

Let  $F$  be a flow network with integer capacities. Your task is to return the minimum cut with the smallest number of edges that cross the cut.

- (a) Consider the new flow network  $\hat{F}$ , defined by the new capacity function

$$\hat{c}(x \rightarrow y) = c(x \rightarrow y) \cdot (m + 1) + 1,$$

where  $m$  denotes the number of edges in  $F$ . Show that if  $(S, T)$  is *not* a minimum cut of  $F$ , then  $(S, T)$  cannot be a minimum cut of  $\hat{F}$ .

*This shows that any minimum cut of  $\hat{F}$  must be a minimum cut of  $F$ .*

- (b) Show that the minimum cut of  $F$  with the minimum number of edges will have the lowest cut capacity in  $\hat{F}$ .

*Hence, the minimum cut of  $\hat{F}$  will be the minimum cut with the smallest number of edges that cross the cut.*

**Rubric.**

- This task will form part of the portfolio.
- Ensure that your argument is clear and keep reworking your solutions until your lab demonstrator is happy with your work.