

Let  $S$  be a string of  $n$  characters. Describe an  $O(n^2)$  algorithm to compute the length of the longest substring that appears both forwards and backwards in  $S$ . The forward and backward substrings should not overlap.

For example, consider the following string

$S = \text{BOBSMAMASEESAUKULELE}.$

The algorithm should return 3 with the substring ASE.

**Hint.** *This is very similar to the palindrome problem from tutorials...*

**Solution.** We use a Dynamic Programming approach by initialising an  $n \times n$  array,  $DP$ , initialised as 0, which still store the length of the longest common substring ending at index  $i$  in the forward direction, and index  $j$  in the backward direction. We will iterate through the list in two nested loops,  $(i, j)$ . For each:

- If  $S[i] = S[j]$  and  $i + j < n - 1$ , set  $DP[i][j] = 1 + DP[i - 1][j + 1]$ . To ensure the substrings don't overlap, we check  $i + j < n - 1$ .
- Otherwise, set  $DP[i][j]$  to 0.

We then retrieve the maximum value from  $DP$  and return the desired information once it is found. We iterate through the string over two nested loops, so clearly our time complexity is  $O(n^2)$ .