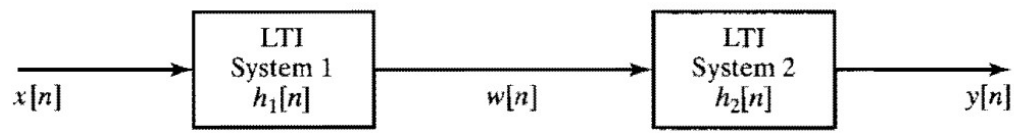
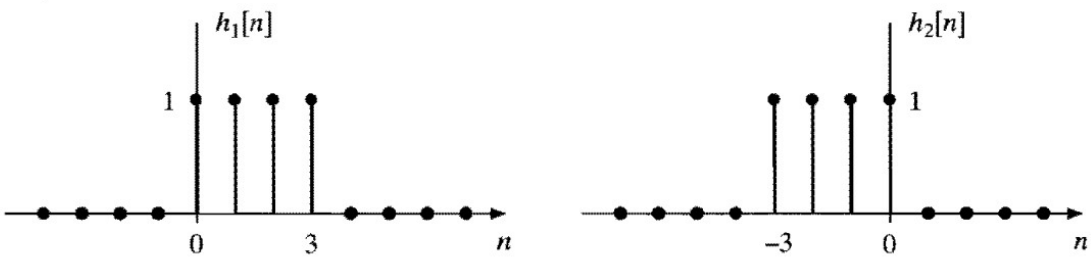
Assignment 3

EE, Signals and Systems 1400-2

Problem 1. Consider the cascade of the two LTI systems in the following figure:





(a) Determine and sketch $w[n]$ if $x[n] = (-1)^n u[n]$. Also, determine the overall output $y[n]$.

(b) Determine and sketch the overall impulse response of the cascade system.

(c) Now consider the the input $x[n] = \delta[n] + 5\delta[n-4] - 2\delta[n-8]$. Sketch $w[n]$.

(d) For the input of part (c), write an expression for the output $y[n]$ in terms of the overall impulse resposne as defined in part (b). Make a carefully labeled sketch of your answer.

Problem 2. Consider a system for which the input $x[n]$ and output y[n] satisfy the difference equation

$$y[n] - \frac{1}{3}y[n-1] = x[n]$$

and for which $y[-2]$ is constrained to be zero for every input. Determine whether or not the system is stable. Provide an argument if you state that the system is stable; otherwise, find a bounded input that results in an unbounded output.

Problem 3. Consider the three sequences:

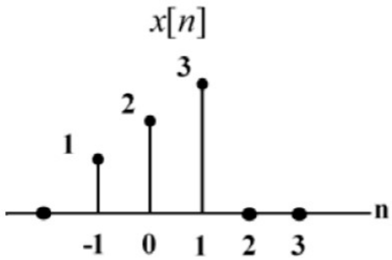
$$\begin{aligned} v[n] &= u[n] - u[n-6] \\ w[n] &= \delta[n] + 2\delta[n-2] + \delta[n-4] \\ q[n] &= \big(v * w\big)[n] \end{aligned}$$

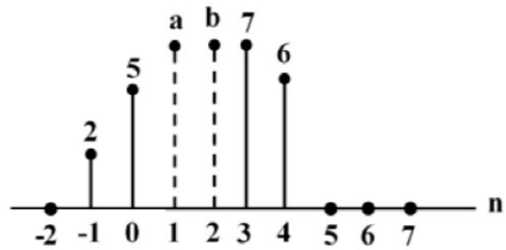
(a) Find and sketch the sequence $q[n]$.

(b) Find and sketch the sequence $r[n]$ such that $(r * v)[n] = \sum_{k=-\infty}^{n-1} q[k]$.

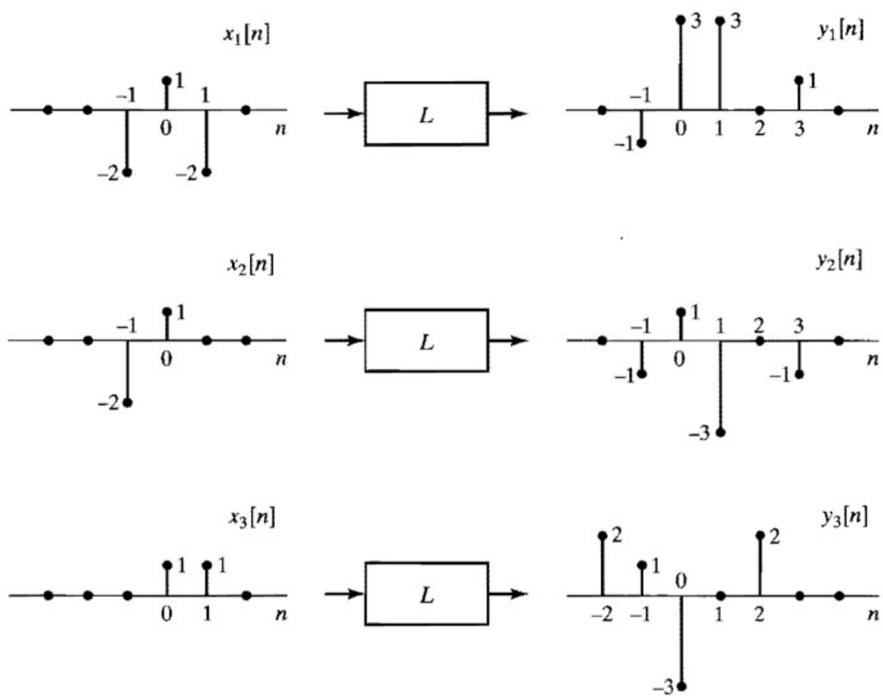
(c) If $\check{x}[n]$ denotes $x[-n]$, is it correct to write $\check{q}[n] = (\check{v} * \check{w})[n]$? Justify your answer.

Problem 4. Consider an LSI discrete-time system with an impulse response with length 4. The output corresponding to the following input is shown in the following figure. Find $y[1] = a$ and $y[2] = b$ in the following sequence.





Problem 5. The system L in the following figure is known to be *linear*. Shown are three output signals $y_1[n]$, $y_2[n]$ and $y_3[n]$ in response to the input signals $x_1[n]$, $x_2[n]$ and $x_3[n]$, respectively.



(a) Determine whether the system L could be shift-invariant.

(b) If the input $x[n]$ to the system L is $\delta[n]$, what is the system response $y[n]$?

Problem 6. Let $y(t) = \big(x * h\big)(t)$. Show the following:

- $\frac{\mathrm{d}}{\mathrm{d}t}y(t) = \big(x * h'\big)(t) = \big(x' * h\big)(t)$
- $\frac{\mathrm{d}}{\mathrm{d}t}y(t) = \int_{-\infty}^{t} \big(x' * h'\big)(\tau)\mathrm{d}\tau$
- $y(t) = \big(x_I * h'\big)(t)$ where $x_I(t) = \int_{-\infty}^{t} x(\tau)\mathrm{d}\tau$
- $y(t) = \big(x' * h_I\big)(t)$ where $h_I(t) = \int_{-\infty}^{t} h(\tau)\mathrm{d}\tau$

Problem 7. The first order differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 2y(t) = x(t)$$

in addition to the *initially at rest* conditions defines a continuous-domain system.

- Verify that the impulse response of this system is $h(t) = \mathrm{e}^{-2t}u(t)$
- Is this system
  (i) memoryless?
  (ii) causal?
  (iii) stable?
  Clearly state your reasoning.