Media Engineering and Technology Faculty
German University in Cairo

A Virtual Environment for Partial-Order Planning

Bachelor Thesis

Author: Mohamed Ayman Tammaa
Supervisors: Assoc. Prof. Haythem Ismail
Submission Date: 19 May, 2024

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This is to certify that

(i) the thesis comprises only my original work toward the Bachelor degree, and

(ii) due acknowledgement has been made in the text to all other material used.

Mohamed Ayman Tammaa
19 May, 2024

Acknowledgements

Text

Abstract

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Chapter 1

Introduction

1.1 Section Name

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1.2 Another Section

Reference to Section 1.1, and reuse of **ac!** nad **ac2!** with also full use of **ac2!** (**ac2!**).

Chapter 2

Background

Planning in Artificial Intelligence (AI) is a fundamental aspect in the field that allows agents to formulate sequences of actions and strategies to achieve a specific goal. It is used in a wide range of fields where agents need to make decisions and take actions based on the current state of the environment.

Classical planning is a type of planning that is used in AI to solve problems that can be represented as a set of states and actions. It deals with straightforward actions & with predictable and deterministic environments, where the agent can predict the outcome of its actions. The challenge in classical planning is to construct a sequence of actions that will transform the initial state of the environment into a desired goal state, while dealing with exponential growth in the search space, and dealing with the actions and steps in chronological order.

Among the different types of planning, we have **state space planning** and **plan space planning**. (They will be discussed in more details in the Key Definition section 2.1) In this thesis, we will focus on plan space planning, and more specifically on Partial Order Planning (POP).

2.1 Key Definitions and Concepts

Here we will tackle some key definitions and concepts that are essential to understand the POP algorithm.

2.1.1 Planning

Planning is the process of formulating a sequence of actions and strategies to achieve a specific goal. It is a fundamental aspect of AI that allows agents to make decisions and take actions based on the current state of the environment. All definitions and concepts in this section are based on the book "Artificial Intelligence: A Modern Approach" by Stuart

Russell and Peter Norvig [9], the book "Automated Planning: Theory & Practice" by Malik Ghallab et al. [7], and on the lecture slides of the course "Introduction to Artificial Intelligence" by Prof. Haythem Ismail at the German University in Cairo [4]. Now, in the following list, we will look at some key definitions and concepts in the field of planning:

- **Agent**: An Agent is an entity that can perceive its environment, make decisions, and take actions to achieve a specific goal. An agent can be a robot, a computer program, or a human being. It can be a simple agent that follows a set of rules, or it can be a complex agent that uses AI to learn and adapt to its environment. For example, a self-driving car is an agent that uses sensors to perceive its environment, a specialized robot is an agent that can sense the environment and perform the required steps to achieve a specific goal.

- **State**: A State is a snapshot of the environment at a specific point in time. It represents the current configuration of the environment, including the location of objects, the status of objects, and the relationships between objects. In planning contexts, it can be represented as a set of propositions. For example, consider a robot that needs to navigate through a maze. The state of the robot can be represented as the location of the robot in the maze, the location of the walls, the location of the obstacles, and the location of the goal. The state of the robot changes as the robot moves through the maze, and the robot needs to update its state to reflect the changes in the environment.

- **State Space Planning**: State Space Planning is a type of planning that is used in AI to solve problems searching through a set of states and actions. In this type of planning, the world is represented as a set of states, and the agent can move from one state to another by applying actions. While the agent is searching, it represents the world as a graph, where the nodes are states and the edges are actions. It has the ability to expand any node using the available actions, and it can backtrack if it reaches a dead-end. When the agent reaches any node, it runs a goal test to check if the current state is the goal state. If the goal test is successful, the agent stops and returns the solution. If the goal test fails, the agent continues searching and expanding until it finds a solution. For example, consider a robot that needs to pickup a package from one location and deliver it to another location. The robot can represent the world as a set of states, where the initial state that the robot is $At(location_A)$, and the goal state is to satisfy the condition Delivered($package, location_B$).The robot can move from one state to another by applying actions like Move($location_A$, $location_B$) and Pickup($package$, $location_C$) and Deliver($package$, $location_B$). Of course, there are some constraints and preconditions that the robot needs to satisfy before applying any action. For example, the robot cannot deliver a package if it has not picked it up first, and the robot cannot pickup a package from a location if the robot is not at that location. One way to solve this problem is to start from the goal state and work backward to the initial state, applying the actions in reverse order, and this is called backward state space planning. To achieve Delivered($package$, $location_B$),

the robot needs to Deliver(*package*, $location_B$), and to deliver the package, the robot needs to Pickup(*package*, $location_C$) & Move($Location_C$, $Location_B$), and to pickup the package, the robot needs to Move($location_A$, $location_C$). So, the robot needs to apply the actions Move($location_A$, $location_C$), Pickup(*package*, $location_C$), Move($Location_C$, $Location_B$), and Deliver(*package*, $location_B$) in this order to achieve the goal state Delivered(*package*, $location_B$).

- **Total-Order Plan**: A Total-Order Plan is a plan that specifies a total order of actions. This means that it gives you a specific sequence of actions that need to be executed in a specific and strict order to achieve a specific goal. It does not give you the freedom or choices while executing the actions. For example, *LeftSock*() → *RightSock*() → *LeftShoe*() → *RightShoe*(), is a total-order plan that specifies a strict order to achieve *wearing both shoes*. Other valid solutions may involve interchanging the order of the socks or the shoes as long as the socks are worn before the shoes. In Figure 2.1, we can see 6 different total-order plans and each one of them can be considered a linearization of the partial order plan in Figure 2.2.

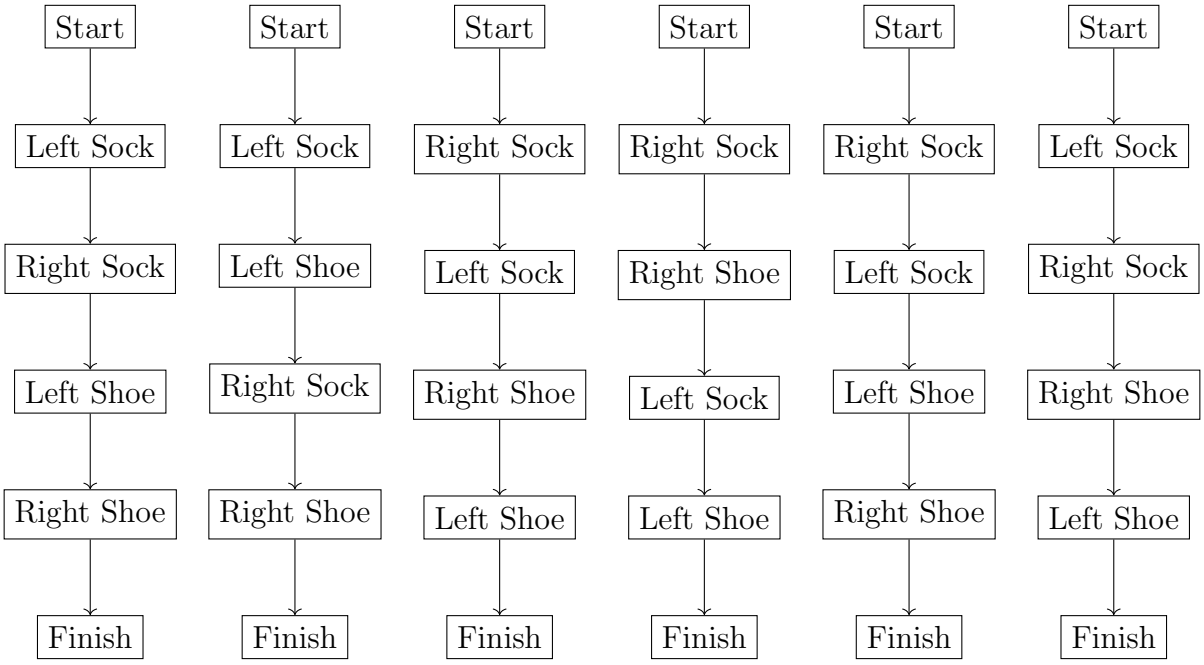

Figure 2.1: Socks & Shoes Total Order Plan solutions.

- **Partial-Order Plan**: A Partial-Order-Plan is a plan that does not specify a total order of actions, but instead specifies a partial order of actions. This means that it gives you a general structure of what needs to be done with some constraints and ordering between them, but some steps/actions are not given a specific order. This gives the agent the freedom to execute some actions in any order, as long as the constraints & general structure are satisfied. For example, consider the problem of putting on socks and shoes. The general structure is to put on the socks first, then

the shoes. But, the order in which you put on the left and right sock and shoe can vary. One person might put on the left sock first, then the right sock, then the left shoe, and finally the right shoe. Another person might interchange the order of the 2 socks or the 2 shoes, and so on. The partial plan does not specify the actual order of the final actions. (See Figure 2.2)

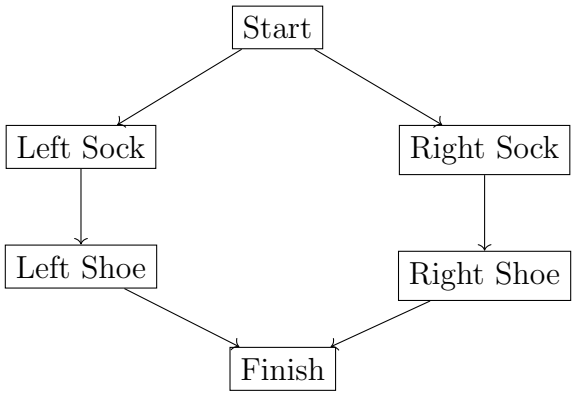

Figure 2.2: Socks & Shoes Partial Order Plan solution.

- **Plan Space Planning**: Plan Space Planning is a type of planning that is used in AI to solve problems searching through a set of partial plans and actions. In this type of planning, the world is represented as some nodes, each node represents a partial plan, and the edges represent the actions that can be applied to move from one partial plan to another. The agent can explore the search space by expanding the nodes and applying the actions, can also backtrack, and it can explore multiple paths in parallel. The agent can also use problem decomposition to break down the problem into smaller subproblems, solve them independently, and then merge the subplans into a final plan. For example, consider the problem of cooking a meal. The agent can represent the world as a set of partial plans, where each partial plan represents a step in the cooking process. The agent can explore the search space by expanding the partial plans and applying the actions to move from one partial plan to another. The agent can use problem decomposition to break down the cooking process into smaller subproblems, such as chopping vegetables, boiling water, and frying meat. The agent can solve each subproblem independently and then merge the subplans into a final plan to cook the meal.

- **Least Commitment Strategy**: Least Commitment Strategy is a strategy that does not require the planner to commit to a specific order of actions. Instead, the planner can choose to leave some actions unordered, and the planner can choose to order actions only when necessary. This allows the planner to explore a larger space of possible plans, and it allows the planner to find plans that are more flexible and more robust.

- **Operator**: An Operator is a tuple $o = (name, preconds, effects)$ where:

  - $name$ is the name of the operator.
  - $preconds$ is a set of preconditions that need to be satisfied before applying the operator.
  - $effects$ is a set of effects that will be achieved after applying the operator.

  Example: $Buy(item, store)$, $preconds = \{At(store)\}$, $effects = \{Has(item)\}$.

- **Action**: An Action is a partially instantiated operator, (i.e. any ground instance of an operator with some of its variables instantiated). Example: $Buy(item, store)$, $Buy(Oranges, Market)$, $Buy(Phone, store)$, are all actions.

- **Achiever**: An Achiever is an operator that can achieve a specific precondition. For example, if we have a precondition p_j that needs to be satisfied, the achiever of p_j is an operator that has p_j in its effects, and their Most General Unifier (MGU) is consistent with the bindings in the plan.

- **Causal Link**: A Causal Link is in the form of $(a_i \xrightarrow{P_j} a_j)$ where:

  - a_i is an action.
  - a_j is another action linked to a_i.
  - P_j is a precondition of a_j, and at the same time an effect of a_i.

  An action a_i is said to achieve a precondition p_j if $p_j \in \text{effects}(a_i)$ and $p_j \in \text{preconds}(a_j)$. A causal link is initiated to ensure that there is another action, before the action with this precondition, that can satisfy it.
  Example: $Go(store) \xrightarrow{At(store)} Buy(item, store)$, where $Go(store)$ achieves the precondition $At(store)$ for the action $Buy(item, store)$.

- **Binding Constraints**: Binding Constraints is a set of constraints that bind variables to values. For example, if we have a binding $B = \{x \leftarrow \{1, 5\}, y \leftarrow 2\}$, this means that the variable x is bound to the values 1 or 5, and the variable y is bound to the value 2.

- **Planning Problem**: A Planning Problem is a problem that can be represented as a set of states and actions. The goal of the planning problem is to find a sequence of actions that will transform the initial state of the environment into a desired goal state. The planning problem can be solved using classical planning algorithms, such as state space planning and plan space planning. A planning problem can be represented as a tuple $P = (O, s_0, g)$ [7] where:

  - O is a set of operators.
  - s_0 is the initial state.
  - g is the goal state.

  Example: $Problem =$ (

  - Operators: $\{Buy(item, store), Go(somewhere)\}$
  - Initial State: $\{At(Home)\}$
  - Goal State: $\{Has(Oranges), At(Home)\}$

).
  This planning problem represents a simple problem where the agent needs to buy some oranges from the store and go back home.

- **Ordering Constraint ($\prec$)**: An Ordering Constraint is a constraint that specifies the order of actions in a partial order plan. It is in the form of $a_i \prec a_j$, which means that action a_i must be executed before action a_j. The ordering constraint is used to ensure that the actions are executed in the correct order and to prevent conflicts between actions.

- **Partial Plan**: A Partial Plan is a tuple $\pi = (A, \prec, B, L)$ where:

  - A is a set of actions, or partially instantiated Operators.
  - $\prec$ is a set of ordering constraints between actions in the form of $a_i \prec a_j$
  - B is a set of bindings.
  - L is a set of causal links.

  are the components of a partial plan.

- **Consistency of Partial Plans**: A partial order plan $\pi = (A, \prec, B, L)$ is consistent if it satisfies the following conditions: [7]

  - The transitive closure of the ordering constraints $\prec$ is a strict partial order.
  - every subsitution σ which binds a variable x to a value in its allowed domain D_x is consistent with the bindings in all other constraints in B.

- **Threat**: A Threat in a partial plan is an action that could potentially undo the effects of another action. A threat is a potential problem in a partial plan that needs to be resolved to make the plan consistent. An action a_k threatens a causal link $(a_i \xrightarrow{P_j} a_j)$ if it has the following properties:

  - $e_k (\in$ effect of $a_k)$ unifies with $\neg p_j$.
  - the MGU of e_k and $\neg p_j$ is consistent with the bindings in B.
  - $\prec \cup \{a_i \prec a_k, a_k \prec a_j\}$ is consistent.

  Once the 3 conditions are met, we can say that a_k threatens the causal link $(a_i \xrightarrow{P_j} a_j)$. Example: $Go(Cinema)$ threatens the causal link $(Go(Store) \xrightarrow{At(Store)} Buy(Oranges, Store))$, as it can negate the effect of being $At(Store)$.

- **Partial Plan Completion**: A partial plan $\pi = (A, \prec, B, L)$ is complete if all the preconditions of the operators or actions in A are satisfied and causally linked, and all causal links in L are not threatened by any action in A.

2.2 Partial Order Planning

Partial Order Planning (POP) is a plan-space search algorithm (See definition in Section 2.1.1) that searches through a set of partial plans. Unlike other planning algorithms, POP gives partial-ordered plans, which means that it does not specify a total order of actions. This gives it the advantage over total order planning algorithms that it can use problem decomposition, work on several subproblems in parallel independently, solve them with several subplans, and then merge the subplans into a final plan.

POP uses least commitment strategy that does not require the planner to commit to a specific order of actions, as defined in Section 2.1.1. In addition, POP uses closed-world assumption, which means that the planner assumes that everything that is not known or not stated is false. This allows the planner to make assumptions about the world and to make decisions based on the available information. POP is also sometimes represented as a Directed Acyclic Graph (DAG), where the nodes represent actions, and the edges represent the causal links or the ordering constraints between actions. (See Figure 2.2 for an example of a DAG representing a partial order plan).

2.2.1 POP Algorithm

The Partial Order Planning (POP) algorithm works by incrementally building a partial order plan, and then refining the plan by adding operators, ordering constraints, bindings, and causal links to resolve threats and make the plan consistent. The planner keeps track of a set of pairs (a_i, p_i), under the name of *agenda*, where a_i is an action and p_i is a precondition of a_i. The *agenda* is used to keep track of the unsatisfied preconditions in the plan.

Overview of the POP Algorithm Execution

The POP algorithm starts by calling the POP function with the initial state s_0 and the goal state g, and some set of operators O. The POP function then initializes the plan with the initial and goal actions (a_0 & a_∞ respectively). The initial action (often referred to as Start) does not have any preconditions, just have effects, and these effects correspond to the initial state of the world. The goal action (often referred to as Finish) does not have any effects, just have preconditions, and these preconditions correspond to the goal state of the world (the conditions you need to meet to finish). Finally, the POP function initializes the agenda with the preconditions of the goal action a_∞. For each precondition p_i in the agenda, $POP()$ adds the pair (a_i, p_i) to the agenda. Then the algorithm calls the POP1 function to start building the plan. The POP1 function selects any action from the agenda, randomly or based on some heuristic, then removes it from the agenda. It then finds the achievers of the selected action, and if there are no achievers, it returns a failure, as there are some condition that cannot be satisfied. If there are achievers, it nondeterministically chooses one of the achievers and adds it to the plan, as in line 14 of

Algorithm 1 POP Algorithm

Function POP(O, s_0, g)
Ensure: a plan
return POP1($\{a_0, a_\infty\}, \{a_0 \prec a_\infty\}, \emptyset, \emptyset, \{a_\infty\} \times$ Preconds(a_∞))

Function POP1($\pi = (A, L, \prec, B)$, agenda)
Ensure: a plan
if agenda $== \emptyset$ **then**
  return π
end if
Select any pair (a_i, p_i) and remove it from agenda
achievers $\leftarrow$ the set of operators achieving (a_i, p_i)
if achievers $== \emptyset$ **then**
  return *failure*
end if
Nondeterministically choose some operator $a_j \in$ achievers
$L \leftarrow L \cup \{\langle a_j \xrightarrow{P_i} a_i \rangle\}$
Update $\prec$ with $a_j \xrightarrow{P_i} a_i$
Update B with binding constraints of this link
if $a_j \notin A$ **then**
  $A \leftarrow A \cup \{a_j\}$
  Update $\prec$ with $a_0 \prec a_j$ and $a_j \prec a_\infty$
  agenda $\leftarrow$ agenda $\cup \{(a_j, p_j) | p_j \in$ preconds$(a_j)\}$
end if
$\pi \leftarrow$ RESOLVE-THREATS(π, a_j, $\langle a_j \prec a_i \rangle$)
return POP1(π, agenda)

Function RESOLVE-THREATS($\pi = (A, L, \prec, B)$, a_l, L)
Ensure: a plan
for each threat a_k on $(a_i \xrightarrow{P_j} a_j)$, where $a_k = a_l$ or $(a_i \xrightarrow{P_j} a_j) = l$ **do**
  Nondeterministically choose one of the following:
  - Update $\prec$ with $a_k \prec a_i$
  - Update with $a_j \prec a_k$
  - Add a binding constraint to B which renders p_i nonunifiable with any threatening effect of a_k.
end for
return π

the algorithm 1 (we will talk about nondeterminism in more details in subsection 3.1.1). It then updates the ordering constraints, bindings, and causal links in the plan in lines 15 to 17. $POP1()$ then checks if the selected action is already in the plan, if not, it does three things: adds the new action to the set of actions A, updates the ordering constraints with the new action to be ordered between the *Start* and *Finish* actions, and adds the preconditions of the new action to the agenda. Finally, it resolves any threats in the plan in line 23 and calls the POP1 function recursively with the updated plan and agenda. The POP1 function continues to build the plan until the agenda is empty, at which point it returns the plan as the solution. If at any point the algorithm encounters a failure, it backtracks and tries a different path. The algorithm continues to explore the search space until it finds a valid plan or exhausts all possible paths.

Threats Resolution

Threats are potential problems in a partial plan that need to be resolved to make the plan consistent. A threat occurs when an action could potentially undo the effects of another action. The POP algorithm uses a threat resolution mechanism to resolve threats and make the plan consistent. The algorithm checks for threats in the plan and resolves them by adding ordering constraints, bindings, or causal links to prevent the threat from occurring. The function **RESOLVE-THREATS** takes a partial plan $\pi = (A, L, \prec, B)$, an action a_l, and a causal link L as input and resolves any threats in the plan. The function gets any threat a_k that threatens the causal link L, or any causal link that a_l itself threatens. Letting a_k be the action that threatens any of the causal links, and $link = (a_i \xrightarrow{P_j} a_j)$ be the causal link that a_k threatens, the function nondeterministically chooses one of the following options to resolve the threat: update the ordering constraints with $a_k \prec a_i$ (called **Demotion**), update with $a_j \prec a_k$ (called **Promotion**), or add a binding constraint to B that renders p_i nonunifiable with any threatening effect of a_k. The function then returns the updated plan with the resolved threats. The threat resolution mechanism is an essential part of the POP algorithm, as it ensures that the plan is consistent and that the actions are executed in the correct order.

2.3 Unity 3D Engine

Unity is a cross-platform game engine developed by Unity Technologies. It was first announced and released in June 2005 at Apple Inc.'s Worldwide Developers Conference as a Mac OS X-exclusive game engine. The engine has since been gradually extended to support more than 25 platforms. It is used to create interactive 2D, 3D, VR, and AR applications. The engine can be used to create both three-dimensional and two-dimensional games as well as simulations for its many platforms. Unity's core advantages are its ease of use, and flexibility. Unity is a powerful engine that is used by developers around the world to create a wide range of games and applications. One of the strongest features of Unity is its built-in physics engine, which allows developers to create realistic physics simulations in their games in a short amount of time.

2.3.1 Virtual Reality (VR) in Unity

Unity is a popular game engine that is used to create VR applications. Unity provides a wide range of tools and features that make it easy to create VR experiences. It supports a wide range of VR devices, including the Oculus Rift, HTC Vive, and PlayStation VR. It also provides a set of tools and APIs that make it easy to create VR applications. Unity's VR support includes features such as stereoscopic rendering, head tracking, and motion controllers. Virtual Reality (VR) now has a wide range of applications, from games and simulations to training and education. It is a powerful tool that allows developers to create immersive and interactive experiences for users.

2.3.2 Visualization of the Partial Order Planning (POP) Algorithm in Unity

In this thesis, Unity was used to visualize the POP algorithm. I created a 3D environment that represents the planning problem, and I used Unity's physics engine to simulate the actions and effects of the operators as a self arranging DAG in the plan. I also used Unity's VR support to create an immersive VR experience that allows users to interact with the planning problem in a virtual environment. The goal of this visualization is to provide a more intuitive and interactive way to understand the POP algorithm and how it works for new learners. By visualizing the algorithm in Unity, we hope to make it more accessible and engaging for users, and to help them better understand the concepts and principles behind the POP algorithm.

Chapter 3

Design and Implementation

In this chapter, we will discuss the design and implementation of the POP algorithm, the design of the Virtual Environment and the Unity components that were used to visualize the POP algorithm. The chapter will also discuss the design of the user interface and the interaction techniques that were used in the VR game.

3.1 POP Algorithm

This section will discuss some of the design decisions that were made during the implementation of the POP algorithm. It will include an overview of the implemented algorithms that were used beside the POP algorithm. In addition, it will discuss the data structures that were used to represent the planning domain and the planning problem.

3.1.1 Nondeterministic Achievers & Threat Search

Partial Order Planning (POP) needs to be able to handle some form of nondeterminism in the planning domain. This is because the planner needs to be able to handle situations where there are multiple ways to achieve a precondition of an action. Nondeterminism is a concept that is used in computer science to describe the occurrence of events without a predictable outcome, where multiple outcomes are possible from a given state. In the context of planning, nondeterminism is used to describe the situation where there are multiple ways to achieve a precondition of an action, for example. This cannot be achieved in practice without trying to search all possible ways of the choices that can be made. So, to model nondeterminism in the planning domain, we need to use some form of graph search algorithm to search for all possible ways to achieve a precondition of an action efficiently. In this project, multiple graph search algorithms were implemented to handle nondeterminism in the planning domain. These algorithms are A-Star (A*) Search, Breadth First Search (BFS), and Depth Limited Search (DLS) which is a variation of Depth First Search (DFS). All of these algorithms are discussed in detail in the following sections.

3.1.2 Graph Search Algorithms

Graph search algorithms are widely used in computer science to solve problems that can be represented as graphs. They have many applications in various fields such as artificial intelligence, computer graphics, and computer vision. In this project, graph search algorithms were used to solve the problem of nondeterminism in the planning domain. The following graph search algorithms were implemented in the project:

- **A-Star (A*) Search Algorithm**
  A* Search is a graph search & traversal algorithm that is widely used in computer science due to its efficiency, optimality, and completeness. It is an informed search algorithm that uses a heuristic function to estimate the cost of reaching the goal from a given state. The heuristic function is used to guide the search towards the goal state by selecting the most promising nodes to explore. A* Search is an extension of Dijkstra's algorithm that uses a heuristic function to estimate the cost of reaching the goal from a given state[1]. A* is a Best First Search algorithm that uses a priority queue to store the nodes to be explored. The priority queue is ordered based on a cost function that combines the cost of reaching the node from the start state and the heuristic estimate of the cost of reaching the goal from the node. The cost function is defined as $f(n) = g(n) + h(n)$, where $g(n)$ is the cost of reaching the node from the start state and $h(n)$ is the heuristic estimate of the cost of reaching the goal from the node. The algorithm selects the node with the lowest f value from the priority queue and explores its neighbors until the goal is reached. The algorithm is guaranteed to find the optimal solution if the heuristic function is admissible, i.e., it never overestimates the cost of reaching the goal from a given state. The algorithm is also complete if the search space is finite and the heuristic function is consistent.

  In this project, $g(n)$ is the path (level) cost from the root node to the current node, and $h(n)$ heuristic was chosen to be the number of unsatisfied preconditions, i.e., the count of pairs in the *agenda* discussed in section 2.2.1.

- **Depth Limited Search (DLS) Search Algorithm**
  First, let's discuss the DFS algorithm. DFS is an uninformed search algorithm that is widely used in computer science to explore a graph or tree data structure. It is a depth-first traversal algorithm that explores the nodes deep in the graph before exploring the nodes at the same level. The algorithm starts at the root node and explores the nodes along each branch before backtracking to explore the other branches. The algorithm uses a stack data structure to store the nodes to be explored. The algorithm is not guaranteed to find the optimal solution, but it is complete if the search space is finite. The algorithm is also efficient in terms of memory usage as it only needs to store the nodes along the current path. However, the algorithm can get stuck in infinite loops if the graph contains an infinite deep path. To avoid this problem, a depth limit can be imposed on the search to limit the depth of the search tree. This is known as the Depth Limited Search (DLS)

algorithm. The algorithm is similar to DFS but with an additional depth limit parameter that specifies the maximum depth of the search tree. The algorithm stops exploring a branch when the depth limit is reached and backtracks to explore other branches. The algorithm is complete if the depth limit is greater than the depth of the optimal solution. The algorithm is also efficient in terms of memory usage. In this project, DLS was also provided as an option to be used in the POP algorithm.

- **Breadth First Search (BFS) Search Algorithm**
  BFS is an uninformed search algorithm that is widely used to explore a graph or tree data structure. It is a breadth-first traversal algorithm, meaning that it explores the nodes at the same level before exploring the nodes at the next level. The algorithm starts at the root node and explores the nodes at the same level before moving to the next level. The algorithm uses a queue data structure to store the nodes to be explored. The algorithm is guaranteed to find a solution if one exist, even if the graph is infinite. However, the algorithm is not guaranteed to always find the optimal solution. The algorithm is also inefficient in terms of memory usage as it needs to store all the nodes at the current level. In this project, BFS was also provided as an option to be used in the POP algorithm.

3.1.3 Unification Algorithm

Finding the Most General Unifier (MGU) of two terms is a crucial step in the POP algorithm. The MGU is used to unify two terms by finding a substitution that makes the two terms equal. The MGU is the most general substitution that can be applied to the two terms to make them equal. For example, consider the following terms: $P(x, B)$ and $P(A, y)$. The MGU of these two terms is $\{x/A, y/B\}$, which means that the terms can be unified by substituting A for x and B for y to make them equal to $P(A, B)$. The MGU is used to unify the preconditions of an action with the current state of the world to determine if the action can be applied. The MGU is also used to unify the effects of an action that can be used, for example, to detect threats in the partial plan.

In this project, the MGU algorithm was implemented using a variation of the unification algorithm discussed in Russell and Norvig's book in Chapter 9 [8], and taken from the lecture slides of the course *Introduction to Artificial Intelligence* by Prof. Haythem Ismail [4]. There is a convention used in the algorithm implementation: the variables are represented as strings starting with a lowercase letter, and the constants are represented as strings starting with an uppercase letter. The algorithm described in Algorithm 2 has three main functions: UNIFY, UNIFY1, and UNIFYVAR. The UNIFY function takes two exrpressions as input, listifies them, and then calls the UNIFY1 function with the two listified expressions and an empty substitution set. An example of listifying an expression is converting the expression $P(x, f(y, z))$ to the list $[P, x, [f, y, z]]$.

Algorithm 2 Unification Algorithm

```
1: function UNIFY(E1, E2) :
2: return  UNIFY1(LISTIFY(E1), LISTIFY(E2), {});
3: ------------------------------------------------------------------------
4:
5: function UNIFY1(E1, E2, μ) :
6: if μ = fail then
7:    return  fail
8: end if
9: if E1 = E2 then
10:   return  μ
11: end if
12: if VAR?(E1) then
13:   return  UNIFYVAR(E1, E2, μ)
14: end if
15: if VAR?(E2) then
16:   return  UNIFYVAR(E2, E1, μ)
17: end if
18: if ATOM?(E1) or ATOM?(E2) then
19:   return  fail
20: end if
21: if LENGTH(E1) != LENGTH(E2) then
22:   return  fail
23: end if
24: return  UNIFY1(REST(E1), REST(E2), UNIFY1(FIRST(E1), FIRST(E2), μ))
25: ------------------------------------------------------------------------
26:
27: function UNIFYVAR(x, e, μ) :
28: if t/x ∈ μ and t ≠ x then
29:   return  UNIFY1(t, e, μ)
30: end if
31: t = SUBST(μ, e)
32: if x occurs in t then
33:   return  fail
34: else
35:   return  μ ∘ {t/x}
36: end if
```

The UNIFY1 function takes two listified expressions and a substitution set as input and returns the most general unifier of the two expressions. The UNIFYVAR function takes a variable, an expression, and a substitution set as input and returns the most general unifier of the variable and the expression. The main algorithm starts in line 6 by checking if the substitution set is a failure, in which it returns a failure. Then, it checks

if the two expressions are equal, in which it returns the curernt substitution set. Then, it checks if $E1$ or $E2$ is a variable, in which it calls the UNIFYVAR function with the first parameter being the variable and the second parameter being the expression. Then, it checks if $E1$ or $E2$ is an atom, in which it returns a failure in line 19. The reason for this is that after making sure that neither $E1$ or $E2$ are variables, and they are not equal, then they must be different atoms, and thus cannot be unified. Then, it checks if the length of $E1$ is not equal to the length of $E2$, in which it returns a failure in line 22, as we cannot unify two expressions with different lengths, like $P(x)$ and $P(x, y)$. Finally, it calls the UNIFY1 function recursively with the rest of the two expressions and the unification of the first elements of the two expressions.

The UNIFYVAR function starts by checking if the variable x is already in the substitution set and bound to a term t that is not equal to x, in which it calls the UNIFY1 function with the term t and the input expression e. If the variable x is not in the substitution set, then it substitutes every term in the expression e with the substitution set μ. Then, it checks if the variable x occurs in the term t, in which it returns a failure, as we cannot unify a variable with an expression that contains the variable. If we had x and $f(x)$, then the unification would fail, since it would result in an infinite recursion. Otherwise, it returns the substitution set μ composite with the substitution $\{t/x\}$, which means that the term t is substituted for the variable x in the substitution set μ.

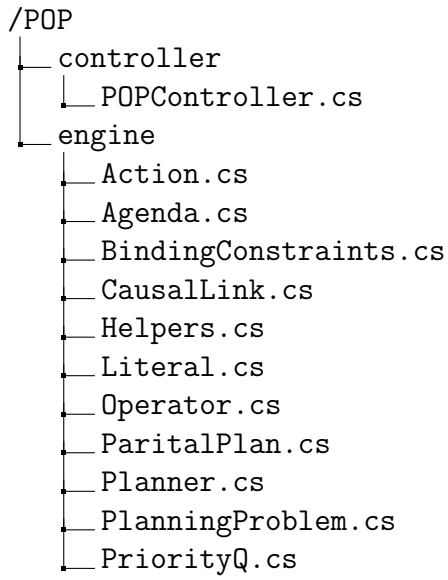

Figure 3.1: Folder Structure of the POP Algorithm Engine

3.1.4 Classes and Folder Structure

The POP algorithm engine was implemented in C#. The algorithm engine was divided into two main folders: the controller folder and the engine folder. The folder structure of the POP algorithm engine is shown in Figure 3.1. The controller folder contains the

POPController class, which is responsible for controlling the execution of the POP algorithm for Unity. The engine folder contains all the classes that implement the POP algorithm. The class diagram of the POP algorithm presented in Figure 3.2 shows the main classes and their relationships in the POP algorithm. In the following list, we will discuss the main classes, their responsibilities, how they are implemented, and what their structure is:

- **PlanningProblem Class:**
  The **PlanningProblem** class is responsible for representing the planning problem, that is the only input to the Partial Order Planning algorithm. The **PlanningProblem** class has the initial state, the goal state, and the set of operators as its members. The initial and goal states are represented as list of *Literal* objects. The operators are represented as a list of *Operator* objects. The class keeps hold of a list of *Literals*, that is not given as an input, but is instantiated automatically once the planning problem is initialized, and it is used to keep track of the existing literals from the initial and goal states in addition to the literals in the preconditions and effects of the operators. This list is to assist the planner.

  The class has only one functional method, which is the **GetListOfAchievers()** method, that takes a *literal* as input. The **GetListOfAchievers()** method is responsible for returning a list of operators that can achieve the input *literal*. The **PlanningProblem** class also has some predefined problems that can be used to test the POP algorithm and the VR game. Among these problems are the *Socks and Shoes problem*, the *Milk, Bananas, and Cordless Drill problem*, the *Groceries Buying problem* which is a simplified variation of the *Milk, Bananas, and Cordless Drill problem*, and some other problems.

- **PartialPlan Class:**
  The **PartialPlan** class is responsible for representing the partial plan that is being constructed by the POP algorithm. The **PartialPlan** class has a set of actions, a set of causal links, an instance of *BindingConstraints* object, and a set of ordering constraints as its members. The class contains some helper functions that are used by the planner like the *GetListOfActionAchievers()* method, and some others that are shown in the class diagram in Figure 3.2.

- **Planner Class:**
  It is the main class that has the core logic of the POP algorithm. The **Planner** class has the Planning Problem, the Partial Plan, and the Agenda as its members. It also has variables counter array to keep track of the generated distinct variables. The following list will discuss the logic and flow of the POP algorithm in the **Planner** class:

  - The **Planner** class has the *POP()* method that implements the POP algorithm. The *POP()* method is responsible for generating the plan by creating a priority queue or stack to store nodes to be used while searching. The *POP()* method also has a loop that iterates over the priority queue or stack until the

goal is reached or no nodes are left. For each node, it checks if the node partial plan is not acyclic, if it is not, then it pops the node from the priority queue or stack and continues to the next node. If the partial plan is acyclic, then it checks if the node is a goal node, if it is, then it returns the plan. If the node is not a goal node, then it expands the node by applying the applicable actions to the node.

- The *EXPAND()* method is responsible for expanding the node. It first chooses a pair of (*action*, *precondition*) from the *agenda* based on some heuristic. The heuristic that I chose is to prioritize the pair with the precondition that has the least number of achievers, and this will be discussed in more details later in this section in the **Agenda** class implementation. Then, after selecting a pair of (*action*, *precondition*), the *POP()* method gets the achievers by gathering the existing actions and new operators that can achieve this precondition, and for each achiever, it creates a new node applies this achiever to it and then adds this node to the main queue or stack.
- The **POP** method also has a helper method called *ApplyAchiever()* that applies the achiever to the node. The *ApplyAchiever()* method works by unifying the achiever's effect with the precondition of the action, then it adds the binding constraints to the partial plan of the node, and then it adds the causal link to the partial plan, as well as updating the ordering constraints with the achiever being ordered before the action with the precondition we are trying to achieve. It also checks whether the achiever is an existing action or a new action, so that if it is a new action, it adds it to the partial plan, update its ordering constraints to be between the *Start* and *Finish* actions, and adds the new action's preconditions to the agenda.
- After applying the achiever, the *EXPAND()* method checks if the new node has threats before adding it to the priority queue or stack through a helper function, *searchResolveThreats()*. The *searchResolveThreats()* method is responsible for searching for threats in the partial plan and resolving them recursively until no threats are left. The *searchResolveThreats()* method has the new added action achiever and the new causal link as inputs. It first starts to check if any of the causal links in the partial plan is threatened by the new action, and if it is, then it resolves the threat by trying promotion and demotion on new cloned nodes. If no other threats are found, then the new node is added to the priority queue or stack.
- The *searchResolveThreats()* method also has a helper method called *isThreat()* that checks if a causal link is threatened by an action. The *isThreat()* method works by checking the conditions, discussed in the definition in Section 2.1.1, to be a threat. The *searchResolveThreats()* then checks if the new causal link input is threatened by any of the existing actions in the partial plan. If it is, then it follows the same steps as before to resolve the threat until there is no threats.
- No node is added to the main queue or stack until it passes all the checks

and has no threats. All nodes outputted from the resolving of threats caused by the new action are then passed to checking whether the new causal link is threatened by any other action. After extensive testing, the planner was found to output wrong results of the all threats are not resolved recursively, or through an iterative approach of the recursive method, as this is the only stage where threats are checked and resolved.

- **Agenda Class:**
  The **Agenda** class is responsible for representing the agenda that is used by the POP algorithm. The agenda is a list of pairs of (*action*, *precondition*) that are used to keep track of the preconditions that need to be achieved. The class internally uses a priority queue to store the pairs of (*action*, *precondition*). The chosen heuristic is to prioritize the pair with the precondition that has the least number of achievers. This is because the precondition with the least number of achievers minimizes the branching factor of the search tree, which leads to a more efficient search [10]. This leads to detecting special cases faster, like the case where the precondition has only one achiever, in which the planner has no any other choice but to apply this achiever. The **Agenda** class has a custom comparer method that compares with the number of achievers of the preconditions. Finally, in the special case where the number of achievers is zero, the class throws an exception, indicating that there is no solution to the problem, and something is wrong with the input problem.

- **Operator Class:**
  The **Operator** class is responsible for representing the operators that are used in the planning domain. The operators has a name, a list of preconditions, a list of effects, and a string variables array to represent the unbonded variables and their relations in the effects and the preconditions of the operator.

- **Action Class:**
  The **Action** class is responsible for representing the actions that are used in the partial plan. The *Operator* class is the superclass of this class. The **Action** class inherits the name, preconditions, effects, and variables array from the *Operator* class. The class also has a method that checks if the action has conflicts in the effects or the preconditions. The conflicts that *hasConflictingPreconditionsOrEffects()* method checks is of of that format: for example if $P(x), \neg P(y) \in$ preconditions, where x and y are bound to the same variable, and P is the same predicate. This is a conflict because it is impossible for x and y to be the same variable and different at the same time.

- **Literal Class:**
  The **Literal** class is responsible for representing the literals, the conditions or the predicates that are used in the planning domain. The literals, in this project, are used to represent the preconditions, the effects, the initial state, and the goal state. The **Literal** class has a name, a list of arguments, and a boolean variable to represent the negation of the literal.

- **BindingConstraints Class:**
  The **BindingConstraints** class is responsible for representing the binding constraints that are used in the partial plan. The binding constraints contains all variables that are bound and equal (or not equal) to a constant term or another variable. As mentioned in subsection 3.1.3, there is a convention that variables are lowercase strings and constants are uppercase strings. In this class, the focus os on four main methods: *SetEqual()*, *SetNotEqual()*, *getBoundEq()*, and *getBoundNE()*. The *SetEqual()* and *SetNotEqual()* methods are used to set the constraints of the variables to be equal or not equal to each other. The *getBoundEq()* and *getBoundNE()* methods are used to get the variables that are bound equal or not equal to a term or another variable.

  In this project, for the **non-equality constraints**, I used a regular *Hash Map* or *Dictionary* data structure to store the constraints. The key of the *Dictionary* is the variable, and the value is a list of variables not equal to the key variable. There is no relation between the variables in the list, and the list is not ordered.

  As for the **equality constraints** between the variables, I used a *Disjoint Sets* representation. **Disjoint Sets** data structure is a set of elements partitioned into a number of non-overlapping subsets[2][3]. The main reason for using a *Disjoint Set* is that it is efficient in terms of time complexity. The implementation used in this project uses the *Union-Find* algorithm to merge the sets and find the representative of the set, in other words, the constant term that is bound to the variable, or a variable if no constant term is bound to it. The *Union-Find* implementation guarantees a time complexity of $O(\log n)$, where n is the number of variables in the partial plan. This number may be large for some problems as the planner uses an incremented counter to generate new variables in action's parameters, as well as in the effects and preconditions of the actions. This number continues to grow as the planner decides to backtrack and generate new actions. Besides the time complexity, the *Disjoint Set* was chosen because for all equal variables in a set, it will always have the same root, which is the representative of the set, which can make things easier to trace for humans.

  Using both data structures, the *Dictionary* and the *Disjoint Set*, the planner can easily check if there exists some logical contradiction in the binding constraints. For example, if there is a variable x that is bound equal to a variable y, and y is bound equal to a third variable z, then another constraint that makes x not equal to z is added direcly or indirectly, then the **BindingConstraints** class can easily detect this contradiction and stop the search.

- **CausalLink Class:**
  The **CausalLink** class is responsible for representing the causal links that are used in the partial plan. The exact definition of a causal link is discussed in the definition in Section 2.1.1. The **CausalLink** class has a *LinkCondition* of type *Literal*, a *Produceri* of type *Action*, and a *Consumerj* of type *Action*. The *Produceri* is the action that has the effect that is the *LinkCondition*, and the *Consumerj* is the

action that has the precondition that is also the *LinkCondition*.

- **PriorityQ Class:**
  The **PriorityQ** is a class taken from the Microsoft documentation and GitHub repository under the MIT license[6][5]. The class is used to mimic the internal original implementation of the *PriorityQueue* in C#. The reason for creating this class is that the latest Unity Long Term Support (LTS) version at the time of writing this thesis uses an older version of C# language (C# 9.0) that does not have the *PriorityQueue* class.

3.1.5 POP Terminal Output

The POP algorithm was tested using the *Socks and Shoes problem*, the *Milk, Bananas, and Cordless Drill problem*, the *Groceries Buying problem*, the *Spare Tires problem*, and some other custom problems. The POP algorithm was able to find the optimal solution for all the problems that were tested.

Milk, Bananas, and Cordless Drill Problem

The *Milk, Bananas, and Cordless Drill problem* is a problem where the agent needs to buy milk and bananas from the supermarket and go to the hardware store to buy a cordless drill. Now let's see the input to the planner in Listing 3.1:

```
PlanningProblem milkBananasCordlessDrill = new PlanningProblem(
    operators: new HashSet<Operator> {
        new Operator("Buy", variables: new []   { "x" },
            preconditions: new List<Literal> { new ("Sells", new
               []{ "store", "x" }), new ("At",new[]{ "store" })
               },
            effects:     new List<Literal> { new ("Have", new[]{"
               x"}) }
        ),
        new Operator("Go", variables: new[]   { "there" },
            preconditions:  new List<Literal> { new ("At", new[]{
                "here" }),       new ("At", new[]{"there"}, false
               ) },
            effects:        new List<Literal> { new ("At", new[]{
                "here" }, false), new ("At", new[]{"there"}) }
        )
    },
    initialState: new List<Literal>{ new("At",new []{"Home"}),
       new("Sells", new []{"SM", "Milk"}), new("Sells", new []{"
       SM", "Bananas"}), new("Sells", new []{"HWS", "Drill"})
                    , new("At",new []{"HWS"}, false), new("At",
                       new []{"SM"}, false)},
```

```
    goalState: new List<Literal> { new("At", new string[] { "Home
       " }), new("Have", new[] { "Milk" }), new("Have", new[] { "
       Bananas" }), new("Have", new[] { "Drill" }) }
);
```

Listing 3.1: Milk, Bananas, and Cordless Drill Problem Input to the Planner

The planner was able to find the optimal solution for the *Milk, Bananas, and Cordless Drill problem* in runtime of 00.260 seconds. The output of the planner is shown in Listing 3.2:

```
    Searching.....................................

Plan  found:


Linearized Steps:
*************
Start() -> Go(HWS) -> Buy(Drill) -> Go(SM) -> Buy(Bananas) -> Buy
   (Milk) -> Go(Home) -> Finish()
************

Actions: Start(), Finish(), Go(Home), Buy(Drill), Buy(Milk), Go(
   SM), Buy(Bananas), Go(HWS)

Links:
Go(Home) --At(Home)--> Finish(),
Buy(Drill) --Have(Drill)--> Finish(),
Start() --Sells(HWS, Drill)--> Buy(Drill),
Buy(Milk) --Have(Milk)--> Finish(),
Start() --Sells(SM, Milk)--> Buy(Milk),
Go(SM) --At(SM)--> Buy(Milk),
Buy(Bananas) --Have(Bananas)--> Finish(),
Start() --Sells(SM, Bananas)--> Buy(Bananas),
Go(SM) --At(SM)--> Buy(Bananas),
Go(HWS) --At(HWS)--> Go(SM),
Go(HWS) --At(HWS)--> Buy(Drill),
Go(SM) --At(SM)--> Go(Home),
Start() --At(Home)--> Go(HWS),
Start() -- ¬At(HWS)--> Go(HWS),
Start() -- ¬At(SM)--> Go(SM),
Go(HWS) -- ¬At(Home)--> Go(Home)

Binding Constraints:
{
Equal:
g0 = Home = a4,
b1 = Drill,
```

```
s1 = HWS = g4 = a2,
b3 = Milk,
s3 = SM = g2 = s4 = a0,
b4 = Bananas,

Not Equal:
}


Ordering Constraints: (Start() < Finish()), (Start() < Go(Home)),
    (Go(Home) < Finish()), (Start() < Buy(Drill)), (Buy(Drill) <
   Finish()), (Start() < Buy(Milk)), (Buy(Milk) < Finish()), (
   Start() < Go(SM)), (Go(SM) < Finish()), (Go(SM) < Buy(Milk)),
   (Go(SM) < Go(Home)), (Buy(Milk) < Go(Home)), (Start() < Buy(
   Bananas)), (Buy(Bananas) < Finish()), (Go(SM) < Buy(Bananas)),
    (Buy(Bananas) < Go(Home)), (Start() < Go(HWS)), (Go(HWS) <
   Finish()), (Go(HWS) < Go(SM)), (Go(HWS) < Buy(Drill)), (Buy(
   Drill) < Go(SM)), (Go(HWS) < Go(Home))

RunTime: 00.260 seconds
```

Listing 3.2: Milk, Bananas, and Cordless Drill Problem Output of the Planner

Spare Tires Problem

The *Spare Tires problem* is a problem where the agent needs to change a flat tire with a spare tire. Now let's see the input to the planner in Listing 3.3:

```
PlanningProblem spareTires = new PlanningProblem(
operators: new HashSet<Operator> {
        new Operator("Remove",
            variables:      new []{"obj", "loc"},
            preconditions:  new List<Literal>{ new ("At", new []{
               "obj", "loc"}), new("Tire", new []{"obj"})},
            effects:        new List<Literal>{ new ("At", new []{
               "obj", "Ground"}), new("At", new []{"obj", "loc"},
                false)}
        ),
        new Operator("PutOn",
            variables:      new []{"t", "Axle"},
            preconditions:  new List<Literal>{ new ("At", new []{
               "t", "Ground"}), new("At", new []{"Flat", "Axle"},
                false), new("Tire", new []{"t"})},
            effects:        new List<Literal>{ new ("At", new []{
               "t", "Axle"}), new("At", new []{"t", "Ground"},
               false)}
```

```
        ),
            new Operator("LeaveOvernight",
            variables:      new string []{},
            preconditions:  new(),
            effects:        new List<Literal>{ new ("At", new []{
               "Spare", "Axle"}, false), new("At", new []{"Spare"
               , "Trunk"}, false), new("At", new []{"Spare", "
               Ground"}, false)
                    ,new("At", new []{"Flat", "Axle"}, false),
                       new("At", new []{"Flat", "Trunk"}, false),
                        new("At", new []{"Flat", "Ground"}, false
                       )}
        )
    },
initialState: new List<Literal> { new("At", new[] { "Flat", "Axle
   " }), new("At", new[] { "Spare", "Trunk" }), new("Tire", new[]
    { "Spare" }), new("Tire", new[] { "Flat" }) },
goalState: new List<Literal> { new("At", new[] { "Spare", "Axle"
   }), new("At", new[] { "Flat", "Ground" }) }
);
```

Listing 3.3: Spare Tires Problem Input to the Planner

The planner was able to find the optimal solution for the *Spare Tires problem* in runtime of 00.520 seconds. The output of the planner is shown in Listing 3.4:

```
Searching............

Plan  found:

Linearized Steps:
*************
Start() -> Remove(Spare, Trunk) -> Remove(Flat, Axle) -> PutOn(
   Spare, Axle) -> Finish()
************


Actions: Start(), Finish(), PutOn(Spare, Axle), Remove(Flat, Axle
   ), Remove(Spare, Trunk)

Links:
PutOn(Spare, Axle) --At(Spare, Axle)--> Finish(),
Start() --Tire(Spare)--> PutOn(Spare, Axle),
Remove(Flat, Axle) --At(Flat, Ground)--> Finish(),
Start() --Tire(Flat)--> Remove(Flat, Axle),
Start() --At(Flat, Axle)--> Remove(Flat, Axle),
Remove(Spare, Trunk) --At(Spare, Ground)--> PutOn(Spare, Axle),
Start() --Tire(Spare)--> Remove(Spare, Trunk),
```

```
Start() --At(Spare, Trunk)--> Remove(Spare, Trunk),
Remove(Flat, Axle) -- ¬At(Flat, Axle)--> PutOn(Spare, Axle)

Binding Constraints:
{
Equal:
aa1 = Ground,
a2 = Flat = r1,
p0 = Spare = r3,
pp0 = Axle = rr1,
rr3 = Trunk,

Not Equal:
}


Ordering Constraints: (Start() < Finish()), (Start() < PutOn(
   Spare, Axle)), (PutOn(Spare, Axle) < Finish()), (Start() <
   Remove(Flat, Axle)), (Remove(Flat, Axle) < Finish()), (Start()
    < Remove(Spare, Trunk)), (Remove(Spare, Trunk) < Finish()), (
   Remove(Spare, Trunk) < PutOn(Spare, Axle)), (Remove(Flat, Axle
   ) < PutOn(Spare, Axle))

RunTime: 00.520 seconds
```

Listing 3.4: Spare Tires Problem Output of the Planner

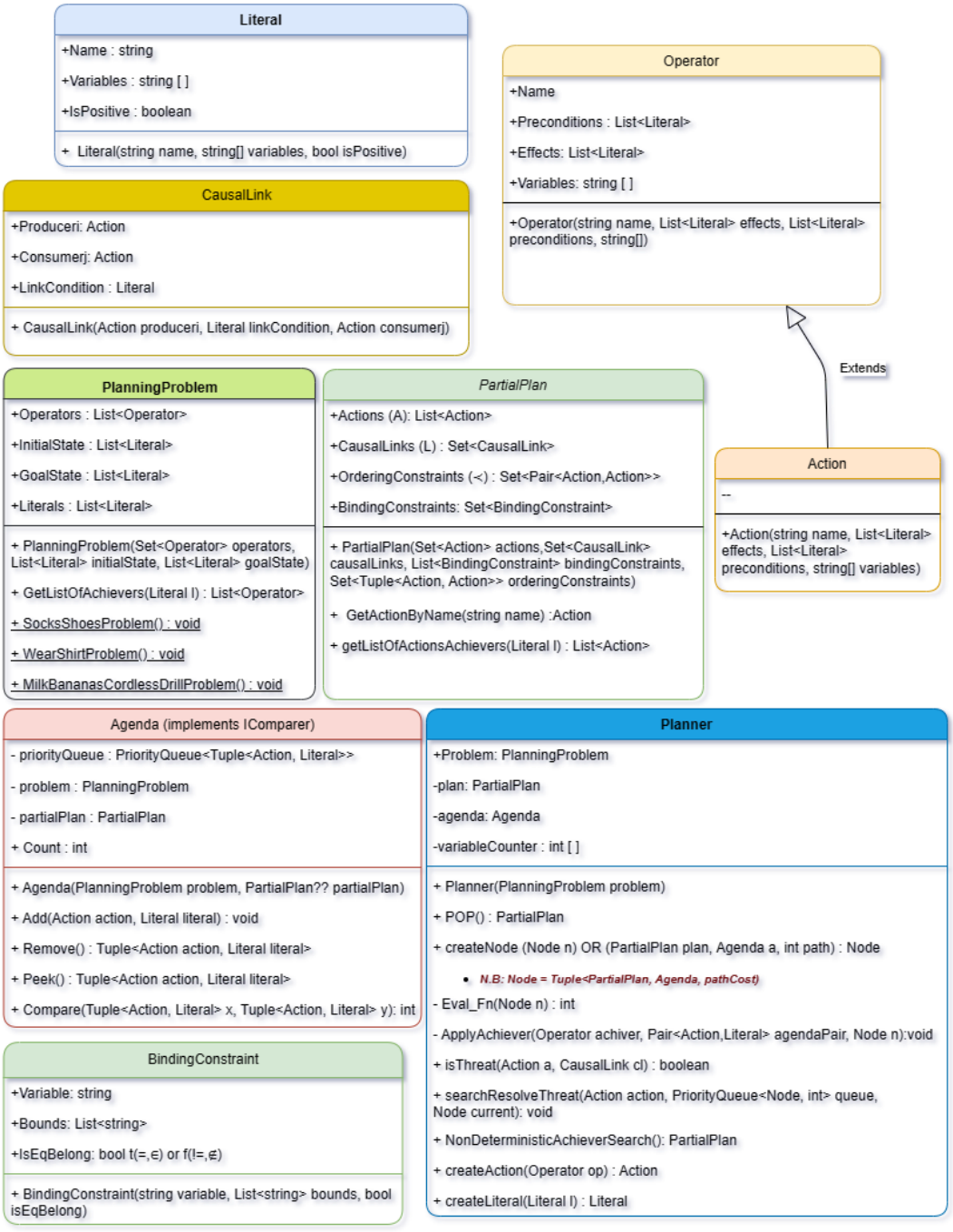

Figure 3.2: Class Diagram of the POP Algorithm

Chapter 4

Conclusion

Conclusion

Chapter 5

Future Work

Text

Appendix

Appendix A

Lists

POP	Partial Order Planning
AI	Artificial Intelligence
MGU	Most General Unifier
DAG	Directed Acyclic Graph
A*	A-Star
DFS	Depth First Search
BFS	Breadth First Search
DLS	Depth Limited Search
VR	Virtual Reality
LTS	Long Term Support

List of Figures

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