Case of post-fire recolonization

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1 Introduction

After studying more than 275 data sets, data bases and data portals of the ecosystems, I found that there are three interesting areas of research which are climate change, water quality change and wildfires. Only the latter was more convenient, as I have to create my own data set and run lots of simulations.

My goal is to model the spread of wildfires and create a domain-specific language (DSL) to define and study the related resilience properties. The results will be supported by an reinforced learning approach.

The DREF is a rigorous conceptual framework to define the concepts of dependability and resilience, created by Professor Nicolas GUELFI who published it as a research paper. The latter was accepted in the Central European Journal of Computer Science as of September 11, 2011. Although it was defined from a software engineering perspective, its generic aspect will allow us to extend it to serve the purpose of this study.

In this first analysis with DREF, the model will be simplified to the extreme and we will assume for example that the study grid is always square of size n, and that there can be no diagonal propagation of fire or propagation between tree clusters, and we don't have any rules for sprouting (that's to say, a burned tree will disappear and leave an empty place or empty quadrant in the grid).

Besides that, we will assume 3 rules extracted from the percolation theorem specific to statistical physics which is based on probabilities and graph theory.

We will start by stating these rules, then we will redefine in an exhaustive way the basic functions and the concepts of DREF while illustrating our mathematical approach.

The main objective is to understand and show the resilience (relying on the DREF) after a forest fire (for example a mixed forest of oak and beech with sprouting, contamination and competition rules) in order to present the recolonization of the forest in a case close to reality.

2 Formalisation in DREF

2.1 Entities, Properties, and Satisfiability

We consider a grid of size n×n and let's assume that:

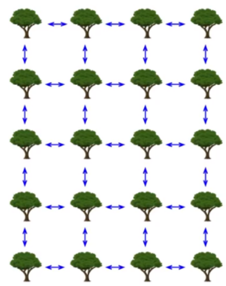


Figure 1: The trees are perfectly aligned and distributed on a square grid.

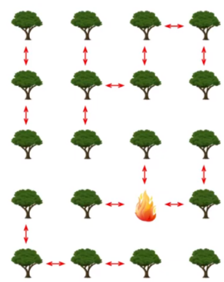


Figure 2: Fire can only spread between a tree and its four closest neighbors.

2.1.1 Evolution axis

The evolution axis is the set discretised by of Tours in $[\![0, N]\!]$, where N is a positive integer. Number of Tours = N+1 ; $(T_0, T_1, T_2, ..., T_{N-1}, T_N)$.

2.1.2 Entities

We are interested in a single entity which will be the configuration of the system at a given time (Tour). It will be noted as e_s, which stands for ecosystem structure.

2.1.3 Change of Entity over time

We will have a configuration at each turn, so N+1 possible configurations in total. This configuration is a triplet of 3 integer numbers included in $[\![0, n]\!]^3$

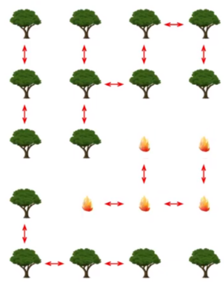


Figure 3: The fire has a probability p between 0 and 1 to spread between each pair of neighboring trees, independently of the other pair of neighboring trees.

and whose sum is n. A triplet will correspond to the number i of trees (TR), j of flames (FL) and k of empty quadrants (EQ).

$$e_s = \{\forall m \in [\![0, N]\!], e_{s_{T_m}} = \{(i_m, j_m, k_m) \in [\![0, n]\!]^3 \ \& \ i_m + j_m + k_m = n^2 \}\}$$

We have N+1 elements in e_s and each of its elements has a maximal number $\frac{(n^2+1)\times(n^2+2)}{2}$ of possible configurations. Therefore, the number of all configurations is: $\frac{(N+1)\times(n^2+1)\times(n^2+2)}{2}$

2.1.4 Properties

What is TCR ? Tree colonization Ratio can be understood simply as the density of trees (ie. total number of trees over the total number of quadrants studied). This will be our main property (called p_0) but we will introduce two others: Maximum theoretical density p_2 and Minimum Theoretical density p_1.

Why it is interesting for applying TCR as a property of resilience using DREF?
In wildfire, if we know the density of trees over time (evolution axis) we can:

- Re-define all functions of resilience introduced in DREF.
- Know exactly whether the system is resilient or not.
- Extend DREF to be used in direct application of Fire studies.

$p_0 = \frac{NumberOfTrees}{NumberofQuadrantsStudied}$

$p_1 = \frac{TheoreticalMinimalNumberOfTrees}{NumberofQuadrantsStudied}$

$p_2 = \frac{TheoreticalMaximalNumberOfTrees}{NumberofQuadrantsStudied}$

In our case and ∀m∈ $[\![0, N]\!]$, we redefine the three properties for every $e_{s_{T_m}}$:

0 flame: $p_{0,m} = p_1 = p_2 = \frac{i_0}{n^2}$

1 flame: $p_{0,m} = \frac{i_{m,1}}{n^2}$; $p_1 = \frac{0}{n^2} = 0$; $p_2 = \frac{i_0 - 1}{n^2}$; $0 \leq i_{m,1} \leq i_0 - 1$

2 flames: $p_{0,m} = \frac{i_{m,2}}{n^2}$; $p_1 = \frac{0}{n^2} = 0$; $p_2 = \frac{i_0 - 2}{n^2}$; $0 \leq i_{m,2} \leq i_0 - 2$

j_0 flames: $p_{0,m} = \frac{i_{m,j_0}}{n^2}$; $p_1 = \frac{0}{n^2} = 0$; $p_2 = \frac{i_0 - j_0}{n^2}$; $0 \leq i_{m,j_0} \leq i_0 - j_0$

p_1 is the minimum theoretical density of trees ∀m∈ $[\![0, N]\!]$. It may or may not be reached during the N rounds, but in theory it is possible. It is always 0 when a fire occurs, because we assume that the spread is global and affects all trees.

p_2 is the maximum theoretical density of trees ∀m∈ $[\![0, N]\!]$. It may or may not be reached during the N rounds, but in theory it is possible. It depends on i_0, j_0 and n (the size of the grid) when a fire occurs, because we assume that the spread is localized and affects just the first j_0 damaged trees out of i_0.

p_0 is a ratio that's always between p_1 and p_2, ∀m∈ $[\![0, N]\!]$. This value is changeable and dependable of the tour T_m, j_0 number of flames and n the size of the grid. This value can be anything among $i_0 - j_0 + 1$ possible scenarios.

Definition : Satisfiability.

Let Prop, Ent be sets of properties and entities; Satisfiability, sat, is a function such that: sat: Prop$\times Ent \rightarrow [0,1] \cup \{\perp\}$

In our forest wildfire model, the satisfiability is always defined:

$\forall$m$\in [\![0, N]\!]$, sat(p_0 , $e_{s_{T_m}}$) $= p_{0,m} \in$ [p$_1$, p_2]$\subseteq [0,1]$ and we define also: sat(p_1 , $e_{s_{T_m}}$) $= p_1$ and sat(p_2 , $e_{s_{T_m}}$) $= p_2$.

We choose our acceptance threshold, 80%, (resp: rejection threshold 20%) arbitrarily.

Satisfied $\rightarrow \forall$m$\in [\![0, N]\!]$, $p_{0,m} \in]20\%, 80\%]$

Unsatisfied $\rightarrow \forall$m$\in [\![0, N]\!]$, $p_{0,m} \in [0, 20\%] \cup]80\%, 1]$

Exactly satisfied $\rightarrow \forall$m$\in [\![0, N]\!]$, $p_{0,m} = 80\%$

Exactly unsatisfied $\rightarrow \forall$m$\in [\![0, N]\!]$, $p_{0,m} = 20\%$

Oversatisfied $\rightarrow \forall$m$\in [\![0, N]\!]$, $p_{0,m}$=p$_1$ or $p_{0,m}$=p$_2$ to satisfy more than p_0

Undersatisfied $\rightarrow \forall$m$\in [\![0, N]\!]$, $p_{0,m} \in [0, p_1[$ Satisfy less than p_0, with $p_1 \leq 20\%$.

2.2 Subjectivity of satisfaction

Let sat be a satisfiability function over Prop and Ent and $<_{prop}$ a partial order defined over Prop. $<_{prop}$ is said strongly sat-compatible because the following property holds:

$\forall$prop$_a$, $prop_b \in \{$p$_0, p_1, p_2\}$, (prop$_a <_{prop}$ $prop_b$) $\Rightarrow \forall$m$\in [\![0, N]\!]$, sat($prop_a$, $e_{s_{T_m}}$) $<$ sat($prop_b$, $e_{s_{T_m}}$).

In fact we have : $<_{prop}$ is the $<$ on $\mathbb{R}$.

$p_1 < p_0 < p_2 \Rightarrow \forall$m$\in [\![0, N]\!]$, sat($p_1$, $e_{s_{T_m}}$) $<$ sat(p_0, $e_{s_{T_m}}$) $<$ sat(p_2, $e_{s_{T_m}}$).

This indicate that a property greater than another must have its satisfiability value always greater than the one of the other for all the entities considered (and for us, we have one entity with N instances).

2.2.1 Observers

Let Obs be a set of observers, and sat_{Obs} an Obs-indexed family of satisfiability functions. If we suppose that the observation is independent then the satisfiability function for Obs is defined as:
$\forall$ o, o' $\in$ Obs , $\forall$ prop $\in \{p_0,\ p_1,\ p_2\}$, sat_{Obs_o}(prop, $e_{s_{T_m}}$) $=$ $sat_{Obs_{o'}}$(prop, $e_{s_{T_m}}$) $=$ sat(prop, $e_{s_{T_m}}$).

2.2.2 Global satisfiabilities

Satisfiability of the ecosystem

Although we have always discussed the satisfiability at tour m $\forall$m$\in [\![0, N]\!]$, it is time to define the satisfiability of e_s, sat(prop, e_s):

$\forall$ prop $\in \{p_0,\ p_1,\ p_2\}$:

$$sat(prop, e_s) = \sum_{m=0}^{N} \frac{sat(prop, e_{s_{T_m}})}{N+1}$$

It's clear that

$$sat(p_1, e_s) = p_1$$

and

$$sat(p_2, e_s) = p_2$$

because the sat for each configuration ($e_{s_{T_m}}$) regarding p_1 and p_2 are independent of m.
Finally,

$$sat(p_0, e_s) = \sum_{m=0}^{N} \frac{sat(p_0, e_{s_{T_m}})}{N+1} = \sum_{m=0}^{N} \frac{p_{0,m}}{N+1}$$

A balanced satisfiability function for Obs: $sat_{w_{Obs}}$

At every tour T, each observation is dependent only on the experimentation (we have only one observer $|Obs| = 1$) and in order to simplify the model we can choose weights to be equal to $\frac{1}{N+1}$. Therefore, we define the global balanced satisfiability given N+1 configurations of the ecosystem structure.Therefore, $w_{Obs} = \frac{1}{N+1}, \frac{1}{N+1}, ..., \frac{1}{N+1}$ (N+1 times) a balancing of Obs and the balanced satisfiability function for Obs is equal to the satisfiability of ecosystem, $sat_{w_{Obs}}$(prop, e$_s$) $= sat(prop, e_s)$.

Global satisfiability : $gsat_{w_{Prop}}$

Let sat be a satisfiability function over Prop and Ent, w_{Prop} a balancing of Prop, then the global balanced satisfiability of sat, is denoted as $gsat_{w_{Prop}}$ and is such that: $gsat_{w_{Prop}} = \frac{\sum_{<p,e>\in dom(sat)} w_{Prop}(p) \times sat(p,e)}{\sum_{p \in Prop} w_{Prop}(p)}$

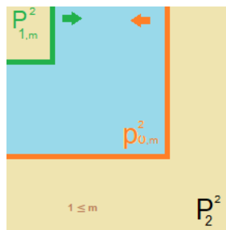


Figure 4: The chosen balancing of Prop, w_{Prop}, can be deduced from this graph

w_{Prop} a balancing of Prop $= \{p_0, p_1, p_2\}$:
$p^2_{1,m} \leq \mathrm{p}^2_{0,m} \leq \mathrm{p}^2_2 \Rightarrow 0 \leq \frac{p^2_{1,m}}{p^2_2} \leq \frac{p^2_{0,m}}{p^2_2} \leq 1$.

We take the p_2 as the origin, and realize that $\frac{p^2_{0,m} - p^2_{1,m}}{p^2_2}$ is interesting as a comparison ratio. Therefore : $w_{Prop} = \{a, b, c\}$ associated to every prop $\in Prop = \{\mathrm{p}_0, p_1, p_2\}$, respectively. (TO BE DISCUSSED).

3 Illustration

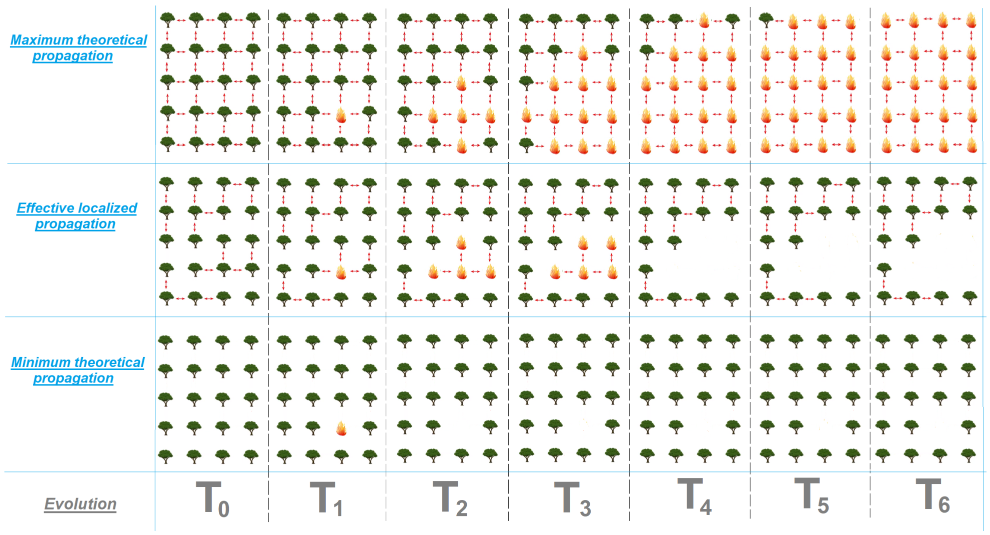
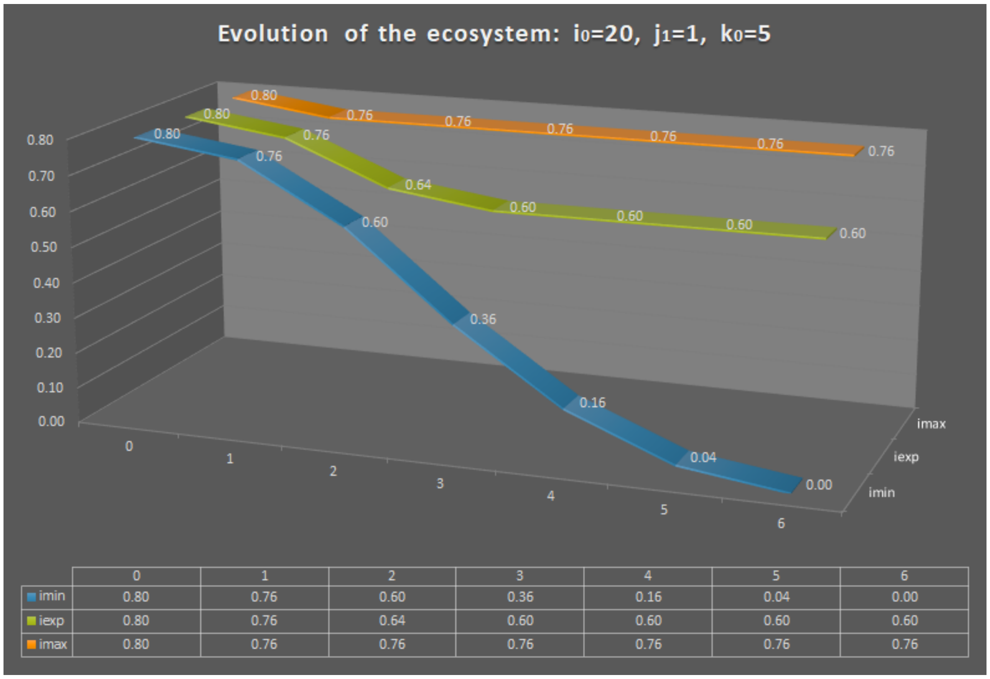


Figure 5: Difference between theoretical and effective propagation considering any grid of size n times n, 1 flame and $i_0 = 20$.

While it's clear that $5 \leq n$, for 1 flame, we will have as expected in the last studied iteration:
1 flame: $p_{0,m} = \frac{i_{m,1}}{n^2} = \frac{15}{n^2}$; $p_1 = \frac{0}{n^2} = 0$; $p_2 = \frac{i_0 - 1}{n^2} = \frac{19}{n^2}$; $0 \leq i_{m,1} \leq i_0 - 1$

Evolution	T_0	T_1	T_2	T_3	T_4	T_5	T_6
Maximum Theoritical propagation	$\frac{20}{n^2}$	$\frac{19}{n^2}$	$\frac{15}{n^2}$	$\frac{9}{n^2}$	$\frac{4}{n^2}$	$\frac{1}{n^2}$	$\frac{0}{n^2}$
Effective localized propagation	$\frac{20}{n^2}$	$\frac{19}{n^2}$	$\frac{16}{n^2}$	$\frac{15}{n^2}$	$\frac{15}{n^2}$	$\frac{15}{n^2}$	$\frac{15}{n^2}$
Minimum Theoritical propagation	$\frac{20}{n^2}$	$\frac{19}{n^2}$	$\frac{19}{n^2}$	$\frac{19}{n^2}$	$\frac{19}{n^2}$	$\frac{19}{n^2}$	$\frac{19}{n^2}$



	0	1	2	3	4	5	6
imin	0.80	0.76	0.60	0.36	0.16	0.04	0.00
iexp	0.80	0.76	0.64	0.60	0.60	0.60	0.60
imax	0.80	0.76	0.76	0.76	0.76	0.76	0.76

Figure 6: Evolution of the ecosystem composed of 20 trees on a 5 x 5 grid and 1 flame, during 7 tours.

Remarks

- If we consider a grid of size 5 x 5,

$$sat(p_0, e_{s_{T_0}}) = \frac{20}{25} = 0.8$$

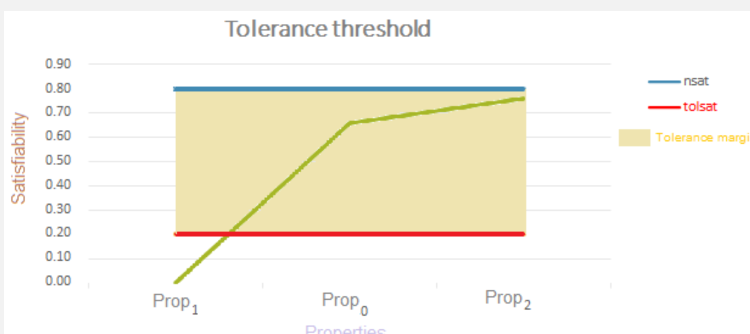
  Thus, $e_{s_{T_0}}$ satisfies exactly p_0 and this will never happen afterwards.

- $e_{s_{T_1}}$ satisfies more than p_0 and we have oversatisfiability at T_1.
- $\forall$ j number of flames in $\mathbb{N}^*$, we consider that the Minimum theoritical propagation is reached at T_1.
- imin : represents the theoritical minimal number of trees associated to the Maximum Theoritical propagation.
- imax : represents the theoritical maximal number of trees associated to the Minimum Theoritical propagation.
- iexp : represents the experimental number of trees that are left after the spread of wildfire at each round.

3.1 Tolerance, Preservation and Improvement

- $\forall < p, e > \in dom(nsat)$ we have a nominal satisfiability function $nsat(r, e) = 80\%$, chosen arbitrarily.
- A requirement for our set of entities, Ent, is a set of properties, Req = p0 , together with a nominal satisfiability function, $nsat = 80\%$, such that $dom(nsat) =< Req, Ent >$.
- Let $rqt =< Req, nsat >$ be a requirement for a set of entities, Ent, and sat a satisfiability function. sat is said to satisfy rqt iff $\forall < r, e > \in dom(nsat)$, $80\% \leq sat(r, e)$.

- Tolerance threshold functions will be denoted as tolsat and it's = 20%. The tolerance margin is the space between nsat and tolsat; that's 60%.
- A preservation is defined as constancy in a satisfiability function w.r.t. an evolution axis.
- An improvement is defined as an increase in a satisfiability function w.r.t. an evolution axis.
- A degradation is defined as a decrease in a satisfiability function w.r.t. an evolution axis.



3.2 Tolerance and Failure:

- Failure:

  Given sat, a satisfiability function, and tolsat, a tolerance threshold, a failure is defined as a tuple $< r, ent >\in dom(sat) \cap dom(tolsat)$ such that: $sat(r, ent) \leq tolsat(r, ent) = 20\%$.

- Tolerance:

  Let sat be a satisfiability function, nsat a nominal satisfiability function and tolsat a tolerance threshold. Then a tolerance is defined as a tuple $< r, ent >$ such that: $< r, ent >\in dom(sat) \wedge < r, ent >\in dom(tolsat) \wedge < r, ent >\in dom(nsat) \wedge sat(r, ent) < nsat(r, ent) \wedge sat(r, ent) > tolsat(r, ent)$.

- Fault Tolerance:

  Let $< Req, nsat >$ be a requirement and tolsat a tolerance threshold, then a requirement with fault-tolerance Req_{FT} is defined as a tuple $< Req, nsat, tolsat >$.

3.3 Resilience as change for improvement

Let ent_v be an entity, and ev = {1, ..., m}, $m \leq N$ be an evolution axis for ent_v. Let sat be a satisfiability function defined over ent_v and over a set of properties Prop= p0, nsat a nominal satisfiability function and tolsat a tolerance threshold defined over ent_v along the evolution axis.

$$\underset{\text{(Maximum tolerance)}}{tolmax^v} = \sum_{\substack{t \in ev \\ p = P_o}} (nsat(p, ent_v^t) - tolsat(p, ent_v^t)) = \frac{60}{100} \times m$$

$$\underset{\text{(Cumulative tolerance)}}{stol^v} = \sum_{\substack{t \in ev \\ p = P_o}} \left(\begin{array}{l} Max(sat(p,ent_v^t),nsat(p,ent_v^t)) \\ -Max(sat(p,ent_v^t),tolsat(p,ent_v^t)) \end{array} \right)$$

$$\underset{\text{(Tolerance level)}}{ltol^v} = \frac{stol^v}{tolmax^v} = \frac{100 \times stol^v}{60 \times m}$$

$$\underset{\text{(Local failure)}}{fail^v} : ev \times P_o \longrightarrow \mathbb{R} \ / \ fail^v(t,p) = \left(\begin{array}{c} tolsat(p,ent_v^t) \\ -Min(sat(p,ent_v^t),tolsat(p,ent_v^t)) \end{array} \right)$$

$$\underset{\text{(Cumulative failure quantity)}}{qfail^v} = |\{< t, p > / fail^v(t,p) > 0\}|$$

$$\underset{\text{Cumulative failure level}}{sfail^v} = \sum_{\substack{t \in ev \\ p = P_o}} fail^v(t,p)$$

Let ent be an entity evolving along the evolution axis $ev = \{1, \ldots, n\}$ (the generative axis). Let sat be a satisfiability function defined over the cited entities and over a set of properties $Prop$, $nsat$ a nominal satisfiability function and $tolsat$ a tolerance threshold defined over the entities along the evolution axis. Let $i, j \in ev$, the properties of **T-resilience**,**F-resilience** and **TF-resilience** (noted $resil_{i,j}^T$, $resil_{i,j}^F$, $resil_{i,j}^{TF}$) are defined as follows:

$$\begin{array}{ll} (i) & resil_{i,j}^T \text{ is true iff } \Delta tol_{i,j} > 0 \\ (ii) & resil_{i,j}^F \text{ is true iff } \Delta qfail_{i,j} > 0 \\ (iii) & resil_{i,j}^{TF} \text{ is true iff } resil_{i,j}^T \wedge resil_{i,j}^F \end{array}$$

The definition of TF-resilience is a strict definition of resilience since it requires that both the level of tolerance and the number of failures if any are reduced.