Assignment 2

EE, Signals and Systems 1400-2

Problem 1. For each one of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, (4) time-invariant, (5) memory-less and justify your answers: (Note that $\mathrm{Even}\{x[n]\} = \frac{x[n]+x[-n]}{2}$)

- $y(t) = \cos\big(x(t)\big)$
- $y(t) = \int_{-\infty}^{t/2} x(\tau)\mathrm{d}\tau$
- $y[n] = n\, x[n]$
- $y[n] = 2x[n]\, u[n]$
- $y[n] = \sum_{k=-\infty}^{n} x[k+2]$
- $y[n] = \mathrm{Even}\{x[n-1]\}$
- $y[n] = \log_{10}(|x[n]|)$
- $y(t) = \frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{e}^{-t}x(t))$
- $y[n] = \sin(\frac{n\pi}{2})x[n] + n\, x[n]\, \sin(\frac{n\pi}{2})$

Problem 2. Consider a system S with the input-output relationship ($x[n]$ is the input and $y[n]$ is the output) as $y[n] = x[n]\big(g[n] + g[n-1]\big)$.

(a) If $g[n] = 1$ for all n, show that S is time-invariant.

(b) If $g[n] = n$, show that S is time-varying.

(c) If $g[n] = 1 + (-1)^n$, show that S is time-invariant.

Problem 3. The energy and power of continuous-domain and discrete-domain signals are defined as:

$$E_x = \begin{cases} \int_{-\infty}^{+\infty} |x(t)|^2 \mathrm{d}t \\ \text{or} \\ \sum_{n=-\infty}^{+\infty} |x[n]|^2 \end{cases} \qquad P_x = \begin{cases} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} |x(t)|^2 \mathrm{d}t \\ \text{or} \\ \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2. \end{cases}$$

Now, a signal signal can be categorized into energy signal, power signal or neither of them, if its every, power or neither of them is finite, respectively. By considering these definitions, determine the type of the following signals and justify your answer.

- $y(t) = \frac{\sin(\pi t)}{\pi t}, \quad t \in \mathbb{R}$
- $y(t) = \mathrm{e}^{-\alpha|t|}, \quad \alpha > 0, \quad t \in \mathbb{R}$
- $y(t) = \mathrm{e}^{-2t^2}, \quad t \in \mathbb{R}$
- $y[n] = \frac{\sin(2n\pi - 5\pi)}{2n\pi - 5\pi}$
- $y(t) = \cos(t) + 3\sin(2t), \quad t \in \mathbb{R}$
- $y(t) = \frac{1}{\sqrt{n}}u(t-1), \quad t \in \mathbb{R}$

Problem 4. Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental period.

- $x(t) = \sqrt{|\sin(\frac{t}{\sqrt{2}})|}$
- $x[n] = \cos[n]$
- $x[n] = \mathrm{e}^{\mathrm{j}\frac{\pi}{3}(n-10)} + \cos(\frac{n\pi}{4})$
- $x(t) = \sinh(\pi t)\cos(3\pi t)$
- $x[n] = |\mathrm{e}^{\mathrm{j}\sin(\frac{2n\pi}{5})}|$
- $x[n] = \cos(\pi n)u[n] + \cos(\pi n)u[-n]$

Problem 5. Is $y(t)$ necessarily periodic under below conditions? If yes, determine the fundamental period

(a) $y(t)$ and $y(t-1)$ are both even. (b) $y(t)$ and $y(t-1)$ are both odd.

Problem 6. Properties of the systems

(a) Check the linearity of the following systems.

(i) $\frac{\mathrm{d}y}{\mathrm{d}t} + 2y = x^2(t)$

(ii) $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \frac{\mathrm{d}y}{\mathrm{d}t} + y = \frac{\mathrm{d}x}{\mathrm{d}t} + 2x(t)$

(iii) $y(t) = \frac{x(t)\mathrm{e}^{\mathrm{j}x(t)}}{\mathrm{j}}$

(iv) $y[n] = (\prod_{i=1}^{n} x[n-i])^{\frac{1}{n}}$

(b) Determine if each of the following systems is invertible. If they are, introduce the inverse system; otherwise, find two distinct input signals to the system that yield the same output.

(i) $y(t) = x(\frac{t}{3})$

(ii) $y[n] = x[2n]$

(iii) $y(t) = \frac{\mathrm{d}x}{\mathrm{d}t}$

(iv) $y[n] = \sum_{k=\infty}^{n} (\frac{1}{2})^{n-k} x[k]$

(v) $y(t) = \int_{-\infty}^{t} x(\tau)\mathrm{d}\tau$

(vi) $y(t) = x(5t-3)$

(vii) $y(t) = 3x(1) + x(2t-1)$

(viii) $y[n] = x[n]\,x[n+1]$

(c) Determine if each of the following systems is time-invariant.

(i) $y(t) = 2x(t) - 5$

(ii) $y(t) = x(\frac{t}{5})$

(iii) $y[n] = x[-n]$

(iv) $y[n] = x[n] - 2n$

(v) $y[n] = n\cos(\frac{n\pi}{5})x[n]$

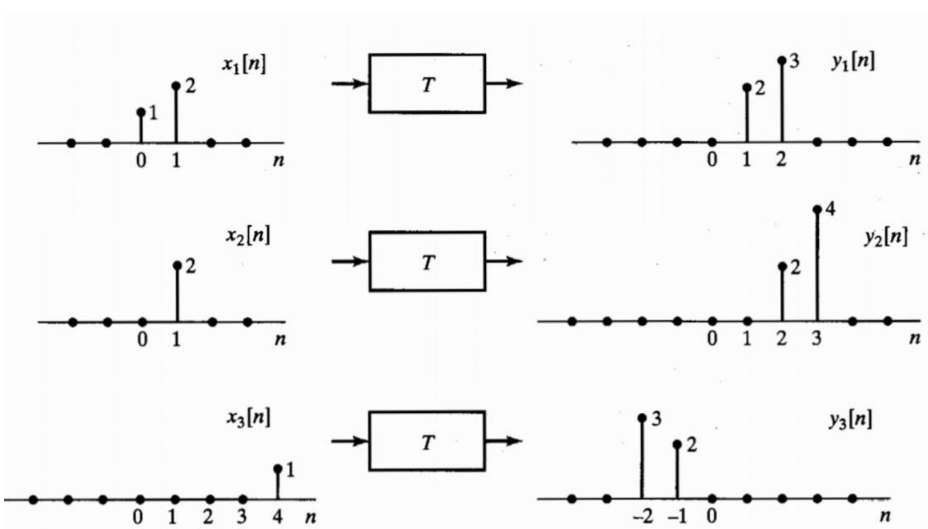
(vi) $y[n] = \sum_{k=-\infty}^{n} x[k]$

(vii) $y(t) = \begin{cases} x(t) + x(t-1) & t \geq 0 \\ 0 & t < 0 \end{cases}$

(viii) $y(t) = \begin{cases} x(t) + x(t-1) & x(t) \geq 0 \\ 0 & x(t) < 0 \end{cases}$

(ix) $y(t) = \begin{cases} x(t) + x(t-1) & x(2t) \geq 0 \\ 0 & x(2t) < 0 \end{cases}$

Problem 8. Consider a time-invariant system T. The output of this system to the three below inputs are depicted.



(a) Can this system be linear? Justify your answer.

(b) Find the output of this system to $x[n] = \delta[n]$.