Assignment 1

EE, Signals and Systems 1400-2

Due: 1400.12.16, 17:00

Problem 1. For each one of the following systems, determine wheter the sysetm is (1) stable, (2) causal, (3) linear, (4) time invariant, (5) memoryless and justify your answers:

(i)

$$y[n] = \begin{cases} n & n \leq x[-n] \\ x[n] & n > x[-n] \end{cases}$$

(ii) $y(t) = \frac{\cos(x(t)+2t)}{x(t-1)}$

(iii) $y[n] = \sum_{k=n-3}^{n+5} x[k]$

(iv) $y(t) = \int_{-\infty}^{t} e^{-\alpha^2} x(\alpha - t) d\alpha$

Problem 2. If the reponse of a linear system to the input $x(t) = t^k$ is $y(t) = \cos(kt)$

(a) Find the reponse of the system to the input below:

$$x(t) = \frac{1+t^9}{1+t}$$

(b) If the output of the system is $y(t) = \cos(t) + 5\cos^3(t) + 8\cos^4(2t)$, find the input corresponding to this ouput.

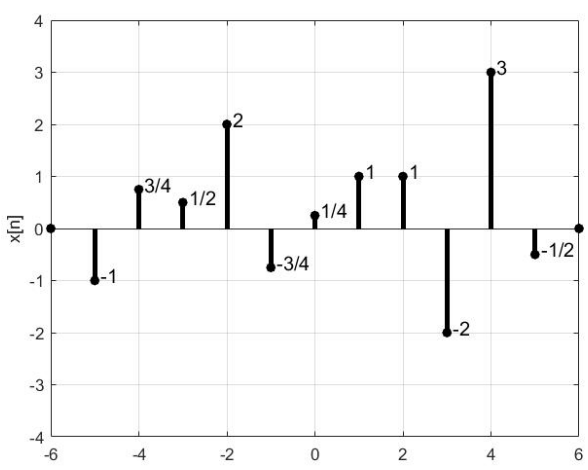
Problem 3. For $x[n]$ indicated in the below, sketch the following:

(a) $x[2-n]$

(b) $x[(n-1)^2]$

(c) $x[n]u[3-n]$

(d) $x[3n+1]$



Problem 4. Let $x[n] = \cos(\omega_x(n + m_x) + \theta_x)$.

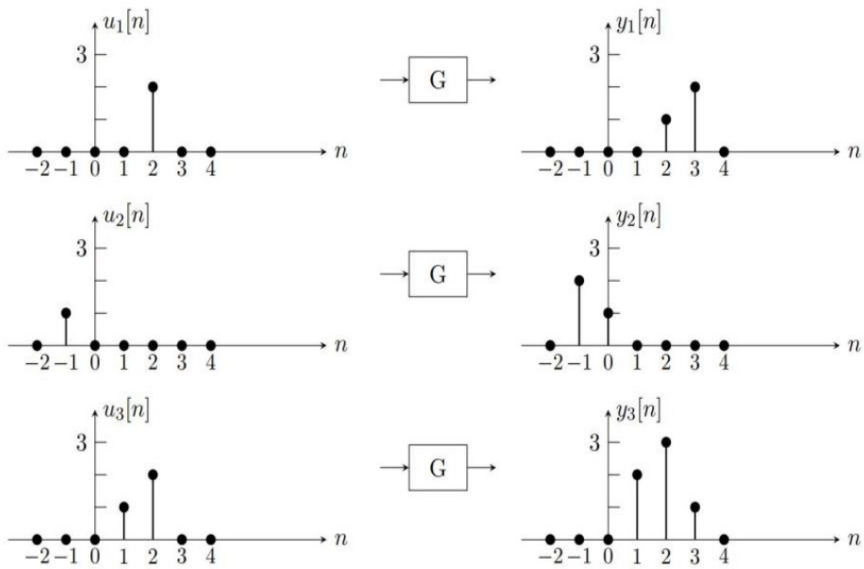
(a) Determine the frequency in hertz and the period of $x[n]$ for each of the following three cases:

	ω_x	m_x	θ_x
(i)	$\frac{\pi}{3}$	0	2π
(ii)	$\frac{3\pi}{4}$	2	$\frac{\pi}{4}$
(iii)	$\frac{3}{4}$	1	$\frac{1}{4}$

(b) With $x[n] = cos\,(\omega_x(n + m_x) + \theta_x)$ and $y[n] = cos\,(\omega_y(n + m_y) + \theta_y)$, determine for which of the following combinations $x[n]$ and $y[n]$ are identically equal.

	ω_x	m_x	θ_x	ω_y	m_y	θ_y
(i)	$\frac{\pi}{3}$	0	2π	$\frac{8\pi}{3}$	0	0
(ii)	$\frac{3\pi}{4}$	2	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	1	-π
(iii)	$\frac{3}{4}$	1	$\frac{1}{4}$	$\frac{3}{4}$	0	1

Problem 5. Consider the discrete-time system G which is time invariant. Using the following 3 input-output diagrams, prove that this system is not linear.



Problem 6. Let $x(t) = \sqrt{2}(1 + \mathrm{j})\mathrm{e}^{\mathrm{j}\frac{3\pi}{4}}\mathrm{e}^{(-1+\mathrm{j}\pi)t}$. Sketch and label the following:

(i) $\Re\{x(t)\}$

(ii) $\Im\{x(t)\}$

(iii) $x(t + 1) + x^*(t + 1)$

Problem 7. Let $x(t)$ be the continuous-domain complex exponential signal $x(t) = \mathrm{e}^{\mathrm{j}\omega_0 t}$ with fundamental frequency ω_0 and fundamental period $T_0 = \frac{2\pi}{\omega_0}$. Consider the discrete-domain signal obtained by taking equally spaced samples of $x(t)$. That is, $x[n] = x(nT) = \mathrm{e}^{\mathrm{j}\omega_0 nT}$.

(a) Show that $x[n]$ is periodic if and only if $\frac{T}{T_0}$ is a rational number, that is, if and only if some multiple of the sampling interval *exactly equals* a multiple of the period $x(t)$.

(b) Suppose that $x[n]$ is periodic, that is, $\frac{T}{T_0}$, where p and q are integers. What are the fundamental period and fundamental frequency of $x[n]$? Express the fundamental frequency as a fraction of $\omega_0 T$.

(c) Again assuming that $\frac{T}{T_0} = \frac{p}{q}$, determine precisely how many periods of $x(t)$ are needed to obtain the samples that form a single period of $x[n]$.