



第九讲 马尔可夫自定位求解方法

李亮

浙江大学控制科学与工程学院

马尔可夫实时自定位计算

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m}) \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m}) p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$



t时刻后验



观测模型



运动模型



t-1时刻后验



马尔可夫实时自定位计算

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m}) \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m}) p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$

求解步骤

- 位姿预估 prediction

$$\bar{p}(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \int \underbrace{p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m})}_{\text{运动模型}} p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$

- 观测更新 updating

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \underbrace{\eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m})}_{\text{观测模型}} \bar{p}(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m})$$

马尔可夫实时自定位主要方法

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m})$$
$$= \underbrace{\eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m})}_{\text{观测模型}} \int \underbrace{p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m})}_{\text{运动模型}} p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$

运算过于复杂



采用不同的假设或近似

经典自定位方法

EKF (Extended Kalman filter) 定位法

PF (Particle filter) 定位法





9.1 扩展卡尔曼滤波定位法

EKF方法

○ 除满足马可夫假设之外，**如果以下三个条件成立**，

- 运动（/状态转移）方程可表示为

$$\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{v}_t, \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$$

- 观测方程可表示为

$$\mathbf{z}_t = h(\mathbf{x}_t) + \omega_t, \omega_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$$

- 初始位姿（/状态）概率 $p(\mathbf{x}_0)$ 服从高斯分布

$$p(\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_0; \boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$$

则状态估计的后验概率满足高斯分布，即 $p(\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_t; \boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$

只需递归计算均值和方差



为什么状态估计的后验概率总是满足高斯分布？

$$\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{v}_t, \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$$

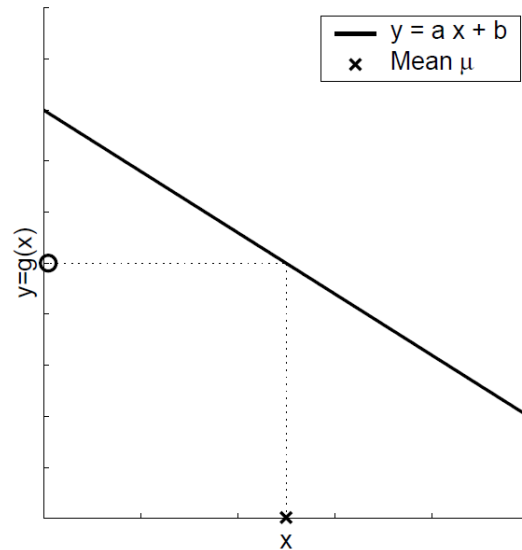
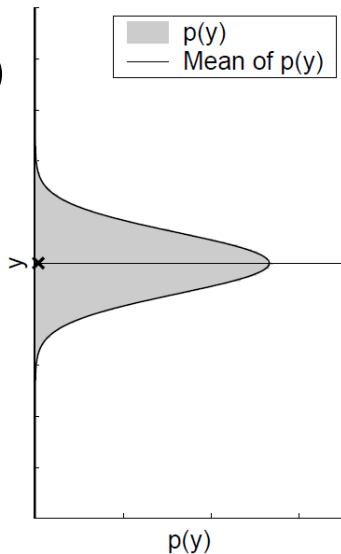


$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{B}_t \mathbf{u}_{t-1} + \mathbf{v}_t$$

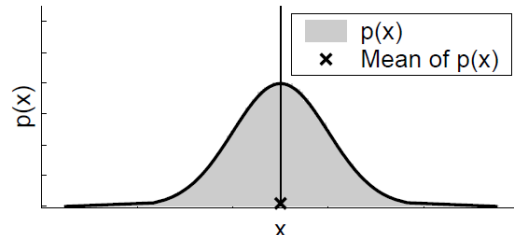
$$\mathbf{z}_t = h(\mathbf{x}_t) + \omega_t, \omega_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$$



$$\mathbf{z}_t = \mathbf{C}_t \mathbf{x}_t + \omega_t$$



对高斯随机变量作任意线性转换，所得到的随机变量仍然满足高斯分布特性，其高斯分布参数能够用封闭形式求解



KF(KALMAN FILTER)

运动方程表示为

$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{B}_t \mathbf{u}_{t-1} + \mathbf{v}_t$$

\mathbf{x}_{t-1} 为 n 维向量

\mathbf{u}_{t-1} 为 m 维向量,

\mathbf{v}_t 为 n 维向量, $\mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$

\mathbf{A}_t 为 $n \times n$ 维矩阵, \mathbf{B}_t 为 $n \times m$ 维矩阵

此时 $p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{B}_t \mathbf{u}_{t-1}, \mathbf{R}_t)$

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m}) \int \mathbf{p}(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m}) p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$

KF

○ 观测方程表示为

$$\mathbf{z}_t = \mathbf{C}_t \mathbf{x}_t + \boldsymbol{\omega}_t$$

\mathbf{C}_t 为 $k \times n$ 维矩阵, k 为观测向量 \mathbf{z}_t 维度

$$\boldsymbol{\omega}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$$

此时

$$p(\mathbf{z}_t | \mathbf{x}_t) = \mathcal{N}(\mathbf{z}_t; \mathbf{C}_t \mathbf{x}_t, \mathbf{Q}_t)$$

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \eta \mathbf{p}(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m}) \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m}) p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$

KF

状态预估

$$\bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1}) = \int \underbrace{p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1})}_{\sim \mathcal{N}(\mathbf{x}_t; \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{B}_t \mathbf{u}_{t-1}, \mathbf{R}_t)} \underbrace{p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2})}_{\sim \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_{t-1}, \boldsymbol{\Sigma}_{t-1})} d\mathbf{x}_{t-1}$$



$$\bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1}) = \mathcal{N}(\mathbf{x}_t; \bar{\boldsymbol{\mu}}_t, \bar{\boldsymbol{\Sigma}}_t)$$

$$\bar{\boldsymbol{\mu}}_t = \mathbf{A}_t \boldsymbol{\mu}_{t-1} + \mathbf{B}_t \mathbf{u}_{t-1}$$

$$\bar{\boldsymbol{\Sigma}}_t = \mathbf{A}_t \boldsymbol{\Sigma}_{t-1} \mathbf{A}_t^T + \mathbf{R}_t$$



KF

观测更新

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}) = \eta \underbrace{p(\mathbf{z}_t | \mathbf{x}_t)}_{\sim \mathcal{N}(\mathbf{z}_t; \mathbf{C}_t \mathbf{x}_t, \mathbf{Q}_t)} \underbrace{\bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1})}_{\sim \mathcal{N}(\mathbf{x}_t; \bar{\boldsymbol{\mu}}_t, \bar{\boldsymbol{\Sigma}}_t)}$$



$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}) = \mathcal{N}(\mathbf{x}_t; \boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

$$\boldsymbol{\mu}_t = \bar{\boldsymbol{\mu}}_t + \mathbf{K}_t (\mathbf{z}_t - \mathbf{C}_t \bar{\boldsymbol{\mu}}_t)$$

$$\boldsymbol{\Sigma}_t = [\mathbf{I} - \mathbf{K}_t \mathbf{C}_t] \bar{\boldsymbol{\Sigma}}_t$$

$$\text{其中 } \mathbf{K}_t = \bar{\boldsymbol{\Sigma}}_t \mathbf{C}_t^T (\mathbf{C}_t \bar{\boldsymbol{\Sigma}}_t \mathbf{C}_t^T + \mathbf{Q}_t)^{-1} \quad \text{称为新信息矩阵}$$

如何理解公式含义?



Algorithm Kalman_filter ($\mu_{t-1}, \Sigma_{t-1}, \mathbf{u}_{t-1}, \mathbf{z}_t$)

$$\bar{\mu}_t = \mathbf{A}_t \mu_{t-1} + \mathbf{B}_t \mathbf{u}_{t-1}$$

$$\bar{\Sigma}_t = \mathbf{A}_t \Sigma_{t-1} \mathbf{A}_t^T + \mathbf{R}_t$$

$$\mathbf{K}_t = \bar{\Sigma}_t \mathbf{C}_t^T (\mathbf{C}_t \bar{\Sigma}_t \mathbf{C}_t^T + \mathbf{Q}_t)^{-1} \longrightarrow O(k^{2.376})$$

$$\mu_t = \bar{\mu}_t + \mathbf{K}_t (\mathbf{z}_t - \mathbf{C}_t \bar{\mu}_t)$$

$$\Sigma_t = [\mathbf{I} - \mathbf{K}_t \mathbf{C}_t] \bar{\Sigma}_t \longrightarrow O(n^2)$$

return μ_t, Σ_t

k 为特征向量维度，即观测到的特征数

n 为状态向量维度，即机器人位姿描述维度



KF

- 实际应用的局限性：要求运动方程和观测方程都是线性函数

一维运动

$$x_t = x_{t-1} + \delta_d \quad \checkmark$$

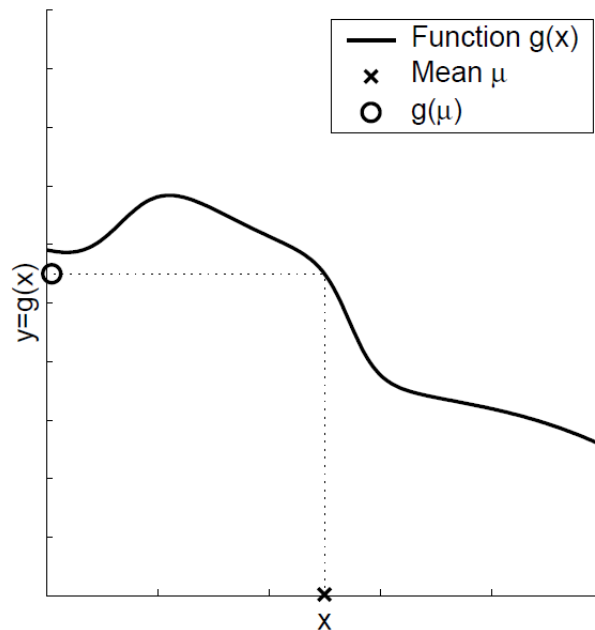
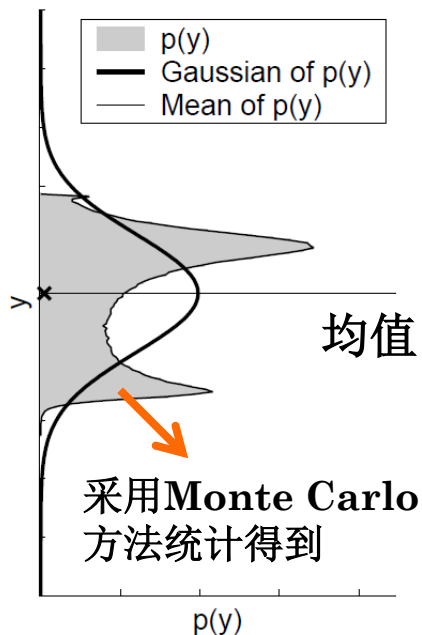
二维运动

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} - \frac{v}{\omega} \sin(\theta_{t-1}) + \frac{v}{\omega} \sin(\theta_{t-1} + \omega \Delta t) \\ x_{t-1} + \frac{v}{\omega} \cos(\theta_{t-1}) - \frac{v}{\omega} \cos(\theta_{t-1} + \omega \Delta t) \\ \theta_{t-1} + \omega \Delta t \end{pmatrix} \quad \times$$

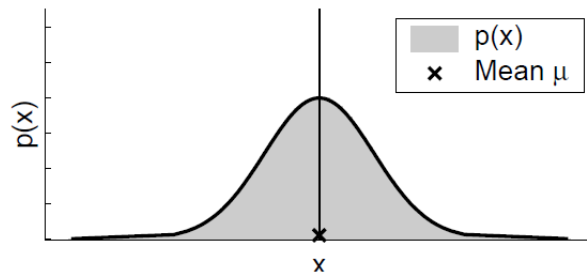
$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} + \delta_d \cos(\theta_{t-1} + \delta_\alpha) \\ x_{t-1} + \delta_d \sin(\theta_{t-1} + \delta_\alpha) \\ \theta_{t-1} + \delta_\alpha + \delta_\beta \end{pmatrix} \quad \times$$



非线性转换



非线性函数破坏了
高斯分布迭代计算



EKF(EXTENDED KALMAN FILTER)

基本思想:

- 采用非线性函数表示运动方程和观测方程

$$\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{v}_t, \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$$

$$\mathbf{z}_t = h(\mathbf{x}_t) + \boldsymbol{\omega}_t, \boldsymbol{\omega}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$$

- 计算非线性转换后实际概率分布的**高斯近似**

KF

计算精确后验概率

EKF

有效估计后验概率的均值和方差



EKF核心思想：线性化近似

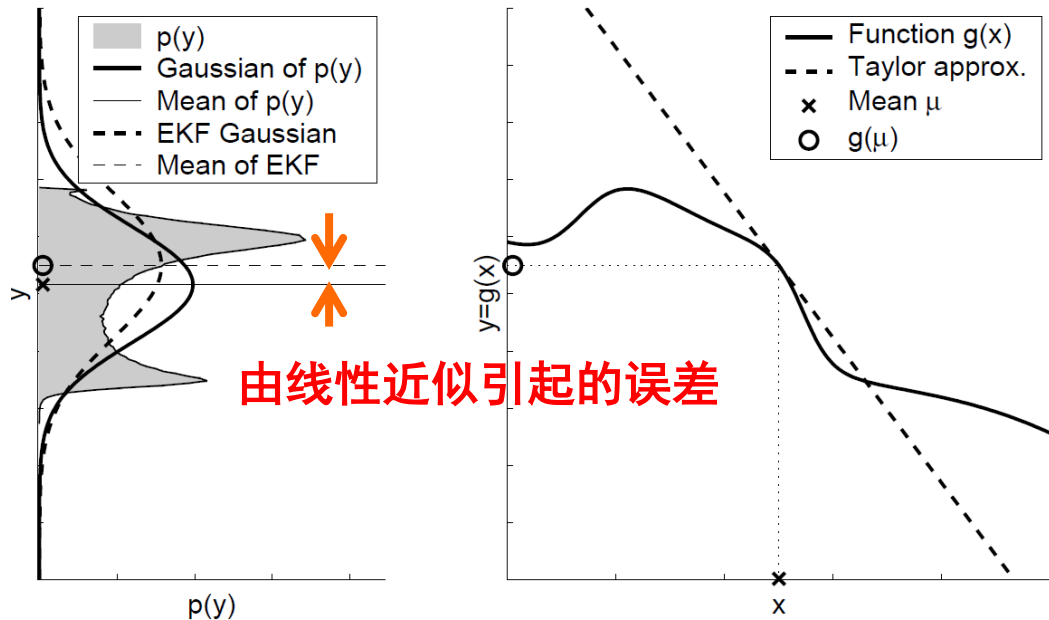
- 一阶泰勒展开(first order taylor expansion)

$$g(x) = g(a) + g'(a)(x - a)$$

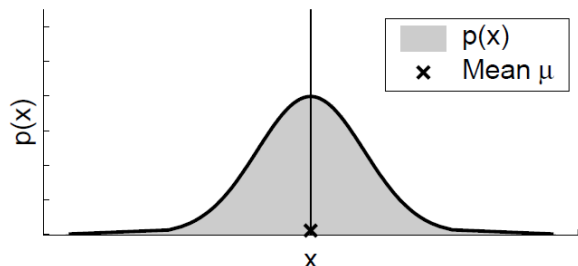
$$g'(a) = \left. \frac{\partial g(x)}{\partial x} \right|_a$$



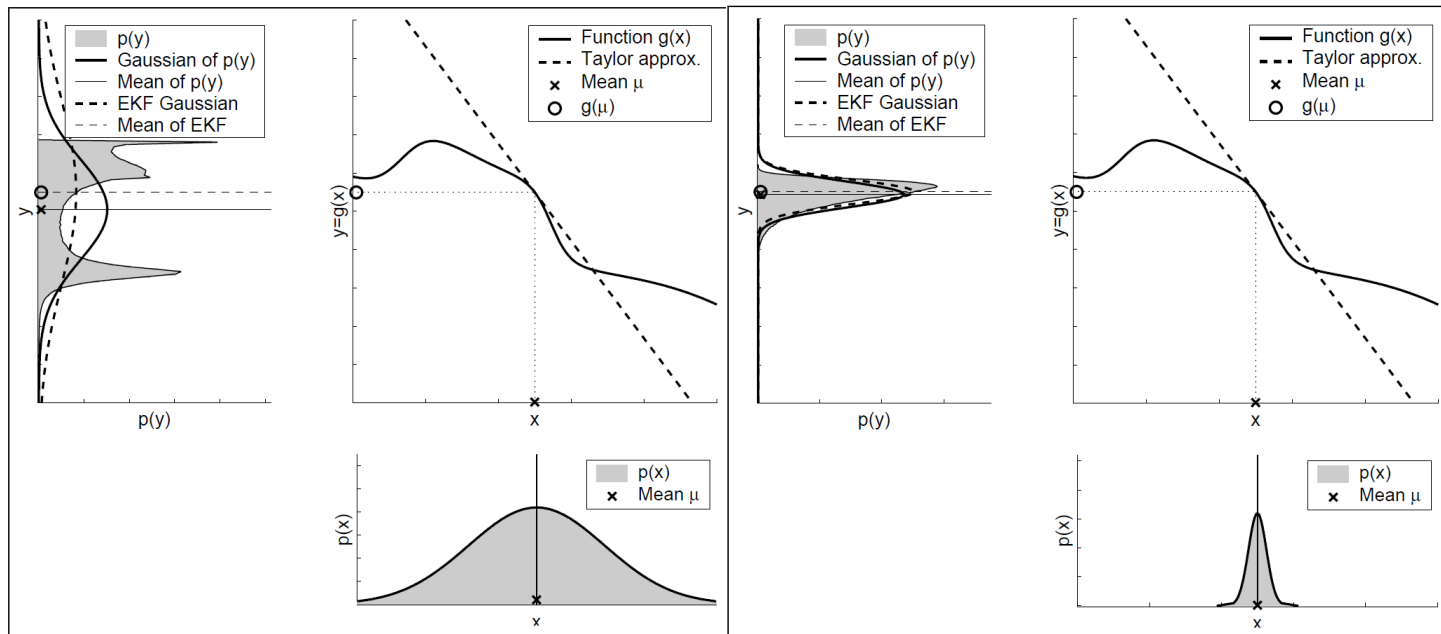
EKF核心思想: 线性化近似



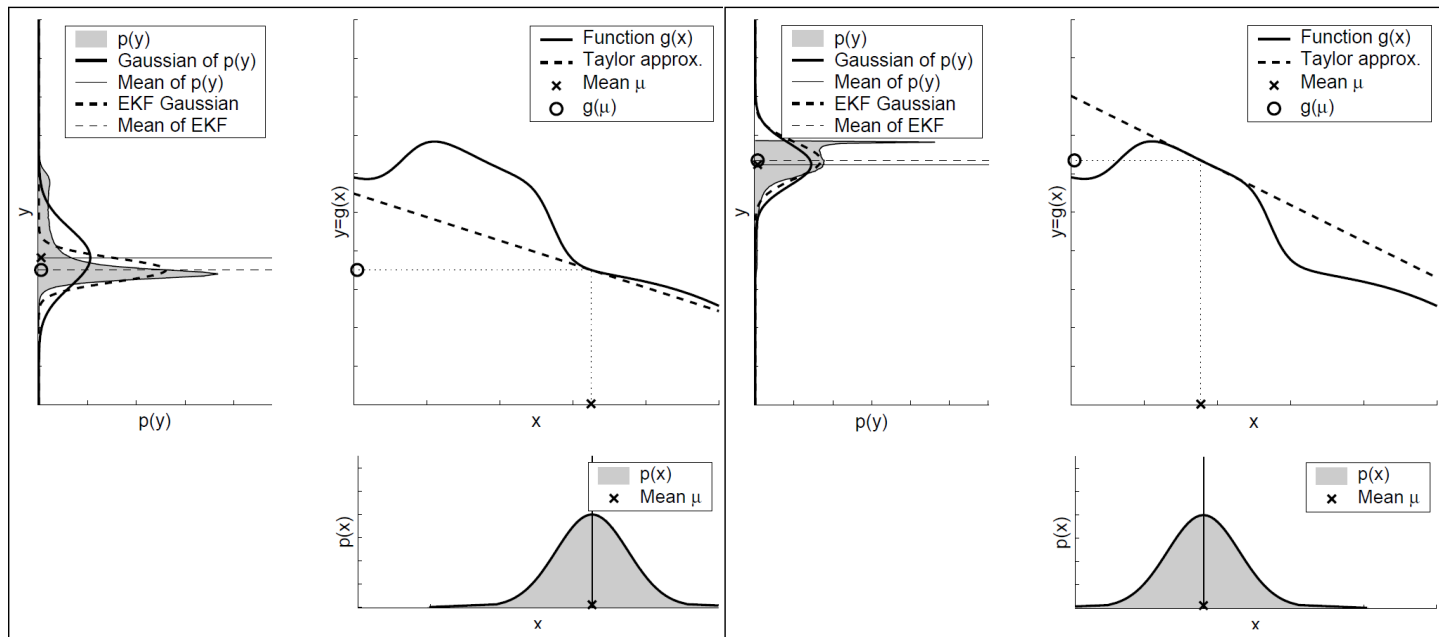
非线性函数 g 由一个在
输入高斯分布均值处
与 g 相切的函数近似



不确定程度的影响



局部非线性程度的影响



EKF

运动方程 $\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{v}_t, \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$

取泰勒展开参数为 $\boldsymbol{\mu}_{t-1}$

$$g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) \approx g(\boldsymbol{\mu}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{G}_t(\mathbf{x}_{t-1} - \boldsymbol{\mu}_{t-1})$$

$$\mathbf{G}_t = \left. \frac{\partial g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})}{\partial \mathbf{x}_{t-1}} \right|_{\mathbf{x}_{t-1} = \boldsymbol{\mu}_{t-1}} \quad \begin{array}{l} \text{Jacobian矩阵} \\ n \times n \end{array}$$

$$p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}) \approx \mathcal{N}(\mathbf{x}_t; g(\boldsymbol{\mu}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{G}_t(\mathbf{x}_{t-1} - \boldsymbol{\mu}_{t-1}), \mathbf{R}_t)$$



EKF

○ 观测方程

$$\mathbf{z}_t = h(\mathbf{x}_t) + \boldsymbol{\omega}_t, \boldsymbol{\omega}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$$

取泰勒展开参数为 $\bar{\boldsymbol{\mu}}_t$

$$h(\mathbf{x}_t) \approx h(\bar{\boldsymbol{\mu}}_t) + \mathbf{H}_t(\mathbf{x}_t - \bar{\boldsymbol{\mu}}_t)$$

$$\mathbf{H}_t = \left. \frac{\partial h(\mathbf{x}_t)}{\partial \mathbf{x}_t} \right|_{\mathbf{x}_t = \bar{\boldsymbol{\mu}}_t} \quad \begin{array}{l} \text{Jacobian矩阵} \\ k \times n \end{array}$$

$$p(\mathbf{z}_t | \mathbf{x}_t) = \mathcal{N}(\mathbf{z}_t; h(\bar{\boldsymbol{\mu}}_t) + \mathbf{H}_t(\mathbf{x}_t - \bar{\boldsymbol{\mu}}_t), \mathbf{Q}_t)$$



EKF

位姿预估

$$\begin{aligned} & \sim \mathcal{N}(\mathbf{x}_t; \mathbf{g}(\bar{\boldsymbol{\mu}}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{G}_t(\mathbf{x}_{t-1} - \bar{\boldsymbol{\mu}}_{t-1}), \mathbf{R}_t) \\ & \sim \mathcal{N}(\mathbf{x}_{t-1}; \bar{\boldsymbol{\mu}}_{t-1}, \bar{\boldsymbol{\Sigma}}_{t-1}) \\ \bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1}) &= \int \underbrace{p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1})}_{\sim \mathcal{N}(\mathbf{x}_t; \mathbf{g}(\bar{\boldsymbol{\mu}}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{G}_t(\mathbf{x}_{t-1} - \bar{\boldsymbol{\mu}}_{t-1}), \mathbf{R}_t)} \underbrace{p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2})}_{\sim \mathcal{N}(\mathbf{x}_{t-1}; \bar{\boldsymbol{\mu}}_{t-1}, \bar{\boldsymbol{\Sigma}}_{t-1})} d\mathbf{x}_{t-1} \end{aligned}$$



$$\bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1}) = \mathcal{N}(\mathbf{x}_t; \bar{\boldsymbol{\mu}}_t, \bar{\boldsymbol{\Sigma}}_t)$$

$$\bar{\boldsymbol{\mu}}_t = \mathbf{g}(\bar{\boldsymbol{\mu}}_{t-1}, \mathbf{u}_{t-1})$$

$$\bar{\boldsymbol{\Sigma}}_t = \mathbf{G}_t \bar{\boldsymbol{\Sigma}}_{t-1} \mathbf{G}_t^T + \mathbf{R}_t$$

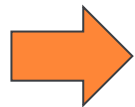


EKF

观测更新

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}) = \eta \underbrace{p(\mathbf{z}_t | \mathbf{x}_t)}_{\downarrow} \underbrace{\bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1})}_{\downarrow}$$

$$\sim \mathcal{N}(\mathbf{z}_t; h(\bar{\boldsymbol{\mu}}_t) + \mathbf{H}_t(\mathbf{x}_t - \bar{\boldsymbol{\mu}}_t), \mathbf{Q}_t) \quad \sim \mathcal{N}(\mathbf{x}_t; \bar{\boldsymbol{\mu}}_t, \bar{\boldsymbol{\Sigma}}_t)$$



$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}) = \mathcal{N}(\mathbf{x}_t; \boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

$$\boldsymbol{\mu}_t = \bar{\boldsymbol{\mu}}_t + \mathbf{K}_t(\mathbf{z}_t - h(\bar{\boldsymbol{\mu}}_t))$$

$$\boldsymbol{\Sigma}_t = [\mathbf{I} - \mathbf{K}_t \mathbf{H}_t] \bar{\boldsymbol{\Sigma}}_t$$

$$\text{其中 } \mathbf{K}_t = \bar{\boldsymbol{\Sigma}}_t \mathbf{H}_t^T (\mathbf{H}_t \bar{\boldsymbol{\Sigma}}_t \mathbf{H}_t^T + \mathbf{Q}_t)^{-1}$$



Algorithm Extended_Kalman_filter $(\boldsymbol{\mu}_{t-1}, \boldsymbol{\Sigma}_{t-1}, \mathbf{u}_{t-1}, \mathbf{z}_t)$

$$\bar{\boldsymbol{\mu}}_t = \mathbf{g}(\boldsymbol{\mu}_{t-1}, \mathbf{u}_{t-1})$$

$$\bar{\boldsymbol{\Sigma}}_t = \mathbf{G}_t \boldsymbol{\Sigma}_{t-1} \mathbf{G}_t^T + \mathbf{R}_t$$

$$\mathbf{K}_t = \bar{\boldsymbol{\Sigma}}_t \mathbf{H}_t^T (\mathbf{H}_t \bar{\boldsymbol{\Sigma}}_t \mathbf{H}_t^T + \mathbf{Q}_t)^{-1} \longrightarrow O(k^{2.376})$$

$$\boldsymbol{\mu}_t = \bar{\boldsymbol{\mu}}_t + \mathbf{K}_t (\mathbf{z}_t - \mathbf{h}(\bar{\boldsymbol{\mu}}_t))$$

$$\boldsymbol{\Sigma}_t = [\mathbf{I} - \mathbf{K}_t \mathbf{H}_t] \bar{\boldsymbol{\Sigma}}_t \longrightarrow O(n^2)$$

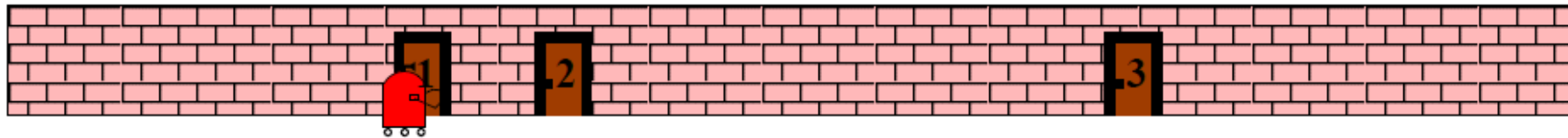
return $\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t$



EKF方法的优缺点

- 优点：简单，计算高效
- 缺点：
 - 线性近似的好坏依赖于
 - 自变量的不确定程度
 - 被近似函数的局部非线性程度
 - EKF采用多变量高斯分布表示估计的概率分布，这种单峰概率分布表示不适合于存在多个假设的情况





EKF LOCALIZATION

采用里程计运动模型

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} + \hat{\delta}_{trans} \cos(\theta_{t-1} + \hat{\delta}_{rot1}) \\ y_{t-1} + \hat{\delta}_{trans} \sin(\theta_{t-1} + \hat{\delta}_{rot1}) \\ \theta_{t-1} + \hat{\delta}_{rot1} + \hat{\delta}_{rot2} \end{pmatrix}$$
$$\begin{aligned} \hat{\delta}_{rot1} &= \delta_{rot1} - \tilde{\delta}_{rot1} \\ \hat{\delta}_{trans} &= \delta_{trans} - \tilde{\delta}_{trans} \\ \hat{\delta}_{rot2} &= \delta_{rot2} - \tilde{\delta}_{rot2} \end{aligned}$$

$$\tilde{\delta} = (\tilde{\delta}_{rot1}, \tilde{\delta}_{trans}, \tilde{\delta}_{rot2})^T \sim N(\mathbf{0}, \mathbf{M}_t)$$

$$\mathbf{M}_t = \text{diag}((\alpha_1 |\delta_{rot1}| + \alpha_2 |\delta_{trans}|)^2, \\ (\alpha_3 |\delta_{trans}| + \alpha_4 |\delta_{rot1} + \delta_{rot2}|)^2, \\ (\alpha_1 |\delta_{rot2}| + \alpha_2 |\delta_{trans}|)^2)$$



需构建EKF下的运动方程

$$\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{v}_t, \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$$

$$\mathbf{x}_{t-1} \sim \mathcal{N}(\boldsymbol{\mu}_{t-1}, \boldsymbol{\Sigma}_{t-1})$$

$$\mathbf{u}_{t-1} = (\delta_{rot1}, \delta_{trans}, \delta_{rot2})^T$$

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \underbrace{\begin{pmatrix} \delta_{trans} \cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans} \sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix}}_{g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})} + \mathbf{v}_t$$

该运动方程中使用的控制量是测量值，不是真实值，通过误差补偿，使得得到的值为近似真值

$$\mathbf{v}_t = (v_x, v_y, v_\theta)^T \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$$

误差由控制变量空间变换到了状态变量空间

由 \mathbf{M}_t 计算 \mathbf{R}_t

在实际控制变量空间下，运动模型为 $\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1} + \tilde{\delta})$

对该方程在 \mathbf{u}_{t-1} 处作一阶泰勒展开，得

$$\mathbf{x}_t = g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1} + \tilde{\delta})$$

$$\begin{aligned} &\approx g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) + g'(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})\tilde{\delta} \\ &= g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) + \mathbf{v}_t \end{aligned}$$

$$\mathbf{R}_t = \mathbf{v}_t \mathbf{v}_t^T = g'(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) \tilde{\delta} \tilde{\delta}^T g'(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})^T = \mathbf{V}_t \mathbf{M}_t \mathbf{V}_t^T$$



$$\mathbf{R}_t = \mathbf{V}_t \mathbf{M}_t \mathbf{V}_t^T$$

$$\mathbf{V}_t = \frac{\partial g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})}{\partial \mathbf{u}_{t-1}} = \begin{pmatrix} \frac{\partial x_t}{\partial \delta_{rot1}} & \frac{\partial x_t}{\partial \delta_{trans}} & \frac{\partial x_t}{\partial \delta_{rot2}} \\ \frac{\partial y_t}{\partial \delta_{rot1}} & \frac{\partial y_t}{\partial \delta_{trans}} & \frac{\partial y_t}{\partial \delta_{rot2}} \\ \frac{\partial \theta_t}{\partial \delta_{rot1}} & \frac{\partial \theta_t}{\partial \delta_{trans}} & \frac{\partial \theta_t}{\partial \delta_{rot2}} \end{pmatrix}$$

$$= \begin{pmatrix} -\delta_{trans} \sin(\mu_{t-1,\theta} + \delta_{rot1}) & \cos(\mu_{t-1,\theta} + \delta_{rot1}) & 0 \\ \delta_{trans} \cos(\mu_{t-1,\theta} + \delta_{rot1}) & \sin(\mu_{t-1,\theta} + \delta_{rot1}) & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta_{trans} \cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans} \sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix} + \mathbf{v}_t$$



位姿预估

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta_{trans} \cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans} \sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix} + \mathbf{v}_t$$

$$\bar{\boldsymbol{\mu}}_t = g(\boldsymbol{\mu}_{t-1}, \mathbf{u}_{t-1})$$

$$\bar{\boldsymbol{\Sigma}}_t = \mathbf{G}_t \boldsymbol{\Sigma}_{t-1} \mathbf{G}_t^T + \mathbf{R}_t$$

求 $g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$ 的Jacobian矩阵

$$\mathbf{G}_t = \left. \frac{\partial g(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})}{\partial \mathbf{x}_{t-1}} \right|_{\mathbf{x}_{t-1} = \bar{\boldsymbol{\mu}}_{t-1}} = \begin{pmatrix} \frac{\partial x_t}{\partial x_{t-1}} & \frac{\partial x_t}{\partial y_{t-1}} & \frac{\partial x_t}{\partial \theta_{t-1}} \\ \frac{\partial y_t}{\partial x_{t-1}} & \frac{\partial y_t}{\partial y_{t-1}} & \frac{\partial y_t}{\partial \theta_{t-1}} \\ \frac{\partial \theta_t}{\partial x_{t-1}} & \frac{\partial \theta_t}{\partial y_{t-1}} & \frac{\partial \theta_t}{\partial \theta_{t-1}} \end{pmatrix} \bigg|_{\mathbf{x}_{t-1} = \bar{\boldsymbol{\mu}}_{t-1}} = \begin{pmatrix} 1 & 0 & -\delta_{trans} \sin(\mu_{t-1, \theta} + \delta_{rot1}) \\ 0 & 1 & \delta_{trans} \cos(\mu_{t-1, \theta} + \delta_{rot1}) \\ 0 & 0 & 1 \end{pmatrix}$$



观测更新



采用特征地图表示 $\mathbf{m} = \{\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_N\}$

\mathbf{m}_i 在地图中的坐标为 $(m_{ix}, m_{iy})^T$

t时刻观测得到的特征为 $\mathbf{z}_t = \{\mathbf{z}_t^1, \mathbf{z}_t^2, \dots\}$

每一个特征有一个 c_t^i 说明 \mathbf{z}_t^i 在地图中对应特征

$$c_t^i \in \{1, \dots, N, N+1\}$$

观测更新

基于所有观测特征的位姿更新

$$p(\mathbf{z}_t \mid \mathbf{x}_t, c_t, \mathbf{m}) = \prod_i p(\mathbf{z}_t^i \mid \mathbf{x}_t, c_t^i, \mathbf{m})$$



$$p(\mathbf{x}_t \mid \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \mathcal{N}(\mathbf{x}_t; \boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

$$\boldsymbol{\mu}_t = \bar{\boldsymbol{\mu}}_t + \sum_i \mathbf{K}_t^i (\mathbf{z}_t^i - h(\bar{\boldsymbol{\mu}}_t, c_t^i, \mathbf{m}))$$

$$\boldsymbol{\Sigma}_t = [\mathbf{I} - \sum_i \mathbf{K}_t^i \mathbf{H}_t^i] \bar{\boldsymbol{\Sigma}}_t$$



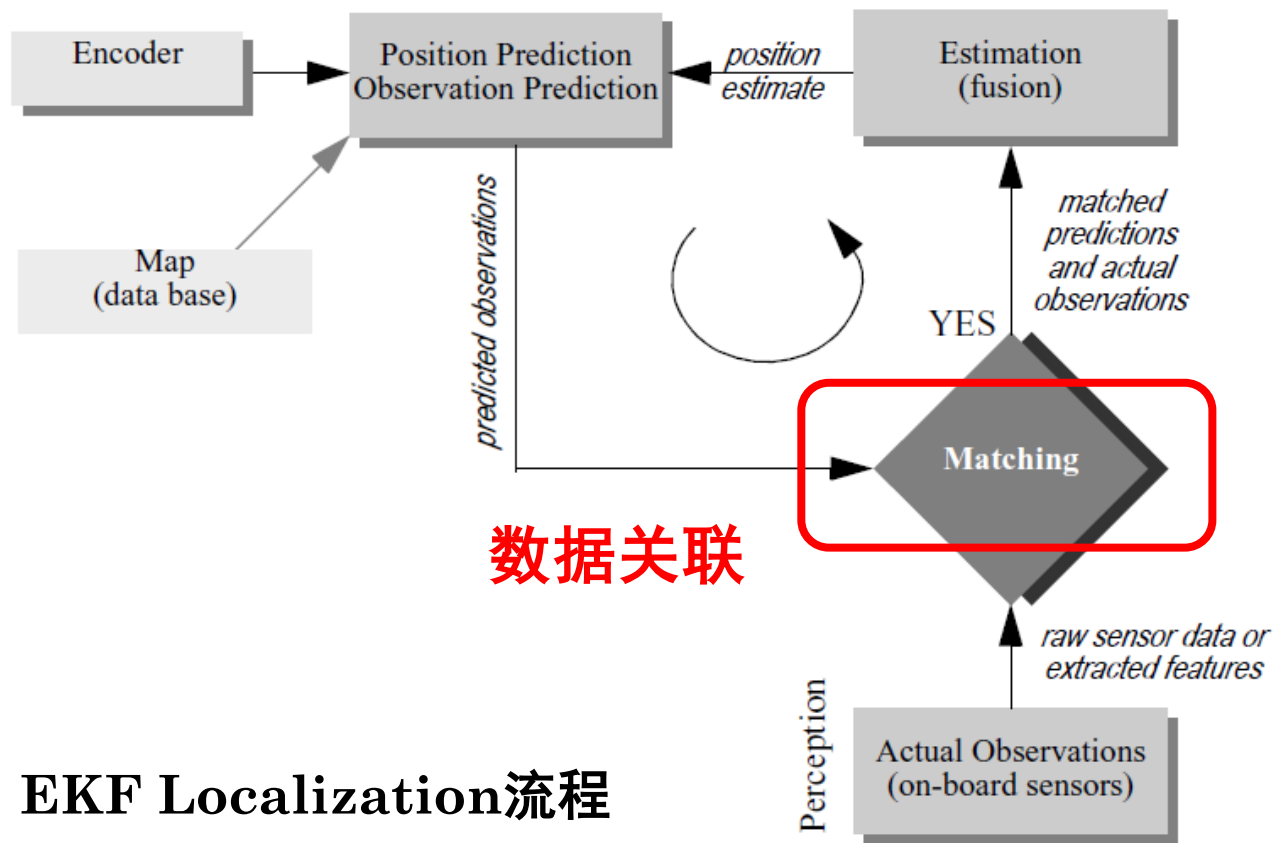
观测更新

设 $c_t^i = j$ **单个**特征观测模型为

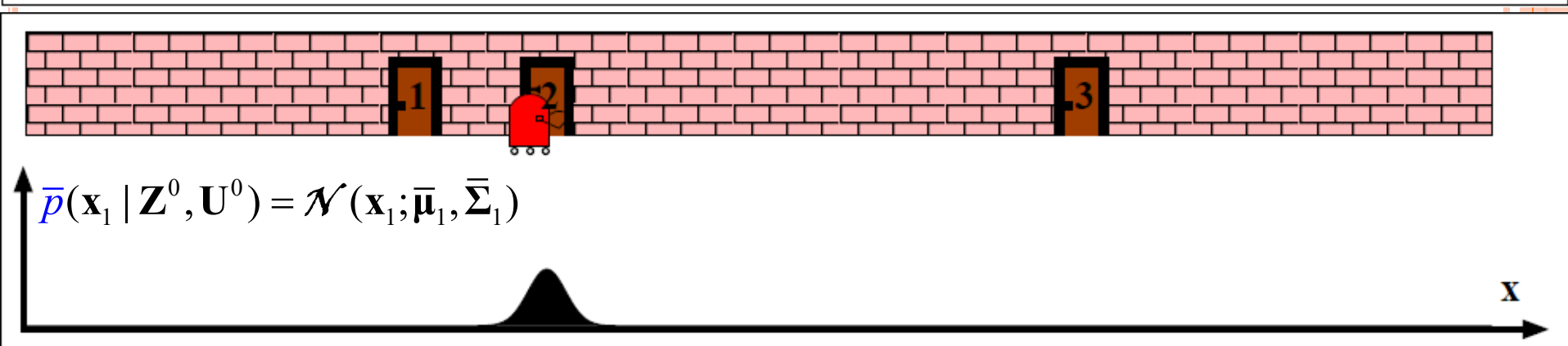
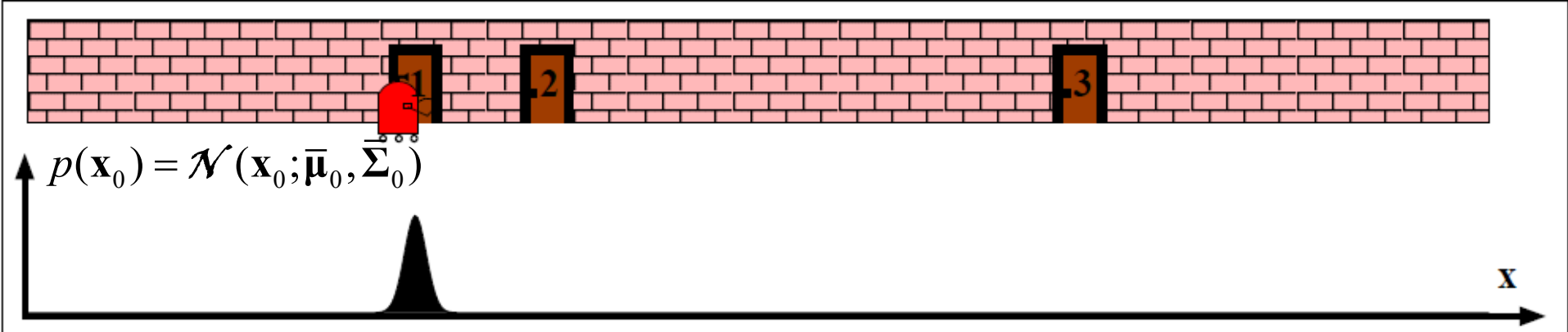
$$\underbrace{\begin{pmatrix} r_t^i \\ \varphi_t^i \\ s_t^i \end{pmatrix}}_{\mathbf{z}_t^i} = \underbrace{\begin{pmatrix} \sqrt{(m_{jx} - x)^2 + (m_{jy} - y)^2} \\ \arctan 2(m_{jy} - y, m_{jx} - x) - \theta \\ s_j \end{pmatrix}}_{\hat{\mathbf{z}}_t^i = h(\mathbf{x}_t, \mathbf{c}_t^i, \mathbf{m})} + \boldsymbol{\omega} \quad \boldsymbol{\omega}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$$

求 $h(\mathbf{x}_t, \mathbf{c}_t^i, \mathbf{m})$ 的Jacobian矩阵 $\mathbf{H}_t^i = \left. \frac{\partial h(\mathbf{x}_t, \mathbf{c}_t^i, \mathbf{m})}{\partial \mathbf{x}_t} \right|_{\mathbf{x}_t = \bar{\boldsymbol{\mu}}_t}$





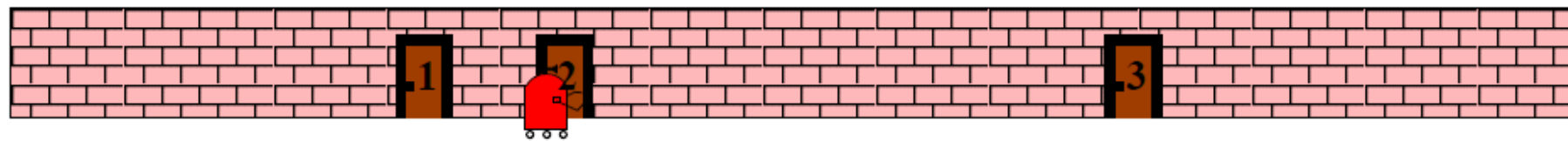
EKF Localization流程



$$\bar{p}(\mathbf{x}_1 | \mathbf{Z}^0, \mathbf{U}^0) = \int p(\mathbf{x}_1 | \mathbf{x}_0, \mathbf{u}_0) p(\mathbf{x}_0) d\mathbf{x}_0$$

$$\approx \mathcal{N}(\mathbf{x}_1; \bar{\boldsymbol{\mu}}_1, \bar{\boldsymbol{\Sigma}}_1) = \mathcal{N}(\mathbf{x}_1; g(\boldsymbol{\mu}_{t-1}, \mathbf{u}_{t-1}), \mathbf{G}_t \boldsymbol{\Sigma}_{t-1} \mathbf{G}_t^T + \mathbf{R}_t)$$





仅观测到一个特征

$$p(\mathbf{z}_1^1 \mid \mathbf{x}_1, c_1^1, \mathbf{m}) \sim \mathcal{N}(\mathbf{z}_1^1; h(\mathbf{x}_1, c_1^1, \mathbf{m}), \mathbf{Q}_1)$$

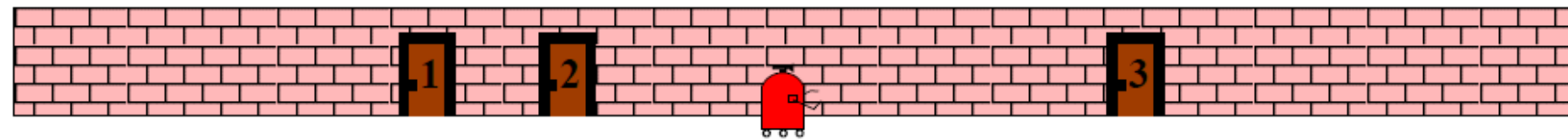


\mathbf{x}

$$p(\mathbf{x}_1 \mid \mathbf{Z}^1, \mathbf{U}^0) = \mathcal{N}(\mathbf{x}_1; \boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$$



\mathbf{x}



$$\bar{p}(\mathbf{x}_2 \mid \mathbf{Z}^1, \mathbf{U}^1) = \mathcal{N}(\mathbf{x}_2; \bar{\boldsymbol{\mu}}_2, \bar{\boldsymbol{\Sigma}}_2)$$



\mathbf{x}

EKF LOCALIZATION的约束

- 定位效果依赖于数据关联的正确性，错误的数据关联将导致定位失败
- 单峰假设和容易受数据关联正确性影响使得EKF不适合实现全局定位



数据关联问题

- 一般情况下 c_t^i 未知
- 解决方法: maximum likelihood correspondence

$$\hat{c}_t = \arg \max_{c_t} p(\mathbf{z}_t | C^t, \mathbf{m}, \mathbf{Z}^{t-1}, \mathbf{U}^{t-1})$$

为避免指数级的巨大计算量，单独估计每个特征的最大可能对应

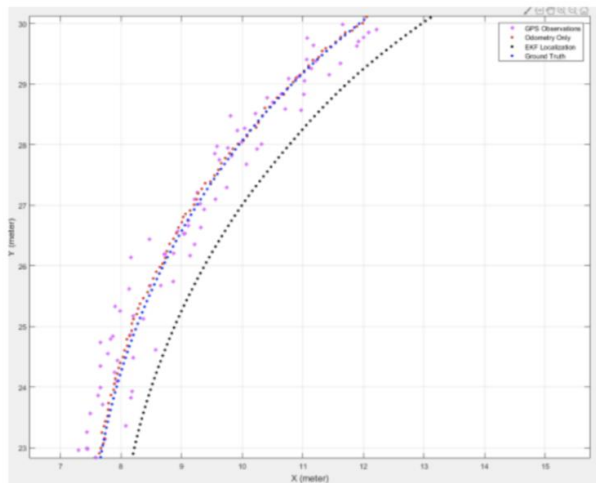
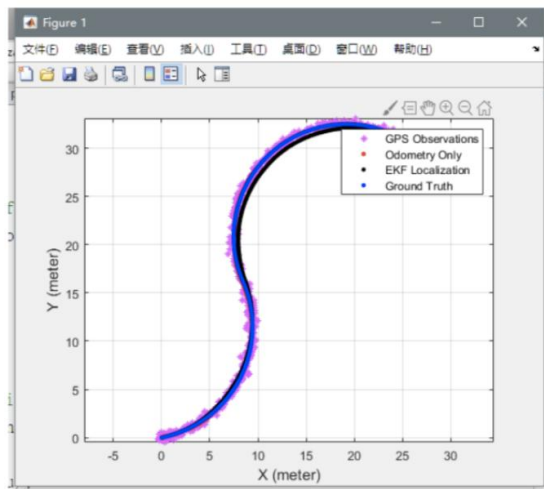
$$\hat{c}_t^i = \arg \max_{c_t^i} p(\mathbf{z}_t^i | C^t, \mathbf{m}, \mathbf{Z}^{t-1}, \mathbf{U}^{t-1})$$

$$\approx \arg \max_{c_t^i} \mathcal{N}(\mathbf{z}_t^i; h(\bar{\boldsymbol{\mu}}_t, c_t^i, \mathbf{m}), \mathbf{H}_t^i \bar{\boldsymbol{\Sigma}}_t \mathbf{H}_t^{iT} + \mathbf{Q}_t)$$



作业

- 在Matlab中，利用ekf_localization.m 文件，实现移动机器人的 EKF 定位算法



助教：贾泽美

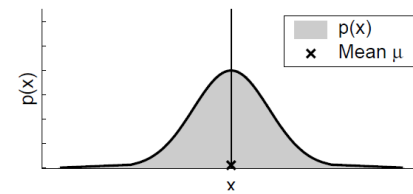
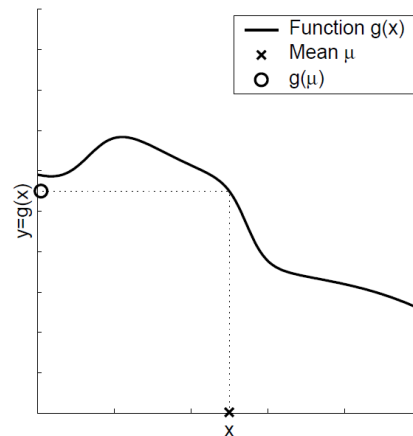
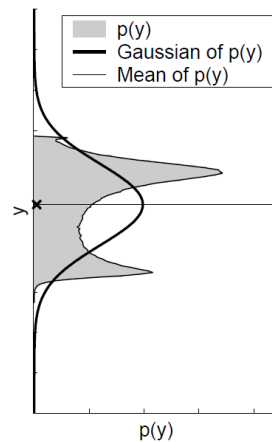




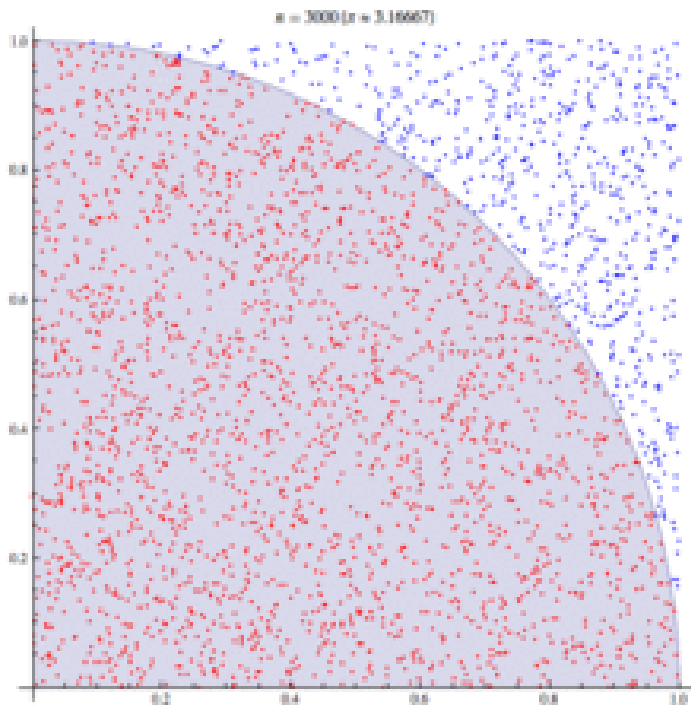
9.2 粒子滤波定位法

蒙特卡洛法 (MONTE CARLO METHOD)

- 粒子滤波的思想来源于Monte Carlo Method (也称为统计模拟方法)，起源于二战中的原子弹设计研究
- 蒙特卡洛法的基本思想：
通过大统计量($N \rightarrow \infty$)的统计实验方法或随机模拟方法来求解某些计算量或随机变量的概率模型



蒙特卡洛法应用实例



利用随机统计法求 π

1. 建立圆与外切正方形面积之间的关系

$$\text{面积比} = \pi / 4$$

2. 在整个区域内随机撒点

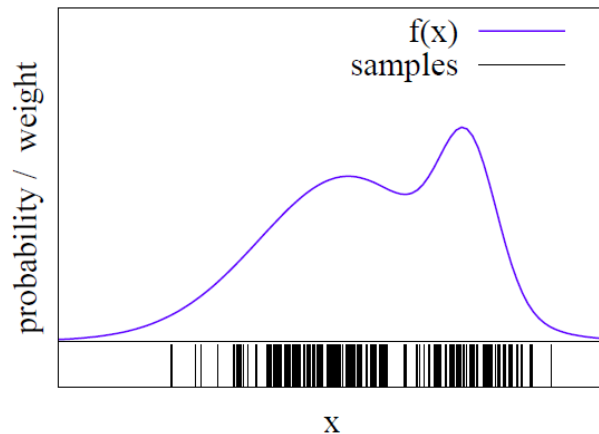
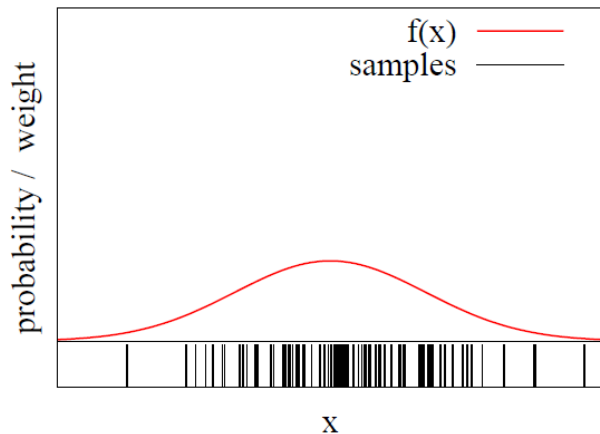
3. 统计圆形内的点数和外切正方形内的点数，
当总点数趋于无穷时，它们之比逼近圆和外切正方形面积之比

$$\text{点数比} \approx \text{面积比} = \pi / 4$$

4. 求得 π

粒子滤波 (PARTICLE FILTER)

- 基本思想：通过一组样本（粒子）分布来近似表示概率分布



某区域内粒子分布越密集，真实状态落在该区域的可能性越大

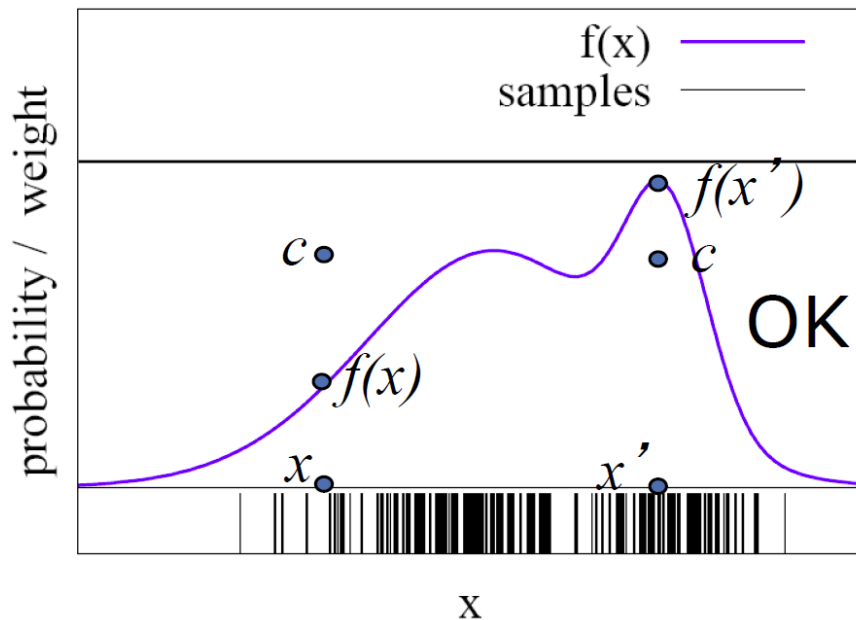
优点：对于任意概率分布都可以通过样本分布来描述

如何采样

- 目标：使样本分布能够近似密度函数/概率分布
- 方法：采用样本排除法

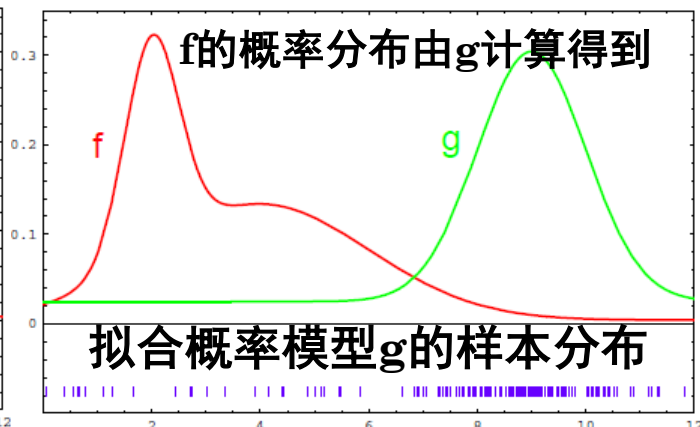
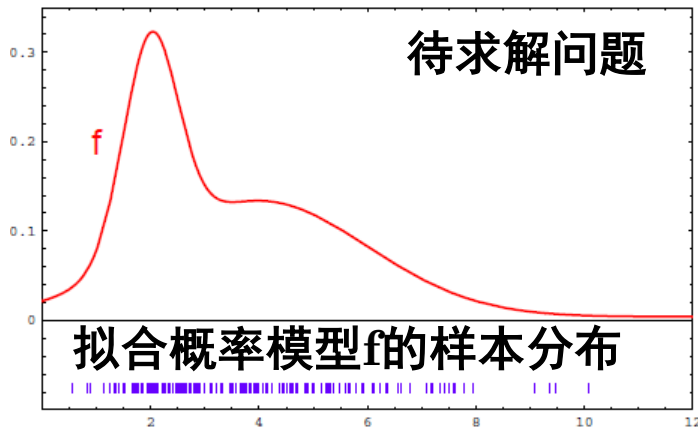
- 从均匀分布中采样 x
- 从 $[0,1]$ 中随机采样 c

$\begin{cases} \text{keep the sample} & \text{if } f(x) > c \\ \text{reject the sample} & \text{otherwise} \end{cases}$



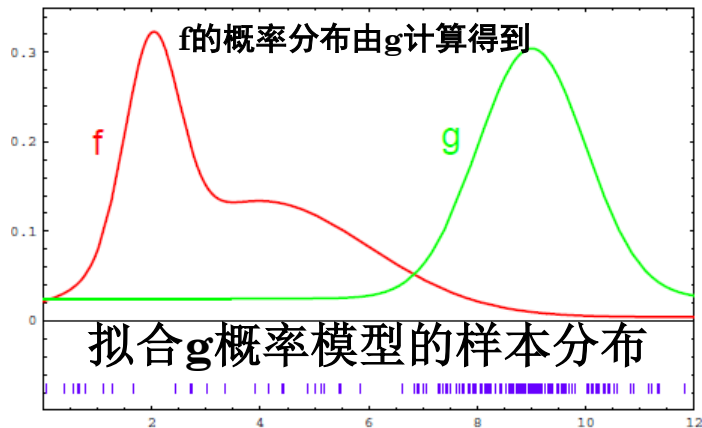
基于重要性评估计算目标样本分布

- 根据一个已知的可以采样的分布采样
- 通过对粒子的重要性评估进行重采样获得目标样本分布



在 g 概率大的地方 f 概率很小，说明这些粒子对于 f 来讲并不重要，而对应于 f 大的地方的那些粒子比较重要，因此可以通过各个样本 f 和 g 比值来评估拟合 g 的样本的重要性

样本重要性评估



f: target distribution
g: proposal distribution

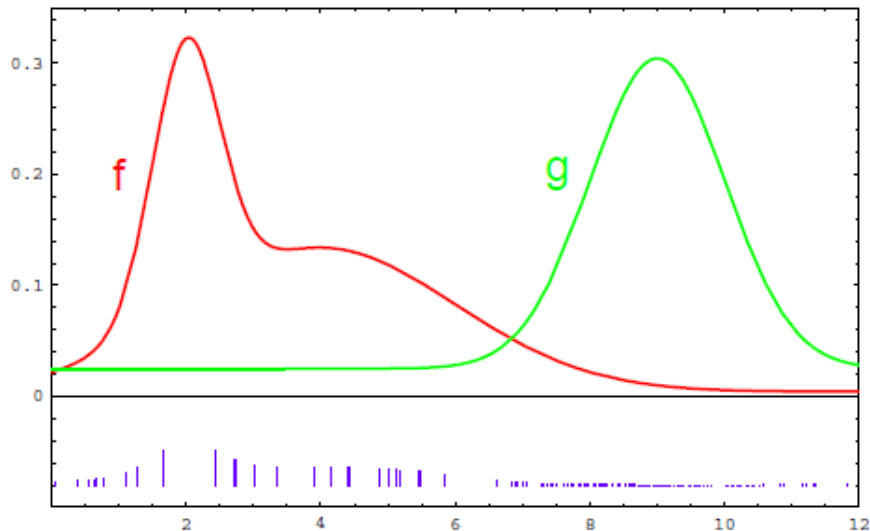
根据每个样本f和g的比值来评估拟合g的样本的重要性

$$\omega(x) = \frac{\text{target distribution}}{\text{proposal distribution}} = \frac{f(x)}{g(x)}$$

$$\text{pre-condition: } f(x) > 0 \rightarrow g(x) > 0$$

样本重要性评估

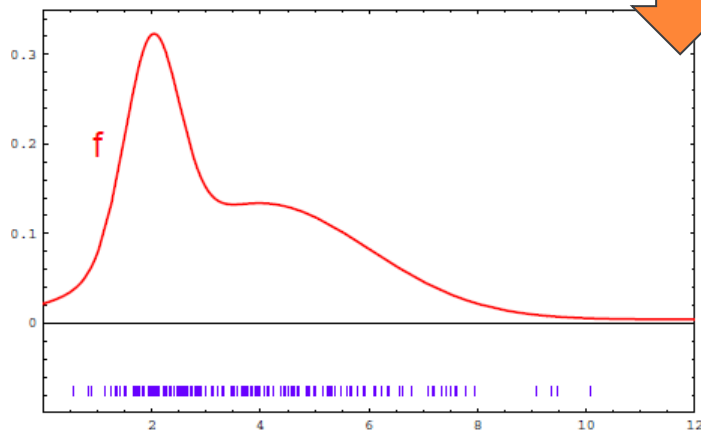
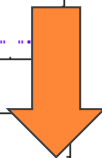
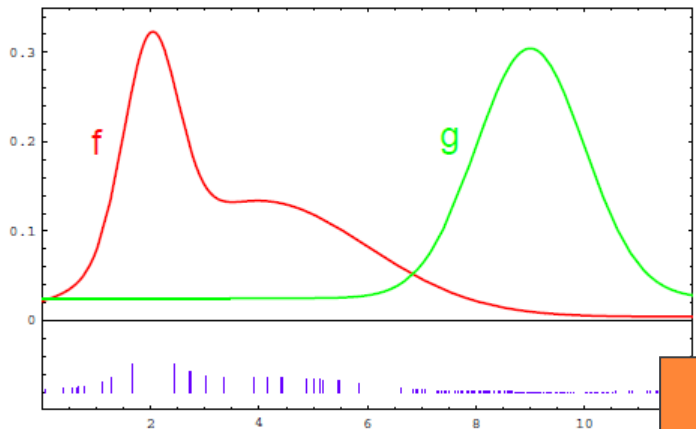
$$\omega(x) = \frac{\text{target distribution}}{\text{proposal distribution}} = \frac{f(x)}{g(x)}$$



$\omega(x)$ Importance weight



基于重要性重采样



$$\bar{\chi} = \left\{ \left\langle \bar{\mathbf{x}}^{[j]}, \bar{\omega}^{[j]} \right\rangle \right\}, j = 1, \dots, M$$

$$\chi = \left\{ \left\langle \mathbf{x}^{[i]}, \omega^{[i]} \right\rangle \right\}, i = 1, \dots, M$$

$$\mathbf{x}^{[i]} \in \{ \bar{\mathbf{x}}^{[j]}, j = 1, \dots, M \}$$

$$\omega^{[i]} = \frac{1}{M}$$



重采样的基本方法

1. 权重归一化 $\tilde{\omega}^{[m]} = \frac{\bar{\omega}^{[m]}}{\sum_{i=1}^M \bar{\omega}^{[i]}}$

2. 根据归一化后的权重在 $\bar{\chi}_t$ 中重新采样

$$\mathbf{x}^{[i]} = \bar{\mathbf{x}}^{[j(i)]}$$

$$p(j(i) = l) = \tilde{\omega}_t^{[l]}, l = 1, \dots, M$$

3. 对于重采样得到的粒子重新赋予权重

$$\omega^{[i]} = \frac{1}{M}$$



重要性准则及重采样举例：根据陆标进行机器人定位

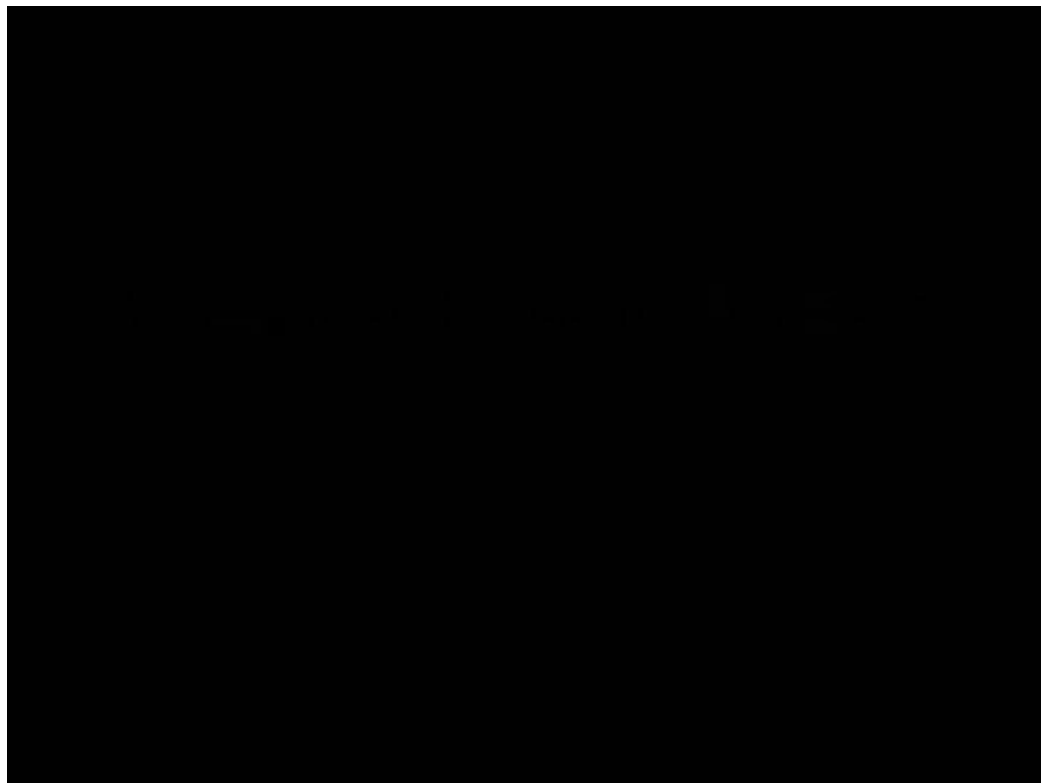


陆标的位置在地图中
是已知的

陆标的观测信息是在机器
人坐标系下的距离信息



重要性准则及重采样举例：根据陆标进行机器人定位



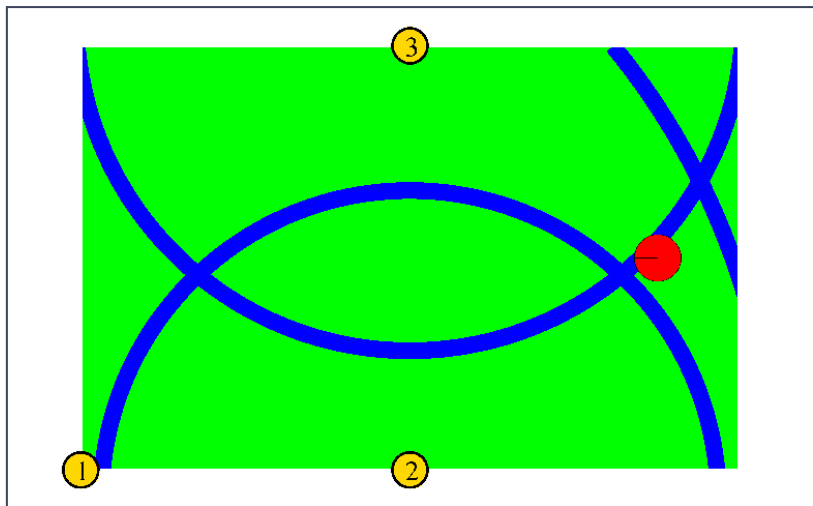
陆标的位置在地图中是已知的

陆标的观测信息是在机器人坐标系下的距离信息





(1)(2)(3)为观测得到的三个路标，
由于地图已知，根据它们在地图
中的位置和测量得到的与机器人
的距离，可以得到机器人所在位
置的推测



因为观测的不确定性，利用三点
法得到的是一个区域



基于多个观测的位置概率分布计算

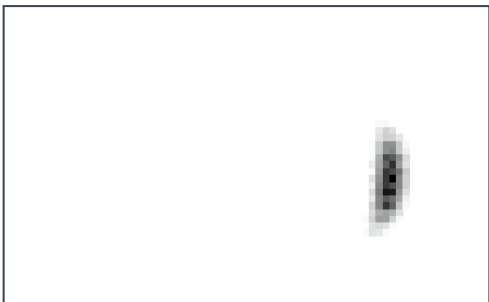
建议分布


$$p(\mathbf{x} | \mathbf{z}_1)$$


$$p(\mathbf{x} | \mathbf{z}_2)$$

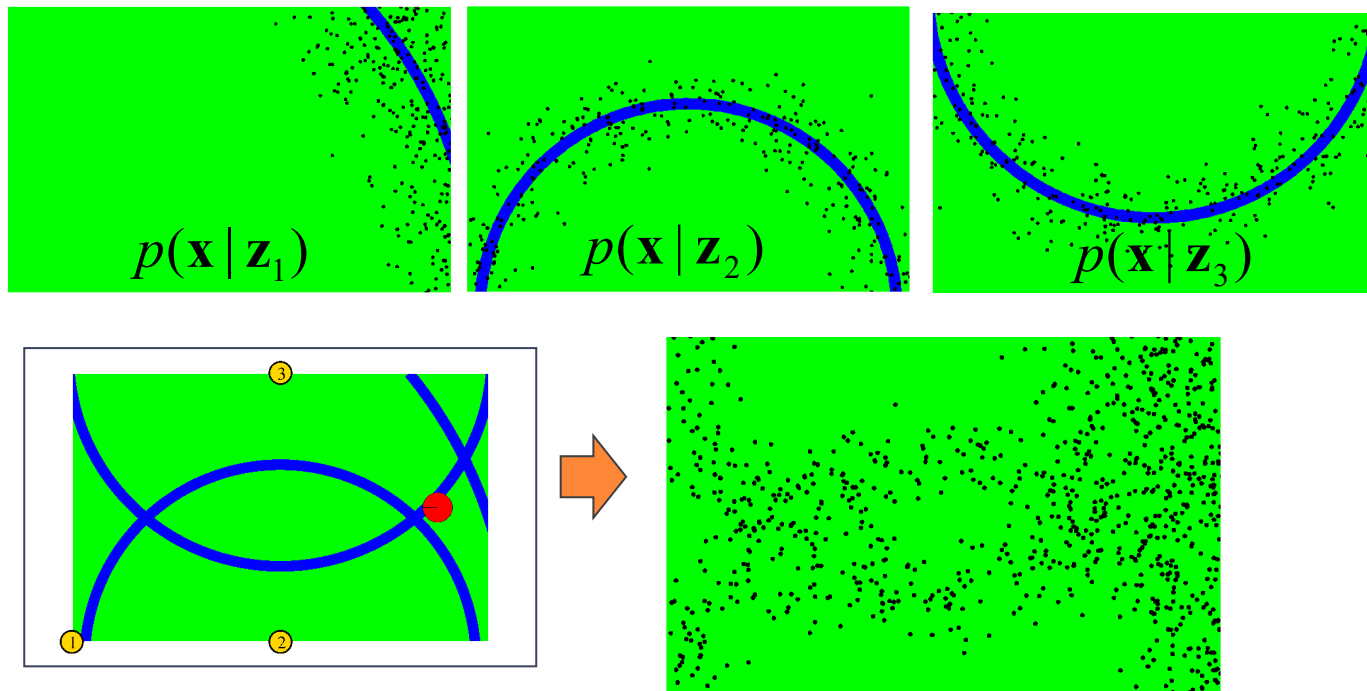

$$p(\mathbf{x} | \mathbf{z}_3)$$

目标分布


$$p(\mathbf{x} | \mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3)$$

问题：利用粒子滤波由建议分布求目标分布

获得建议分布样本： 通过对观测参数增加噪声进行样本采样

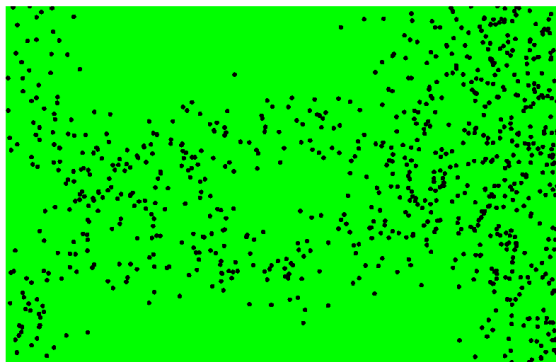


总样本集合，每个样本由三个观测中的一个产生

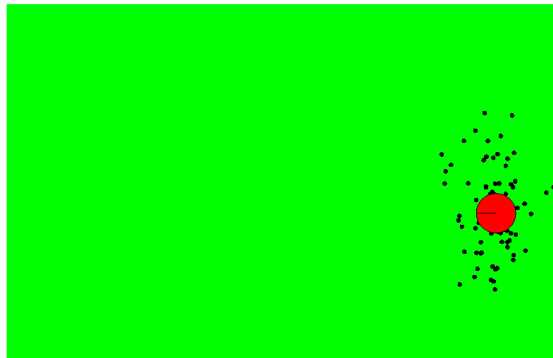
样本的重要性计算

目标分布

$$p(\mathbf{x} | \mathbf{z}_1, \dots, \mathbf{z}_n) = \frac{\prod_k p(\mathbf{z}_k | \mathbf{x}) p(\mathbf{x})}{p(\mathbf{z}_1, \dots, \mathbf{z}_n)}$$



建议分布粒子样本



目标分布粒子样本



样本的重要性计算

目标分布

$$p(\mathbf{x} | \mathbf{z}_1, \dots, \mathbf{z}_n) = \frac{\prod_k p(\mathbf{z}_k | \mathbf{x}) p(\mathbf{x})}{p(\mathbf{z}_1, \dots, \mathbf{z}_n)}$$

陆标 \mathbf{z}_l 建议分布

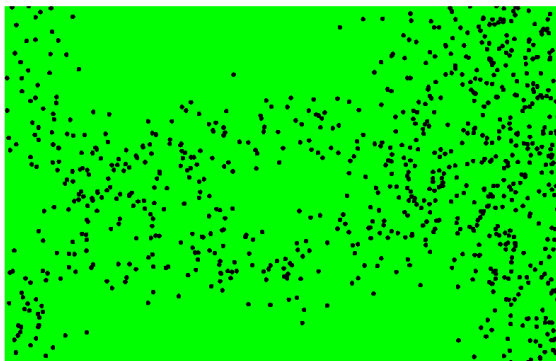
$$p(\mathbf{x} | \mathbf{z}_l) = \frac{p(\mathbf{z}_l | \mathbf{x}) p(\mathbf{x})}{p(\mathbf{z}_l)}$$

权重计算

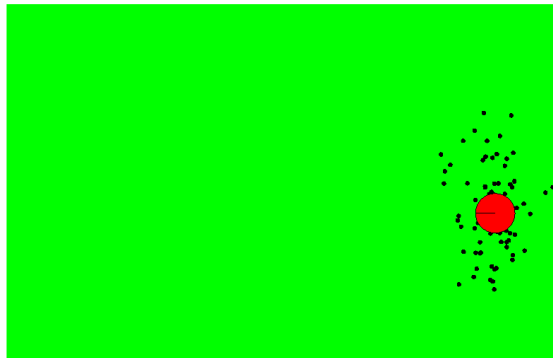
$$\omega = \frac{p(\mathbf{x} | \mathbf{z}_1, \dots, \mathbf{z}_n)}{p(\mathbf{x} | \mathbf{z}_l)} = \frac{p(\mathbf{z}_l) \prod_{k \neq l} p(\mathbf{z}_k | \mathbf{x})}{p(\mathbf{z}_1, \dots, \mathbf{z}_n)}$$



根据样本权重进行重采样



Weighted samples



After resampling



PARTICLE FILTER LOCALIZATION

- 记t时刻的粒子集合为

$$\mathcal{X}_t = \{\mathbf{x}_t^{[1]}, \mathbf{x}_t^{[2]}, \dots, \mathbf{x}_t^{[M]}\} \quad \text{上标为粒子索引, M为粒子总数}$$

每一个粒子 $\mathbf{x}_t^{[i]}$ 是t时刻状态的一个假设

$$\mathbf{x}_t^{[i]} \sim p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m})$$

某一粒子从0时刻到当前时刻的序列 $\mathbf{x}_{0:t}^{[i]} = \mathbf{x}_0^{[i]}, \mathbf{x}_1^{[i]}, \dots, \mathbf{x}_t^{[i]}$

表示了对某一路径的假设

粒子滤波实际是计算所有状态序列的后验概率



PARTICLE FILTER LOCALIZATION

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m}) \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m}) p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$



t时刻后验



观测模型



运动模型



t-1时刻后验



t时刻粒子集



粒子评价



粒子传递



t-1时刻粒子集



PARTICLE FILTER LOCALIZATION

○ 算法

- 以表示上一时刻概率分布的粒子集合为起点 χ_{t-1}
- 利用运动模型对粒子进行更新并采样
- 利用观测模型来计算粒子的权重
- 根据权重重新采样，生成表示当前时刻信度的粒子集合 χ_t

○ 也称为Monte Carlo Localization (MCL)



利用运动模型对粒子进行更新及采样

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m}) \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m}) p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$



基于运动模型的
粒子更新与采样



t-1时刻粒子集

$$\mathcal{X}_{t-1} = \{\mathbf{x}_{t-1}^{[1]}, \mathbf{x}_{t-1}^{[2]}, \dots, \mathbf{x}_{t-1}^{[M]}\}$$

$$\bar{\mathbf{x}}_t^{[i]} = \mathbf{x}_{t-1}^{[i]} + \begin{pmatrix} \delta_{trans} \cos(\theta + \delta_{rot1}) \\ \delta_{trans} \sin(\theta + \delta_{rot1}) \\ \theta + \delta_{rot1} + \delta_{rot2} \end{pmatrix} + \mathbf{v}_t, \mathbf{v}_t \sim N(\mathbf{0}, \mathbf{R}_t)$$

$$\bar{\mathcal{X}}_t = \{\bar{\mathbf{x}}_t^{[1]}, \bar{\mathbf{x}}_t^{[2]}, \dots, \bar{\mathbf{x}}_t^{[M]}\}$$



利用运动模型对粒子进行更新及采样

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m}) \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m}) p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$



基于运动模型的
粒子更新与采样



t-1时刻粒子集

$$\mathcal{X}_{t-1} = \{\mathbf{x}_{t-1}^{[1]}, \mathbf{x}_{t-1}^{[2]}, \dots, \mathbf{x}_{t-1}^{[M]}\}$$

$$\bar{\mathcal{X}}_t = \{\bar{\mathbf{x}}_t^{[1]}, \bar{\mathbf{x}}_t^{[2]}, \dots, \bar{\mathbf{x}}_t^{[M]}\} \sim \bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1}, \mathbf{m})$$



采用观测模型计算粒子权重

为什么？

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m}) \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m}) p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$

目标分布



基于运动模型的
粒子更新与采样



t-1时刻粒子集

$$\mathcal{X}_{t-1} = \{\mathbf{x}_{t-1}^{[1]}, \mathbf{x}_{t-1}^{[2]}, \dots, \mathbf{x}_{t-1}^{[M]}\}$$

$$\bar{\mathcal{X}}_t = \{\bar{\mathbf{x}}_t^{[1]}, \bar{\mathbf{x}}_t^{[2]}, \dots, \bar{\mathbf{x}}_t^{[M]}\} \sim \bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1}, \mathbf{m})$$

建议分布



采用观测模型计算粒子权重

为什么？

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \underbrace{\eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m})}_{\text{权重}} \int \underbrace{p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m})}_{\text{基于运动模型的粒子更新与采样}} \underbrace{p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m})}_{\text{t-1时刻粒子集}} d\mathbf{x}_{t-1}$$

目标分布

权重

基于运动模型的
粒子更新与采样

t-1时刻粒子集

$$\mathbf{x}_{t-1} = \{\mathbf{x}_{t-1}^{[1]}, \mathbf{x}_{t-1}^{[2]}, \dots, \mathbf{x}_{t-1}^{[M]}\}$$

$$\bar{\mathbf{x}}_t = \{\bar{\mathbf{x}}_t^{[1]}, \bar{\mathbf{x}}_t^{[2]}, \dots, \bar{\mathbf{x}}_t^{[M]}\} \sim \bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1}, \mathbf{m})$$

建议分布



采用观测模型计算粒子权重

$$\omega_t^{[i]} = p\left(\mathbf{z}_t \mid \bar{\mathbf{x}}_t^{[i]}, \mathbf{m}\right)$$

$$p\left(\mathbf{z}_t \mid \bar{\mathbf{x}}_t^{[i]}, \mathbf{m}\right) = p\left(f_t^1, f_t^2, \dots, f_t^K \mid \bar{\mathbf{x}}_t^{[i]}, \mathbf{m}\right) = \prod_{k=1}^K p\left(f_t^k \mid \bar{\mathbf{x}}_t^{[i]}, \mathbf{m}\right)$$

$$p\left(f_t^k \mid \bar{\mathbf{x}}_t^{[i]}, \mathbf{m}\right) = N(f_t^i; \hat{f}_t^j, \sigma_f^2)$$

$$\bar{\mathcal{X}}_t = \{\bar{\mathbf{x}}_t^{[1]}, \bar{\mathbf{x}}_t^{[2]}, \dots, \bar{\mathbf{x}}_t^{[M]}\} \sim \bar{p}(\mathbf{x}_t \mid \mathbf{Z}^{t-1}, \mathbf{U}^{t-1}, \mathbf{m})$$

$$[\langle \bar{\mathbf{x}}_t^{[i]}, \omega_t^{[i]} \rangle] \sim p(\mathbf{x}_t \mid \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m})$$



Algorithm Particle_filter($\mathcal{X}_{t-1}, \mathbf{u}_{t-1}, \mathbf{z}_t$)

$$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$$

for $i=1$ *to* M *do*

Sample $\bar{\mathbf{x}}_t^{[i]} \sim \bar{p}(\mathbf{x}_t | \mathbf{Z}^{t-1}, \mathbf{U}^{t-1}, \mathbf{m})$

$$\omega_t^{[i]} = p(\mathbf{z}_t | \bar{\mathbf{x}}_t^{[i]}, \mathbf{m})$$

$$\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle \bar{\mathbf{x}}_t^{[i]}, \omega_t^{[i]} \rangle$$

endfor

for $i=1$ *to* M *do*

draw I *with probability* $\propto \omega_t^{[i]}$

$\mathbf{x}_t^{[i]} = \bar{\mathbf{x}}_t^{[i]}$, *add* $\mathbf{x}_t^{[i]}$ *to* \mathcal{X}_t

endfor

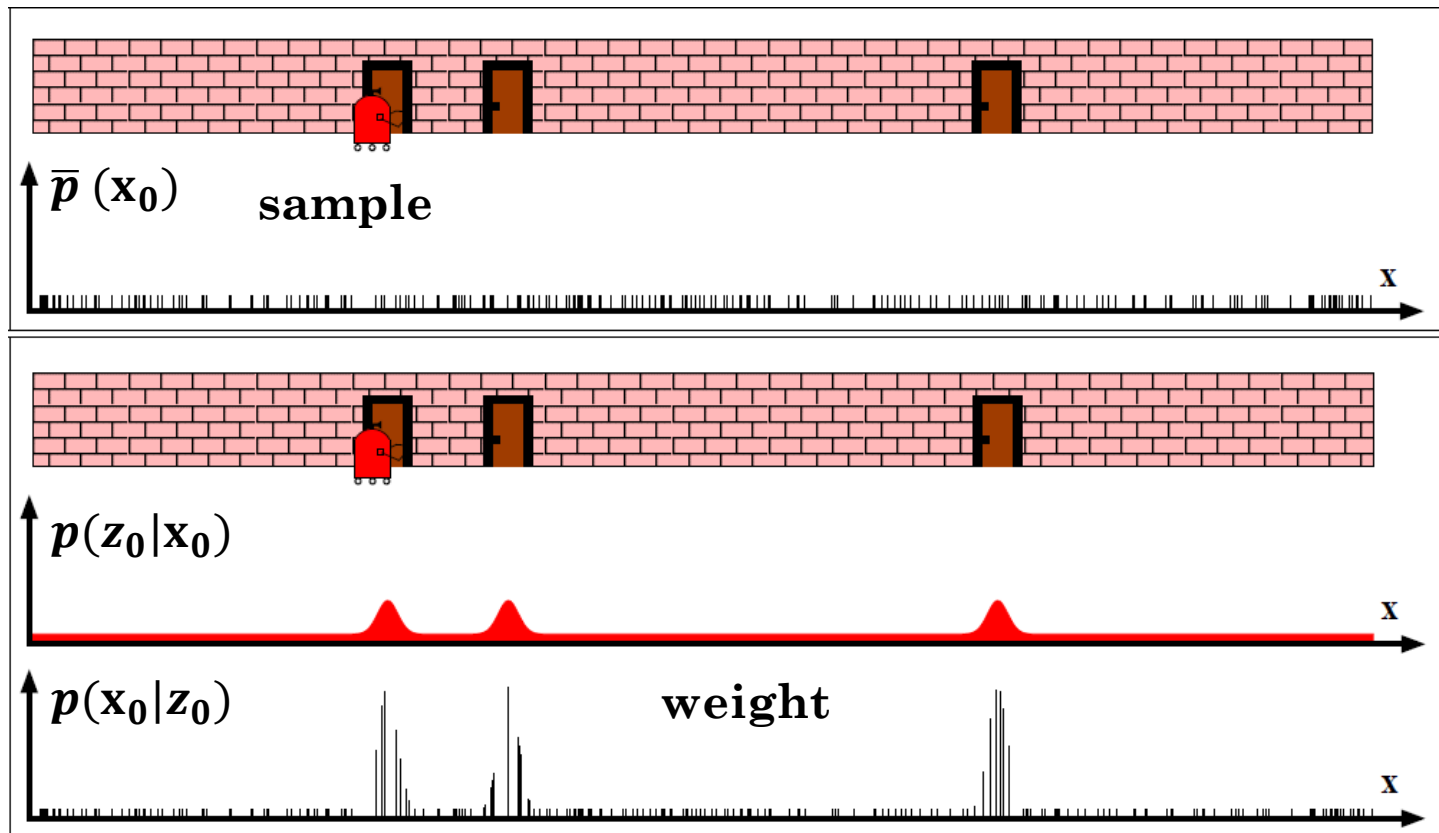
return \mathcal{X}_t

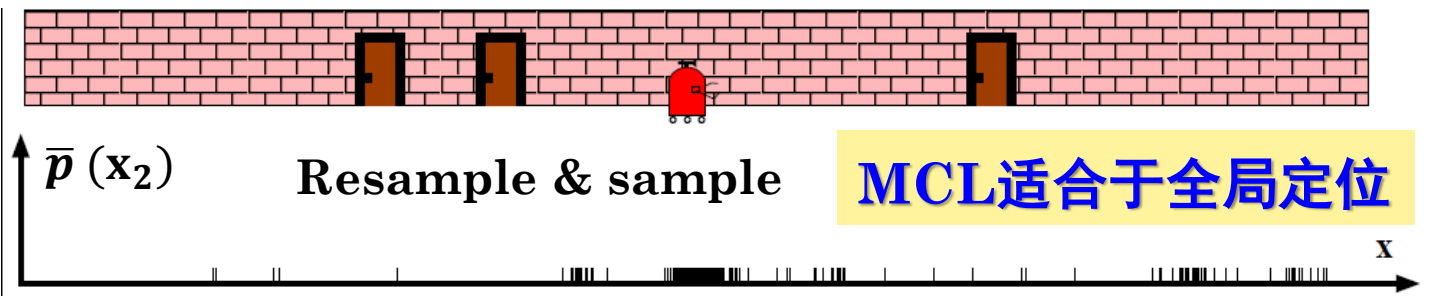
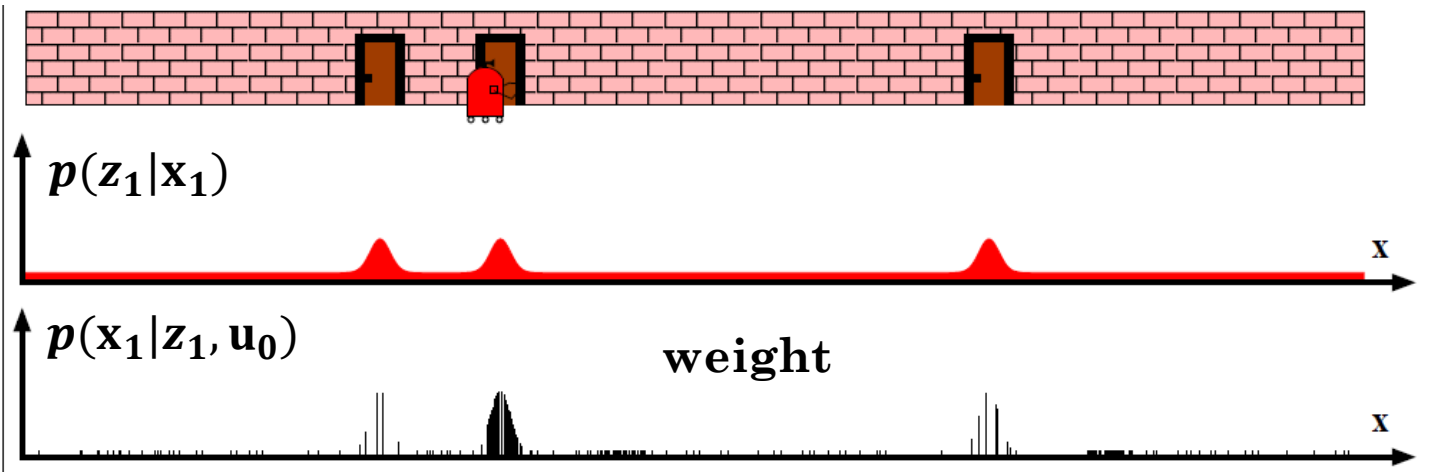
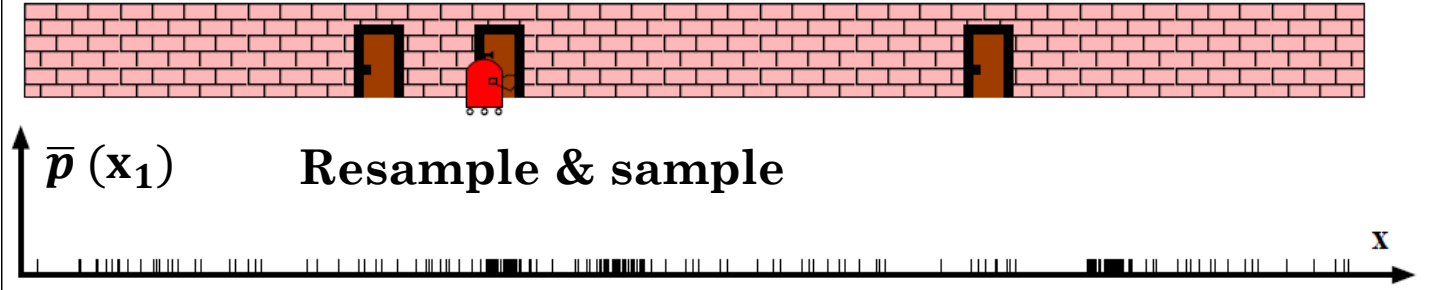
重采样 resampling

根据粒子的重要性重新采样



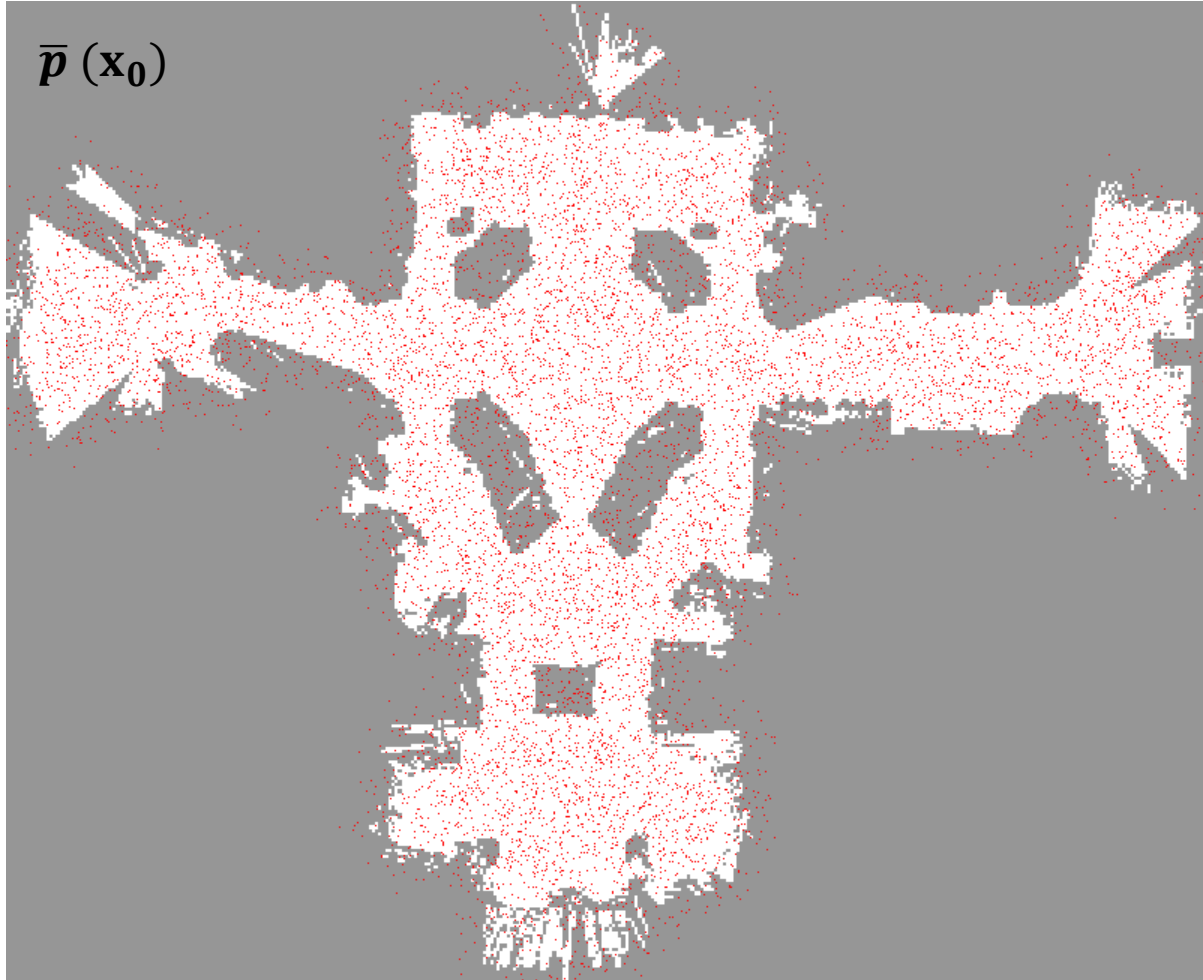
Monte Carlo 定位法

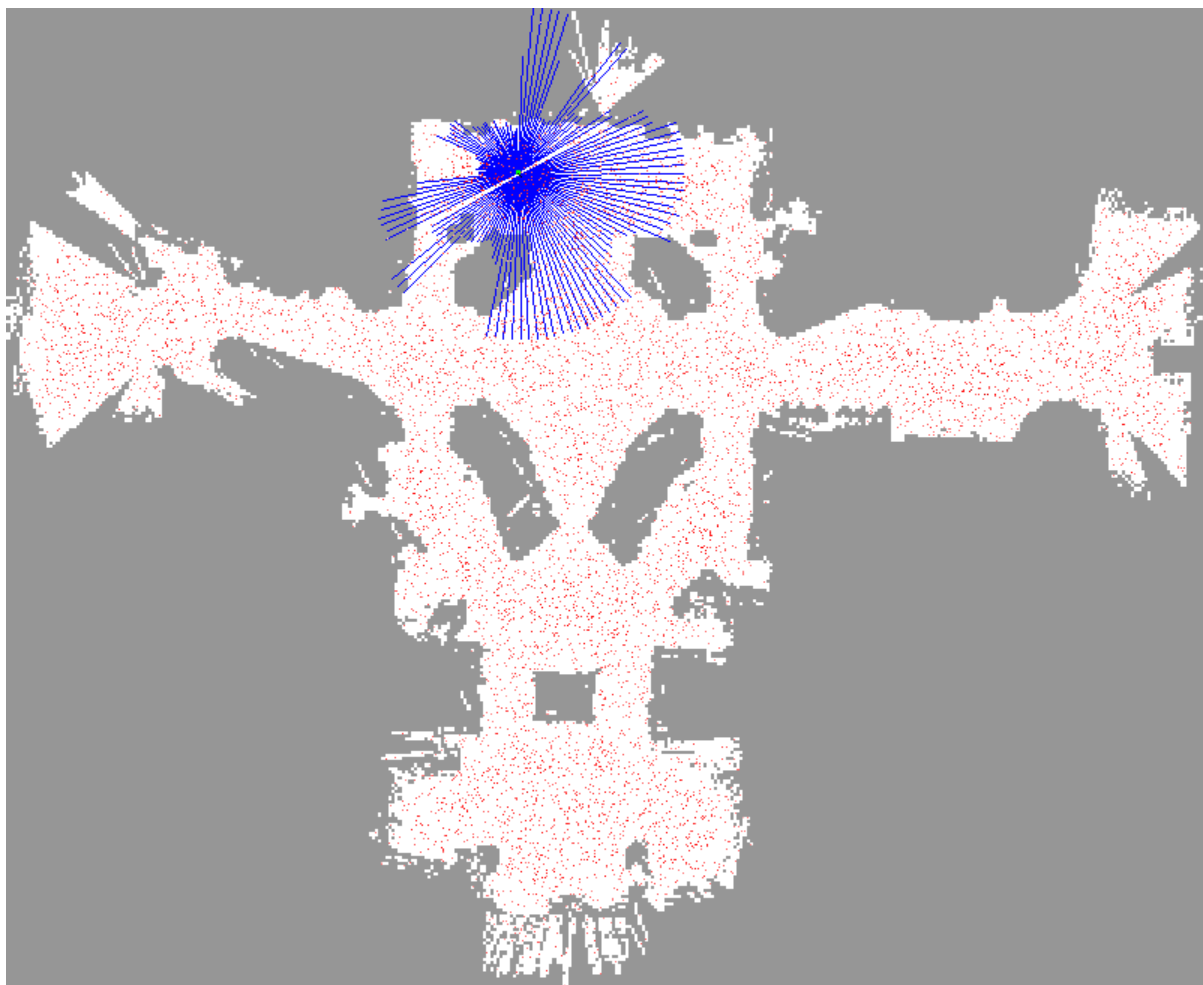




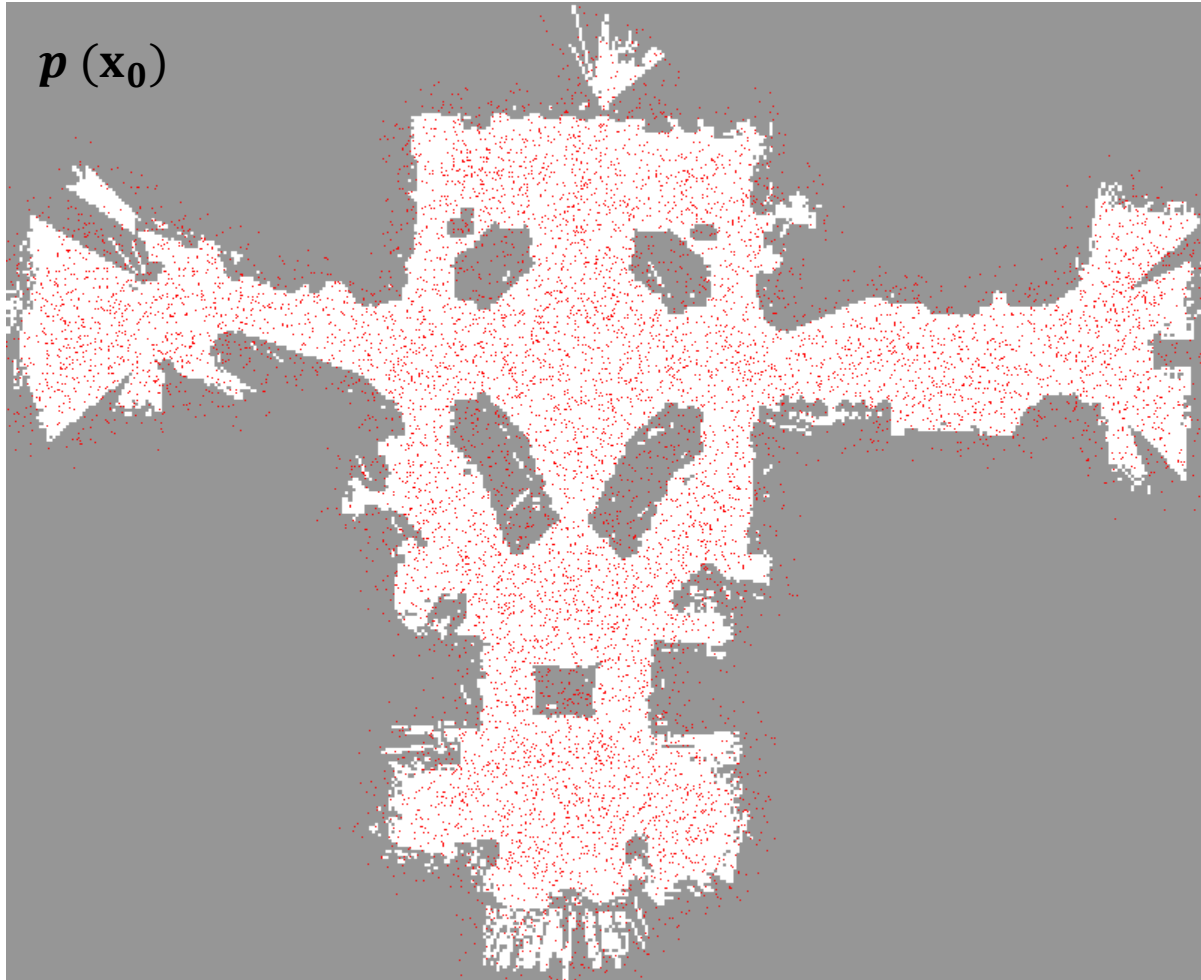
MCL适合于全局定位

$\bar{p}(\mathbf{x}_0)$

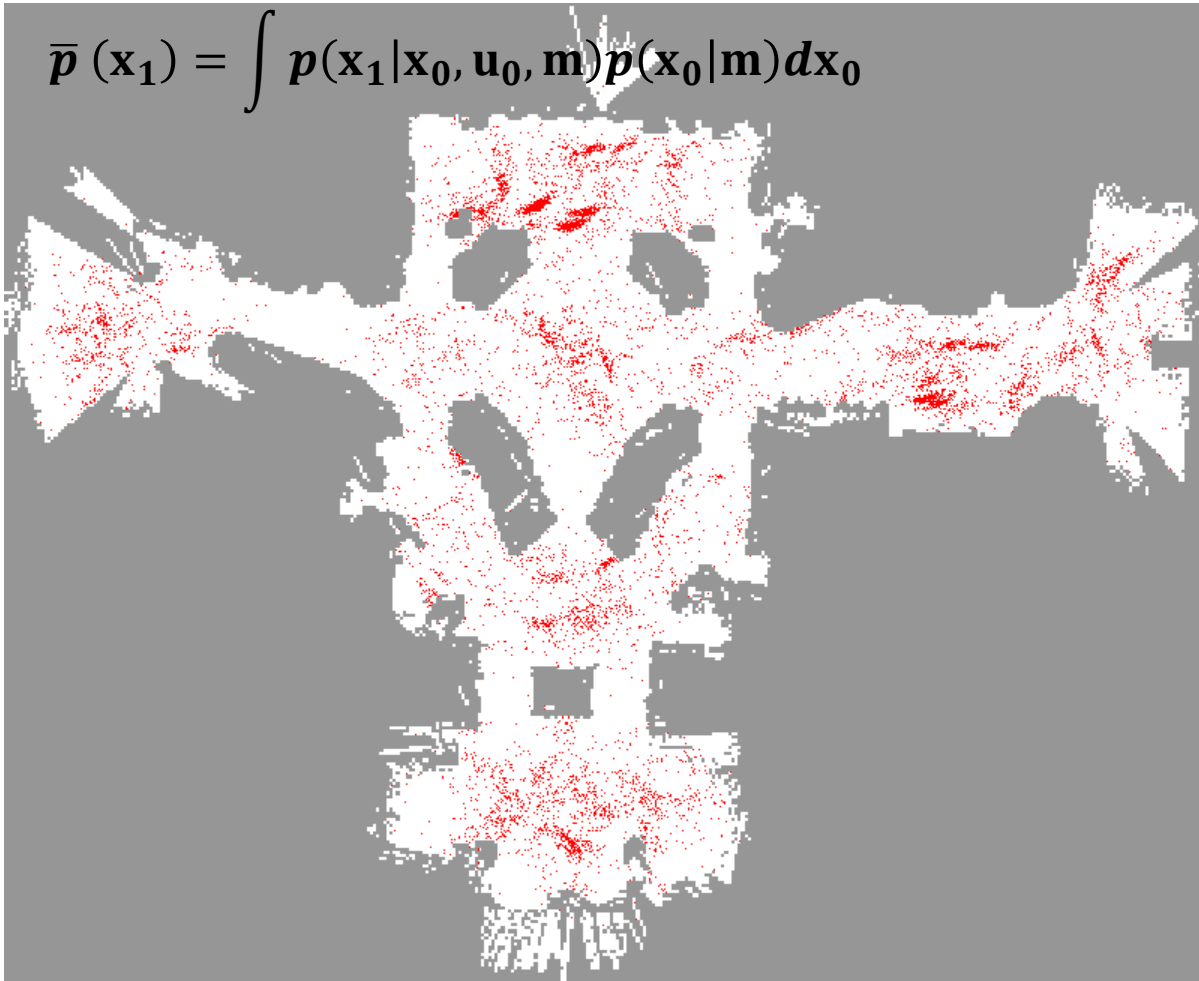


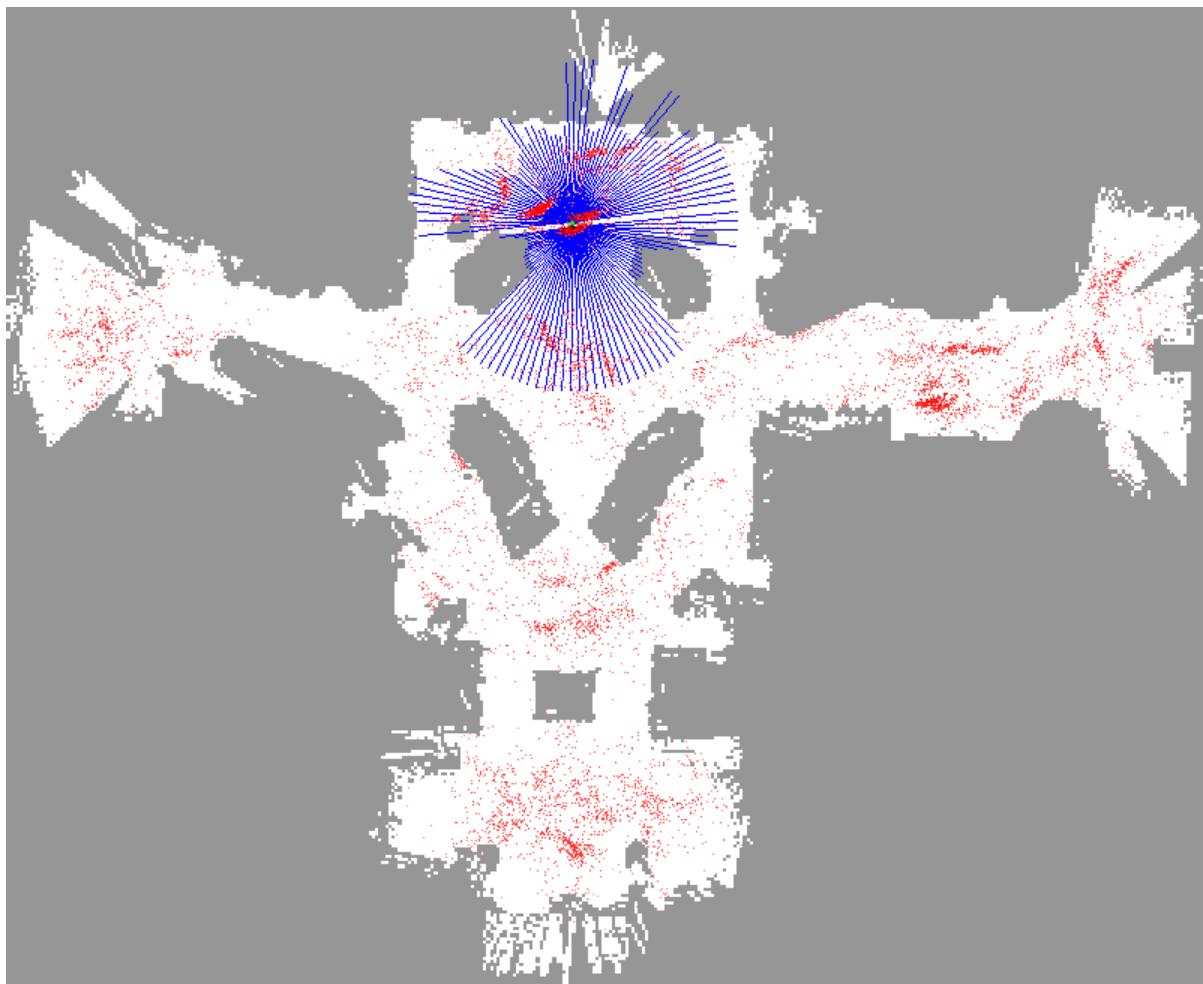


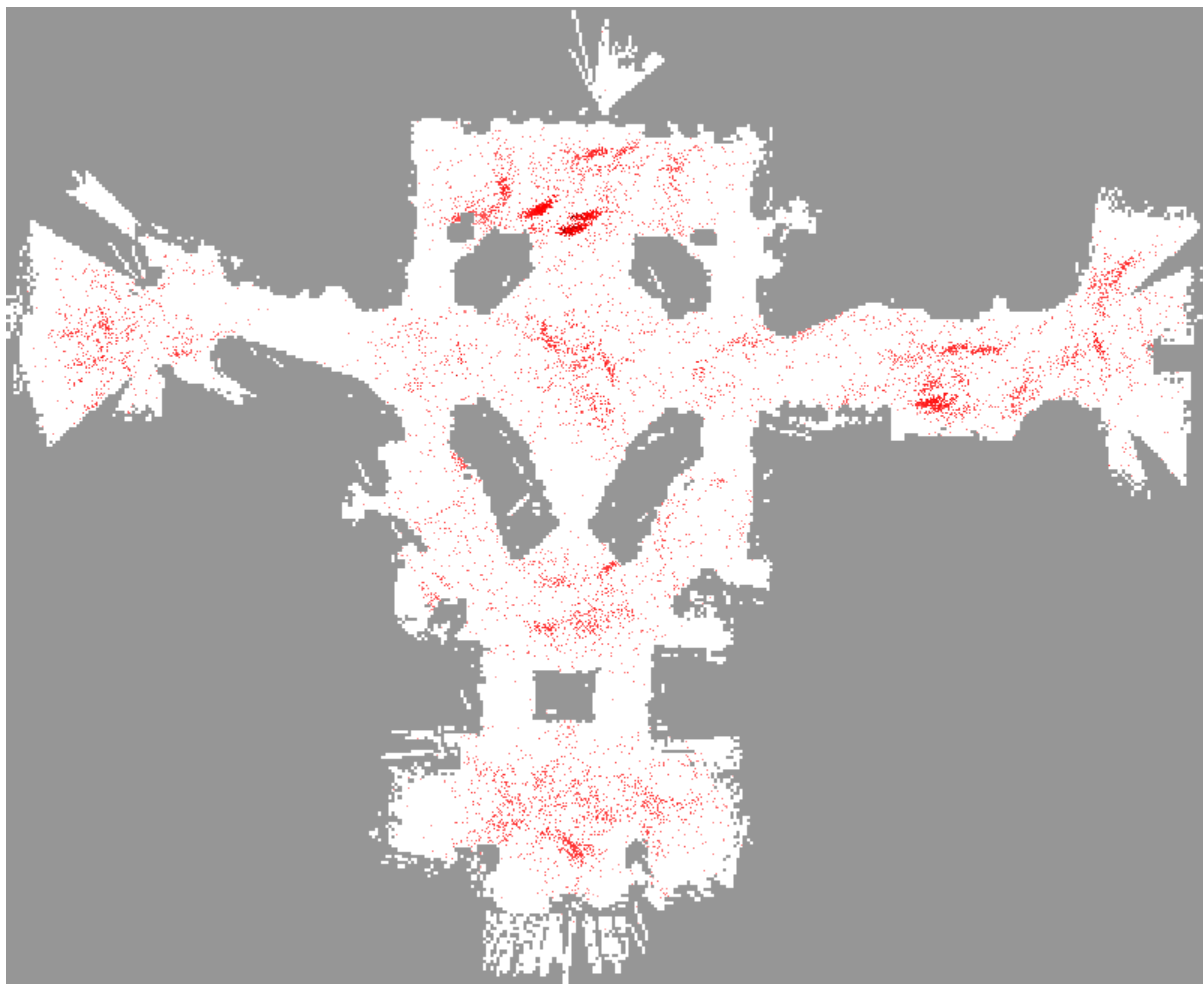
$p(\mathbf{x}_0)$

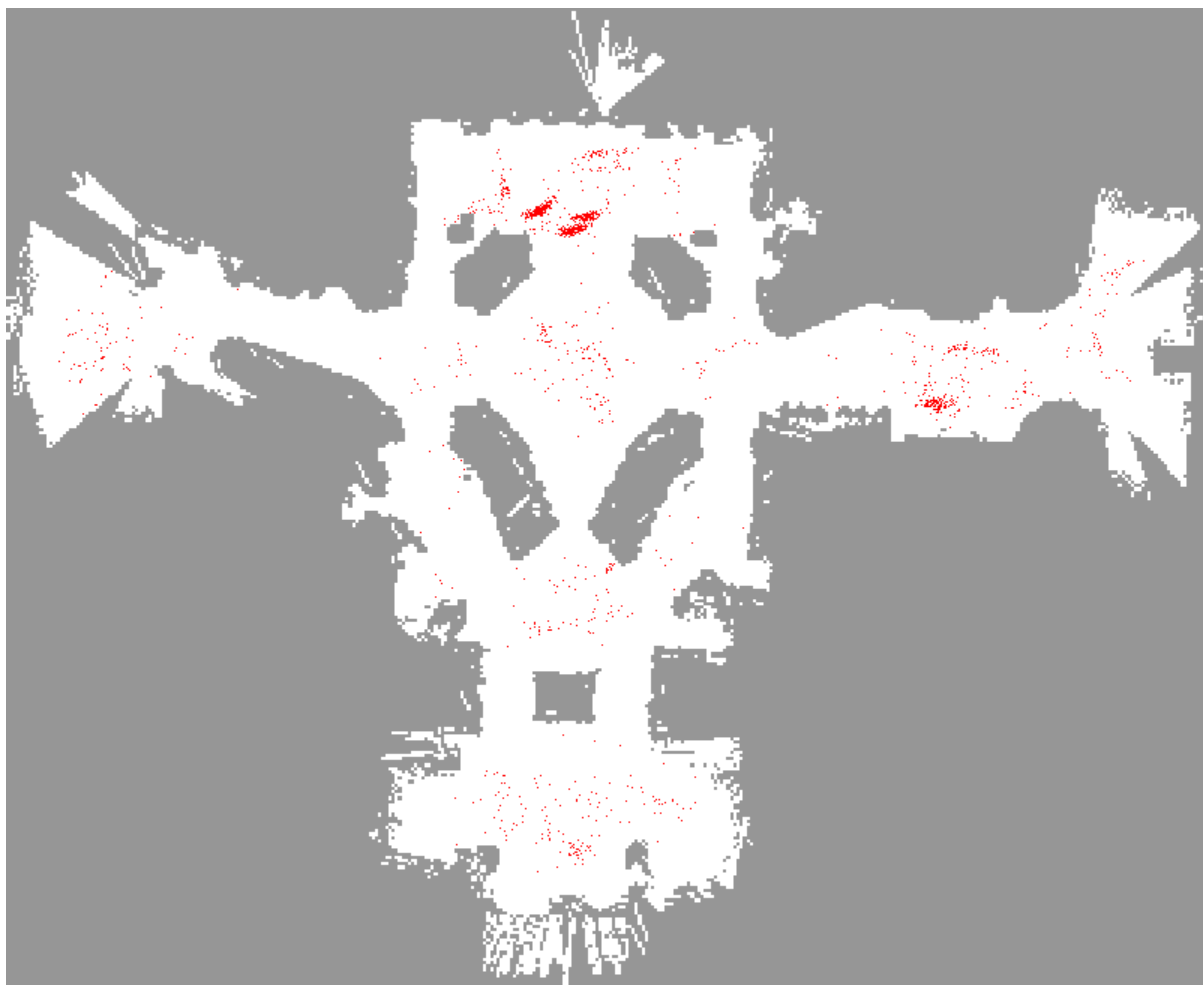


$$\bar{p}(\mathbf{x}_1) = \int p(\mathbf{x}_1|\mathbf{x}_0, \mathbf{u}_0, \mathbf{m})p(\mathbf{x}_0|\mathbf{m})d\mathbf{x}_0$$

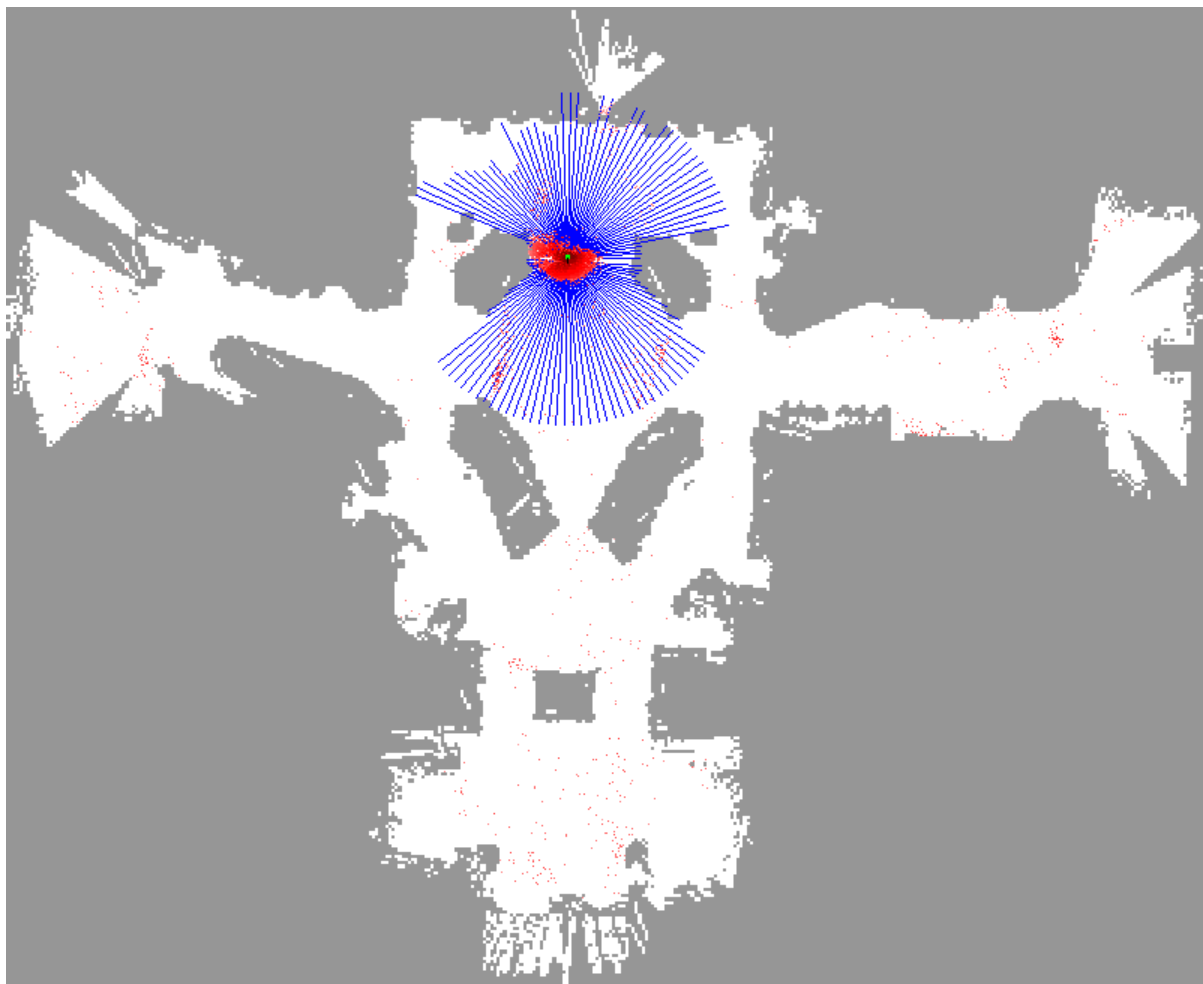




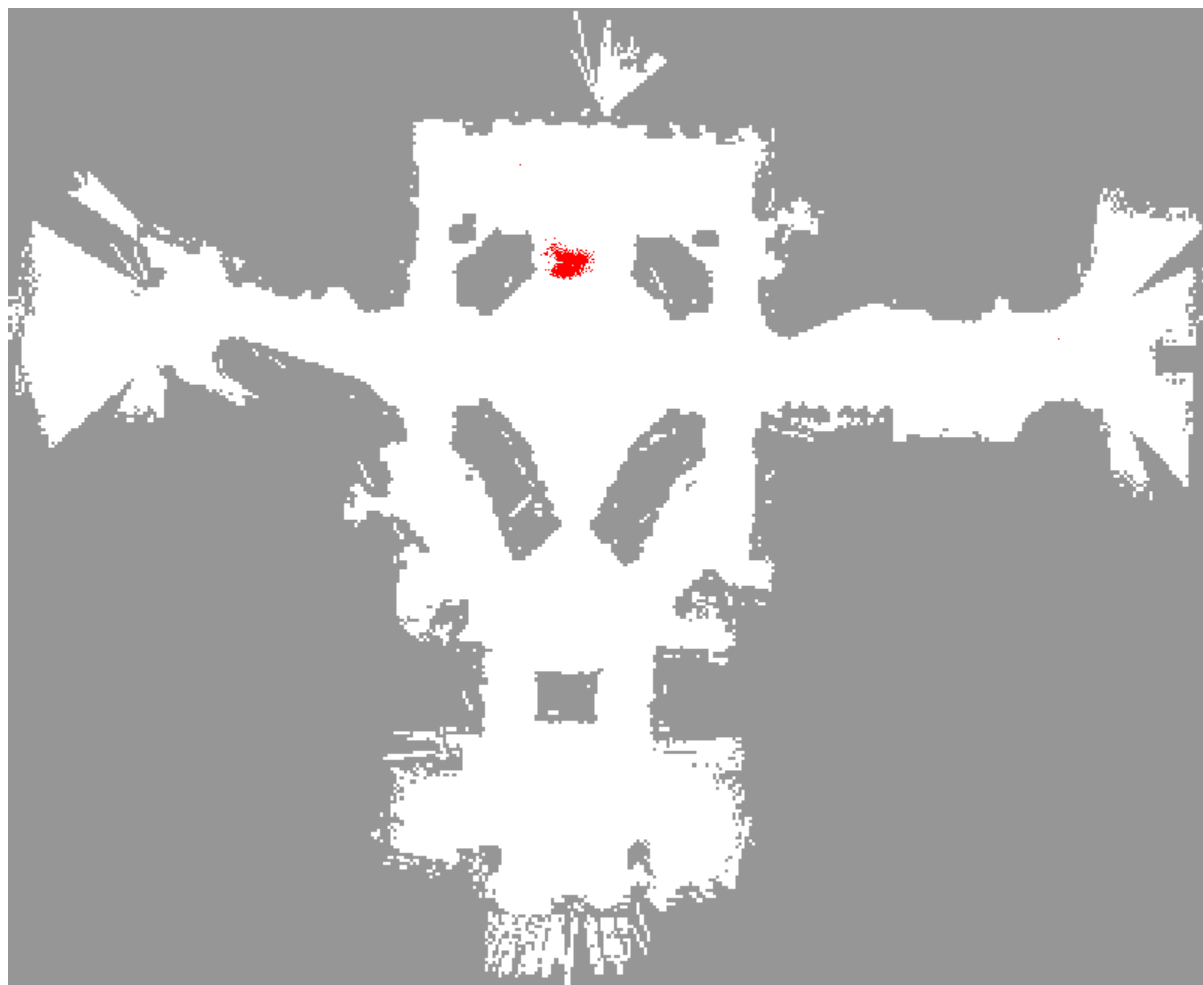


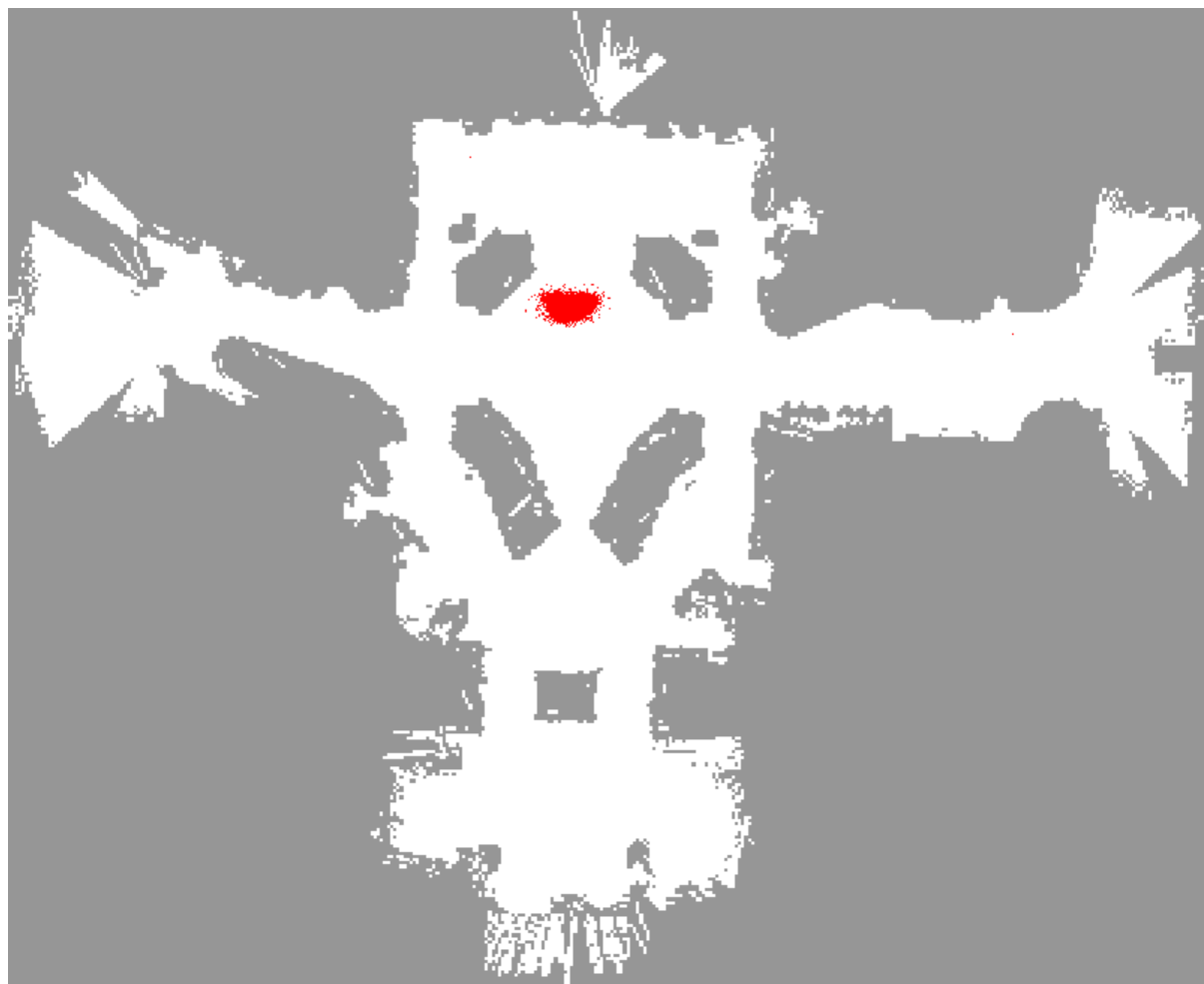


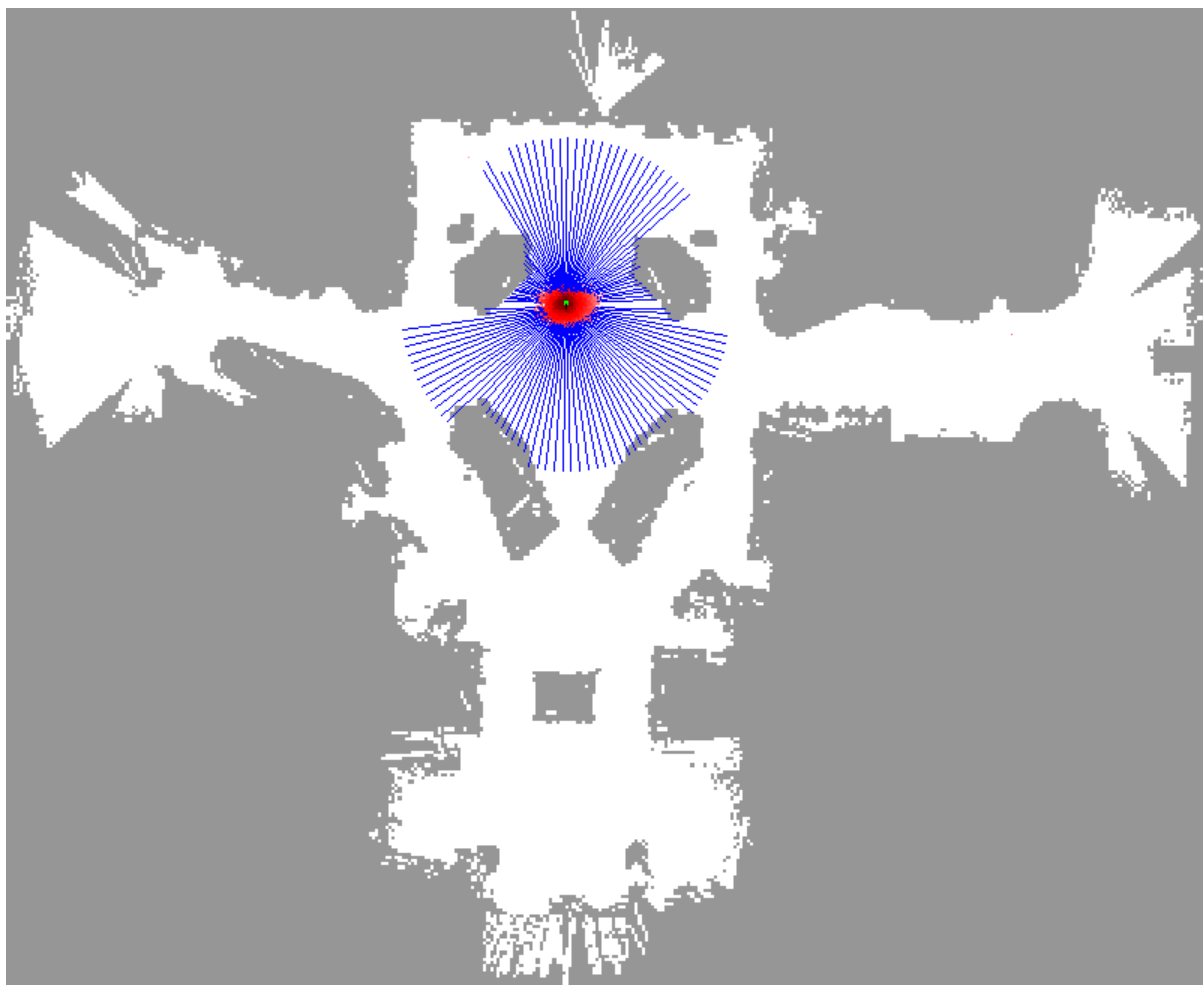


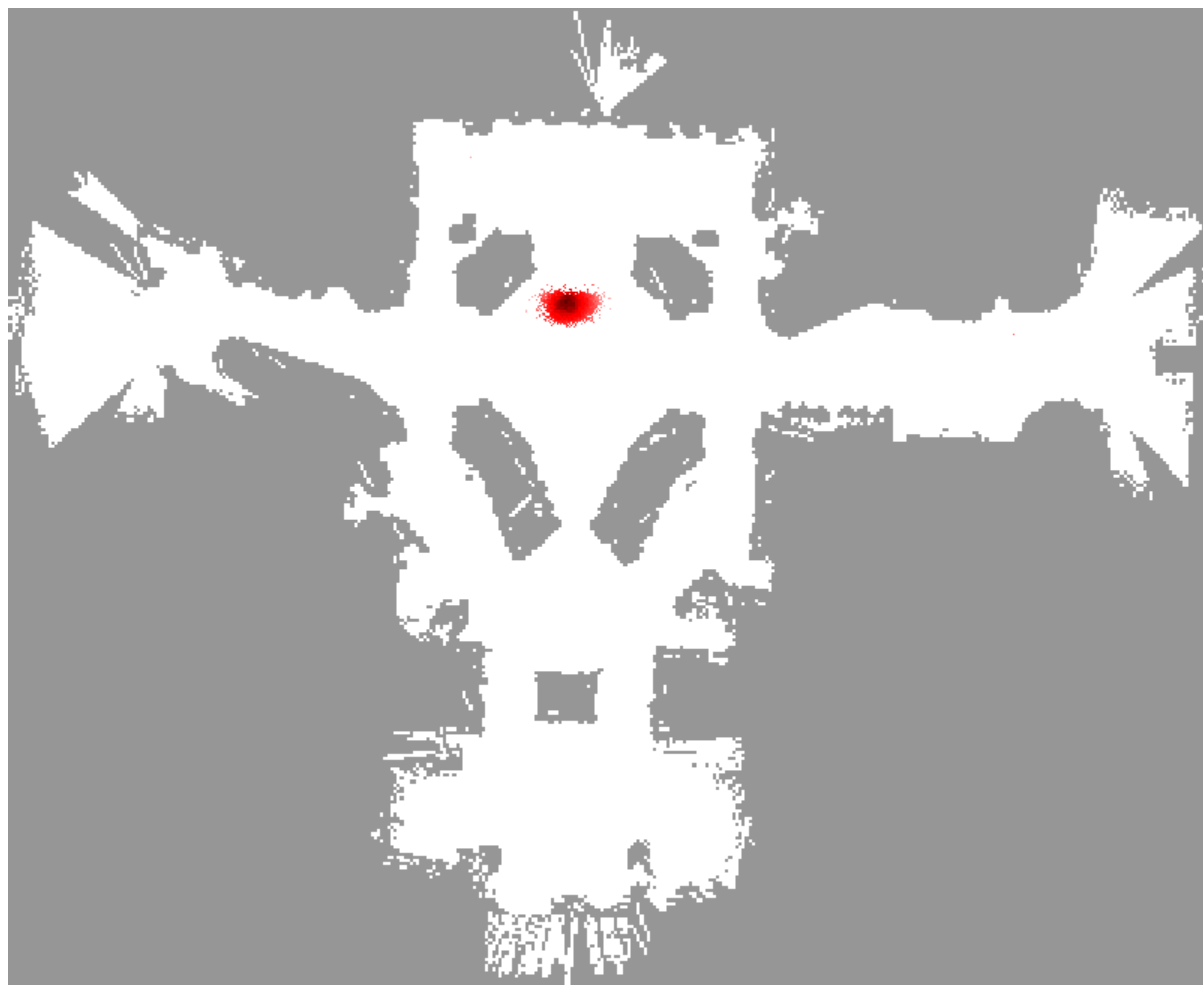


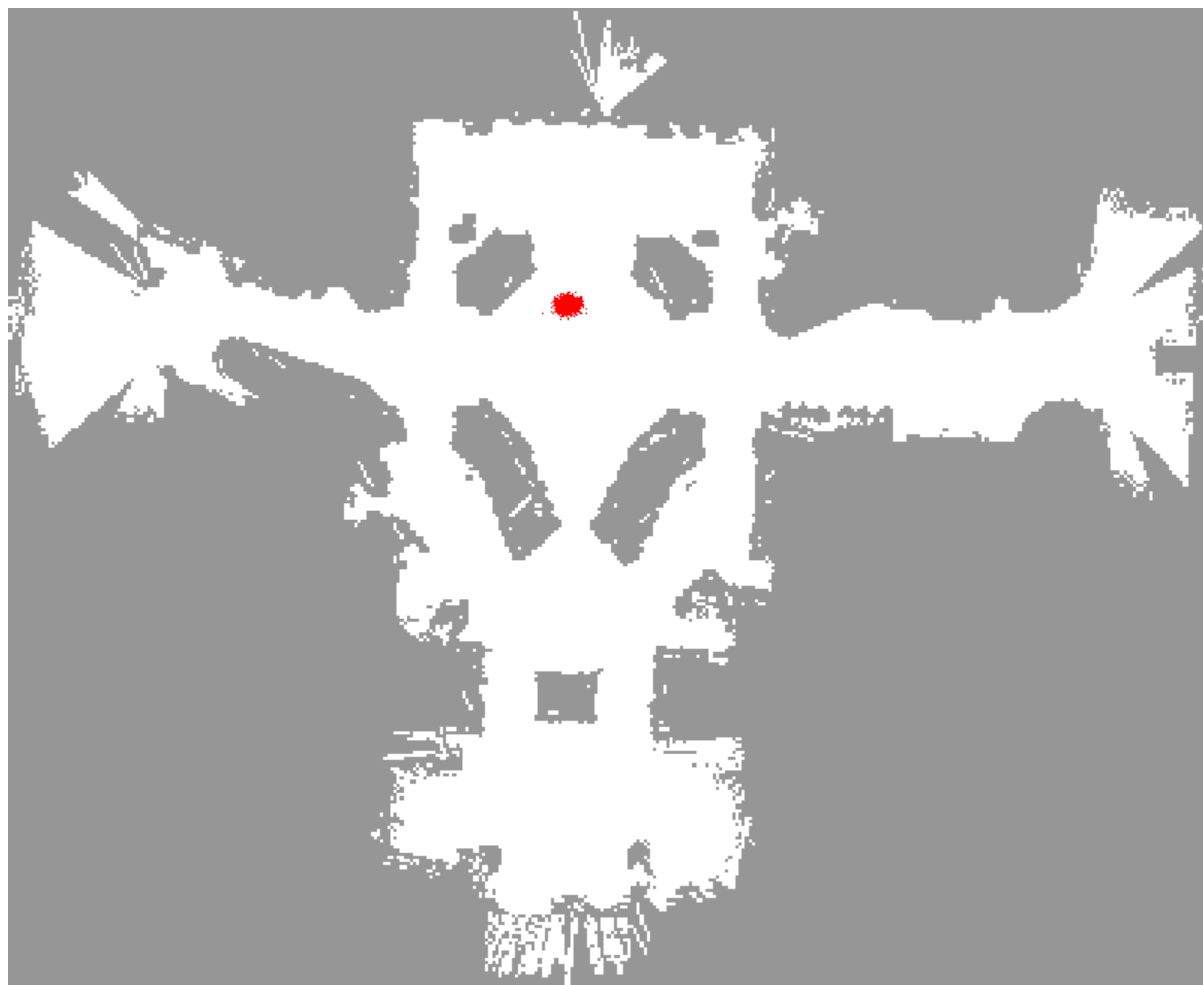


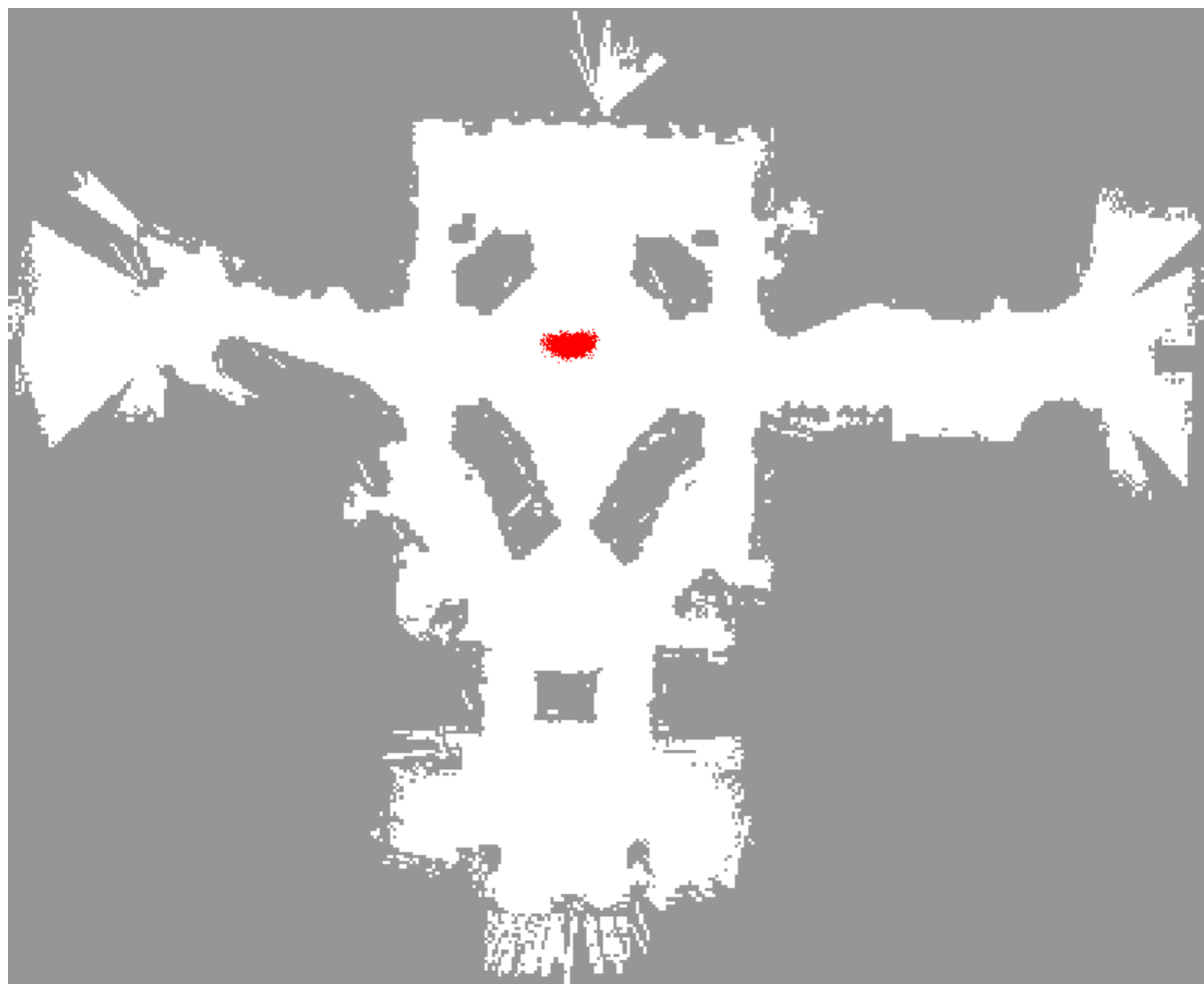


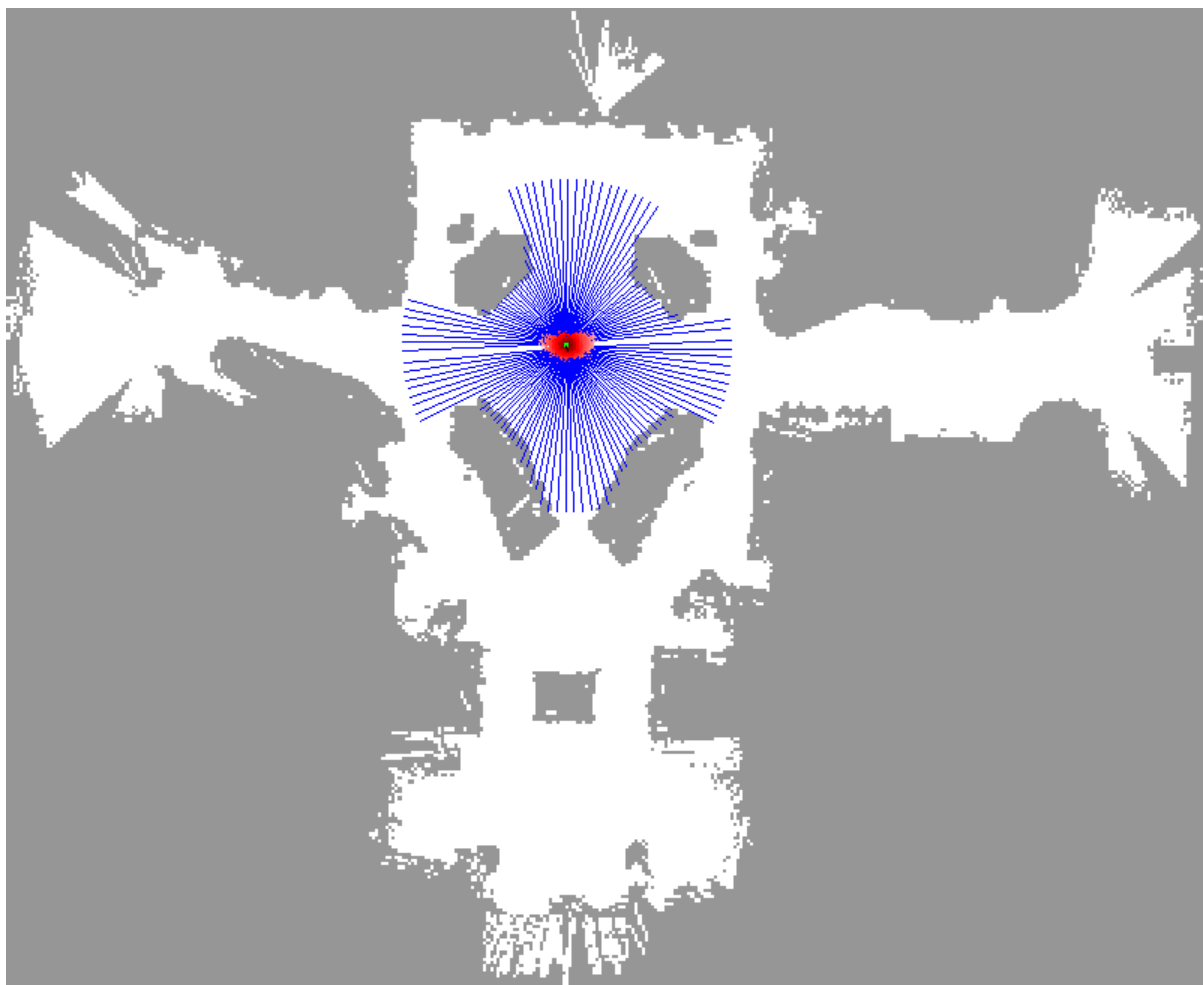












PF方法中的误差

来源于三个方面

- 由样本集合近似带来的系统偏差
- 由随机采样带来的偏差
- 由重采样带来的偏差



样本集合近似带来的系统偏差



M要足够地大

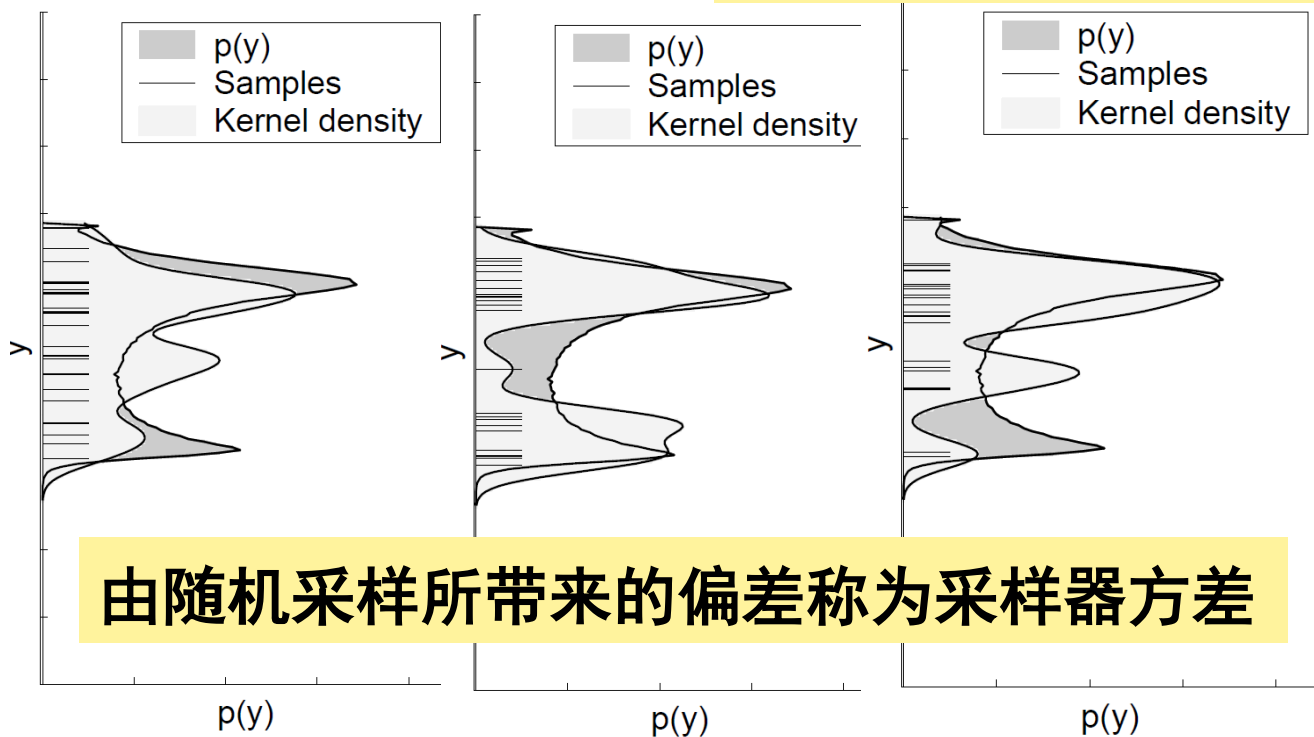
- 问题的根源：重采样时的归一化

$$\tilde{\omega}_t^{[m]} = \frac{\omega_t^{[m]}}{\sum_{i=1}^M \omega_t^{[i]}}$$

- 从M维空间计算得到的 $\omega_t^{[m]}$ 在归一化后，增加了一个约束，即 $\sum_m \tilde{\omega}_t^{[m]} = 1$ ，使得样本权重的维度变为M-1维
- 极端情况 $M=1$ ，观测模型将失去作用

由随机采样带来的误差

对同一个概率密度函数，作多次采样，样本数均为25

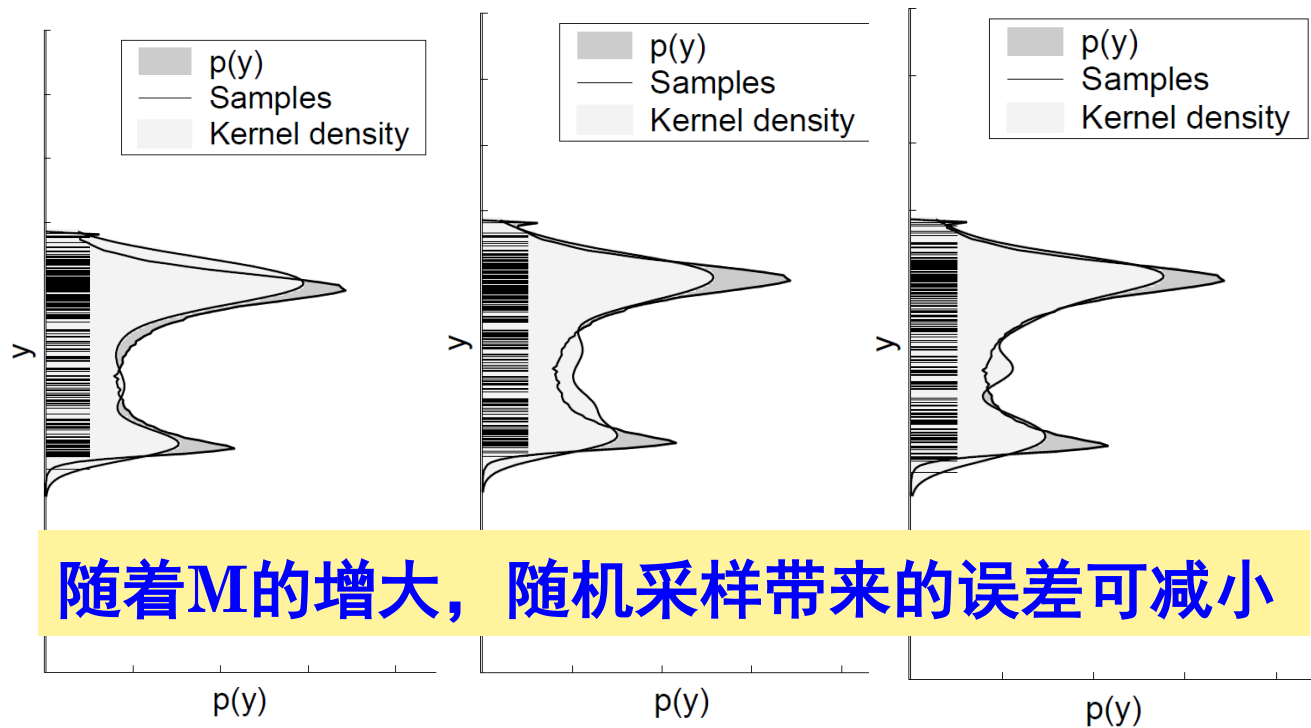


由随机采样所带来的偏差称为采样器方差

由不同的采样恢复得到的概率密度与原始概率密度比较

由随机采样带来的误差

250个粒子



由重采样带来的偏差

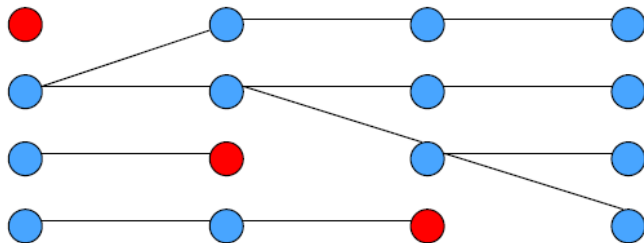
- 例1: $\mathbf{x}_t = \mathbf{x}_{t-1}$ 机器人没有运动且没有观测

理论上, \mathbf{x}_t 的概率分布 = \mathbf{x}_{t-1} 的概率分布

但重采样步骤无法生成相同的样本分布

- 例2: 不断重采样可能会使所有粒子聚集在同一状态估计上

每次淘汰25%的粒子, 则4次后所有粒子归于同一假设



粒子分布失去多样性

粒子分布失去多样性带来的问题和解决方案

- 问题：尽管粒子集合本身方差在减少，但作为真值估计器的粒子集合的方差在增大
- 解决方案：减少重采样次数

$$\omega_t^{[m]} = \begin{cases} 1 / M & \text{if resampling took place} \\ p(\mathbf{z}_t | \mathbf{x}_t^{[m]}) \omega_{t-1}^{[m]} & \text{if no resampling took place} \end{cases}$$



避免粒子分布失去多样性的重采样策略

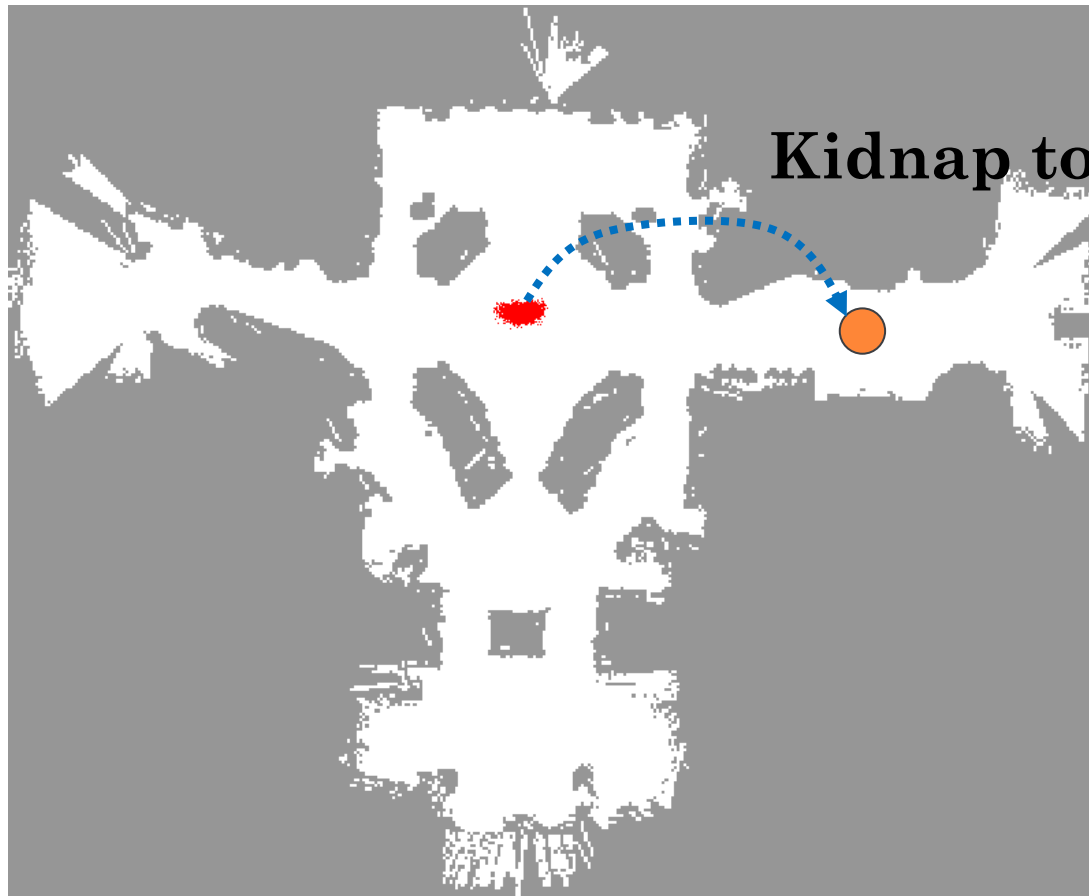
- 根据权重方差来决定是否重采样

重采样次数过少 → 大量粒子分布在概率低的区域

$$\hat{N}_{eff} = \frac{1}{\sum_{i=1}^M (\omega_t^{[i]})^2} \quad \begin{cases} resample & \hat{N}_{eff} > \bar{N}_{thre} \\ not\ resample & otherwise \end{cases}$$



绑架问题



Kidnap to here



绑架检测

- 检测一段时间内的观测模型概率分布

$$p(\mathbf{z}_t \mid \mathbf{Z}^{t-1}, \mathbf{u}_{t1:t-1}, \mathbf{m}) \propto \prod_{t'=t1}^t \frac{1}{M} \prod_{m=1}^M \omega_{t'}^{[m]}$$

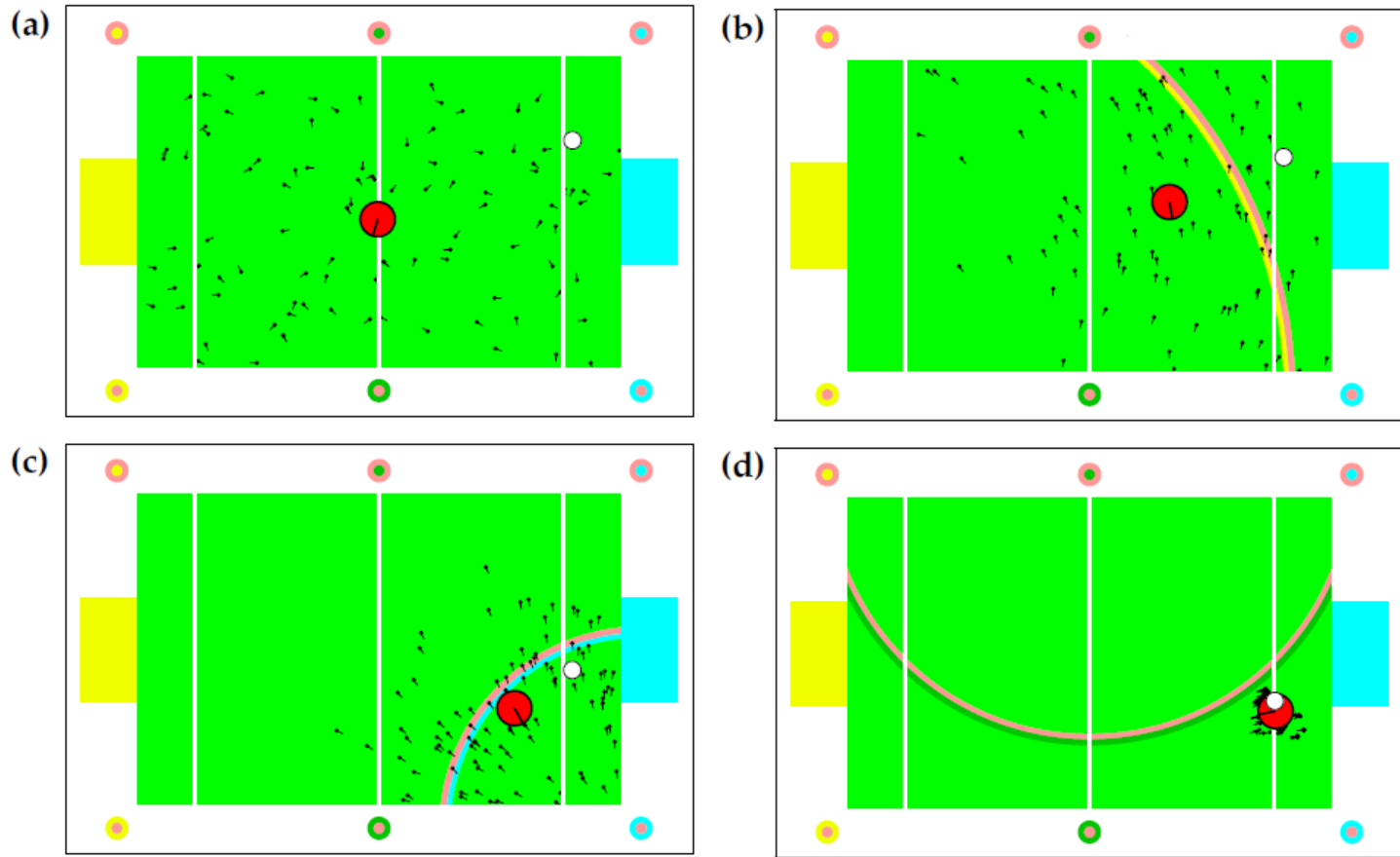
如果概率都偏低，说明粒子都偏离了真值



调整粒子分布

- 在位姿空间均匀分布，利用当前观测评估权重
- 根据观测可能性分布
 - 见前面对重要性评估和重采样的举例

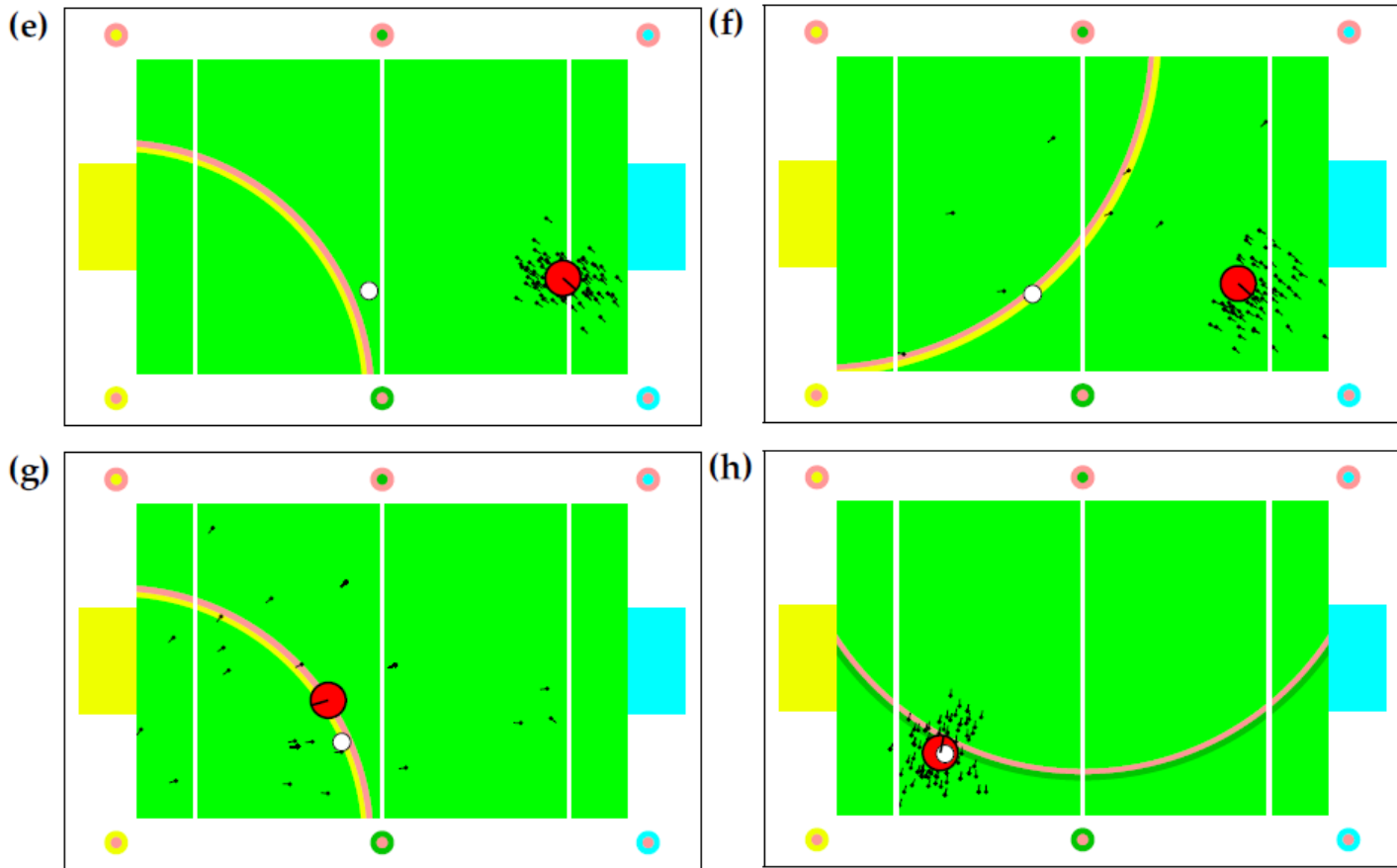




小的白色圆圈表示机器人实际所在位置，大的红色圆圈表示粒子均值
所得观测用检测到的图标为圆心以所检测到的距离为半径的圆弧表示



被绑架后重新定位，根据观测发现粒子概率都很小，说明定位出现了错误，
在下面的定位过程中，根据观测的可能性增加粒子



粒子数量问题

- 粒子数过少→不能覆盖状态可能区域
- 粒子数过多→计算浪费
- 全局定位时需要较多粒子
- 全局定位收敛后转为位置跟踪，只需少量粒子



粒子滤波定位方法存在的问题

- 难以推广到大范围环境6DoF位姿估计：粒子集必须足够大，才能覆盖 $t-1$ 时刻后验的分布
- 为解决绑架问题，始终需要保持一部分粒子全局分布，避免完全收敛到局部，但导致全局也很难快速收敛到新位置



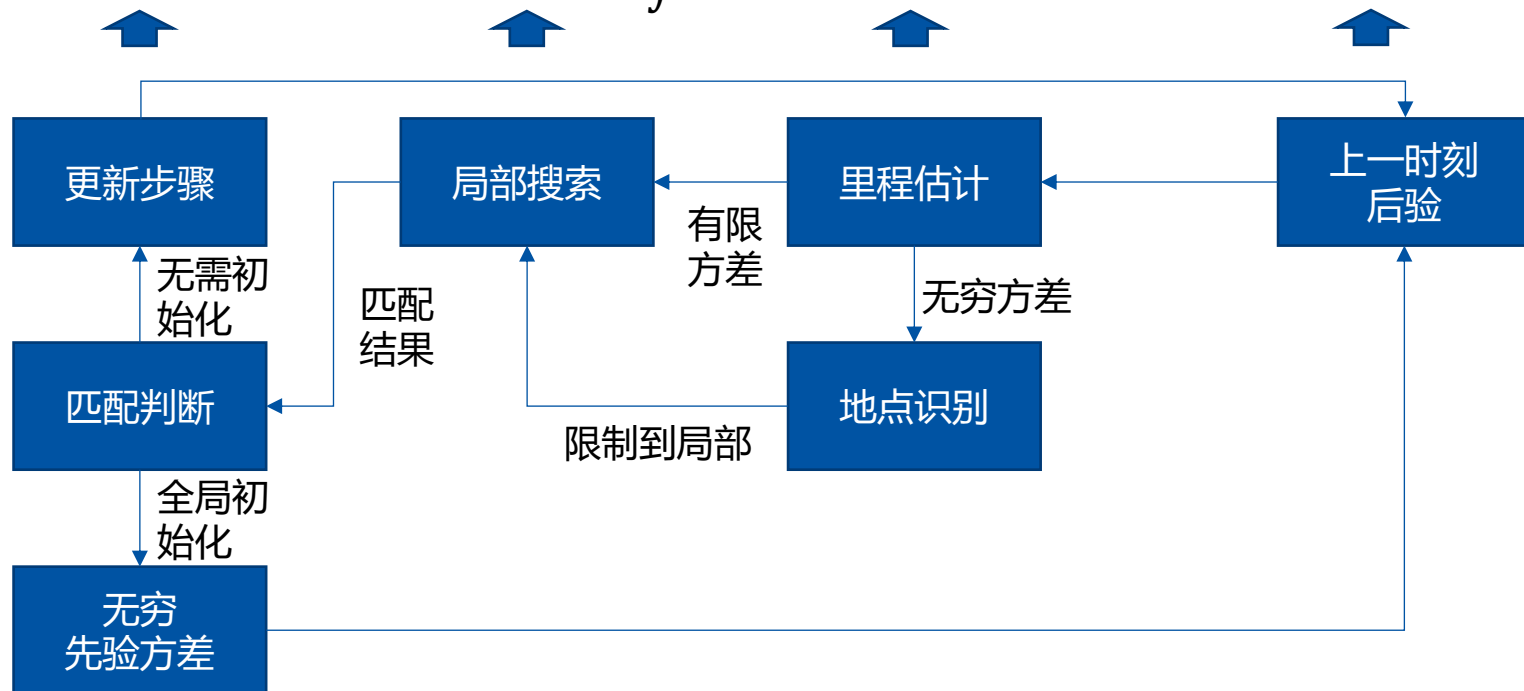
3D-ICP, ORB-SLAM-loc, ROVIO-loc

总体仍然是贝叶斯滤波+马尔科夫架构



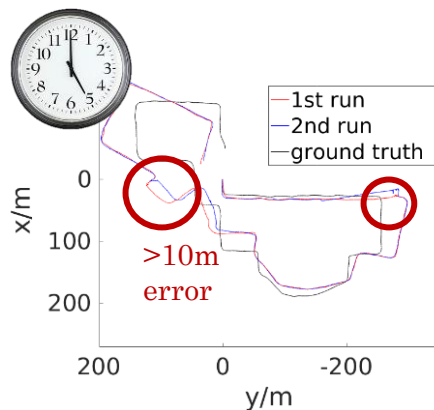
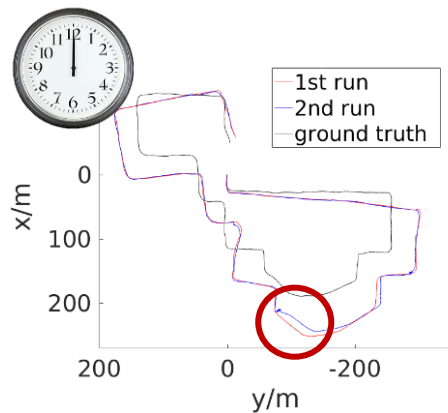
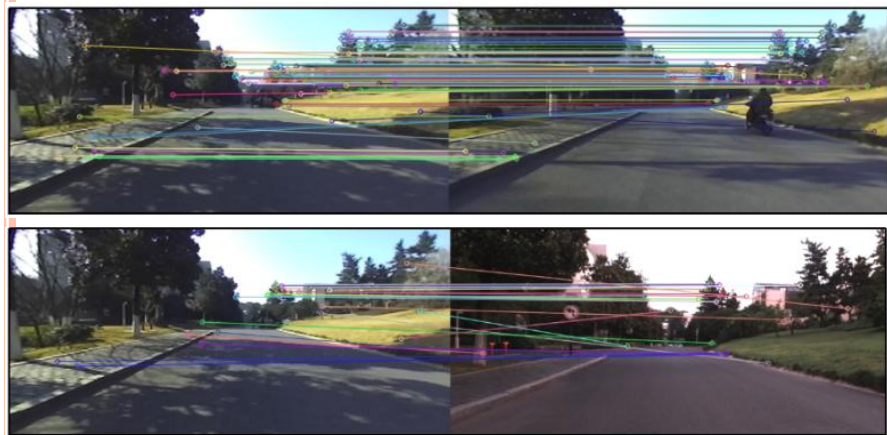
目前常用定位系统架构

$$p(\mathbf{x}_t | \mathbf{Z}^t, \mathbf{U}^{t-1}, \mathbf{m}) = \eta p(\mathbf{z}_t | \mathbf{x}_t, \mathbf{m}) \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{m}) p(\mathbf{x}_{t-1} | \mathbf{Z}^{t-1}, \mathbf{U}^{t-2}, \mathbf{m}) d\mathbf{x}_{t-1}$$

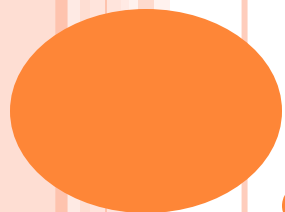


目前定位存在的问题

○ 环境存在相似性和动态性



动态变化导致可匹配特征量少、描述鲁棒性差，大范围环境匹配工作量大



END

