

Consider the following problem.

Let $[n] := \{1, \dots, n\}$. What is the size of the largest subset $S \subseteq [n]$ such that every pair of elements in S is coprime?

Let P_n denote the set of primes in $[n]$. It is clear that $S := \{1\} \cup P_n$ is a subset that satisfies the condition that every pair of elements in S is coprime. Prove that $|S|$ is correct.

Note. *This implies that the natural greedy algorithm of “include x iff. x is prime” returns a subset of maximum cardinality.*

Hint. *There are a few ways to prove this; one way is to define an injection. Another way is to use a standard exchange argument.*

Rubric.

- Your argument should show that S has the largest cardinality; specifically, show that if there exist another subset $S' \subseteq [n]$ with the same property, then $|S'| \leq |S|$.
- This task will form part of the portfolio.
- Ensure that your argument is clear and keep reworking your solutions until your lab demonstrator is happy with your work.