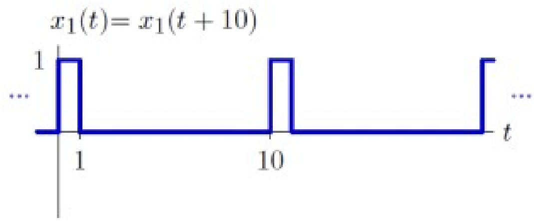
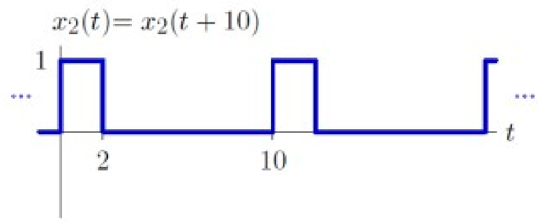
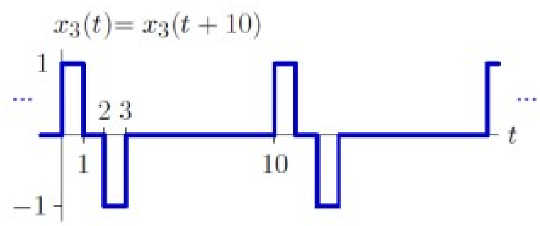
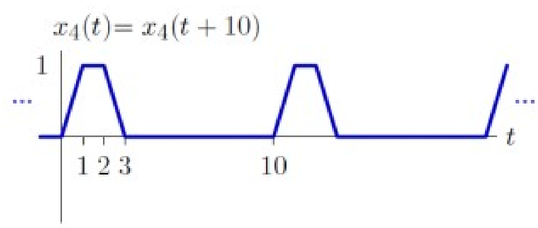
Assignment 4

Problem 1. Determine the Fourier series expansion of the signals depicted below.









Problem 2. Determine the Fourier series expansion of the following signals/functionals:

1. $x_1(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t - nT)$
2. $x_2(t) = \sum_{n=-\infty}^{\infty} \delta'(t - nT)$
3. $x_3(t) = \cos(t) + \cos(2.5t)$
4. $x_4(t) = \mathrm{e}^{t-n}$ for $n \leq t < n+1$
5. $x_5(t) = |\cos(2\pi f_0 t)|$
6. $x_6(t) = |\cos(2\pi f_0 t)| + \cos(2\pi f_0 t)$

Problem 3. Consider the signal $x(t)$ with fundametnal period $T = 4$ and Fourier series coefficients a_k. If the signal is presented as below in one period, find the result of the following expressions:

$$x(t) = 1 - \frac{t^2}{2}, \quad -2 < t < 2$$

- $\sum_{k=-\infty}^{+\infty} \mathrm{j}^k a_k$
- $\sum_{k=-\infty}^{+\infty} (-1)^k a_k$
- $\sum_{k=-\infty}^{+\infty} |a_k|^2$
- $\sum_{k=-\infty}^{+\infty} a_{2k}$

Problem 4. Let $x(t)$ and $y(t)$ be continuous-domain periodic signals with period T_0 and Fourier series representations given by

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k \mathrm{e}^{\mathrm{j}k\omega_0 t}, \quad y(t) = \sum_{k=-\infty}^{+\infty} b_k \mathrm{e}^{\mathrm{j}k\omega_0 t}$$

(a) Show that the Fourier series coefficients of the signal

$$z(t) = x(t)y(t) = \sum_{k=-\infty}^{+\infty} c_k \mathrm{e}^{\mathrm{j}k\omega_0 t}$$

are given by the discrete convolution $c_k = \sum_{n=-\infty}^{+\infty} a_n b_{k-n}$

(b) Use part (a) to compute the Fourier series coefficients of the following signal:

$$x_1(t) = \sin(20\pi t). \sum_{k=-\infty}^{+\infty} \Pi\big(\tfrac{1}{2}(t-3k)\big),$$

where $\Pi(t)$ is

$$\Pi(t) = \begin{cases} 1 & |t| \leq \frac{1}{2} \\ 0 & |t| > \frac{1}{2} \end{cases}$$

(c) Prove the following expression.

$$\frac{1}{T_0} \int_{\alpha}^{\alpha+T_0} x(t)\overline{y(t)}\mathrm{d}t = \sum_{k=-\infty}^{+\infty} a_k b_k^* \tag{1}$$

Problem 5. Show that for all periodic signals with finite power, the Fourier series coefficients a_k tend to zero as $k \to \infty$.

Problem 6. We know the following information about signal $x(t)$:

1. $x(t)$ is real-valued,

2. $x(t)$ is periodic with period $T = 6$ and Fourier coefficients a_k,

3. $a_k = 0$ for $k = 0$ and $k > 2$,

4. $x(t) = -x(t-3)$,

5. $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 \mathrm{d}t = \frac{1}{2}$, and

6. a_1 is a positive real number.

Show that $x(t) = A\cos(Bt + C)$, and find the constants A, B and C.