

A *forest* is a collection of disjoint trees. In other words, a forest is an undirected and acyclic graph, where all of its components are trees. Prove that a forest with k edges has $n - k$ connected components, where n denotes the number of vertices in the forest.



Figure 1: A forest with 4 edges, 6 vertices, and 2 connected components.

Solution. Consider each vertex on its own without any connection. Any connection between the vertices will reduce the total connected components by one. Since these components are trees, adding an edge must combine two separate components. If the edge is added on the same component, then the figure is no longer a tree. Therefore, if there are k edges, we have eliminated k components and therefore we must have $n - k$ total connected components.