Assignment 10

Problem 1. Determine the inverse $\mathcal{Z}$-transform of

$$X(z) = \ln(1 - 2z), \quad |z| < \tfrac{1}{2}$$

(a) by using tie power series $\ln(1 - x) = -\sum_{m=1}^{\infty} \frac{x^m}{m}, \quad |x| < 1,$

(b) by first differentiating $X(z)$ and then using the derivative to recover $x[n]$.

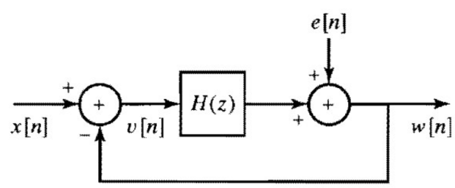
Problem 2. $H(z)$ is the system function of the causal LSI system in the below figure.

(a) Using the $\mathcal{Z}$-transform of signals in the figure, obtain an expression for $W(z)$ in the form

$$W(z) = H_1(z)X(z) + H_2(z)E(z),$$

where $H_1(z)$ and $H_2(z)$ are expressed in terms of $H(z)$.

(b) Is $H(z)$ stable? Are $H_1(z)$ and $H_2(z)$ stable?



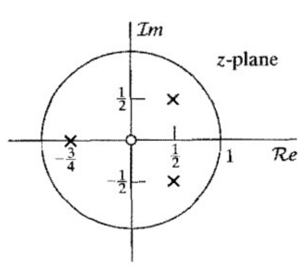
Problem 3. For each of the following signals, determine the $\mathcal{Z}$-transform and its ROC, and sketch the pole-zero diagram:

(a) $x[n] = a^n\, u[n] + b^n\, u[n] + c^n\, u[-n-1], \quad |a| < |b| < |c|$

(b) $x[n] = n^2 a^n\, u[n]$

(c) $e^{n^4}[\cos(\frac{\pi}{12}n)]u[n] - e^{n^4}[\cos(\frac{\pi}{12}n)]u[n-1]$

Problem 4. The pole-zero diagram in the following figure corresponds to the $\mathcal{Z}$-transform $X(z)$ of a causal signal $x[n]$. Sketch the pole-zero diagram of $Y(z)$, where $y[n] = x[-n+3]$. Also, specify the ROC for $Y(z)$.



Problem 5. The LSI system $\mathcal{S}$ is such that

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n] + 2^n u[-n-1] \xmapsto{\mathcal{S}} y[n] = 6\left(\tfrac{1}{2}\right)^n u[n] - 6\left(\tfrac{3}{4}\right)^n u[n]$$

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(a) Find the system function $H(z)$ of the $\mathcal{S}$. Plot the poles and zeros of $\mathcal{S}$, and indicate its ROC.

(b) Find the impulse response $h[n]$ of the system.

(c) Write a difference equation that characterizes the system.

(d) Is the system stable? Is it causal?

Problem 6. A causal and stable LSI system $\mathcal{S}$ has its input $x[n]$ and output $y[n]$ related by the linear constant-coefficient difference equation

$$y[n] + \sum_{k=1}^{10} \alpha_k y[n-k] = x[n] + \beta x[n-1].$$

Let $h[n]$ be the impulse response of $\mathcal{S}$.

(a) Show that $h[0]$ is nonzero.

(b) Show that α_1 can be determined by knowing β, $h[0]$, and $h[1]$.

(c) If $h[n] = (0.9)^n \cos(\pi n/4)$ for $0 \leq n \leq 10$, sketch the pole-zero plot for $\mathcal{S}$, and indicate its ROC.