B2- Mathematics

B-MAT-200

110borwein

Saving years of calculations...

Borwein integrals

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> (!)
> - Your repository must contain the totality of your source files, but no useless files (binary, temp files, obj files,...).
> - All the bonus files (including a potential specific Makefile) should be in a directory named *bonus*.
> - Error messages have to be written on the error output, and the program should then exit with the 84 error code (0 if there is no error).

In 2001, the Borwein brothers studied integrals that now bear their name, beautifully defined by :

$$\forall n \in \mathbb{N}, I_n = \int_0^{+\infty} \prod_{k=0}^{n} \frac{sin\left(\frac{x}{2k+1}\right)}{\frac{x}{2k+1}} dx$$

All these integrals seem equal to $\frac{\pi}{2}$ (I guess you noticed it).

Some decades ago, an old-school mathemacian would have had to hand-calculate the values of the first integrals (it would need some months, or even years), then suppose all the integrals equal $\frac{\pi}{2}$, and finally try and demonstrate this conjecture.
You are not that kind of person.
Today, we use numerical calculus to know a many values of these integrals as possible before getting into a demonstration ; this is the goal of this project.

You have to compute Borwein integrals, using the rectangle method, the trapezoidal rule and the Simpson rule, and print both the I_n and the difference between this value and $\frac{\pi}{2}$.

> 💡 Since it is impossible to compute the integral between 0 et $+\infty$, the uppper bound will be limited to 5000.

> 💡 Integration interval is to be divided into 10000 sub-intervals.

```
Terminal                                                                      -  +  x
~/B-MAT-200> ./110borwein -h
USAGE
          ./110borwein n

DESCRIPTION
          n     constant defining the integral to be computed
```

> ⚠ Your program output has to be strictly identical to the one below.

```
Terminal                                                                      -  +  x
~/B-MAT-200> ./110borwein 0
Rectangles:
I0 = 1.8208154789
diff = 0.2500191521

Trapezoids:
I0 = 1.5707660806
diff = -0.0000302462

Simpson:
I0 = 1.5707654320
diff = -0.0000308948
```