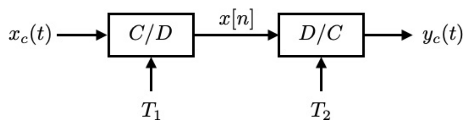
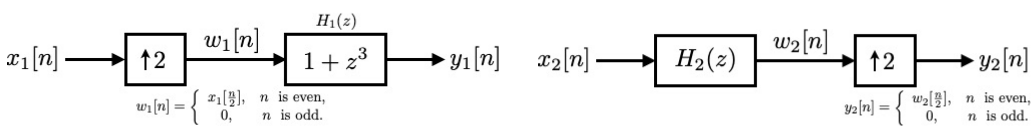
Assignment 11

Problem 1. In the following figure, assume that $\widehat{x}_c(\omega) = 0, |\omega| \geq \pi/T_1$. For the general case, in which $T_1 \neq T_2$ in the system, express $y_c(t)$ in terms of $x_c(t)$. Is the basic relationship different for $T_1 > T_2$ and $T_1 < T_2$?

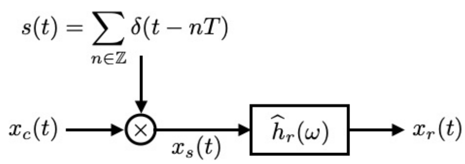


Problem 2. For the systems shown below, determine whether or not it is possible to specify $H_2(z)$ in System 2 so that $y_2[n] = y_1[n]$ whenever $x_2[n] = x_1[n]$; if yes, specify $H_2(z)$.



System 1 System 2

Problem 3. Consider the sampling procedure as shown below: where the frequency



response of the reconstruction filter is

$$\widehat{h}_r(\omega) = \begin{cases} T & \text{if } |\omega| \leq \frac{\pi}{T} \\ 0 & \text{if } |\omega| > \frac{\pi}{T} \end{cases}$$

Now, for the input signal

$$x_c(t) = 2\cos\left(100\pi t - \tfrac{\pi}{4}\right) + \cos\left(300\pi t + \tfrac{\pi}{3}\right) \quad -\infty < t < \infty$$

(a) Determine the continuous-domain Fourier transform $\widehat{x}_c(\omega)$ and plot it as a function of ω.

(b) Plot the Fourier transform $\widehat{x}_s(\omega)$ as a function of ω for $-\frac{2\pi}{T} \leq \omega \leq \frac{2\pi}{T}$ given $f_s = \frac{1}{T} = 500$ samples/sec. What is the output $x_r(t)$ in this case? (You should be able to give an exact expression for $x_r(t)$)

(c) Repeat part (b) for $f_s = \frac{1}{T} = 250$ samples/sec.

(d) Is it possible to choose the sampling rate such that

$$x_r(t) = A + 2\cos\left(100\pi t - \tfrac{\pi}{4}\right),$$

where A is a constant? If yes, what are the sampling rate $f_s = \frac{1}{T}$ and the parameter A?

Problem 4. In the below system, $\widehat{x}_c(\omega)$ and $\widehat{h}\left(e^{j\omega}\right)$ are as shown. Sketch and label the Fourier transform of $y_c(t)$ for each of the following cases:

(a) $\frac{1}{T_1} = \frac{1}{T_2} = 10^4$

(a) $\frac{1}{T_1} = \frac{1}{T_2} = 2 \times 10^4$

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