Let $F = (V, E, s, t, w)$ be an (s, t)-flow network. In this problem, our task is to be able to classify vertices in V by a special kind of property that we will define.

- A vertex v is *saucy* if v appears on the source side of *every* minimum cut; that is, if (S, T) is a minimum cut of F, then $v \in S$.
- A vertex v is *sinky* if v appears on the sink side of *every* minimum cut; that is, if (S, T) is a minimum cut of F, then $v \in T$.
- A vertex v is *saunky* if v is neither saucy nor sinky. In other words, there exist at least one minimum cut (S, T) for which $v \in T$ and there exist at least one minimum cut (S', T') for which $v \in S'$.

Describe an $O(mn^2)$ algorithm to classify each vertex as either saucy, sinky, or saunky.

Note. *There can exist exponentially many minimum cuts in a flow network, so do not try and enumerate over all possible minimum cuts.*

Rubric.

- You should additionally prove that your algorithm correctly classifies the vertices.
- This task will form part of the portfolio.
- Ensure that your argument is clear and keep reworking your solutions until your lab demonstrator is happy with your work.