

**Solution.** We will consider a binary search on  $k$ , with an upper bound of  $n$  and a lower bound of 0. For each value of  $k$ , we will use a modified directed graph, say  $G_k$ . In this graph, each vertex  $v$  is split into  $v_{\text{in}}$  and  $v_{\text{out}}$ , and each edge  $(u, v) \in E$  is connected in both directions with a capacity of 1. For each vertex, we will add an edge between  $v_{\text{in}}$  and  $v_{\text{out}}$  with a capacity of  $k$ . We will also initialize a source  $s$ , which connects to each of the  $v_{\text{out}}$  vertices, and a sink  $t$ , which connects to the  $v_{\text{in}}$  vertices. All of these new edges should have capacities equal to the degree of the corresponding vertices in  $G$ . For each of these values of  $k$ , we can then find the maximum flow from  $s$  to  $t$ .

The  $v_{\text{in}}$  vertices can receive at most  $k$  units of flow, so if we can find a maximum flow that matches the total capacity going into each  $v_{\text{in}}$  vertex, then we have found a valid  $k$ -orientation. If the maximum flow equals the sum of the degrees of all vertices (which is the total capacity going out of the source  $s$ ), then the flow successfully represents a  $k$ -orientation. Our binary search will take  $O(\log n)$  iterations, and for each  $k$ , we will use the Ford-Fulkerson algorithm to find the maximum flow, which takes  $O(m^2)$  time in the worst case. Therefore, the overall time complexity of the algorithm is  $O(m^2 \log n)$ .