Solution.

Consider a standard exchange argument using the following conditions:

- If $x = 1$, it's already in S.
- If x is prime, then it's already in S.
- If x is composite, then $\exists p \in P_n$ such that $p|x$ by the fundamental theorem of arithmetic, so including x in S would violate the condition that every pair of elements in S is comprime.

  If we were to find an alternative solution which contains a composite number, we would be able to swap it with at least one other prime number, so any solution with a comprime value is not necessarily optimal, and hence $|S|$ is the maximum cardinality for such a set.