

**Solution.**

- (a) If  $P = NP$ , then it is implied that every problem in  $NP$  is as hard as every other problem in  $NP$ , including those that were considered  $NP$ -complete.  $NP-C = NP = P$  since. So, clearly  $P \cup NP-C = P \cup P = P = NP$ , and since  $NP \setminus NP = \emptyset$ , we have completed the proof.
- (b) If  $NP \setminus (P \cup NP-C) = \emptyset$ , then it must be the case that  $NP \subseteq P \cup NP-C$ . This implies that each element of  $NP$  either belongs to either  $P$  or  $NP-C$ . An  $NP-I$  (intermediate) problem would be one from  $NP$  that belongs to neither  $P$  nor  $NP-C$ . The assumption  $NP \subseteq P \cup NP-C$  implies that  $NP-I = \emptyset$ , and, by Ladner's theorem, it must follow that  $P = NP$ .