第七讲 里程估计

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里程估计(ODOMETRY)

- 根据传感器感知信息推导机器人位姿（位置和角度）变化
- 用途：
  - 航位推算 (Dead-reckoning)：基于已知位置，利用里程估计，推算现在位置

为什么不用发送给机器人的运动控制指令进行航位推算？

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x - \frac{v}{\omega}\sin(\theta) + \frac{v}{\omega}\sin(\theta + \omega\Delta t) \\ y + \frac{v}{\omega}\cos(\theta) - \frac{v}{\omega}\cos(\theta + \omega\Delta t) \\ \theta + \omega\Delta t \end{pmatrix}$$

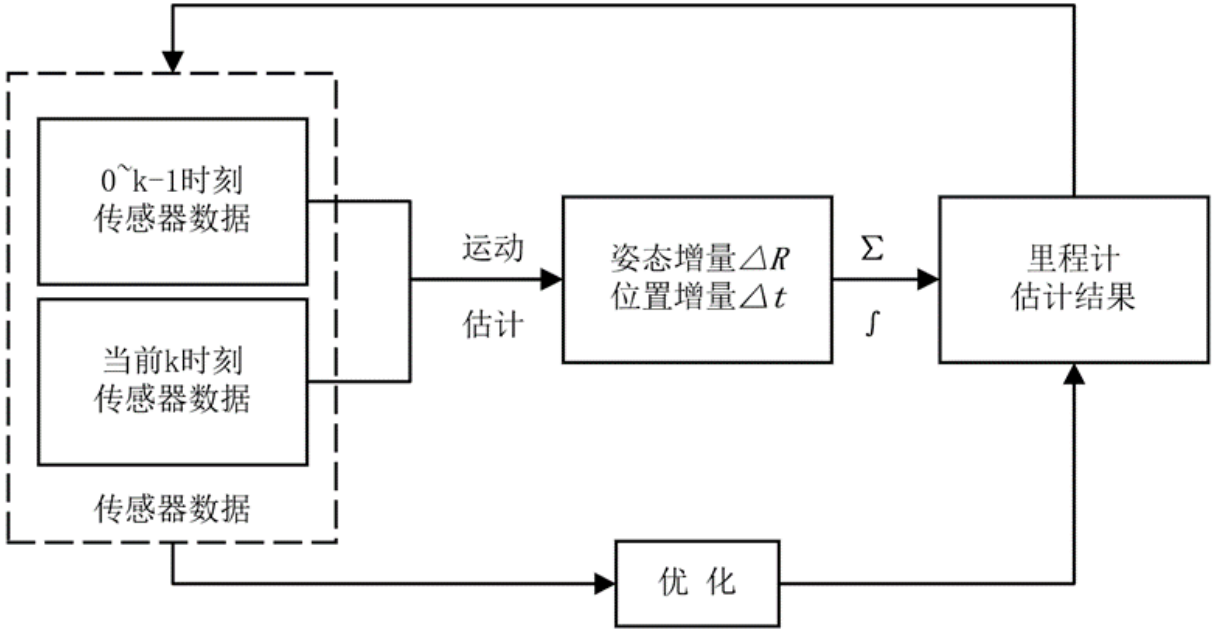
由于控制的滞后性和可能存在超调，

机器人实际执行控制指令与发送控制指令存在偏差

最早的里程估计：根据码盘获得实际执行速度进行位姿变化估计

里程估计方法

- 基本思路



里程估计方法

- 基于机器人运动感知信息，结合运动学模型
  - 电机码盘（轮式里程计）
  - IMU（惯性单元，加速度计+陀螺仪）（惯性里程计）
- 基于环境感知传感器信息，通过最佳匹配估计
  - 激光里程计（LO）
  - 视觉里程计（VO）

位姿变化的数学描述

二维平面运动　　$(\Delta x, \Delta y, \Delta \theta)$

三维空间运动　　$(\Delta x, \Delta y, \Delta z, \Delta \alpha, \Delta \beta, \Delta \gamma)$

一般统一表示为旋转矩阵和平移向量形式　R, t

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}, t = [\Delta x, \Delta y, \Delta z]^T$$

$$x' = Rx + t \qquad RR^T = I$$

旋转矩阵与姿态变化关系

绕x轴旋转(roll, 滚动角)

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

绕y轴旋转(pitch, 俯仰角)

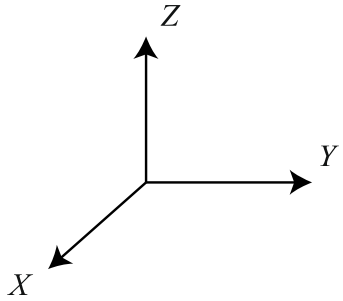
$$R_y = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$$

绕z轴旋转(yaw,偏航角)

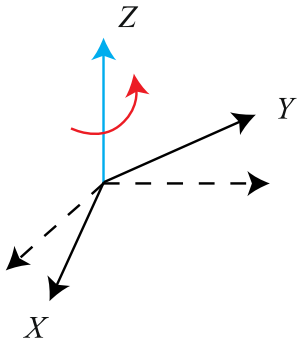
$$R_z = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = R_z(\Delta\gamma)R_y(\Delta\beta)R_x(\Delta\alpha)$$

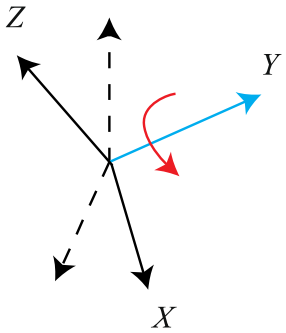
旋转矩阵与姿态变化关系



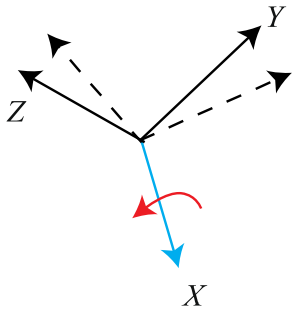
原始坐标系



第一次旋转



第二次旋转



第三次旋转

$$\boldsymbol{R = R_x(\Delta\gamma)R_y(\Delta\beta)R_z(\Delta\alpha)}$$

里程估计问题

根据感知信息求旋转矩阵和平移向量 R, t

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}, t = [\Delta x, \Delta y, \Delta z]^T$$

根据旋转矩阵求姿态变化

由旋转矩阵求欧拉角

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} c_y c_z & c_z s_x s_y - c_x s_z & s_x s_z + c_x c_z s_y \\ c_y s_z & c_x c_z + s_x s_y s_z & c_x s_y s_z - c_z s_z \\ -s_y & c_y s_x & c_x c_y \end{bmatrix}$$

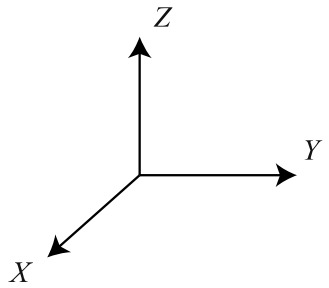
求解方程可得

$$\Delta\alpha = atan2(r_{32}, r_{33})$$

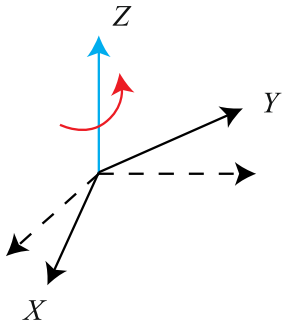
$$\Delta\beta = atan2\left(-r_{31}, \sqrt{r_{32}^2 + r_{33}^2}\right)$$

$$\Delta\gamma = atan2(r_{21}, r_{11})$$

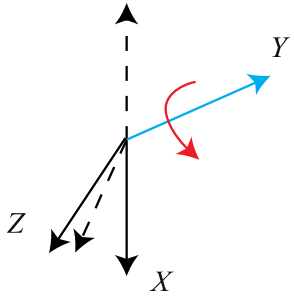
欧拉角表示存在奇异性



原始坐标系



第一次旋转



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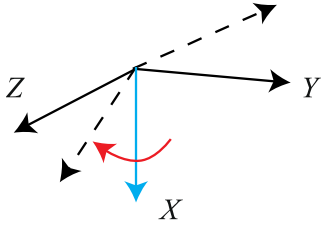


第三次旋转

当俯仰角为±90°时，第一次旋转与第三次旋转将使用同一个轴，使系统丢失了一个自由度，也称为万向锁问题

四元数表示法

- 实际应用时通常采用四元数表示法
- 爱尔兰数学家William Rowan Hamilton于1843年提出
- 出发点：三维空间的任意旋转，都可以用绕三维空间的某个轴旋转过某个角度来表示，即表示为旋转向量

$$(\theta, n_x, n_y, n_z)^T$$

$n = (n_x, n_y, n_z)$ 为旋转轴单位向量

θ 为旋转角度

四元数表示法

- 用复数集表示复平面上的向量，复数的乘法可表示复平面上的旋转
- 四元数

$$q = (q_0, q_1, q_2, q_3)^T$$

$$q = q_0 + q_1 i + q_2 j + q_3 k$$

实部为零时，称为虚四元数

$$\begin{cases} i^2 = j^2 = k^2 = -1 \\ ij = k, ji = -k \\ jk = i, kj = -i \\ ki = j, ik = -j \end{cases}$$

四元数下的旋转矩阵

- 采用单元四元数表示旋转轴

$$q = (q_0, q_1, q_2, q_3)^T = (\cos\frac{\theta}{2}, n_x\sin\frac{\theta}{2}, n_y\sin\frac{\theta}{2}, n_z\sin\frac{\theta}{2})^T$$

- 把三维空间点用一个虚四元数表示　$p = (0, x, y, z)$

- 旋转后的点p'可表示为　$p' = qpq^{-1}$

- 利用四元数可避免对旋转欧拉角的解算，直接进行旋转变换

四元数下的旋转矩阵

$$R = \begin{bmatrix} 1-2q_2^2-2q_3^2 & 2q_1q_2+2q_0q_3 & 2q_1q_3-2q_0q_2 \\ 2q_1q_2-2q_0q_3 & 1-2q_1^2-2q_3^2 & 2q_2q_3+2q_0q_1 \\ 2q_1q_3-2q_0q_2 & 2q_2q_3-2q_0q_1 & 1-2q_1^2-2q_2^2 \end{bmatrix}$$

根据旋转矩阵R求解四元数

$$q_0 = \frac{\sqrt{tr(R)+1}}{2}$$

$$q_1 = \frac{r_{23} - r_{32}}{4q_0}$$

$$q_2 = \frac{r_{31} - r_{13}}{4q_0}$$

$$q_3 = \frac{r_{12} - r_{21}}{4q_0}$$

其中 $q_0 \neq 0$，

$1 + r_{11} + r_{22} + r_{33} > 0$

四元数表示的优点

- 表达更紧凑
- 求解无奇异
- 计算更快速

7.1 基于运动感知的里程估计

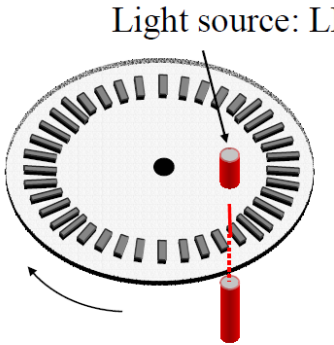
基于电机码盘的轮式移动机器人里程估计

- （1）根据电机码盘获得轮子转速

减速器

电机

码盘



光电编码器

磁编码器

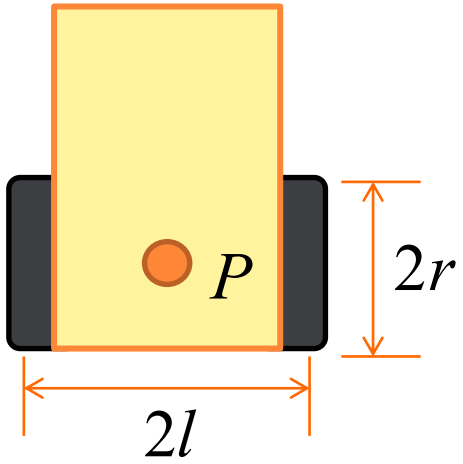
$$\dot{\varphi} = \frac{2\pi n}{\eta}$$

n 码盘测量得到的电机转速（转/分）

η 齿轮减速比

基于电机码盘的轮式移动机器人里程估计

- （2）结合运动学模型计算参考点速度
- （3）假设短时间片内为匀速运动，计算位姿变化

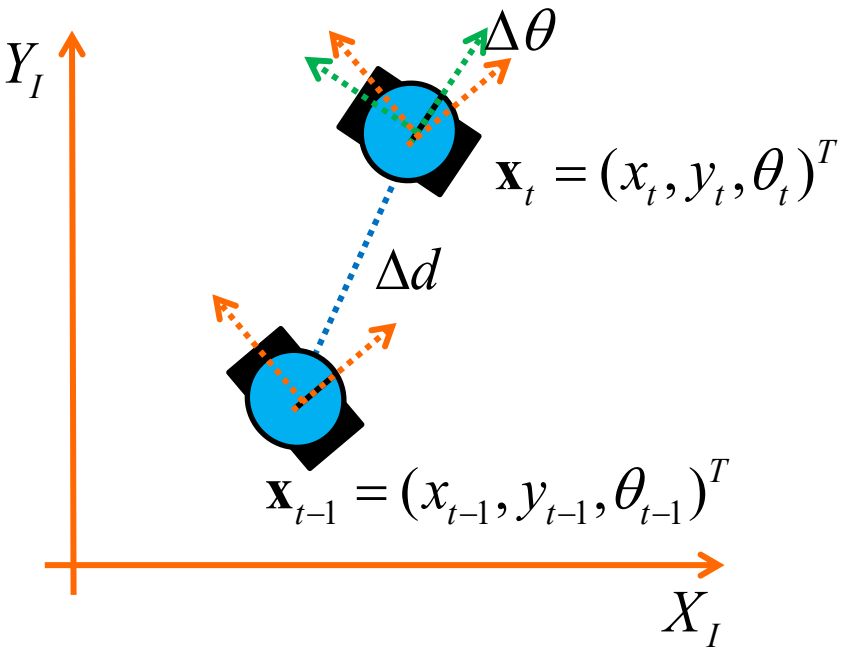


$$v = \frac{r\dot{\varphi}_l}{2} + \frac{r\dot{\varphi}_r}{2}$$

$$w = \frac{r\dot{\varphi}_l}{2l} - \frac{r\dot{\varphi}_r}{2l}$$

$$\Delta d = v\Delta t, \Delta\theta = w\Delta t$$

基于位姿变化的航位推算



$$\begin{cases} x_t = x_{t-1} + \Delta d \cos(\theta_{t-1} + \Delta\theta) \\ y_t = y_{t-1} + \Delta d \sin(\theta_{t-1} + \Delta\theta) \\ \theta_t = \theta_{t-1} + \Delta\theta \end{cases}$$

轮式里程估计误差

- 系统误差
  - 轮半径误差
  - 轮子安装精度误差（不平行，两边距离不相等）
  - 编码器精度误差
  - 采样精度误差
  - 齿轮减速比精度
- 偶然误差
  - 地面不平
  - 轮子打滑
  - ……

$$v = \frac{r\dot{\varphi}_l}{2} + \frac{r\dot{\varphi}_r}{2}$$

$$w = \frac{r\dot{\varphi}_l}{2l} - \frac{r\dot{\varphi}_r}{2l}$$

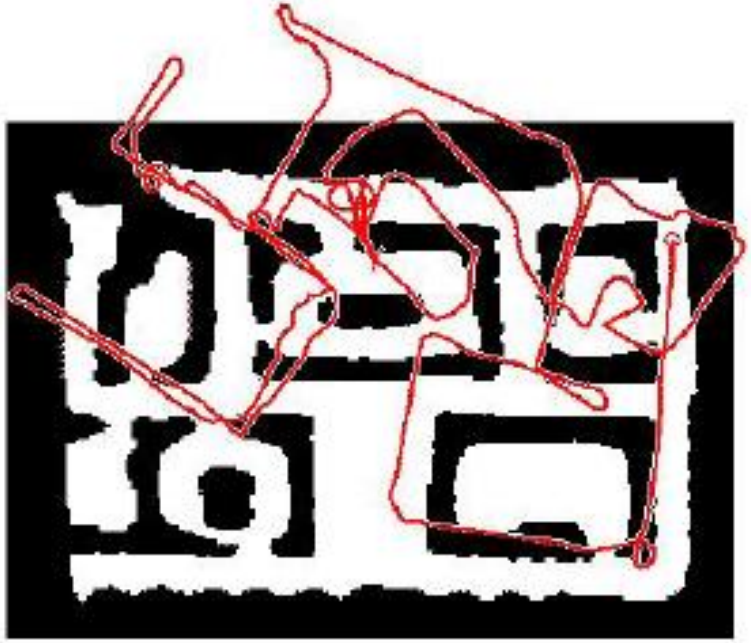
$$\Delta d = v\Delta t, \Delta\theta = w\Delta t$$

$$\dot{\varphi} = \frac{2\pi n}{\eta}$$

里程计估计误差导致问题

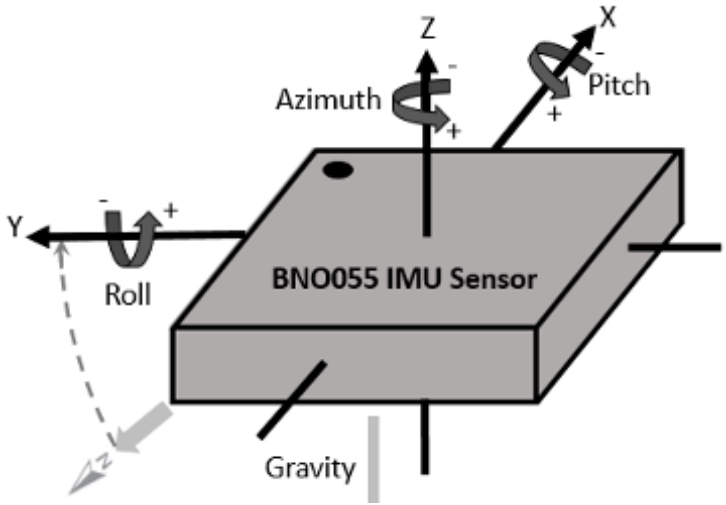
- 在航位推算时，里程计误差被累加，推算随着时间而增长

$$\begin{cases} x_t = x_{t-1} + \Delta d \cos(\theta_{t-1} + \Delta\theta) \\ y_t = y_{t-1} + \Delta d \sin(\theta_{t-1} + \Delta\theta) \\ \theta_t = \theta_{t-1} + \Delta\theta \end{cases}$$



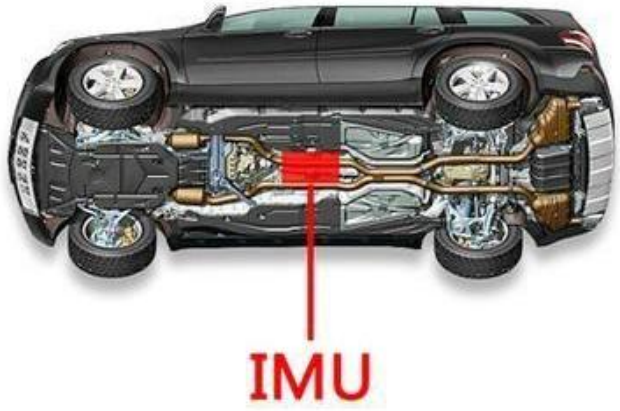
基于惯性单元的里程估计

- 惯性单元IMU
  - Inertial Measurement Unit
  - 一般含有三轴的加速度计和三轴的陀螺仪
  - 通常集成一个三轴磁力计用于校正 IMU 的姿态估计
- 通过积分运算可得载体在导航坐标系中的姿态、速度和位置等信息



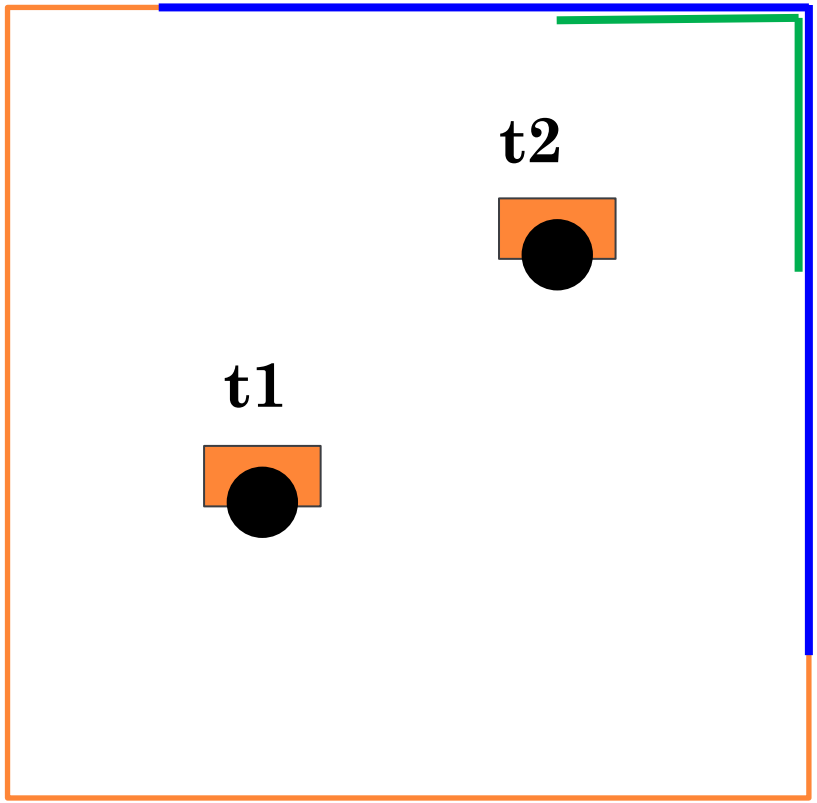
基于惯性单元的里程估计

- 优点：
  - 全天候
  - 采样频率高
  - 短时精度较好
- 缺点：
  - 随着时间的增长累积误差较大，无法满足移动机器人长距离精确定位的要求，需要融合其它传感器进行组合导航

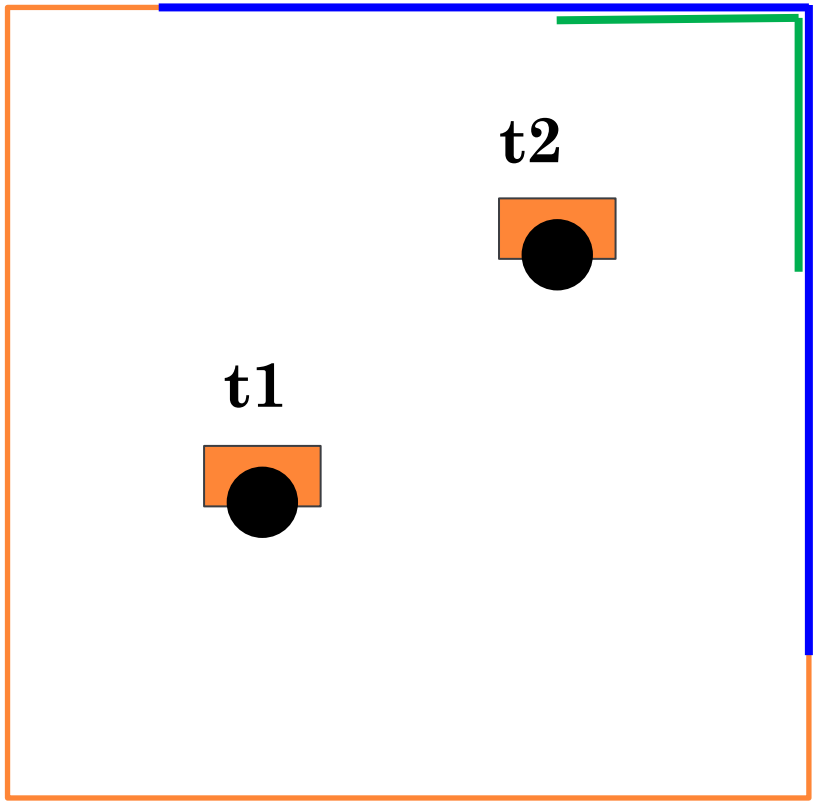


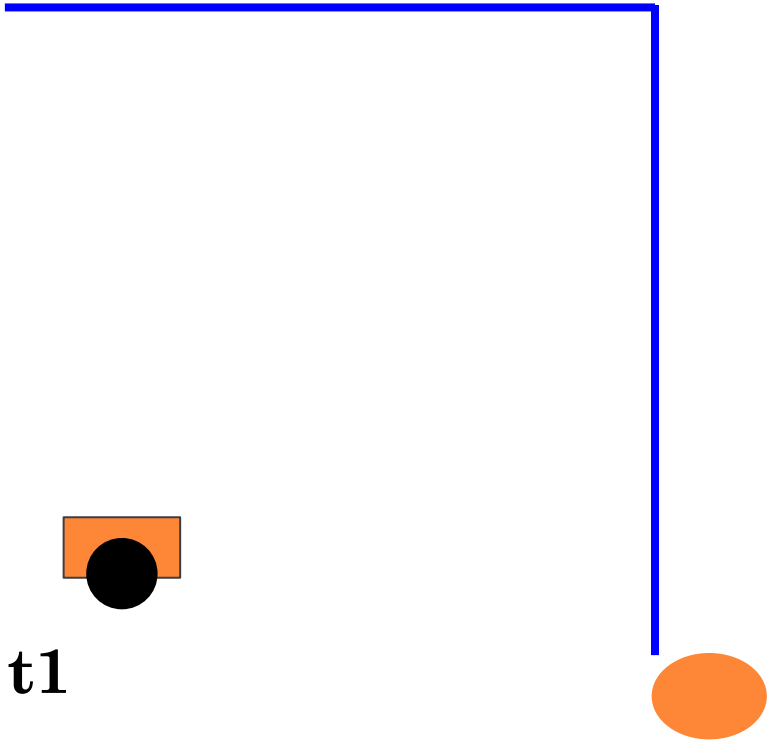
7.2 激光里程计

激光里程计

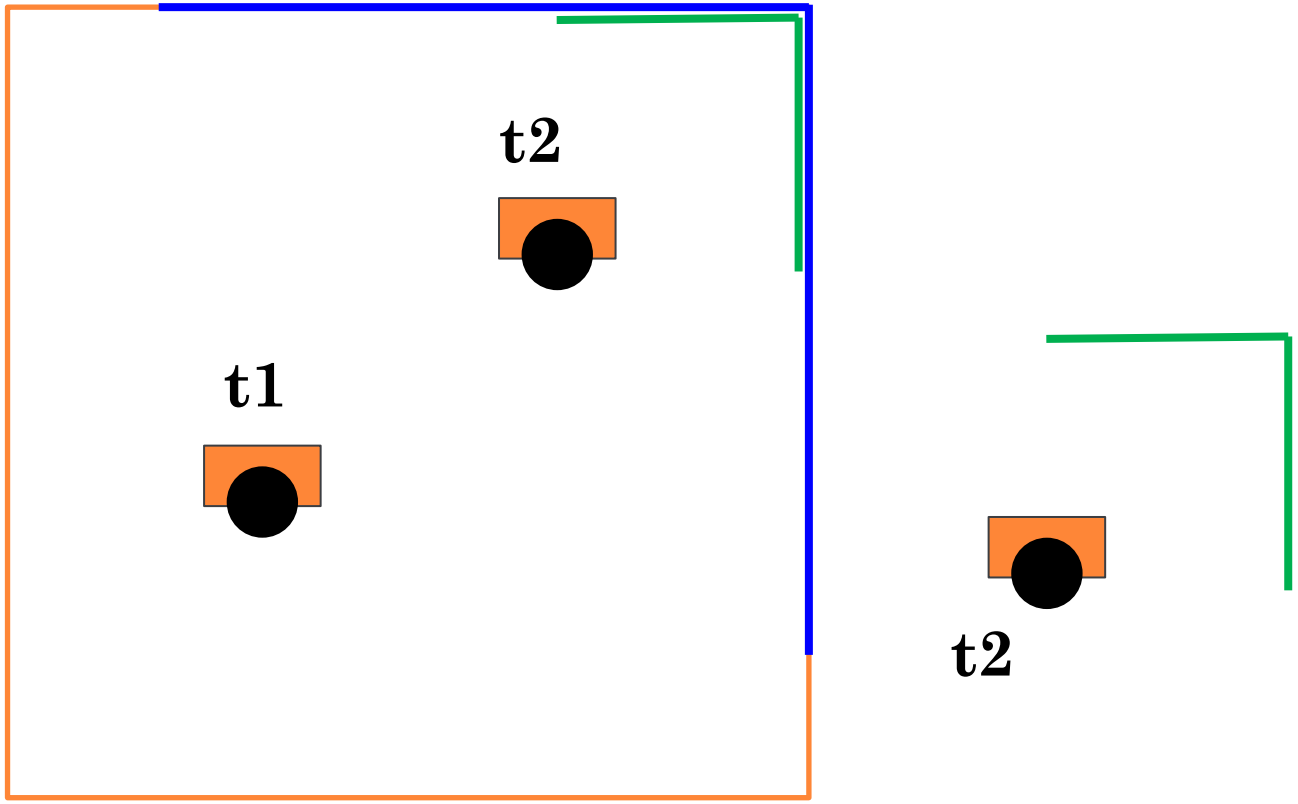


激光里程计





激光里程计



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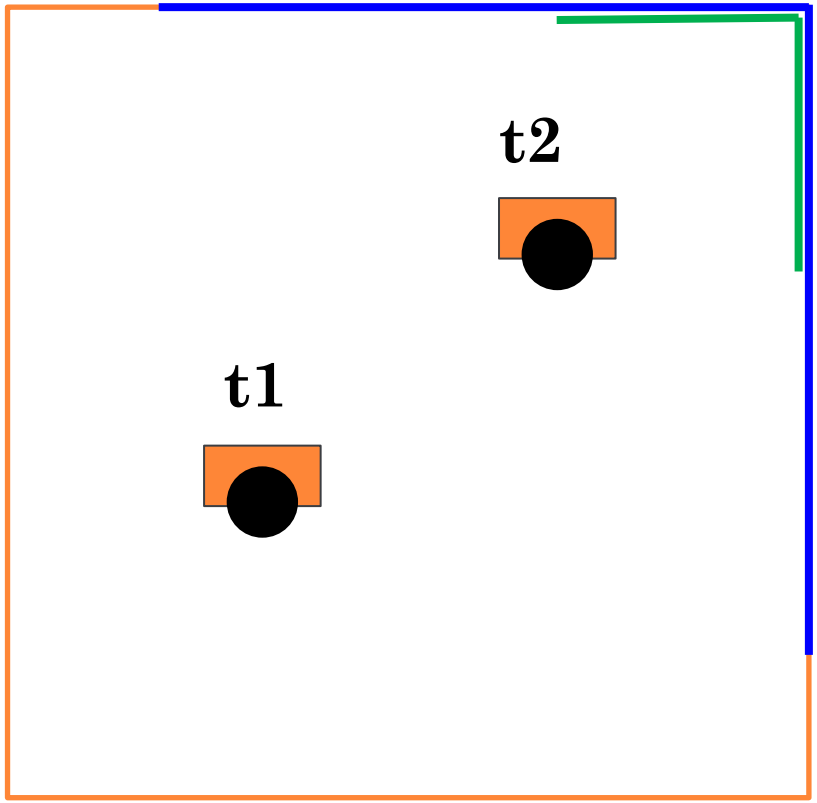

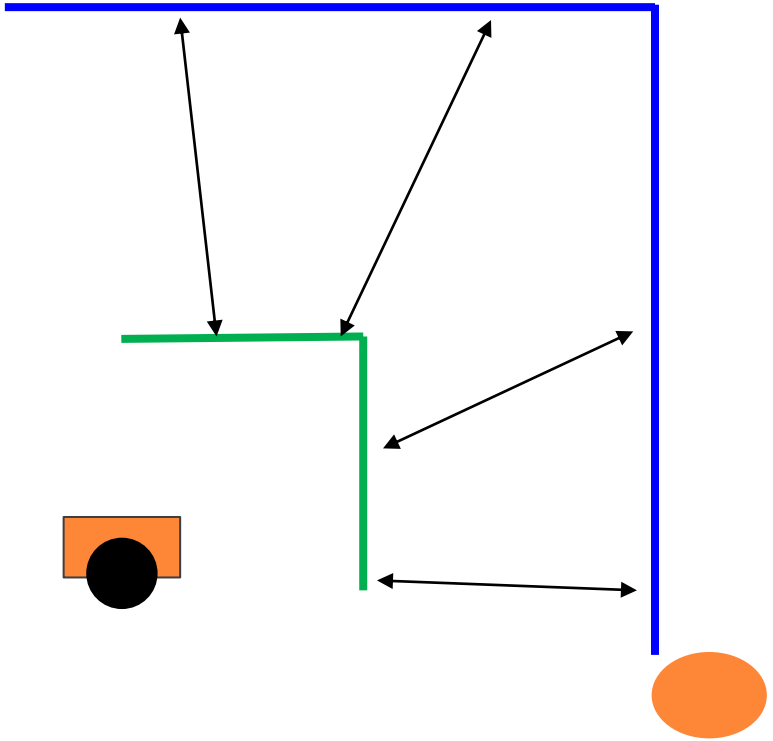

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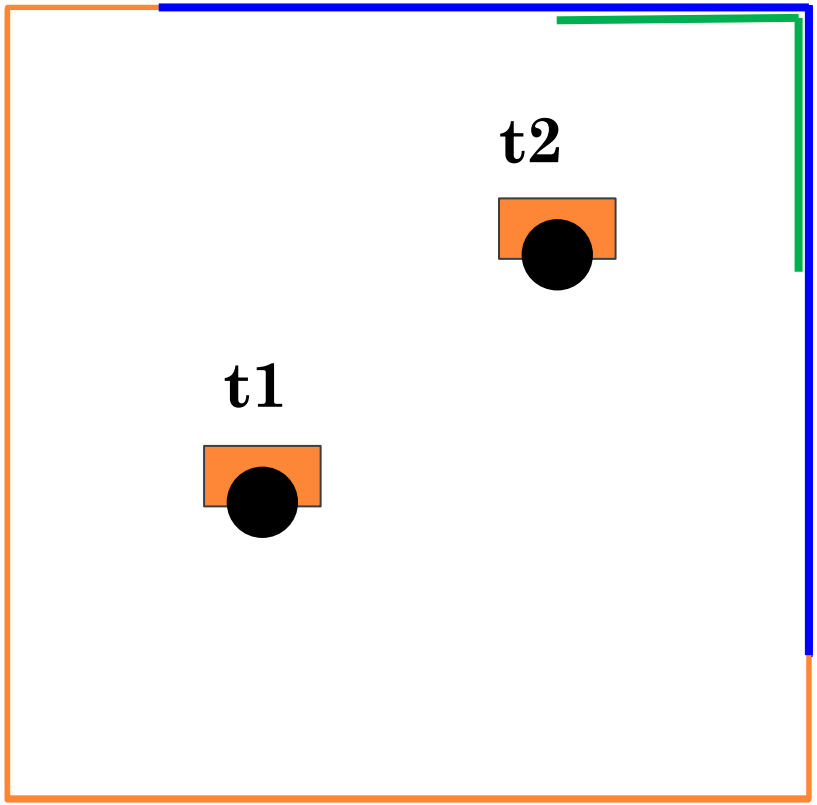
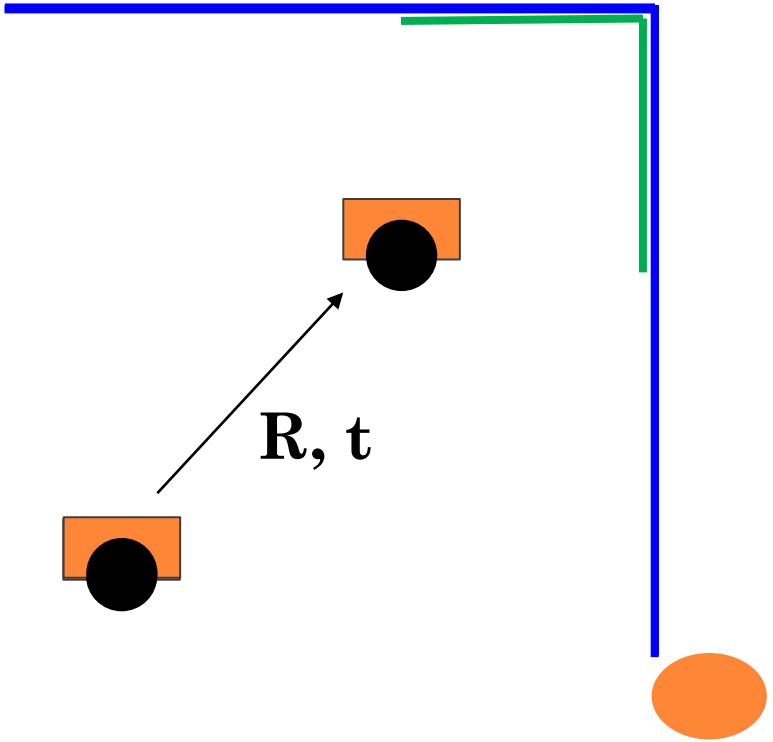
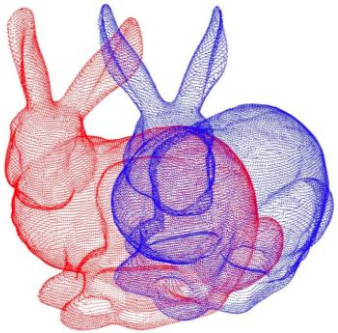


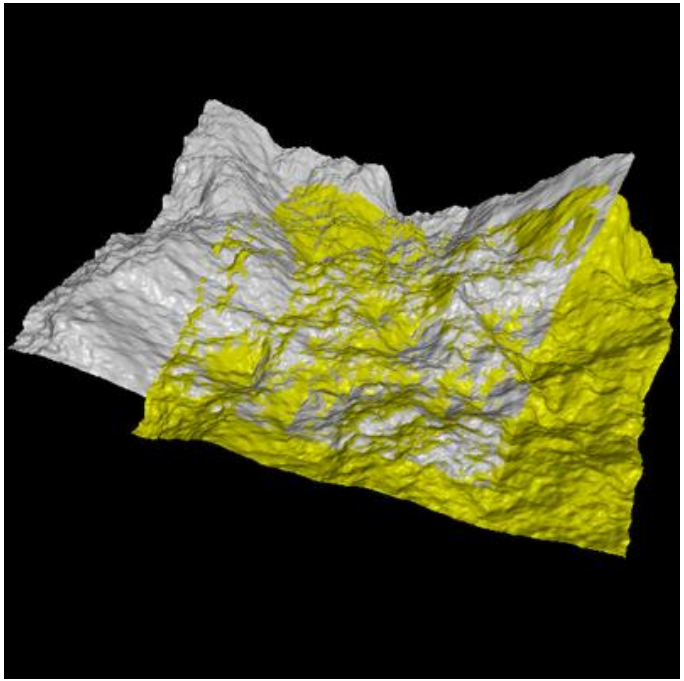


激光里程计

- 采用ICP(Iterative Closest Point)算法
  - 估计P’集合点与P集合点的初始位姿关系
  - 根据最近邻域规则建立P’集合点与P集合点的关联
  - 利用线性代数/非线性优化的方式估计旋转平移量
  - 对点集合P’的点进行旋转平移
  - 如果旋转平移后重新关联的均方差小于阈值，则结束
  - 否则迭代重复上述步骤



ICP实例



ICP

- Point to Point
- Line to Line
- Plane to Plane
- Point to Line
- Point to Plane
- Line to Plane

POINT-POINT ICP

- 输入：点集合$\mathbf{P}, \mathbf{P} = \{\mathbf{p_1}, \dots, \mathbf{p_n}\}$

  点集合$\mathbf{P'}, \mathbf{P'} = \{\mathbf{p'_1}, \dots, \mathbf{p'_n}\}$

- 目标：计算两组数据之间的旋转平移量$\mathbf{R}, \mathbf{t}$，使得两组数据形成最佳匹配，即两组数据的距离误差最小

Point-Point ICP

- 第i个匹配对点的误差为

$$\mathbf{e_i} = \mathbf{p_i} - (\mathbf{R}\mathbf{p'}_\mathbf{i} + \mathbf{t})$$

- 构建成最小二乘问题，求使得误差平方和达到最小的R,t

$$min\ \frac{1}{2}\sum_{i=1}^{n}\left\|\mathbf{p_i} - (\mathbf{R}\mathbf{p'}_\mathbf{i} + \mathbf{t})\right\|^2$$

线性代数求解方法

1. 定义两组点集合的质心位置$\mathbf{p}, \mathbf{p}'$

$$\mathbf{p} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{p_i} \qquad \mathbf{p}' = \frac{1}{n}\sum_{i=1}^{n} \mathbf{p}'_{\mathbf{i}}$$

2. 计算每个点的去质心坐标

$$q_{\mathbf{i}} = \mathbf{p_i} - \mathbf{p} \qquad q'_{\mathbf{i}} = \mathbf{p}'_{\mathbf{i}} - \mathbf{p}'$$

3. 根据以下优化问题计算旋转矩阵 $\mathbf{R}^* = \operatorname{argmin} \frac{1}{2}\sum_{\mathbf{i=1}}^{\mathbf{n}} \left\| \mathbf{q_i} - \mathbf{R}\mathbf{q}'_{\mathbf{i}} \right\|^2$

4. 根据R计算t $\mathbf{t}^* = \mathbf{p} - \mathbf{R}^*\mathbf{p}'$

R,t 分解计算说明

$$\frac{1}{2}\sum_{i=1}^{n}\left\|\mathbf{p_i}-(\mathbf{R}\mathbf{p'_i}+\mathbf{t})\right\|^2 = \frac{1}{2}\sum_{i=1}^{n}\left\|\mathbf{p_i}-\mathbf{R}\mathbf{p'_i}-\mathbf{t}-\mathbf{p}+\mathbf{R}\mathbf{p'}+\mathbf{p}-\mathbf{R}\mathbf{p'}\right\|^2$$

$$= \frac{1}{2}\sum_{i=1}^{n}\left\|(\mathbf{p_i}-\mathbf{p}-\mathbf{R}(\mathbf{p'_i}-\mathbf{p'}))+(\mathbf{p}-\mathbf{R}\mathbf{p'}-\mathbf{t})\right\|^2$$

$$= \frac{1}{2}\sum_{i=1}^{n}\left\|(\mathbf{p_i}-\mathbf{p}-\mathbf{R}(\mathbf{p'_i}-\mathbf{p'}))\right\|^2+\|\mathbf{p}-\mathbf{R}\mathbf{p'}-\mathbf{t}\|^2$$

$$+2(\mathbf{p_i}-\mathbf{p}-\mathbf{R}(\mathbf{p'_i}-\mathbf{p'}))^T(\mathbf{p}-\mathbf{R}\mathbf{p'}-\mathbf{t})$$

$$\mathbf{p}=\frac{1}{n}\sum_{i=1}^{n}\mathbf{p_i} \quad \mathbf{p'}=\frac{1}{n}\sum_{i=1}^{n}\mathbf{p'_i}$$

$$\sum_{i=1}^{n}(\mathbf{p_i}-\mathbf{p}-\mathbf{R}(\mathbf{p'_i}-\mathbf{p'}))^T(\mathbf{p}-\mathbf{R}\mathbf{p'}-\mathbf{t}) = 0$$

R,t 分解计算说明

优化目标函数简化为

$$min\ \frac{1}{2}\sum_{i=1}^{n}\left\|(\mathbf{p_i}-\mathbf{p}-\mathbf{R}(\mathbf{p'_i}-\mathbf{p'}))\right\|^2+\|\mathbf{p}-\mathbf{R}\mathbf{p'}-\mathbf{t}\|^2$$

左边只跟旋转矩阵 R相关

右边既有R 也有 t，但是只跟质心相关

因此问题可以分解为两步法解决

求解R

$$\frac{1}{2}\sum_{i=1}^{n}\left\|(\mathbf{p_i}-\mathbf{p}-\mathbf{R}(\mathbf{p'_i}-\mathbf{p'}))\right\|^2=\frac{1}{2}\sum_{\mathbf{i=1}}^{\mathbf{n}}\left\|(\mathbf{q_i}-\mathbf{R}\mathbf{q'_i})\right\|^2$$

$$=\frac{1}{2}\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{q}_i^T\mathbf{q}_i+\mathbf{q'}_i^T\underbrace{\mathbf{R}^T\mathbf{R}}_{=\mathbf{I}}\mathbf{q'}_i-2\mathbf{q}_i^T\mathbf{R}\mathbf{q'}_i$$

与R 无关

优化目标函数变为 $max\sum_{\mathbf{i=1}}^{\mathbf{n}}\mathbf{q}_i^T\mathbf{R}\mathbf{q'}_i$

求解R

$$\sum_{i=1}^{n} \mathbf{q}_i^T \mathbf{R} \mathbf{q}'_i = \sum_{i=1}^{n} tr(\mathbf{q}_i^T \mathbf{R} \mathbf{q}'_i) = \sum_{i=1}^{n} tr(\mathbf{R} \mathbf{q}'_i \mathbf{q}_i^T) = tr\left(\mathbf{R} \sum_{i=1}^{n} \mathbf{q}'_i \mathbf{q}_i^T\right)$$

定义矩阵W $\mathbf{W} = \sum_{i=1}^{n} \mathbf{q}'_i \mathbf{q}_i^T$

对 W 进行SVD分解，可得 $\mathbf{W} = \mathbf{U}\mathbf{S}\mathbf{V}^T$

S 为奇异值组成的对角矩阵，对角线元素从大到小排列，其余为0

U 和 V 满足 $\mathbf{U}^T\mathbf{U} = \mathbf{I}$ $\mathbf{V}^T\mathbf{V} = \mathbf{I}$

求解R

- 目标函数上界

$$tr(RUSV^T) = tr(SV^TRU) \sim tr(SH)$$

$$H = \begin{pmatrix} h_1^T \\ h_2^T \\ h_3^T \end{pmatrix}$$

h是模为1的向量，
因此每个元素的模必小于1

$$tr(RUSV^T) = tr\begin{pmatrix} s_1h_1^T \\ s_2h_2^T \\ s_3h_3^T \end{pmatrix} = s_1h_{11} + s_2h_{22} + s_3h_{33}$$

$$\leq s_1 + s_2 + s_3$$

求解R

- 等号成立时

$$H = V^T R U = I$$

- 可得R

$$R = V U^T$$

- 进一步得到t

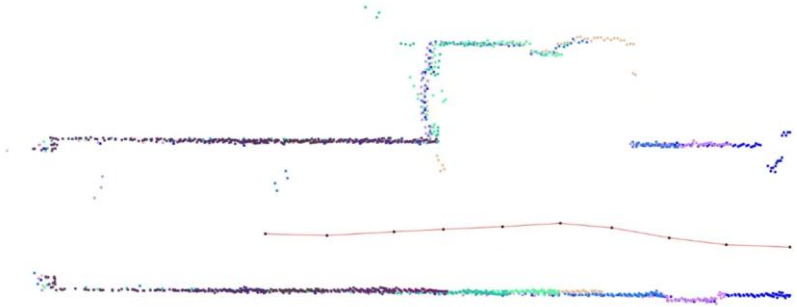
$$t = p - R p'$$

关于ICP的思考

- ICP计算效率主要受什么影响？
- ICP迭代寻优会存在什么问题？
- 对于移动机器人来讲，两帧点云数据做ICP的初值如何获得？

作业

- 给定10帧移动机器人的二维激光点云，实现相邻之间的 ICP 点云匹配，进而航位推算得到机器人这10帧的轨迹，并进行局部点云地图合成
- 实现工具或语言不限制(C++，Python，Matlab 等选一种)
- ICP 类型不限制(点到点， 点到面等)



END