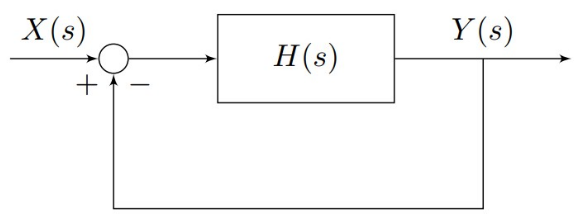
Assignment 8

Problem 1. Find the LSI system with real impulse response and transfer function $H(s)$ that satisfies:

- The response of the system to e^t is e^t.
- The system has three finite poles and no finite zero.
- The system has a pole at $s = \mathrm{j}$.
- The step response of the below system is asymptotically 1 at $t \to +\infty$.



Problem 2. The signal $y(t) = \mathrm{e}^{-2t}u(t)$ is the output of a causal all-pass system for which the transfer function is $H(s) = \frac{s-1}{s+1}$

(a) Find and sketch at least two possible inputs $x(t)$ that result in $y(t)$ as the output.

(b) What is the input $x(t)$ if it is known that

$$\int_{-\infty}^{+\infty} |x(t)|\mathrm{d}t < \infty$$

(c) What is the input $x(t)$ if it is known that a stable (but not necessarily causal) system exists that outputs $x(t)$ for the input $y(t)$? Find the impulse response $h(t)$ of this filter, and show by direct convolution that it has the claimed property (i.e., $\big(y * h\big)(t) = x(t)\big)$.

Problem 3. Find the impulse response of the following systems defined by differential equations. Assume that all systems are initially at rest.

(a) $\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 11\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 24y(t) = 5\frac{\mathrm{d}}{\mathrm{d}t}x(t) + 3x(t)$

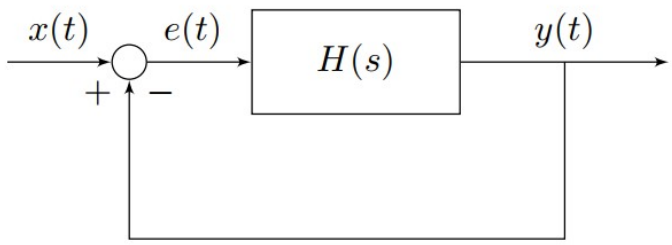
(b) $\frac{\mathrm{d}^4}{\mathrm{d}t^4}y(t) + 4\frac{\mathrm{d}}{\mathrm{d}t}y(t) = 3\frac{\mathrm{d}}{\mathrm{d}t}x(t) + 2x(t)$

Problem 4. We have an LSI system with transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s}$$

where ω_n and ξ are constants. We add a unit-gain feedback to this system as shown in the following figure. Assume that the input to the overall system is the unit-step signal and define the error signal as:

$$e(t) = x(t) - y(t)$$



(a) Evaluate $I_1 = \int_0^{+\infty} te'(t)\mathrm{d}t$.

(b) Assuming $\xi > 1$, find $I_2 = \int_0^{+\infty} |te'(t)|\mathrm{d}t$.

Problem 5. $X(s)$ is the Laplace transform of $x(t)$ about which we know

(1) $x(t)$ is real and even.

(2) $X(s)$ has four poles and no zeros in the finite s-plane.

(3) $X(s)$ has a pole at $s = \frac{1}{2}\mathrm{e}^{\mathrm{j}\pi/4}$.

(4) $\int_{-\infty}^{+\infty} x(t)\mathrm{d}t = 4$.

Determine $x(t)$ and its ROC.

Problem 6. $x(t)$ and $y(t)$ are right-sided signals that satisfy

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = -2y(t) + \delta(t),$$
$$\frac{\mathrm{d}}{\mathrm{d}t}y(t) = 2x(t).$$

Determine $Y(s)$ and $X(s)$, along with their regions of convergence.