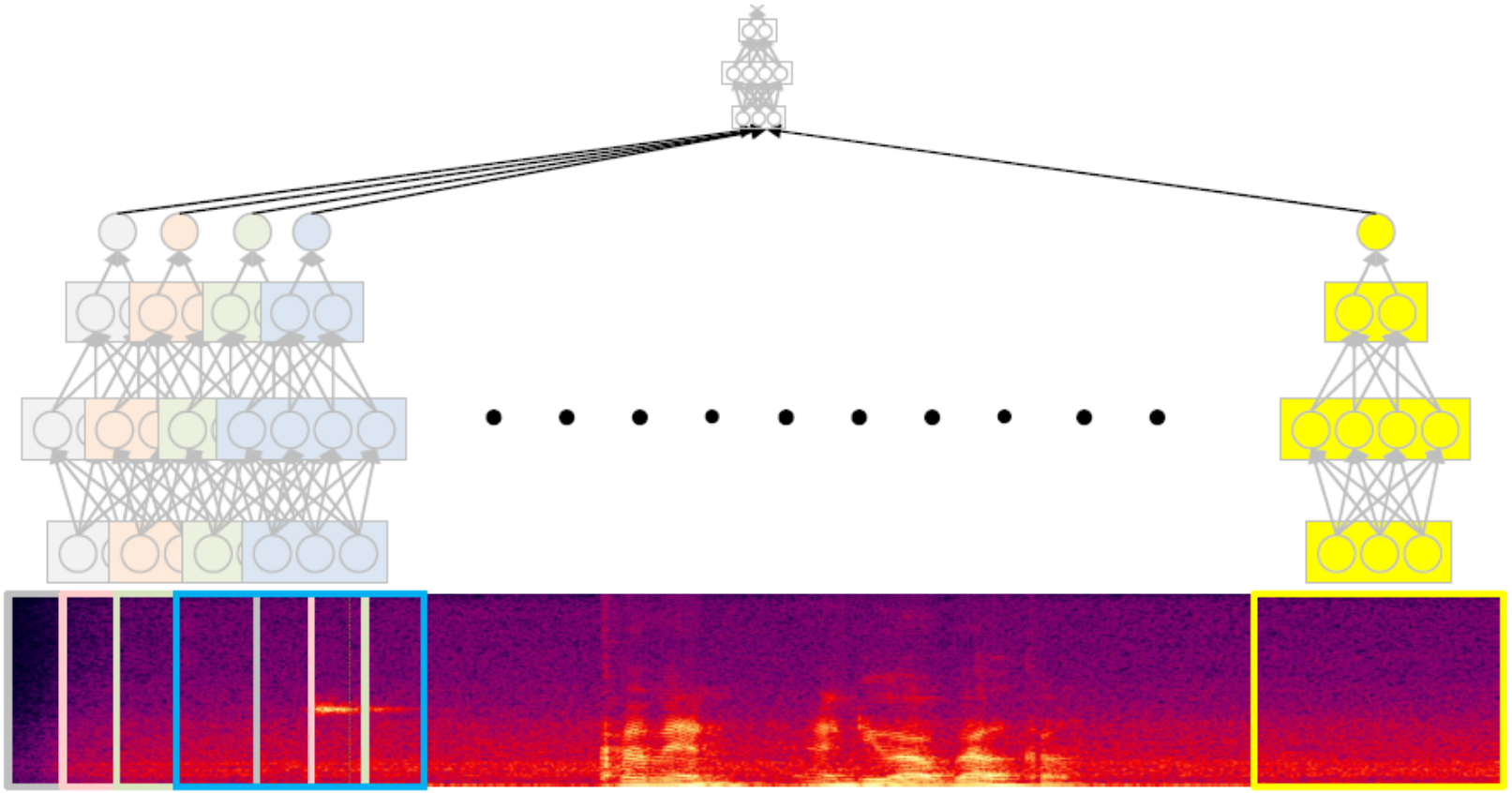
Solution: Scan

- "Does welcome occur in this recording?"
  - Maximum of all the outputs (Equivalent of Boolean OR)
  - Or a proper softmax/logistic
    - Finding a welcome in adjacent windows makes it more likely that we didn't find noise
    - Adjacent windows can combine their evidence



Solution: Scan

- "Does welcome occur in this recording?"
  - Maximum of all the outputs (Equivalent of Boolean OR)
  - Or a proper softmax/logistic
  - Or even an MLP



Scanning with an MLP

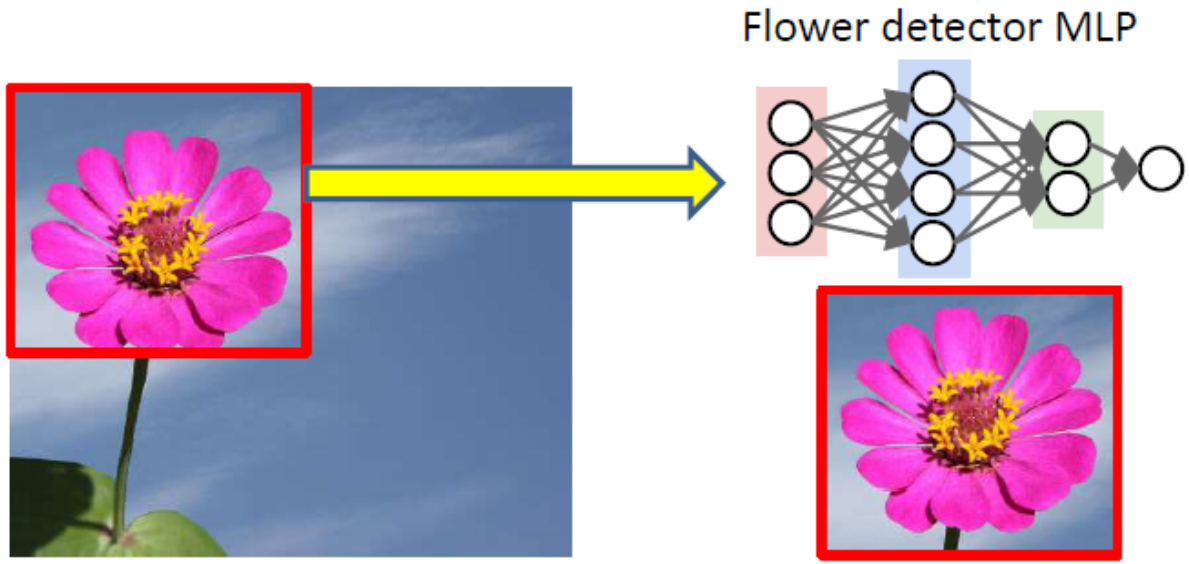
- K = width of "patch" evaluated by MLP

```
For t = 1:T-K+1
  XSegment = x(:, t:t+K-1)
  y(t) = MLP(XSegment)

Y = softmax(y(1)..y(T-K+1))
```

Scanning the picture to find a flower

- Scan for the desired object
- At each location, the entire region is sent through the MLP



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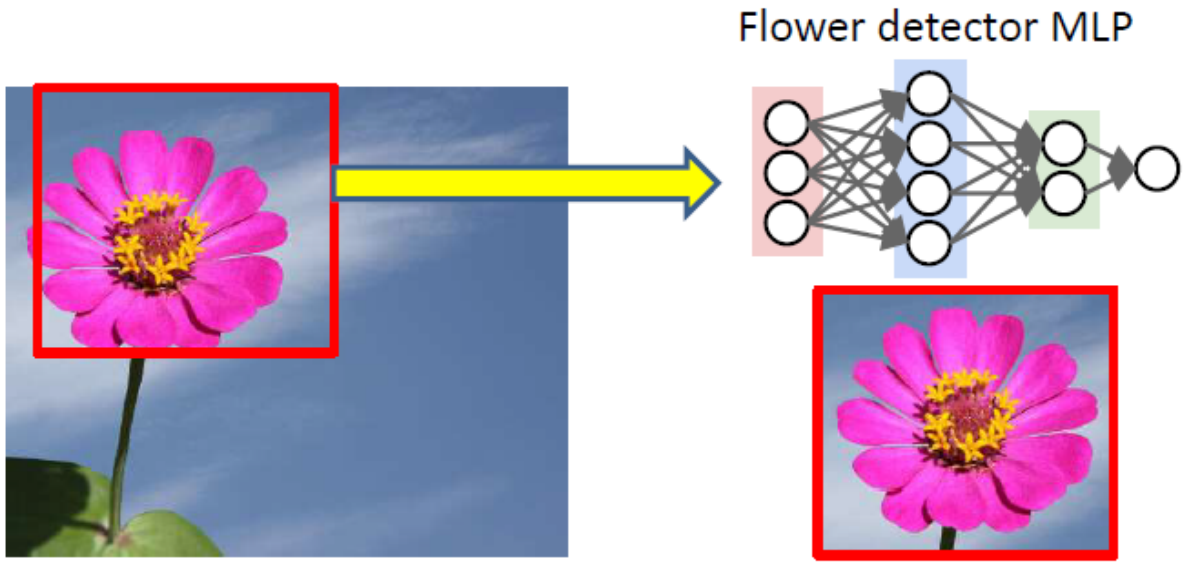

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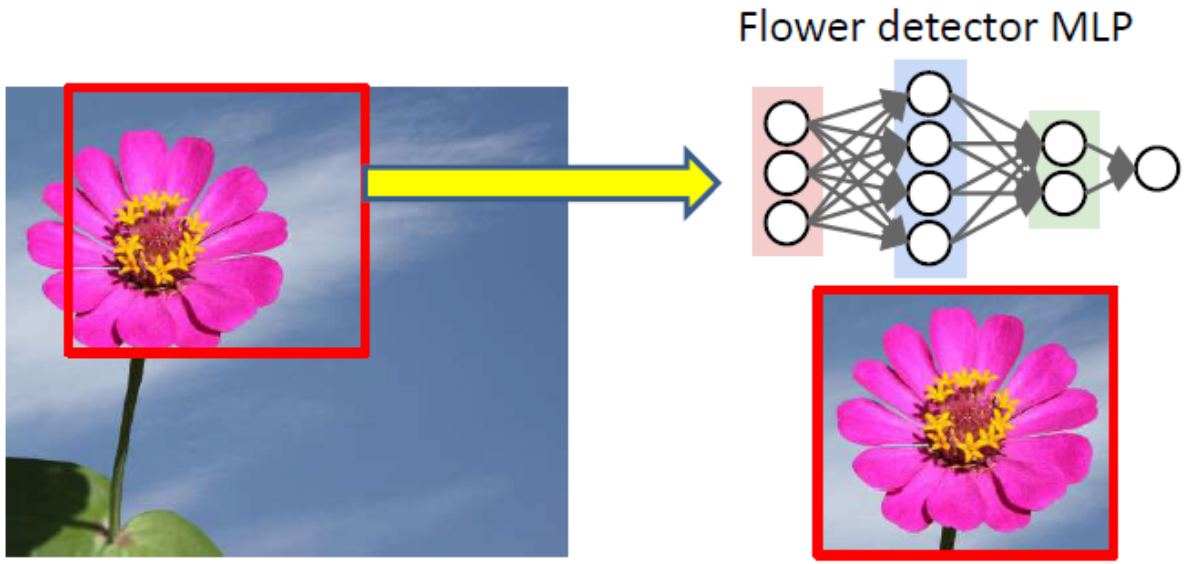

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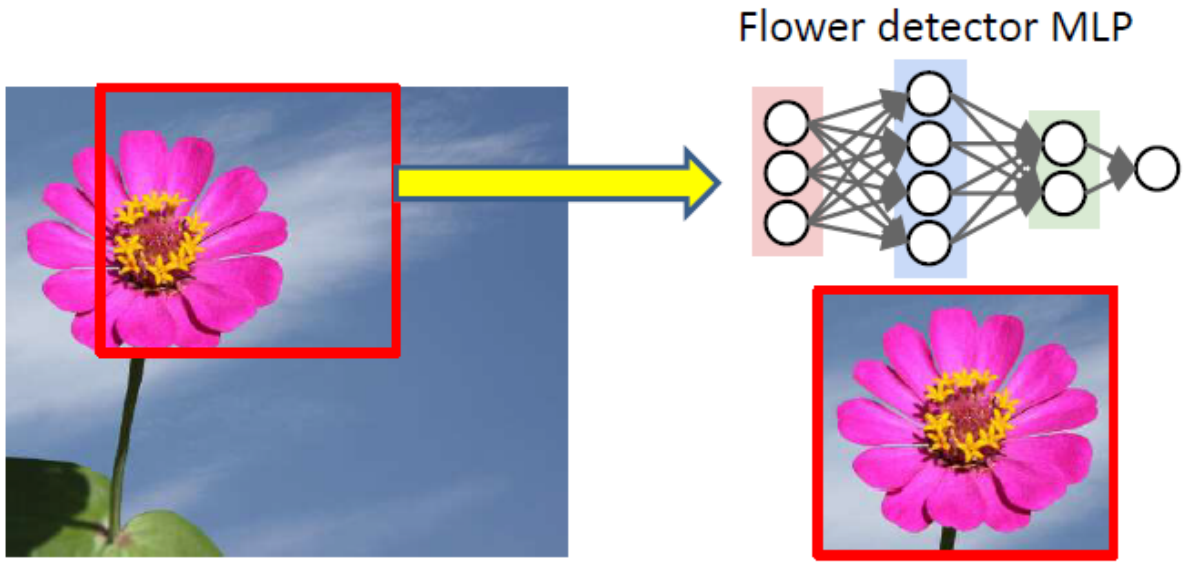

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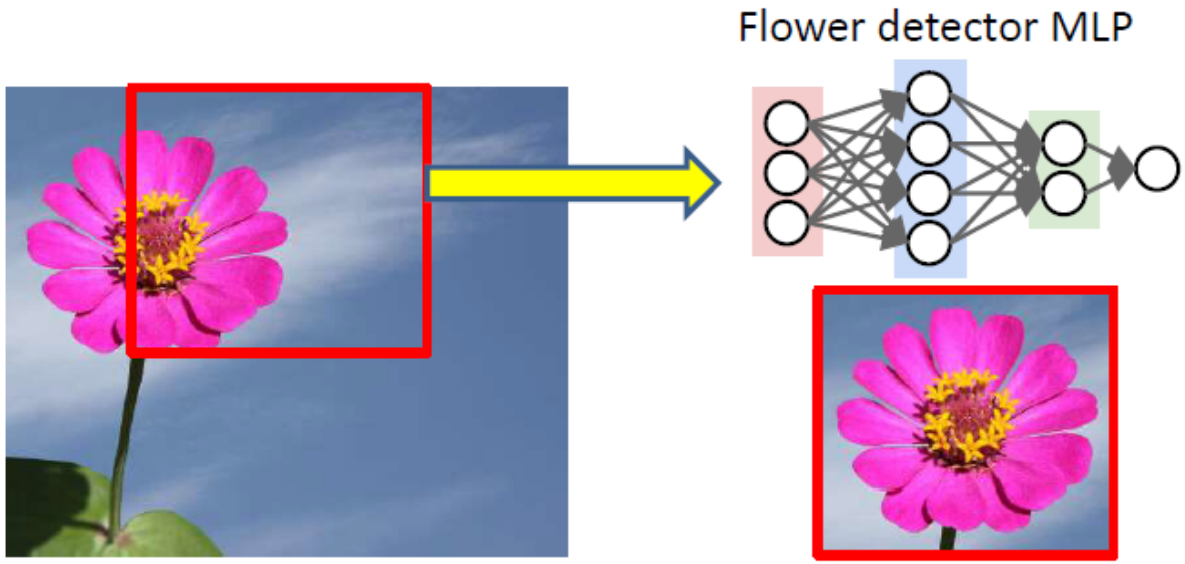

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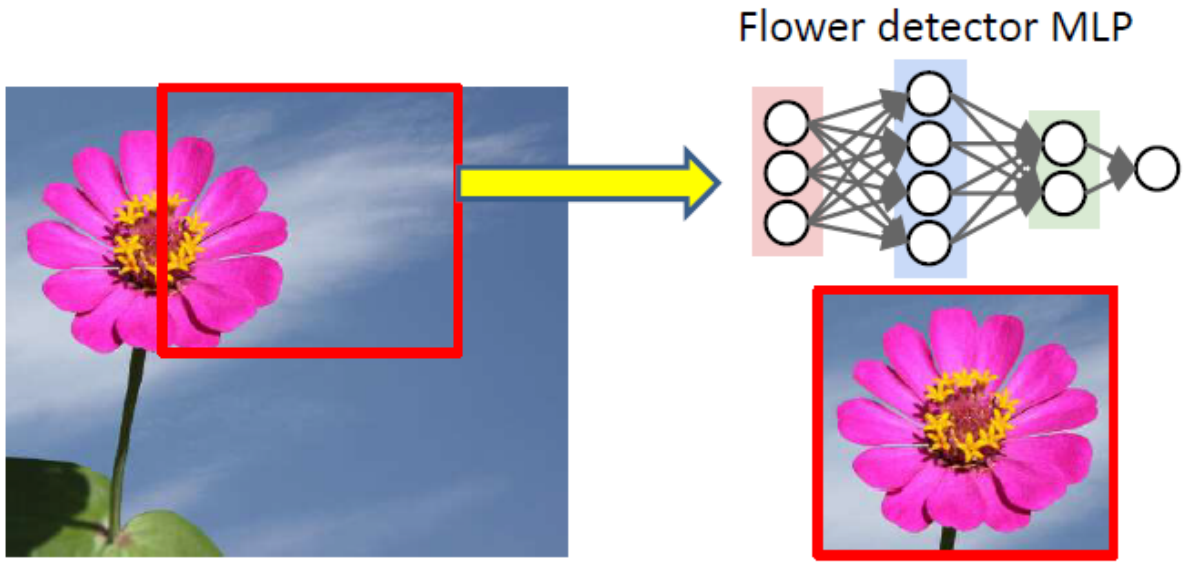

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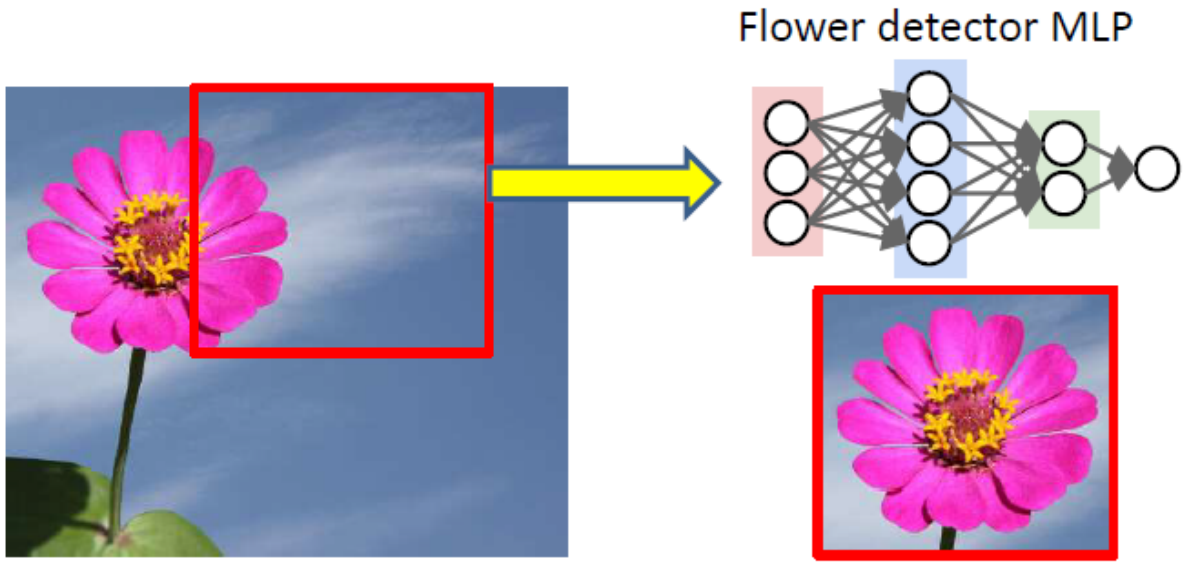

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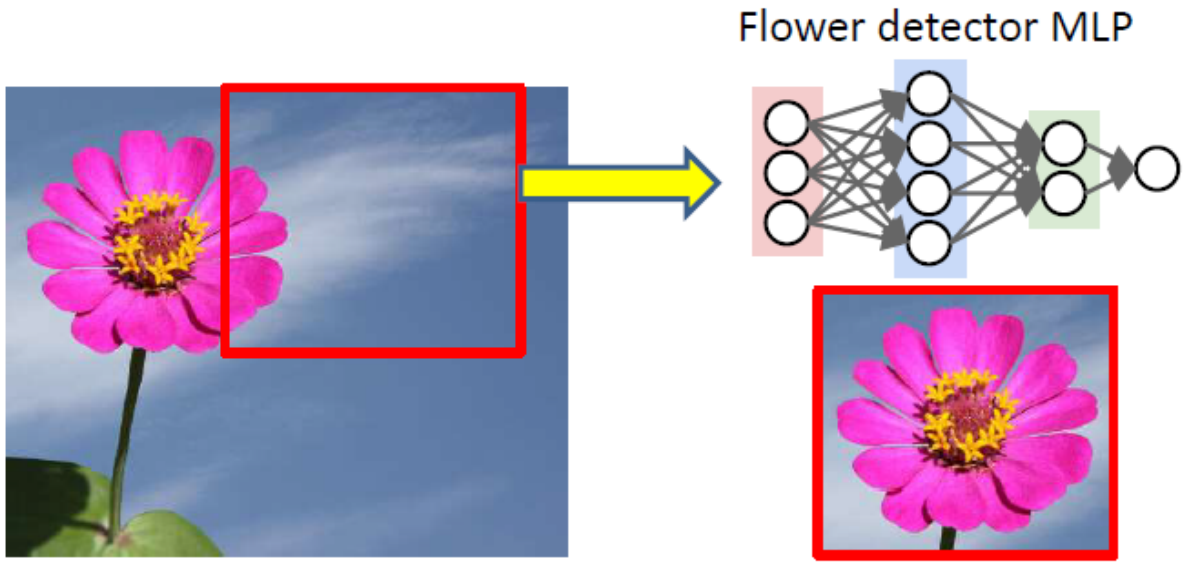

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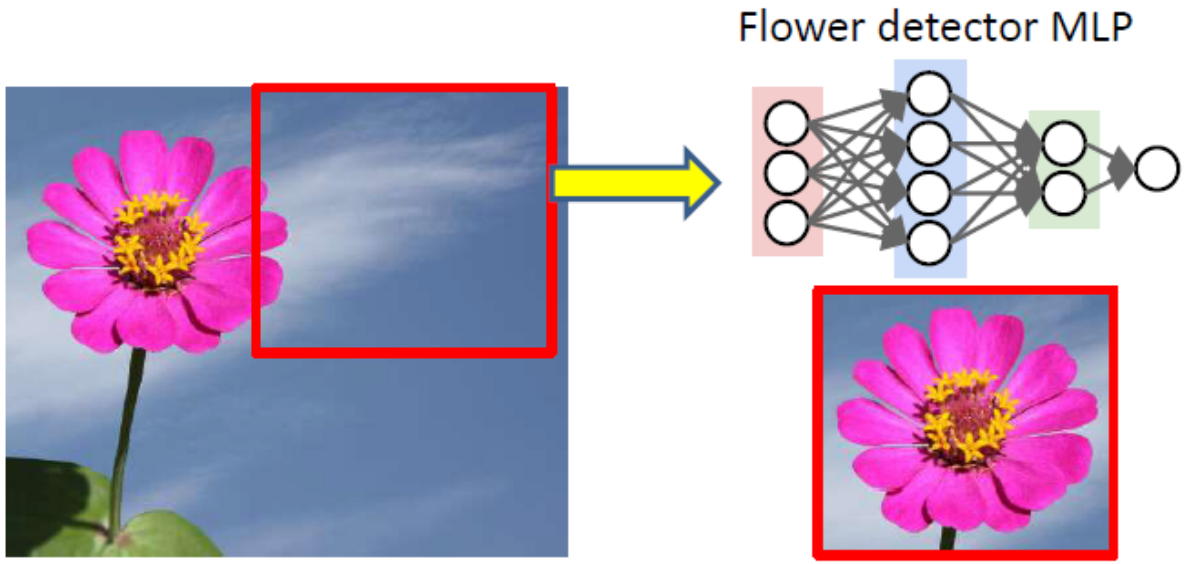

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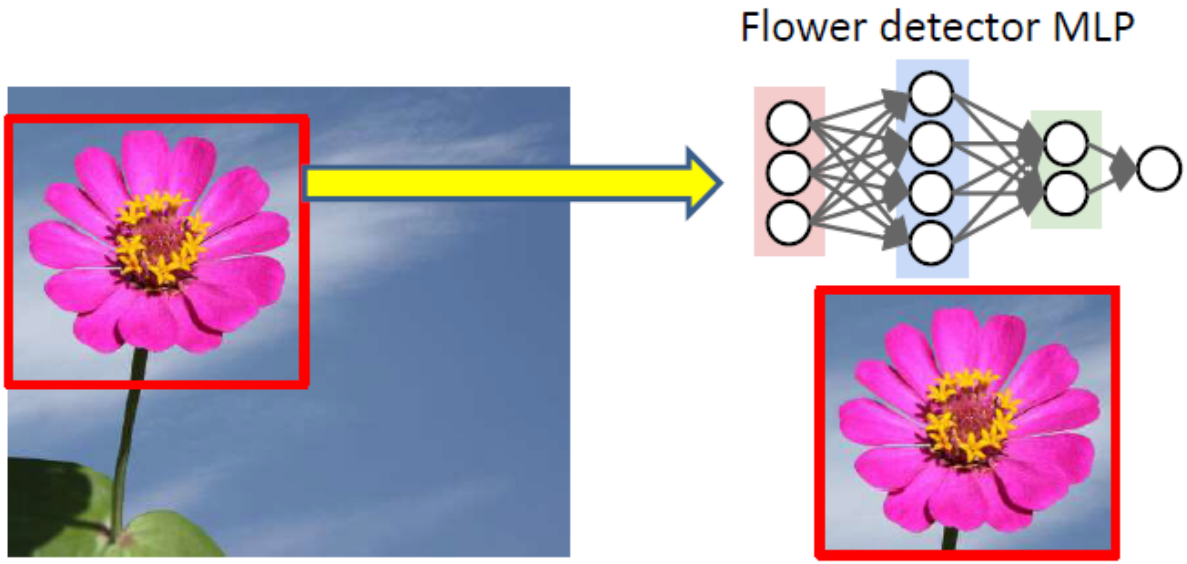

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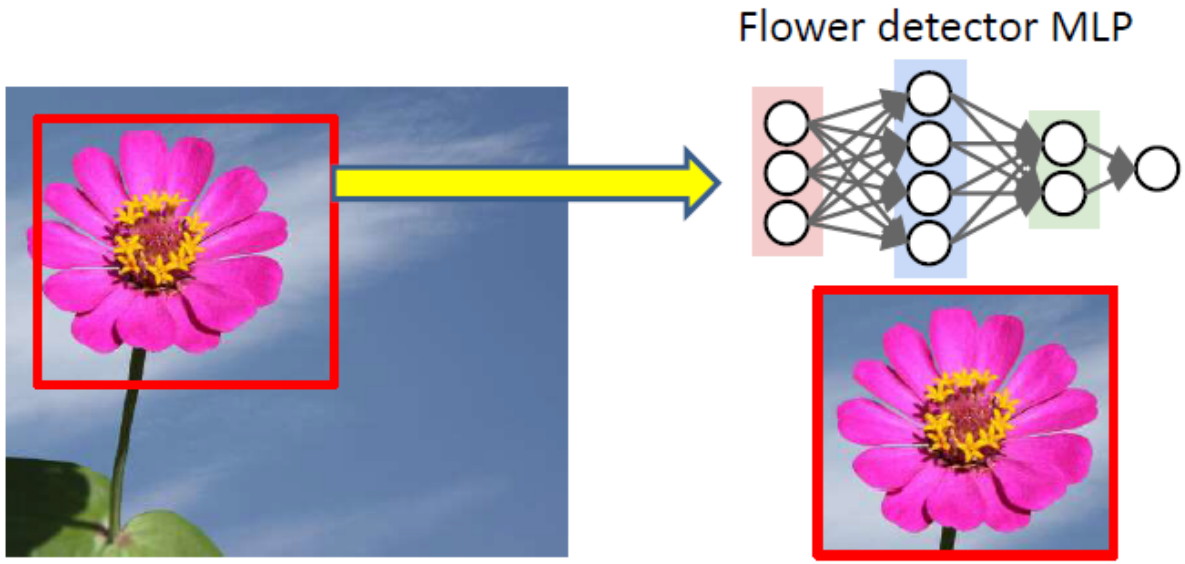

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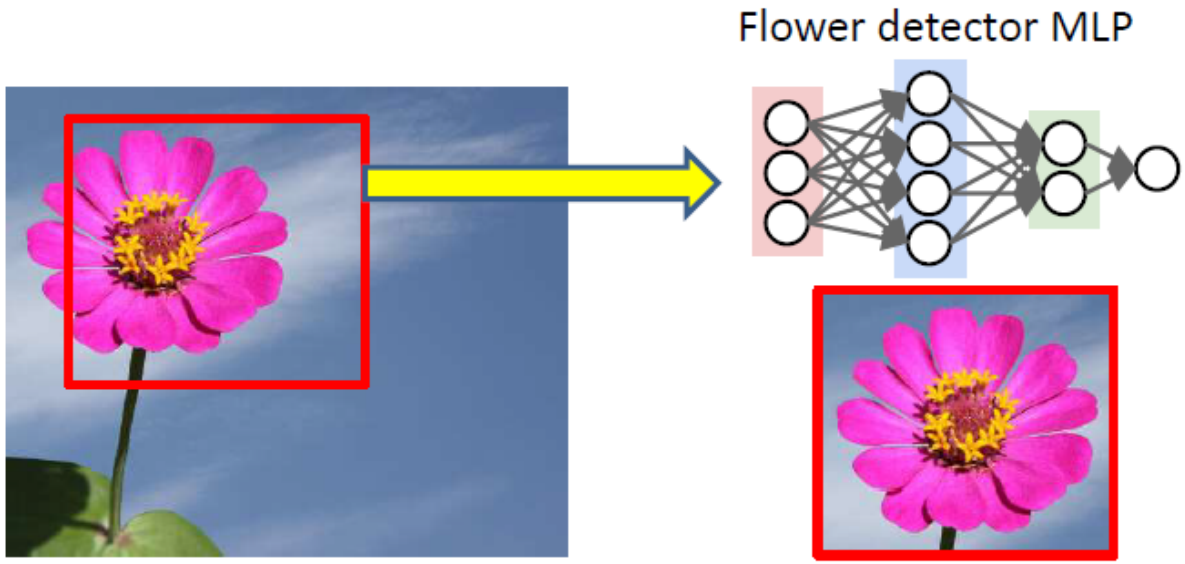

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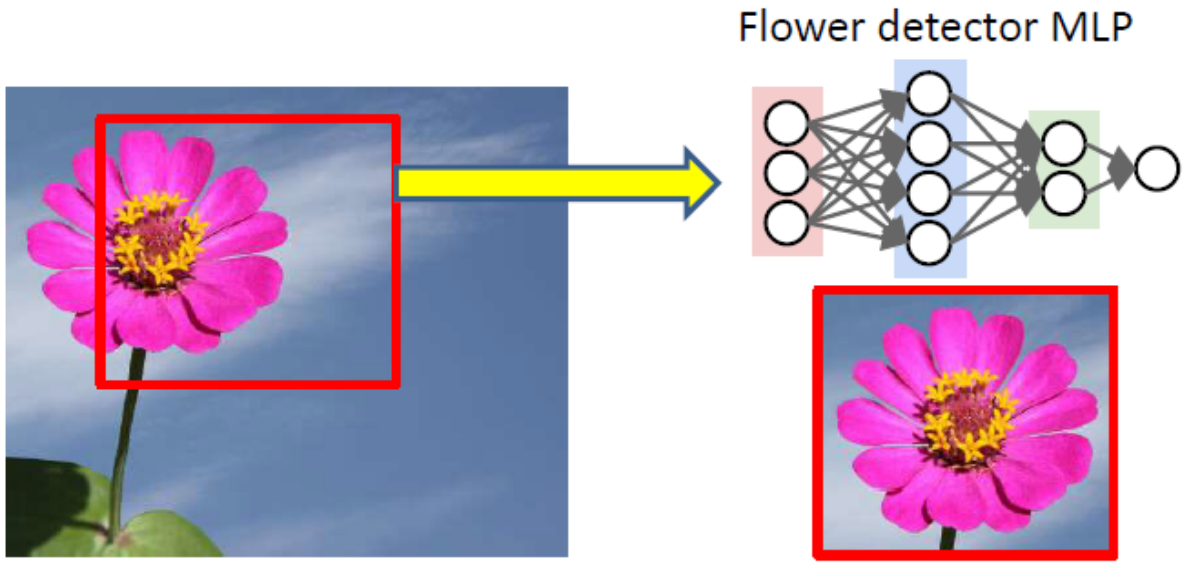

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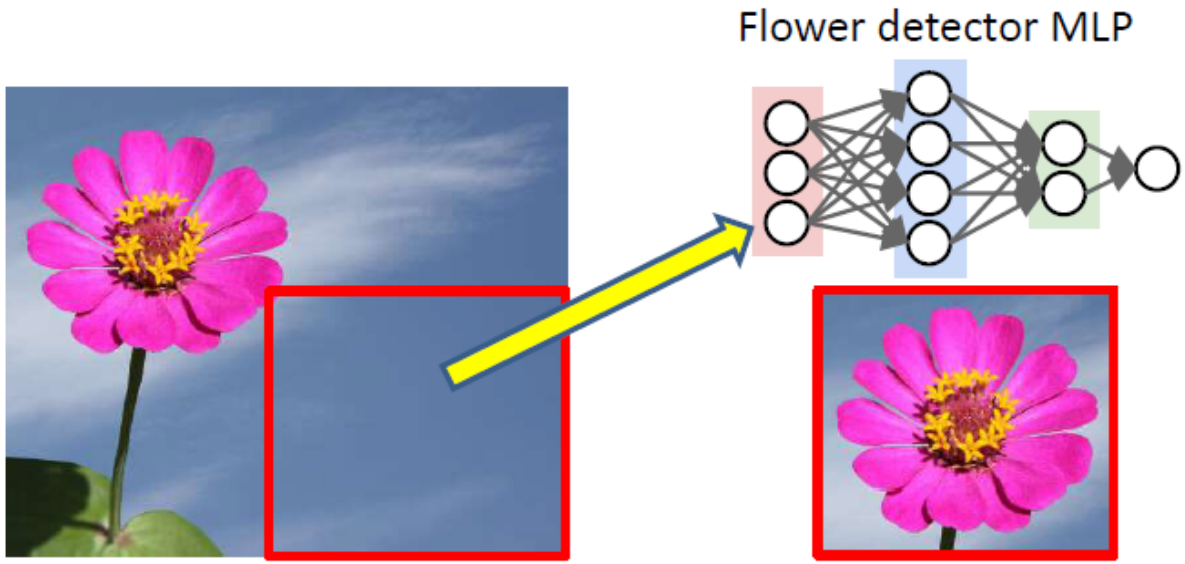
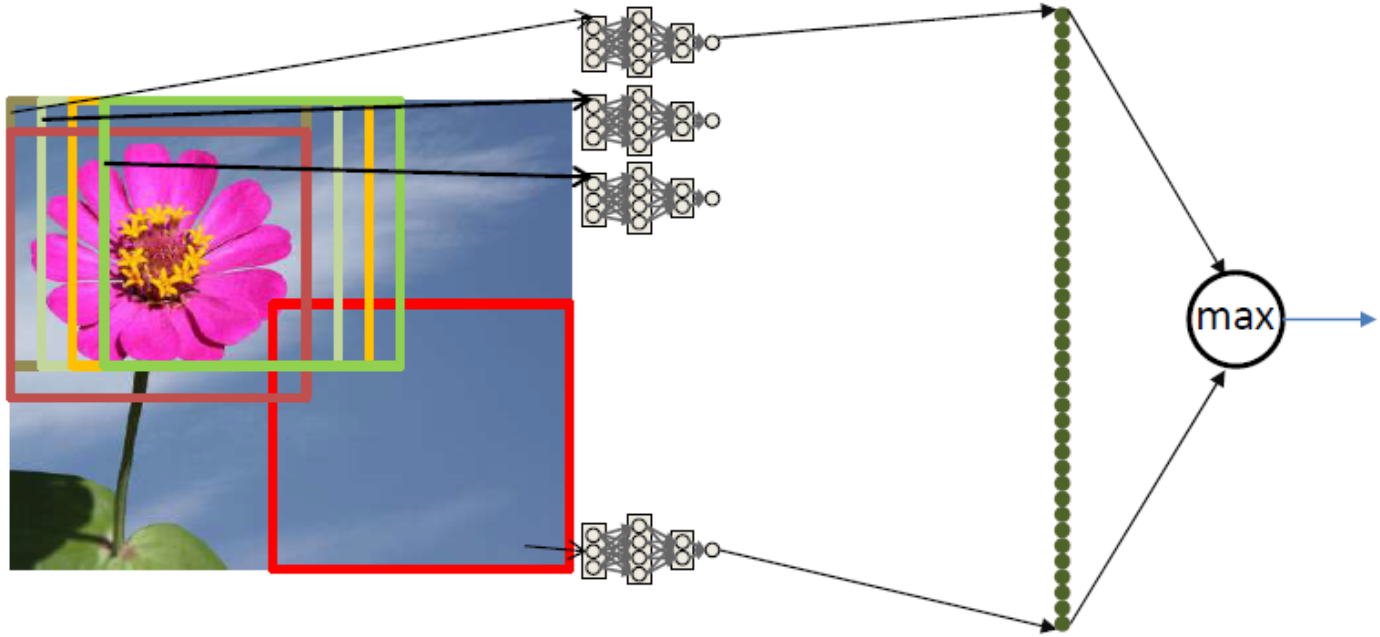


Scanning the picture to find a flower

- Determine if any of the locations had a flower
  - We get one classification output per scanned location
    - Each dot in the right represents the output of the MLP when it classifies one location in the input figure
    - The score output by the MLP
  - Look at the maximum value
    - If the picture has a flower, the location with the flower will result in high output value



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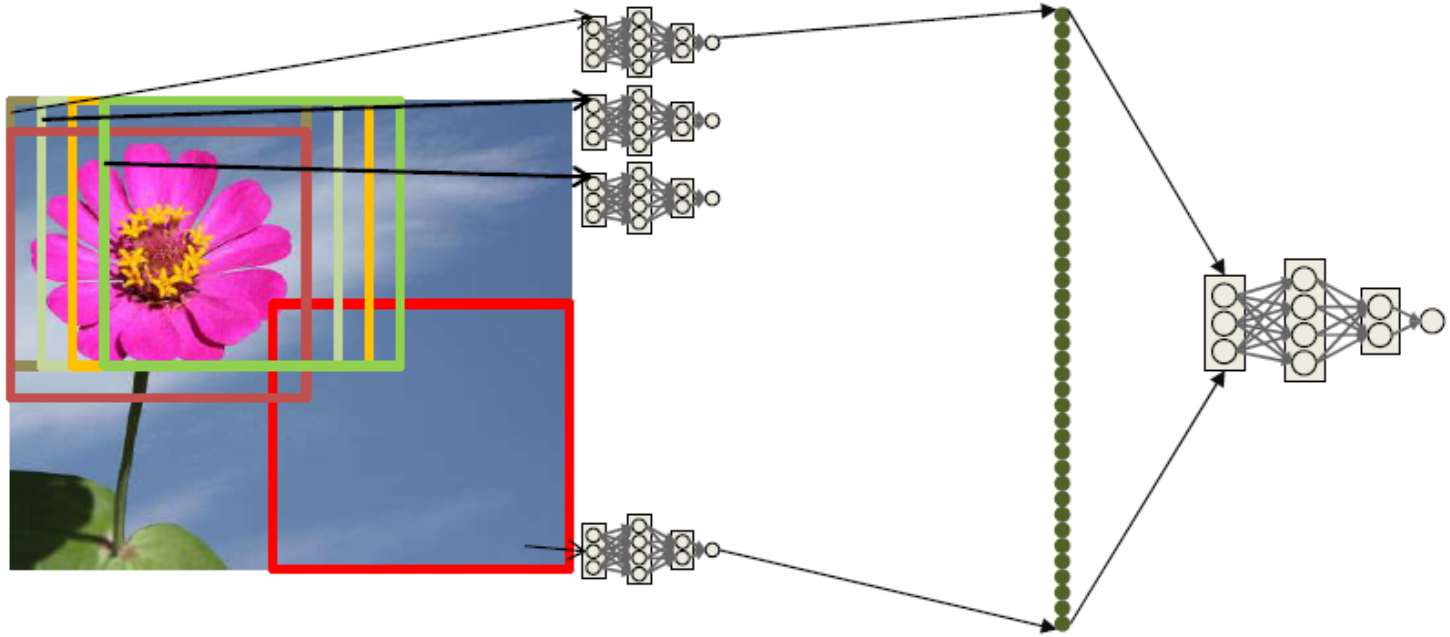
    - The score output by the MLP
  - Look at the maximum value
    - If the picture has a flower, the location with the flower will result in high output value
    - Or pass it through a softmax or even an MLP



Scanning with an MLP

- $K \times K$ = size of "patch" evaluated by MLP
- W is width of image
- H is height of image

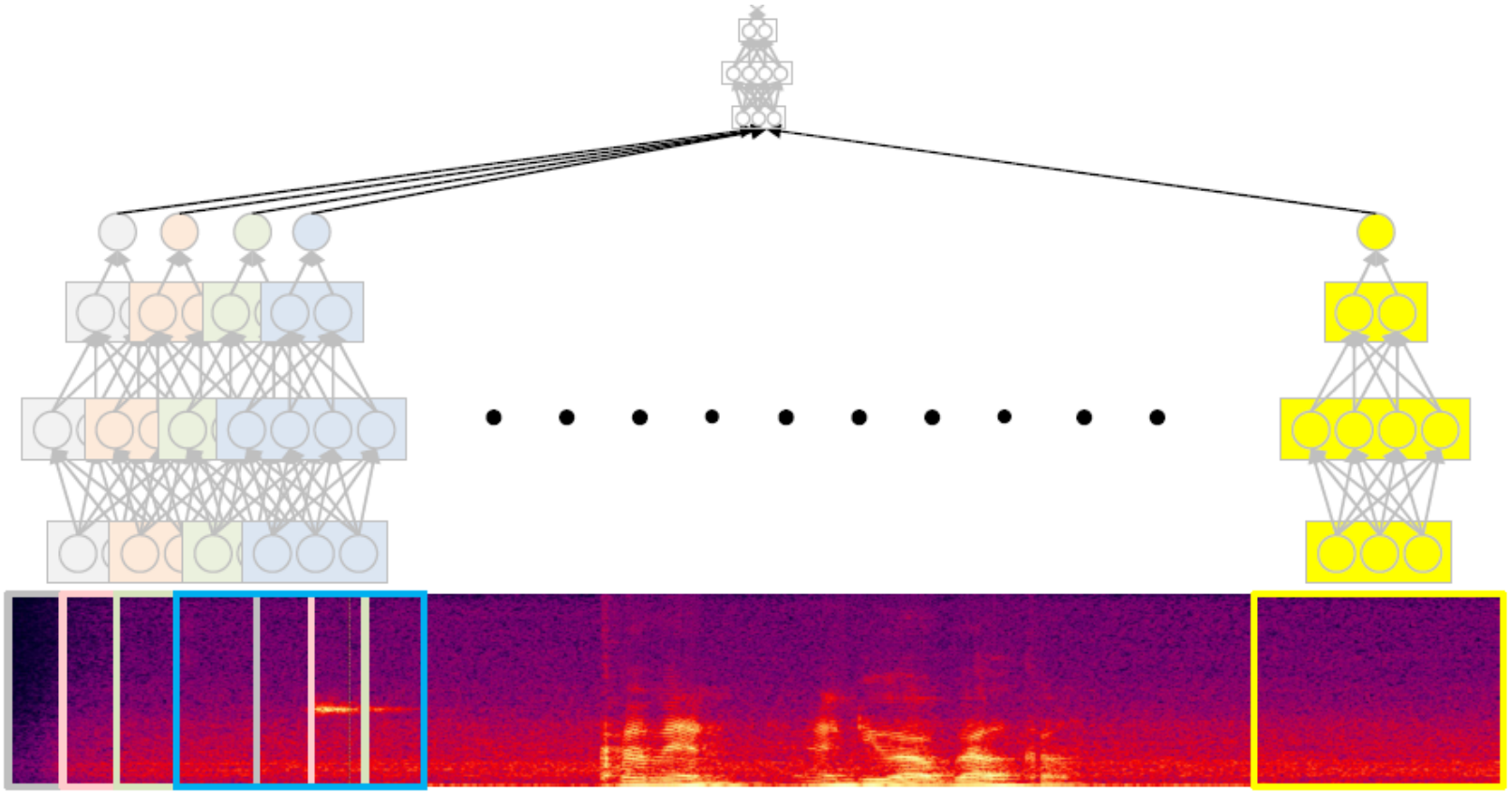
```
For i = 1:W-K+1
    For j = 1:H-K+1
        ImgSegment = Img(i:i+K-1, j:j+K-1)
        y(i,j) = MLP(ImgSegment)

Y = softmax( y(1,1)..y(W-K+1,H-K+1) )
```

Convolutional Neural Networks

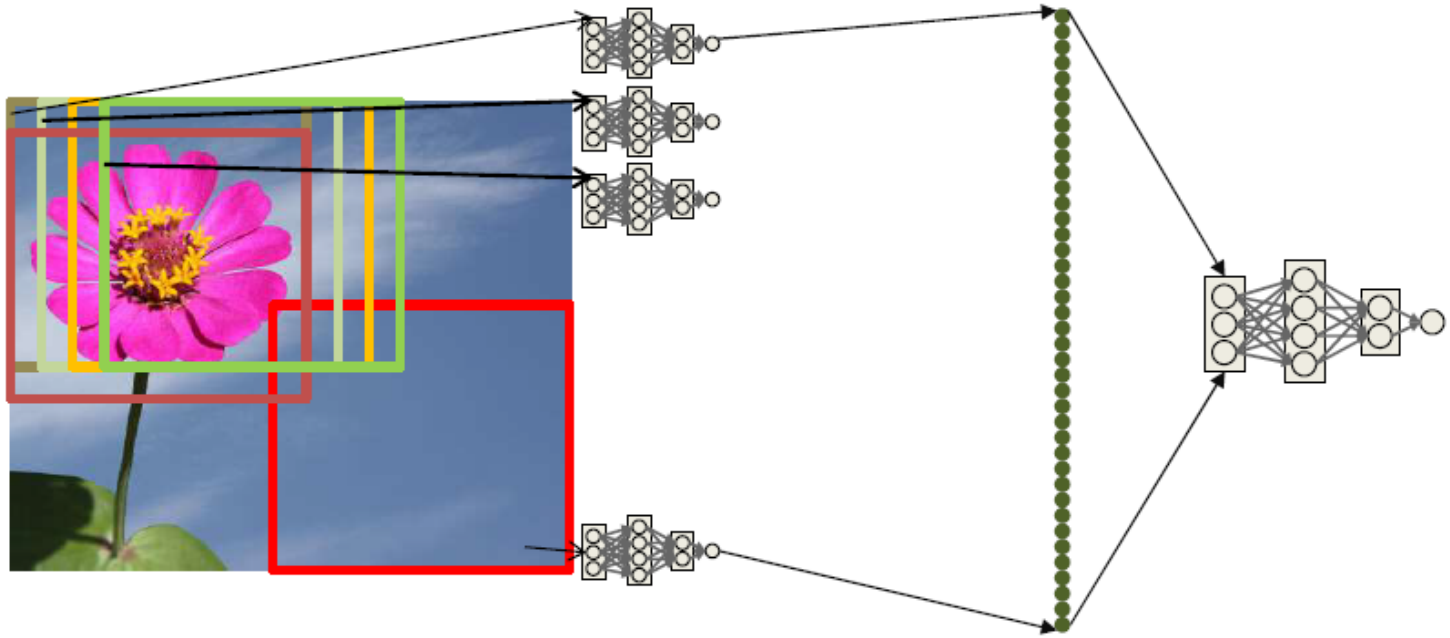
Its just a giant network with common subnets

- The entire operation can be viewed as one **giant network**
  - With many subnetworks, one per window
  - Restriction: **All subnets are identical and have an identical set of parameters**
    - This is what we call a shared parameter network
- The network is shift-invariant!



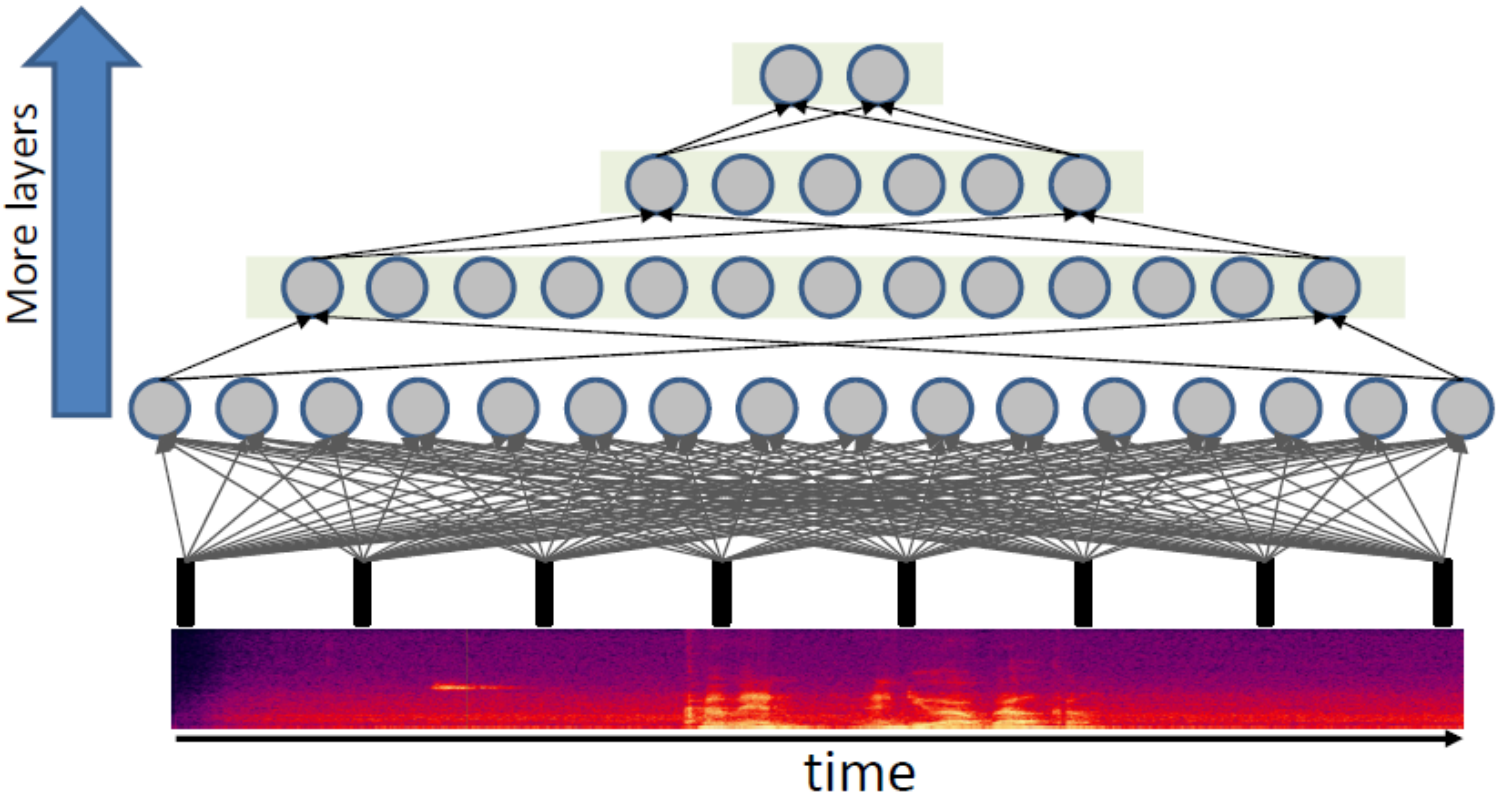
Its just a giant network with common subnets

- The entire operation can be viewed as one giant network
  - With many subnetworks, one per window
  - With one key feature: **all subnets are identical**
- The network is shift-invariant!



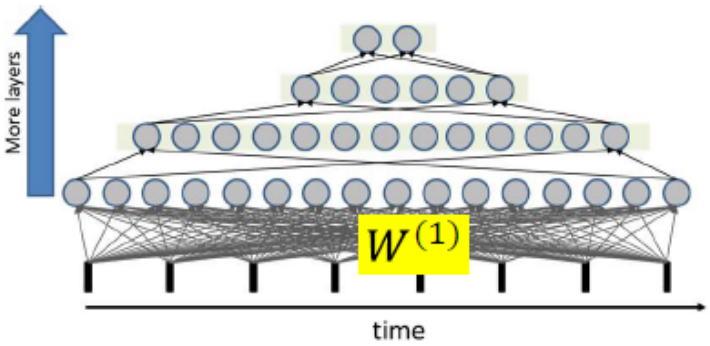
Regular networks vs. scanning networks

- In a regular MLP every neuron in a layer is connected by a unique weight to every unit in the previous layer
  - All entries in the weight matrix are unique
  - The weight matrix is (generally) full



Regular network

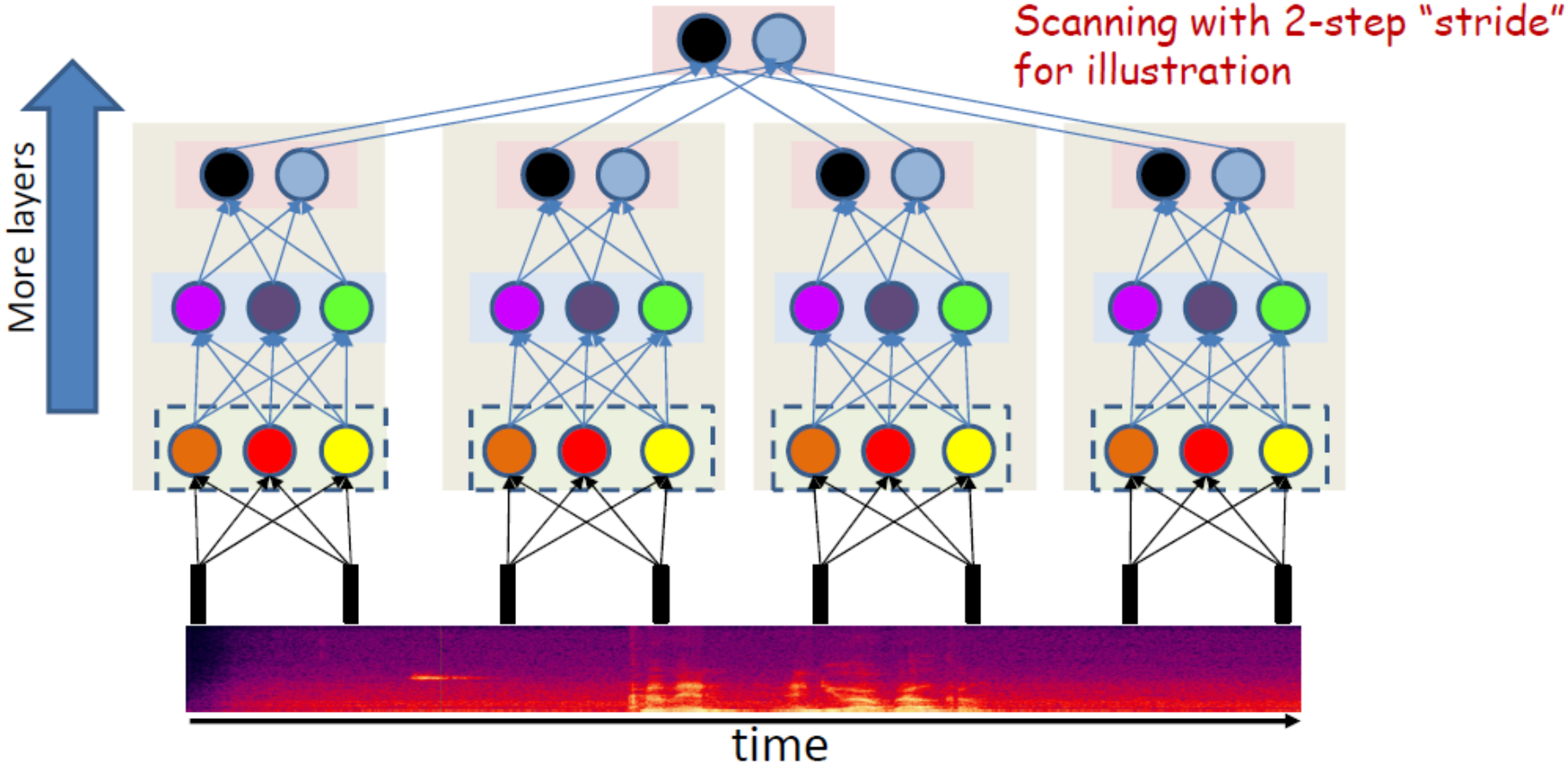
- Consider the first layer
  - Assume N inputs and M outputs
- The weights matrix is a full $M \times N$ matrix
  - Requiring MN unique parameters



$$W^{T}_{(1)} = \begin{bmatrix} w_{11} & w_{21} & w_{31} & w_{41} & \cdots & \cdots & \cdots & w_{N1} \\ w_{12} & w_{22} & w_{32} & w_{42} & \cdots & \cdots & \cdots & w_{N2} \\ w_{13} & w_{23} & w_{33} & w_{43} & \cdots & \cdots & \cdots & w_{N3} \\ w_{14} & w_{24} & w_{34} & w_{44} & \cdots & \cdots & \cdots & w_{N4} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_{1M} & w_{2M} & w_{3M} & w_{4M} & \cdots & \cdots & \cdots & w_{NM} \end{bmatrix}$$

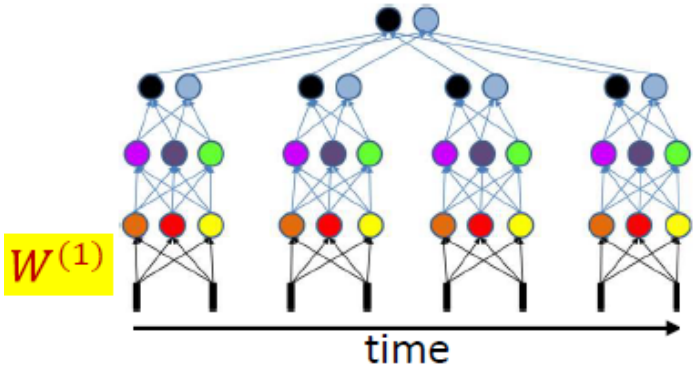
Scanning networks

- In a scanning MLP each neuron is connected to a subset of neurons in the previous layer
  - The weights matrix is sparse
  - The weights matrix is block structured with identical blocks



Scanning networks

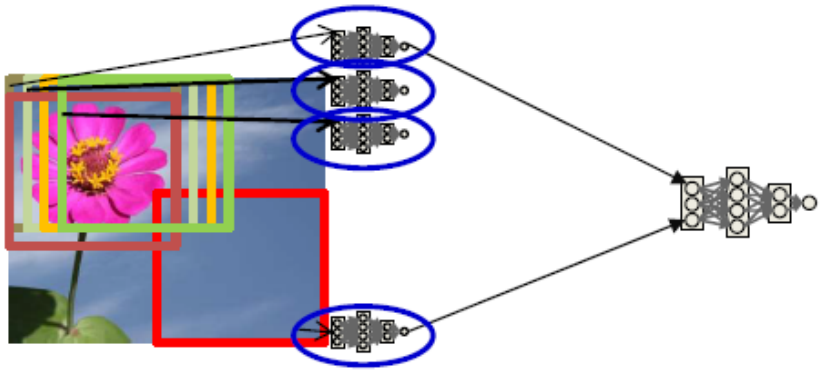
- In a scanning MLP each neuron is connected to a subset of neurons in the previous layer
  - The weights matrix is sparse
  - The weights matrix is block structured with identical blocks
  - **The network is a shared parameter model**
  - **Also, far fewer parameters**

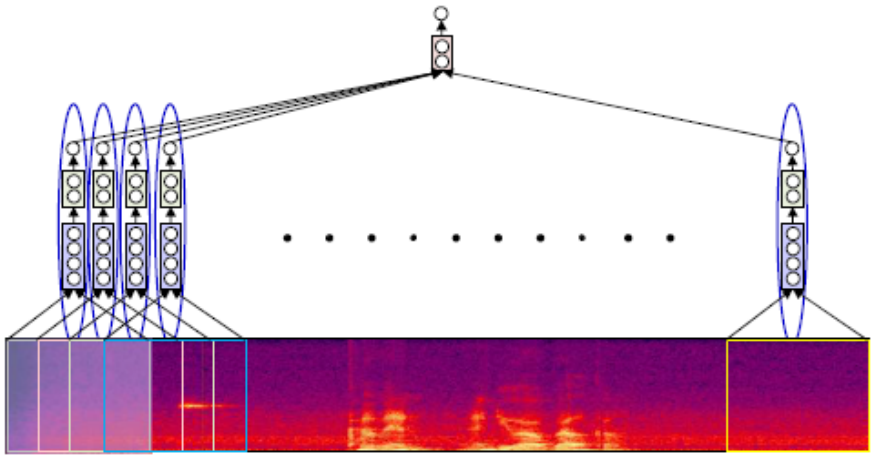


$$W^{(1)} = \begin{bmatrix} w_{11} & w_{12} & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ w_{21} & w_{22} & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ w_{31} & w_{32} & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & w_{11} & w_{12} & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & w_{21} & w_{22} & 0 & 0 & 0 & \vdots & \vdots \\ 0 & 0 & w_{31} & w_{32} & 0 & 0 & 0 & \vdots & \vdots \\ 0 & 0 & 0 & 0 & w_{11} & w_{12} & 0 & 0 & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & w_{31} & w_{32} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ . \\ . \\ X_N \end{bmatrix}$$

Training the network: constraint

- These are shared parameter networks
  - All lower-level subnets are identical
    - Are all searching for the same pattern
  - Any update of the parameters of one copy of the subnet must equally update all copies





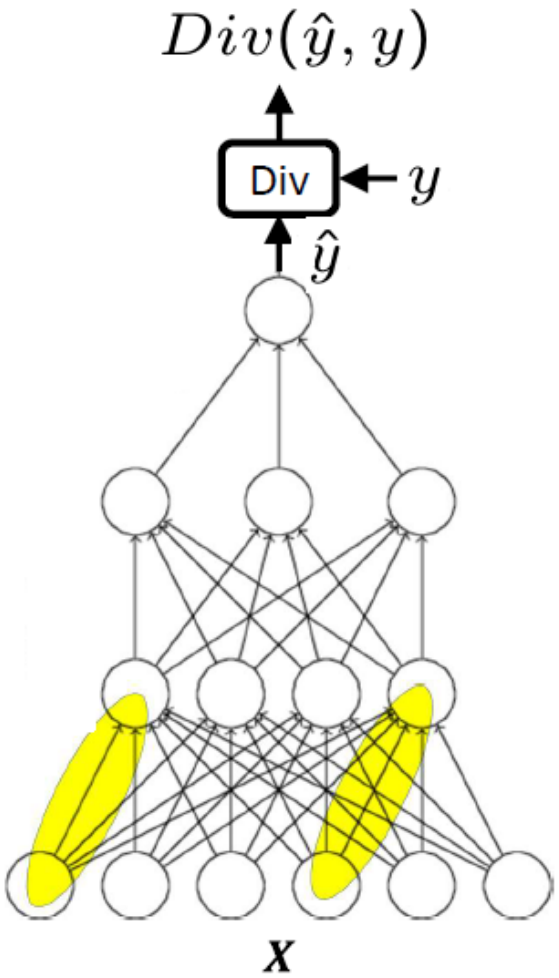
Learning in shared parameter networks

- Consider a simple network with shared weights

$$w_{ij}^{k} = w_{mn}^{\ell} = w^{s}$$

  - A weight w_{ij}^{k} is required to be identical to the weight w_{mn}^{ℓ}

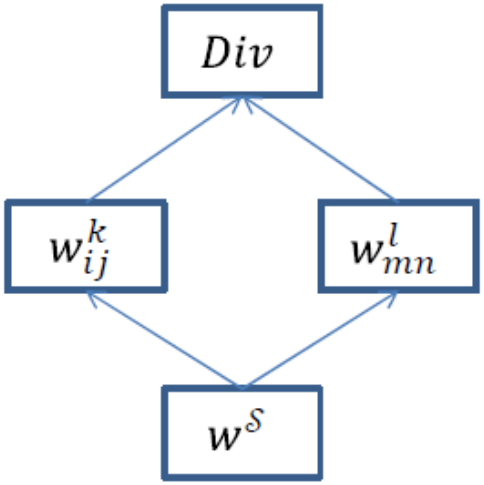
- For any training instance X, a small perturbation of w^{s} perturbs both w_{ij}^{k} and w_{mn}^{ℓ} identically
  - Each of these perturbations will individually influence the loss $Div(\hat{y}, y)$



Computing the divergence of shared parameters

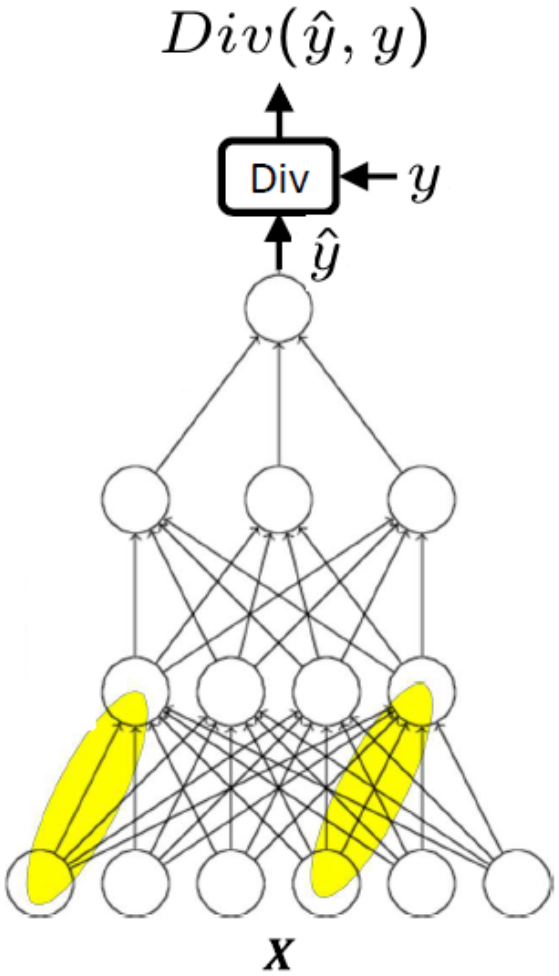
- Each of the individual terms can be computed via backpropagation

Influence diagram



$$\frac{dDiv}{dw^S} = \frac{\partial Div}{\partial w_{ij}^k}\frac{dw_{ij}^k}{dw^S} + \frac{\partial Div}{\partial w_{mn}^l}\frac{dw_{mn}^l}{dw^S}$$

$$= \frac{\partial Div}{\partial w_{ij}^k} + \frac{\partial Div}{\partial w_{mn}^l}$$

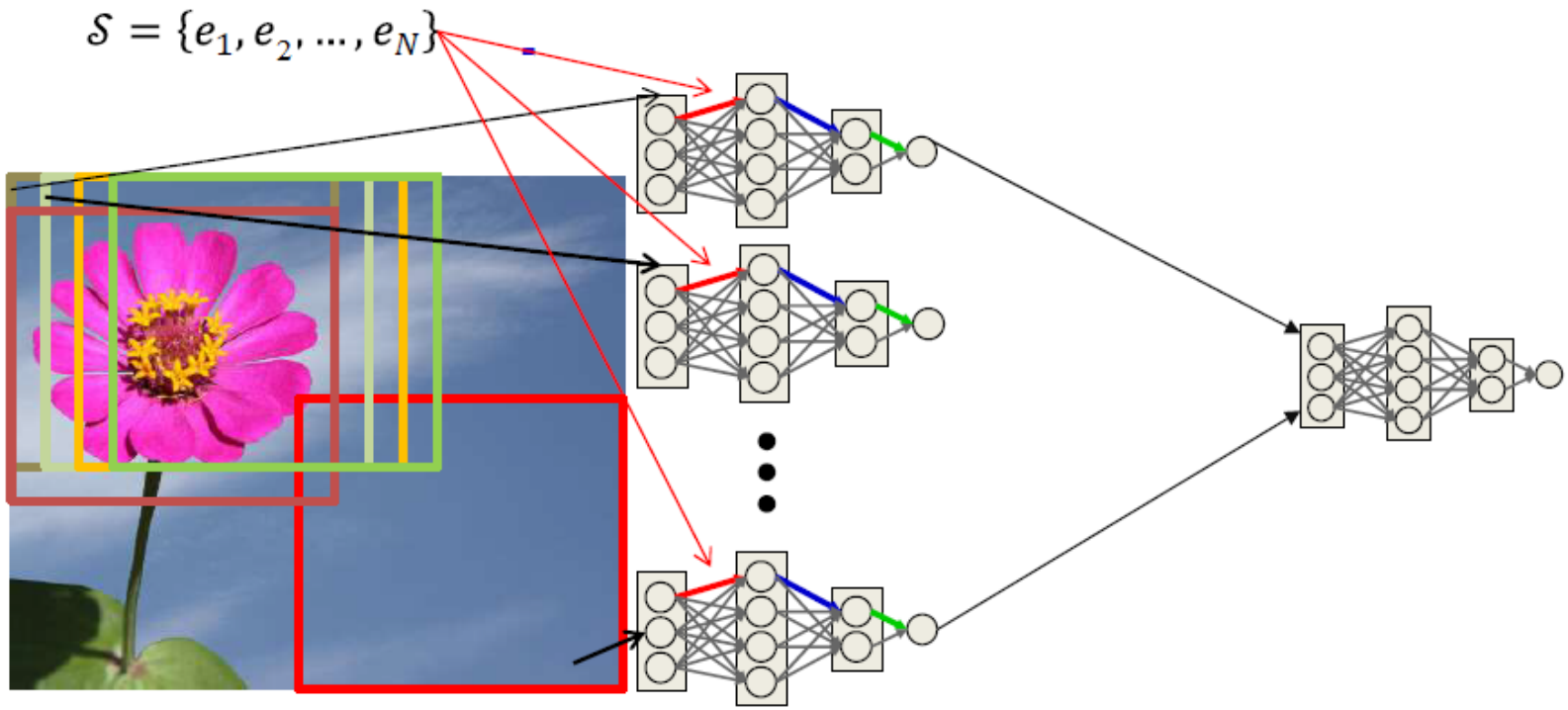


Computing the divergence of shared parameters

- More generally, let S be any set of edges that have a common value, and w^S be the common weight of the set
  - E.g. the set of all red weights in the figure

$$\frac{d\,Div}{d\,w^{\mathcal{S}}} = \sum_{i=1}^{N} \frac{\partial Div}{\partial w^{e_i}}$$

- The individual terms in the sum can be computed via backpropagation



Training networks with shared parameters

- Gradient descent algorithm:
- Initialize all weights $\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_K$
- Do:
  - For every set $\mathcal{S}$:
    - Compute:

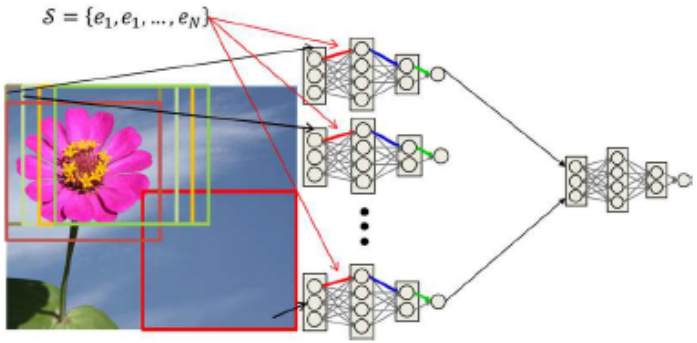
$$\nabla_{\mathcal{S}} Loss = \frac{dLoss}{dw^{\mathcal{S}}}$$

$$w^{\mathcal{S}} = w^{\mathcal{S}} - \eta \nabla_{\mathcal{S}} Loss^T$$

    - For every $(k, i, j) \in \mathcal{S}$ update:

$$w_{i,j}^{(k)} = w^{\mathcal{S}}$$

- Until $Loss$ has converged



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Training networks with shared parameters

- Gradient descent algorithm:
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- Do:
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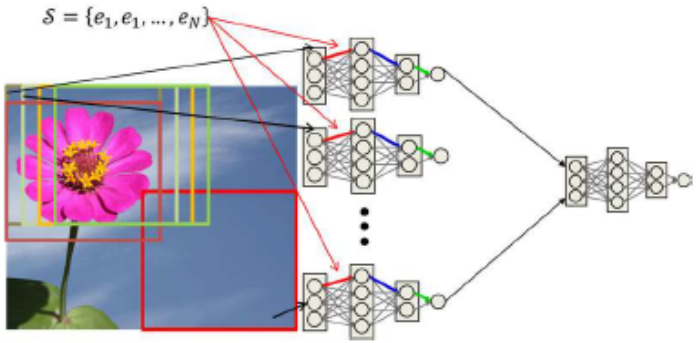

64

Training networks with shared parameters

- For every training instance X
  - For every set $\mathcal{S}$:
    - For every $(k, i, j) \in \mathcal{S}$:

$$\frac{dLoss}{dw^{\mathcal{S}}} \mathrel{+}= \frac{\partial Div}{\partial w_{i,j}^{(k)}}$$

  - $\nabla_{\mathcal{S}} Loss = \frac{dLoss}{dw^{\mathcal{S}}}$
    - Compute:

$$\nabla_{\mathcal{S}} Loss = \frac{dLoss}{dw^{\mathcal{S}}}$$

$$w^{\mathcal{S}} = w^{\mathcal{S}} - \eta \nabla_{\mathcal{S}} Loss^{T}$$

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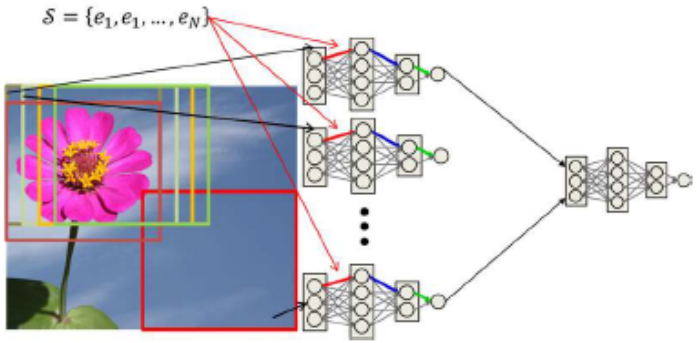


65

Training networks with shared parameters

- For every training instance X
  - For every set $\mathcal{S}$:
    - For every $(k, i, j) \in \mathcal{S}$:

$$\frac{dLoss}{dw^{\mathcal{S}}} \mathrel{+}= \frac{\partial Div}{\partial w_{i,j}^{(k)}}$$

Computed by Backprop

  - $\nabla_{\mathcal{S}} Loss = \frac{dLoss}{dw^{\mathcal{S}}}$
  - Compute:

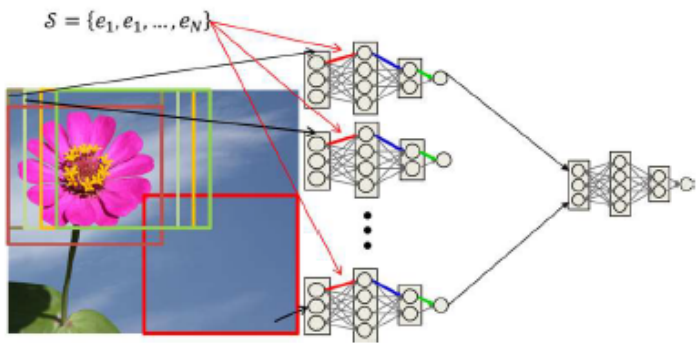
$$\nabla_{\mathcal{S}} Loss = \frac{dLoss}{dw^{\mathcal{S}}}$$

$$w^{\mathcal{S}} = w^{\mathcal{S}} - \eta \nabla_{\mathcal{S}} Loss^T$$

  - For every $(k, i, j) \in \mathcal{S}$ update:

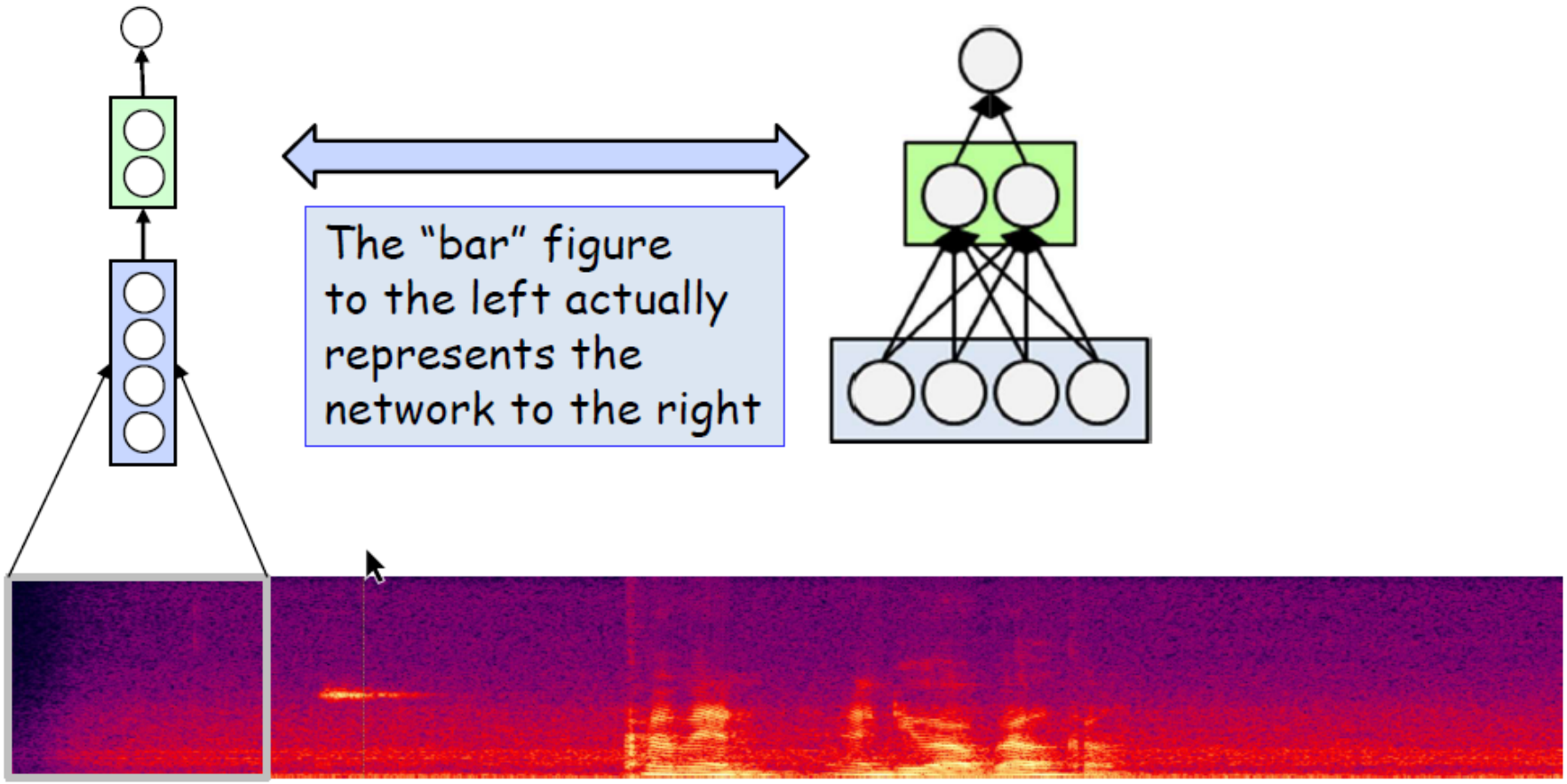
$$w_{i,j}^{(k)} = w^{\mathcal{S}}$$

- Until $Loss$ has converged



66

Let's do it in a different order



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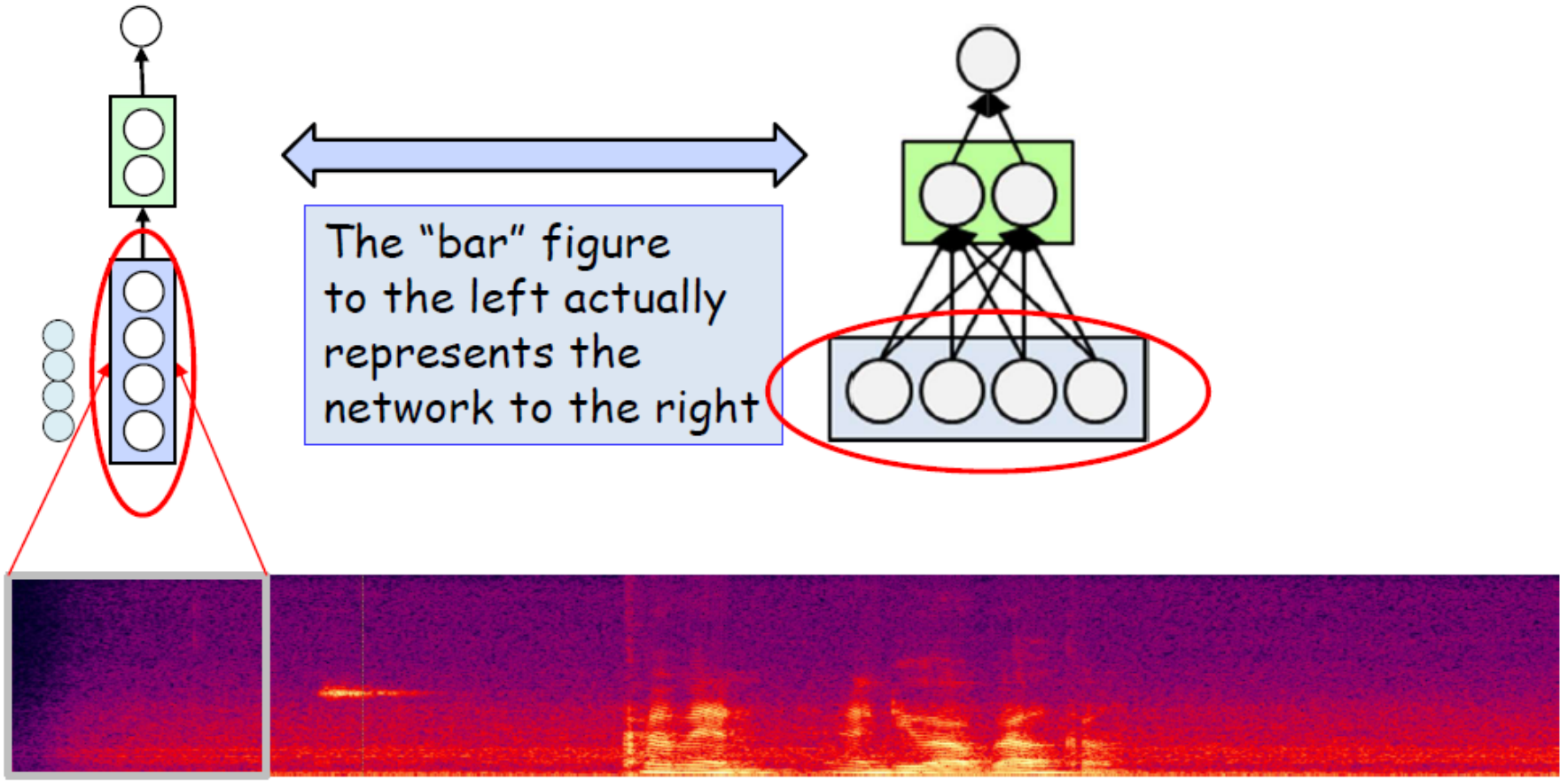

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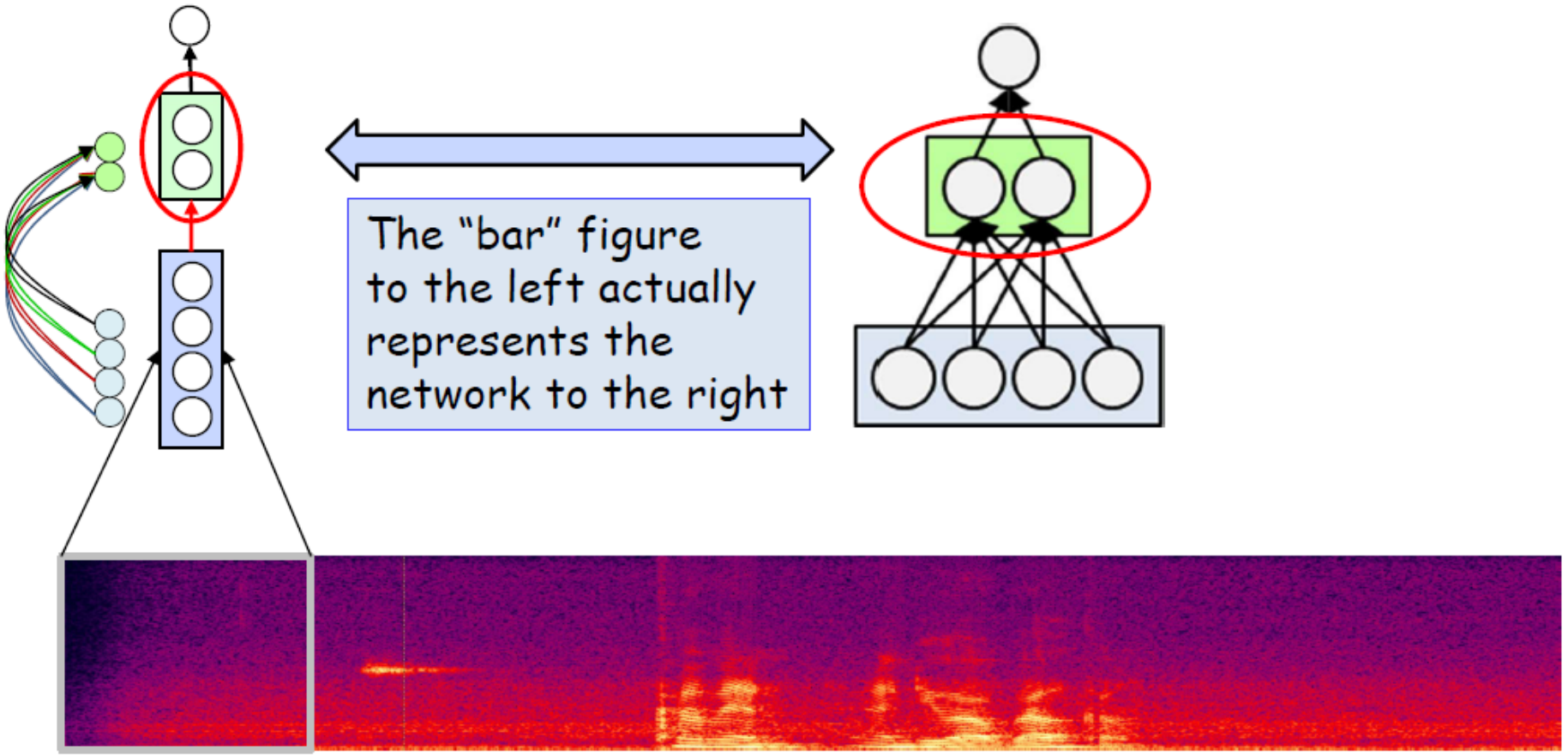

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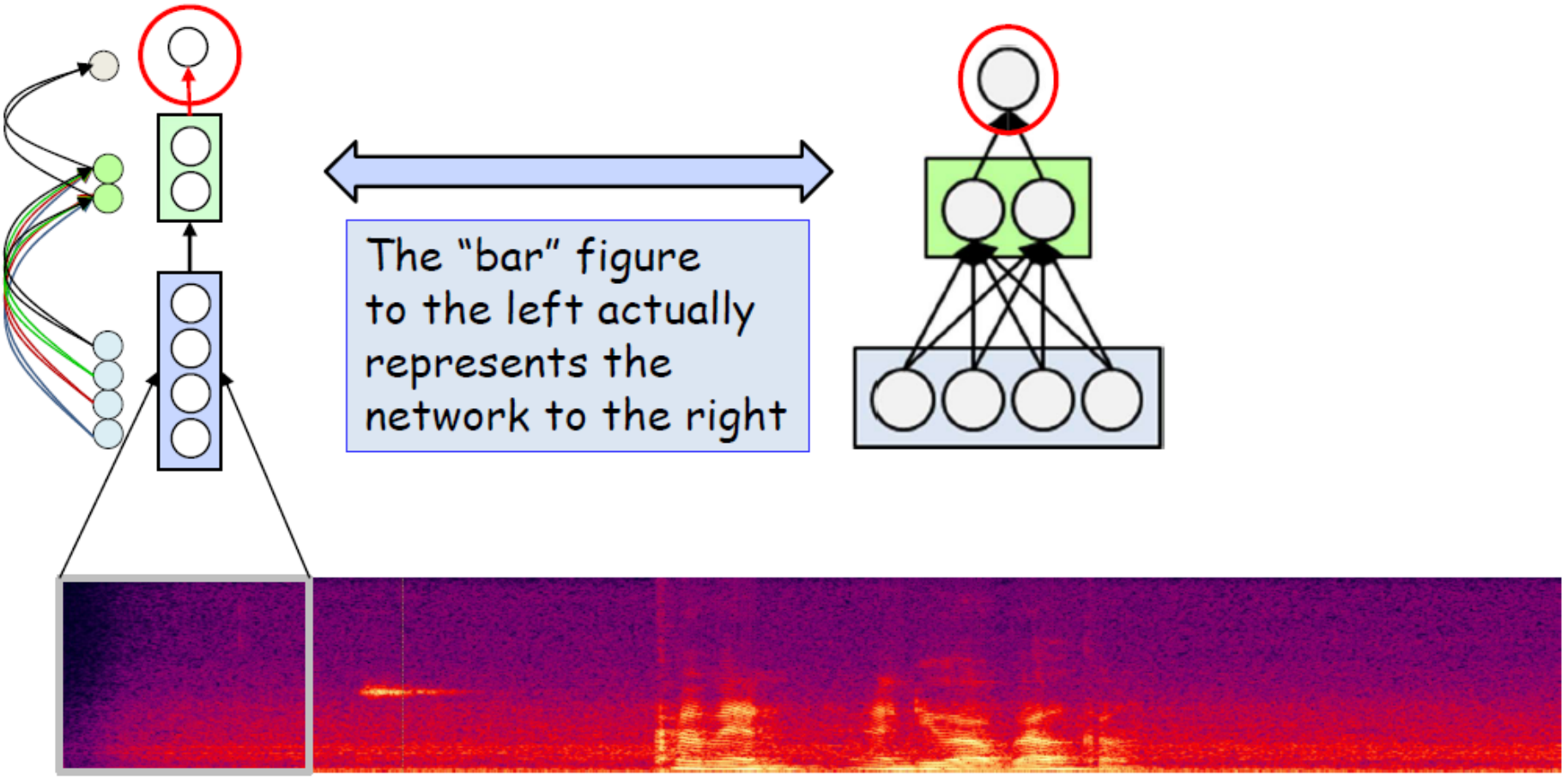

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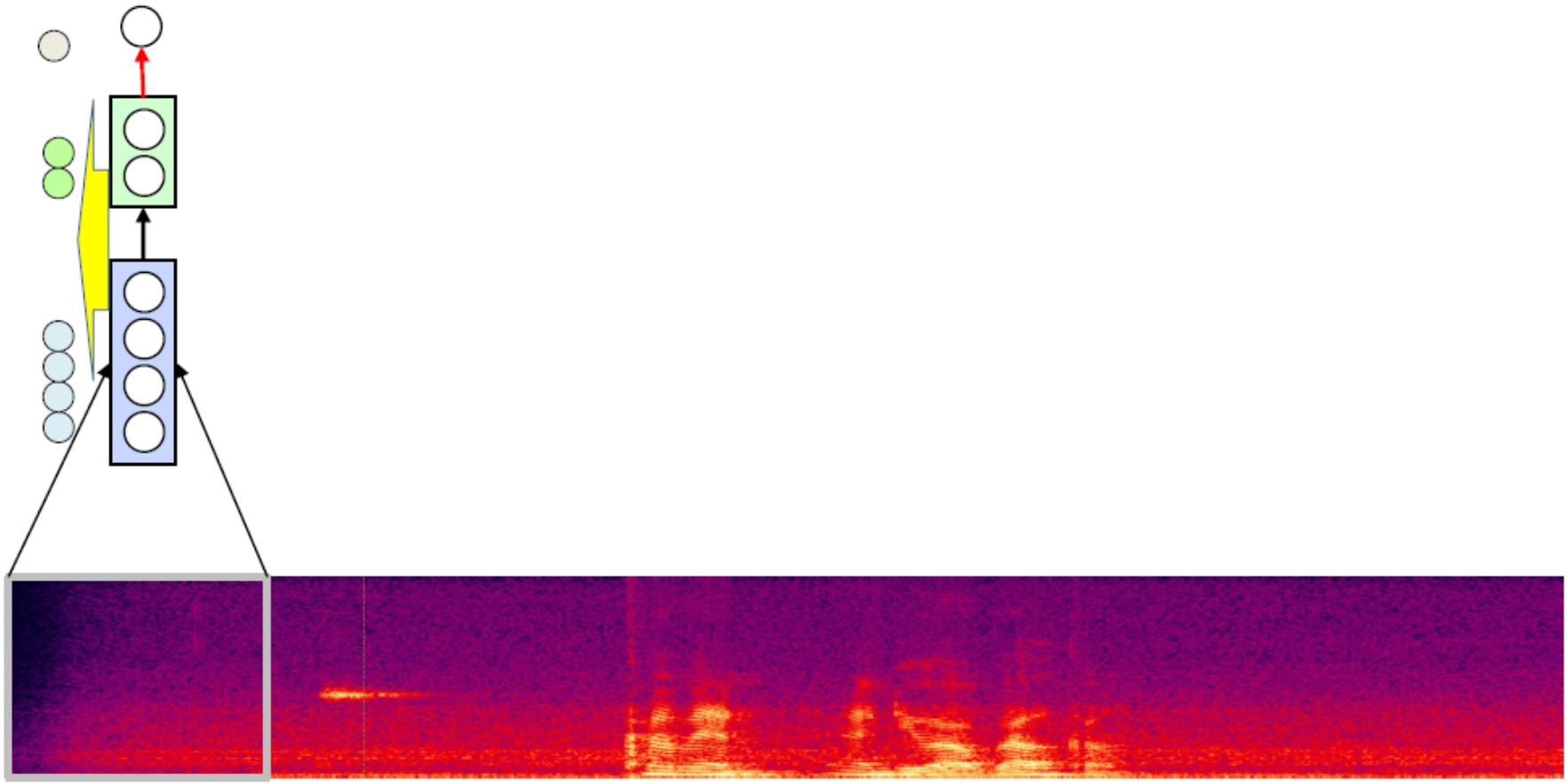

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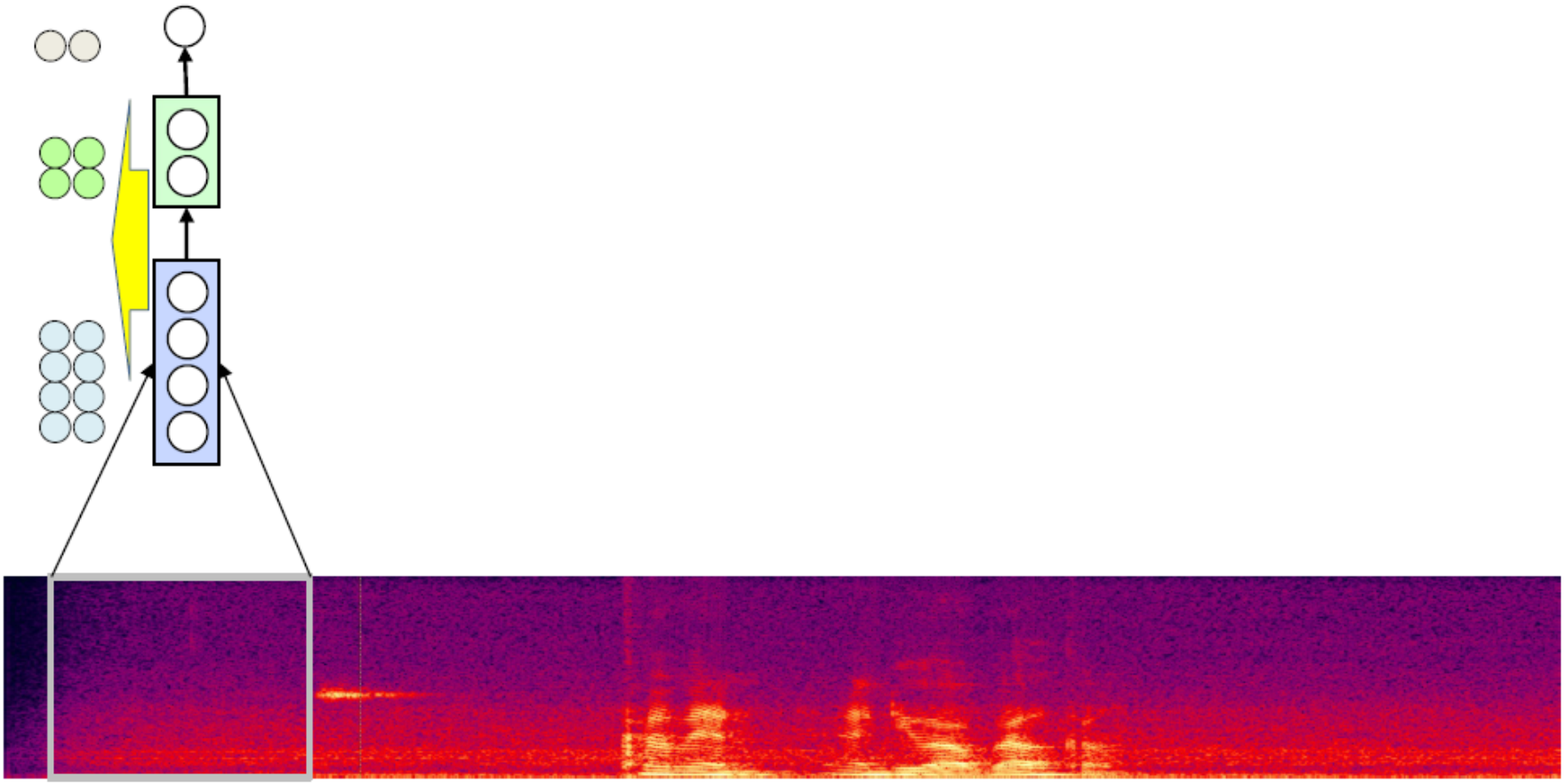

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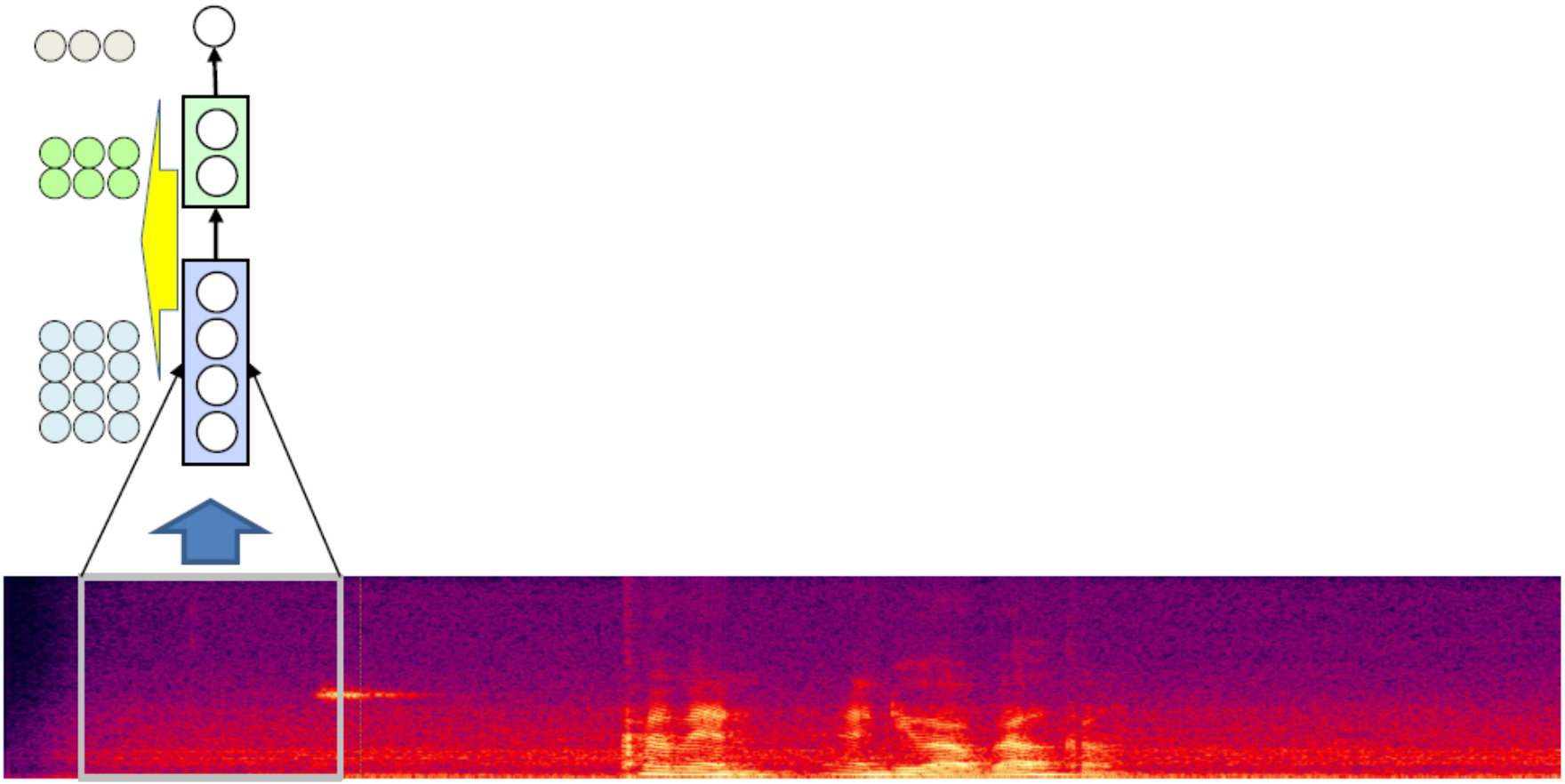

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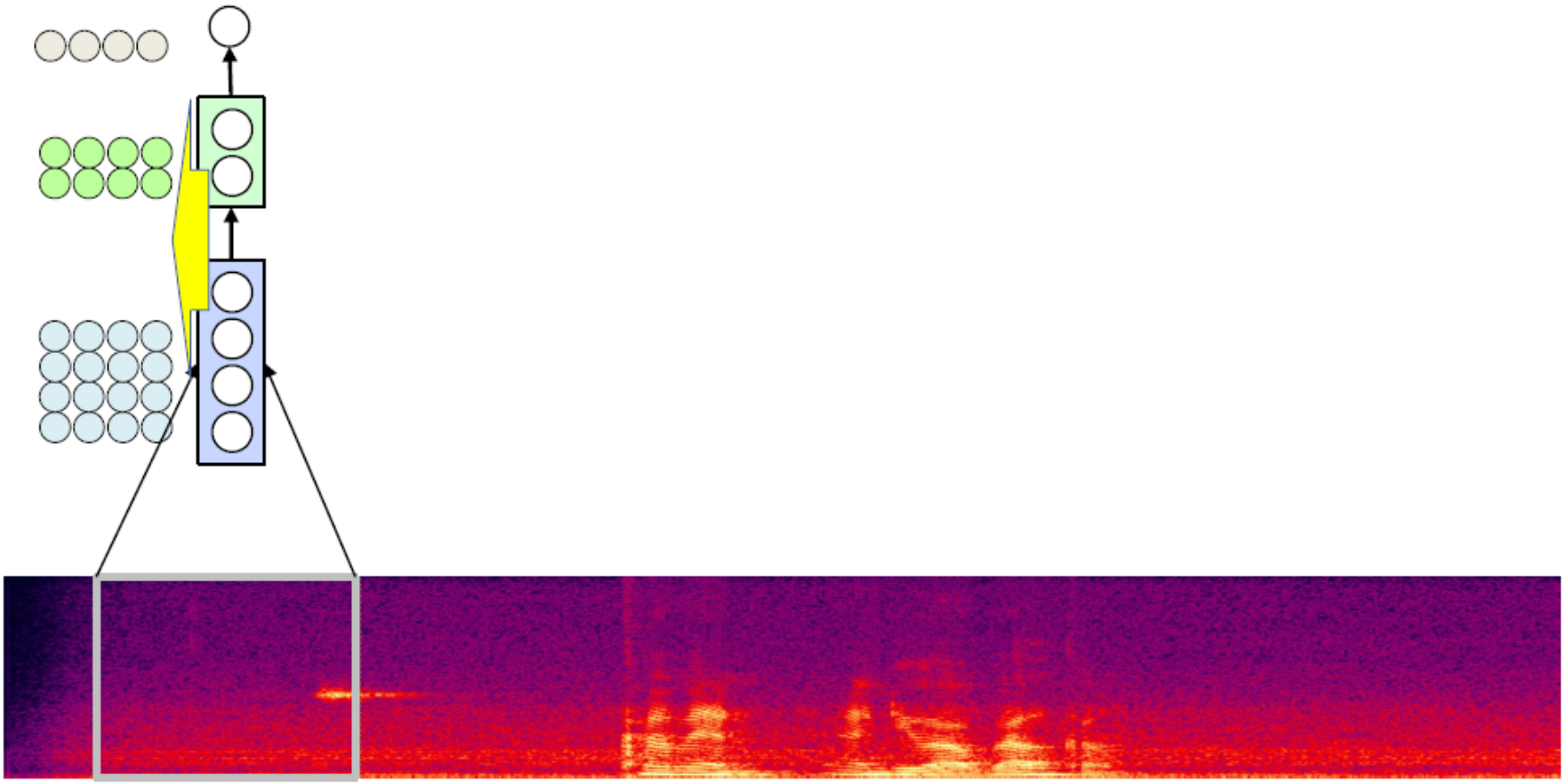

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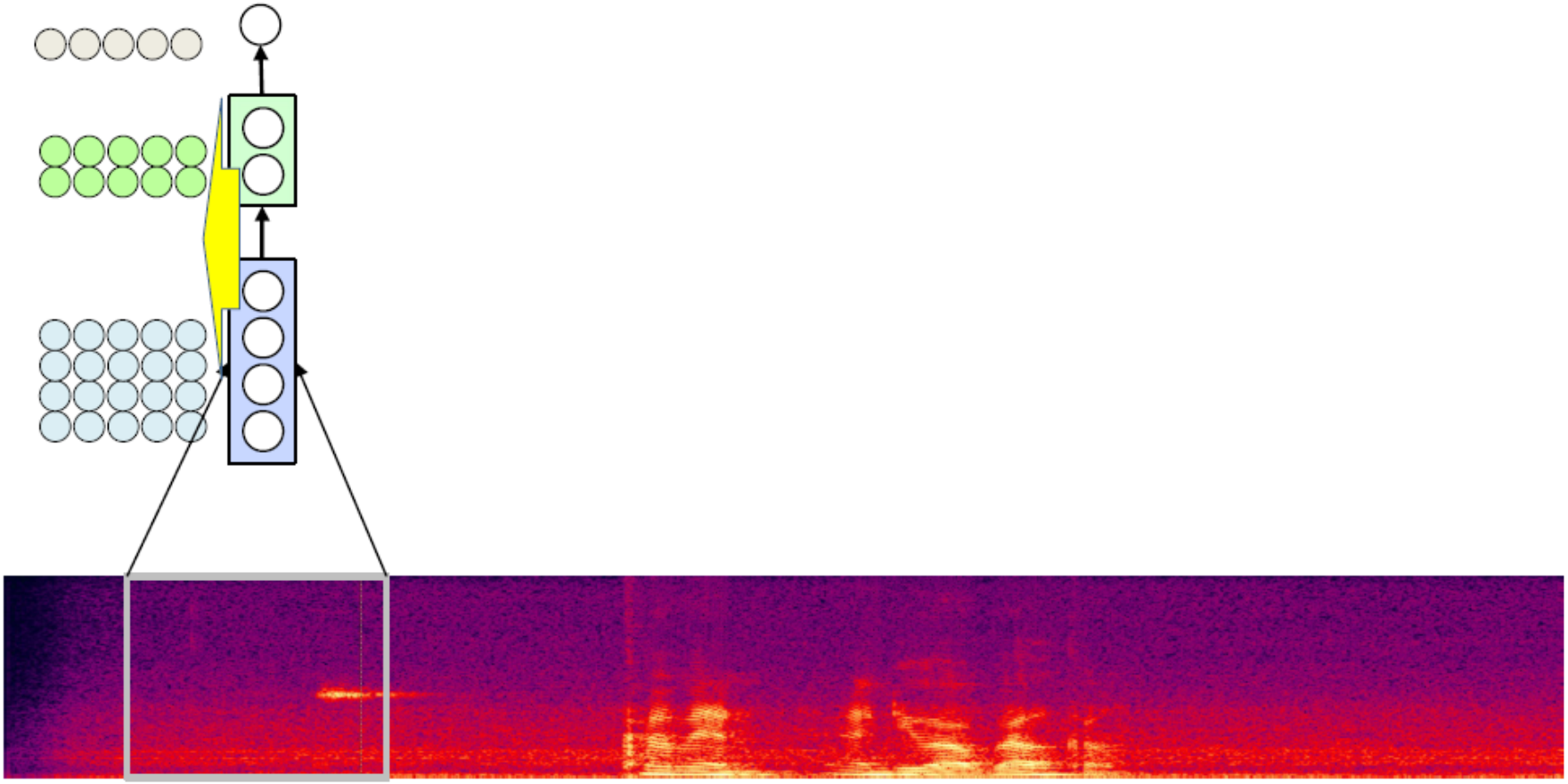

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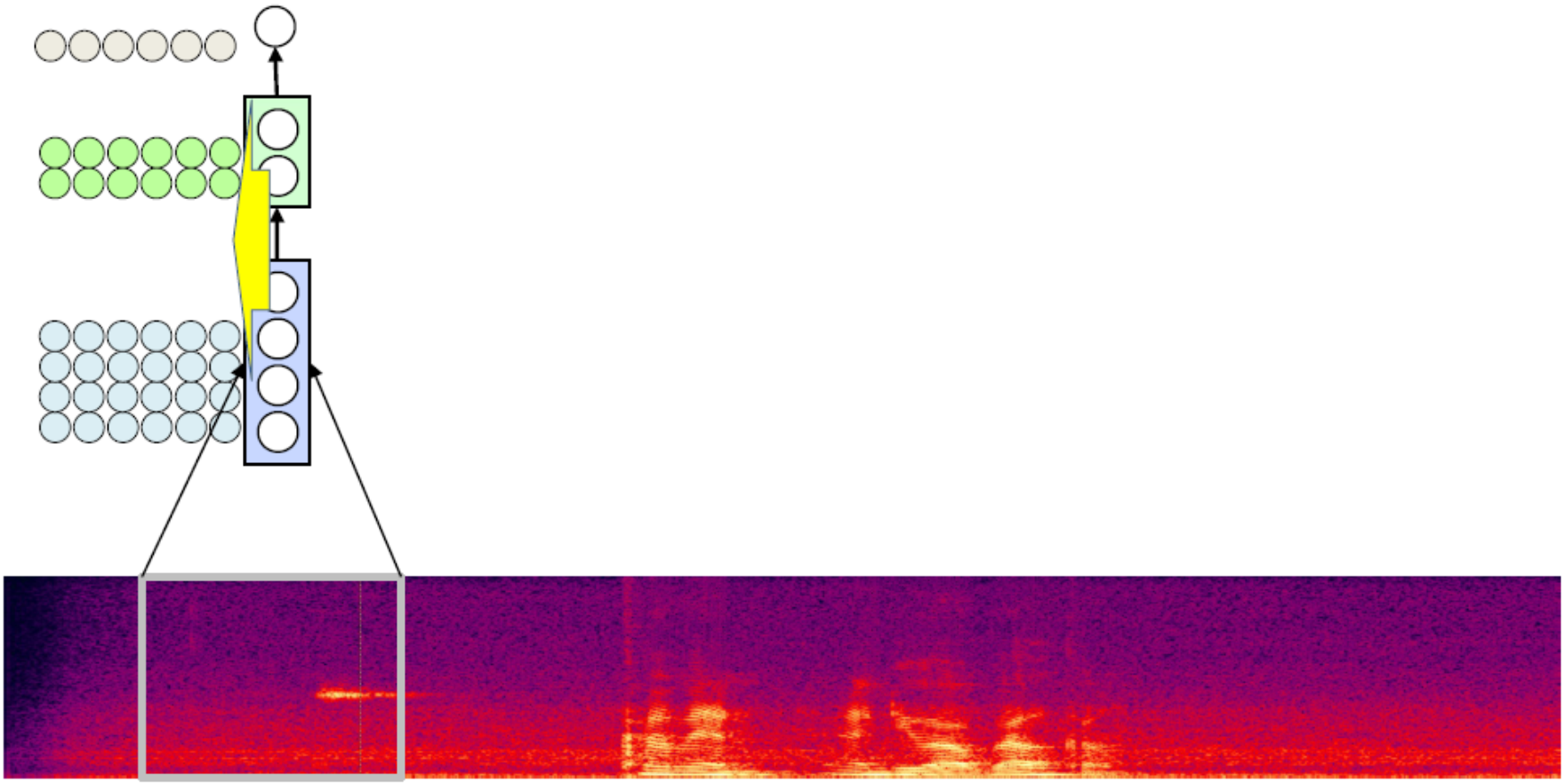

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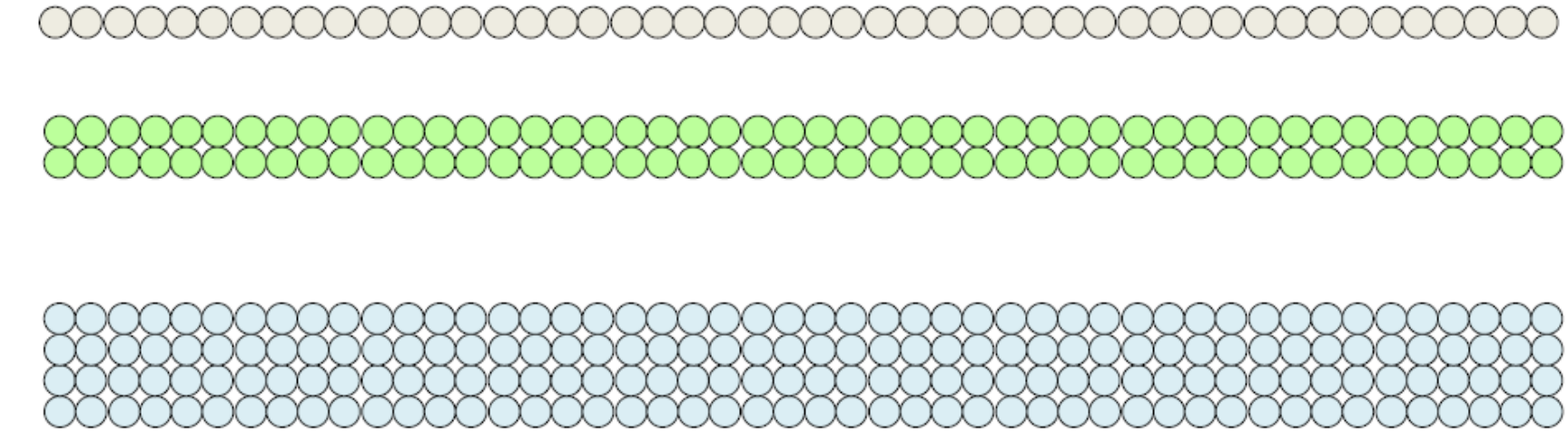

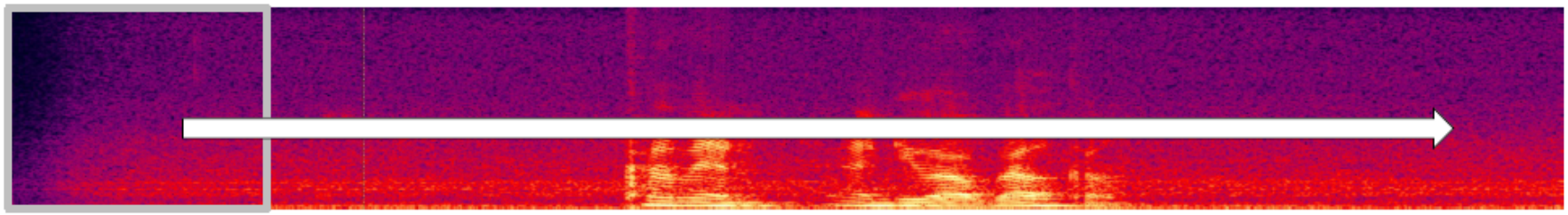

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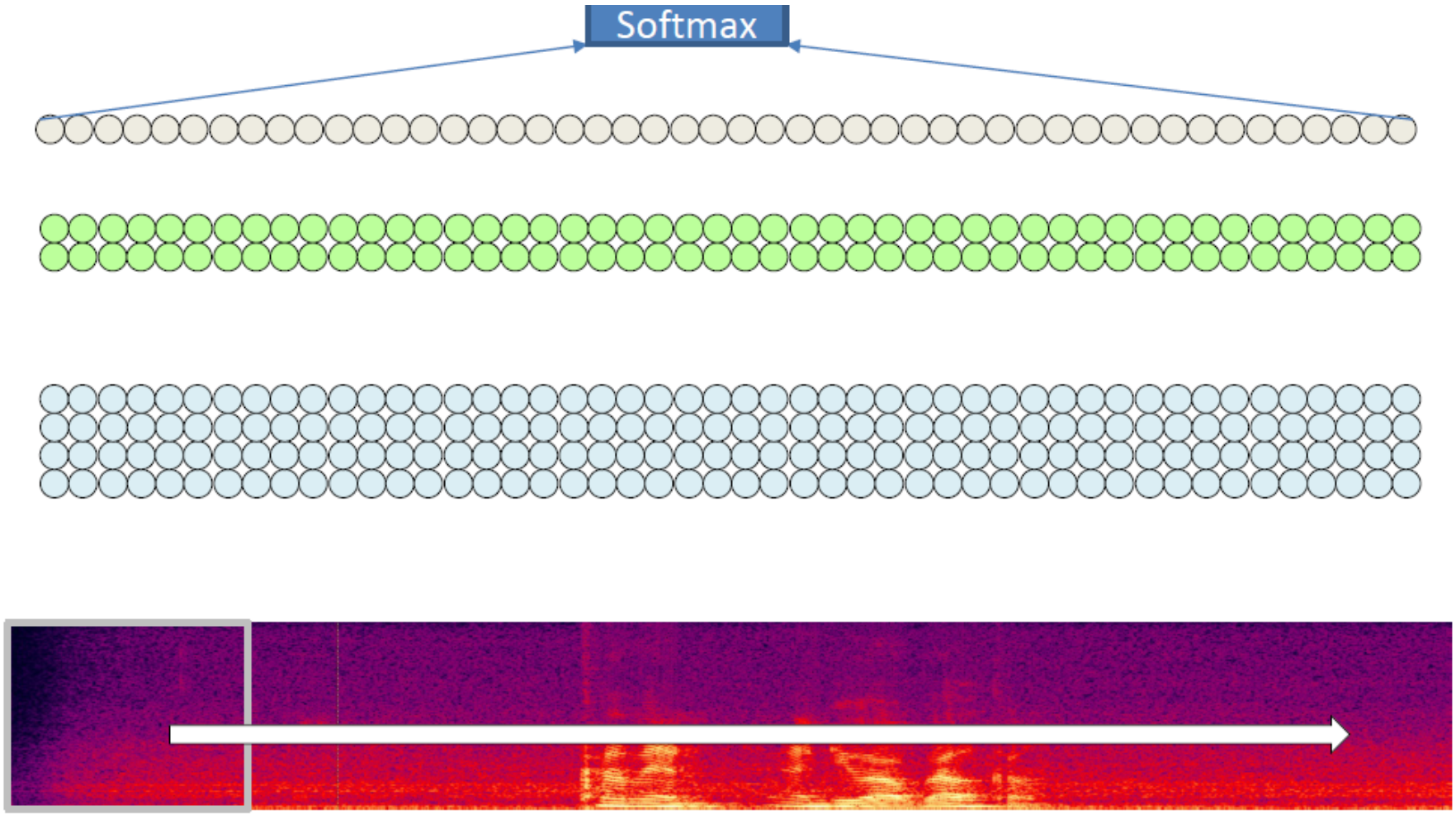

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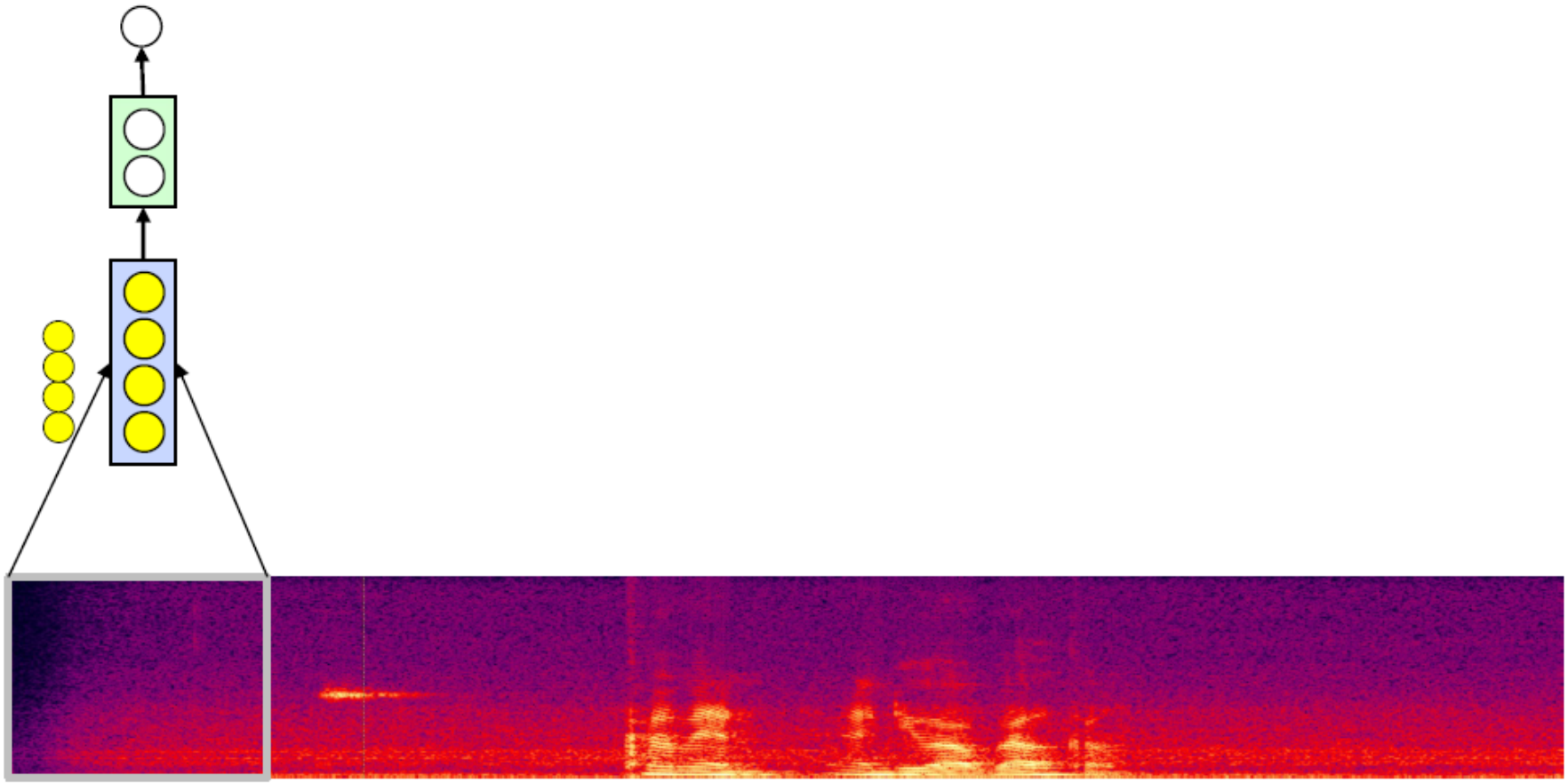

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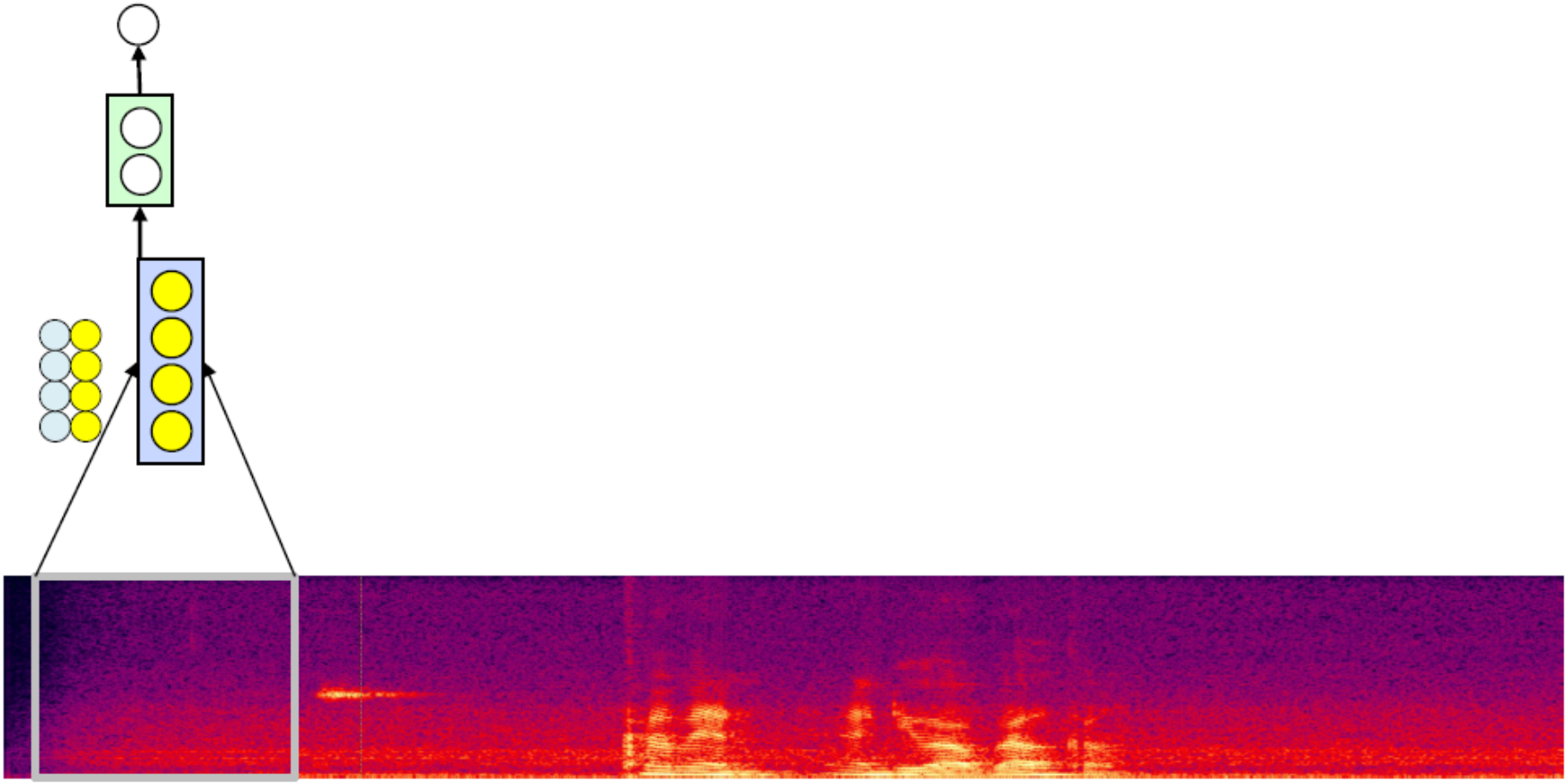

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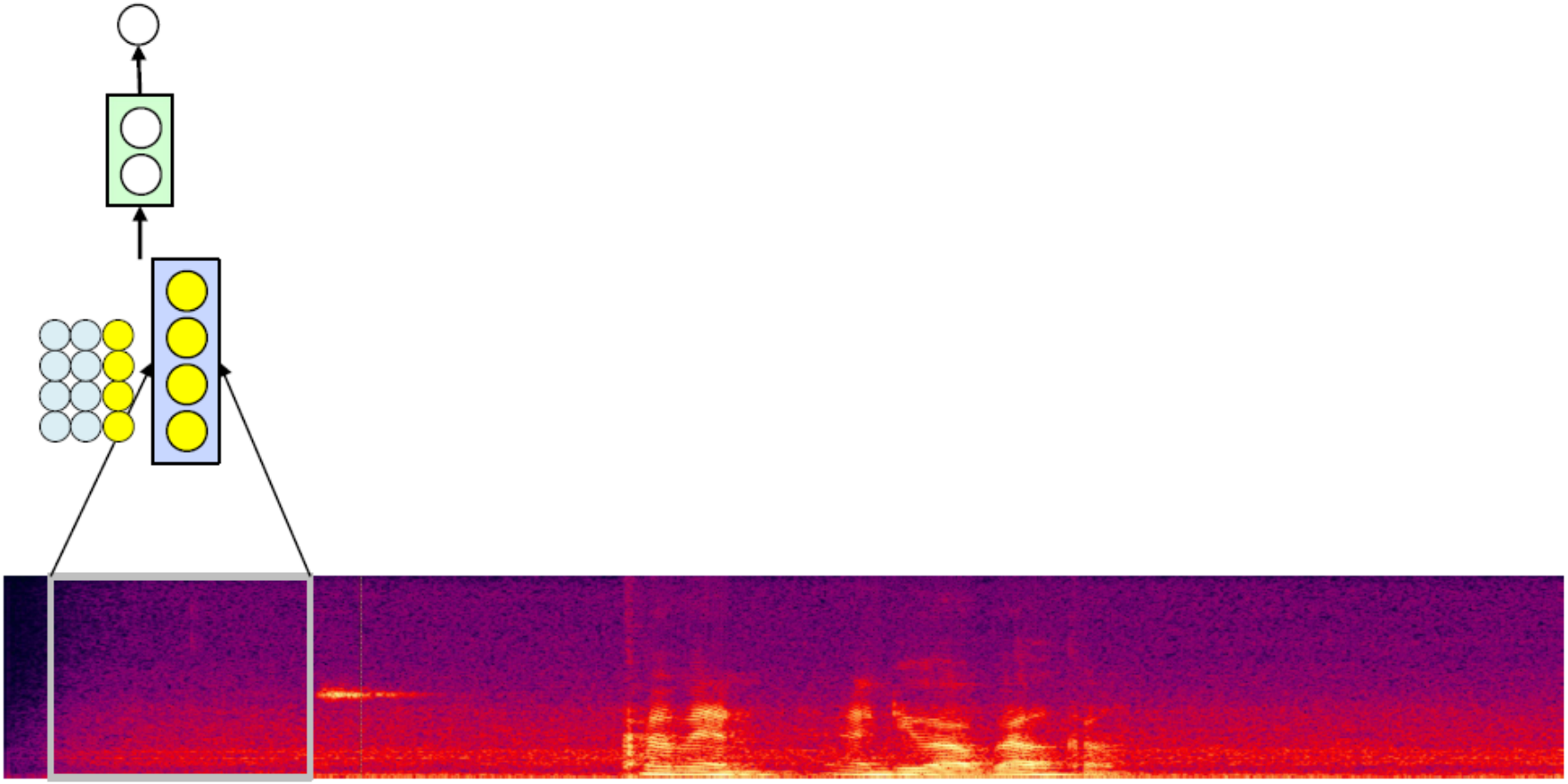

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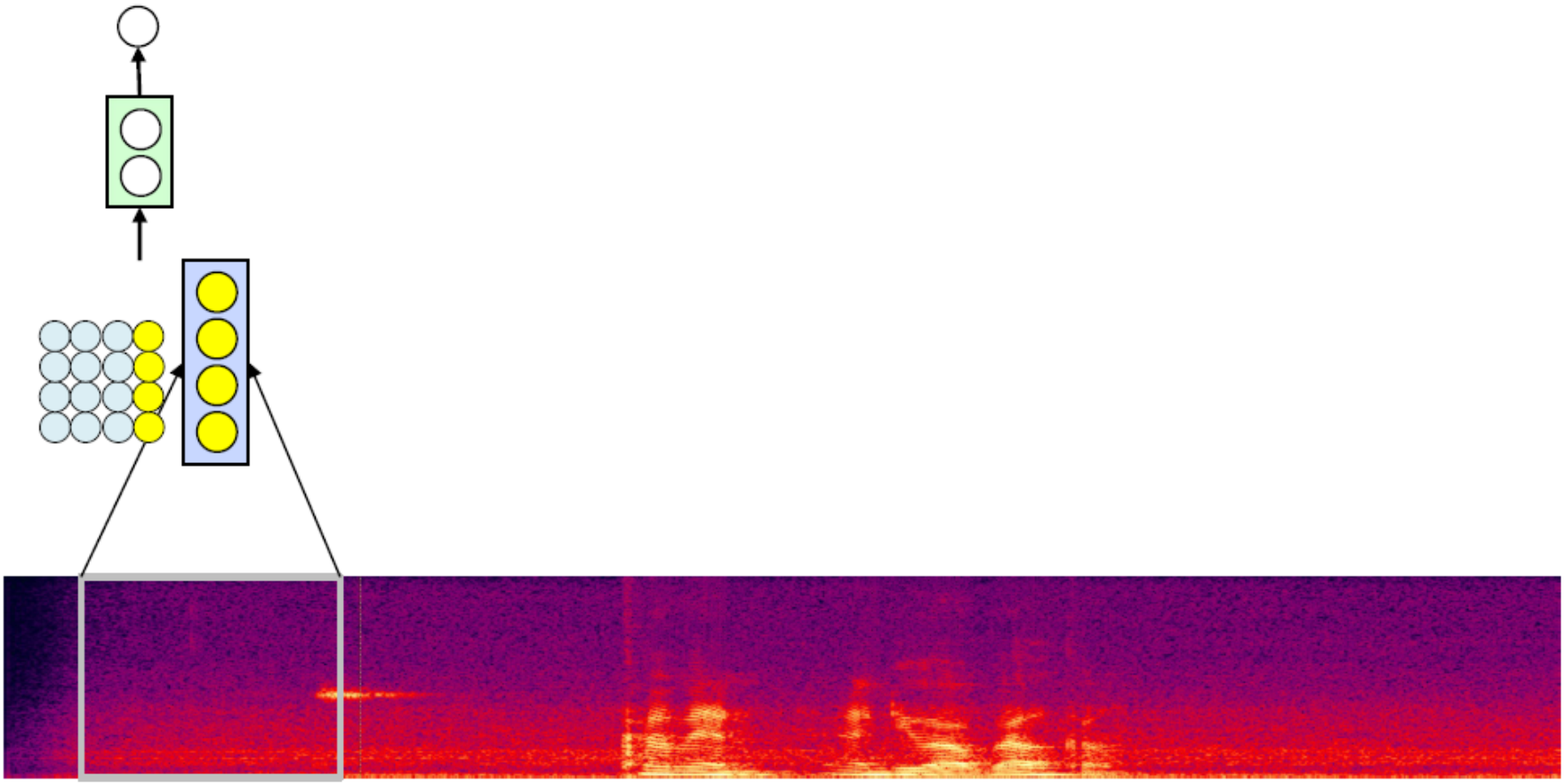

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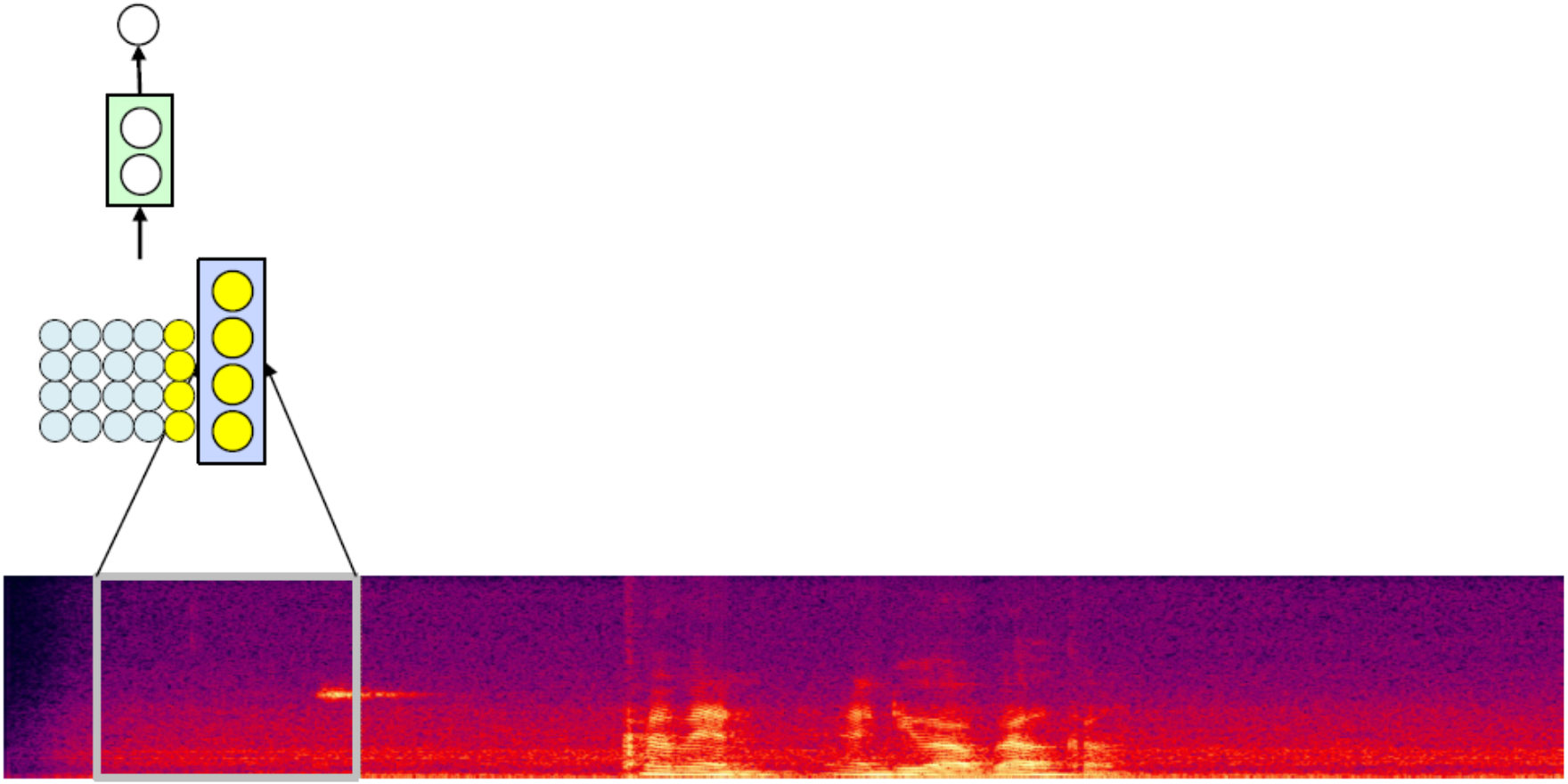

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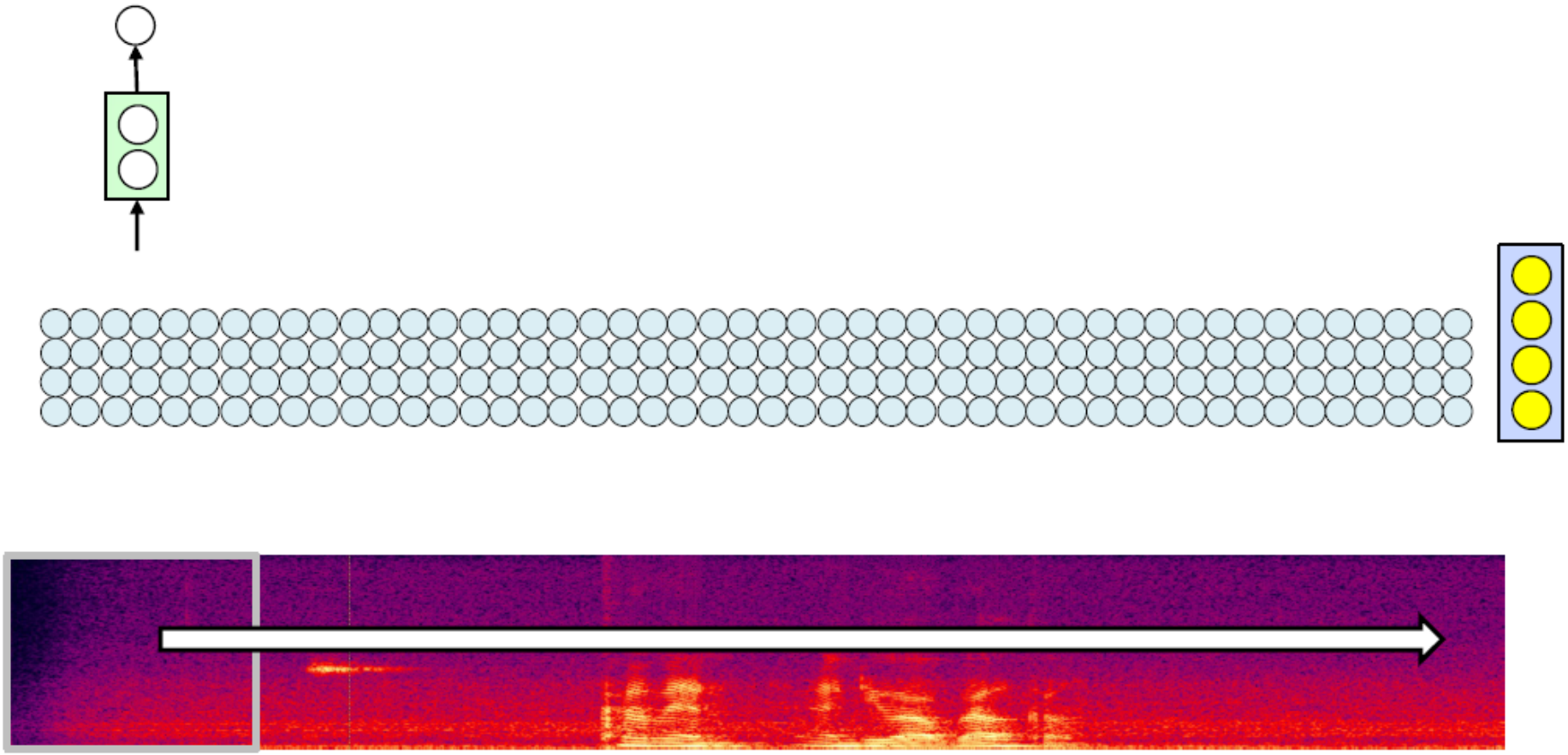

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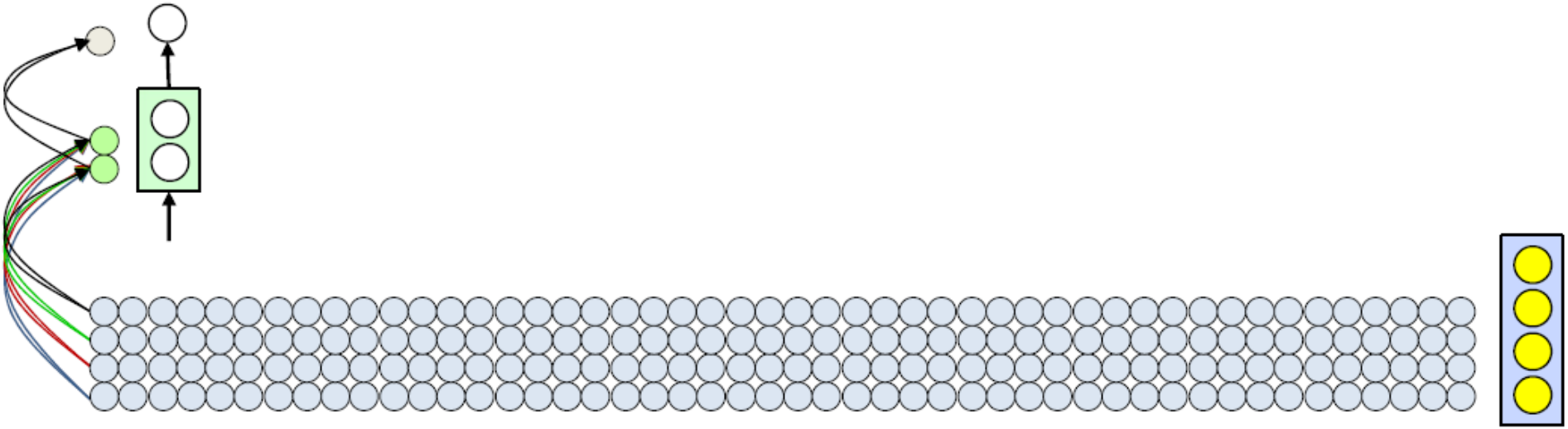

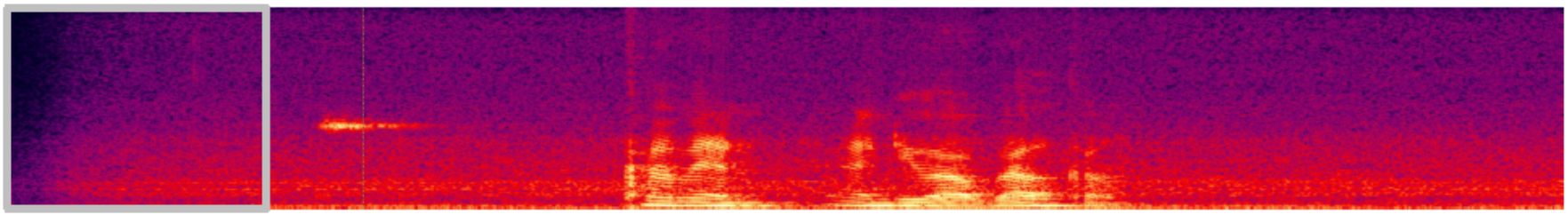

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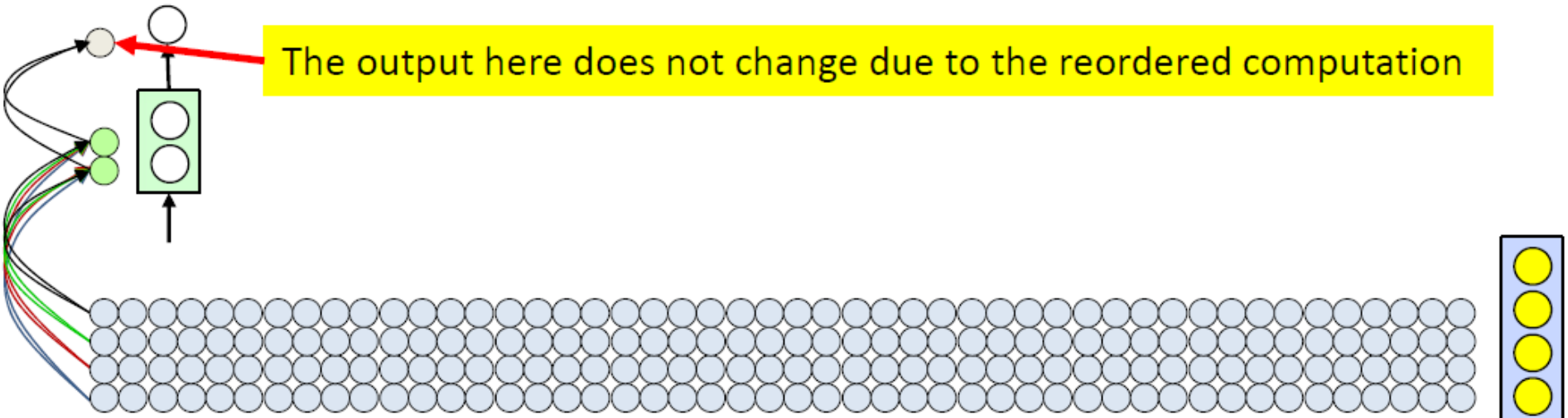

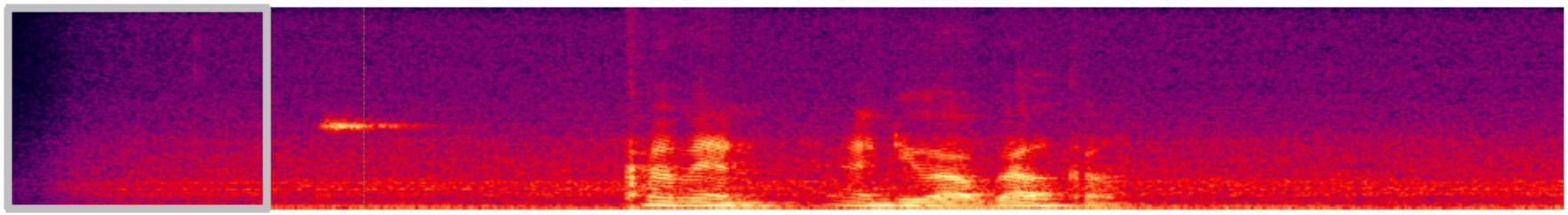

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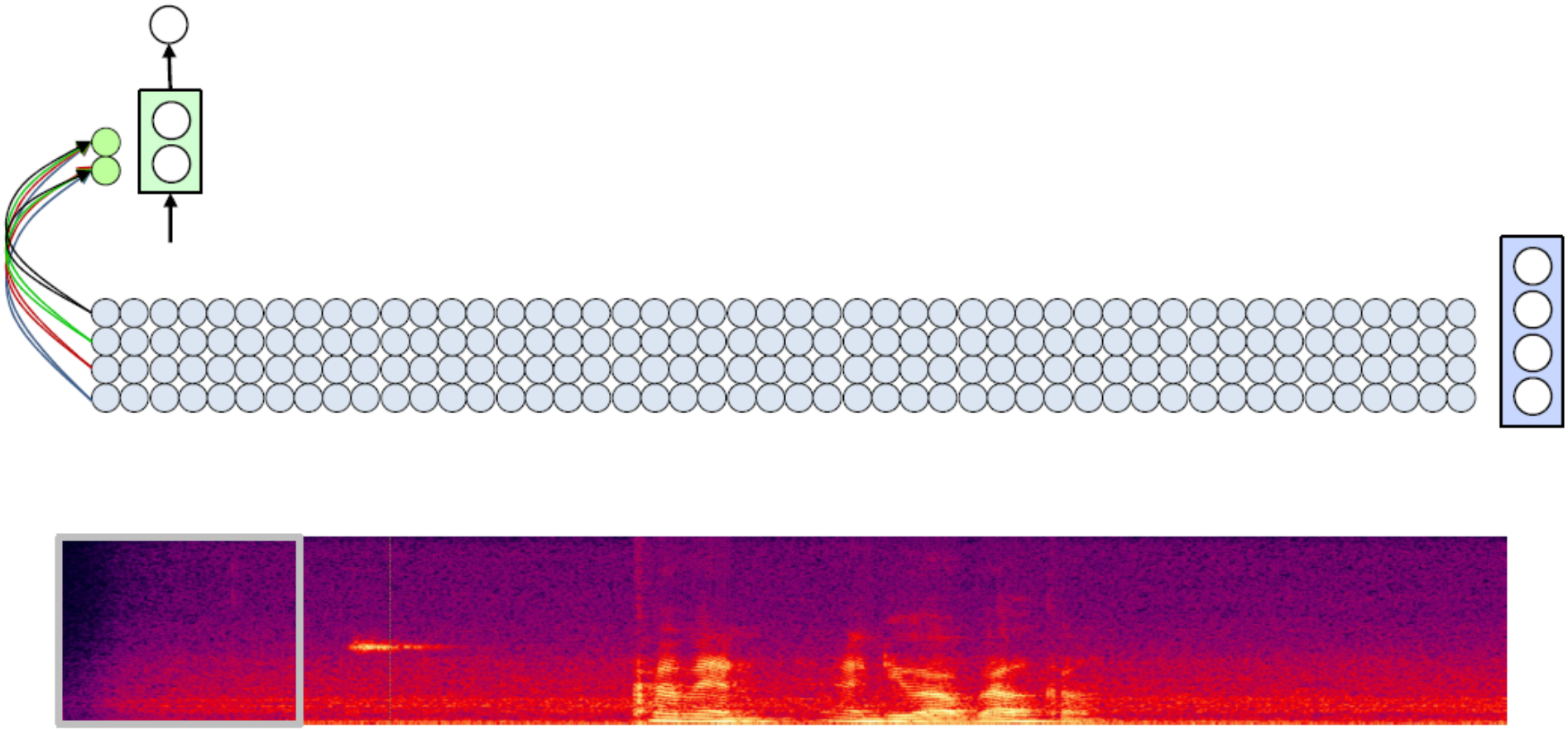

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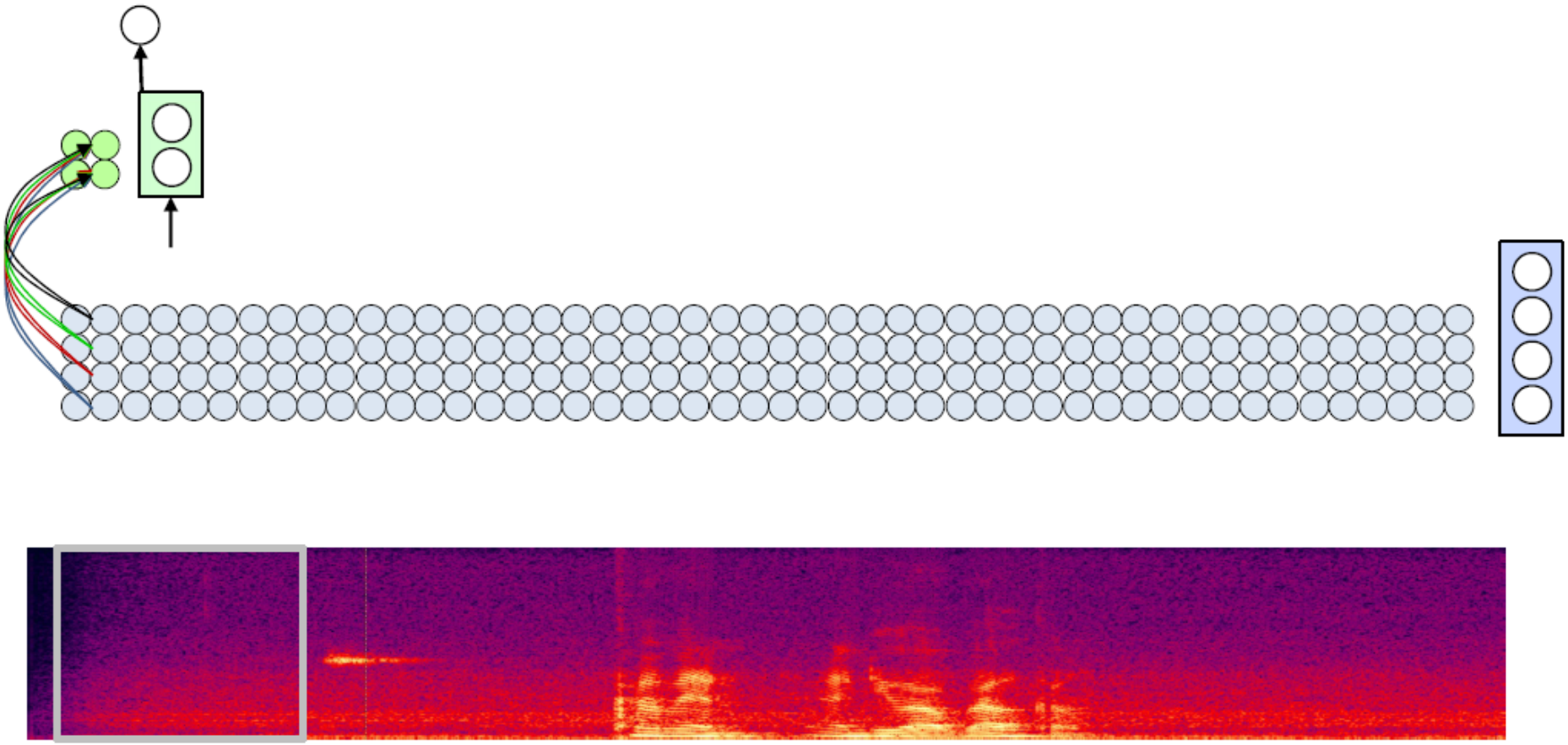

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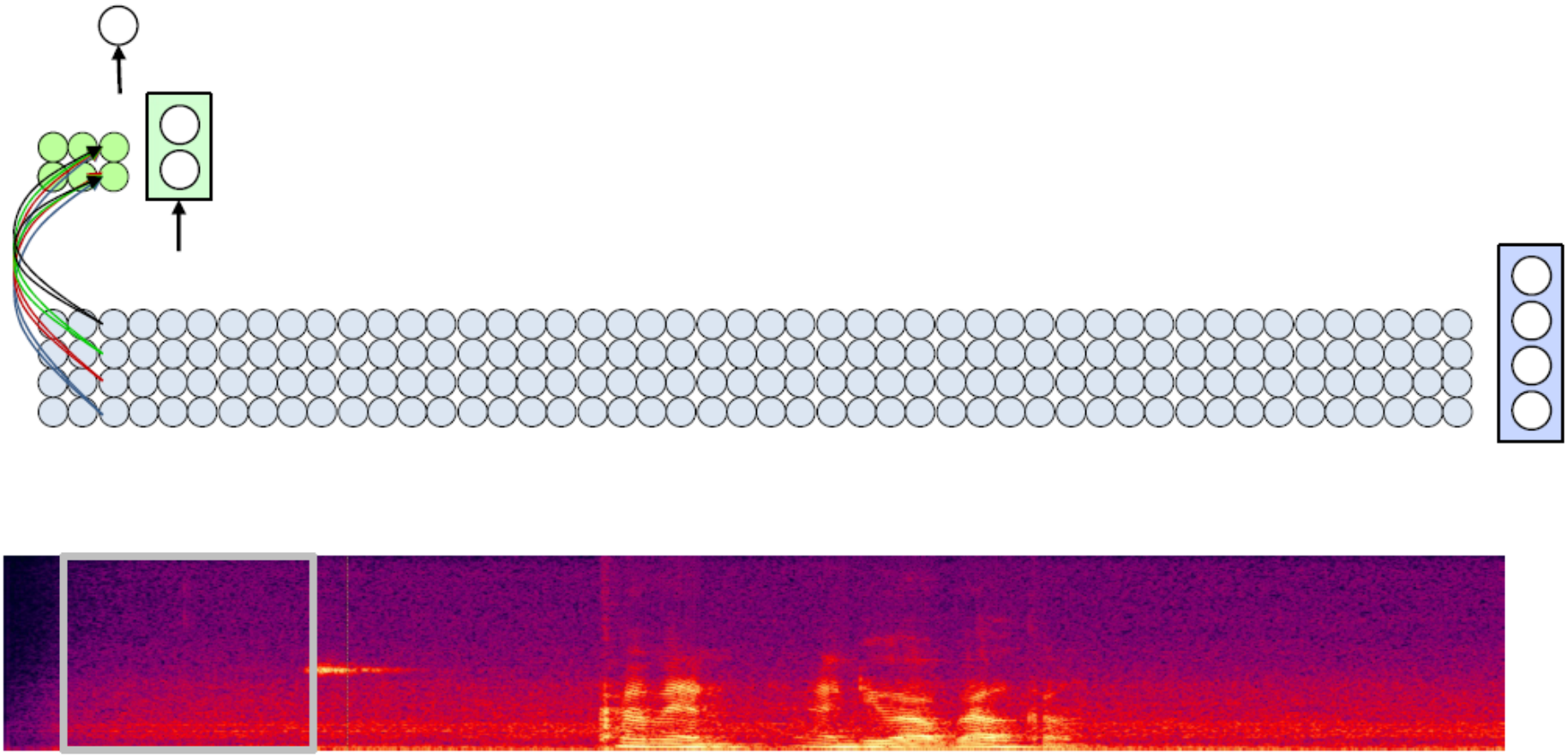

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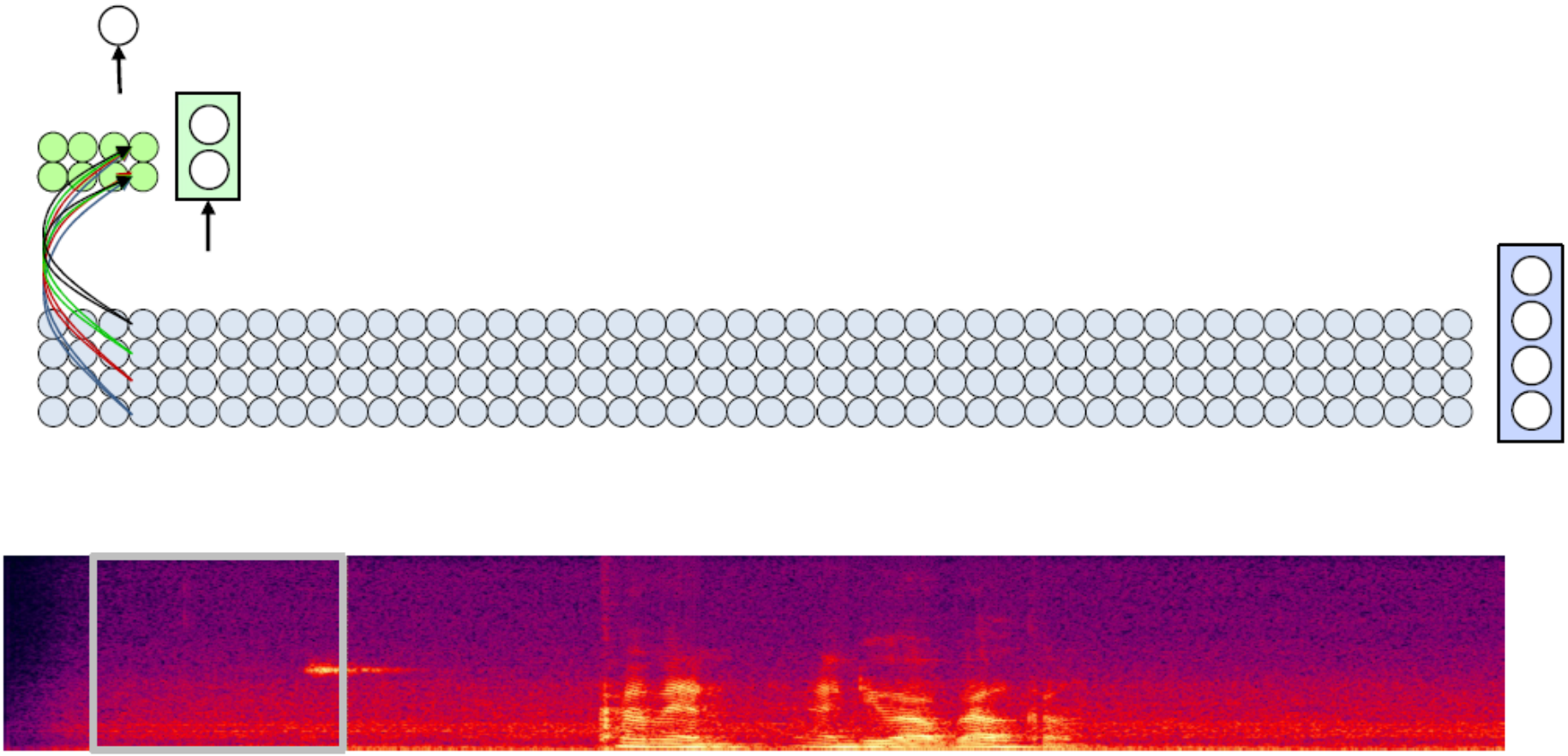

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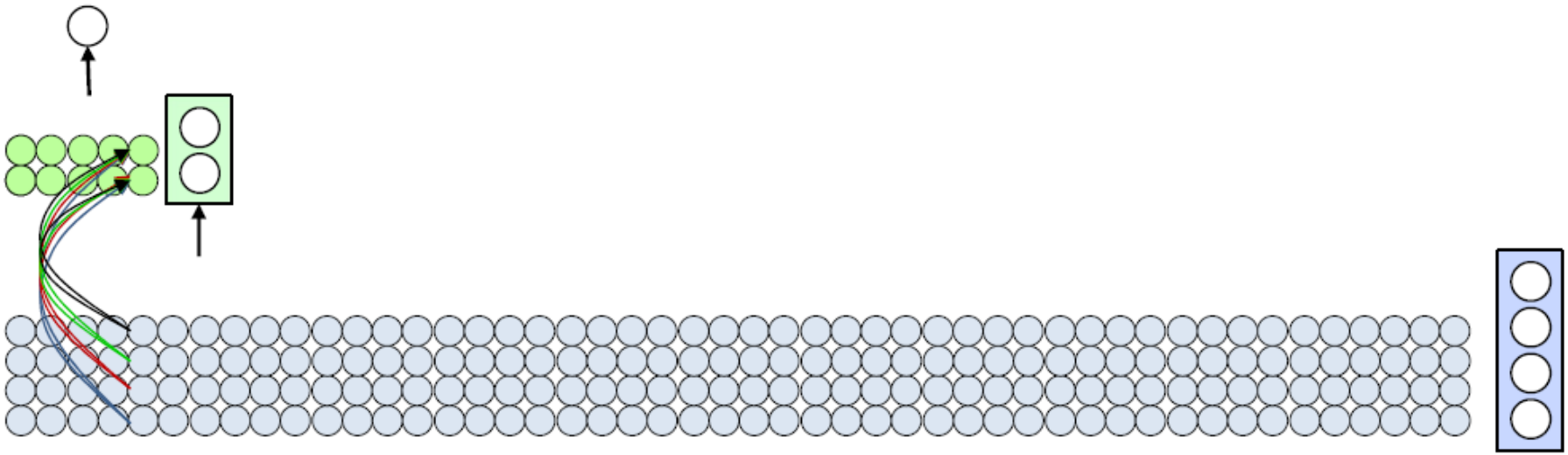

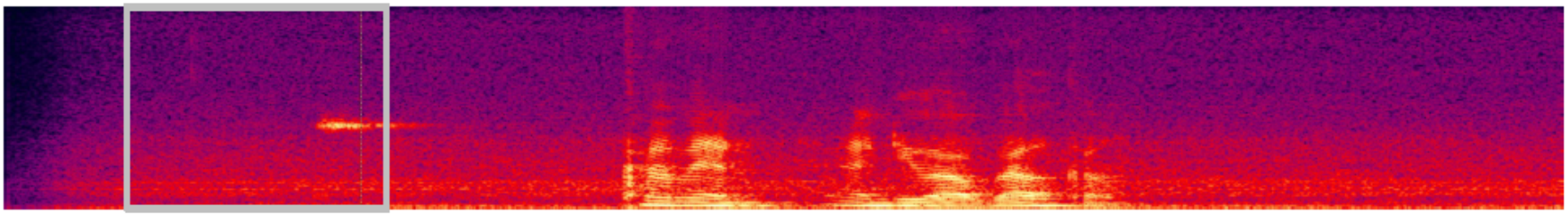

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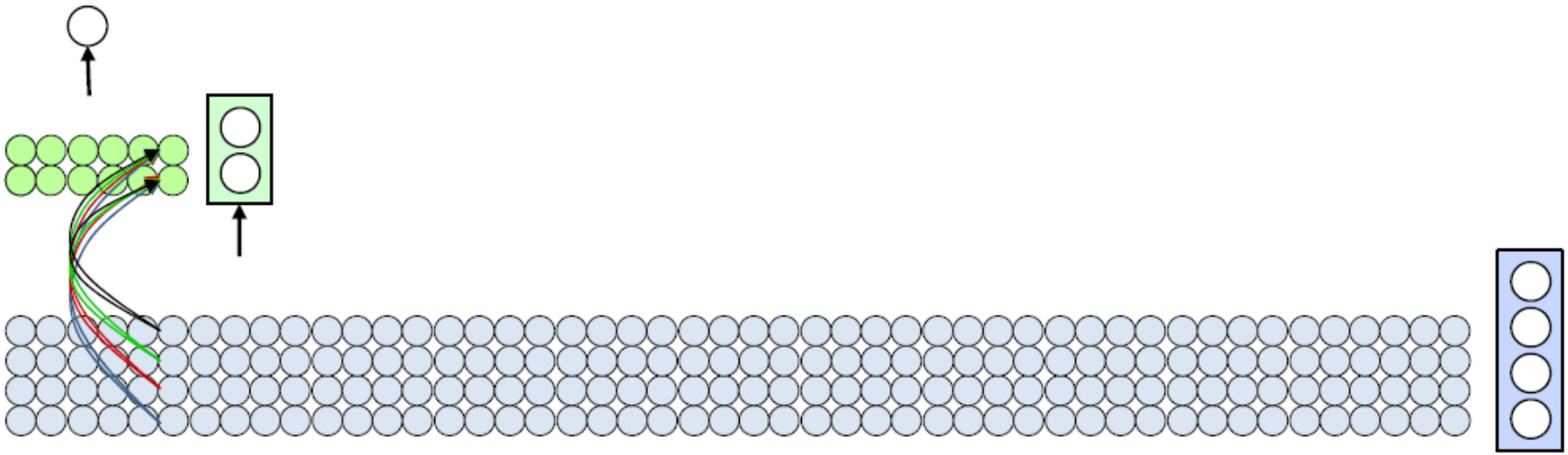

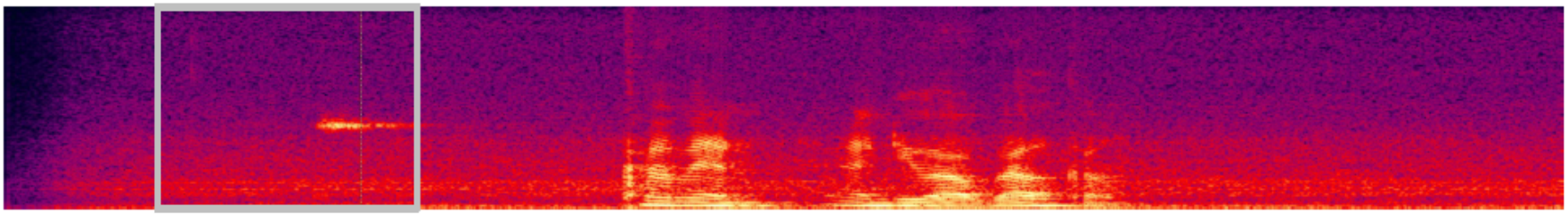

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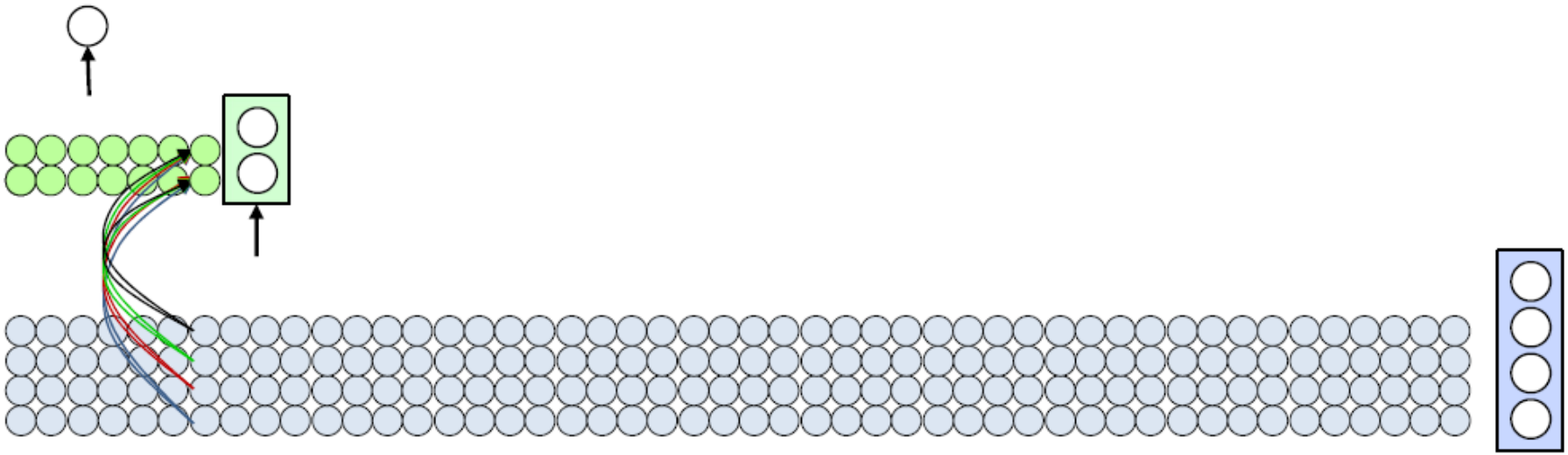

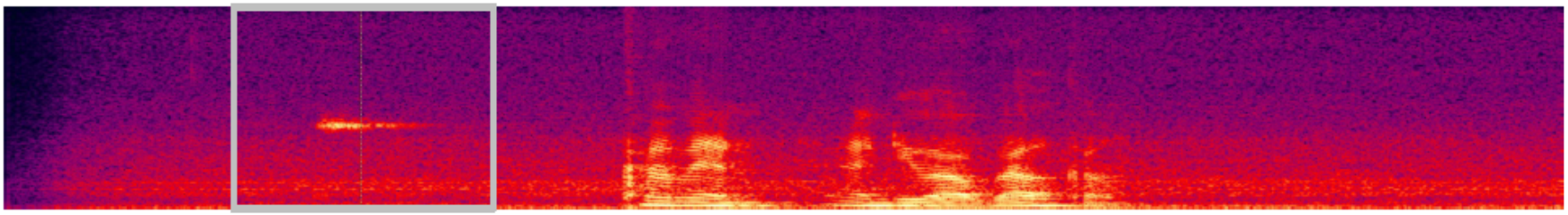

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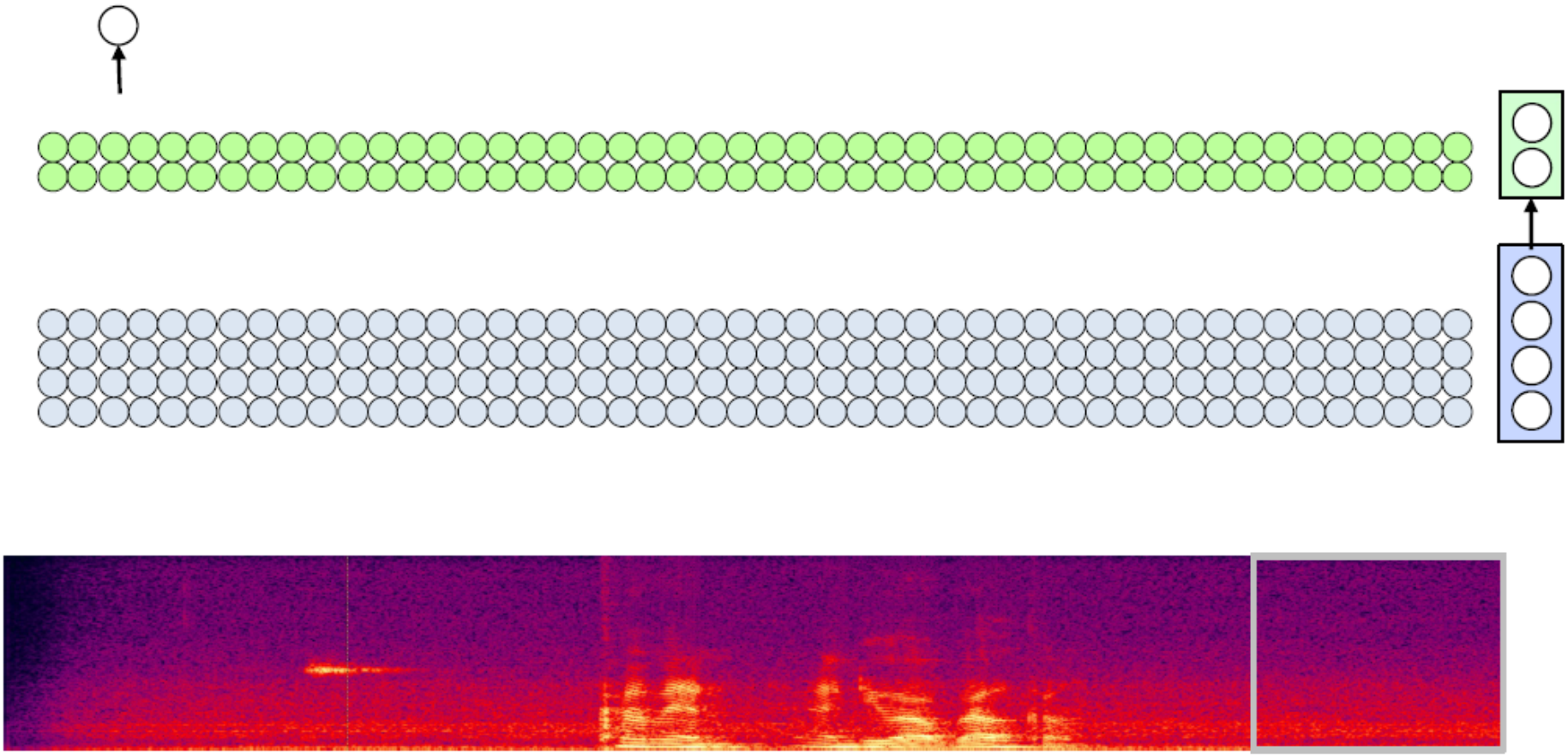

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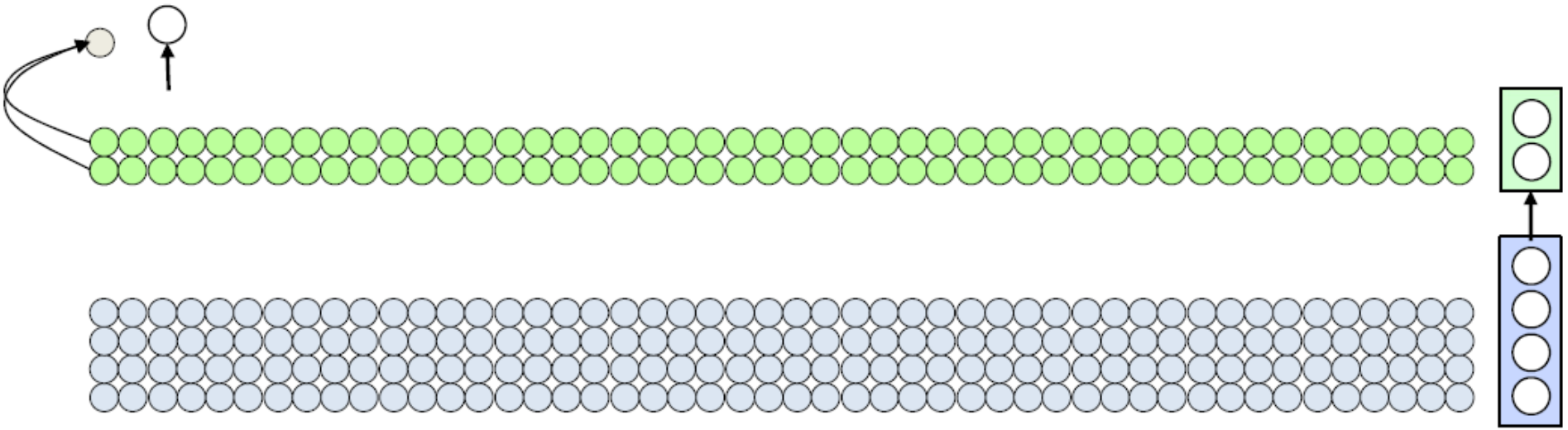

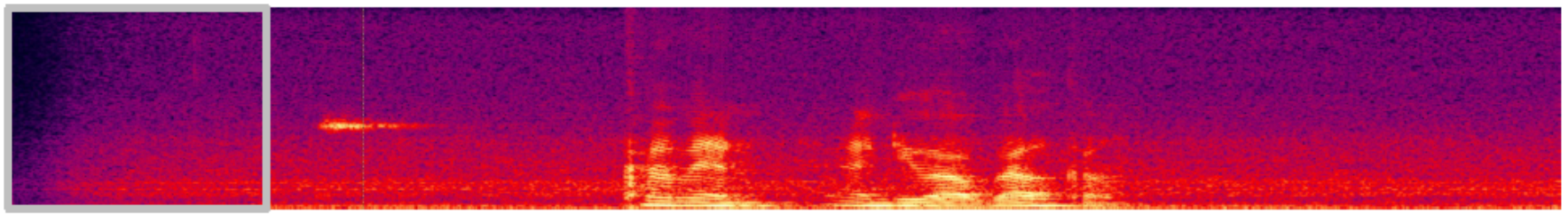

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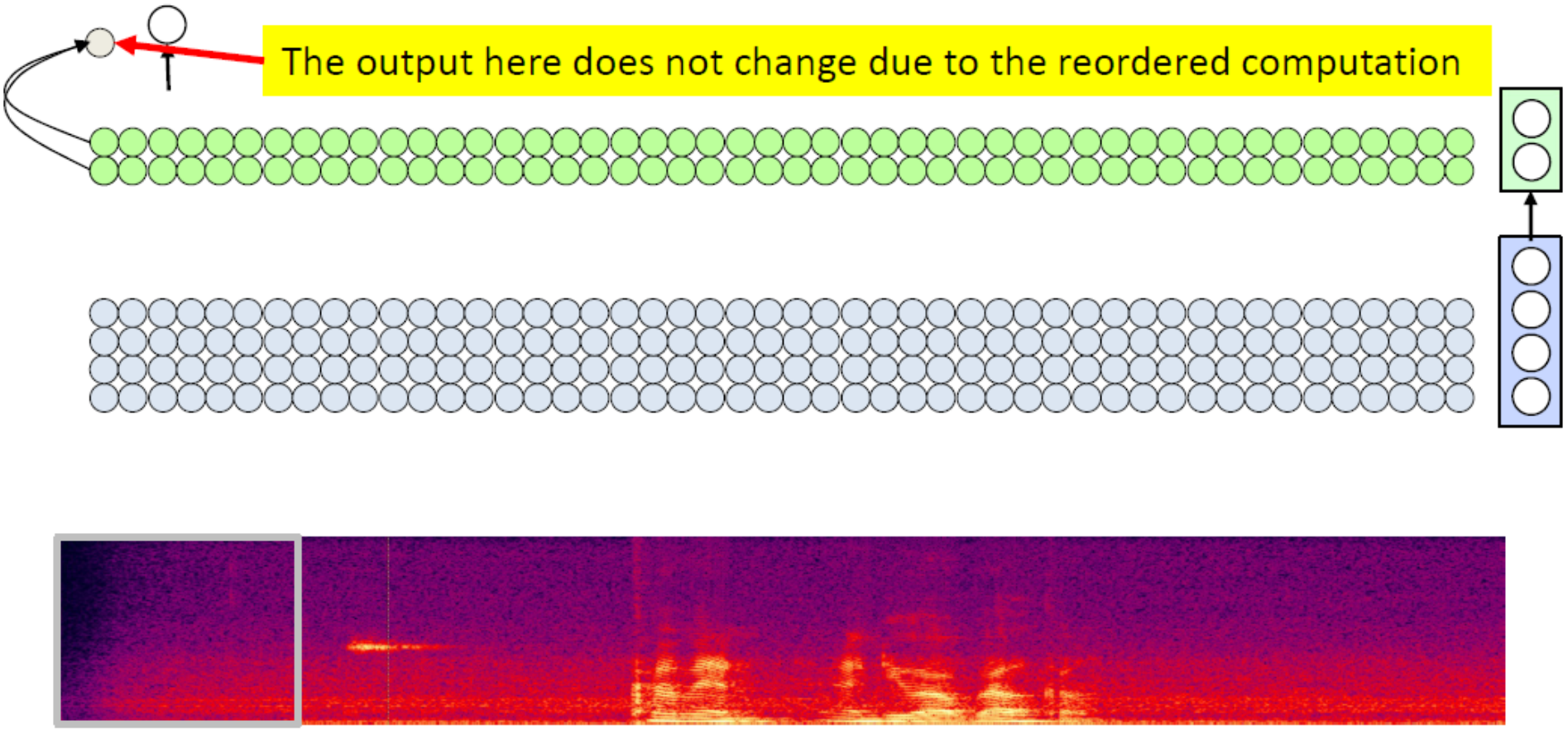

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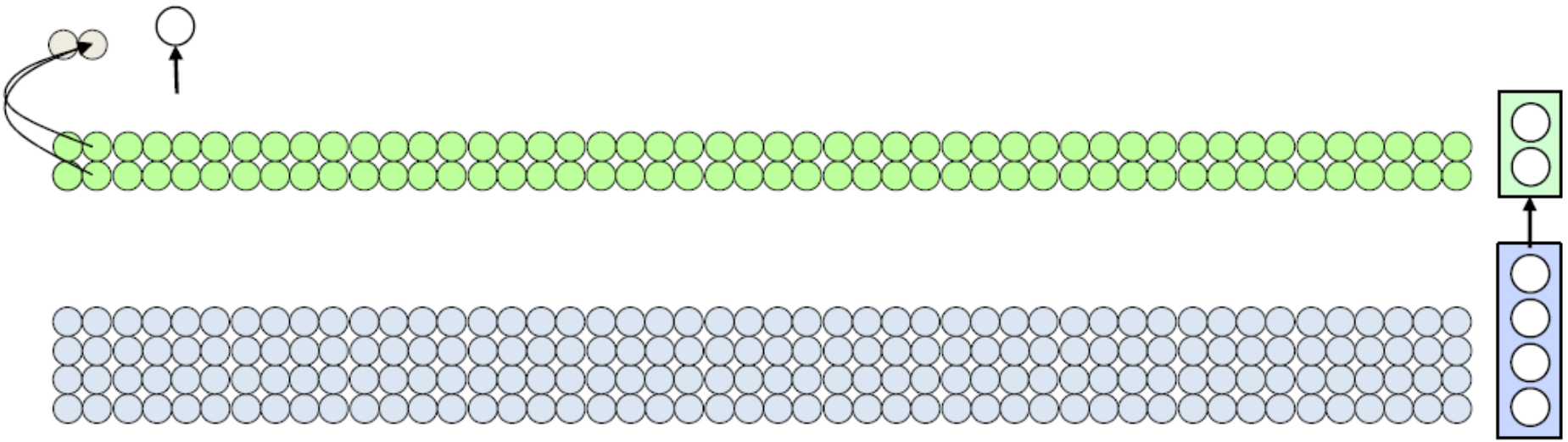

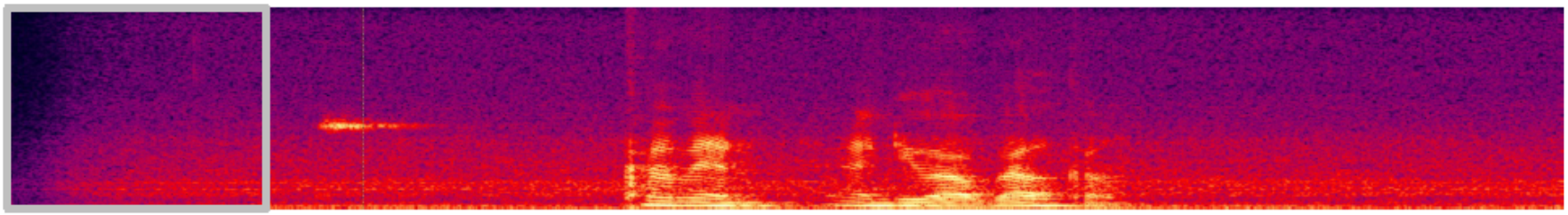

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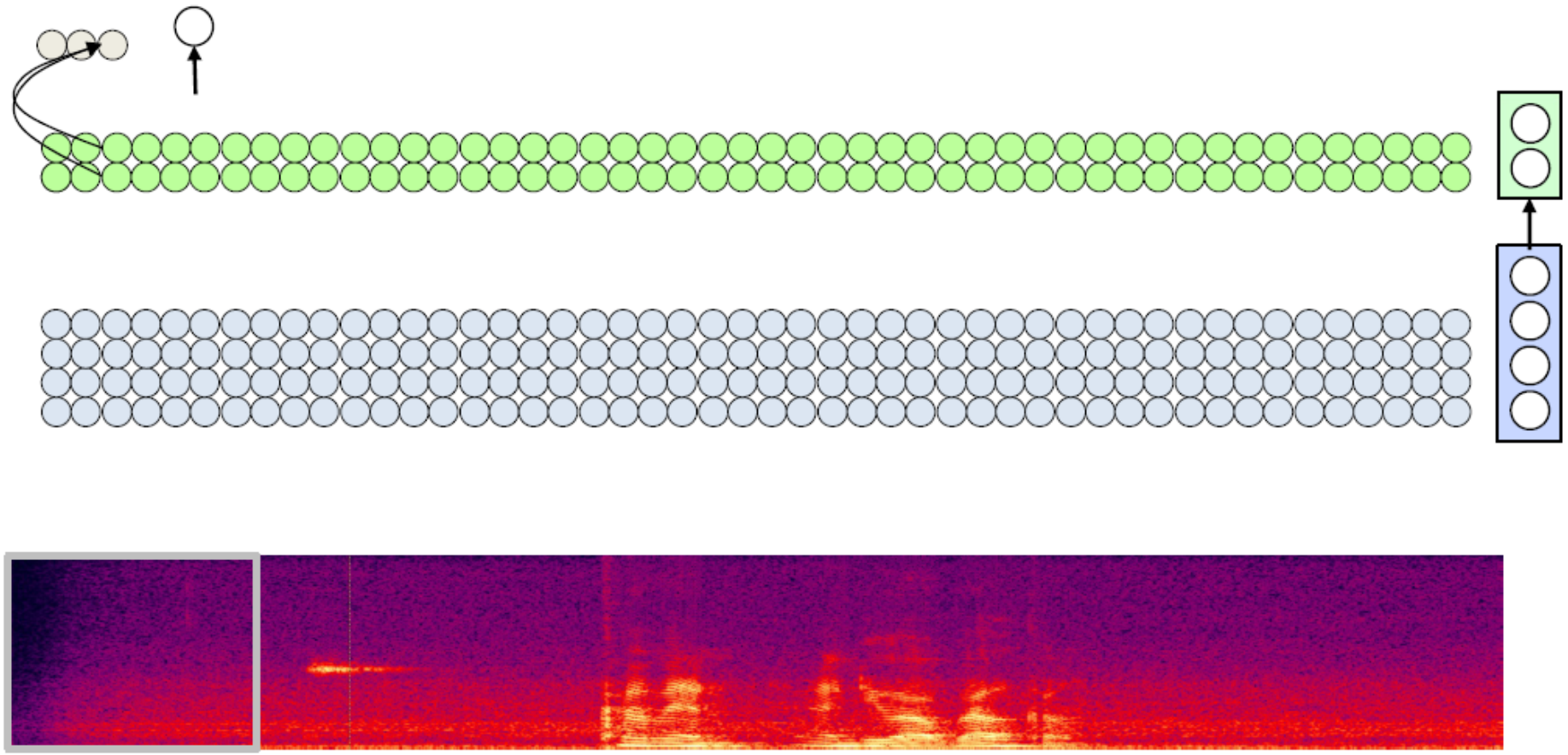

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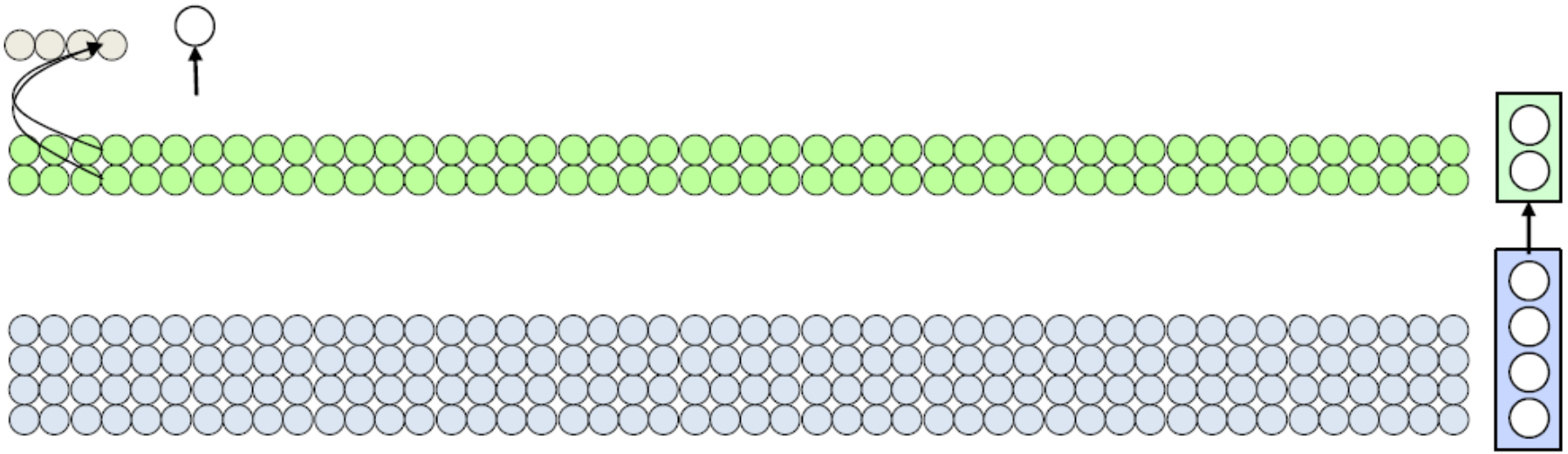

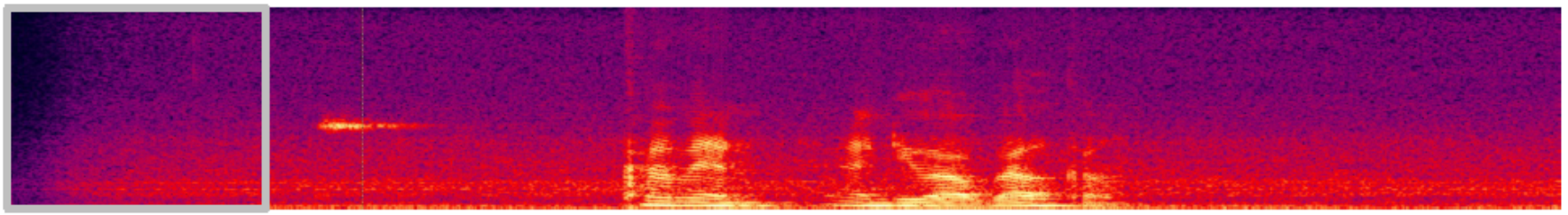

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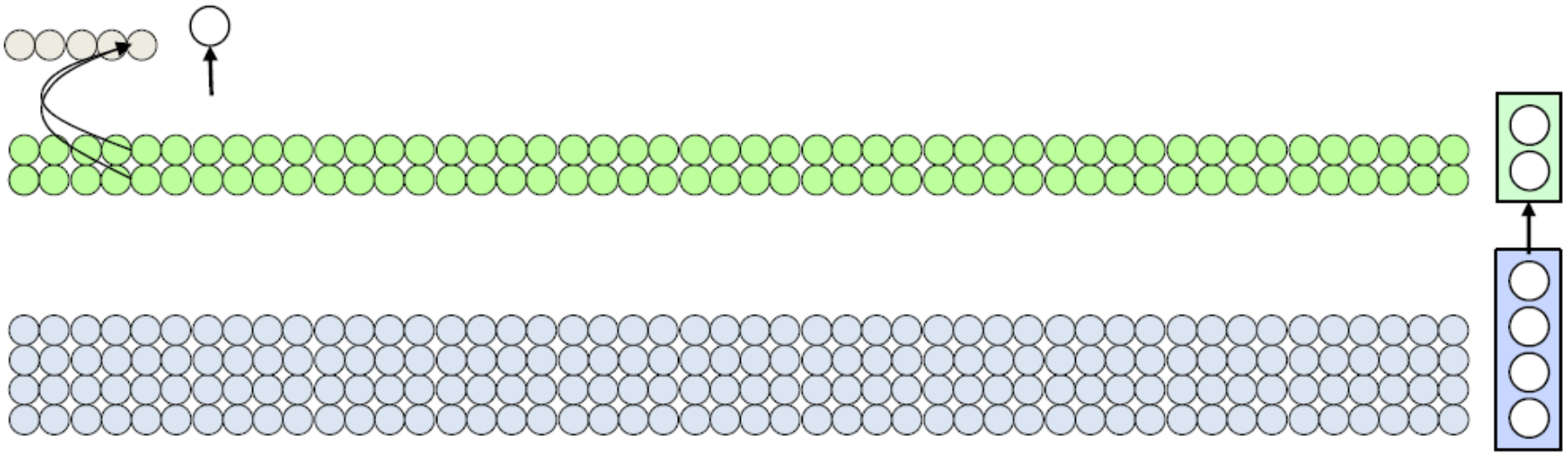

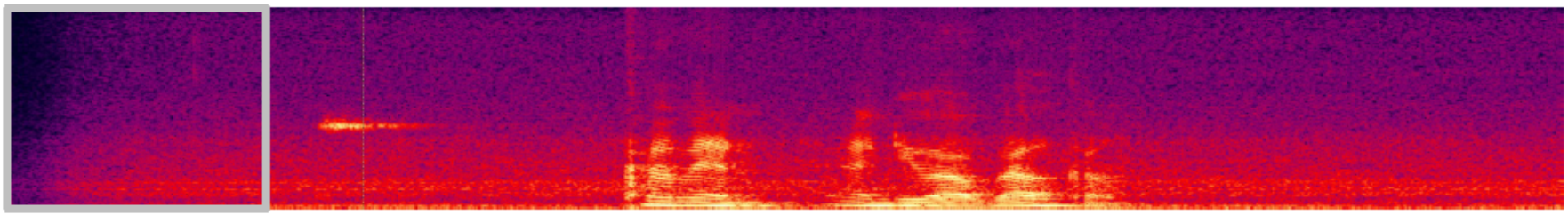

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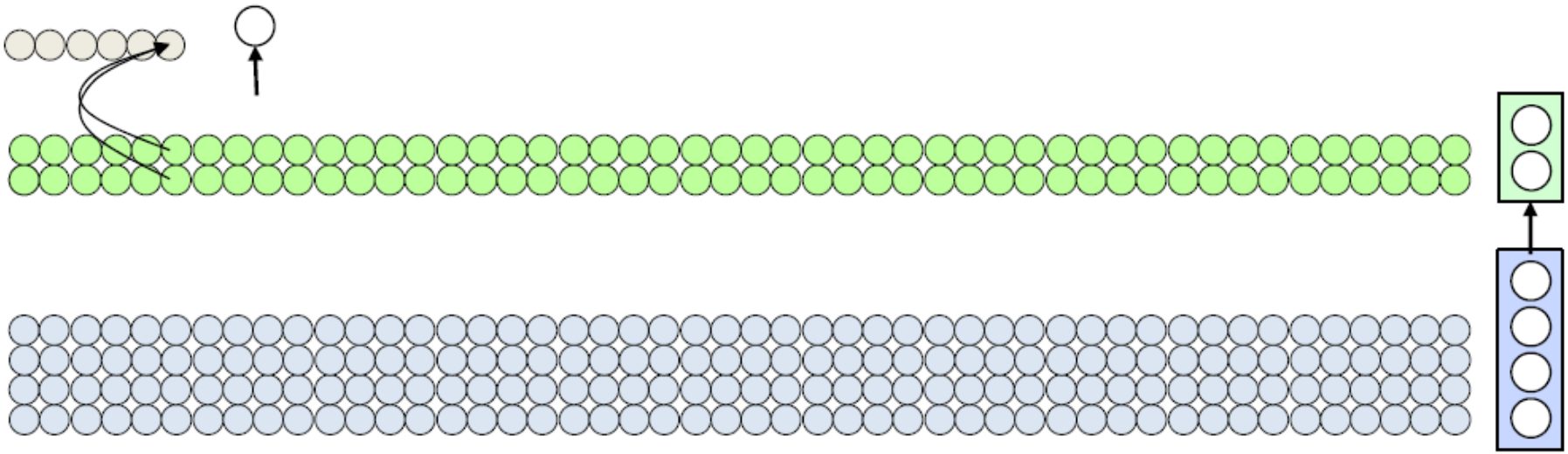

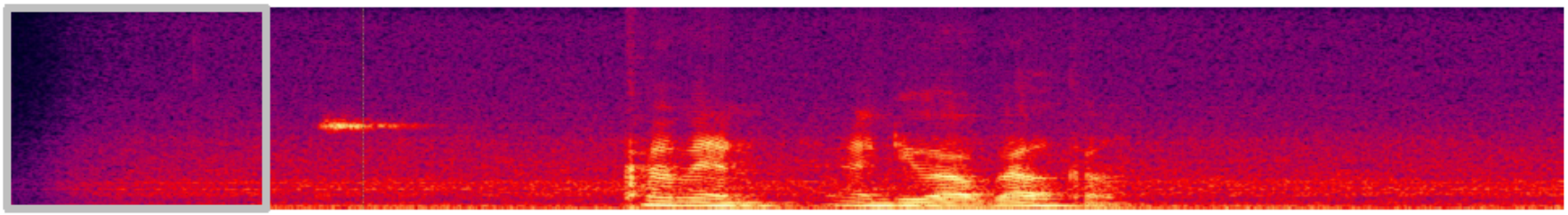

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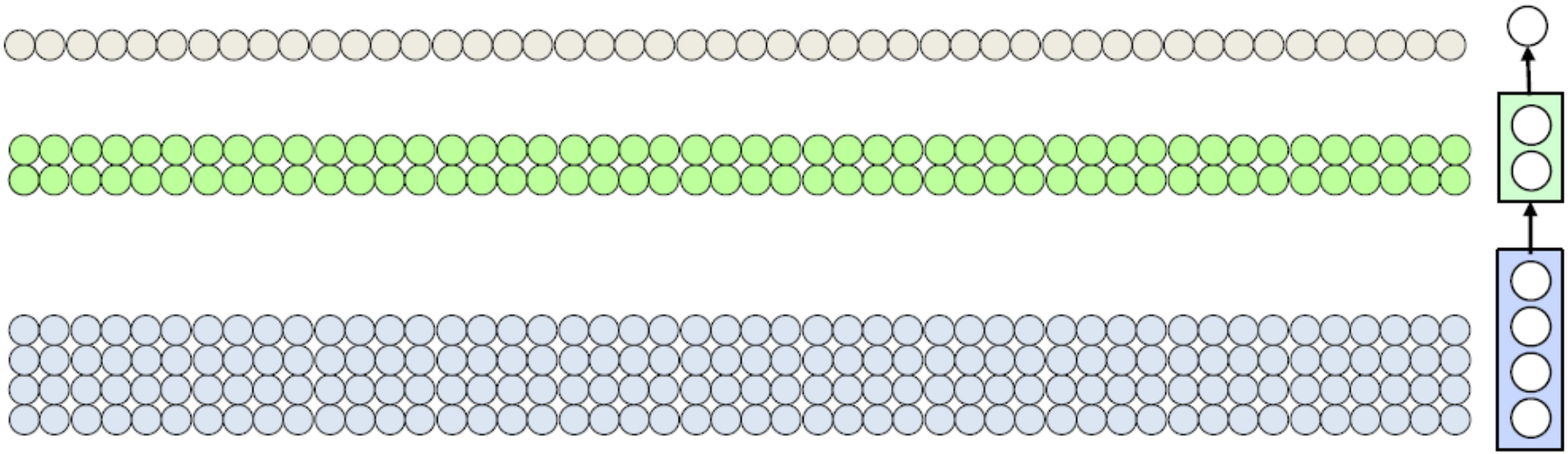

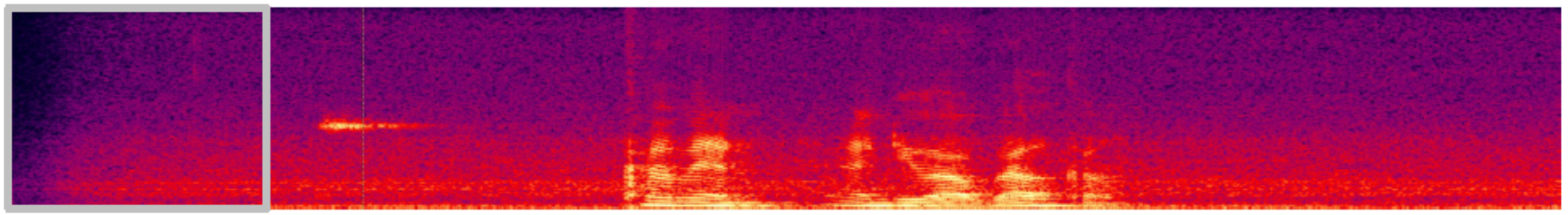

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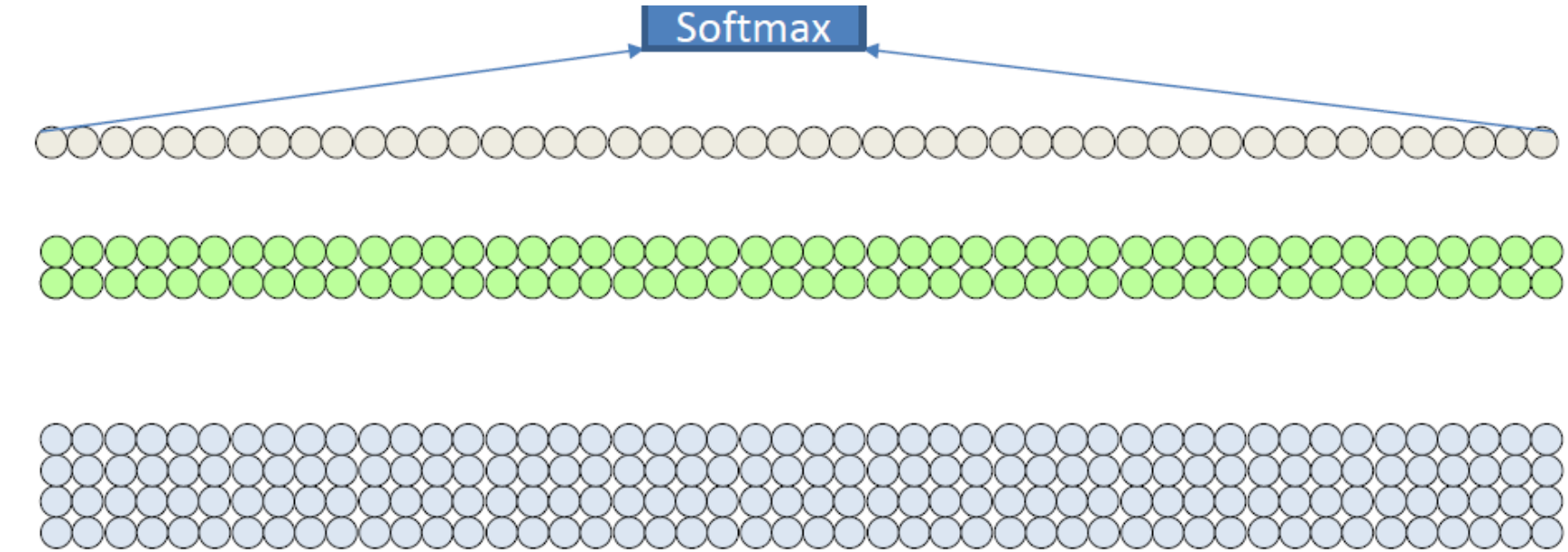
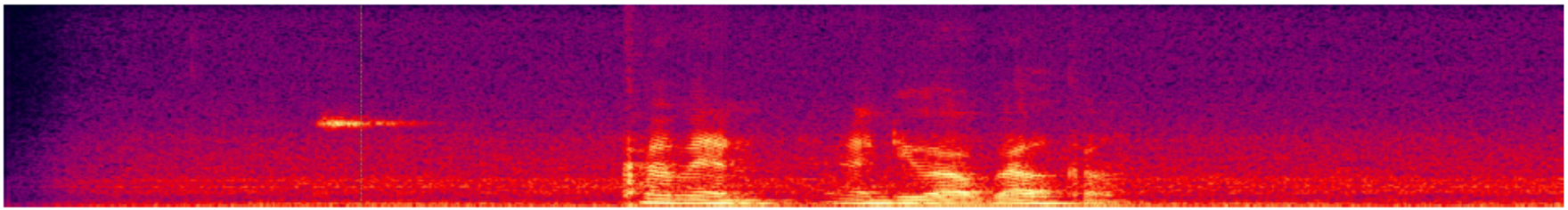
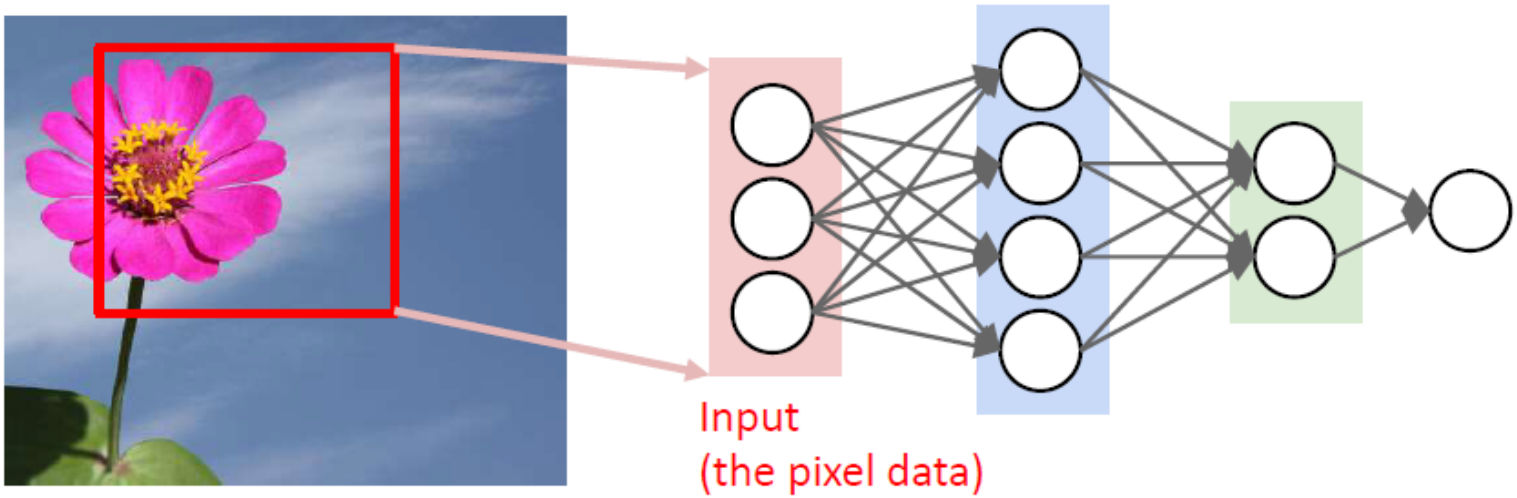




Scanning in 2D: A closer look

- Scan for the desired object
- At each location, the entire region is sent through an MLP
- The “input layer” is just the pixels in the image connecting to the hidden layer



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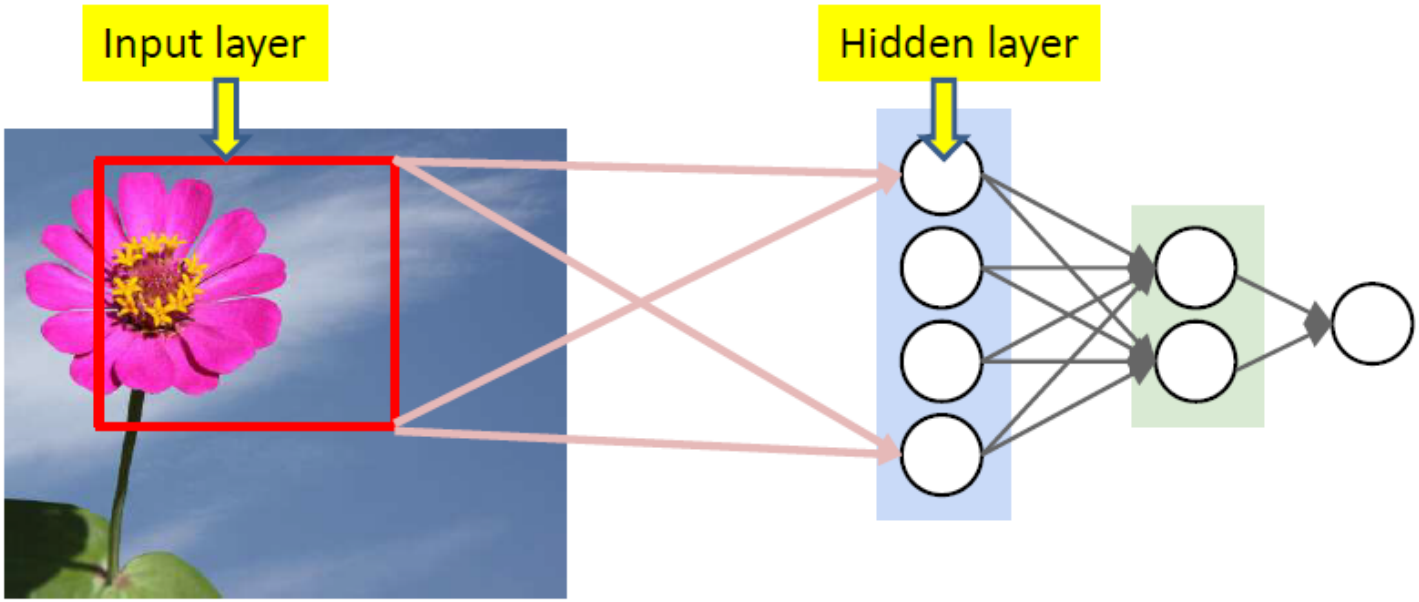

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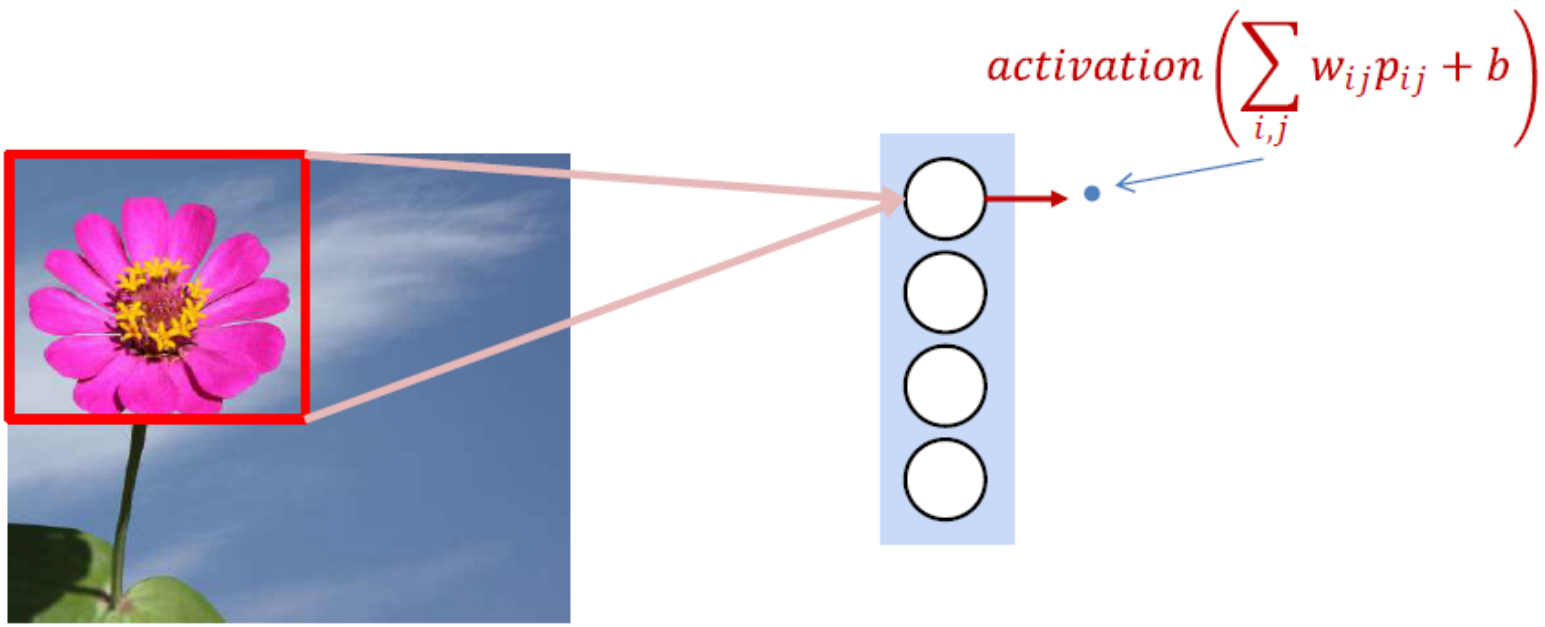
- Consider **a single neuron (filter)** in the first layer
  - At each position of the box, the neuron is evaluating a “window” of the picture at that location, as part of the classification for that region
- Let us compute the output of the first neuron for all the windows in the picture before computing the rest of the neurons
  - “Scanning” the image with just the neuron
  - We could arrange the outputs in correspondence to the original picture



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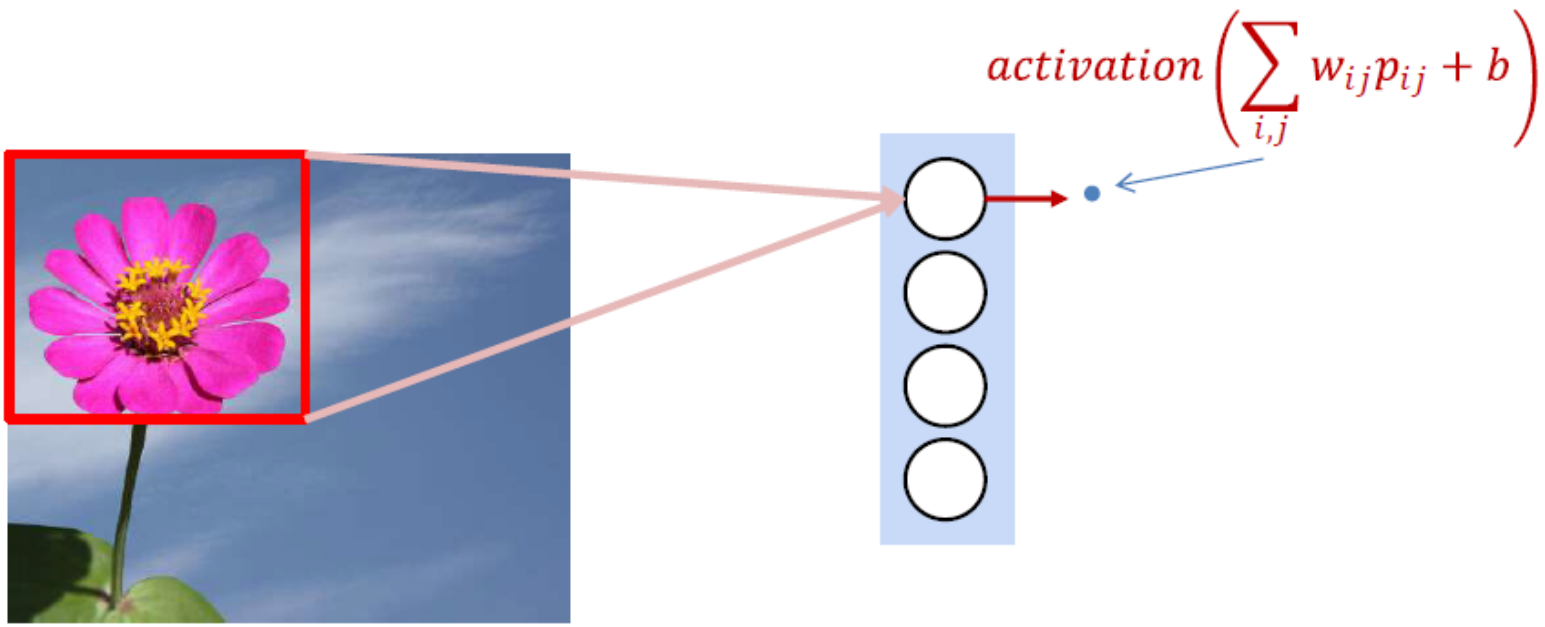

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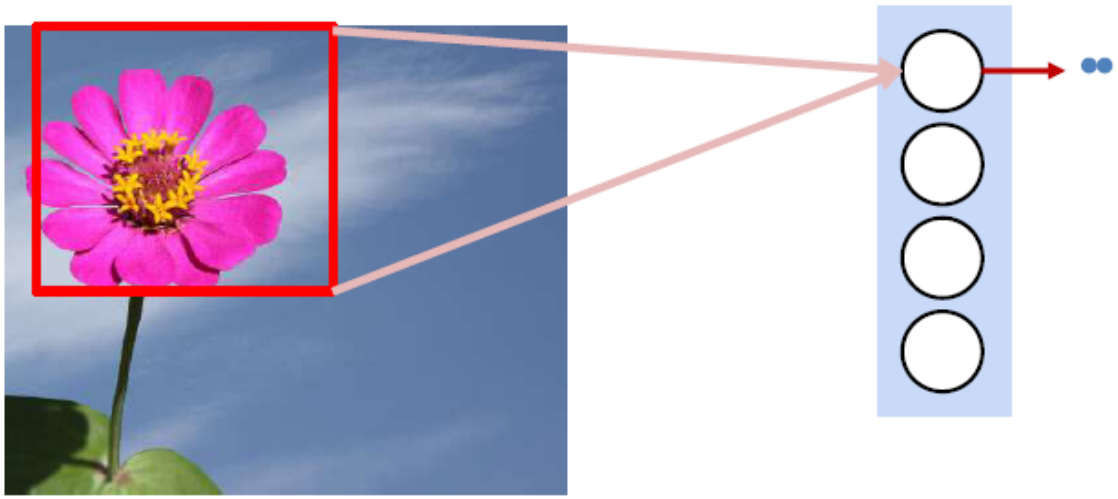

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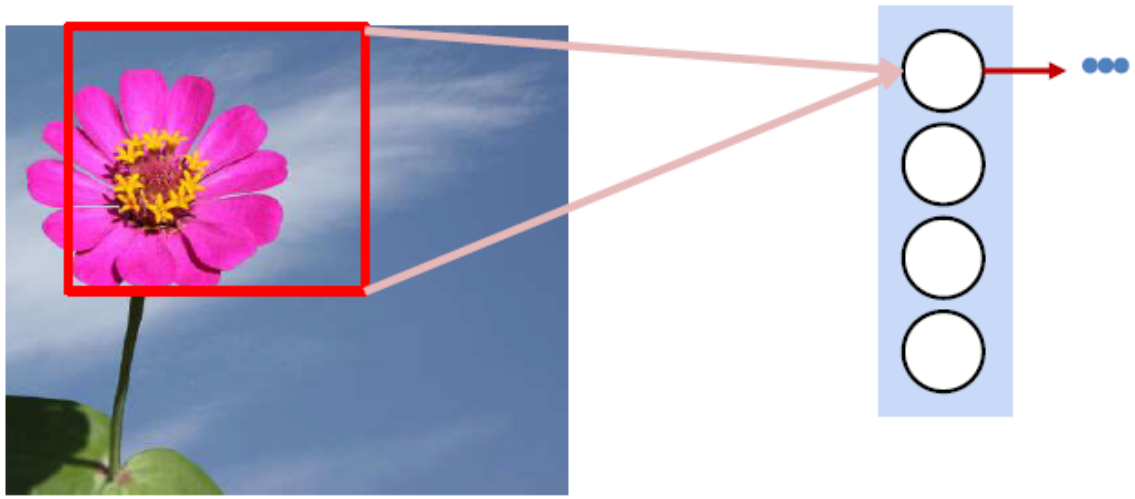

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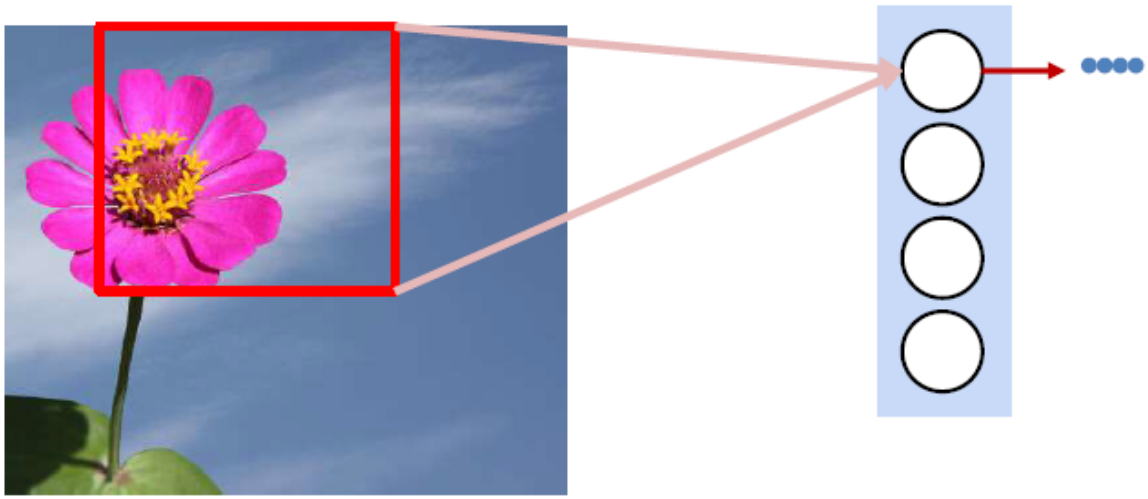

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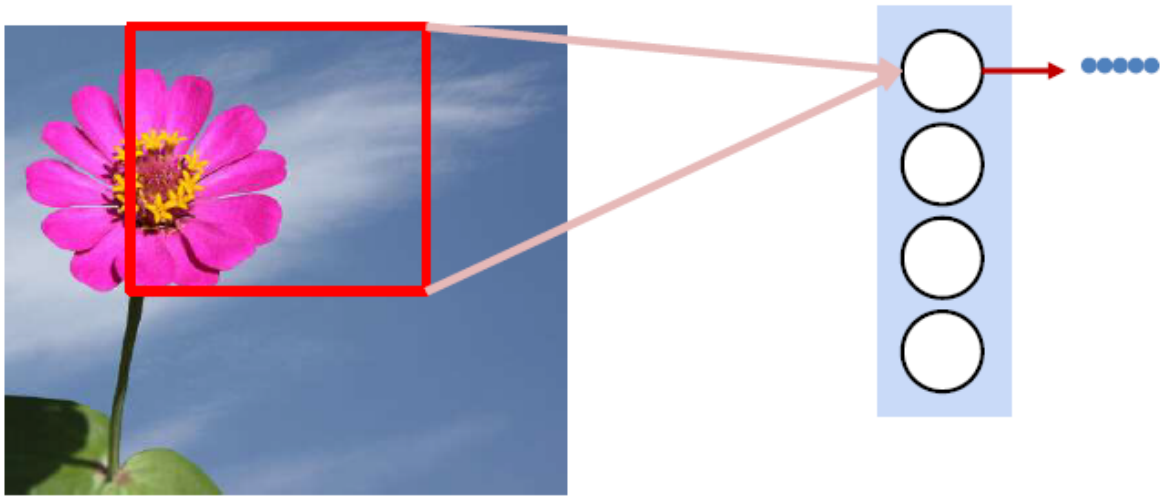

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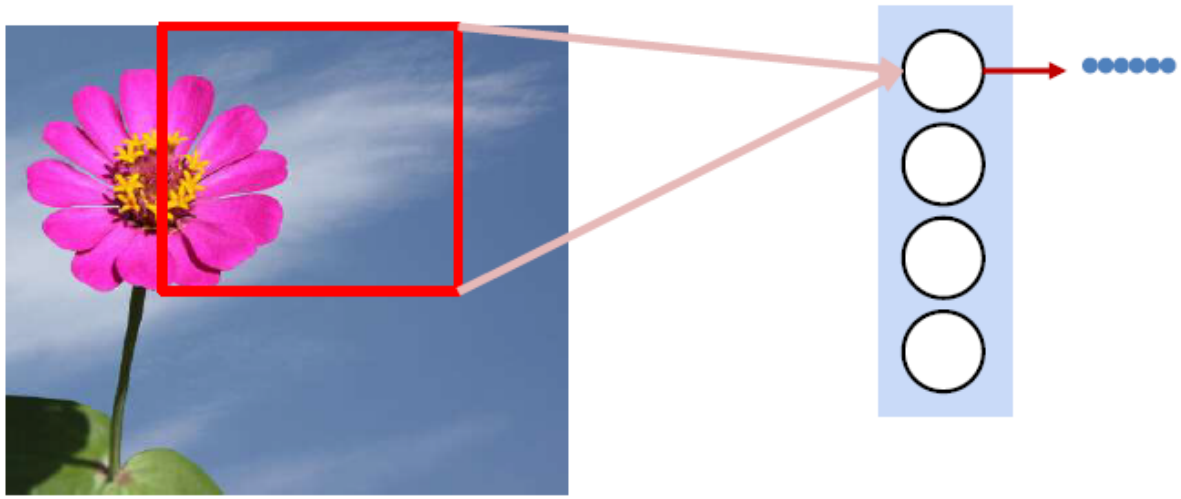

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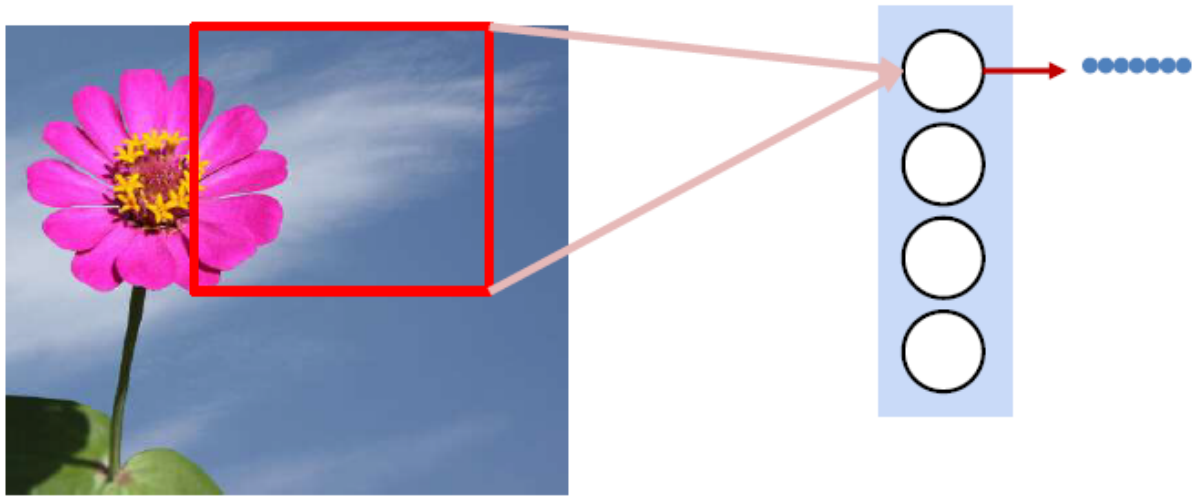

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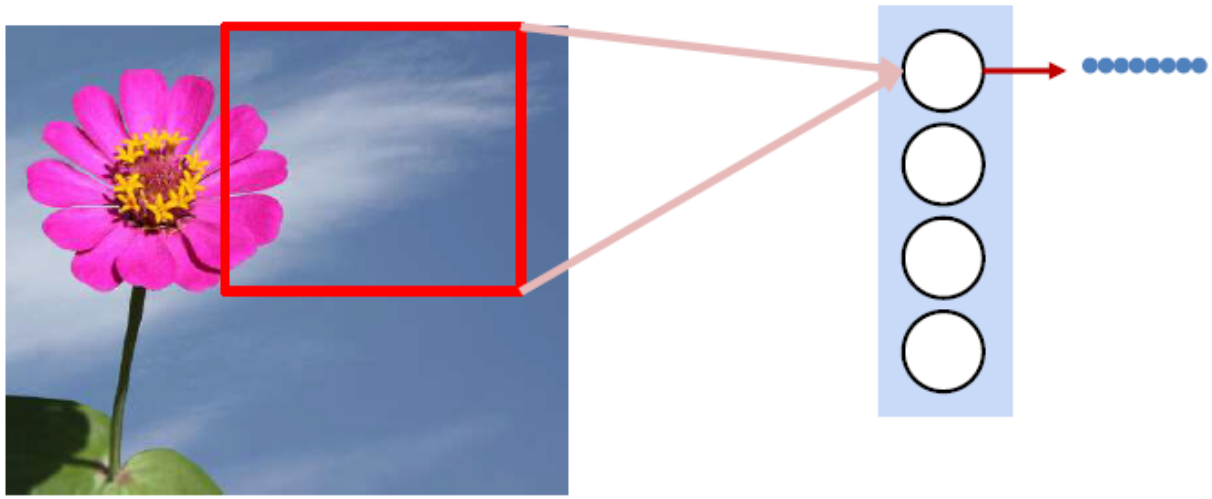

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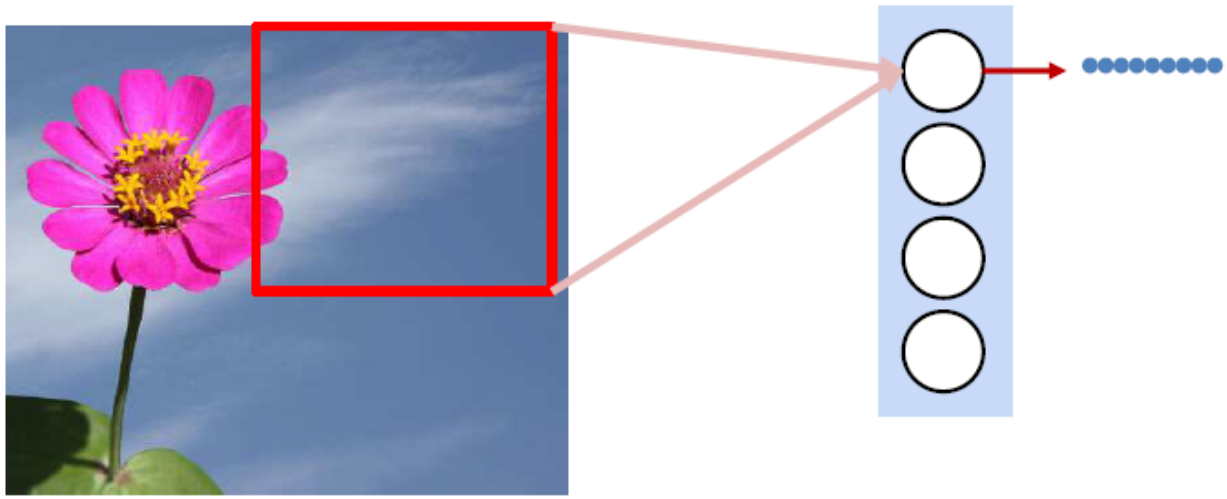

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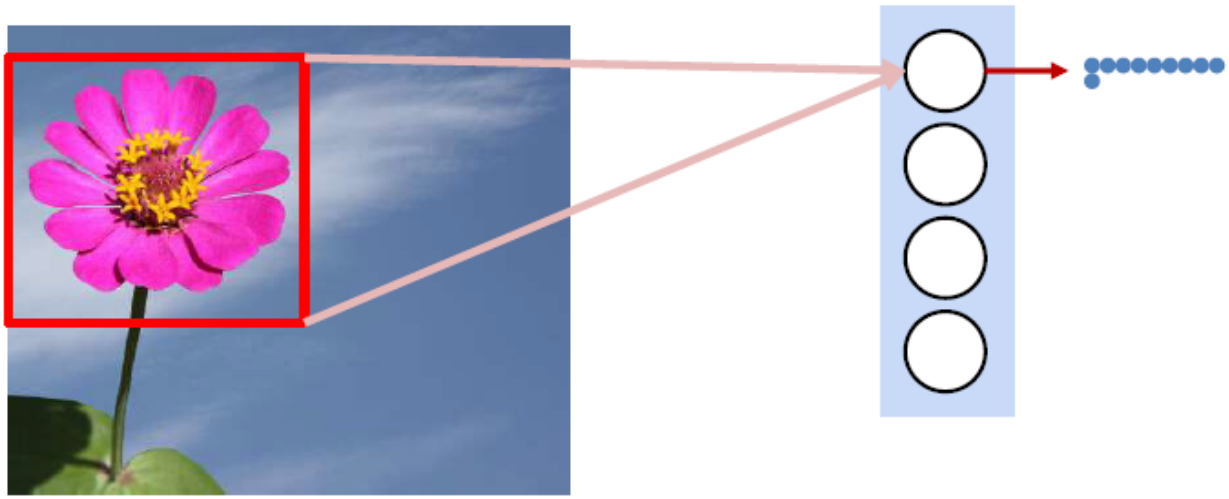

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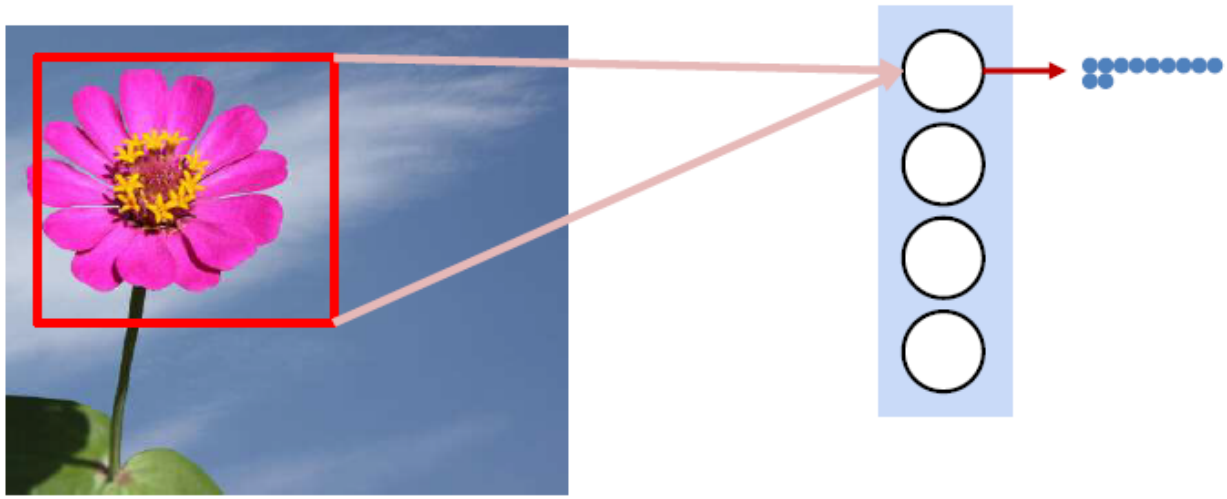

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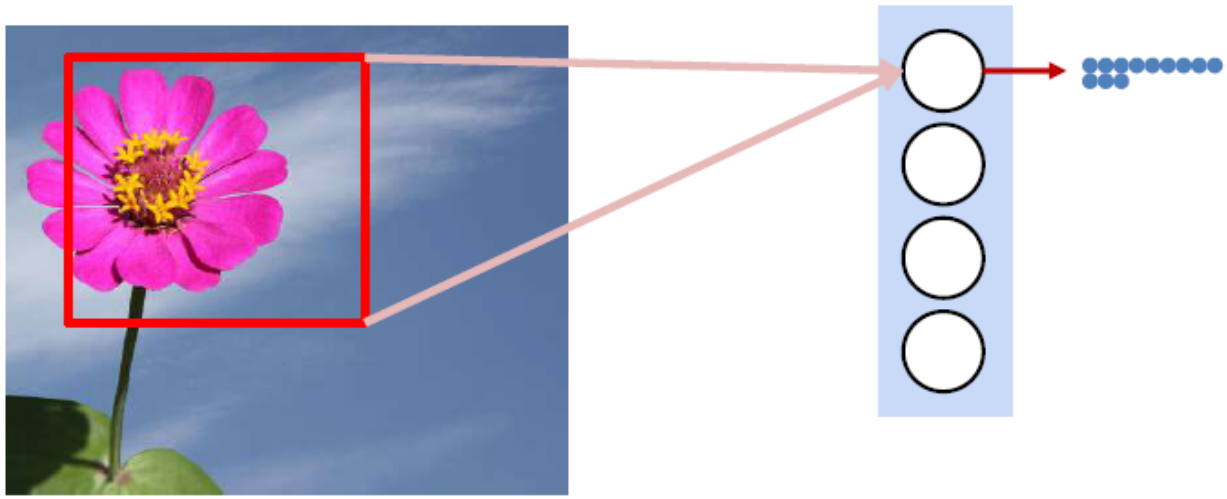

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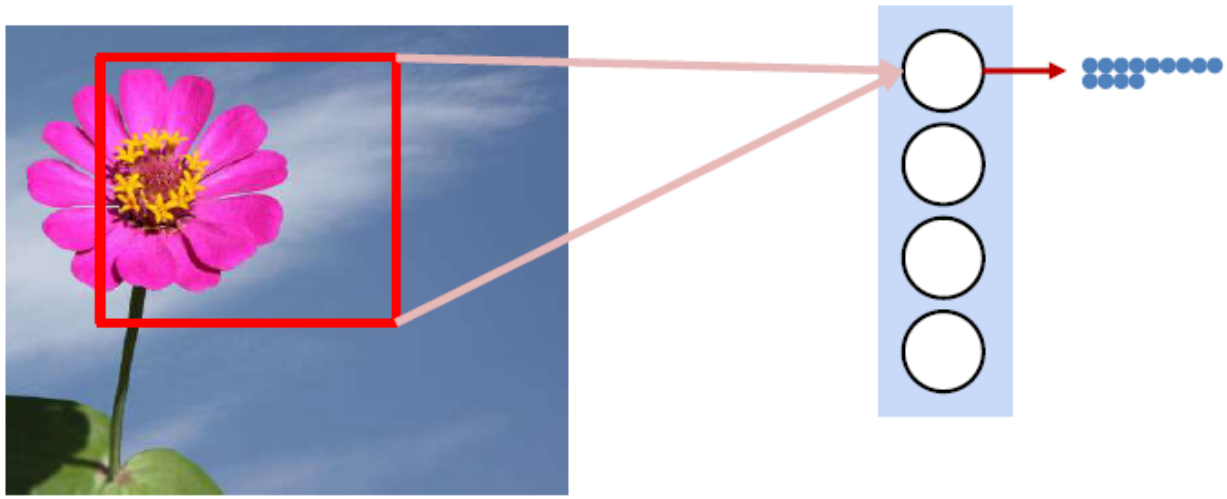

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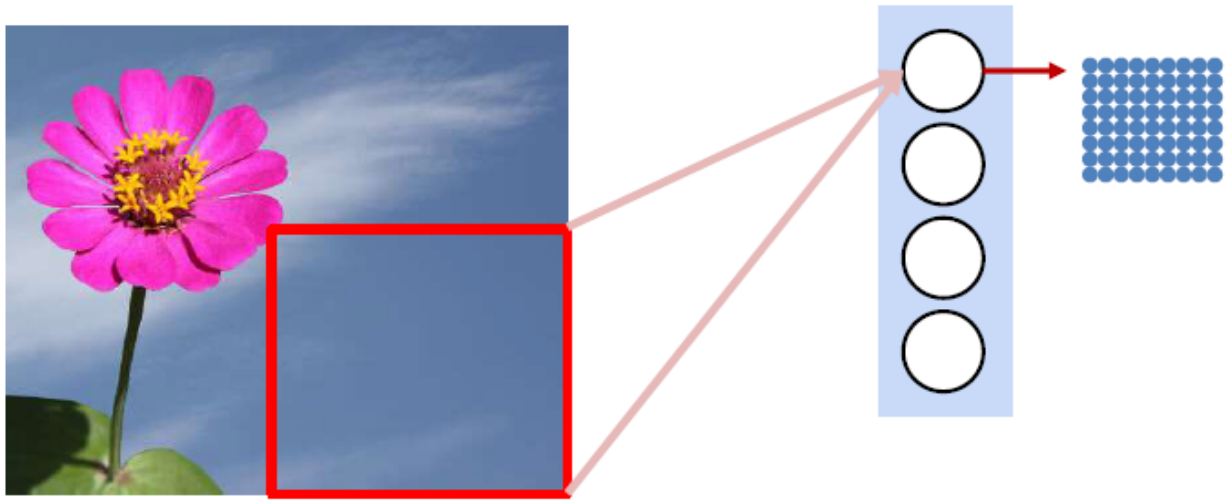
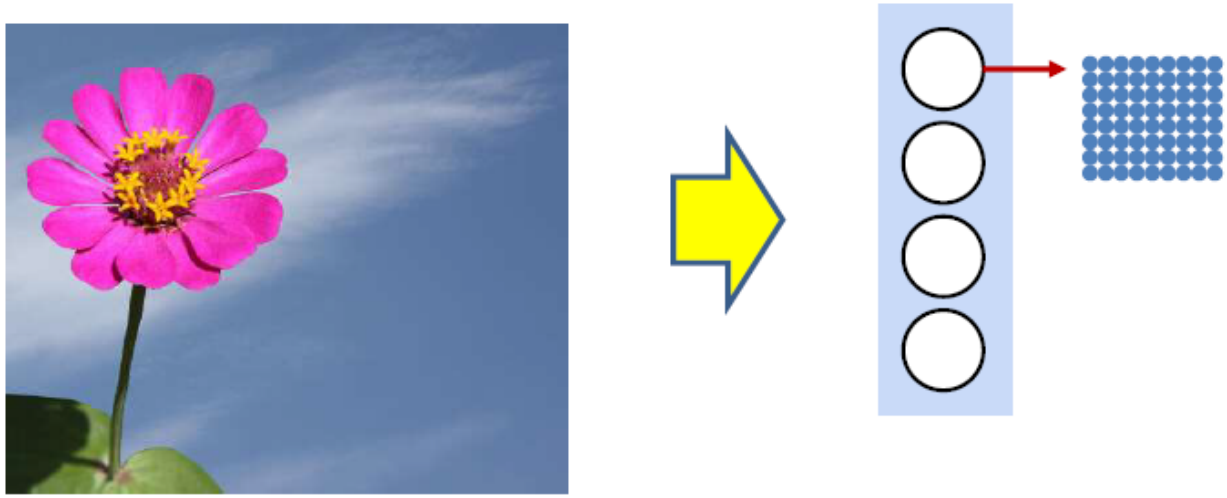


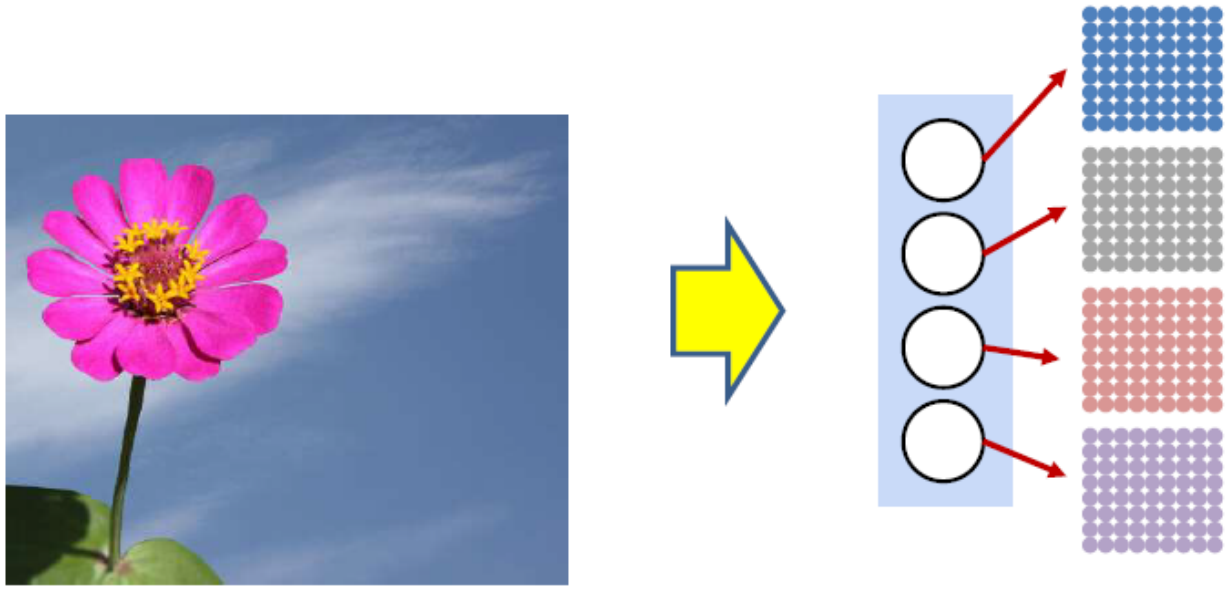
Scanning in 2D: A closer look

- Let us compute the output of the first neuron for all the windows in the picture before computing the rest of the neurons
- Eventually, we can arrange the outputs from the response at the scanned positions into a rectangle that's proportional in size to the original picture



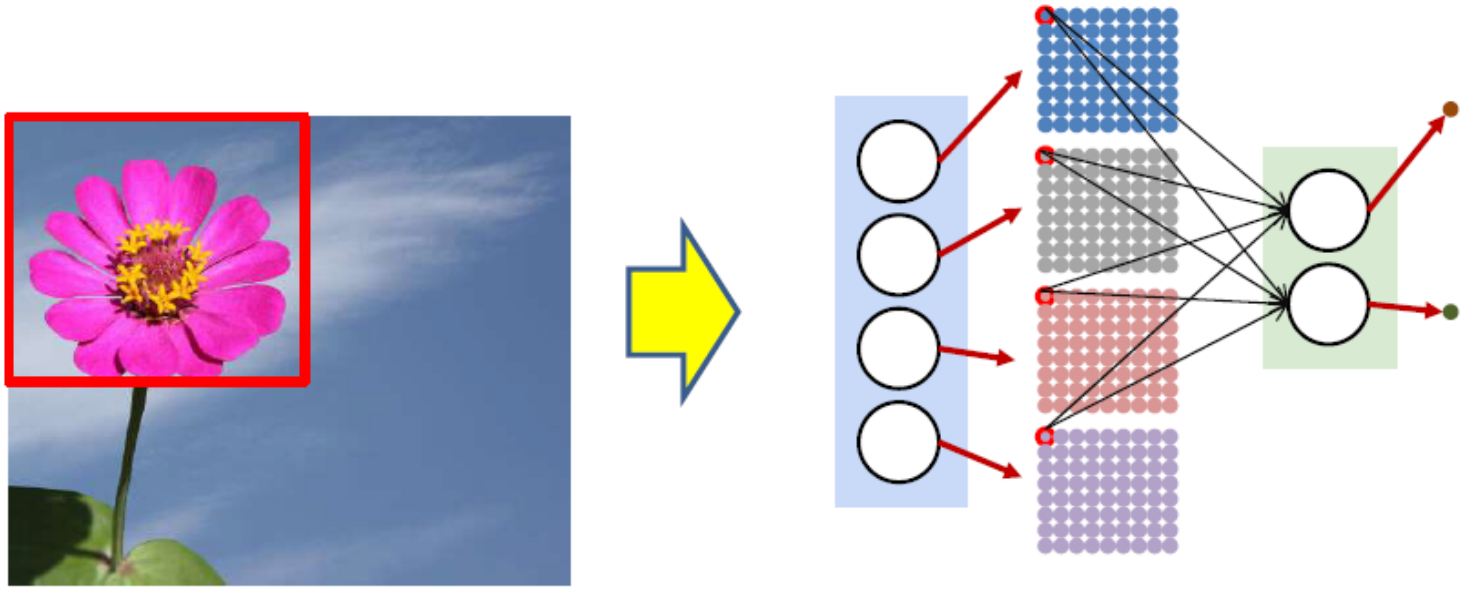
Scanning in 2D: A closer look

- We can repeat the process for each of the first-layer neurons
  - "Scan" the input with the neuron
  - Arrange the neuron's outputs from the scanned positions according to their positions in the original image (**Activation Maps**)



Scanning in 2D: A closer look

- We can recurse the logic
  - The second level neurons too can “scan” the rectangular outputs of the first-level neurons before computing subsequent layers
  - (Un)like the first level, they must jointly scan multiple “**activation maps**”
    - Each location in the output of the second level neuron considers the corresponding locations from the output maps of all the first-level neurons



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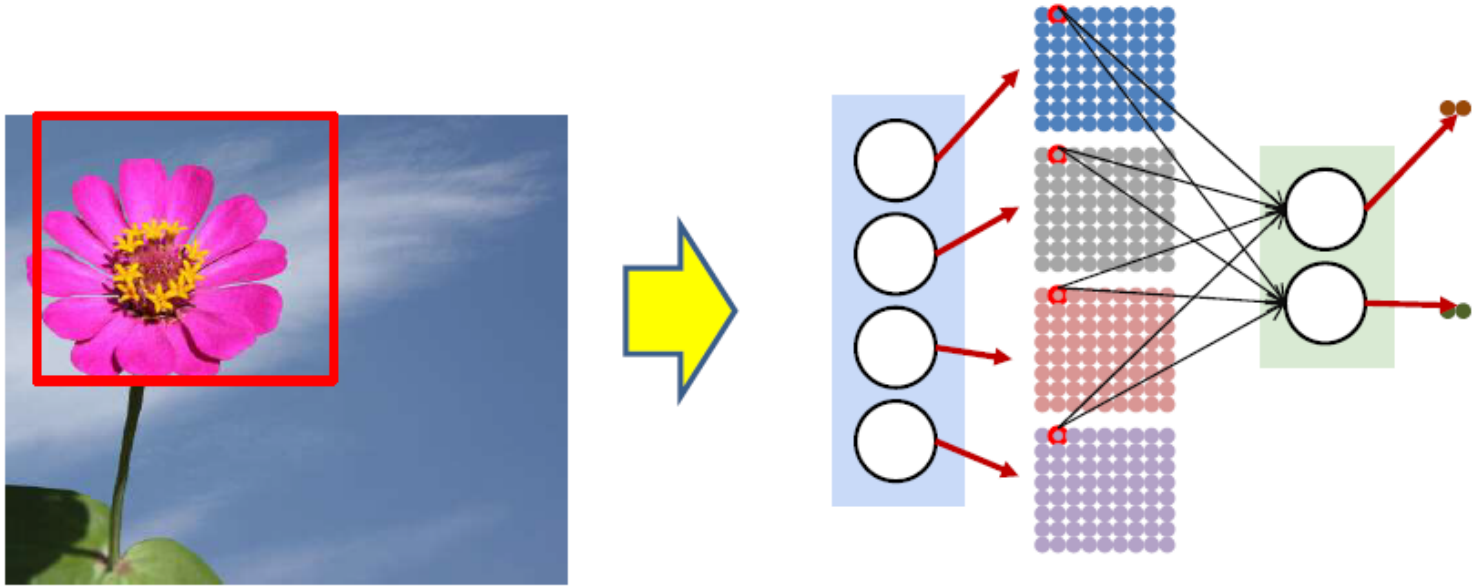

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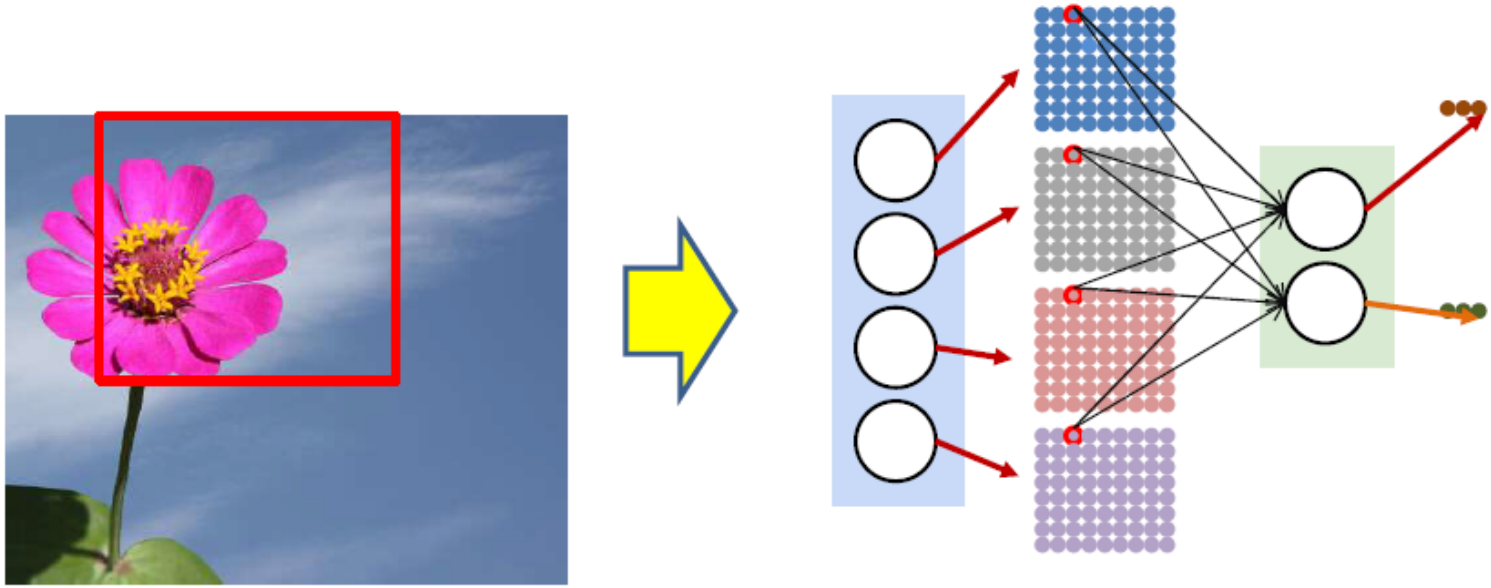

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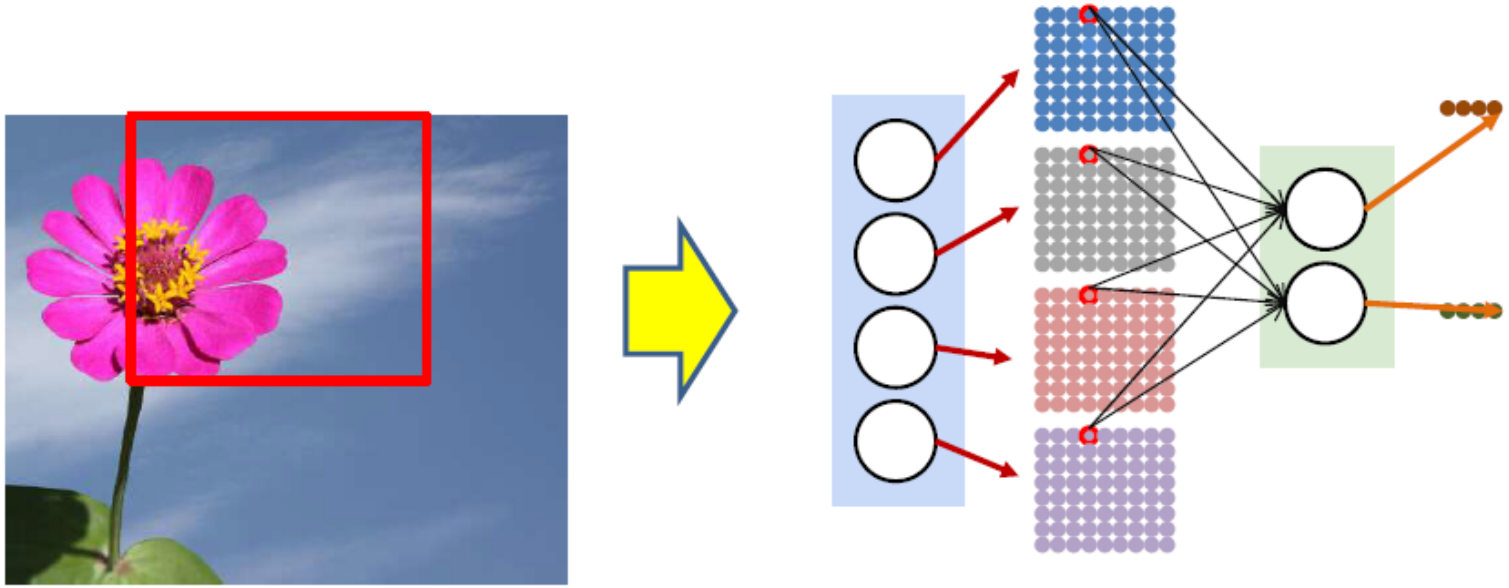

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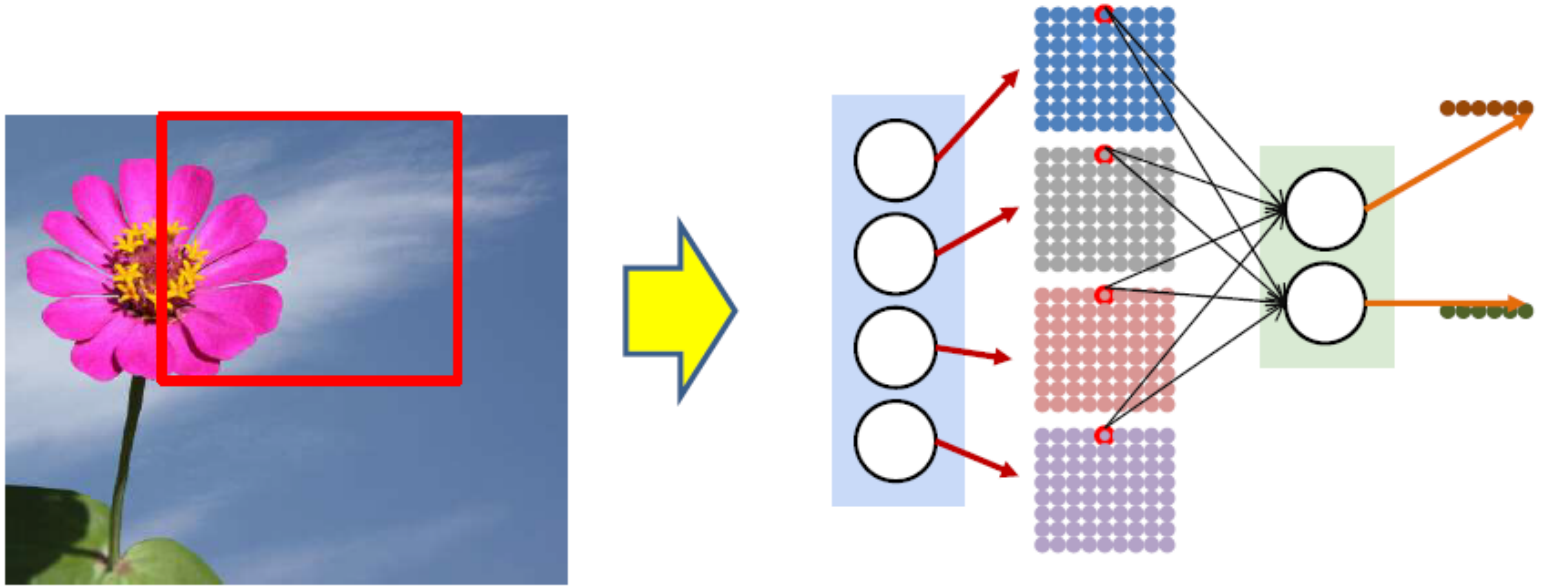

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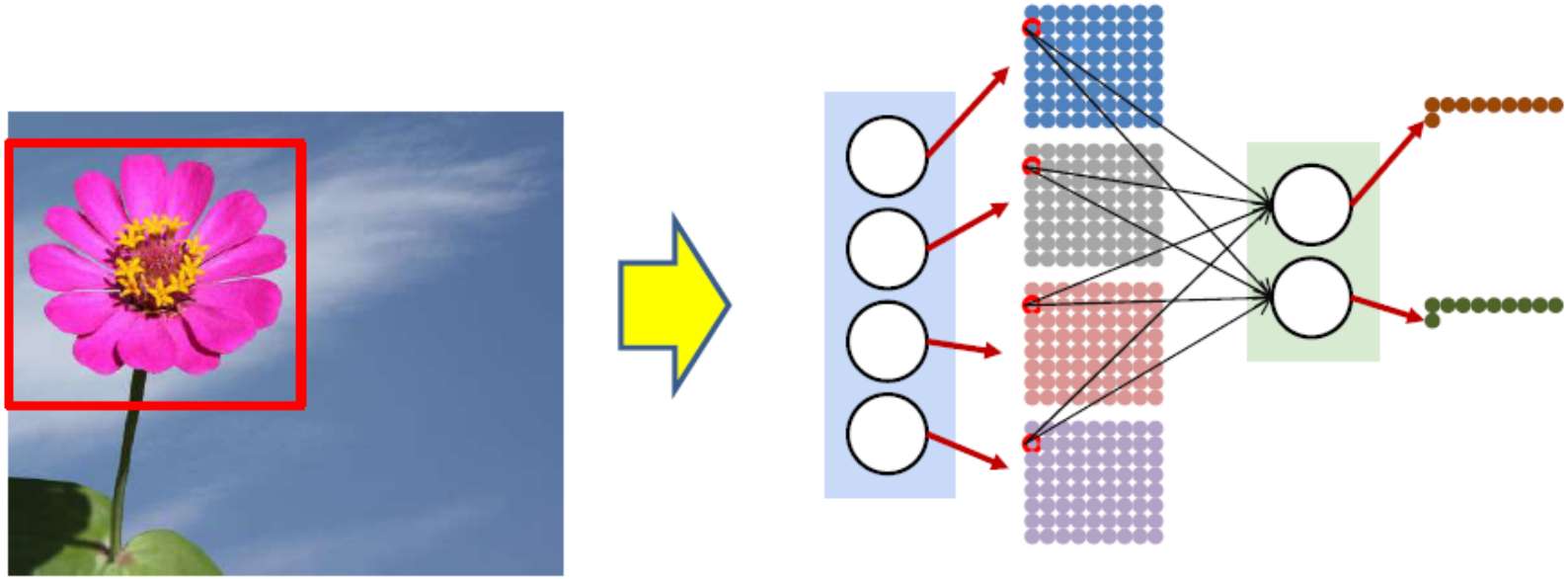

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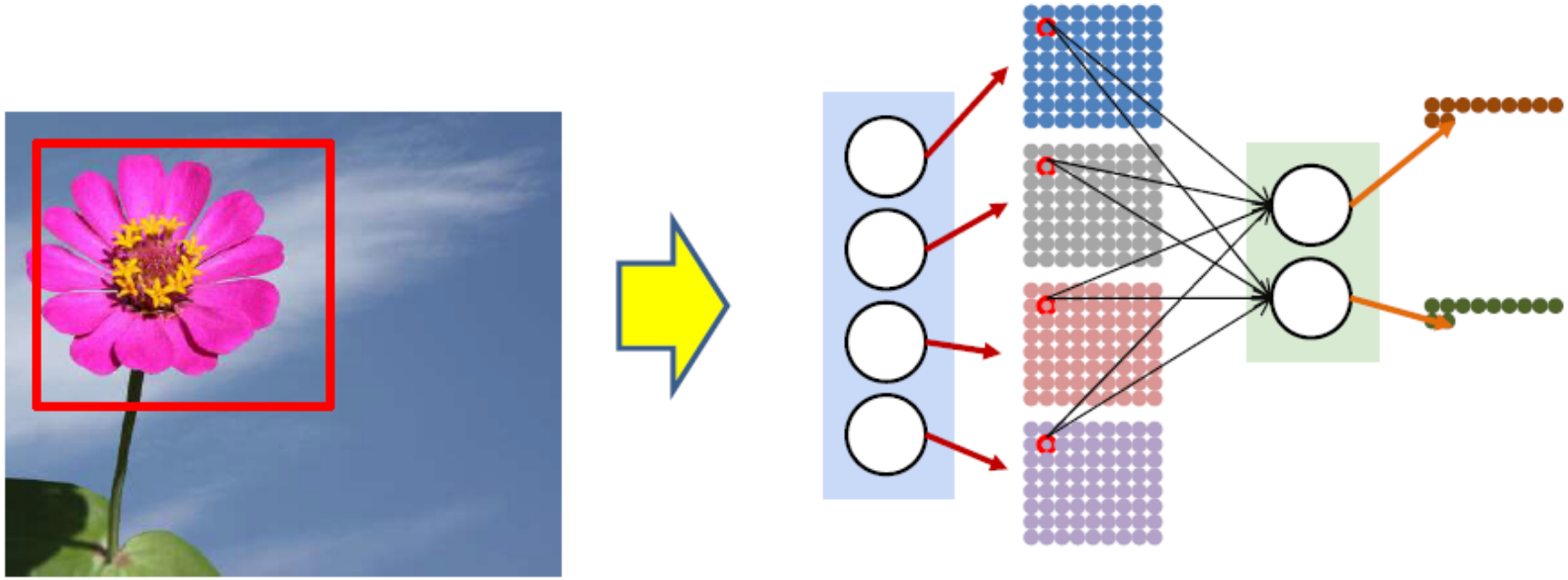

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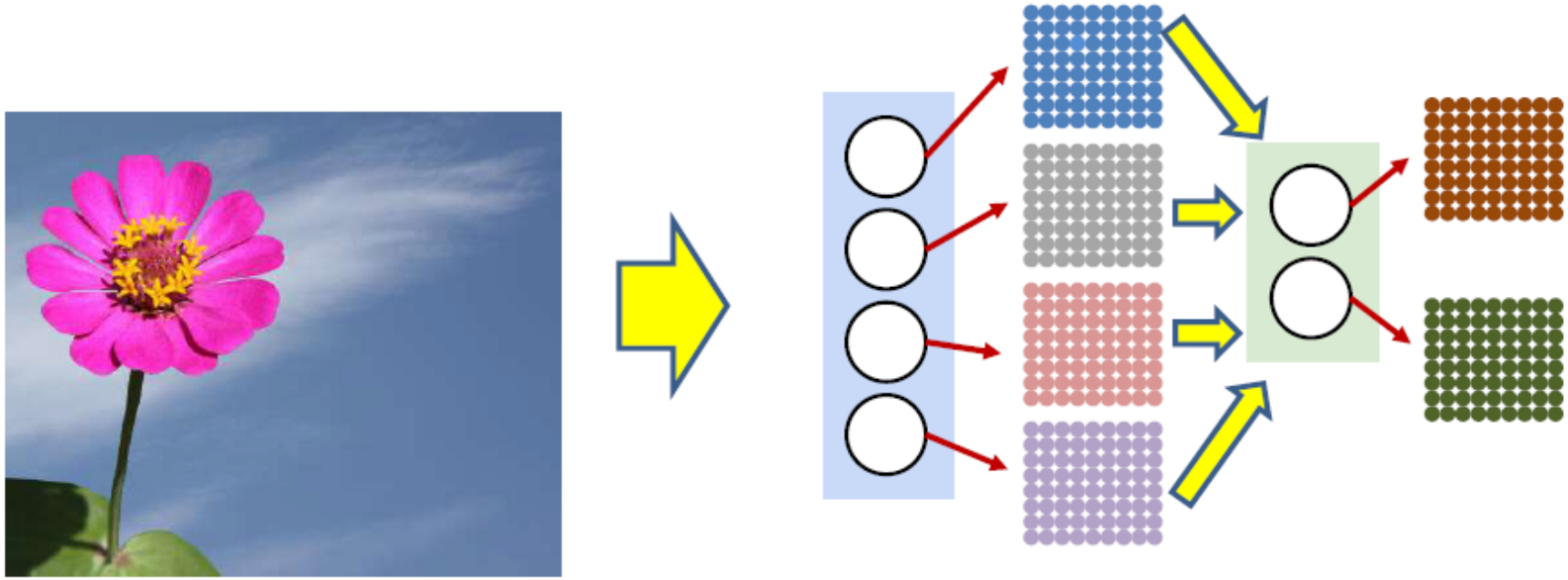

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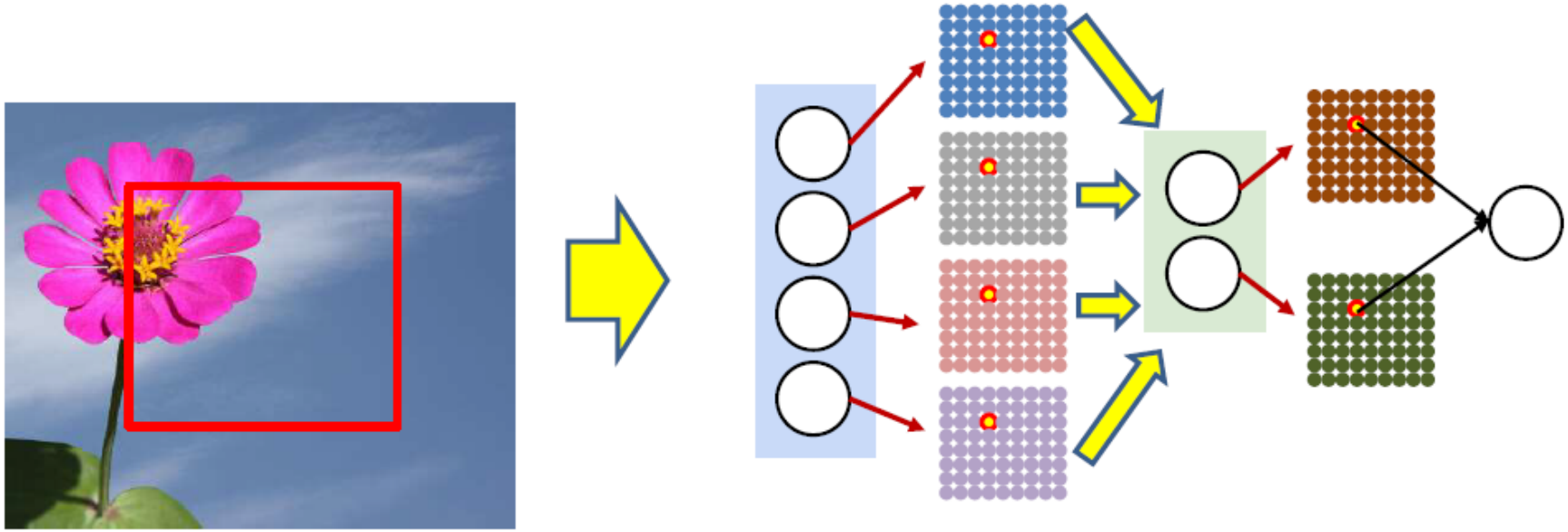
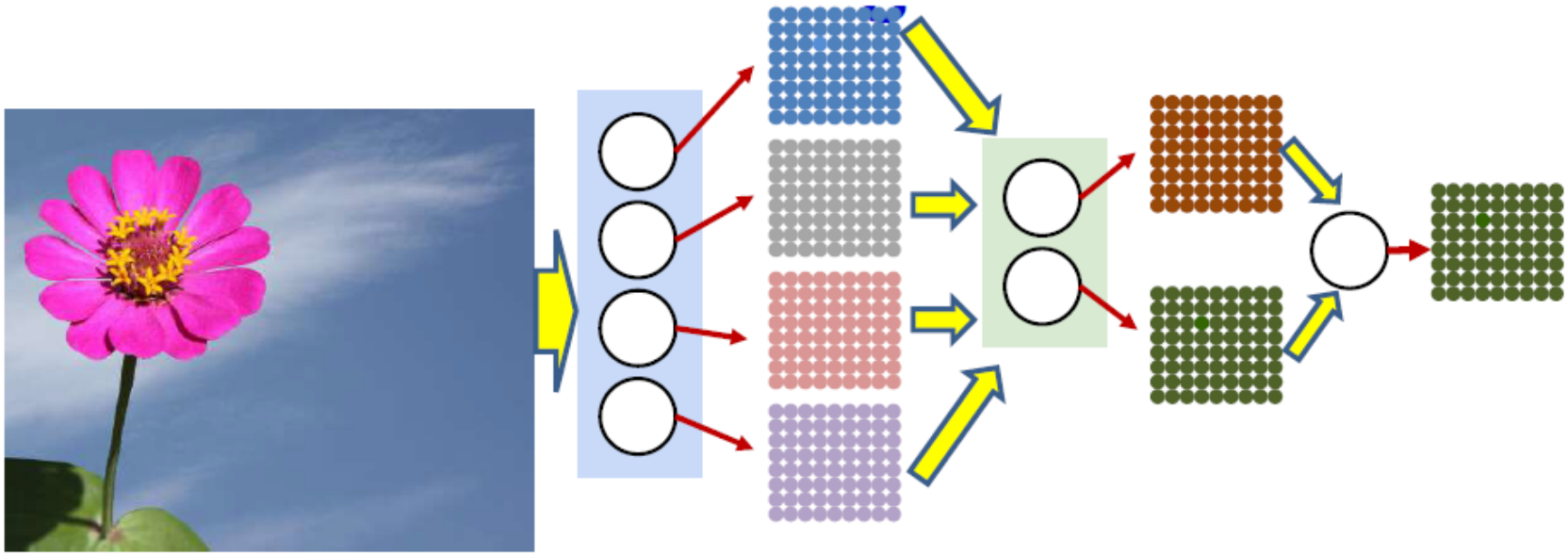


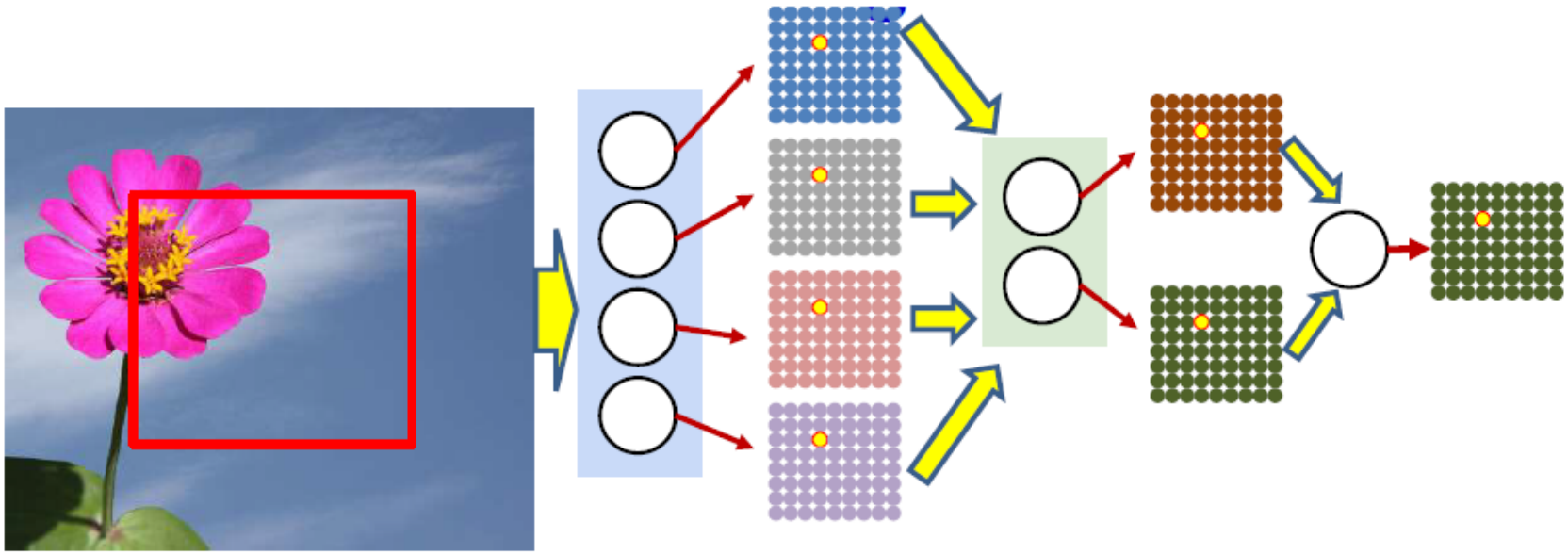
Detecting a picture anywhere in the image

- Recursing the logic, we can create a map for the neurons in the next layer as well
  - The map is a flower detector for each location of the original image



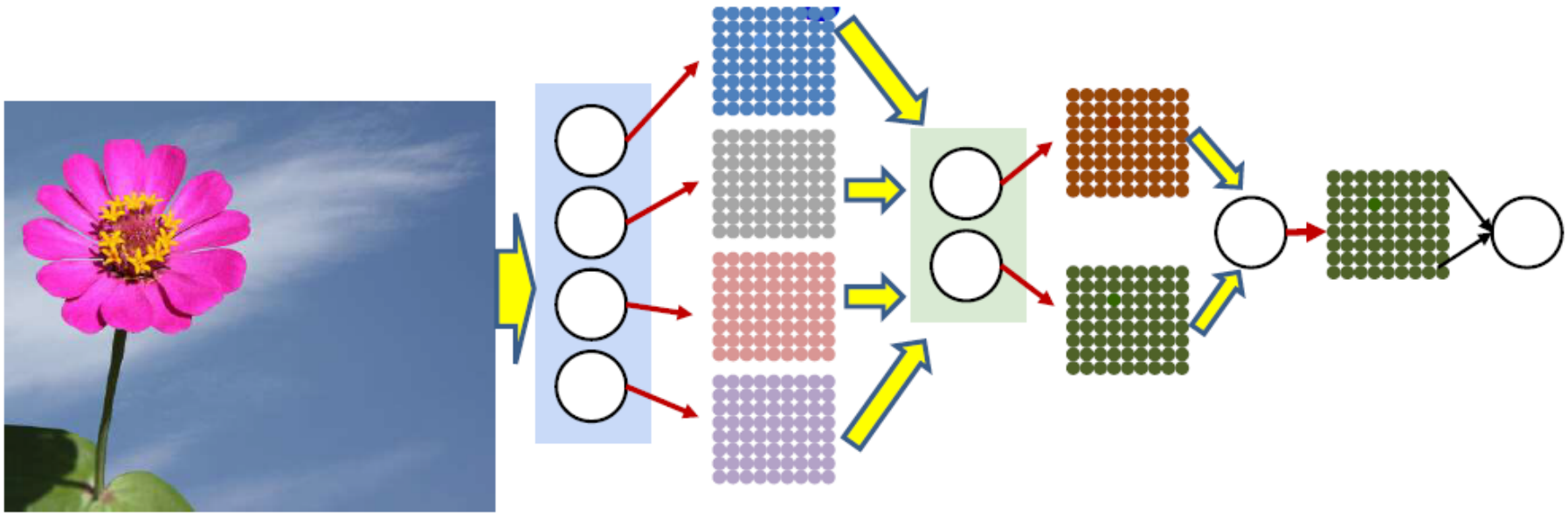
Detecting a picture anywhere in the image

- To detect a picture at any location in the original image, only need to consider the corresponding location of the output map
- Actual problem? Is there a flower in the image
  - Not “detect the location of a flower”



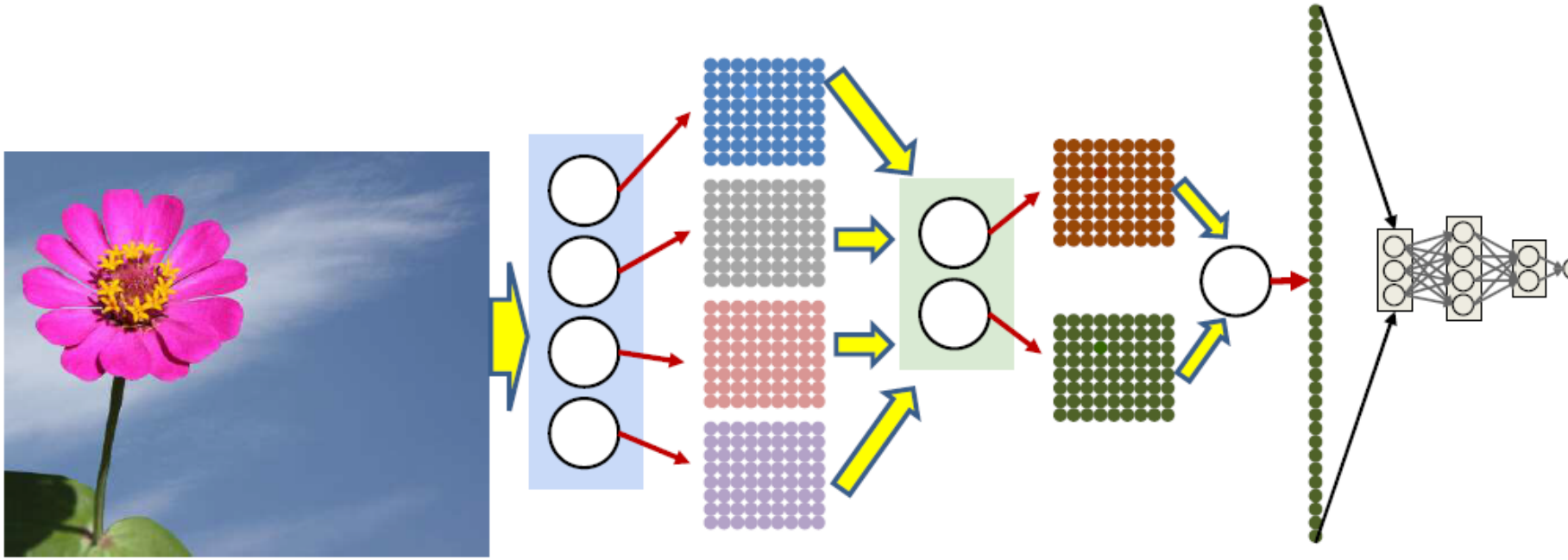
Detecting a picture anywhere in the image

- Is there a flower in the picture?
- The entire output map can be sent into a final **max** to detect a flower in the full picture
  - Or a softmax, or a full MLP. . .



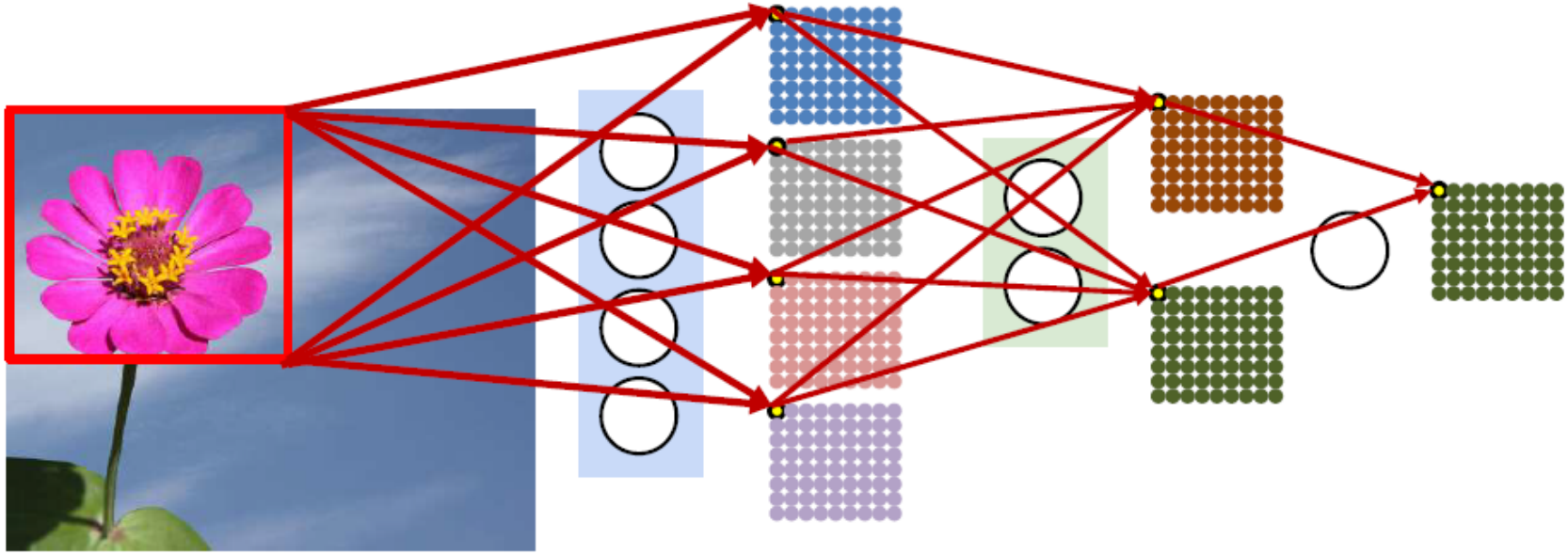
Detecting a picture anywhere in the image

- Redrawing the final layer
  - "Flatten" the output of the neurons into a single block, since the arrangement is no longer important
  - Pass that through a **max/softmax/MLP**



The behavior of the layers

- The first layer neurons “look” at the entire “window” to extract window level features
  - Subsequent layers only perform classification over these window-level features
- We don’t know the size of flower!
  - **Distributing the scan**



Distributing the scan

- Distribution forces localized patterns in lower layers
  - **More generalizable**
- We can distribute the pattern matching over two layers and still achieve the same block analysis at the second layer
  - The first layer evaluates smaller blocks of pixels
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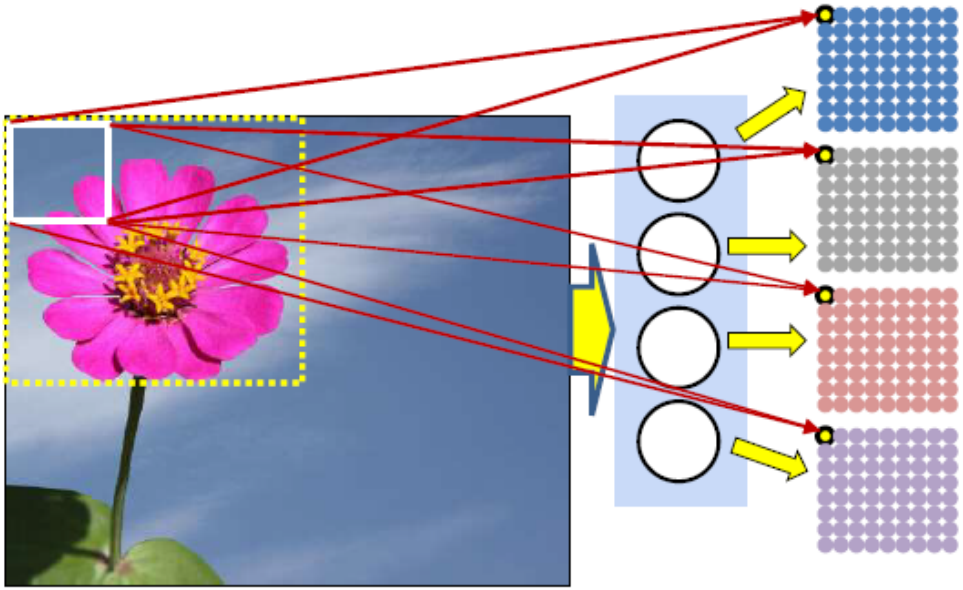
- The higher layer implicitly learns the arrangement of sub-patterns that represents the larger pattern (the flower in this case)



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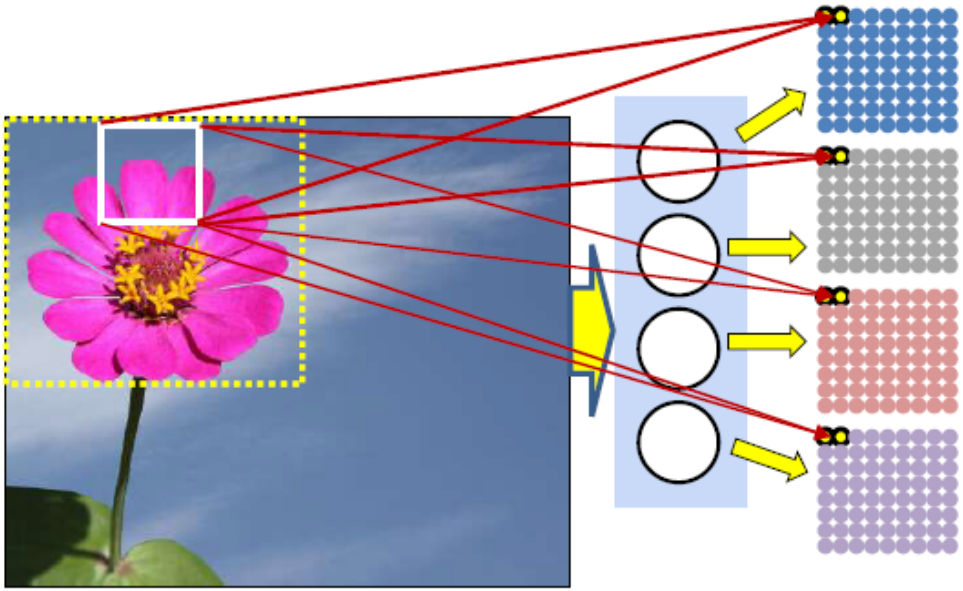

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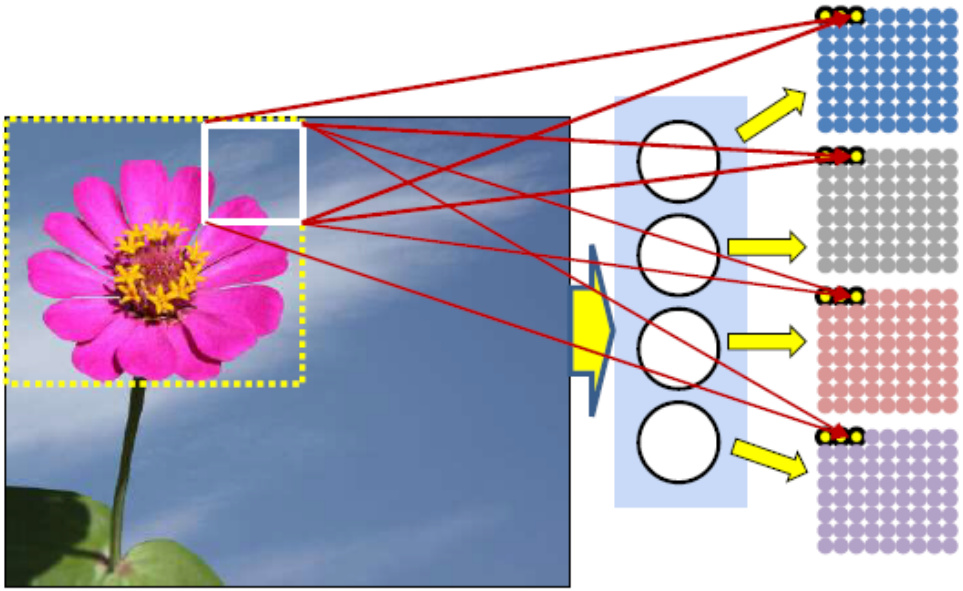

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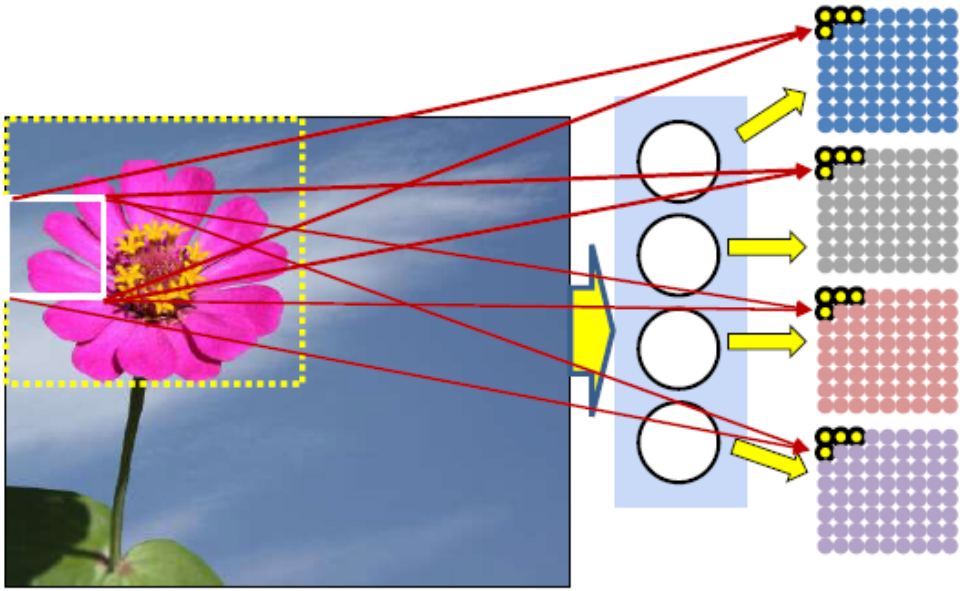

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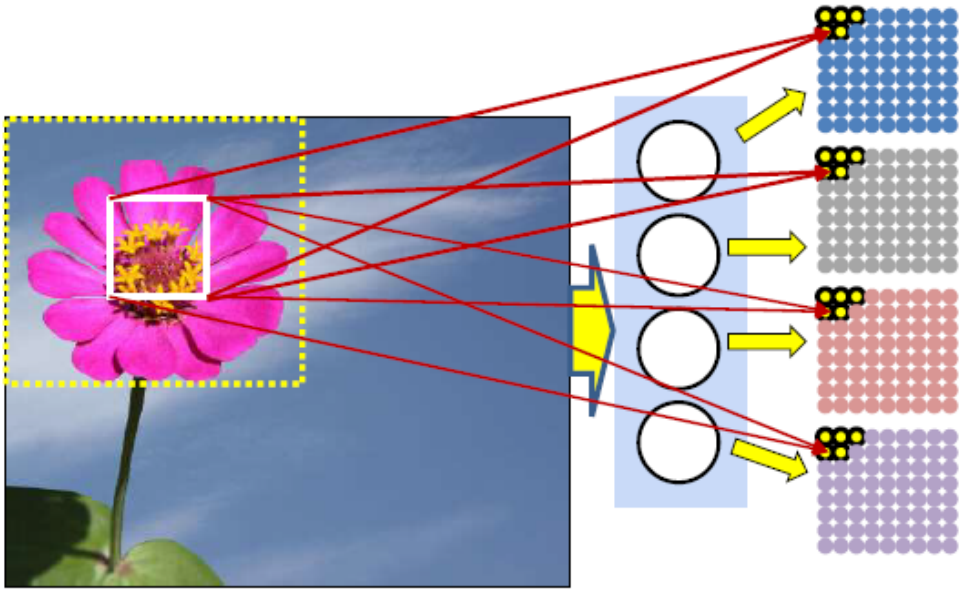

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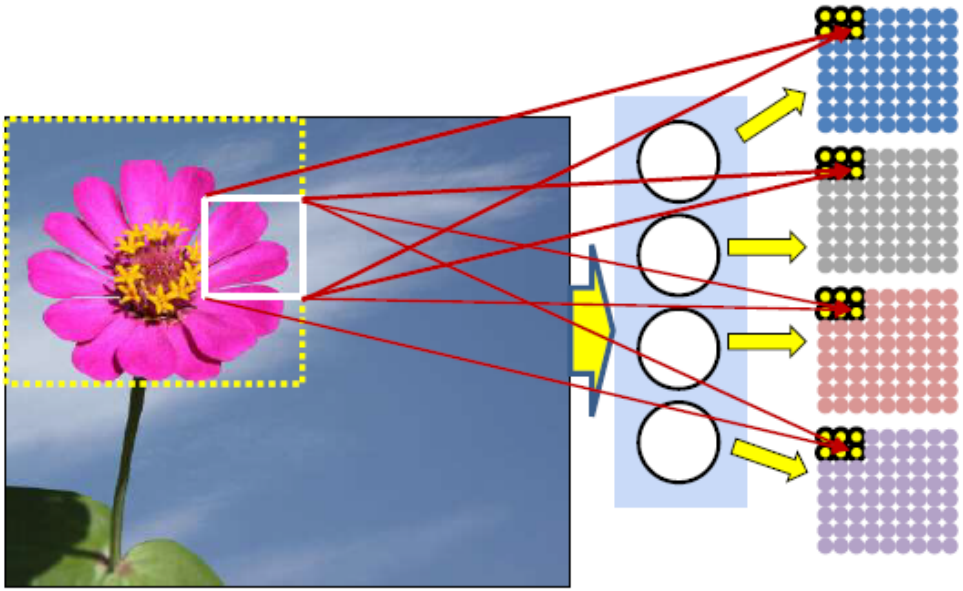

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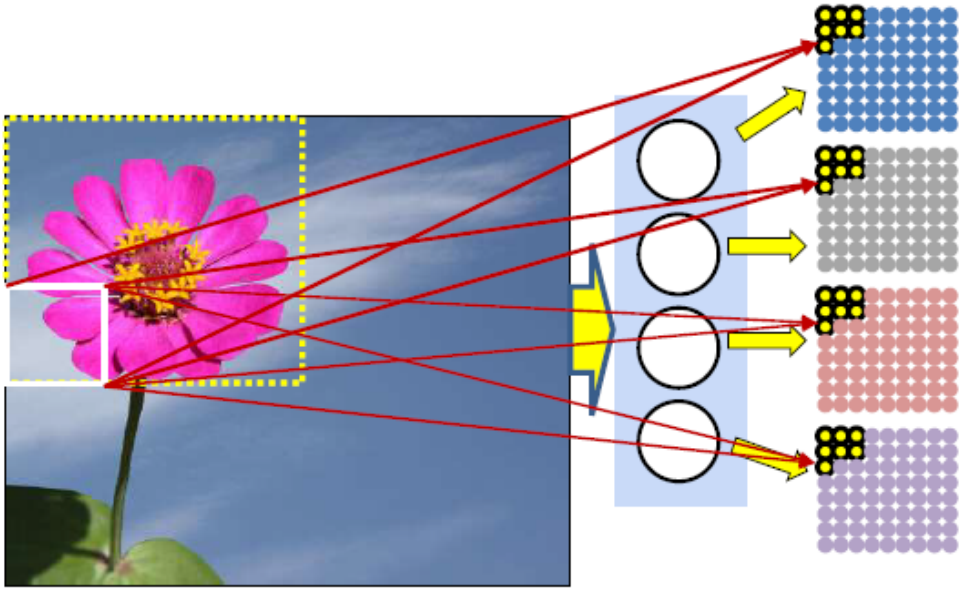

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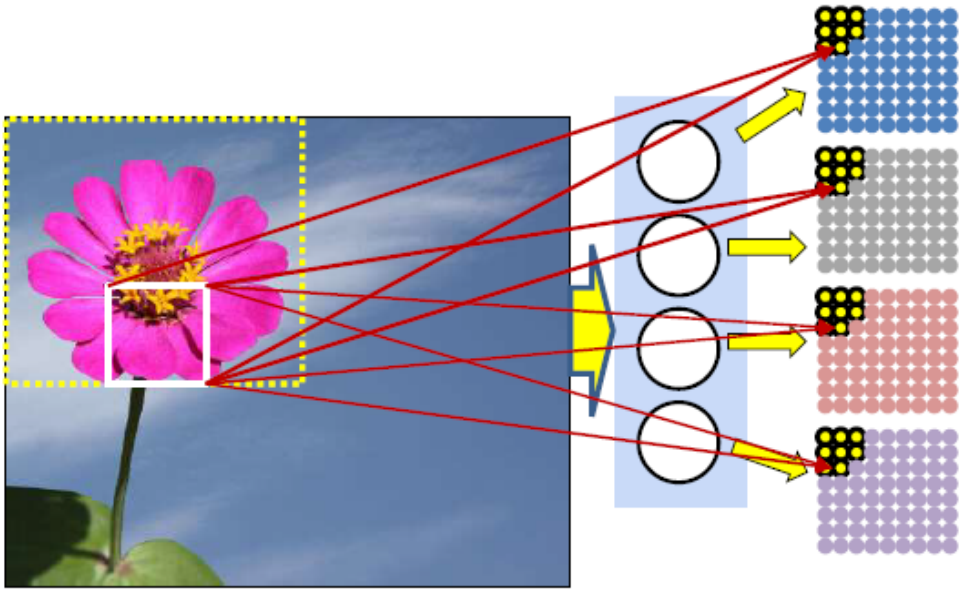

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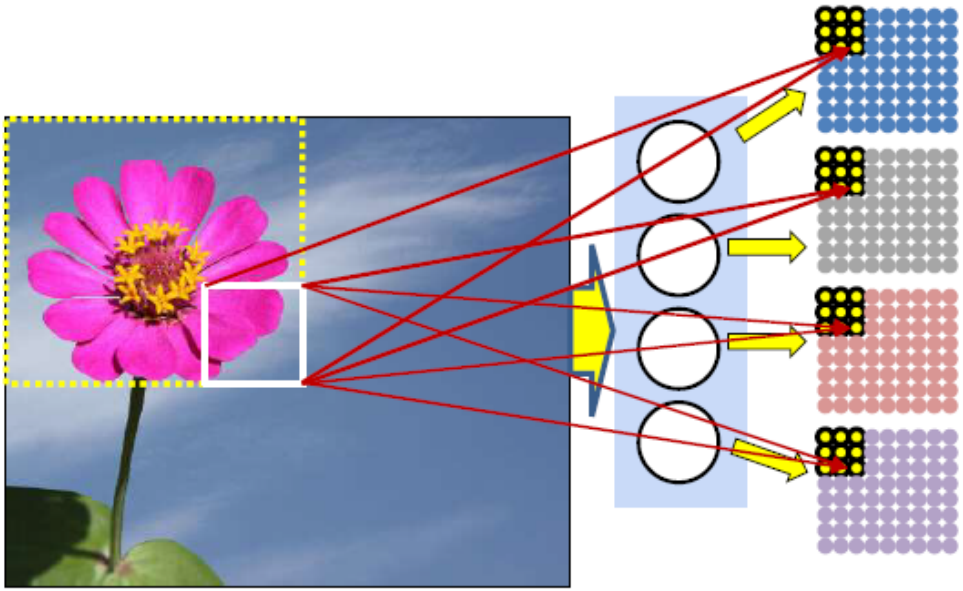

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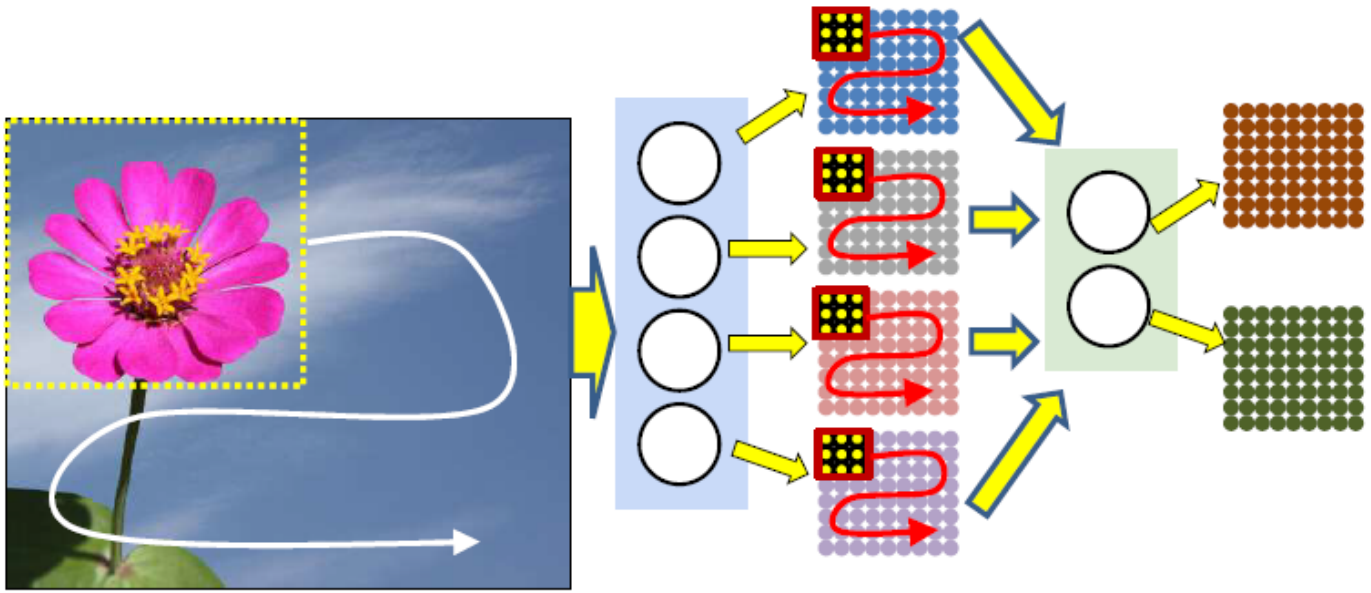

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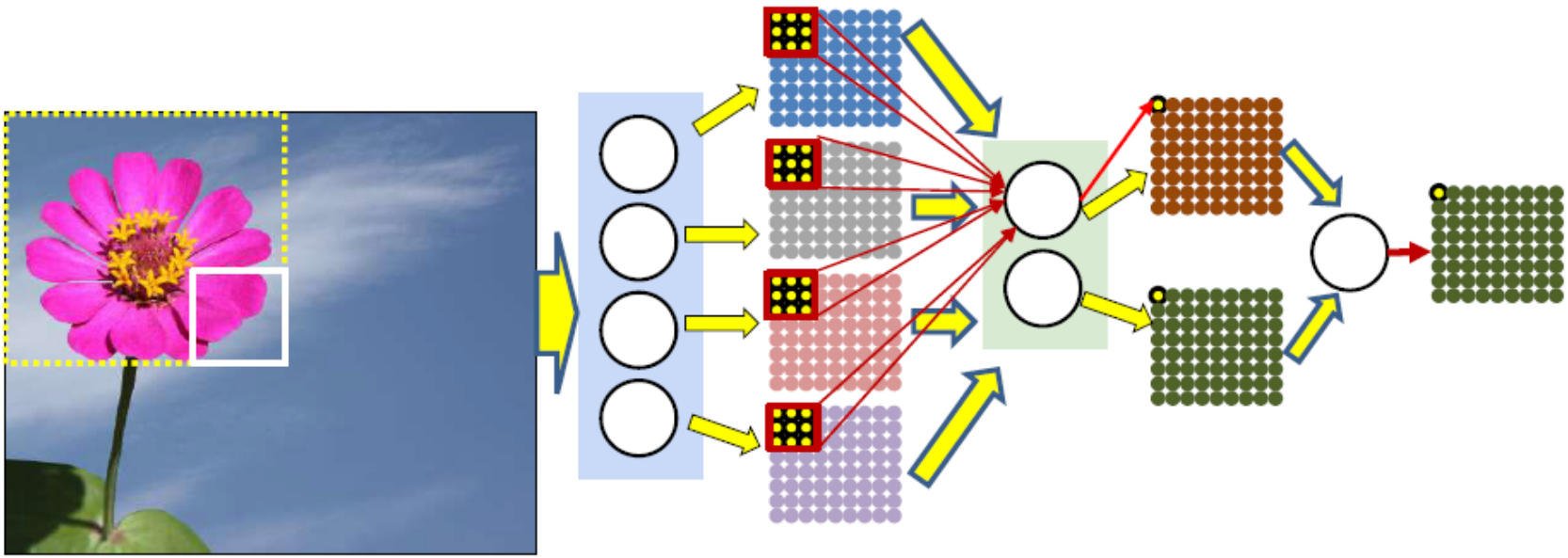

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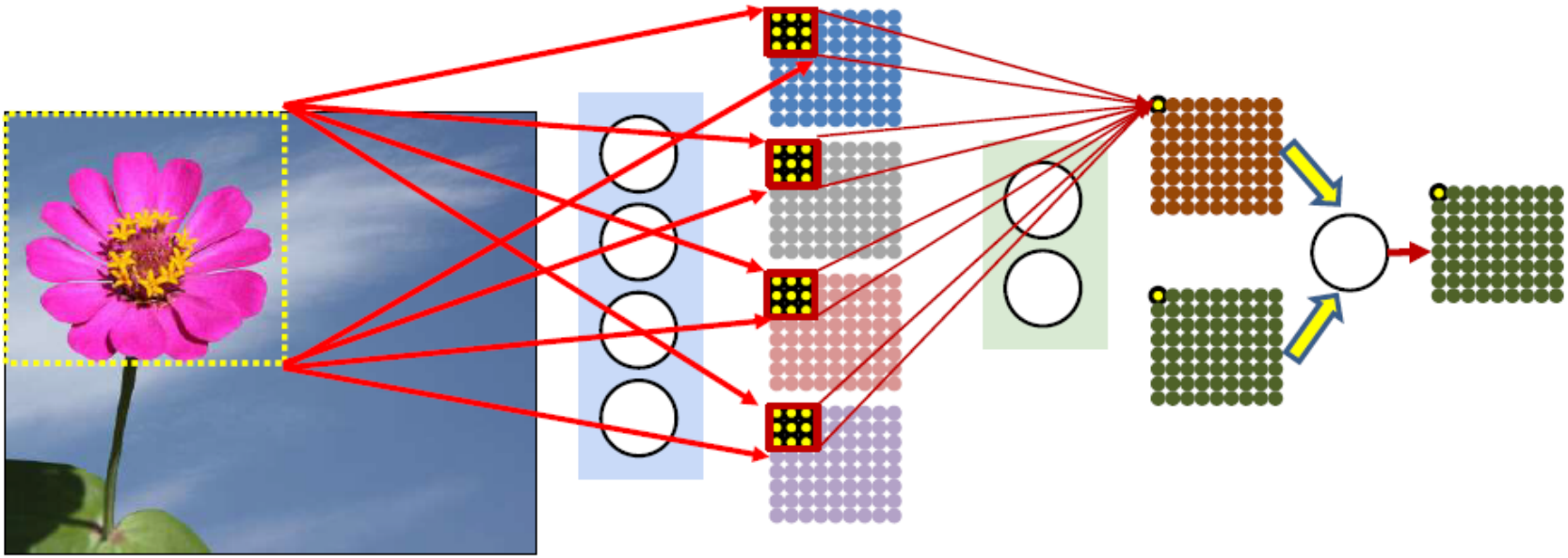

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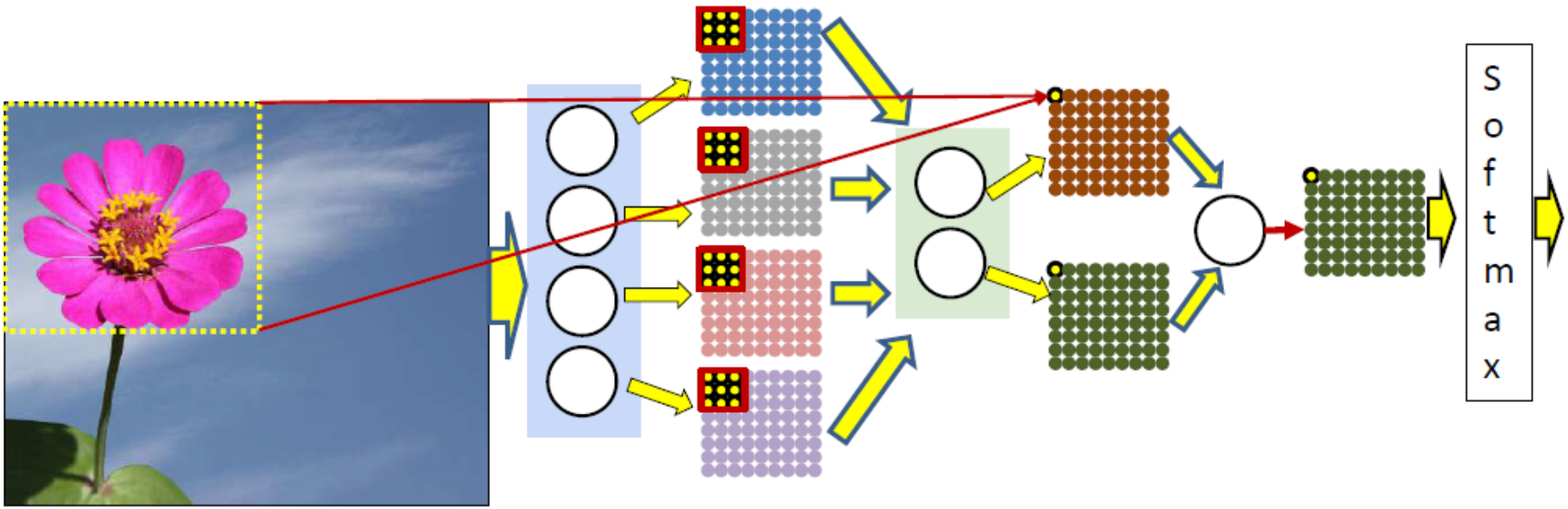
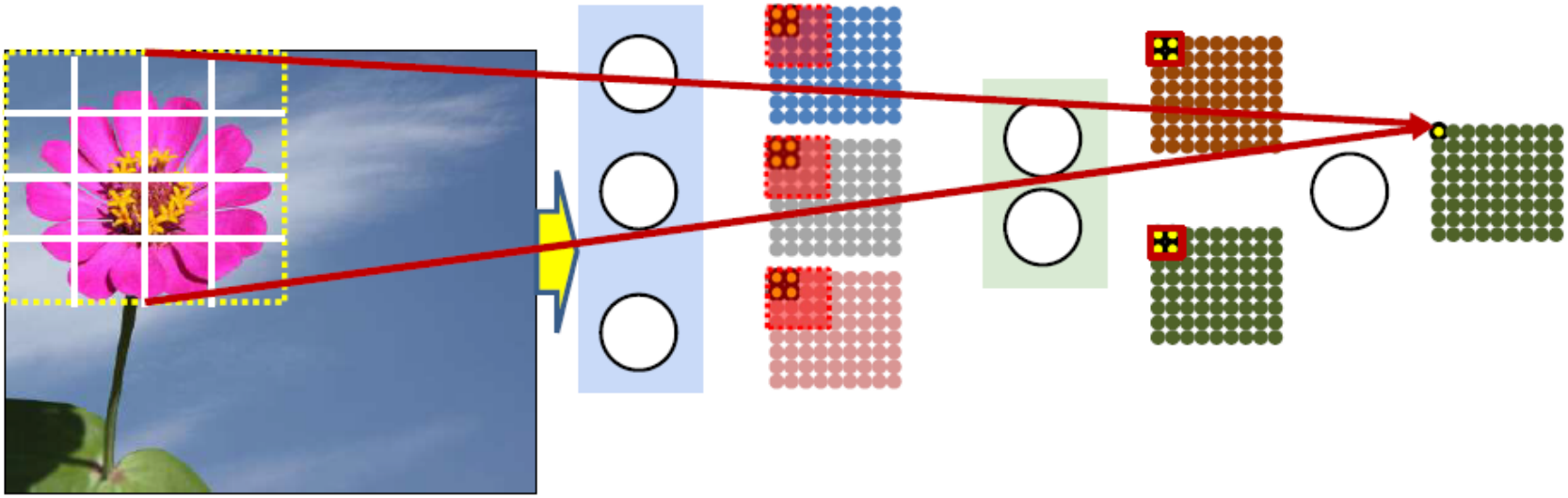


This logic can be recursed

- Building the pattern over 3 layers



What is a convolution

- Scanning an image with a “filter”
  - Note: a filter is really just a perceptron, with weights and a bias

Example 5x5 image with binary pixels

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Example 3x3 filter

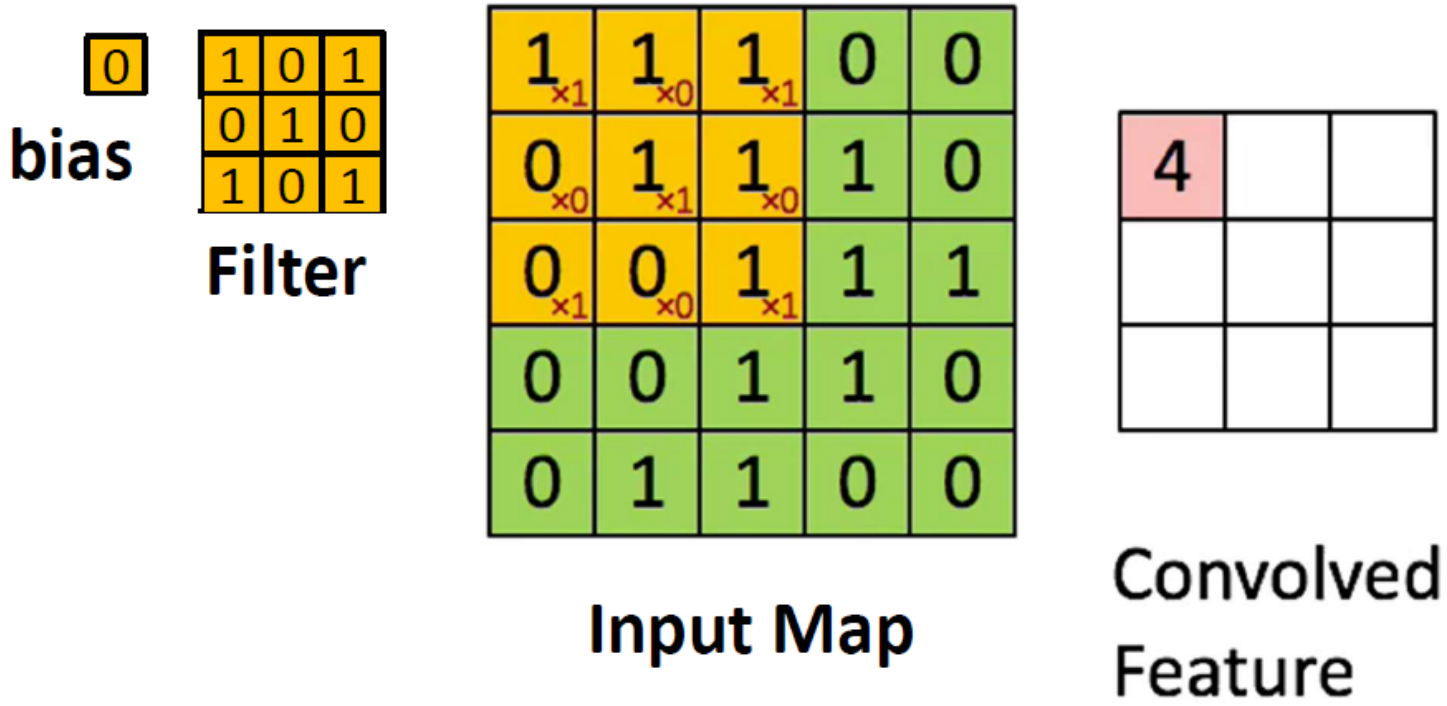
1	0	1
0	1	0
1	0	1

bias

0

What is a convolution

- Scanning an image with a “filter”
  - At each location, the filter and the underlying map values are multiplied component wise, and the products are added along with the bias



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  - E.g. with a “stride” of two pixels per shift

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0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Example 3x3 filter

1	0	1
0	1	0
1	0	1

bias

0

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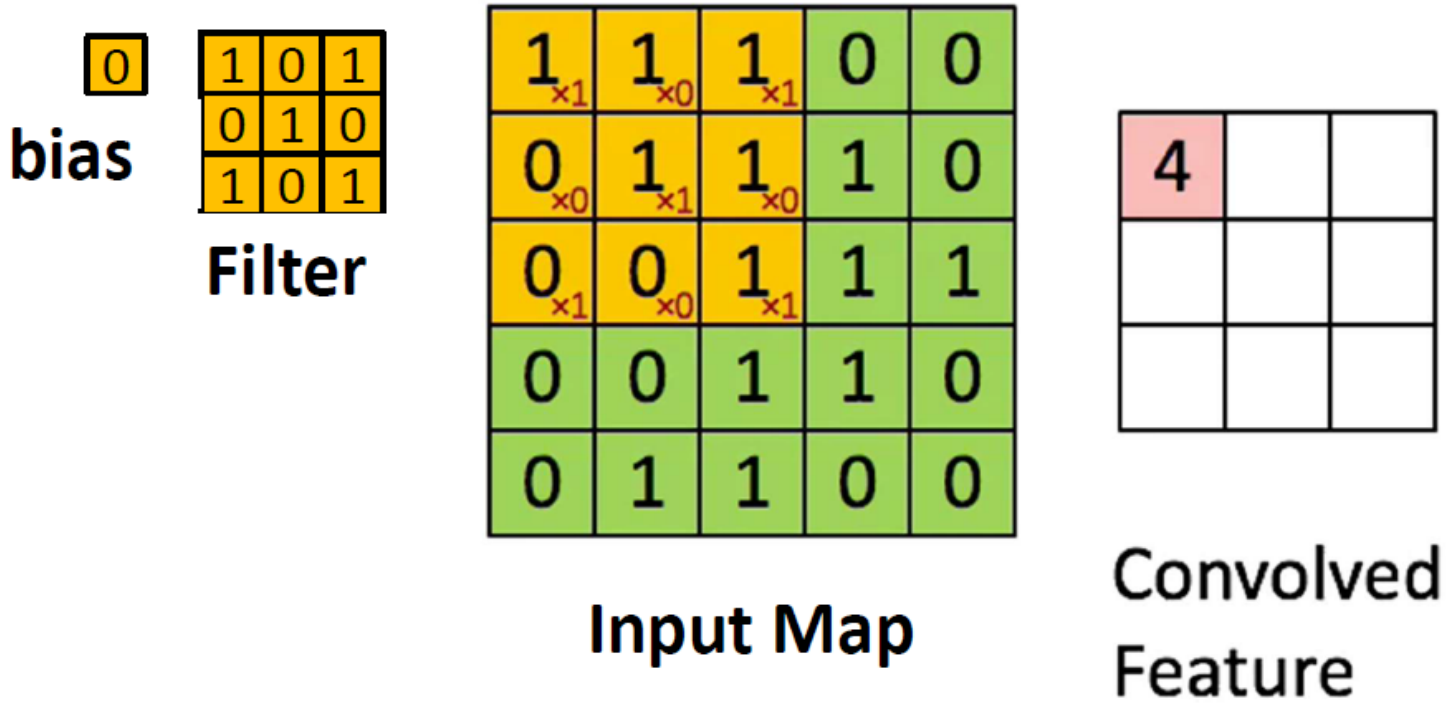

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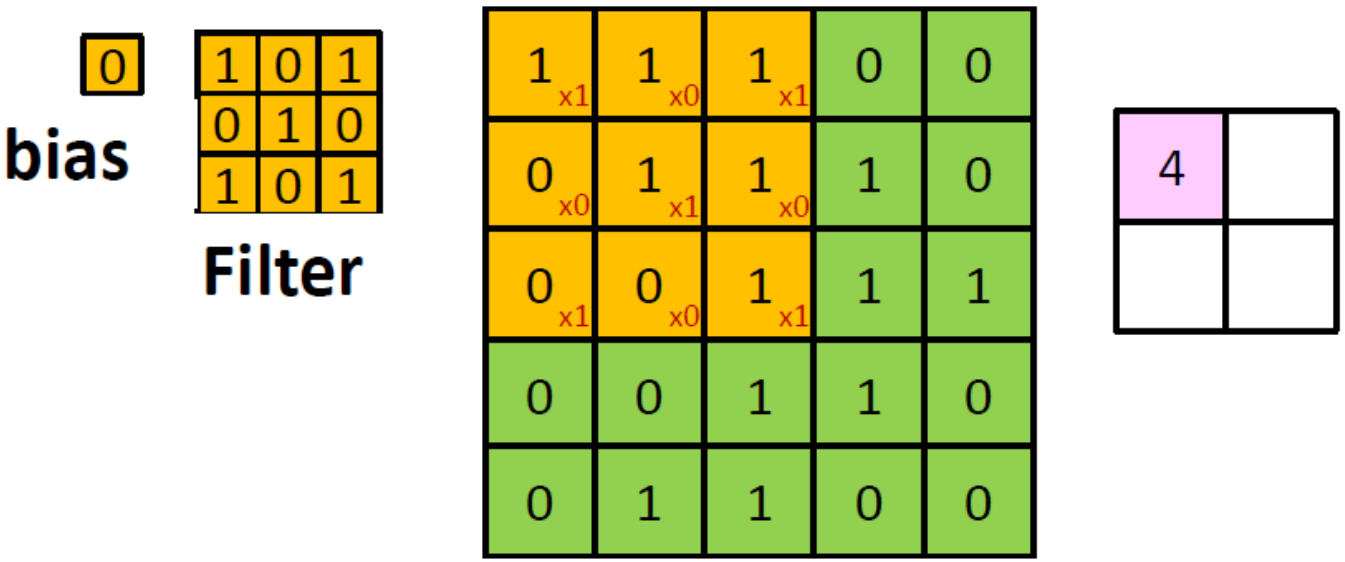

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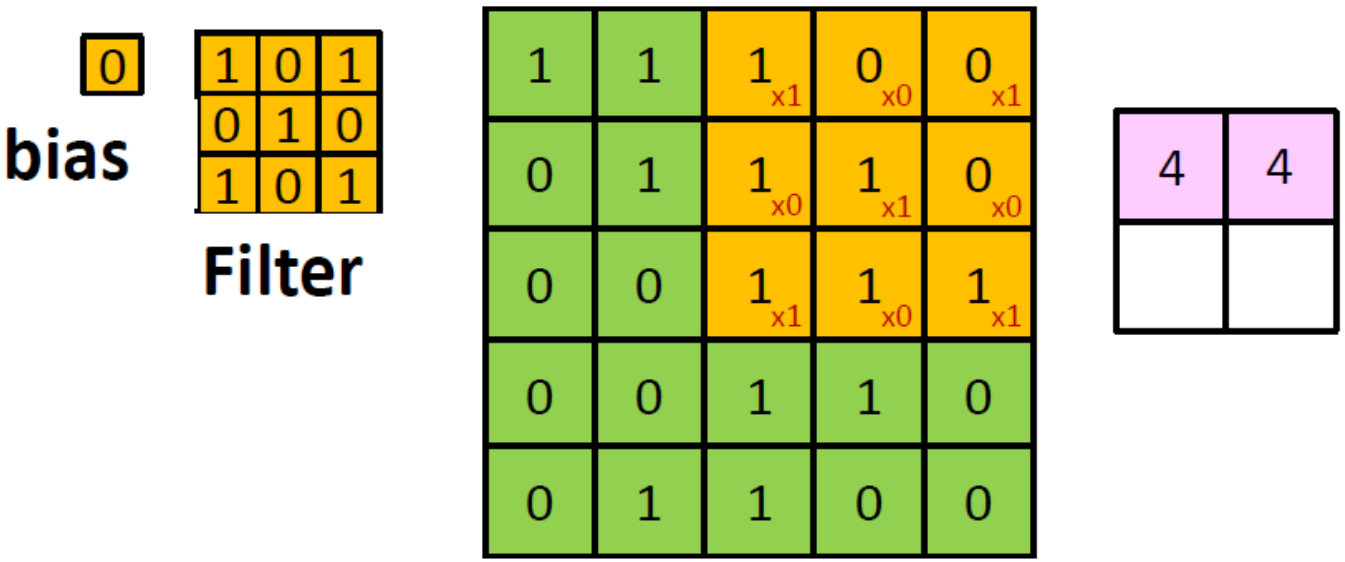
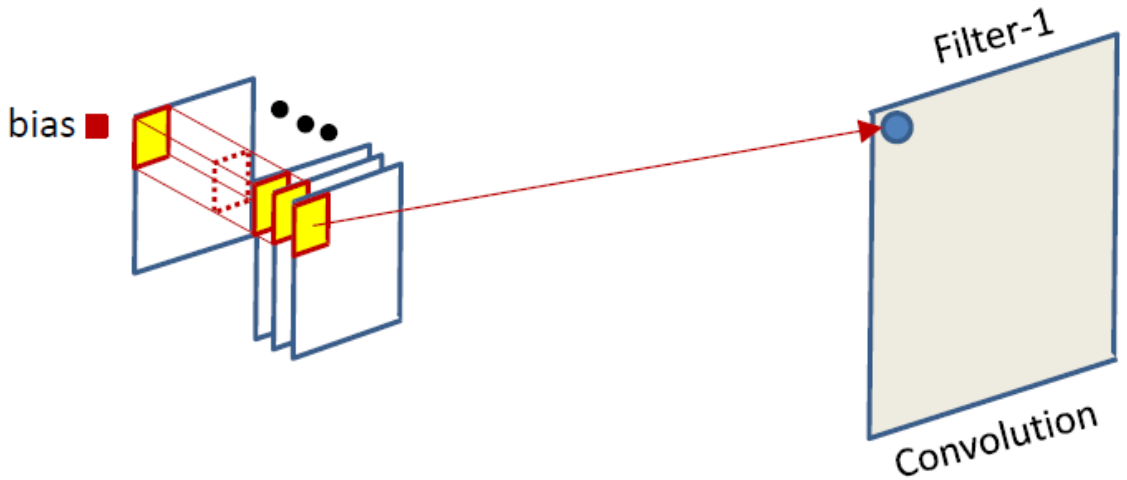


The “cube” view of input

- The computation of the convolutional map at any location sums the convolutional outputs at all planes



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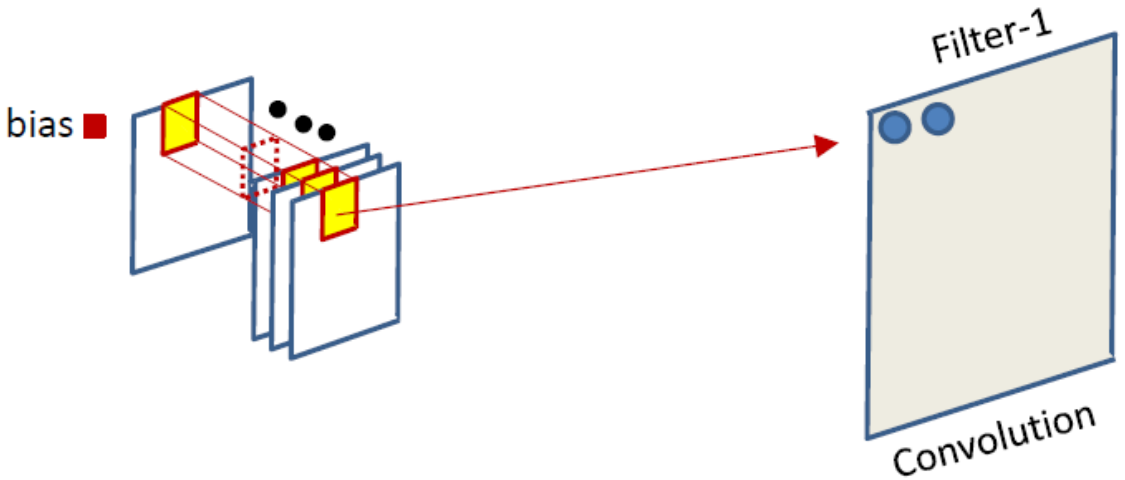

The “cube” view of input

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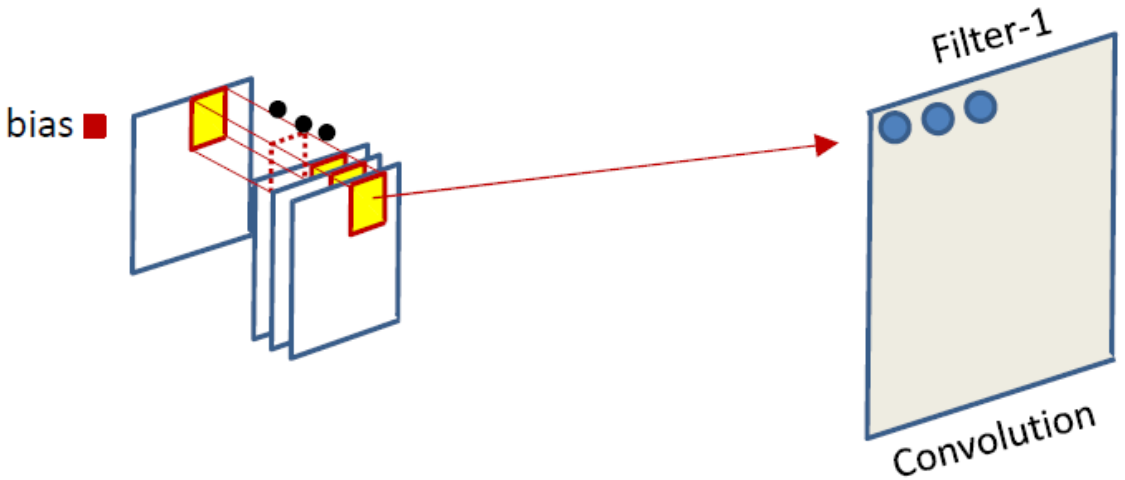

The “cube” view of input

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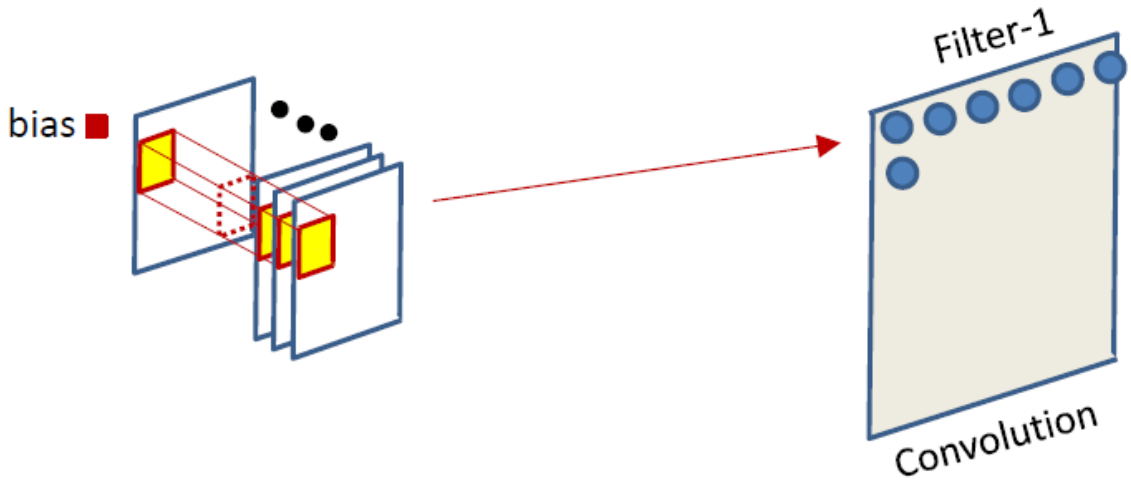

The “cube” view of input

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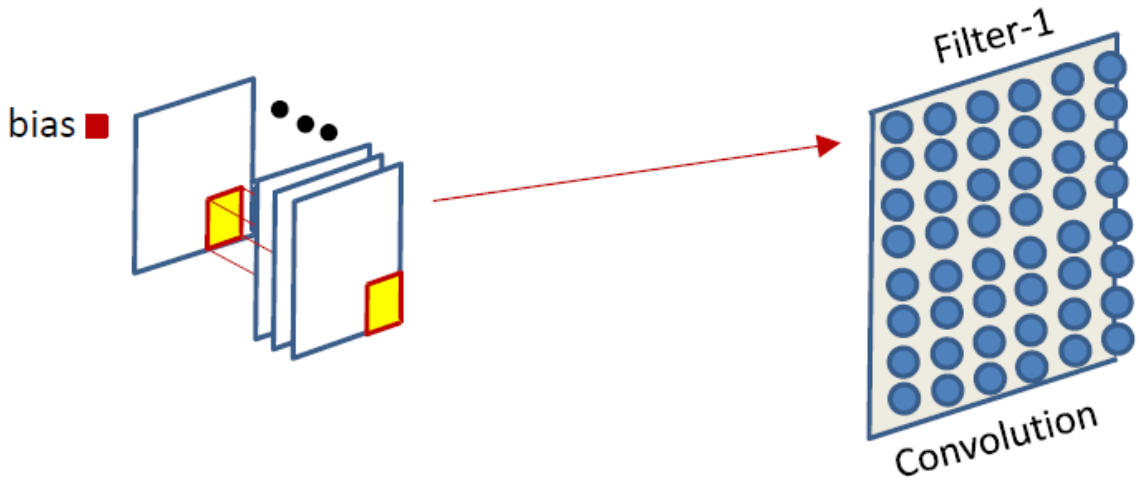
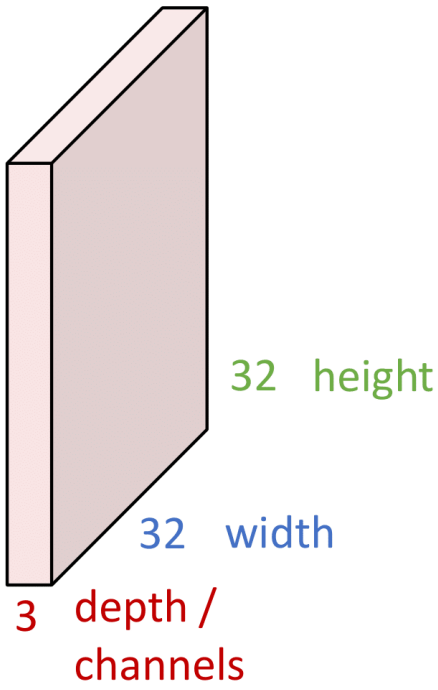
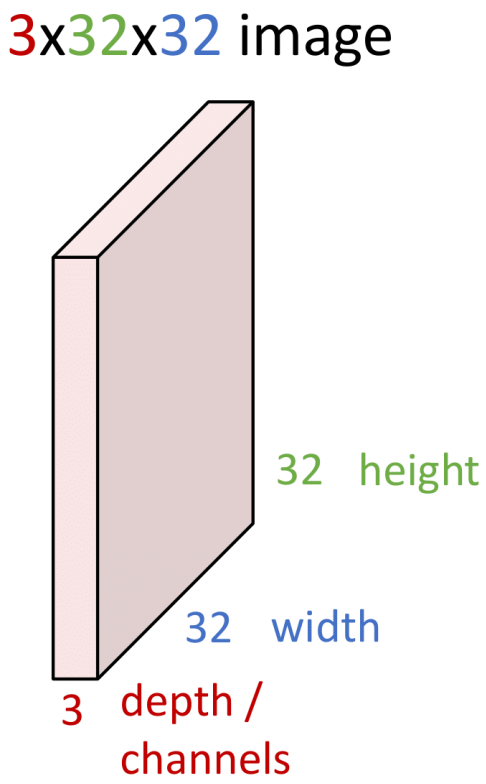


Convolution Layer

3x32x32 image: preserve spatial structure

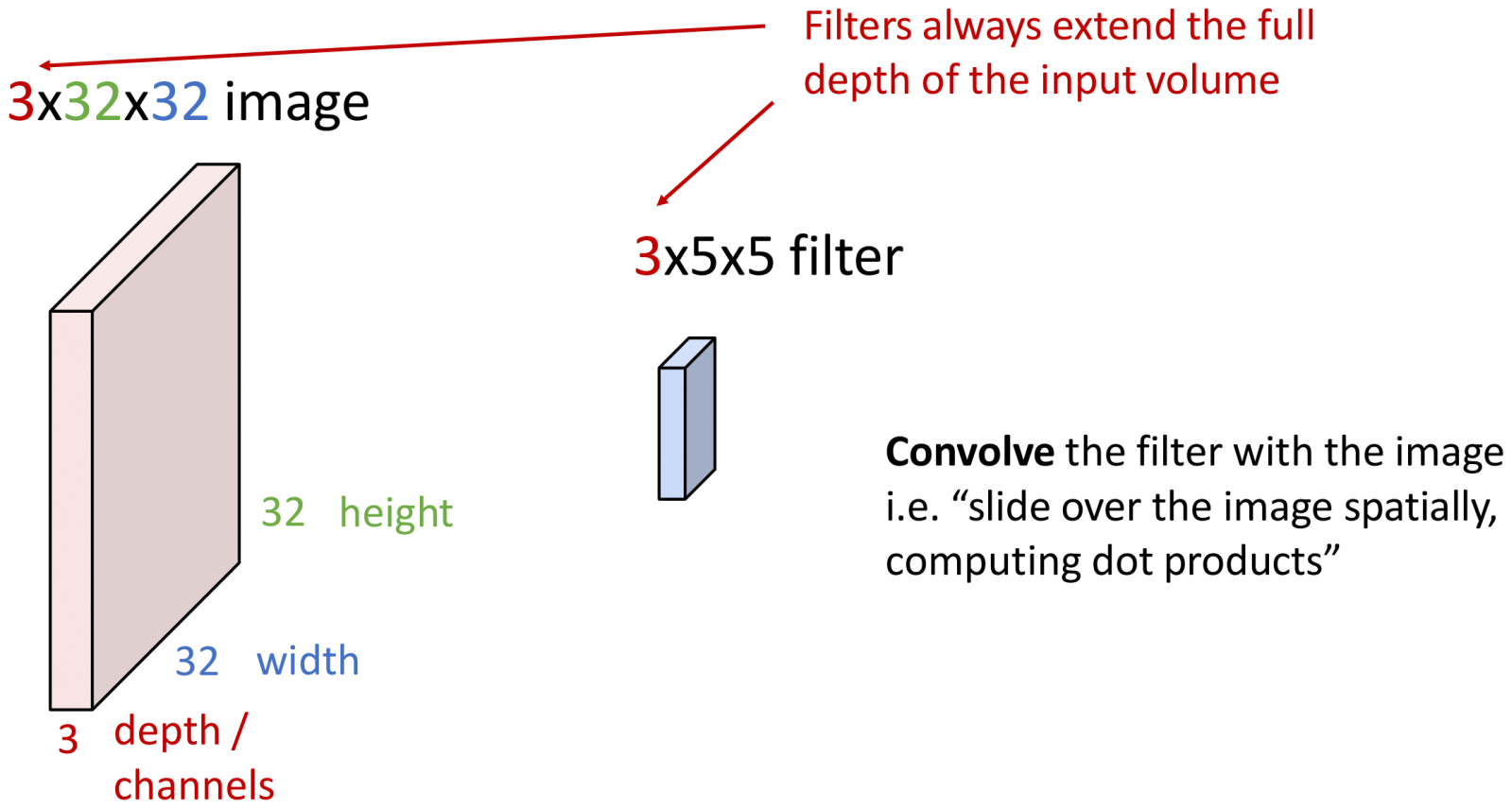


Convolution Layer

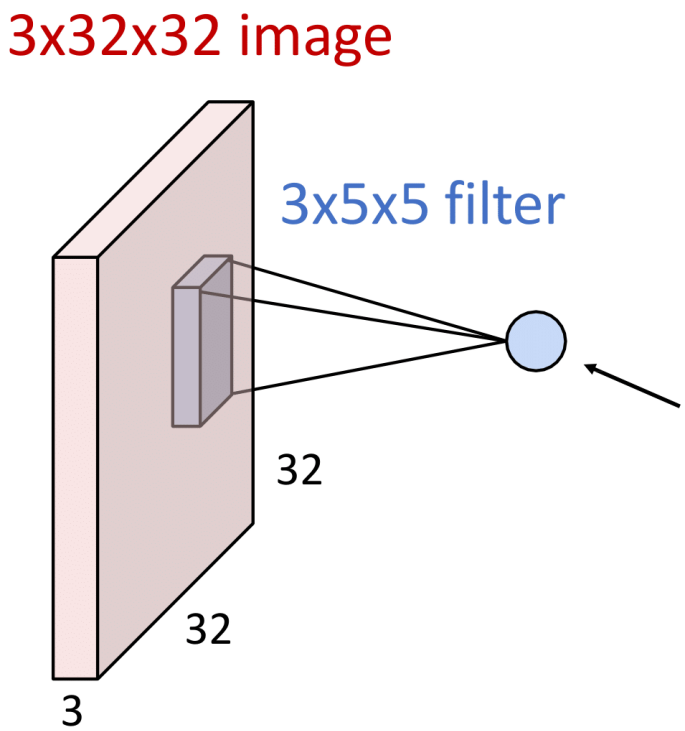


Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

Convolution Layer



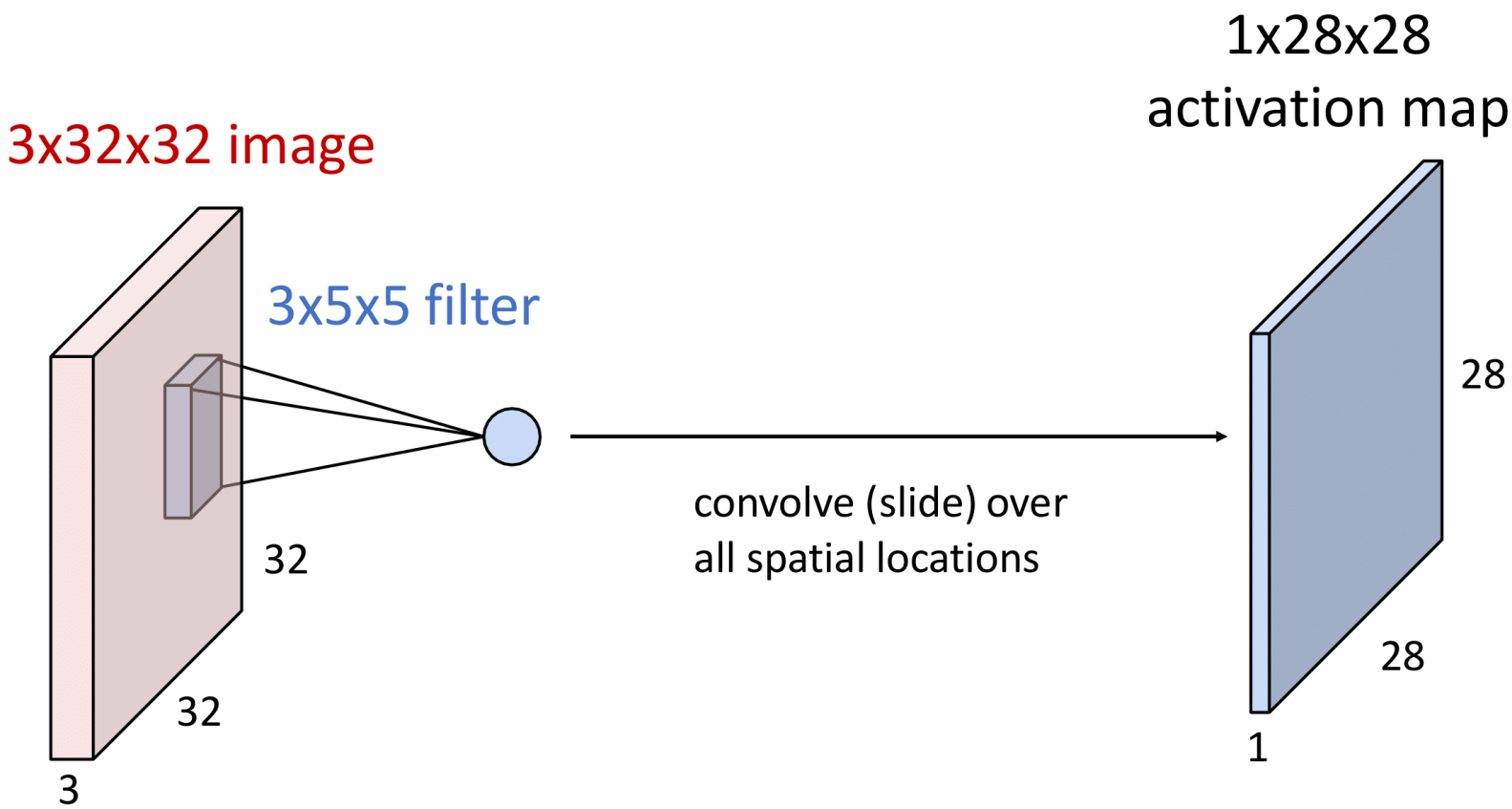
Convolution Layer



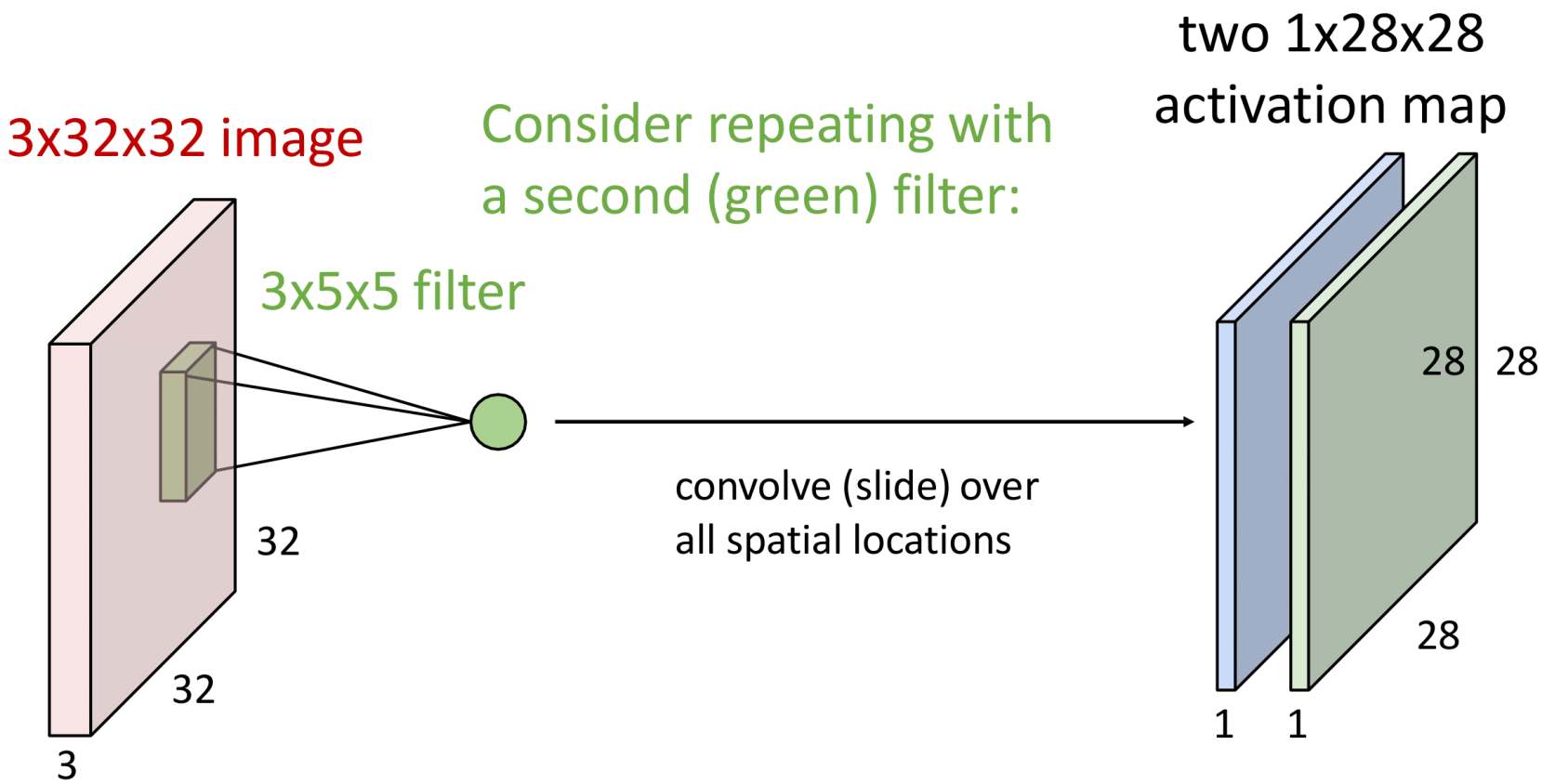
1 number:
the result of taking a dot product between the filter and a small 3x5x5 chunk of the image
(i.e. 3*5*5 = 75-dimensional dot product + bias)

$$w^T x + b$$

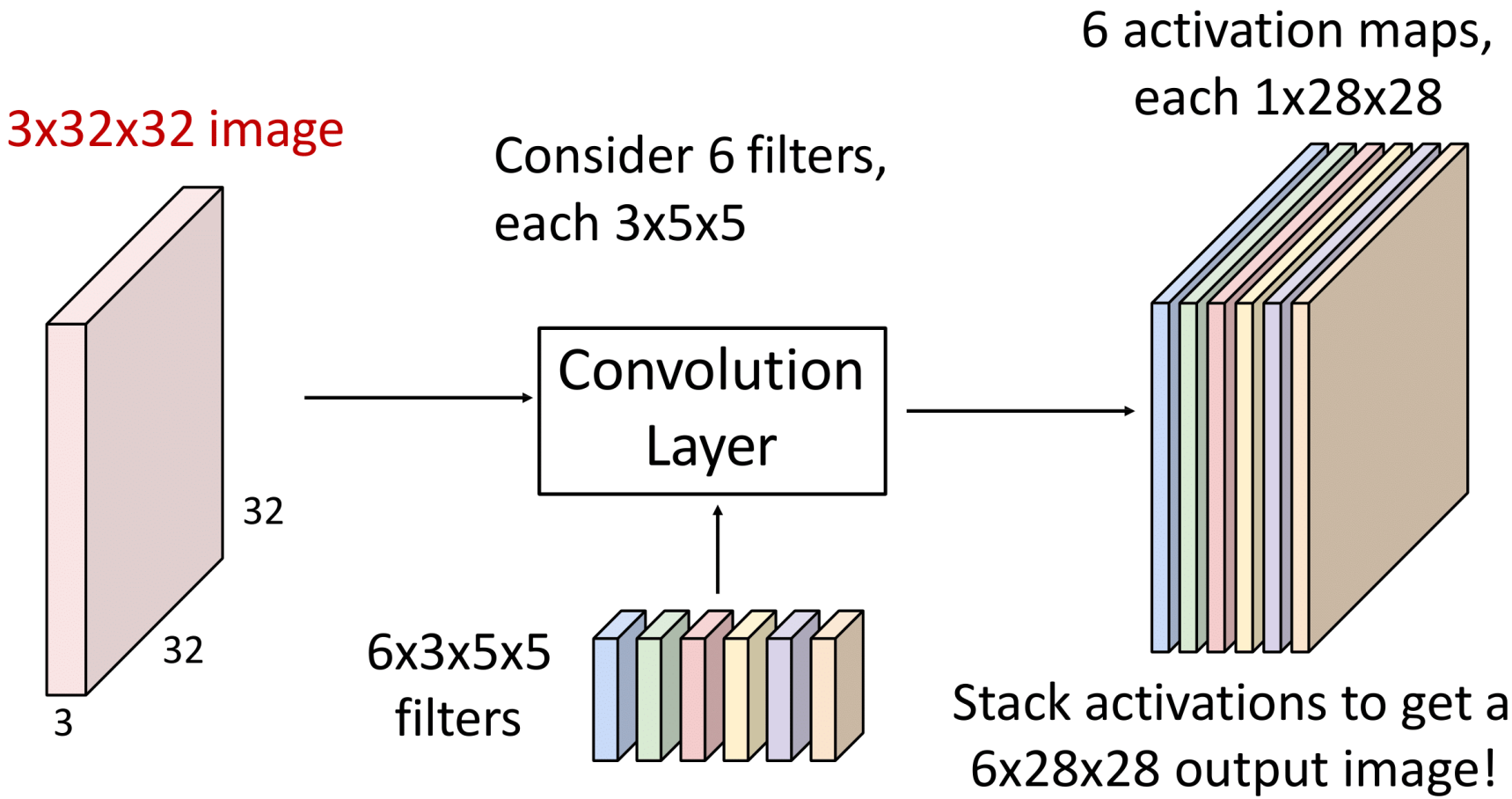
Convolution Layer



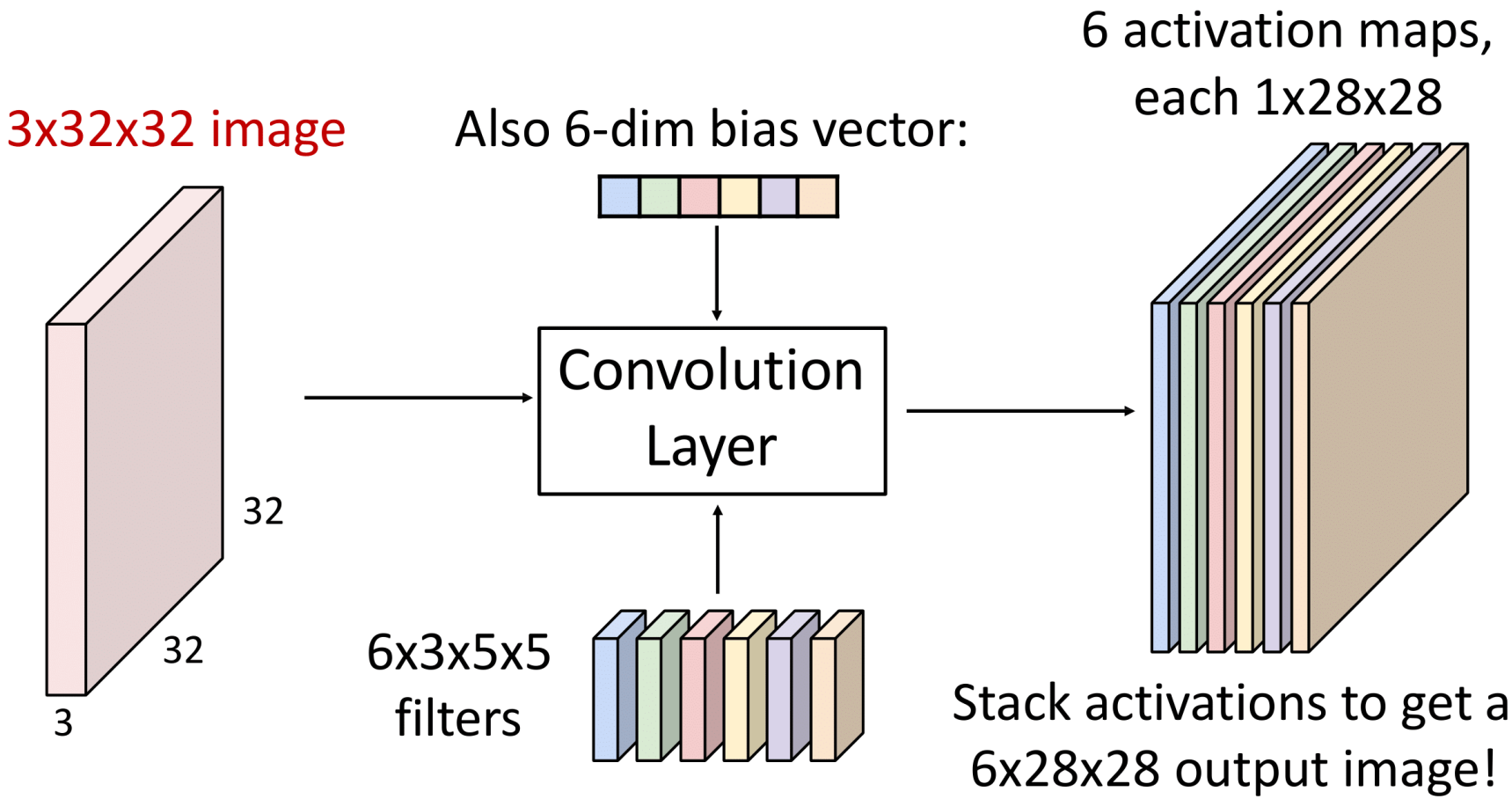
Convolution Layer



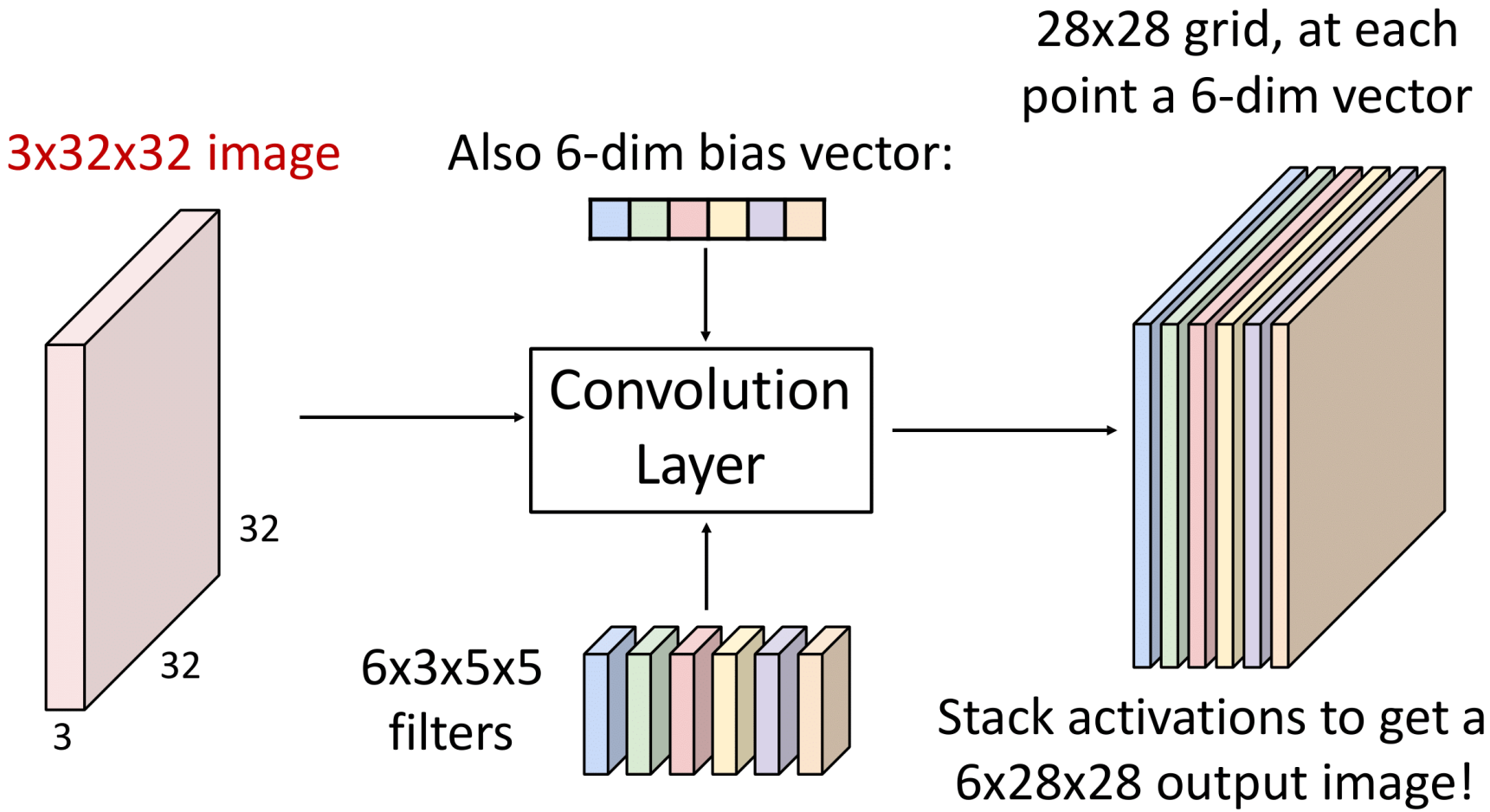
Convolution Layer



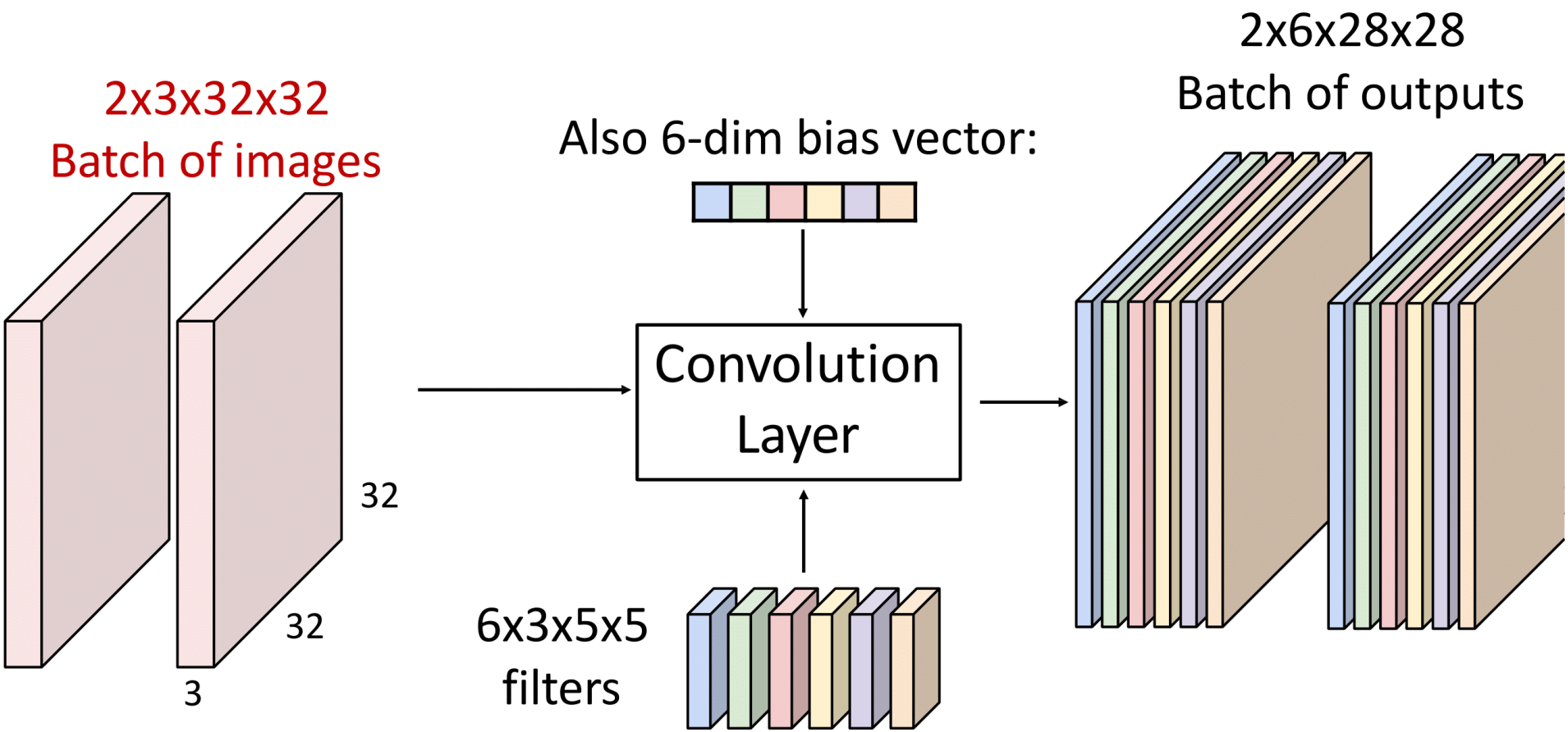
Convolution Layer



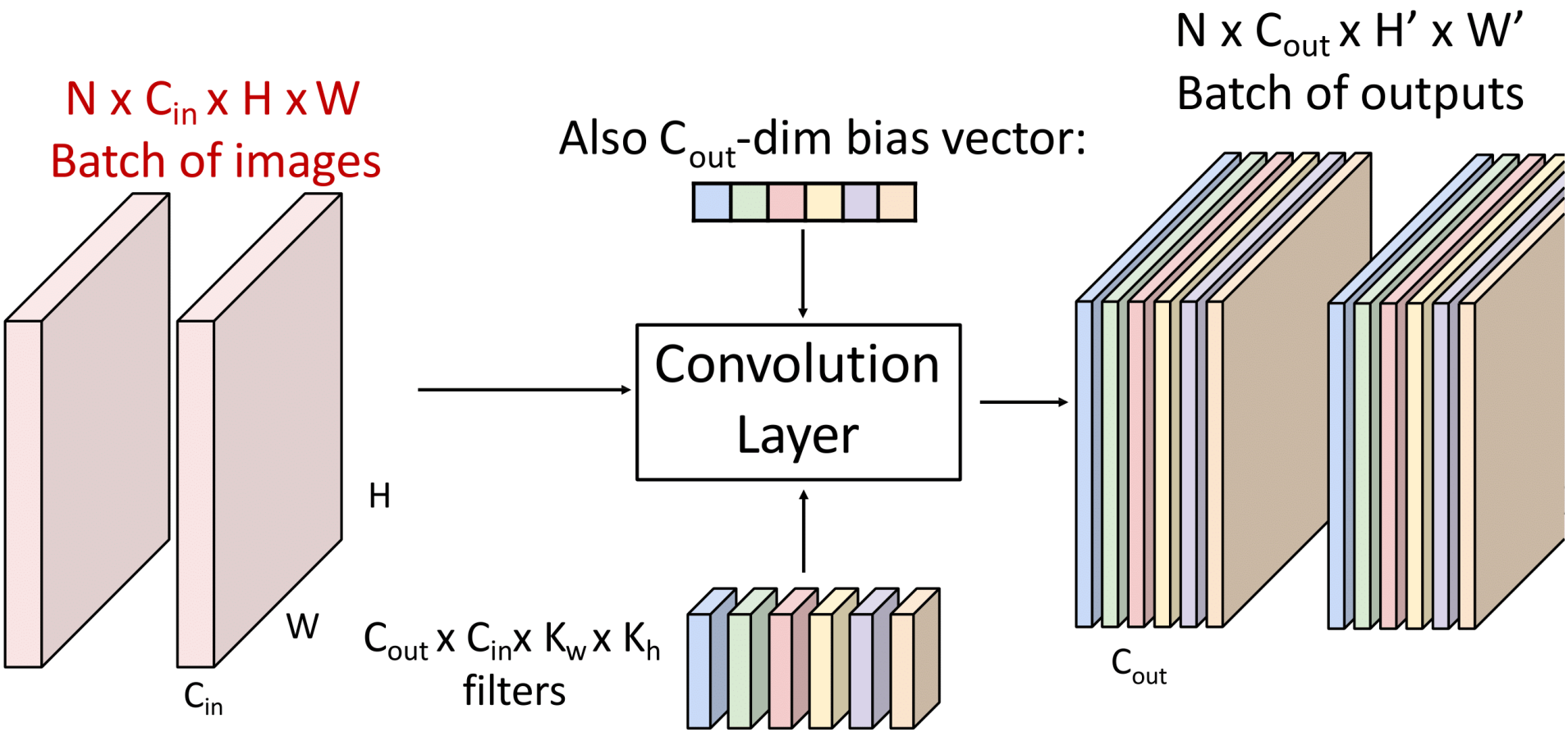
Convolution Layer



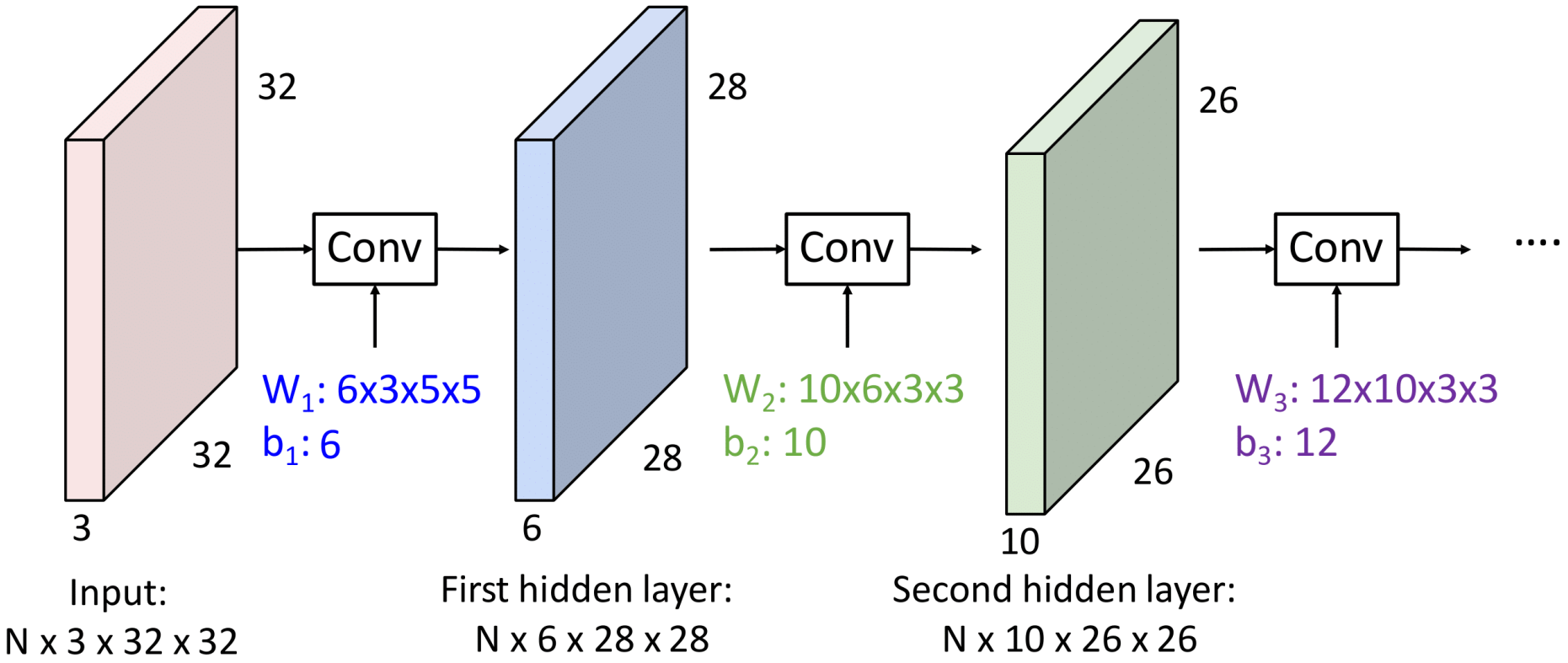
Convolution Layer



Convolution Layer

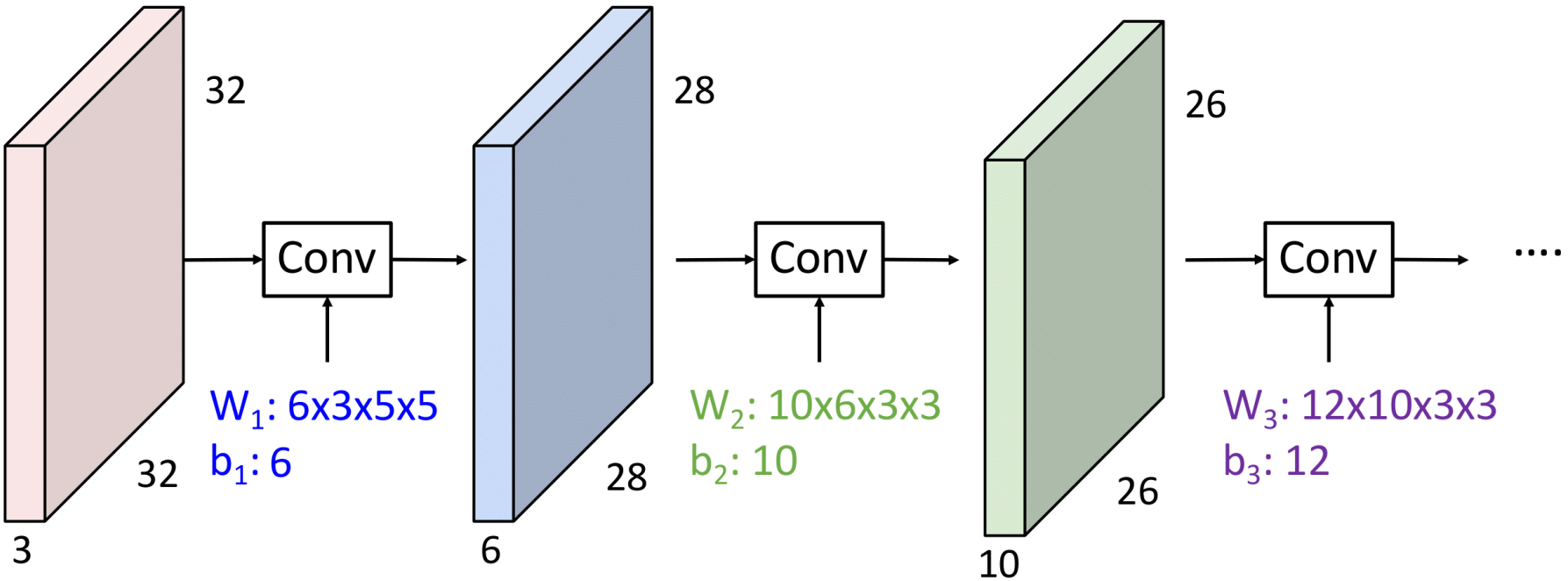


Stacking convolutions



Stacking convolutions

Q: What happens if we stack two convolution layers?



Input:
N x 3 x 32 x 32

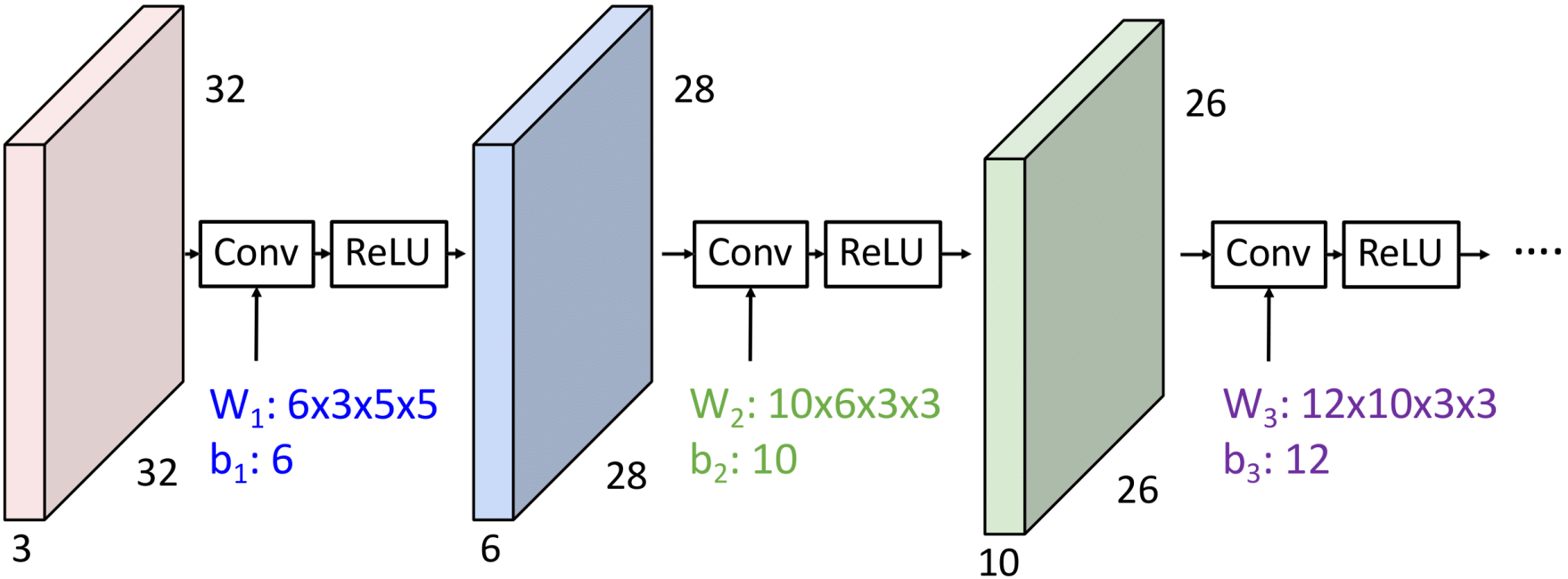
First hidden layer:
N x 6 x 28 x 28

Second hidden layer:
N x 10 x 26 x 26

Stacking convolutions

Q: What happens if we stack two convolution layers? (Recall $y=W_2W_1x$ is a linear classifier)

A: We get another convolution!

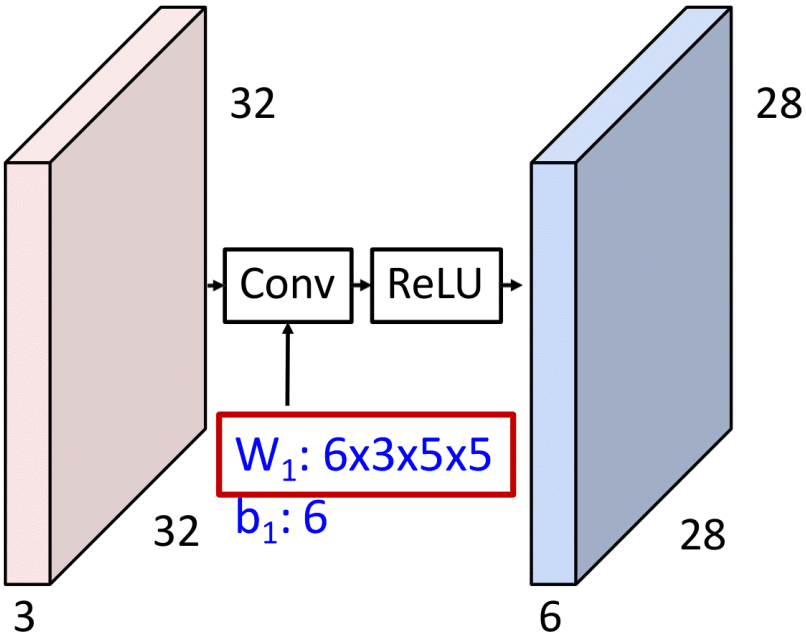


Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

Second hidden layer:
N x 10 x 26 x 26

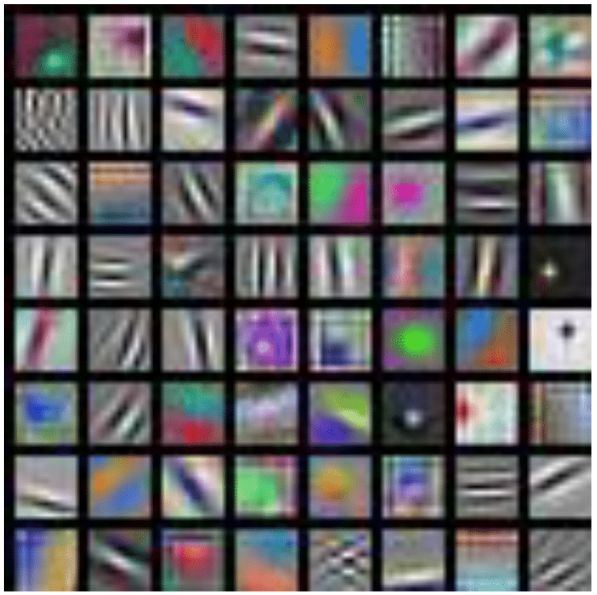
What do convolutional filters learn?



Input:
N x 3 x 32 x 32

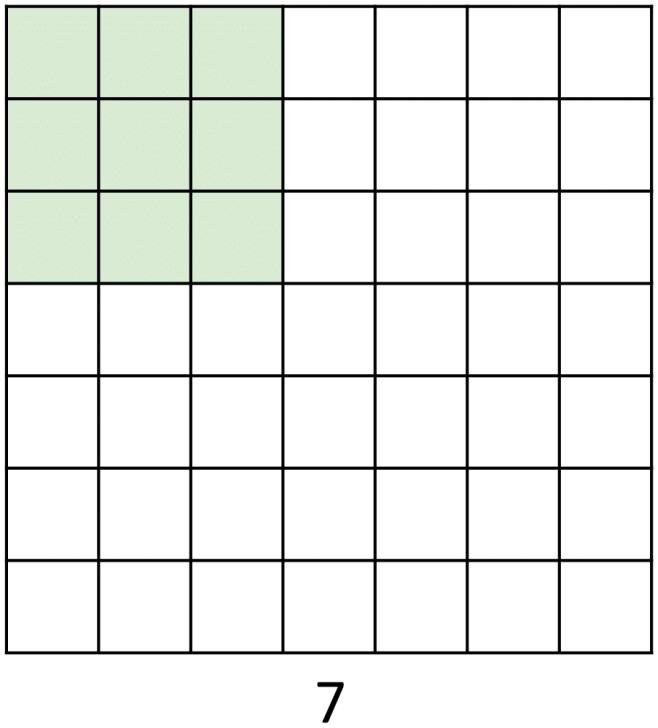
First hidden layer:
N x 6 x 28 x 28

First-layer conv filters: local image templates
(Often learns oriented edges, opposing colors)



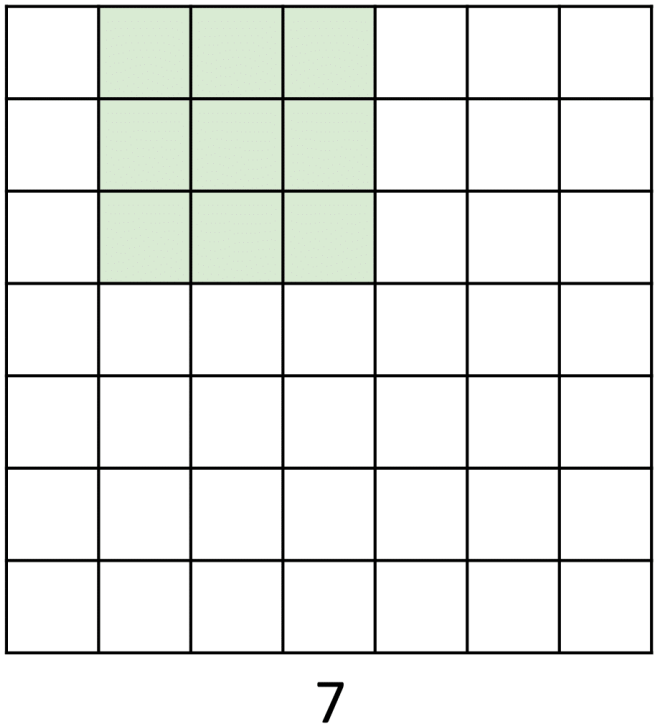
AlexNet: 64 filters, each 3x11x11

A closer look at spatial dimensions



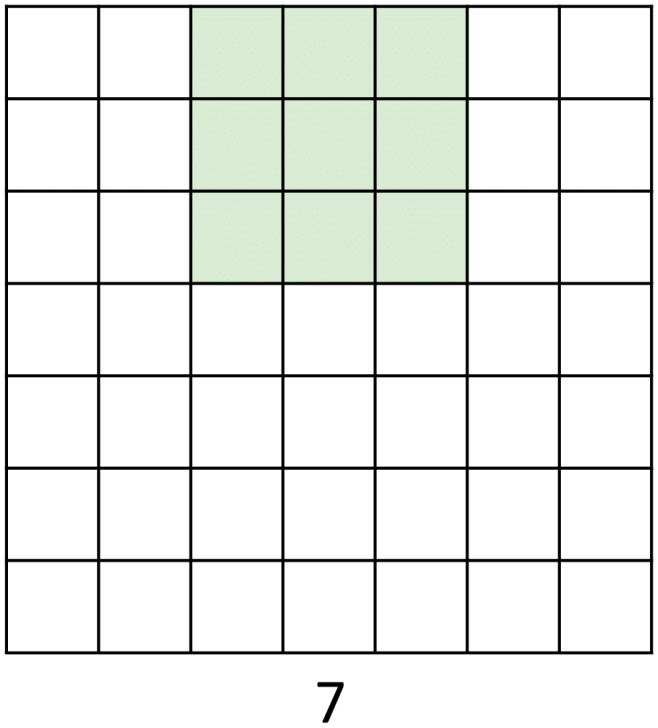
Input: 7x7
Filter: 3x3

A closer look at spatial dimensions



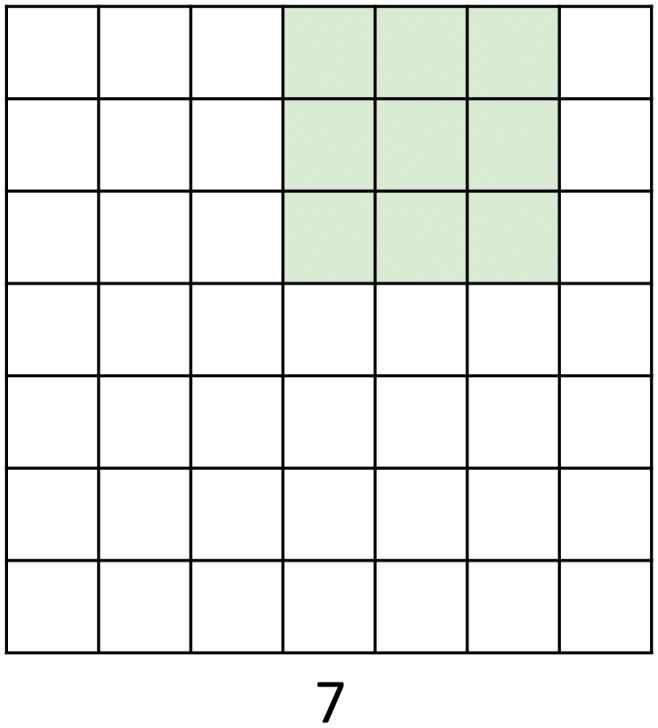
Input: 7x7
Filter: 3x3

A closer look at spatial dimensions



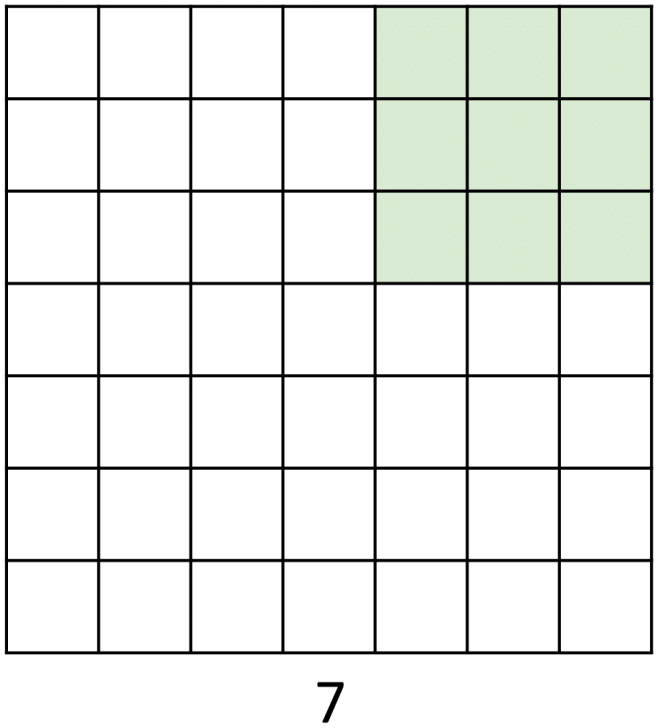
Input: 7x7
Filter: 3x3

A closer look at spatial dimensions



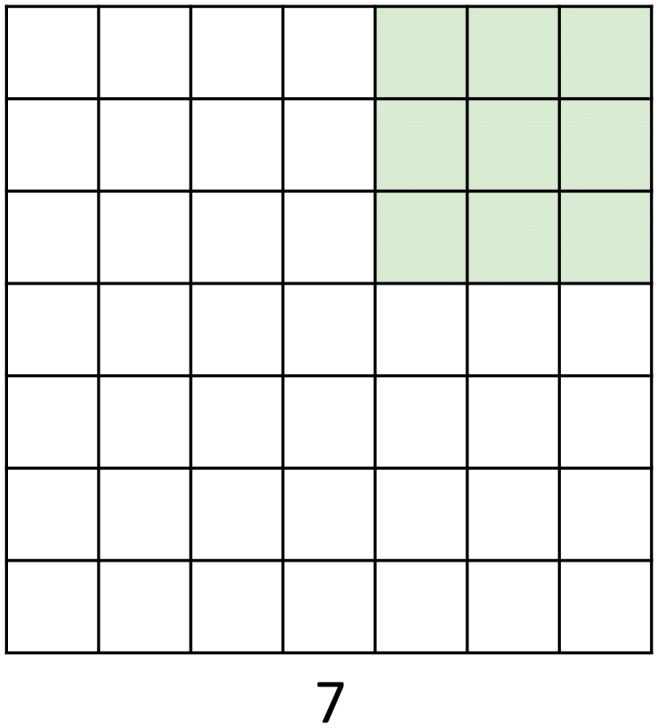
Input: 7x7
Filter: 3x3

A closer look at spatial dimensions



Input: 7x7
Filter: 3x3
Output: 5x5

A closer look at spatial dimensions



Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Output: W – K + 1

Problem: Feature maps "shrink" with each layer!

A closer look at spatial dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

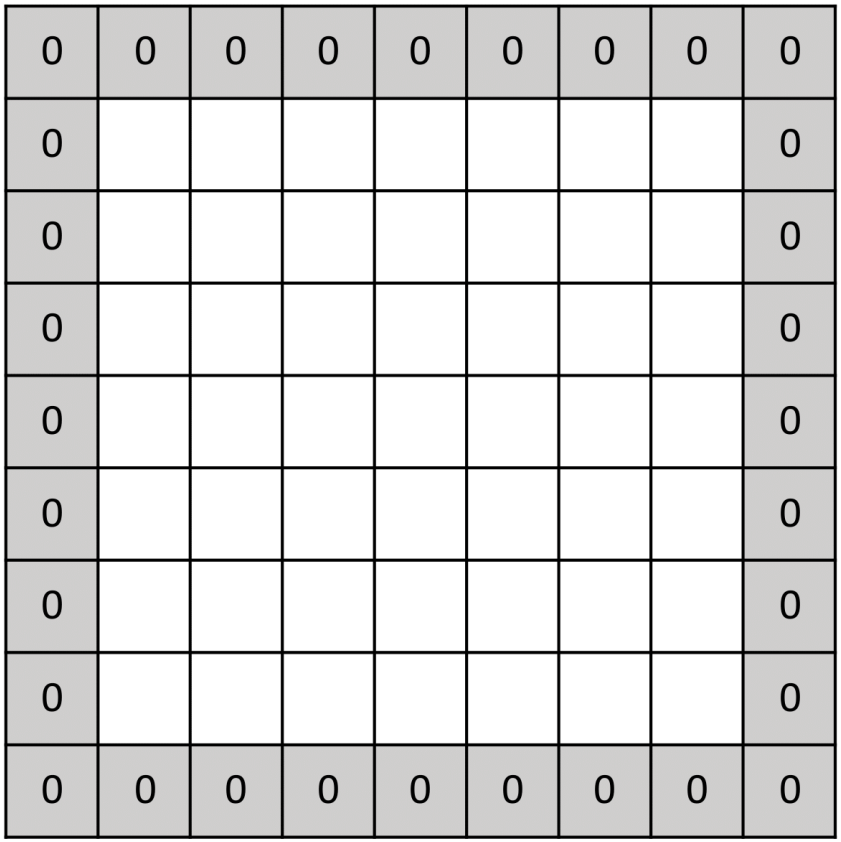
Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Output: W – K + 1

Problem: Feature maps "shrink" with each layer!

Solution: **padding**
Add zeros around the input

A closer look at spatial dimensions



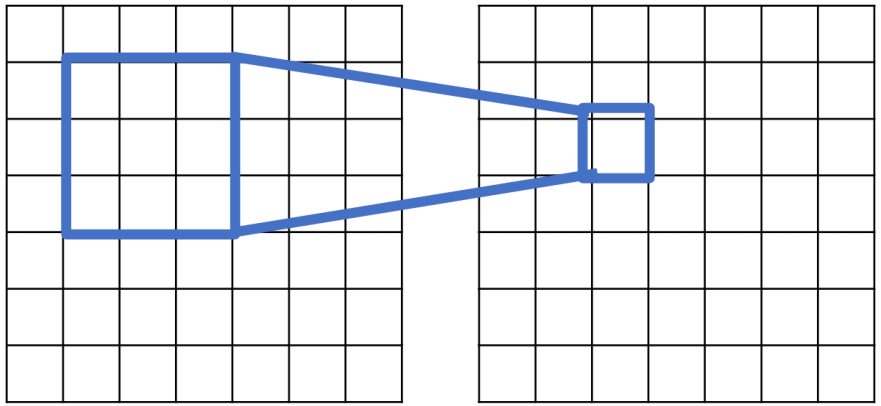
Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Padding: P
Output: W – K + 1 + 2P

Very common:
Set P = (K – 1) / 2 to make output have same size as input!

Receptive Fields

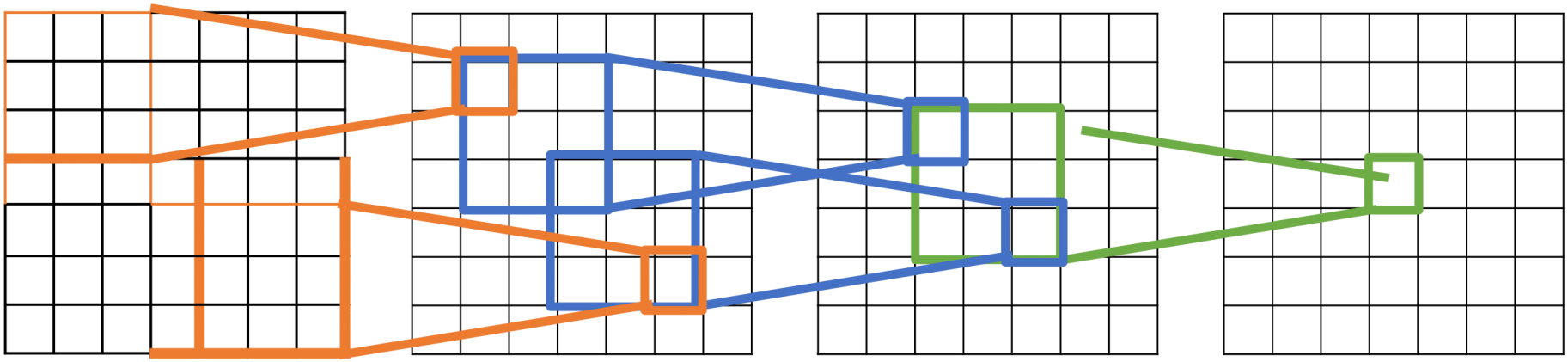
For convolution with kernel size K, each element in the output depends on a K x K **receptive field** in the input



Input Output

Receptive Fields

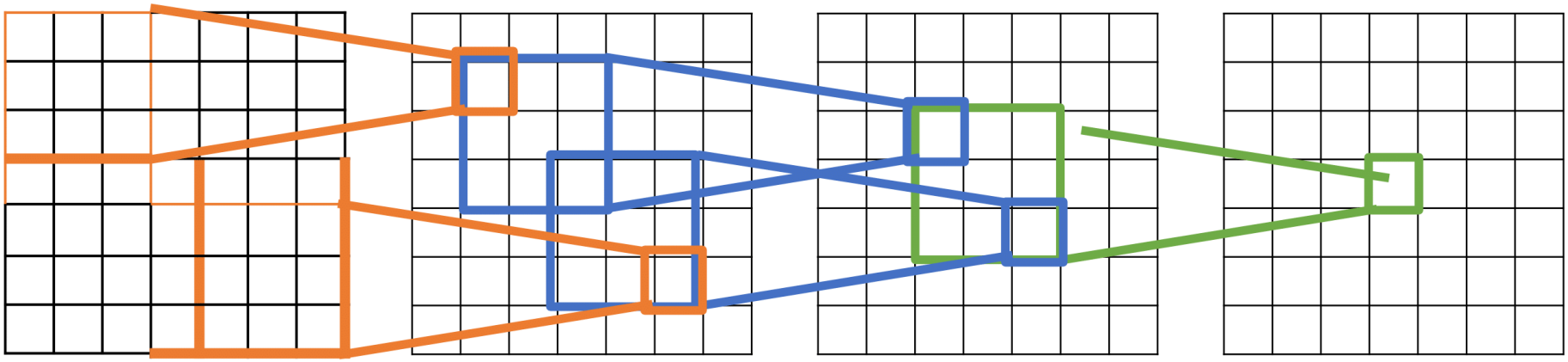
Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



Be careful – ”receptive field in the input” vs “receptive field in the previous layer”
Hopefully clear from context!

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



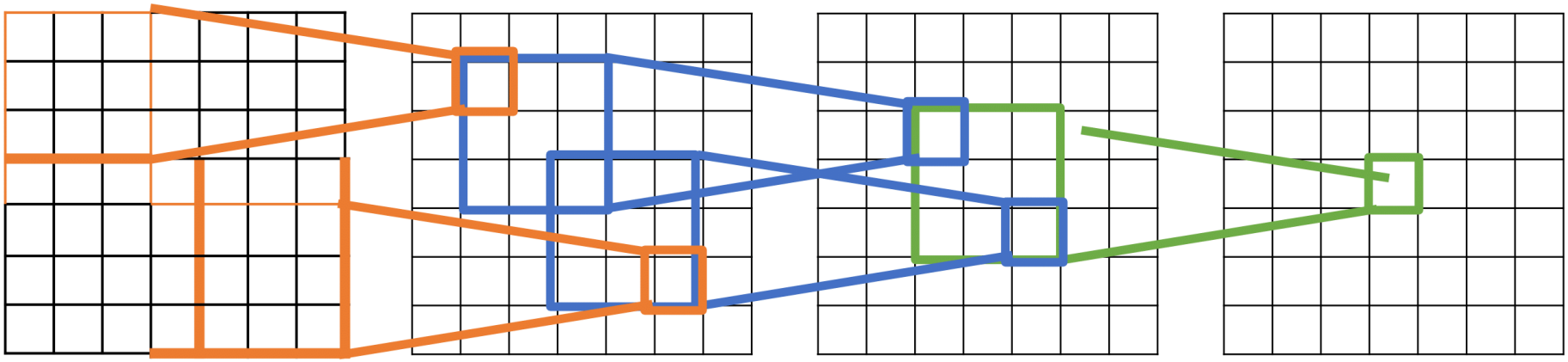
Input

Problem: For large images we need many layers for each output to “see” the whole image image

Output

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



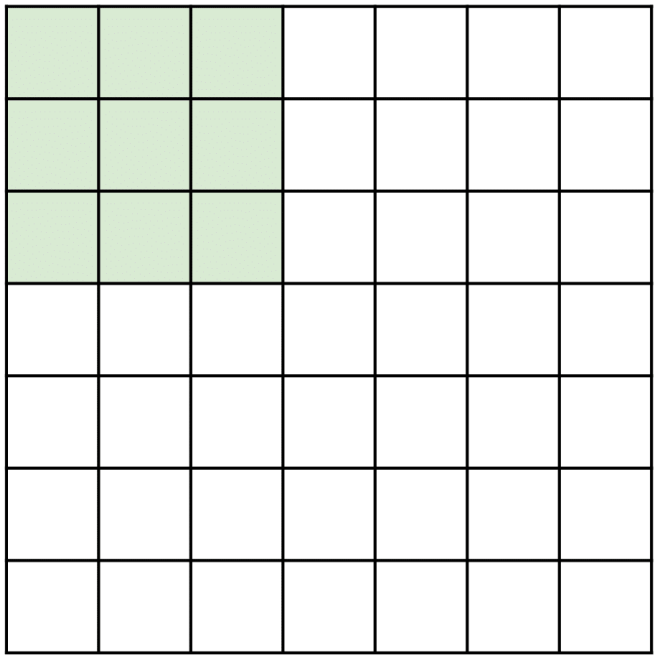
Input

Output

Problem: For large images we need many layers for each output to “see” the whole image image

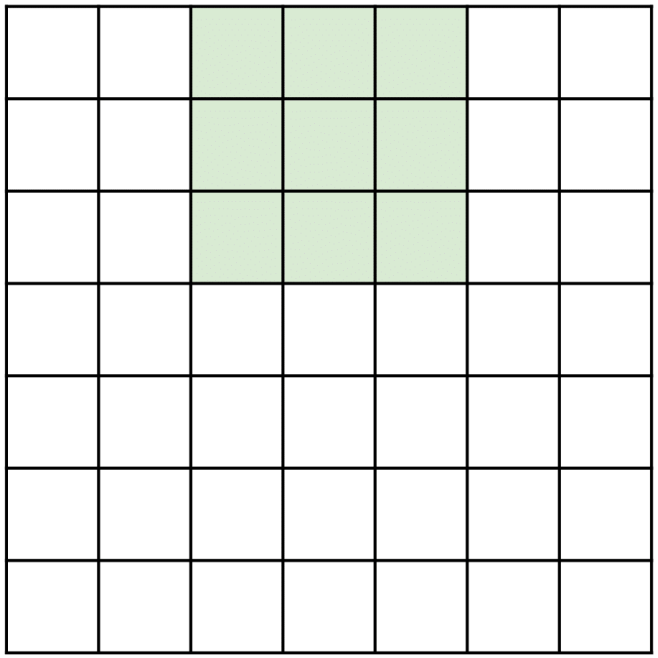
Solution: Downsample inside the network

Strided Convolution



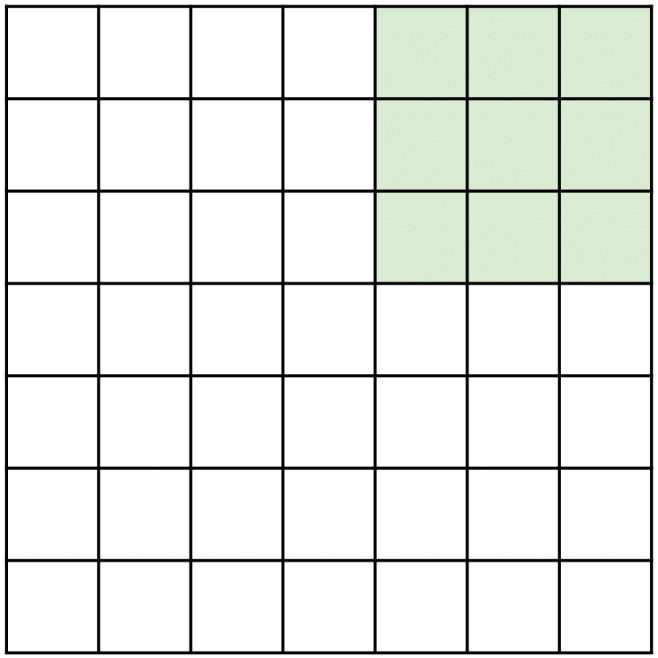
Input: 7x7
Filter: 3x3
Stride: 2

Strided Convolution



Input: 7x7
Filter: 3x3
Stride: 2

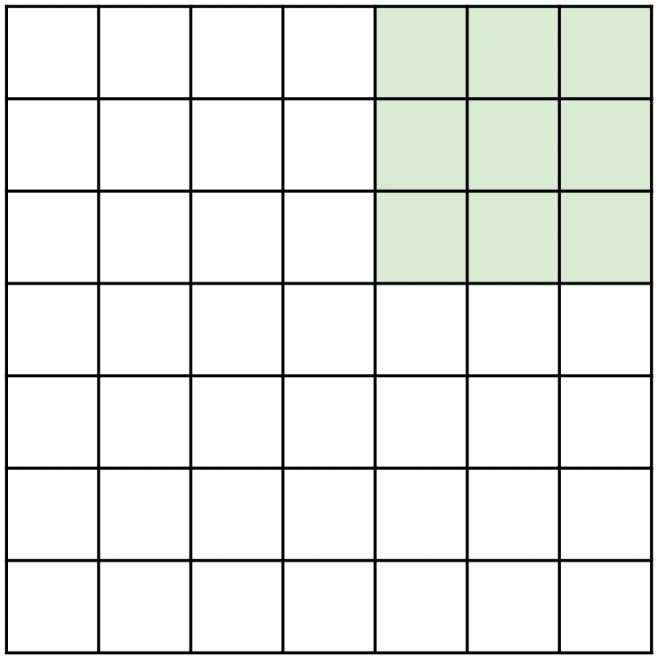
Strided Convolution



Input: 7x7
Filter: 3x3
Stride: 2

Output: 3x3

Strided Convolution



Input: 7x7
Filter: 3x3
Stride: 2

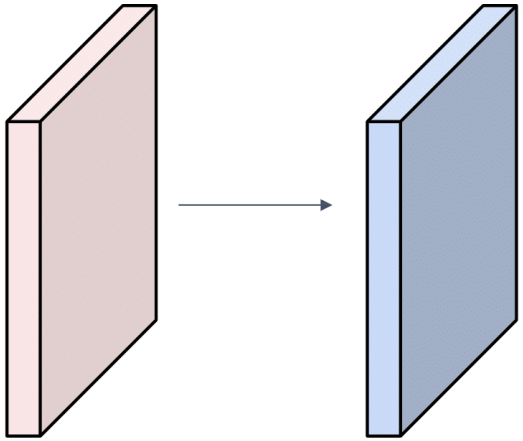
Output: 3x3

In general:
Input: W
Filter: K
Padding: P
Stride: S
Output: (W – K + 2P) / S + 1

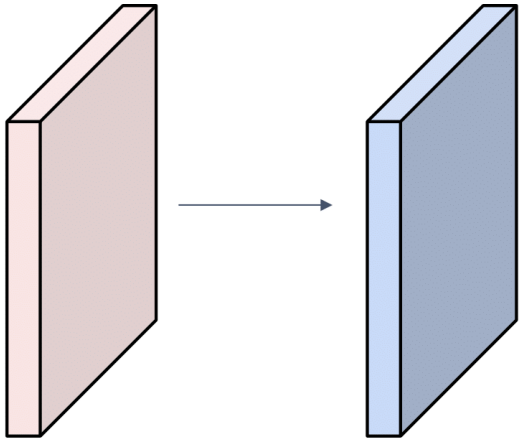
Convolution example

Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: ?



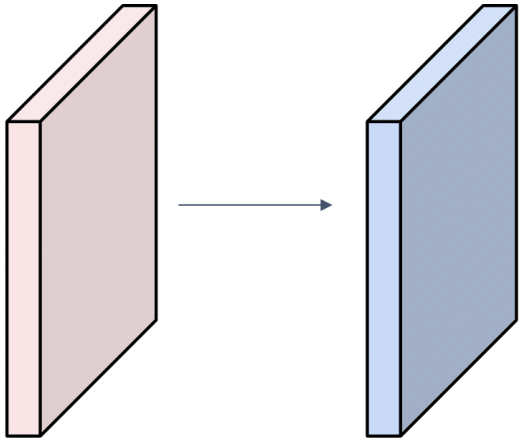
Convolution example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size:
(32+2*2-5)/1+1 = 32 spatially, so
10 x 32 x 32

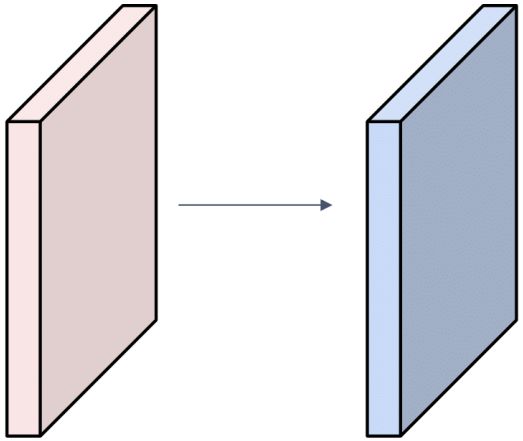
Convolution example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: ?

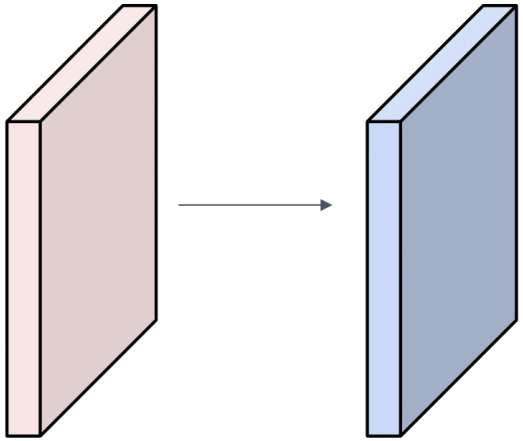
Convolution example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: **760**
Parameters per filter: 3*5*5 + 1 (for bias) = 76
10 filters, so total is 10 * 76 = **760**

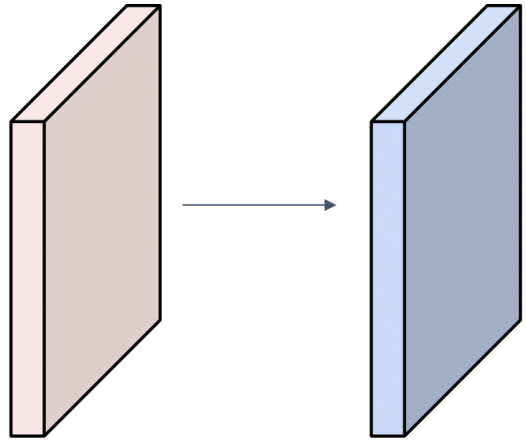
Convolution example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: 760
Number of multiply-add operations: ?

Convolution example

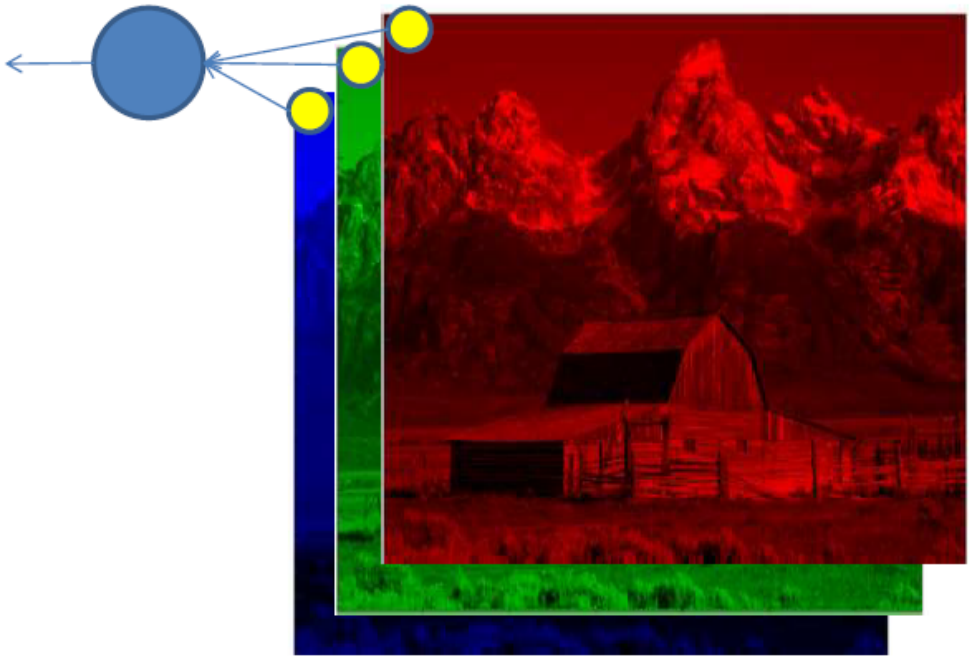


Input volume: **3** x 32 x 32
10 **5x5** filters with stride 1, pad 2

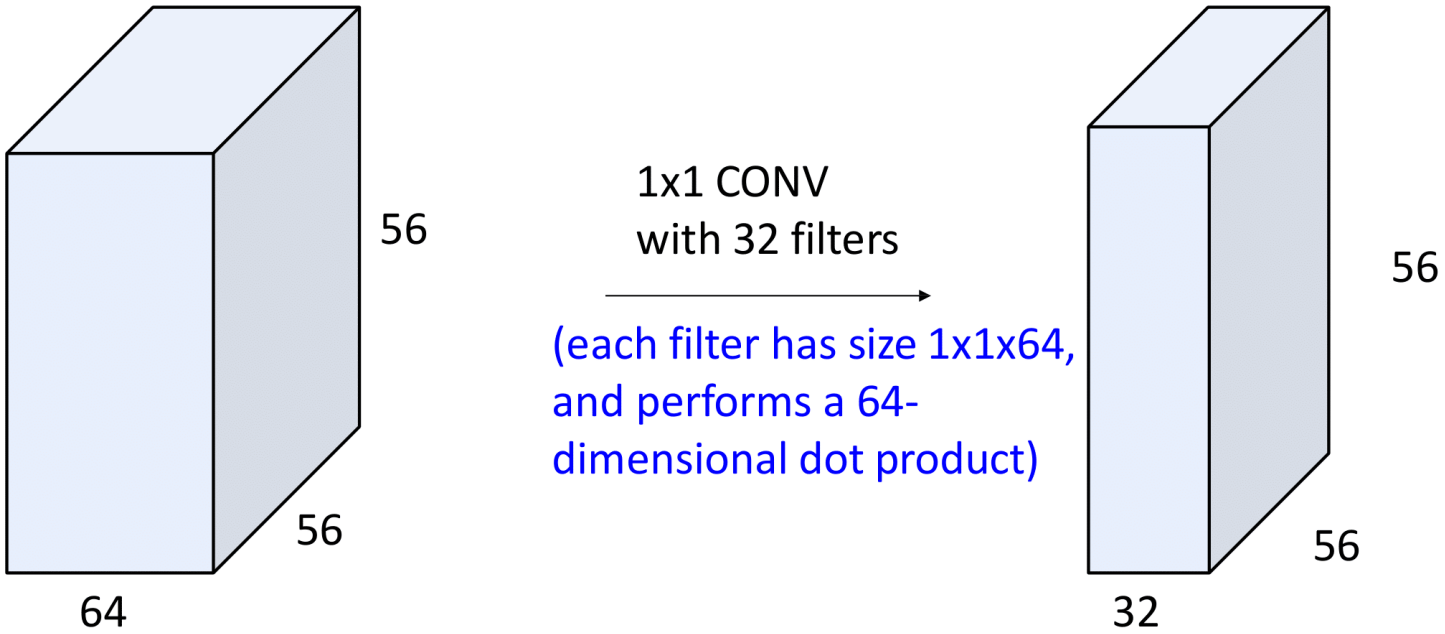
Output volume size: **10 x 32 x 32**
Number of learnable parameters: 760
Number of multiply-add operations: **768,000**
10*32*32 = 10,240 outputs; each output is the inner product of two **3**x**5x5** tensors (75 elems); total = 75*10240 = **768K**

The 1x1 filter

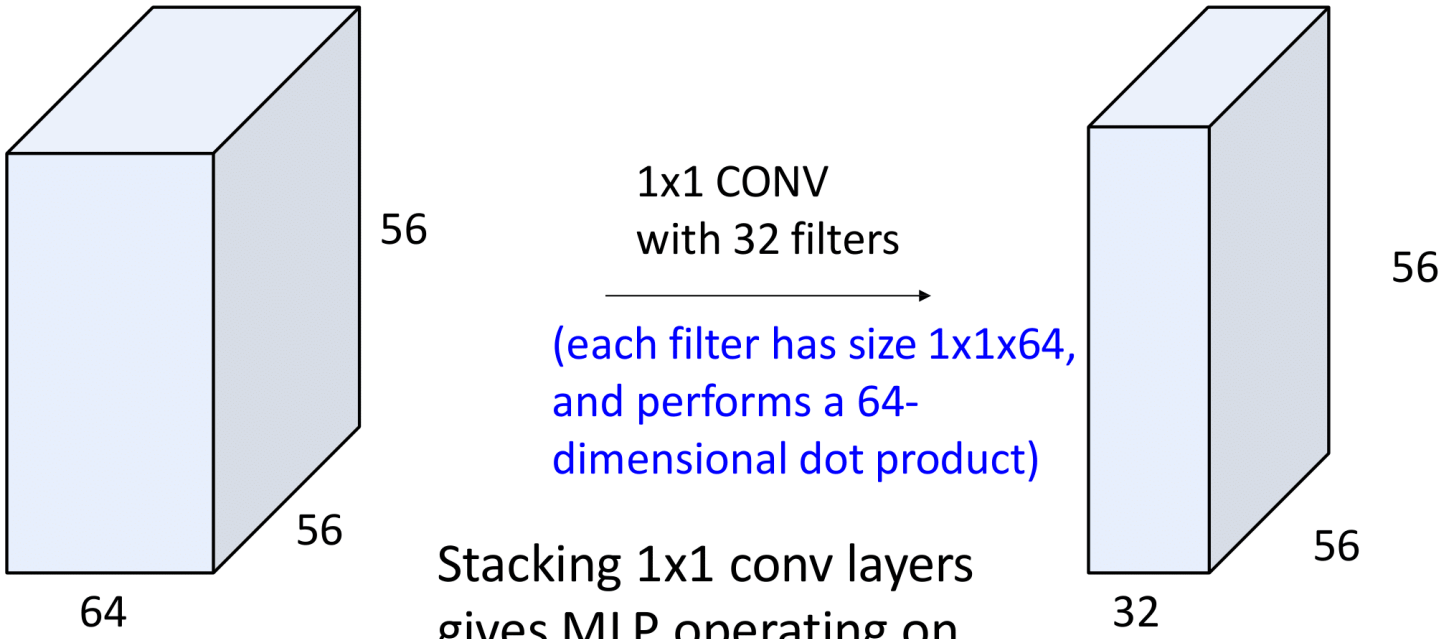
- A 1x1 filter is simply a perceptron that operates over the depth of the stack of maps, but has no spatial extent
  - Takes one pixel from each of the maps (at a given location) as input



Example: 1x1 convolution



Example: 1x1 convolution



Stacking 1x1 conv layers gives MLP operating on each input position

Lin et al, “Network in Network”, ICLR 2014

Convolution summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W

giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:

- H' = (H – K + 2P) / S + 1
- W' = (W – K + 2P) / S + 1

Convolution summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W
giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:

- H' = (H – K + 2P) / S + 1
- W' = (W – K + 2P) / S + 1

Common settings:

$K_H = K_W$ (Small square filters)

P = (K – 1) / 2 ("Same" padding)

C_{in}, C_{out} = 32, 64, 128, 256 (powers of 2)

K = 3, P = 1, S = 1 (3x3 conv)

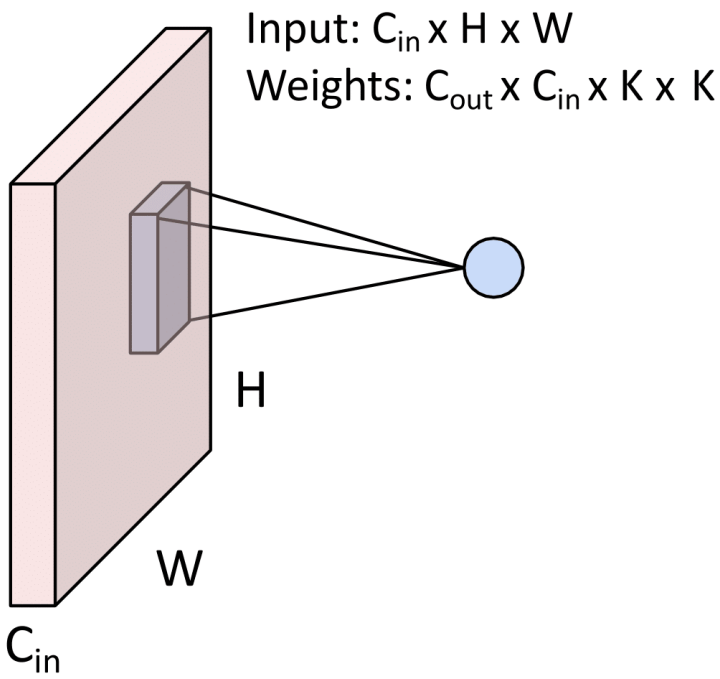
K = 5, P = 2, S = 1 (5x5 conv)

K = 1, P = 0, S = 1 (1x1 conv)

K = 3, P = 1, S = 2 (Downsample by 2)

Other types of convolution

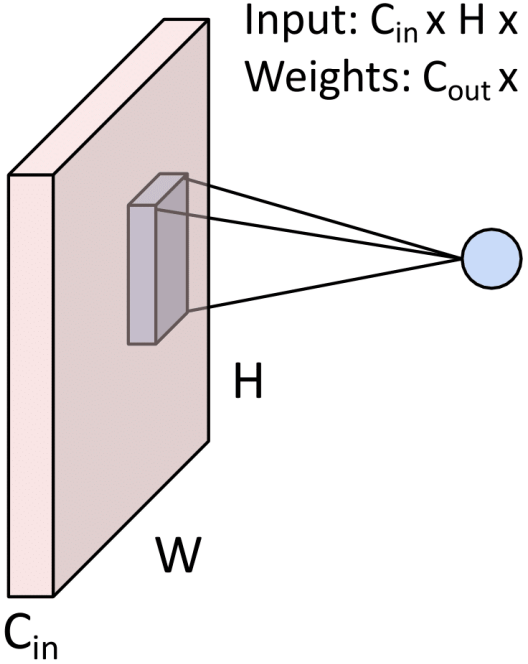
So far: 2D Convolution



Other types of convolution

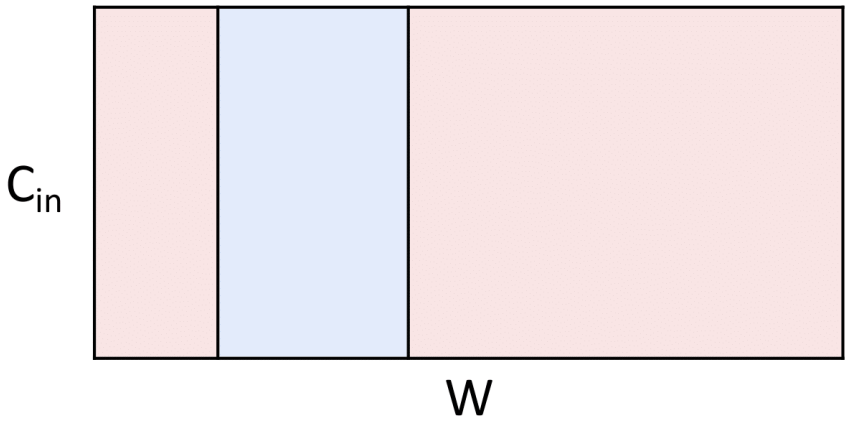
So far: 2D Convolution

Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



1D Convolution

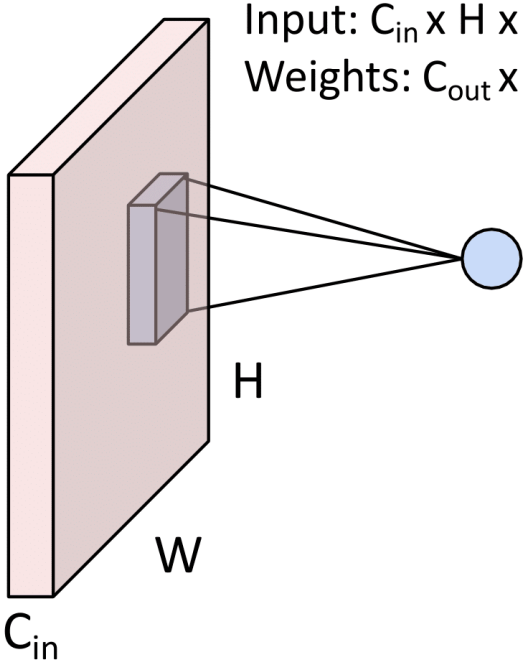
Input: C_{in} x W
Weights: C_{out} x C_{in} x K



Other types of convolution

So far: 2D Convolution

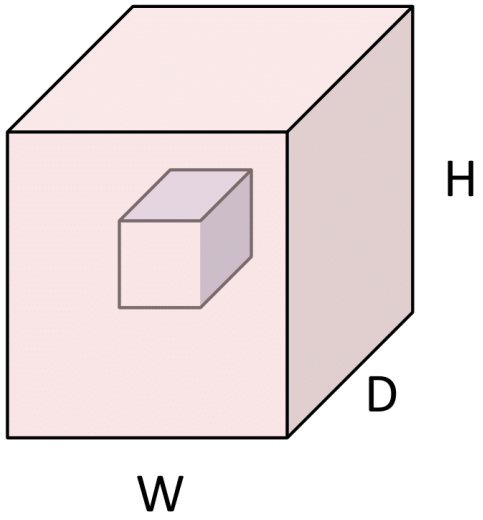
Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



3D Convolution

Input: C_{in} x H x W x D
Weights: C_{out} x C_{in} x K x K x K

C_{in}-dim vector
at each point
in the volume



PyTorch Convolution Layers

Conv2d

CLASS `torch.nn.Conv2d(`*`in_channels`*`, `*`out_channels`*`, `*`kernel_size`*`, `*`stride=1`*`, `*`padding=0`*`, `*`dilation=1`*`, `*`groups=1`*`, `*`bias=True`*`, `*`padding_mode='zeros'`*`)` [SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$

PyTorch Convolution Layers

Conv2d

CLASS `torch.nn.Conv2d(`*in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'*`)` [SOURCE]

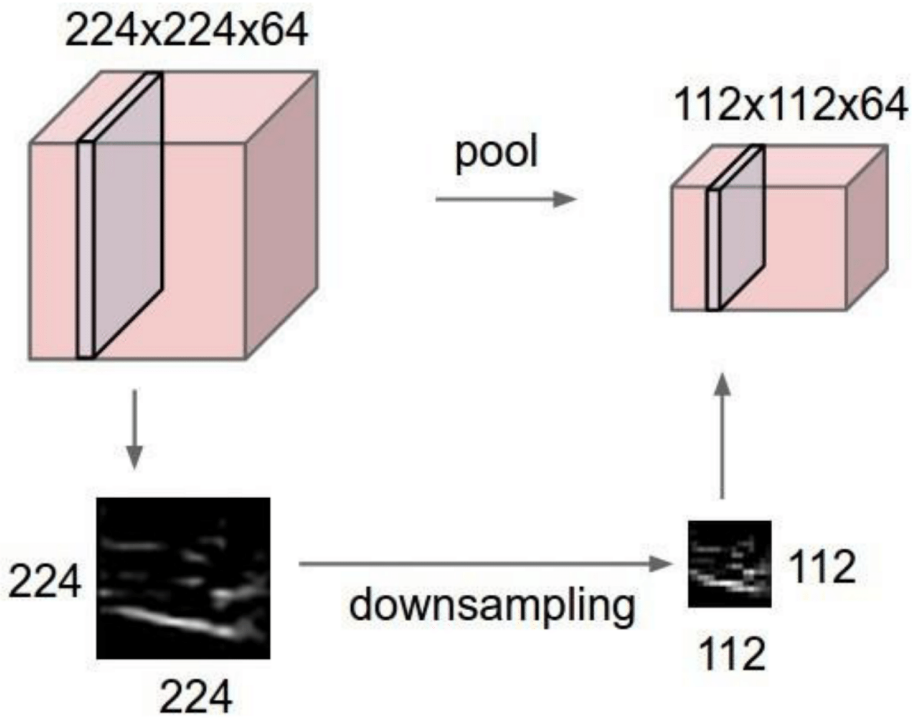
Conv1d

CLASS `torch.nn.Conv1d(`*in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'*`)` [SOURCE]

Conv3d

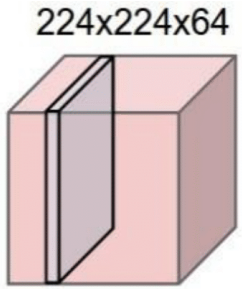
CLASS `torch.nn.Conv3d(`*in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros'*`)` [SOURCE]

Pooling Layers: Another way to downsample

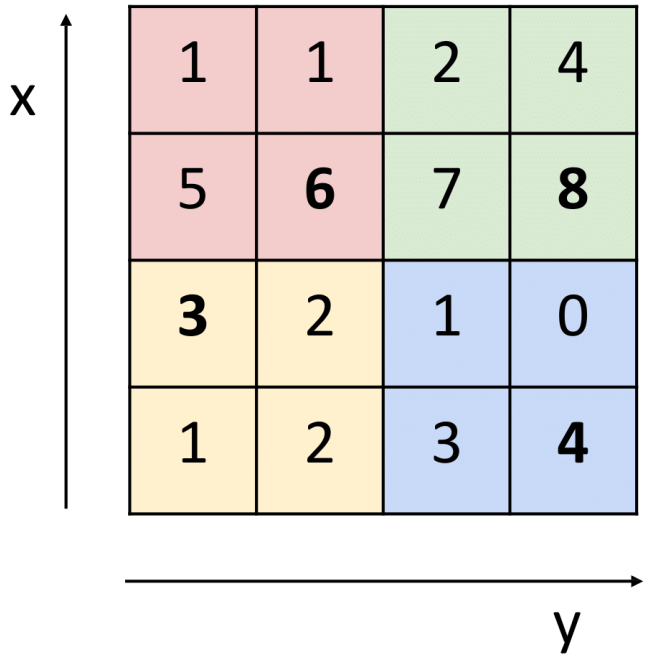


Hyperparameters:
Kernel Size
Stride
Pooling function

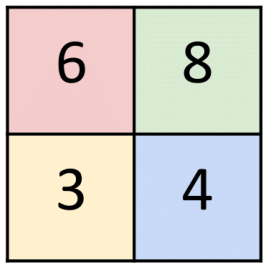
Max Pooling



Single depth slice



Max pooling with 2x2 kernel size and stride 2



Introduces **invariance** to small spatial shifts

No learnable parameters!

Pooling Summary

Input: C x H x W

Hyperparameters:

- Kernel size: K
- Stride: S
- Pooling function (max, avg)

Output: C x H' x W' where

- H' = (H – K) / S + 1
- W' = (W – K) / S + 1

Learnable parameters: None!

Common settings:

max, K = 2, S = 2

max, K = 3, S = 2 (AlexNet)

References

- Deep Learning, Ian Goodfellow, MIT Press, Ch. 9