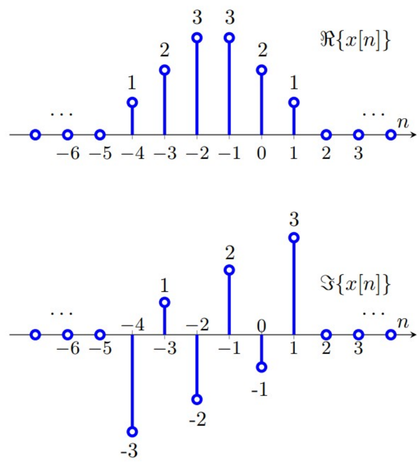
Assignment 9

EE, Signals and Systems 1400-2

Problem 1. $x[n]$ is a complex-valued signal with Fourier transform $\widehat{x}(e^{j\omega})$. The real and imaginary parts of this signal are shown in the below figure (the signal is zero outside the depicted interval). Respond to the following questions without explicitly deriving the Fourier transform of the signal.



(a) Find $\widehat{x}(e^{j\omega})|_{\omega=0}$.

(b) Find $\widehat{x}(e^{j\omega})|_{\omega=\pi}$.

(c) Find $\int_{-\pi}^{\pi} \widehat{x}(e^{j\omega})d\omega$.

(d) Find and plot the signal whose Fourier transform is $\widehat{x}(e^{-j\omega})$.

(d) Find and plot the signal whose Fourier transform is $j\Im\{x(e^{-j\omega})\}$.

Problem 2. Consider an LSI system defined by the difference equation

$$y[n] = -2x[n] + 4x[n-1] - 2x[n-2].$$

(a) Determine the impulse response of this system.

(b) Determine the frequency response of this system in the form

$$\widehat{h}(e^{j\omega}) = a(\omega)e^{-j\omega n_d},$$

where $a(\omega)$ is a real-valued function of ω. Explicitly specify $a(\omega)$ and the parameter n_d.

(c) Plot of the magnitude and phase of $\widehat{h}(e^{j\omega})$, separately.

(d) Suppose that the input to the system is

$$x_1[n] = 1 + e^{j\frac{\pi}{2}n} \quad -\infty < n < +\infty.$$

Use the frequency response to determine the corresponding output $y_1[n]$.

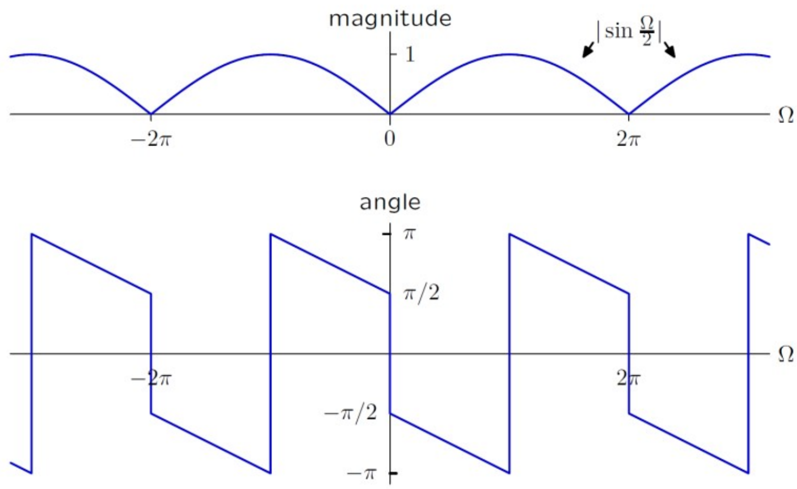


Figure 1: Figure for Problem 6.

(e) For the input

$$x_2[n] = (1 + \mathrm{e}^{\mathrm{j}\frac{\pi}{2}n})u[n] \qquad -\infty < n < +\infty$$

use the difference equation or the discrete-domain convolution to determine the output $y_2[n]$ for $-\infty < n < +\infty$. Compare $y_1[n]$ and $y_2[n]$. For which values of n do we have $y_1[n] = y_2[n]$?

Problem 3. The frequency response of a discrete-domain LSI system is given as

$$\widehat{h}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{(1-\mathrm{j}\mathrm{e}^{-\mathrm{j}\omega})(1+\mathrm{j}\mathrm{e}^{-\mathrm{j}\omega})}{1-0.8\mathrm{e}^{-\mathrm{j}\omega}} = \frac{1+\mathrm{e}^{-\mathrm{j}2\omega}}{1-0.8\mathrm{e}^{-\mathrm{j}\omega}} = \frac{1}{1-0.8\mathrm{e}^{-\mathrm{j}\omega}} + \frac{\mathrm{e}^{-\mathrm{j}2\omega}}{1-0.8\mathrm{e}^{-\mathrm{j}\omega}}.$$

(a) Using one of the above forms, find the impulse response $h[n]$ of the system.

(b) Find a difference equation that expresses the input-output relation of this system.

(c) For which value of ω_0, the input

$$x[n] = 4 + 2\cos(\omega_0 n) \quad \text{for} -\infty < n < \infty,$$

results in the constant output $y[n] = A$? What is A in this case?

Problem 4. The input-output relation of an LSI system is as follows

$$y[n] = x[n] + 2x[n-1] + x[n-2].$$

(a) Determine the impulse response $h[n]$ of the system.

(b) Is this a stable system?

(c) Determine the frequency response $\widehat{h}(\mathrm{e}^{\mathrm{j}\omega})$ of the system in the simplest form.

(d) Plot the magnitude and phase of the frequency response.

(e) Find $h_1[n]$ if $\widehat{h}_1(\mathrm{e}^{\mathrm{j}\omega}) = \widehat{h}(\mathrm{e}^{\mathrm{j}(\omega+\pi)})$.

Problem 5. An LSI system has the frequency response

$$\widehat{h}(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1 - 1.25\mathrm{e}^{-\mathrm{j}\omega}}{1 - 0.8\mathrm{e}^{-\mathrm{j}\omega}} = 1 - \frac{0.45\mathrm{e}^{-\mathrm{j}\omega}}{1 - 0.8\mathrm{e}^{-\mathrm{j}\omega}}$$

(a) Specify a difference equation describing the input-output relation of this system.

(b) Show that $|\widehat{h}(\mathrm{e}^{\mathrm{j}\omega})|$ is constant.

(c) If the input to the above system is $x[n] = \cos(0.2\pi n)$, then, the output is of the form $y[n] = A\cos(0.2\pi n + \theta)$. Find A and θ?

Problem 6. The magnitude and phase of the Fourier transform of $x[n]$ are shown in Fig. 1 Find and plot the signal $x[n]$.