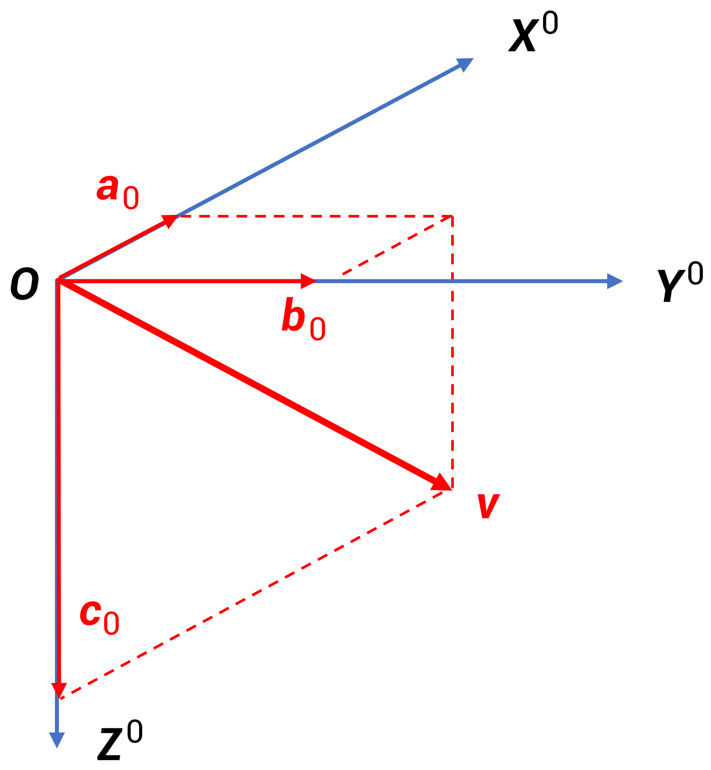
四旋翼无人机的动力学模型

向量在不同坐标系下的表达



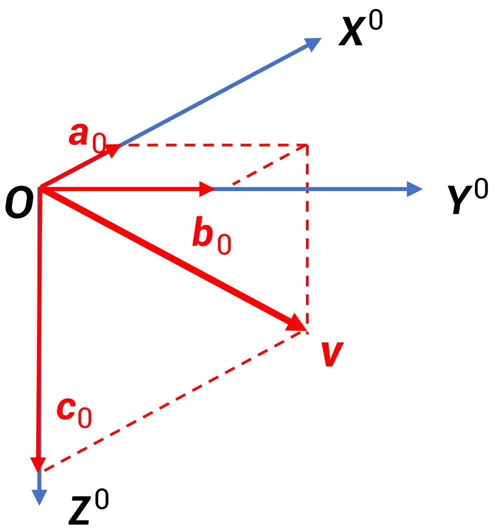
$$\boldsymbol{v} = \boldsymbol{a}_0 + \boldsymbol{b}_0 + \boldsymbol{c}_0$$

$$|\boldsymbol{a}_0| = \alpha \qquad |\boldsymbol{b}_0| = \beta \qquad |\boldsymbol{c}_0| = \gamma$$

$$\boldsymbol{a}_0^0 = \begin{bmatrix}\alpha\\0\\0\end{bmatrix} \qquad \boldsymbol{b}_0^0 = \begin{bmatrix}0\\\beta\\0\end{bmatrix} \qquad \boldsymbol{c}_0^0 = \begin{bmatrix}0\\0\\\gamma\end{bmatrix}$$

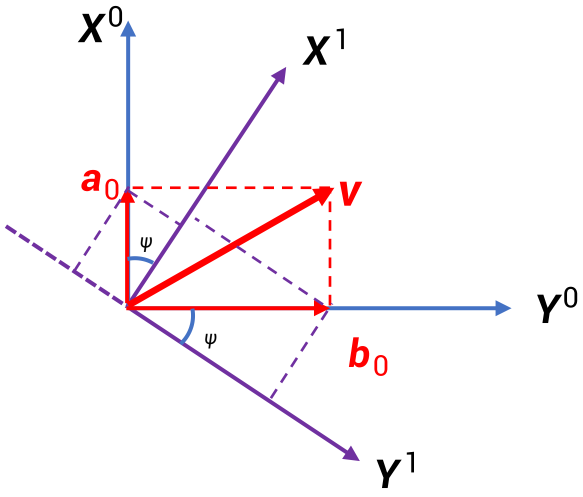
$$\boldsymbol{v}^0 = \begin{bmatrix}\alpha\\0\\0\end{bmatrix} + \begin{bmatrix}0\\\beta\\0\end{bmatrix} + \begin{bmatrix}0\\0\\\gamma\end{bmatrix} = \begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}$$

向量在不同坐标系下的表达——绕z轴旋转ψ



$$\boldsymbol{v} = \boldsymbol{a}_0 + \boldsymbol{b}_0 + \boldsymbol{c}_0$$

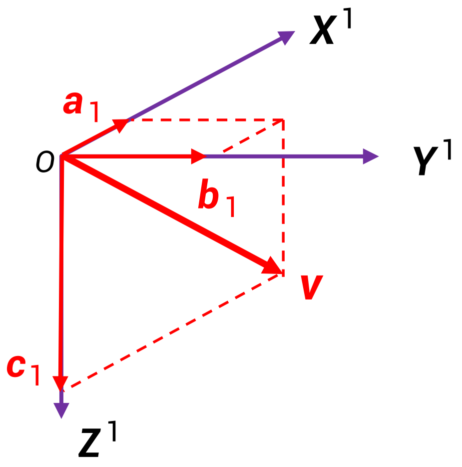
$$\boldsymbol{a}_0^1 = \begin{bmatrix} \alpha\cos\psi \\ -\alpha\sin\psi \\ 0 \end{bmatrix} \qquad \boldsymbol{b}_0^1 = \begin{bmatrix} \beta\sin\psi \\ \beta\cos\psi \\ 0 \end{bmatrix} \qquad \boldsymbol{c}_0^1 = \begin{bmatrix} 0 \\ 0 \\ \gamma \end{bmatrix}$$

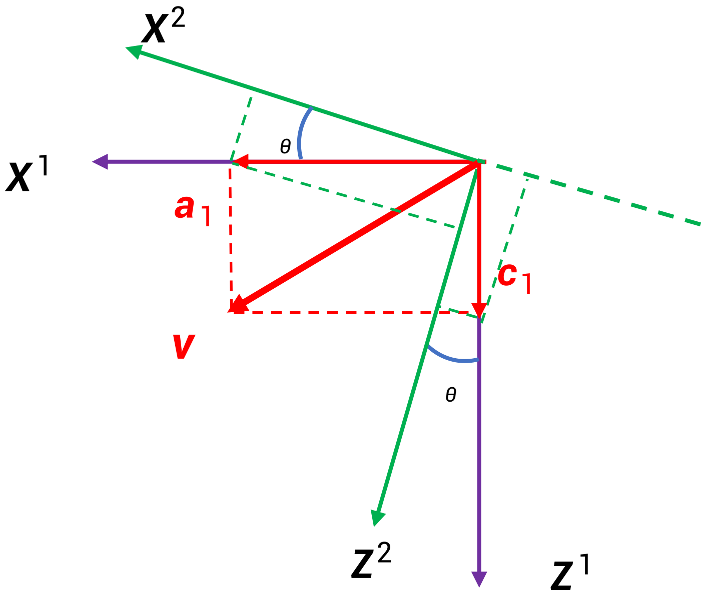


$$\boldsymbol{v}^1 = \begin{bmatrix} \alpha\cos\psi + \beta\sin\psi \\ -\alpha\sin\psi + \beta\cos\psi \\ \gamma \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{旋转矩阵}} \underbrace{\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}}_{\boldsymbol{v}^0}$$

$$\boldsymbol{v}^1 = \Re(\psi)\boldsymbol{v}^0$$

向量在不同坐标系下的表达——绕y轴旋转θ





$$\boldsymbol{v} = \boldsymbol{a}_1 + \boldsymbol{b}_1 + \boldsymbol{c}_1$$

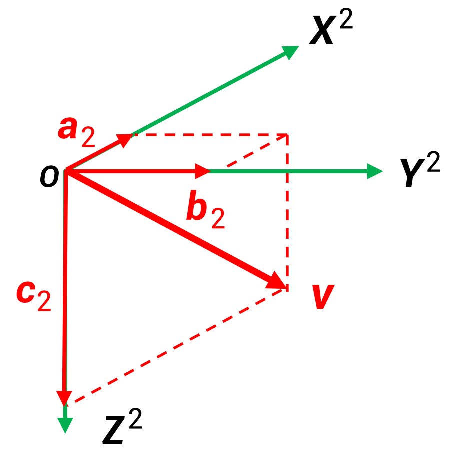
$$\boldsymbol{a}_1^2 = \begin{bmatrix} |\boldsymbol{a}_1|\cos\theta \\ 0 \\ |\boldsymbol{a}_1|\sin\theta \end{bmatrix} \quad b_1^2 = \begin{bmatrix} 0 \\ |\boldsymbol{b}_1| \\ 0 \end{bmatrix} \quad \boldsymbol{c}_1^2 = \begin{bmatrix} -|\boldsymbol{c}_1|\sin\theta \\ 0 \\ |\boldsymbol{c}_1|\cos\theta \end{bmatrix}$$

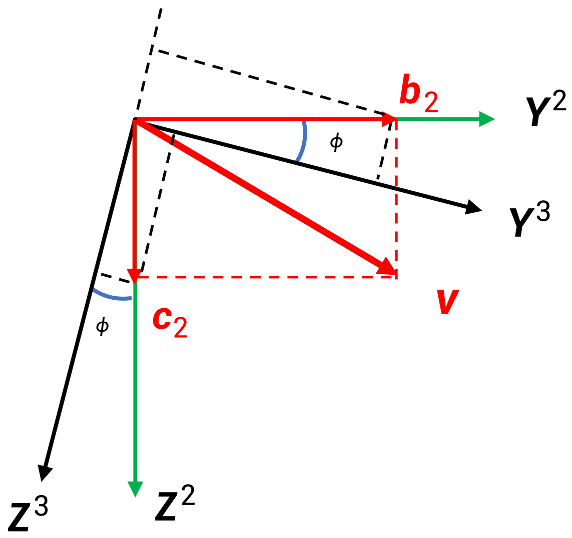
$$\boldsymbol{v}^2 = \begin{bmatrix} |\boldsymbol{a}_1|\cos\theta - |\boldsymbol{c}_1|\sin\theta \\ |\boldsymbol{b}_1| \\ |\boldsymbol{a}_1|\sin\theta + |\boldsymbol{c}_1|\cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} |\boldsymbol{a}_1| \\ |\boldsymbol{b}_1| \\ |\boldsymbol{c}_1| \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$\boldsymbol{v}^2 = \Re(\theta)\Re(\psi)\boldsymbol{v}^0$$

向量在不同坐标系下的表达——绕x轴旋转ϕ





$$\boldsymbol{v} = \boldsymbol{a}_2 + \boldsymbol{b}_2 + \boldsymbol{c}_2$$

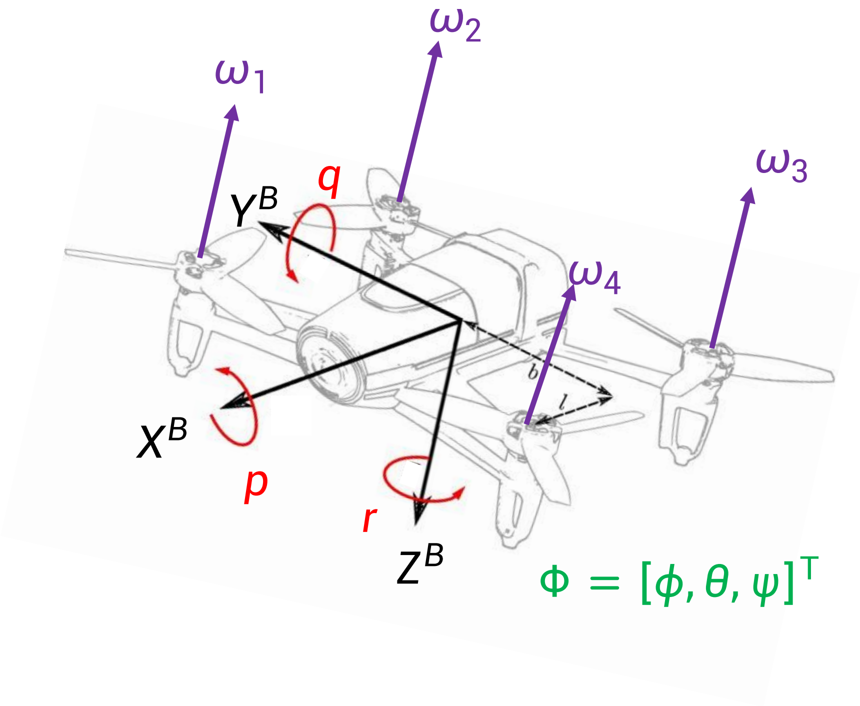
$$\boldsymbol{a}_2^3 = \begin{bmatrix} |\boldsymbol{a}_2| \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{b}_2^3 = \begin{bmatrix} 0 \\ |\boldsymbol{b}_2|\cos\phi \\ -|\boldsymbol{b}_2|\sin\phi \end{bmatrix} \quad \boldsymbol{c}_2^3 = \begin{bmatrix} 0 \\ |\boldsymbol{c}_2|\sin\phi \\ |\boldsymbol{c}_2|\cos\phi \end{bmatrix}$$

$$\boldsymbol{v}^3 = \begin{bmatrix} |\boldsymbol{a}_2| \\ |\boldsymbol{b}_2|\cos\phi + |\boldsymbol{c}_2|\sin\phi \\ -|\boldsymbol{b}_2|\sin\phi + |\boldsymbol{c}_2|\cos\phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} |\boldsymbol{a}_2| \\ |\boldsymbol{b}_2| \\ |\boldsymbol{c}_2| \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$\boldsymbol{v}^2 = \Re(\phi)\Re(\theta)\Re(\psi)\boldsymbol{v}^0$$

四旋翼无人机动力学模型总结



$$\begin{bmatrix} \dot{x}^E \\ \dot{y}^E \\ \dot{z}^E \end{bmatrix} = \begin{bmatrix} v_x^E \\ v_y^E \\ v_y^E \end{bmatrix}$$

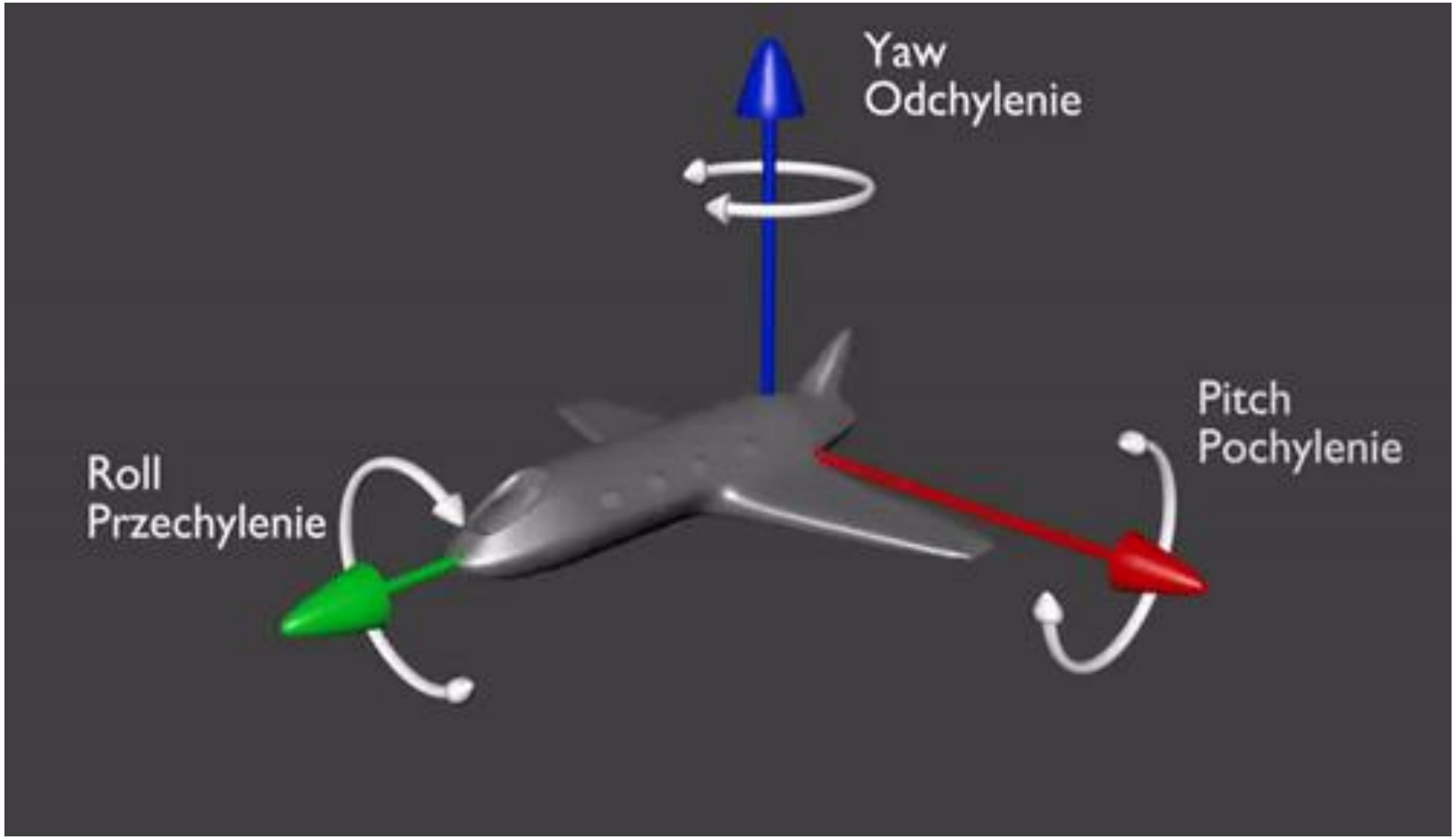
$$\begin{bmatrix} \dot{v}_x^E \\ \dot{v}_y^E \\ \dot{v}_z^E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \Re_B^E(\phi, \theta, \psi) \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix} + \mathbf{D}(\mathbf{v}, \phi, \theta, p, q, r \ldots)$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta \sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

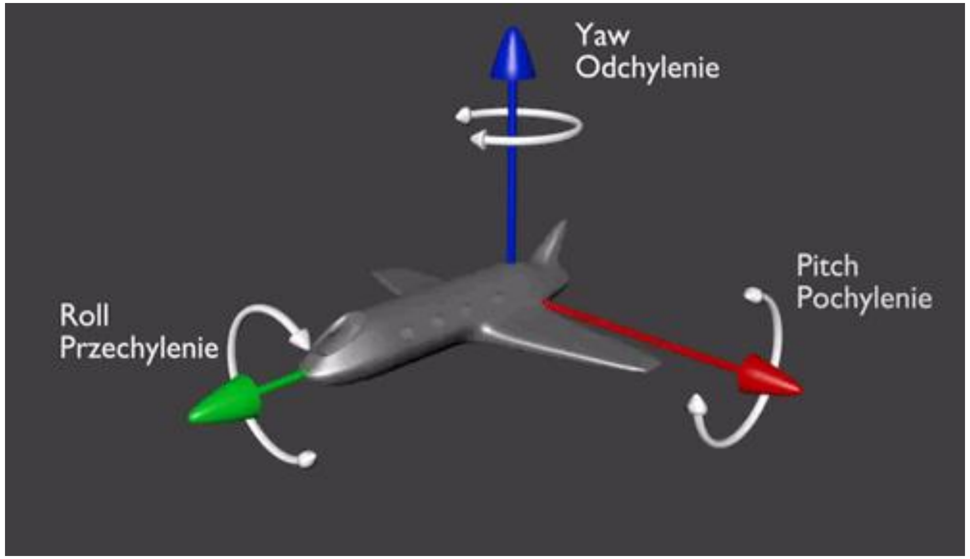
$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = (\mathbf{I}^B)^{-1} \left(\begin{bmatrix} L \\ M \\ N \end{bmatrix}^B - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbf{I}^B \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right)$$

四旋翼无人机的飞行控制

四旋翼无人机解耦



四旋翼无人机解耦



$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & \tan\theta\sin\phi & \tan\theta\cos\phi\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix}$$

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = (\mathbf{I}^B)^{-1}\left(\begin{bmatrix}L\\ M\\ N\end{bmatrix}^B - \begin{bmatrix}p\\ q\\ r\end{bmatrix}\times \mathbf{I}^B\begin{bmatrix}p\\ q\\ r\end{bmatrix}\right)$$

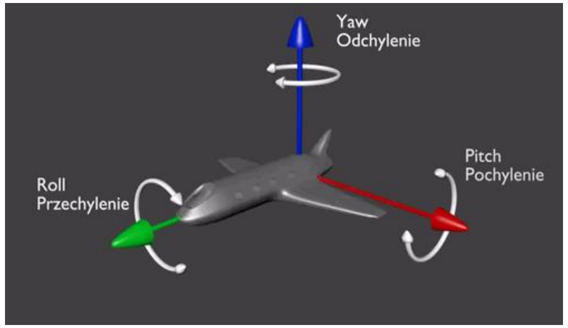
$\phi = 0,\ \ \psi = 0$ $\quad$ $p = 0,\ \ r = 0$

$L = 0,\ \ N = 0$ $\quad$ 解偶、化简

$$\begin{cases}\dot{\theta} = q\\ \dot{q} = \dfrac{M}{I_{yy}}\end{cases} \qquad \begin{cases}\dot{\phi} = p\\ \dot{p} = \dfrac{L}{I_{xx}}\end{cases} \qquad \begin{cases}\dot{\psi} = r\\ \dot{r} = \dfrac{N}{I_{zz}}\end{cases}$$

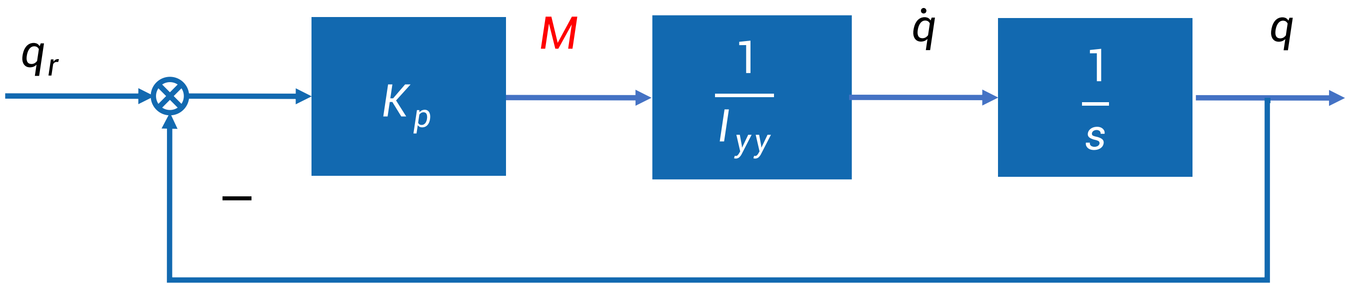
独立、单入单出（SISIO）、线性系统

四旋翼俯仰速率控制器



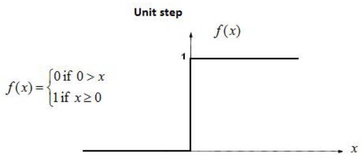
$$\dot{\theta} = q$$

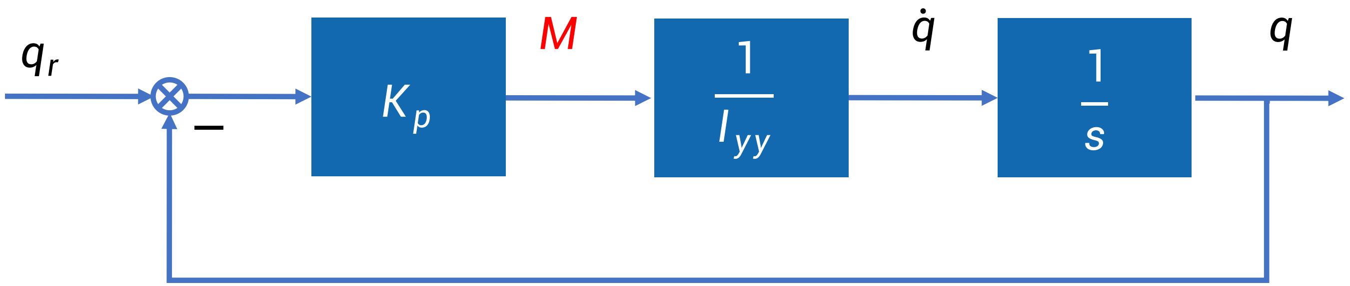
$$\dot{q} = \frac{M}{I_{yy}}$$



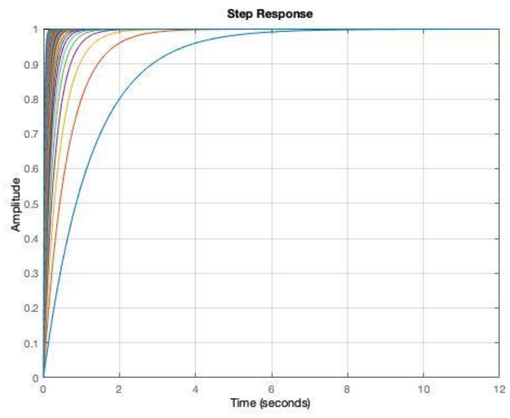
$$\frac{q(s)}{q_r(s)} = \frac{K_p}{I_{yy}s + K_p} \approx 1$$

四旋翼俯仰速率控制器

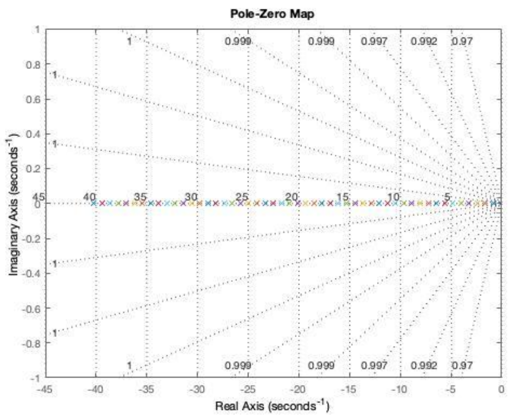




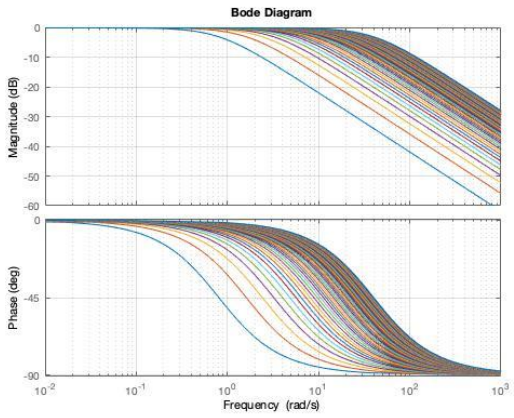
$$\frac{q(s)}{q_r(s)} = \frac{K_p}{I_{yy}s + K_p} \approx 1$$



阶跃响应



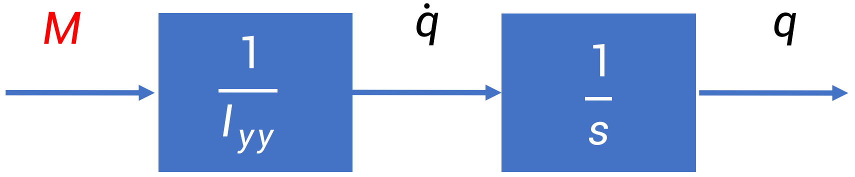
极点位置



波特图

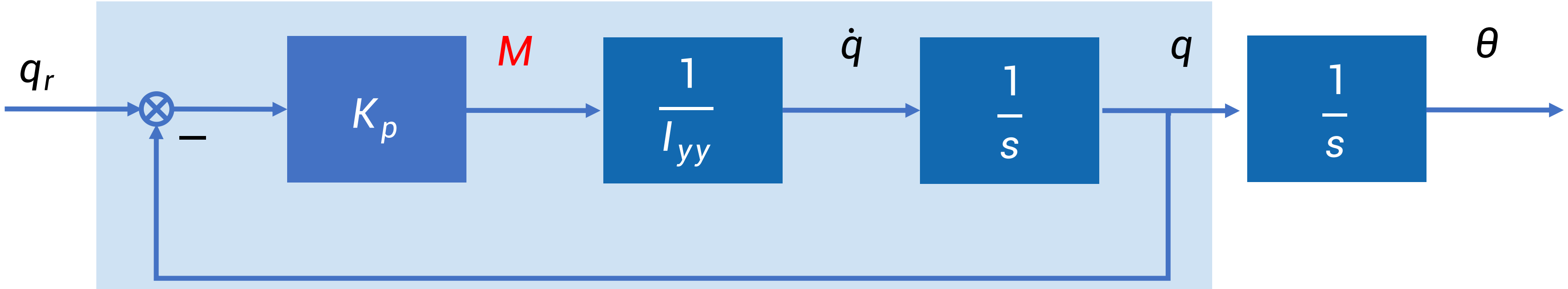
有速率控制器后的模型

$$\begin{cases} \dot{\theta} = q \\ \dot{q} = \dfrac{M}{I_{yy}} \end{cases}$$



有速率控制器后

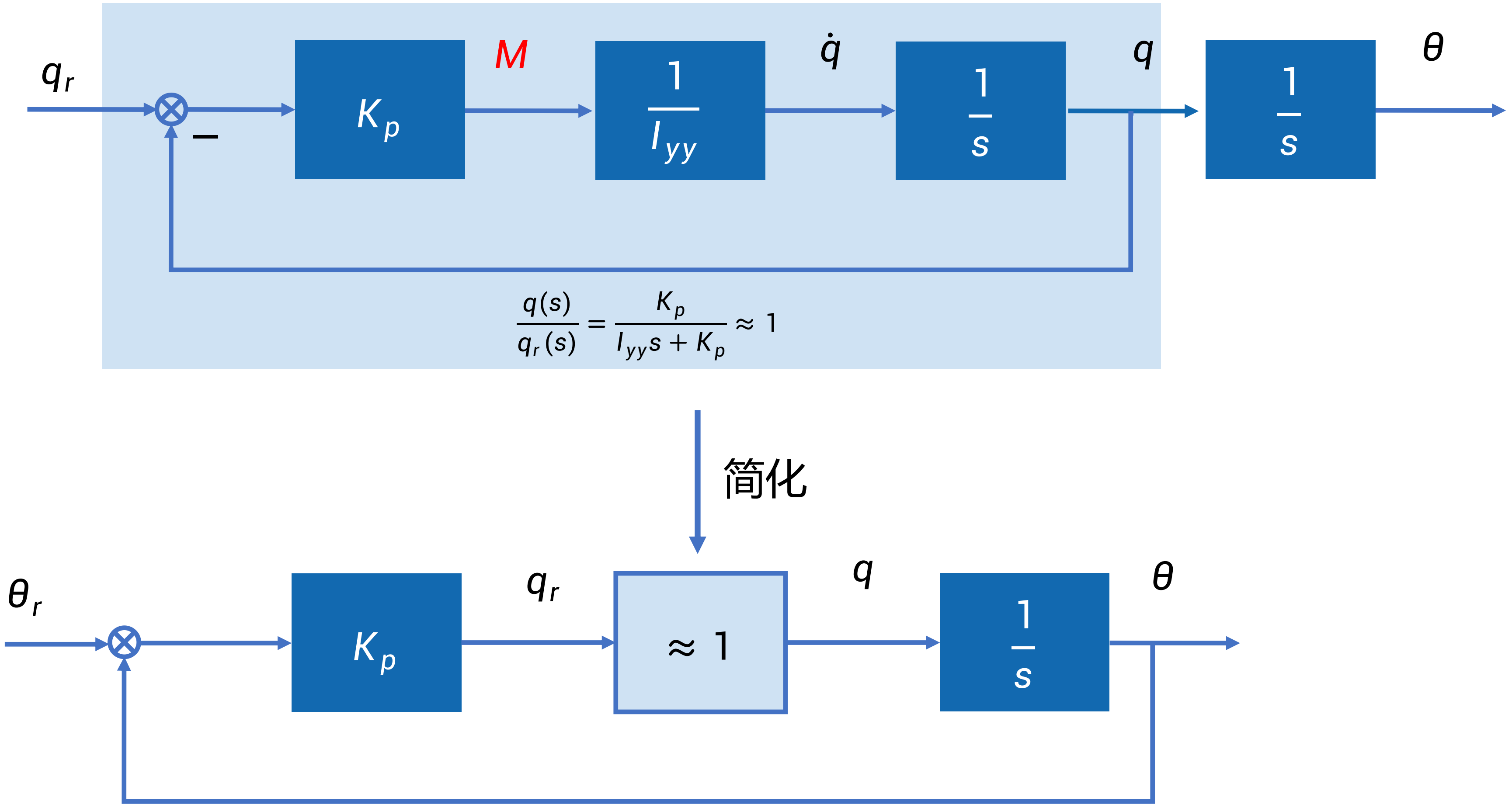
$$\begin{cases} \dot{\theta} = q \\ \dot{q} = q_r \end{cases}$$



$$\frac{q(s)}{q_r(s)} = \frac{K_p}{I_{yy}s + K_p} \approx 1$$

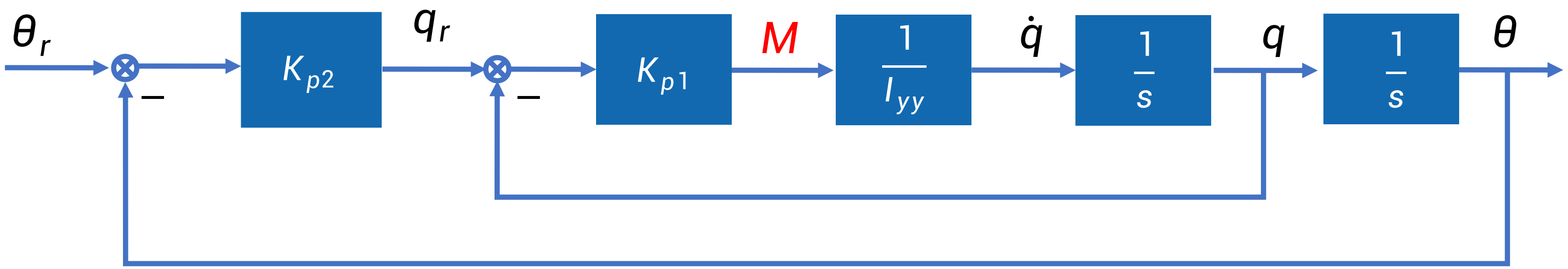
姿态控制器

$$\begin{cases} \dot{\theta} = q \\ \dot{q} = q_r \end{cases}$$



底层控制器（速率控制器）已经设计好，不再需要考虑它怎么工作，**要多少速率有多少速率**

姿态控制器/串级控制器

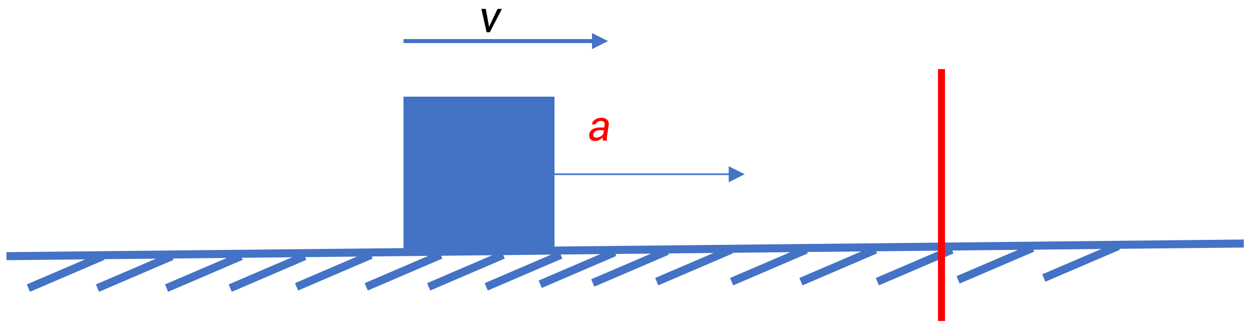


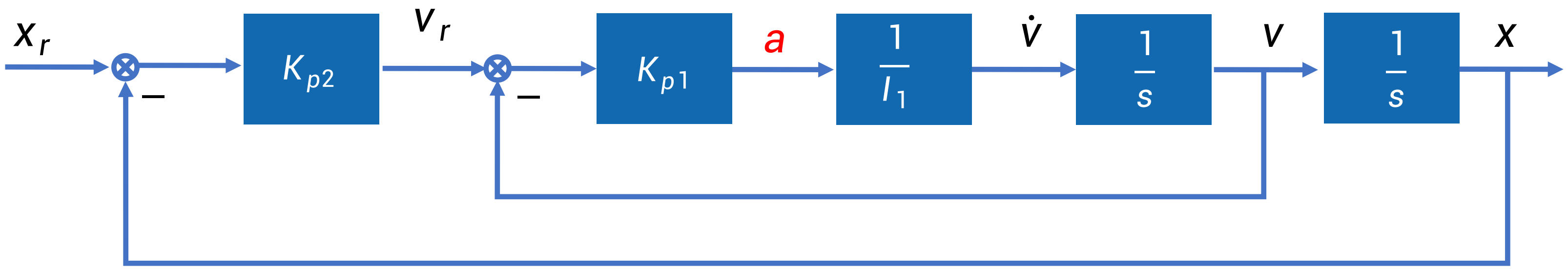
$$G(s) = \frac{K_{p1}K_{p2}}{I_{yy}s^2 + K_{p1}s + K_{p1}K_{p2}}$$

- 外环控制器计算需要多少角速率来跟踪姿态指令
- 内环控制器计算需要多少力矩来跟踪角速率指令

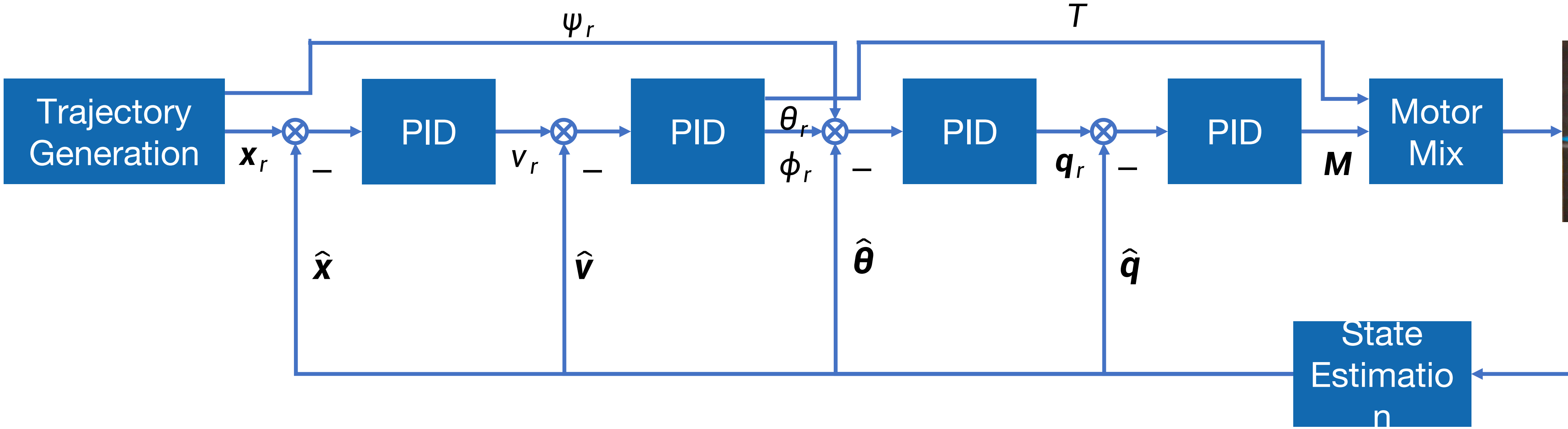
为什么不直接用P控制器?

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a$$





无人机控制回路总结



Trajectory Generation
PID
PID
PID
PID
Motor Mix
State Estimation
ψ_r
T
$\boldsymbol{x}_r$
v_r
θ_r
ϕ_r
$\boldsymbol{q}_r$
$\boldsymbol{M}$
$-$
$-$
$-$
$-$
$\hat{\boldsymbol{x}}$
$\hat{\boldsymbol{v}}$
$\hat{\boldsymbol{\theta}}$
$\hat{\boldsymbol{q}}$

TUDelft