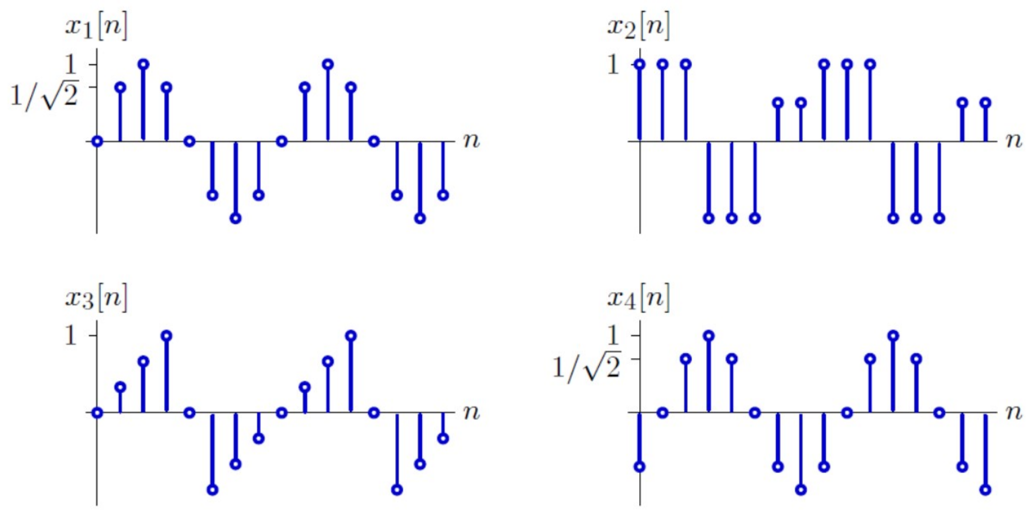
Assignment 5

Problem 1. Determine the Fourier series coefficients of the following discrete-time signals by assume the fundamental period is N=8.



Problem 2. The following periodic functional

$$x(t) = \sum_{m=-\infty}^{\infty} \delta(t-3m) + \delta(t-1-3m) - \delta(t-2-3m)$$

is the input to an LSI system that maps e^{st} into $\left(\mathrm{e}^{s/4} - \mathrm{e}^{-s/4}\right)\mathrm{e}^{st}$ (for all $s \in \mathbb{C}$). If the Fourier series coefficients of the output signal, $y(t)$, are denoted by b_k, find b_3.

Problem 3. For each set of Fourier series coefficients below, find the explicit form of the continuous-domain periodic signal with period $T = 4$ in time (determine the signal within $0 \leq t < 4$).

(1) $a_k = \begin{cases} \mathrm{j}k & |k| < 3 \\ 0 & \text{otherwise} \end{cases}$ (2) $b_k = \begin{cases} 1 & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$

Problem 4. $x[n]$ is a periodic signal with period N.

(1) Show that if N is even and $x[n] = -x\left[n + \frac{N}{2}\right]$, then, a_k would be zero for even ks. (a_k is the kth Fourier series coefficient of $x[n]$.)

(2) Show that if N is divisible by an integer m and $\sum_{r=0}^{(N/m)-1} x\left[n + r\frac{N}{m}\right] = 0$, then, a_k would be zero if k is a mutiple of m.

Problem 5.

(1) $x[n]$ and $y[n]$ are two periodic signals with period N. If the Fourier series coefficients of these signals are denoted by a_k and b_k, respectively, find (with proof) the Fourier series coefficients of the signal $z[n] = x[n]y[n]$.

(2) If $N = 5$ and for $3 \leq k < 8$ we have $a_k = (-1)^k b_k = k$, find the Fourier series coefficients of the signal $z[n]$.

Problem 6. $x(t)$ is a real periodic continuous-domain signal with period $T = 6$, Fourier series coefficients $\{a_k\}_k$, and the following properties:

- $x(t) = -x(t-3)$
- $\forall |k| > 3, \quad a_k = 0$
- $a_3 a_{-3}^* = 25$
- $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 \mathrm{d}t = 50$

Determine $x(t)$ based on the given information.