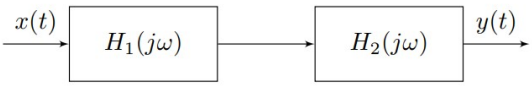
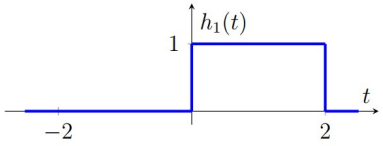
Assignment 7

EE, Signals and Systems 1400-2

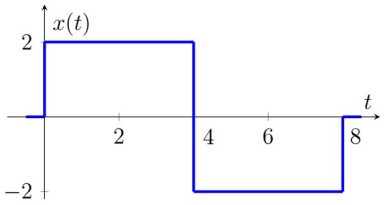
Problem 1. Consider the following LSI system.



If the impulse response of the first system is:



and the frequency response of the second system is $\widehat{h}_2(\omega) = \mathrm{j}\omega$, answer the following questions for the below input.



(a) Find $\widehat{y}(\omega)$.

(b) Find the overall impulse response of system.

(c) Find $y(t)$ and plot it.

Problem 2. The output $y(t)$ of a causal LSI system is related to the input $x(t)$ by the equation

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 20y(t) = \int_{-\infty}^{+\infty} x(\tau)z(t-\tau)\mathrm{d}\tau \; - \; x(t),$$

where $z(t) = \mathrm{e}^{-t}u(t) + 2\delta(t)$.

(a) Find the frequency response $\widehat{h}(\omega) = \widehat{y}(\omega)/\widehat{x}(\omega)$ of this system.

(b) Determine the impulse response of the system.

Problem 3. For each of the following Laplace transforms, find all the possible inverses.

- $X_1(s) = \frac{s+2}{(s+1)^2}$
- $X_2(s) = \frac{1}{s}\mathrm{e}^{-sT}$
- $X_3(s) = \frac{s+1}{(s+1)^2+4}$
- $X_4(s) = \frac{(s+1)^2-1}{[(s+1)^2+1]^2}$

Problem 4. The transfer function of an LSI system is $H(s) = k\frac{s-a}{s+b}$, where a and b are real and positive constants. We know that the response of this system to the input $x(t) = \cos(t)$ is $y(t) = \sin(t)$. What is the relation between a and b?

Problem 5.

(a) Find the Fourier transform of the following signal

$$x(t) = \frac{2}{(2-t)^2 + 3\mathrm{j}t}$$

(b) Find the Fourier series coefficients of the following continuous-domain and periodic signal

$$x(t) = \frac{1}{(2 + \mathrm{e}^{-\mathrm{j}t})^2}$$

Problem 6. Find the Laplace transform of the following signals and determine the associated region of convergence for each.

- $x_1(t) = |t|\mathrm{e}^{-t}$
- $x_2(t) = \cos(\omega_0 t + b)u(t)$
- $x_3(t) = \sin(t)u\big(\sin(t)\big)u(t)$
- $x_4(t) = \mathrm{e}^{-4t}u(t) + \mathrm{e}^{-5t}\sin(5t)u(t)$

Problem 7. The following figures represent the poles and zeros of Laplace transform of some signals. Which one of the following diagrams could represent an even signal?