

Solution.

- (a) For each vertex $x \in V$, split them into x_{in} and x_{out} . Then, add connections for each edge in G . For example, take $(x, y) \in E$, we would add a directed edge from x_{out} to y_{in} , setting their capacity to 1. We will introduce a new node, say t to be a sink vertex and connect it to u and v with a weight of 1 each. We will let w be a source vertex. We can then run the Ford-Fulkerson algorithm on this new graph to find the maximum flow from w to t . If the maximum flow is 2, then we know that the only way that could happen is if there exists a simple path between w and t through u and another through v . The Ford-Fulkerson algorithm runs in $O(fm)$ time, for maximum flow f , which would be 2 in our case. Accounting for creating the transformed graph, our time complexity is $O(n + m)$.