

A battle takes place between n heroes and m monsters. Let h_j denote the starting amount of health points of monster j , where $1 \leq j \leq m$. During one round of the battle, each hero attacks a monster of their own choice, causing one damage to the monster's health points. If a monster's number of health points reduces to zero at any time, they are killed. At the end of each round, all monsters still alive will gain k health points; therefore, a monster can exceed h_j health points. The monsters will not attack any of the heroes.

The heroes win the battle if they manage to kill all of the monsters.

- (a) Suppose that $k < n$. Describe a strategy that chooses which monster(s) to attack in each round, in order to guarantee victory for the heroes in the fewest number of rounds as possible.
- (b) Prove that your algorithm guarantees that the heroes win.
- (c) Briefly explain why your strategy guarantees that the heroes win in the fewest number of rounds possible.
- (d) Now, suppose that $k \geq n$. Under what circumstance(s) can the heroes win the battle? Briefly explain why.

Rubric.

- In part (a), you do not have to analyse the running time; just a strategy suffices. In parts (b) and (c), you will prove the correctness of the strategy.
- In part (d), you should provide one or more conditions and then briefly explain why this guarantees victory for the heroes.
- This task will form part of the portfolio.
- Ensure that your argument is clear and keep reworking your solutions until your lab demonstrator is happy with your work.