CHAPTER Two

FORCE VECTORS

Chapter Objectives

- To show how to add forces and resolve them into components using the Parallelogram Law.
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction.

FORCE VECTORS

2.1 Scalars and Vectors

Most of the physical quantities in mechanics can be expressed mathematically by means of scalars and vectors.

Scalar. A quantity characterized by a positive or negative number is called a *scalar*. For example, mass, volume, and length are scalar quantities often used in statics. In this book, scalars are indicated by letters in italic type, such as the scalar *A*.

Vector. A *vector* is a quantity that has both a magnitude and a direction. In statics the vector quantities frequently encountered are position, force, and moment. For handwritten work, a vector is generally represented by a letter with an arrow written over it, such as $\vec{A}$ The magnitude is designated or simply $|A|$. In this book vectors will be symbolized in boldface type; for example, *A* is used to designate the vector 'A'. Its magnitude, which is always a positive quantity, is symbolized in italic type, written as $|A|$, or simply *A* when it is understood that *A* is a positive scalar.

A vector is represented graphically by an arrow, which is used to define its magnitude, direction, and sense. The *magnitude* of the vector is the length of the arrow, the *direction* is defined by the angle between a reference axis and the arrow's line of action, and the *sense* is indicated by the arrowhead. For example, the vector **A** shown in Fig. 2-1 has a magnitude of 4 units, a direction which is 20° measured counterclockwise from the horizontal axis, and a sense which is upward and to the right. The point *O* is called the *tail* of the vector, the point *P* the *tip* or *head.*

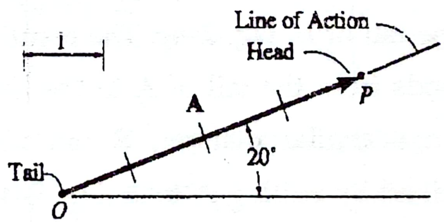
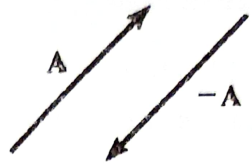


Fig. 2.1

2.2 Vector Operations

Multiplication and Division of a Vector by a Scalar. The product of vector **A** and scalar a, yielding a**A**, is defined as a vector having a magnitude $|aA|$ The *sense* of a**A** is the *same* as **A** provided a is *positive;* it is *opposite* to **A** if a is *negative.* In particular, the negative of a vector is formed by multiplying the vector by the scalar (-1), Fig. 2.2. Division of a vector by a scalar can be defined using the laws of multiplication, since $\mathbf{A}/a = (1/a)\mathbf{A}, a \neq 0$. Graphic examples of these operations are shown in Fig. 2-3.

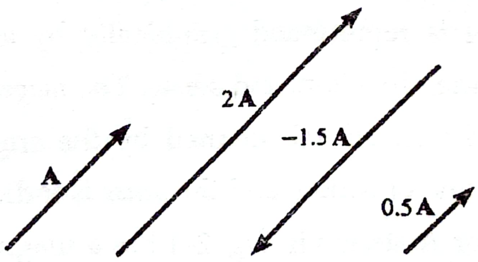


Vector **A** and its negative counterpart

Fig. 2.2

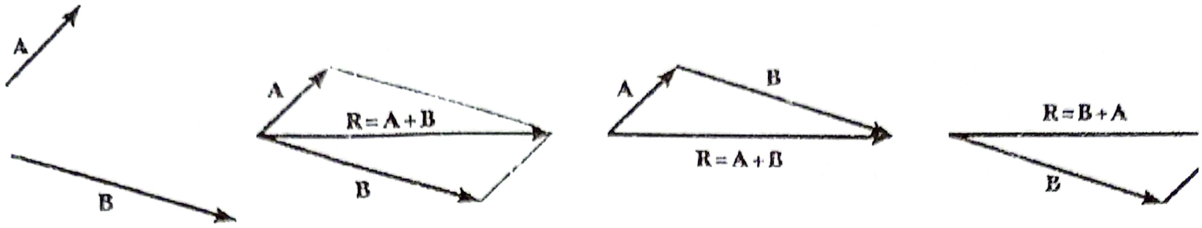
Vector Addition. Two vectors **A** and **B** such as force or position, Fig. 2-4*a*, may be added to form a "resultant" vector **R** = **A** + **B** by using the *parallelogram law.* To do this, **A** and **B** are joined at their tails, Fig. 2-4*b*. Parallel lines drawn from the head of each vector intersect at a common point, thereby forming the adjacent sides of a parallelogram. As shown, the resultant **R** is the diagonal of the parallelogram, which extends from the tails of **A** and **B** to the intersection of the lines.

We can also add **B** to **A** using a *triangle construction,* which is a special case of the parallelogram law, whereby vector **B** is added to vector **A** in a "head-to-tail" fashion, i.e., by connecting the head of **A** to the tail of **B**, Fig. 2-4*c*. The resultant **R** extends from the tail of **A** to the head of **B**. In a similar manner, **R** can also be obtained by adding **A** to **B**, Fig. 2-4*d*. By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either order, i.e., **R** = **A** + **B** = **B** + **A**.



Scalar Multiplication and Division

Fig. 2.3



(a)

Parallelogram Law
(b)

Triangle construction
(c)

Triangle constructio
(d)

Fig. 2.4

As a special case, if the two vectors A and B are *collinear*, i.e., both have the same line of action, the parallelogram law reduces to an *algebraic* or *scalar addition* **R** = A + **B** as shown in Fig. 2–5.

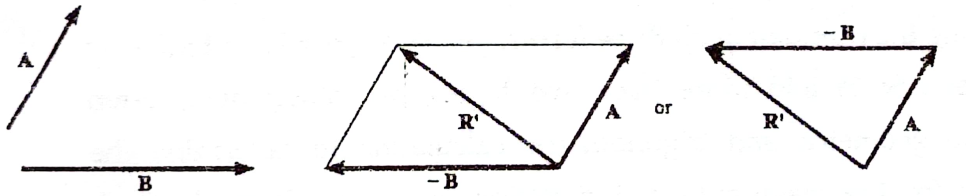
Addition of collinear vectors

Fig. 2.5

Vector Subtraction. The resultant *difference* between two vectors **A** and **B** of the same type may be expressed as

$$\mathbf{R'} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

This vector sum is shown graphically in Fig. 2–6. Subtraction is therefore defined as a special case of addition, so the rules of vector addition also apply to vector subtraction.



Parallelogram law

Triangle construction

Vector Subtraction

Fig. 2.6

Resolution of Vector. A vector may be resolved into two "components" having known lines of action by using the parallelogram law. For example, if **R** in Fig. 2–7*a* is to be resolved into components acting along the lines *a* and *b*, one starts at the *head* of **R** and extends a line *parallel* to *a* until it intersects *b*. Likewise, a line parallel to *b* is drawn from the *head* of **R** to the point of intersection with *a*, Fig. 2–7*a*.The two components **A** and **B** are then drawn such that they extend from the tail of **R** to the points of intersection, as shown in Fig. 2–7*b*.

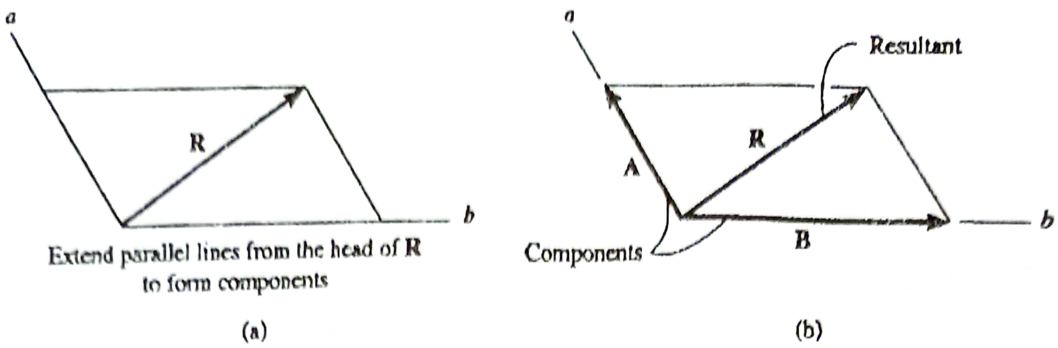


Fig. 2.7: Resolution of a vector

2.3 Vector Addition of Forces

Experimental evidence has shown that a force is a vector quantity since it has a specified magnitude, direction, and sense and it adds according to the parallelogram law. Two common problems in statics involve either finding the resultant force, knowing its components, or resolving a known force into two components. As described in Sec. 2.2, both of these problems require application of the parallelogram law.

If more than two forces are to be added, successive applications of the parallelogram law can be carried out in order to obtain the resultant force. For example, if three forces $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{F_3}$ act at a point O, Fig. 2-8, the resultant of any two of the forces is found - say, $\mathbf{F_1} + \mathbf{F_2}$ and then this resultant is added to the third force, yielding the resultant of all three forces; i.e., $\mathbf{F_R} = (\mathbf{F_1} + \mathbf{F_2}) + \mathbf{F_3}$. Using the parallelogram law to add more than two forces, as shown here, often requires extensive geometric and trigonometric calculation to determine the numerical values for the magnitude and direction of the resultant. Instead, problems of this type are easily solved by using the "rectangular-component method," which is explained later.

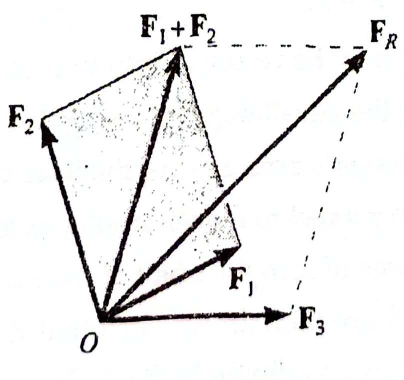


Fig. 2.8

PROCEDURE FOR ANALYSIS

Problems that involve the addition of two forces can be solved as follows:

Parallelogram Law.

• Make a sketch showing the vector addition using the parallelogram law.

• Two "component" forces add according to the parallelogram law, yielding a *resultant* force that forms the diagonal of the parallelogram.

• If a force is to be resolved into *components* along two axes directed from the tail of the force, then start at the head of the force and construct lines parallel to the axes, thereby forming the parallelogram. The sides of the parallelogram represent the components.

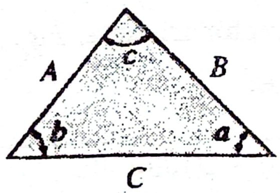
• Label all the known and unknown force magnitudes and the angles on the sketch and identify the two unknowns.

Trigonometry.

• Redraw a half portion of the parallelogram to illustrate the triangular head-to-tail addition of the components.

• The magnitude of the resultant force can be determined from the law of cosines, and its direction is determined from the law of sines, Fig. 2-9.

• The magnitude of two force components are determined from the law of sines, Fig. 2-9.



Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Fig. 2.9

E X A M P L E 2.1

The "eyebolt" in Fig. 2-10a is subjected to two forces F_1 and F_2 Determine the magnitude and direction of the resultant force acting on the "eyebolt."

Solution:

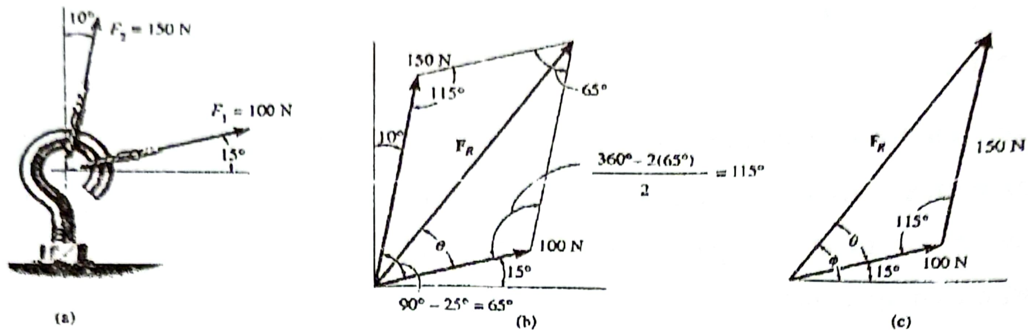


Fig. 2.10

Graphical Procedure. Choosing an arbitrary scale of 50 N = ½in., the force parallelogram is constructed as shown in Fig. 2-25 b. Using a scale and protractor, we can measure the force resultant F_R directly from this drawing. It is found that

F_R= 213 Ans.

θ = 55^0 Ans.

Parallelogram Law. The parallelogram law of addition is shown in Fig. 2–10b. The two unknowns are the magnitude of F_R and the angle θ (theta)

Trigonometry. From Fig. 2–10*b*, the vector triangle, Fig. 2–10*c*, is constructed. is determined by using the law of cosines:

$$F_R = \sqrt{(100)^2 + (150)^2 - 2(100)150\cos 115^\circ}$$
$$= \sqrt{10{,}000 + 22{,}500 - 30{,}000(-0.423)}$$
$$= 212.6$$

Applying the law of sines, using the computed value of F_R, we find the angle θ:

$$\frac{150}{\sin\theta} = \frac{212.6}{\sin 115^\circ}$$
$$\sin\theta = \frac{150}{212.6}(0.906) = 0.639$$
$$\theta = 39.8$$

Thus,

α = 39.8° + 15.0° = 54.8° **Ans.**

EXAMPLE 2.2

The force F acting on the frame shown in Fig. 2-11a has a magnitude of 500 N and is to be resolved into two components acting along members AB and AC. Determine the angle θ measured below the horizontal, so that the component F_{AC} is directed from A toward C and has a magnitude of 400 N.

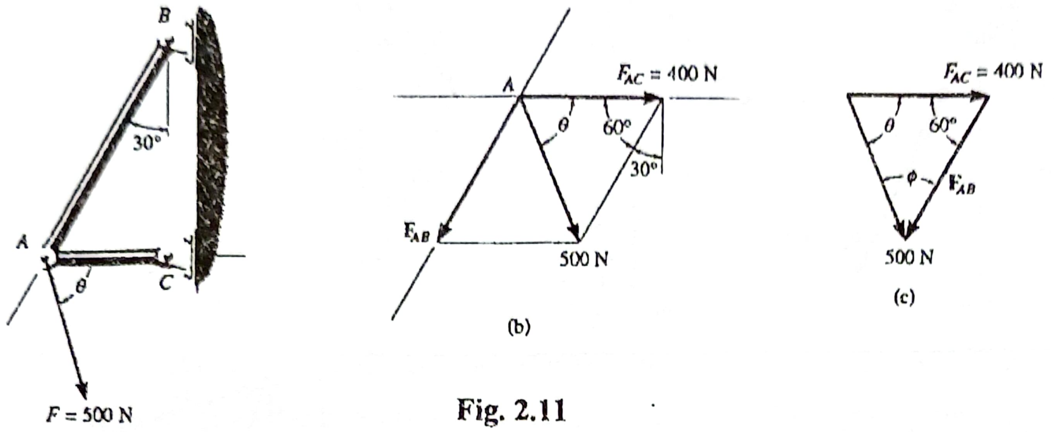


Fig. 2.11

Solution

By using the parallelogram law, the vector addition of the two components yielding the resultant is shown in Fig. 2-11*b*. Note carefully how the resultant force is resolved into the two components F_{AB} and F_{AC} which have specified lines of action. The corresponding vector triangle is shown in Fig. 2-11*c*.

The angle φ can be determined by using the law of sines:

$$\frac{400\text{ N}}{\sin\phi} = \frac{500\text{ N}}{\sin 60^\circ}$$

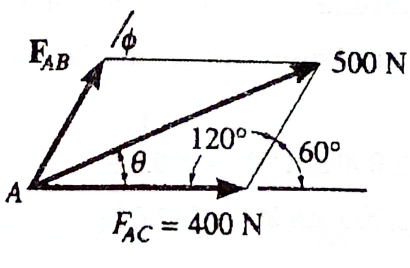
$$\sin\phi = \left(\frac{400\text{ N}}{500\text{ N}}\right)\sin 60^\circ = 0.6928$$

$$\phi = 43.9^\circ$$

Hence $\theta = 180^\circ - 60^\circ - 43.9^\circ = 76.1^\circ$ **Ans.**

Using this value θ for apply the law of cosines or the law of sines and show that F_{AB} has a magnitude of 561 N.

Notice that F can also be directed at an angle θ above the horizontal, as shown in Fig. 2-11d, and still produce the required component F_{AC}. Show that in this case $\theta = 16.1^\circ$ and $F_{AB} = 161$ N.



(d)

EXAMPLE 2.3

The ring shown in Fig. 2-12*a* is subjected to two forces, F_1 and F_2. If it is required that the resultant force have a magnitude of 1 kN and be directed vertically downward, determine (a) the magnitudes of F_1 and F_2 provided $\theta = 30°$, and (b) the magnitudes of F_1 and F_2 if F_2 is to be a minimum.

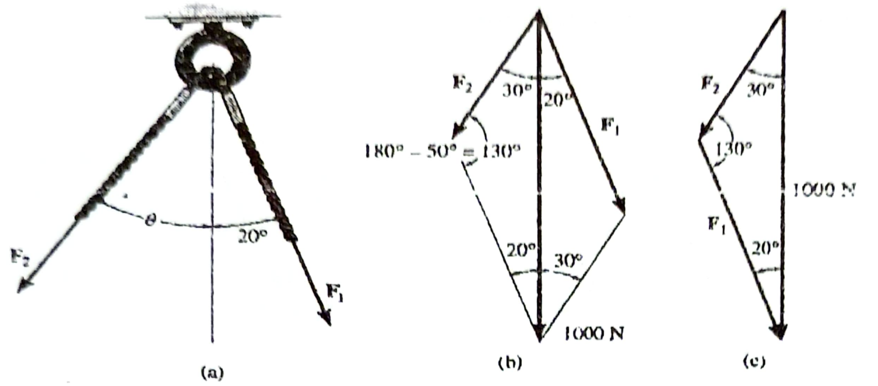
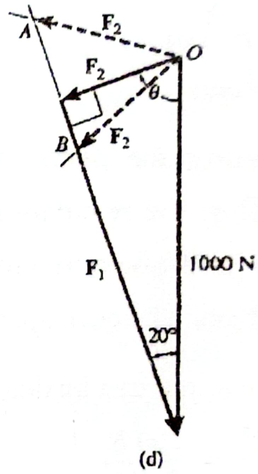


Fig. 2.11

Solution:

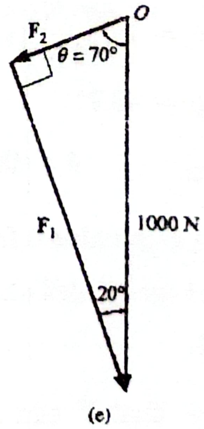
Part (a). A sketch of the vector addition, using the parallelogram law, is given in Fig.2.12b. The unknown magnitudes F_1 and F_2 can be found using the law of sines; namely,



$$\frac{F_1}{\sin 30^\circ} = \frac{1{,}000\text{N}}{\sin 130^\circ}, \qquad F_1 = 652.7\text{ N} \qquad \text{Ans.}$$

$$\frac{F_2}{\sin 20^\circ} = \frac{1{,}000\text{N}}{\sin 130^\circ}, \qquad F_2 = 446.5\text{ N} \qquad \text{Ans.}$$

Part (b). As shown in Fig. 2.12d using the triangle law, vector F_2 may be added to F in various ways to yield the net resultant F_R. In particular, the minimum length or magnitude of F_2 will occur when the line of action of F_2 is perpendicular to F_1. Any other direction, such as OA or OB, yields a larger value for the magnitude of F_2. Hence, when $\theta = 90° - 20° = 70°$, the value of F_2 is minimum. From the triangle shown in Fig. 2-12e, it is seen that



$F_1 = 1{,}000 \sin 70° = 939.7$ N **Ans.**

$F_2 = 1{,}000 \sin 20° = 342.0$ N **Ans.**

EXAMPLE 2.4

The gusset plate in Fig. 2.13a is subjected to four forces which are concurrent at point O. Determine the resultant of these forces using the polygon method of vector addition.

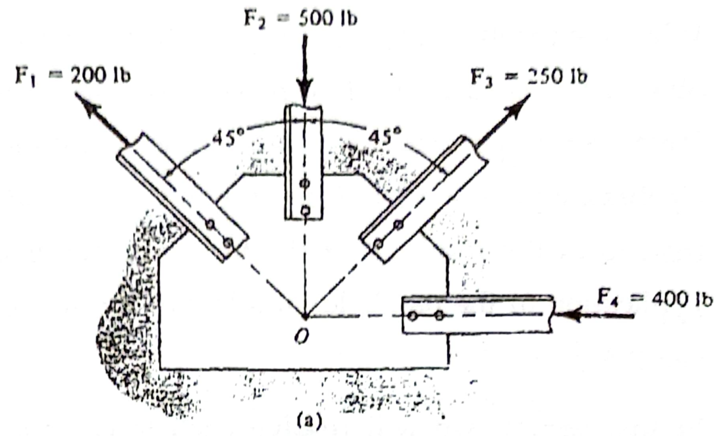
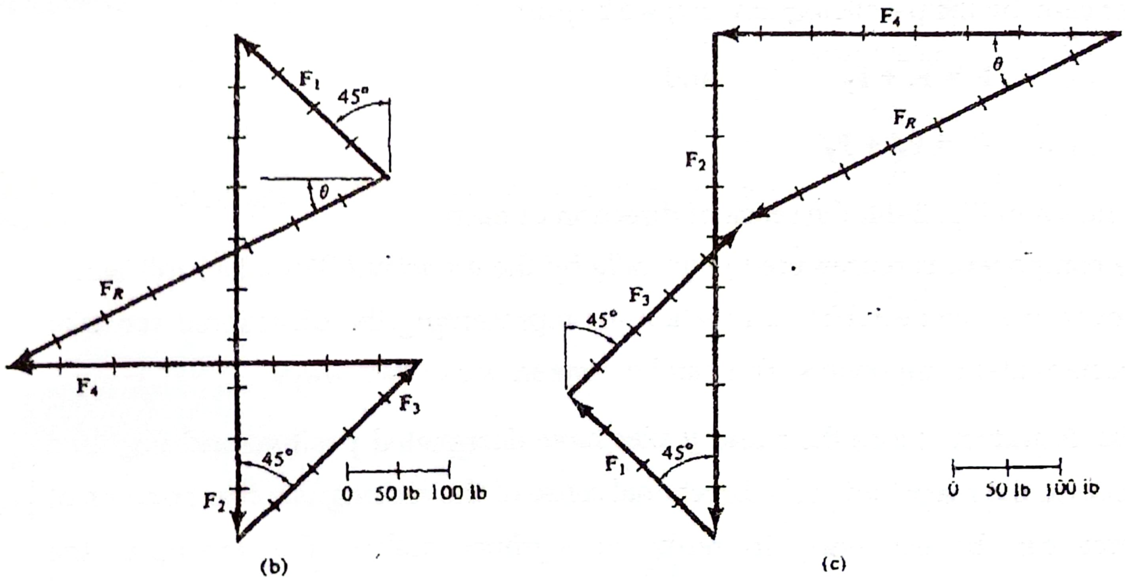


Fig. 2.13

Solution:

Choosing an arbitrary scale of 50 lb = in. and starting with F_1, we add all the vectors together in a "tip-to-tail" fashion, as shown in Fig. 2.13b. The resultant force F_R is then drawn from the initial point of F_1 to the head of F_4 Direct measurements from the figure give



F_R = 408 lb Ans.

θ = 26° Ans.

Since vector addition is commutative, the same results are obtained from the construction shown in Fig. 2.13c. In this case $\mathbf{F_R = F_4+F_2+F_1+F_3}$.

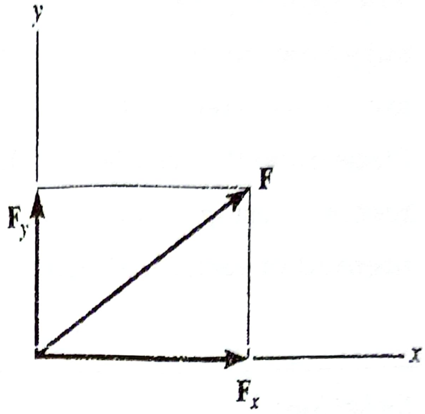
2.4 Addition of a System of Coplanar Forces

When the resultant of more than two forces has to be obtained, it is easier to find the components of each force along specified axes, add these components algebraically, and then form the resultant, rather than form the resultant of the forces by successive application of the parallelogram law as discussed in Sec. 2.3 .

In this section we will resolve each force into its rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$ which lie along the x and y axes, respectively, Fig. 2–14*a*. Although the axes are horizontal and vertical, they may in general be directed at any inclination, as long as they remain perpendicular to one another, Fig. 2–14*b*. In either case, by the parallelogram law, we require

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y \quad \text{and}$$

$$\mathbf{F}' = \mathbf{F}_x' + \mathbf{F}_y'$$



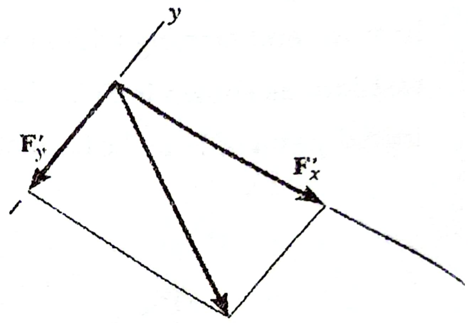


Fig. 2.14

As shown in Fig. 2–14, the sense of direction of each force component is represented *graphically* by the *arrowhead*. For *analytical* work, however, we must establish a notation for representing the directional sense of the rectangular components. This can be done in one of two ways.

Scalar Notation. Since the x and y axes have designated positive and negative directions, the magnitude and directional sense of the rectangular components of a force can be expressed in terms of *algebraic scalars*. For example, the components of **F** in Fig. 2–14*a* can be represented by positive scalars F_x and F_y since their sense of direction is along the *positive* x and y axes, respectively. In a similar manner, the components of **F'** in Fig. 2–14*b* are F_x' and $- F_y'$ Here the y component is negative, since F_y' is directed along the negative y axis.

It is important to keep in mind that this scalar notation is to be used only for computational purposes, not for graphical representations in figures. Throughout the book, the *head of a vector arrow* in any figure indicates the sense of the vector *graphically*; algebraic signs are not used for this purpose. Thus, the vectors in Figs. 2–14*a* and 2–14*b* are designated by using boldface (vector) notation. Whenever italic symbols are written near vector arrows in figures, they indicate the *magnitude* of the vector, which is *always* a *positive* quantity.

Cartesian Vector Notation. It is also possible to represent the components of a force in terms of Cartesian unit vectors. When we do this the methods of vector algebra are easier to apply, and we will see that this becomes particularly advantageous for solving problems in three dimensions.

In two dimensions the *Cartesian unit vectors* **i** and **j** are used to designate the *directions* of the x and y axes, respectively, Fig. 2–15*a*. These vectors have a dimensionless magnitude of unity, and their sense (or arrowhead) will be described analytically by a plus or minus sign, depending on whether they are pointing along the positive or negative x or y axis.

As shown in Fig. 2–15*a*, the *magnitude* of each component of **F** is *always a positive quantity*, which is represented by the (positive) scalars F_x and F_y Therefore, having established notation to represent the magnitude and the direction of each vector component, we can express **F** in Fig. 2–15*a* as the *Cartesian vector*,

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

And in the same way, **F′** in Fig. 2–15*b* can be expressed as

$$\mathbf{F} = F_x'\mathbf{i} + F_y'(-\mathbf{j})$$

Or simply $$\mathbf{F} = F_x'\mathbf{i} - F_y'\,\mathbf{j}$$

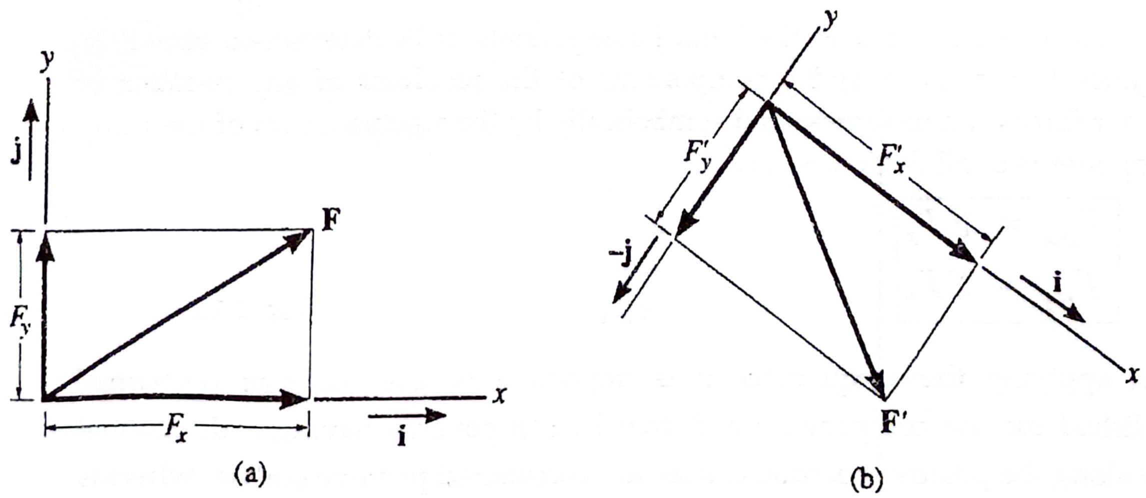
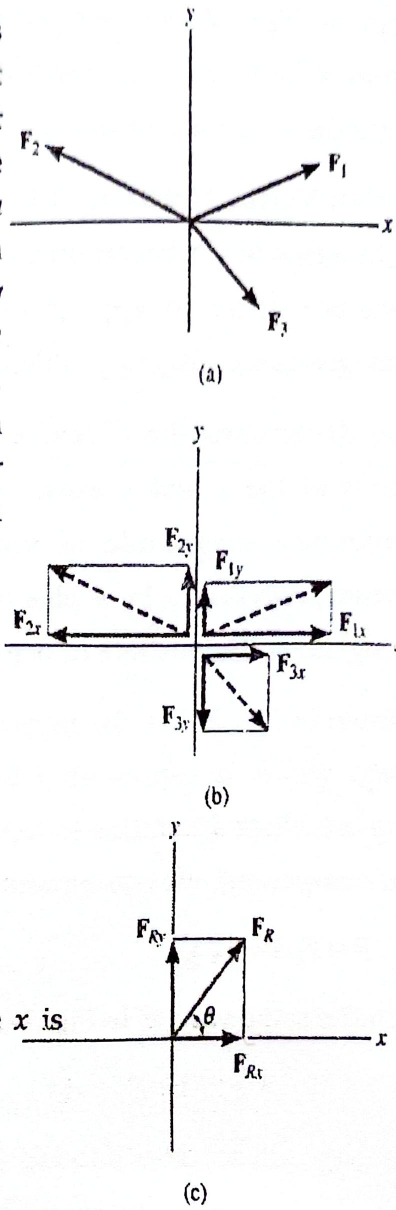


Fig. 2.15

Coplanar Force Resultants. Either of the two methods just described can be used to determine the resultant of several *coplanar forces*. To do this, each force is first resolved into its x and y components, and then the respective components are added using *scalar algebra* since they are collinear. The resultant force is then formed by adding the resultants of the x and y components using the parallelogram law. For example, consider the three concurrent forces in Fig. 2-16*a*, which have x and y components as shown in Fig. 2-16*b*. To solve this problem using *Cartesian vector notation*, each force is first represented as a Cartesian vector, i.e.,

$$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\mathbf{i} + F_{2y}\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$$



The vector resultant is therefore

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

If *scalar notation* is used, then, from Fig. 2-16*b*, since x is positive to the right and y is positive upward, we have

$$F_{Rx} = F_{1x} - F_{2x} + F_{3x}$$
$$F_{Ry} = F_{1y} + F_{2y} - F_{3y}$$

These results are the *same* as the i and j components of $\mathbf{F_R}$ determined above. In the general case, the x and y components of the resultant of any number of coplanar forces can be represented symbolically by the algebraic sum of the x and y components of all the forces, i.e.,

$$\boxed{\begin{aligned}F_{Rx} &= \Sigma F_x \\ F_{Ry} &= \Sigma F_y\end{aligned}} \qquad (2\text{-}1)$$

Fig. 2.16

When applying these equations, it is important to use the *sign convention* established for the components; and that is, components having a directional sense along the positive coordinate axes are considered positive scalars, whereas those having a directional sense along the negative coordinate axes are considered negative scalars. If this convention is followed, then the signs of the resultant components will specify the sense of these components. For example, a

EXAMPLE 2.5

A man pulls with a force of 300 N on a rope attached to a building, as shown in Fig. 2.17a. What are the horizontal and vertical components of the force exerted by the rope at point A?

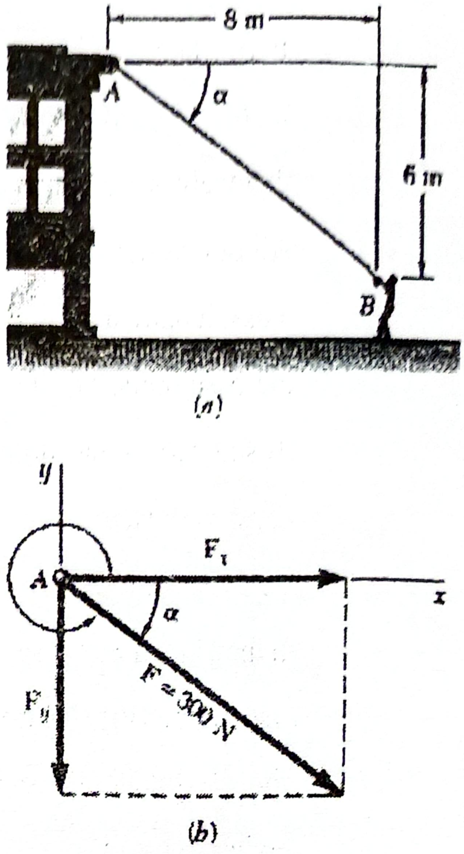


Fig. 2.17

Solution: It is seen from Fig. 2.23b that

$F_x = +(300\text{ N})\cos\alpha$ $\qquad$ $F_y = -(300\text{ N})\sin\alpha$

Observing that AB = 10 m, we find from Fig. 2.17a

$$\cos\alpha = \frac{8m}{AB} = \frac{8m}{10m} = \frac{4}{5}$$

$$\sin\alpha = \frac{6m}{AB} = \frac{6m}{10m} = \frac{3}{5}$$

We thus obtain

$$F_x = +(300N)^{\frac{4}{5}} = +240\text{ N} \quad , F_y = -(300N)^{\frac{3}{5}} = -180\text{ N}$$

and write $\qquad$ $\mathbf{F} = (240\text{N})\mathbf{i} - (180\text{N})\mathbf{j}$

When a force F is defined by its rectangular components F_x and F_y (see Fig. 2.17), the angle θ defining its direction can be obtained by writing

$$\tan\theta = \frac{F_y}{F_x}$$

The magnitude F of the force may be obtained by applying the Pythagorean theorem and writing

$$F = \sqrt{F_x^2 + F_y^2}$$

However, once θ has been found, it is usually easier to determine the magnitude of the force by the process of solving one of the formulas (2.8) for F.

EXAMPLE 2.6

A force F = (700 lb)i + (1500 lb)j is applied to a bolt A. Determine the magnitude of the force and the angle θ it forms with the horizontal.

Solution:

First we draw a diagram showing the two rectangular components of the force and the angle θ (Fig. 2.18). From Eq. (2.9), we write

$$\tan\theta = \frac{F_y}{F_x} = \frac{1500\text{lb}}{700\text{lb}}$$

Using a calculator, we enter 1500 lb and divide by 700 lb; computing the arc tangent of the quotient, we obtain θ = 65.0°. Solving the second of Eqs. (2.8) for F, we have

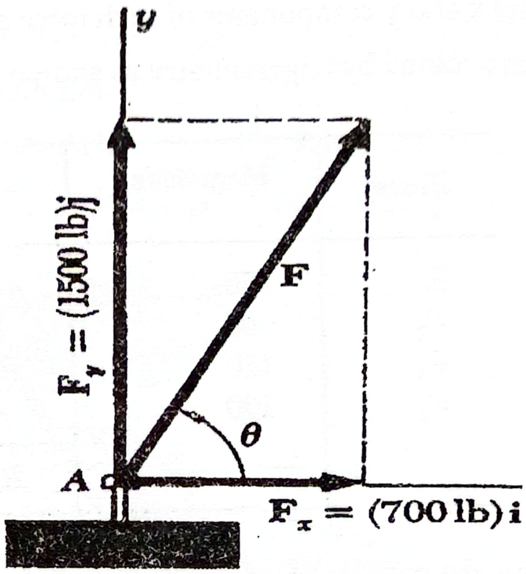


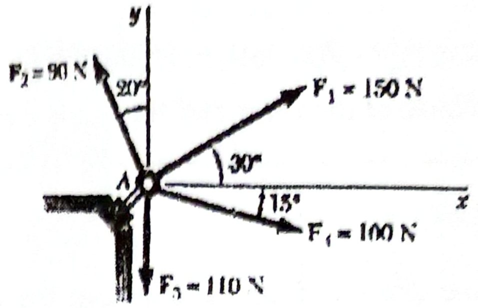
Fig. 2.18

$$F = \frac{F_y}{\sin\theta} = \frac{1500\text{ lb}}{\sin 65.0°} = 1655\text{ lb}$$

The last calculation is facilitated if the value of F is stored when originally entered; it may then be recalled to be divided by sin θ.

EXAMPLE 2.7

Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.



Solution:

The x and y components of each force are determined by trigonometry as shown in the following table

Force	Magnitude, N	x Component, N	y Component, N
F_1	150	+129.9	+75.0
F_2	80	−27.4	+75.2
F_3	110	0	−110.0
F_4	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$

thus the resultant R of the four forces is

$$R = R_x i + R_x j$$

$$R = (199.1\ N)i + (14.3\ N)j$$

The magnitude and direction of the resultant may now be determined from the triangle shown, we have

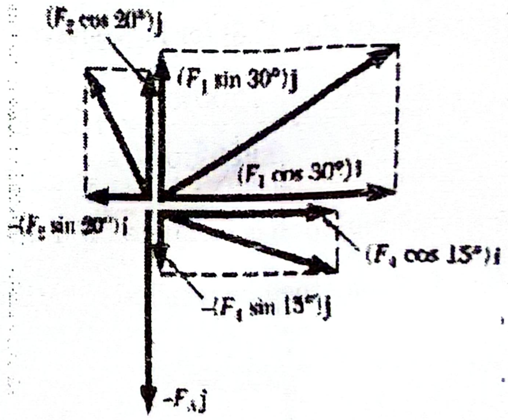


Fig. 2.19

$$\tan\alpha = \frac{R_y}{R_x} = \frac{14.3N}{199.1N} \qquad \alpha = 4.1°$$

$$R = \frac{14.3N}{\sin\alpha} = 199.6N \qquad R = 199.6N\angle 4.1°$$

With a calculator, the last computation may be facilitated if the value of R is stored when originally entered; it may then be recalled to be divided by sin α.

2.5 Equilibrium of a Particle.

In the preceding sections, we discussed the methods for determining the resultant of several forces acting on a particle. Although this has not occurred in any of the problems considered so far, it is quite possible for the resultant to be zero. In such a case, the net effect of the given forces is zero, and the particle is said to be in equilibrium. We thus have the following definition: When the resultant of all the forces acting on a particle is zero, the particle is in equilibrium.

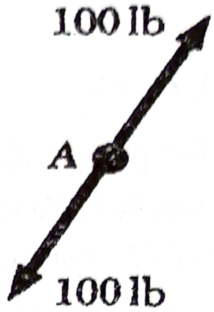


Fig. 2.20

A particle which is acted upon by two forces will be in equilibrium if the two forces have the same magnitude, same line of action, and opposite sense. The resultant of the two forces is then zero. Such a case is shown in Fig. 2.20.

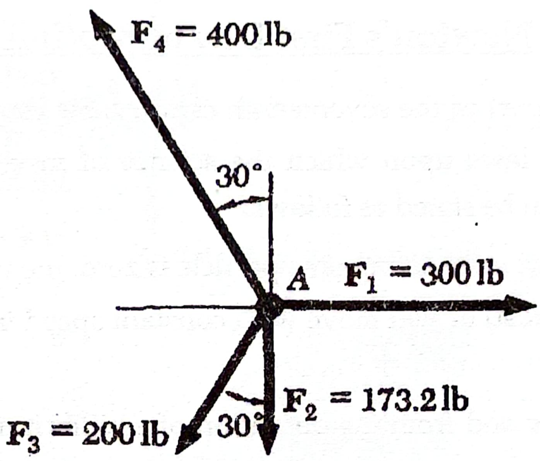


Fig. 2.21

Another case of equilibrium of a particle is represented in Fig. 2.21, where four forces are shown acting on A. In Fig. 2.33, the resultant of the given forces is determined by the polygon rule. Starting from point 0 with F_1 and arranging the forces in tip-to-tail fashion, we find that the tip of F_4 coincides with the starting point 0. Thus the resultant R of the given system of forces is zero, and the particle is in equilibrium.

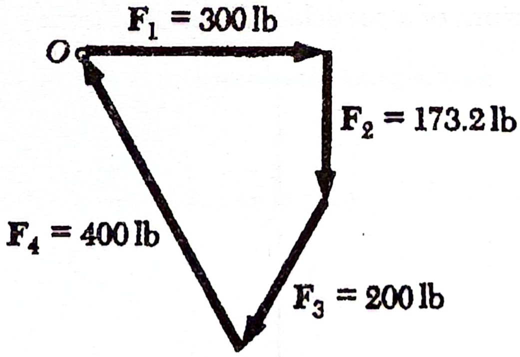


Fig. 2.22

The closed polygon drawn in Fig. 2.22 provides a graphical expression of the equilibrium of A. To express algebraically the conditions for the equilibrium of a particle, we write

$$R = \sum F = 0 \qquad (2.14)$$

Resolving each force F into rectangular components, we have

$$\sum(F_x\mathrm{i}+F_y\mathrm{j})=0 \qquad \text{or} \qquad (\sum F_x)\mathrm{i}+(\sum F_y)\mathrm{j}=0$$

We conclude that the necessary and sufficient conditions for the equilibrium of a particle are

$$\sum F_x=0 \qquad \sum F_y=0 \qquad (2.15)$$

Returning to the particle shown ,in Fig. 2.21 we check that the equilibrium conditions are satisfied. We write

$\sum F_x$ = 300 lb - (200 lb) sin 30° - (400 lb) sin 30°

= 300 lb - 100 lb - 200 lb = 0

$\sum F_y$ = -173.2 lb - (200 lb) cos 30° + (400 lb) cos 30°

= - 173.2 lb - 173.2 lb + 346.4 lb = 0

2.6 Newton's First Law of Motion.

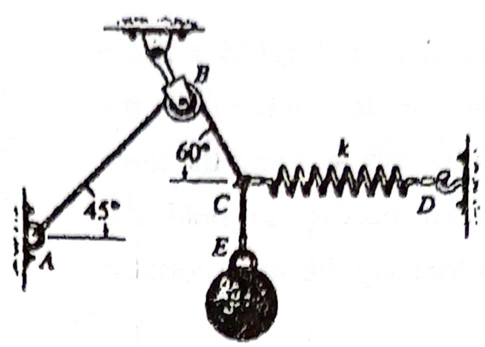
In the latter part of the seventeenth century, Sir Isaac Newton formulated three fundamental laws upon which the science of mechanics is based. The first of these laws can be stated as follows:

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

From this law and from the definition of equilibrium given in Sec. 2.8, it is seen that a particle in equilibrium either is at rest or is moving in a straight line with constant speed. In the following section, various problems concerning the equilibrium of a particle will be considered.

EXAMPLE 2.8

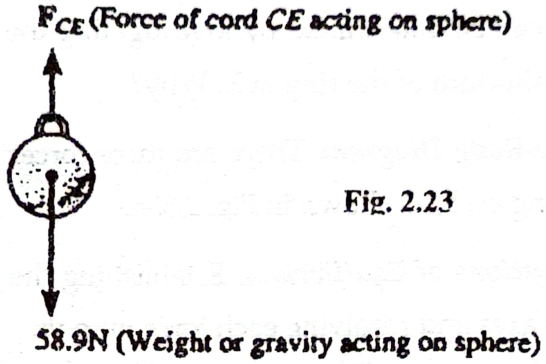
The sphere in Figure 2.23a has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord CE, and the knot at C.



(a)

Solution:

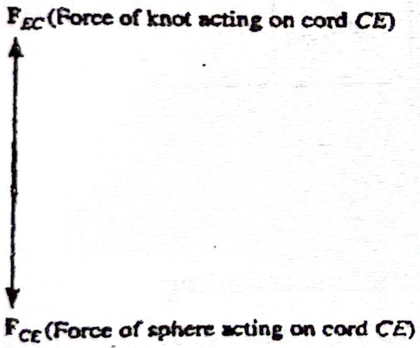
Sphere. By inspection, there are only two forces acting on the sphere, namely, its weight and the force of cord CE. The sphere has a weight of 6 kg (9.81 m/s = 58.9 N. Fig. 2.23b



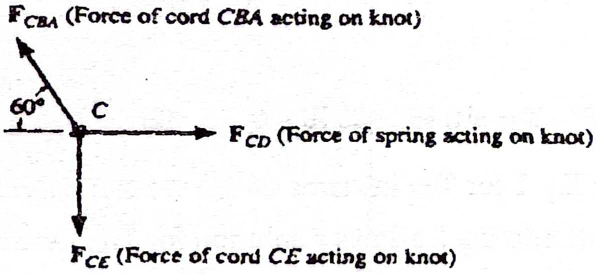
(b)

Fig. 2.23

Cord CE. When the cord CE is isolated from its surroundings, its free- body diagram shows only two forces acting on it, namely, the force of the sphere and the force of the knot, Fig 2.23c. Also, F_{CE} and F_{EC} pull on the cord and keep it in tension so that it doesn't collapse. For



(c)



(d)

Fig. 2.23

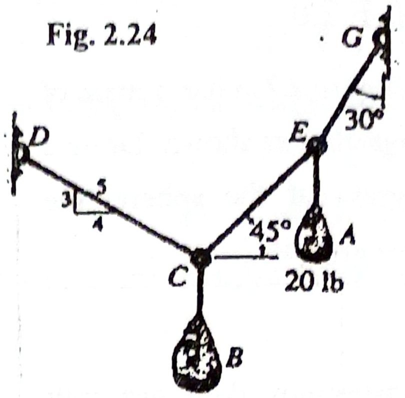
equilibrium, $F_{CE} = F_{EC}$.

Knot. The knot at C is subjected to three forces, Fig.2.23d. They are caused by the cords CBA and CE and the spring CD. As required the free-body diagram shows all these forces labeled with their magnitudes and directions.

EXAMPLE 2.9

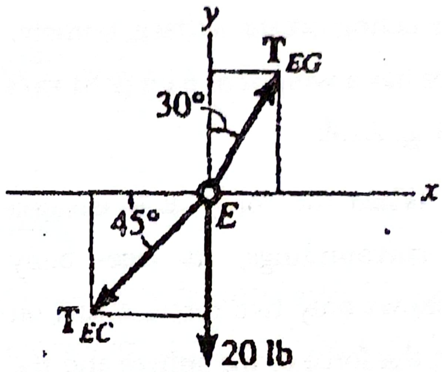
If the sack at A in Fig.2.24 a has a weight of 20 lb, determine the weight of the sack at B and the force in each cord needed to hold the system in the equilibrium position shown.

Fig. 2.24



Solution:

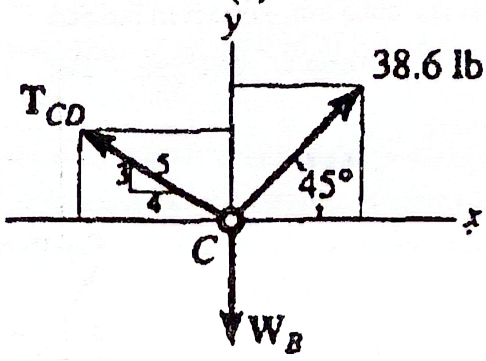
Since the weight of A is known, the unknown tension in the two cords EG and EC can be determined by investigating the equilibrium of the ring at E. Why?



(b)

Free-Body Diagram. There are three forces acting on E, as shown in Fig. 2.35b.

Equations of Equilibrium. Establishing the x, y axes and resolving each force onto its x and y components using trigonometry, we have



$\Sigma F_x = 0;$

$T_{EG} \sin 30° - T_{EC} \cos 45° = 0$ (1)

$\Sigma F_y = 0;$

$T_{EG} \cos 30° - T_{EC} \sin 45° - 20 \text{ lb} = 0$ (2)

Solving Eq. 1 for T_{EG} in terms of T_{EC} we can now investigate and substituting the result into Eq. 2 allows a solution for T_{EC}. One then obtains T_{EG} from Eq. 1. The results are

$T_{EC} = 38.6$ lb Ans.

$T_{EG} = 54.6$ lb Ans.

Using the calculated result for T_{EC}, the equilibrium of the ring at C can now be investigated to determine the tension in CD and the weight of B.

Free-Body Diagram. As shown in Fig.2.24 c, TEC = 38.6 lb "pulls" on C. The reason for this becomes clear when one draws the free-body diagram of cord CE and applies both equilibrium and the principle of action, equal but opposite force reaction (Newton's third law), Fig. 2.24d.

Equations of Equilibrium. Establishing the x, y axes and noting the components of T_{CD} are proportional to the slope of the cord as defined by the 3 - 4 - 5 triangle, we have

$\Sigma F_x = 0;$

$38.6 \cos 45° \text{ lb} - (4/5)\, T_{CD} = 0$ (3)

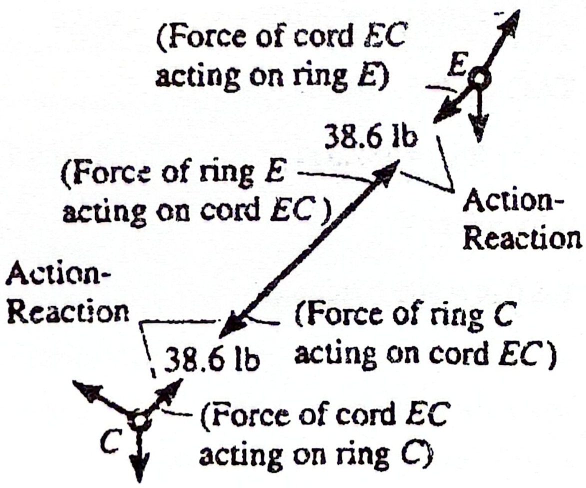
$\Sigma F_y = 0;$

$(3/5)\, T_{CD} + 38.6 \sin 45° \text{ lb} - W_B = 0$ (4)

Solving Eq. 3 and substituting the result into Eq. 4 yields

$T_{CD} = 34.2$ lb Ans.

$W_B = 47.8$ lb Ans.



(d)

Fig. 2.24

E X A M P L E 2.10

Determine the required length of cord AC in Fig. 2.36a so that the 8-kg lamp is suspended in the position shown. The undeformed length of spring AB is l'_{AB} = 0.4 m, and the spring has a stiffness of k_{AB} = 300 N/m.

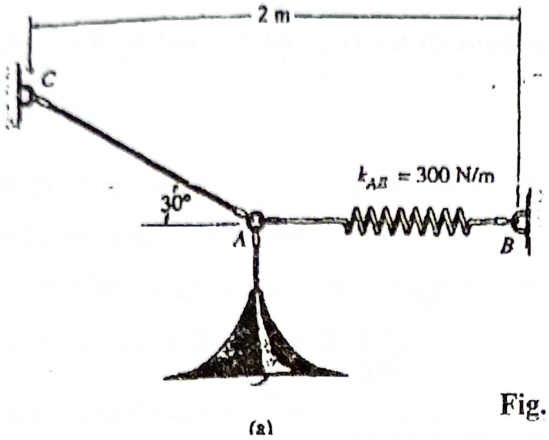
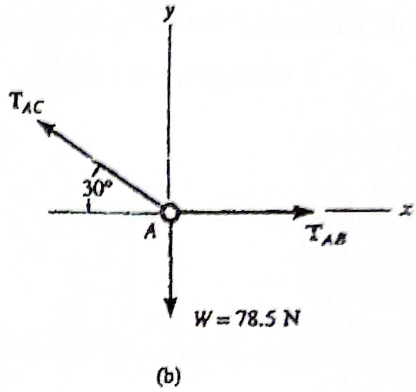


Fig. 2.25



Solution: If the force in spring AB is known, the stretch of the spring can be found using F = ks. From the problem geometry, it is then possible to calculate the required length of AC. The free-body diagram of the ring is as shown.

Equations of Equilibrium. Using the x, y axes,

$\Sigma F_x = 0;$ $\quad T_{AB} - T_{AC} \cos 30° = 0$

$\Sigma F_y = 0;$ $\quad T_{AC} \sin 30° - 78.5\text{ N} = 0$

Solving, we obtain $\quad T_{AC} = 157.0\text{ N} \quad T_{AB} = 136.0\text{ N}$

The stretch of spring AB is therefore

$T_{AB} = k_{AB}s_{AB};$ $\quad 136.0\text{ N} = 300\text{ N/m}\,(s_{AB})$

$s_{AB} = 0.453\text{ m}$

so the stretched length is

$l_{AB} = l'_{AB} + s_{AB}$

$l_{AB} = 0.4\text{ m} + 0.453\text{ m} = 0.853\text{ m}$

The horizontal distance from C to B, Fig. a, requires

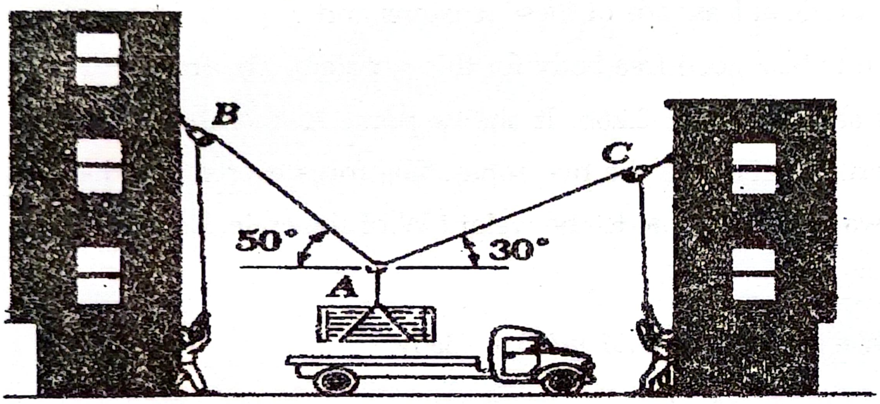
$2\text{ m} = l_{AC} \cos 30° + 0.853\text{ m}$

$l_{AC} = 1.32\text{ m}$ $\quad$ *Ans.*

2.7 Problems Involving the Equilibrium of a Particle. Free-Body Diagram.

In practice, a problem in engineering mechanics is derived from an actual physical situation. A sketch showing the physical conditions of the problem is known as a space diagram.

The methods of analysis discussed in the preceding sections apply to a system of forces acting on a particle. A large number of problems involving actual structures, however, may be reduced to problems concerning the equilibrium of a particle. This is done by choosing a significant particle and drawing a separate diagram showing this particle and all the forces acting on it. Such a diagram is called a free-body diagram.



(a) Space diagram

Fig. 2.26

As an example, consider the 75-kg crate shown in the space diagram of Fig. 2.26a. This crate was lying between two buildings, and it is now being lifted onto a truck, which will remove it. The crate is supported by a vertical cable, which is joined at A to two ropes which pass over pulleys attached to the buildings at B and, C. It is desired to determine the tension in each of the ropes AB and AC.

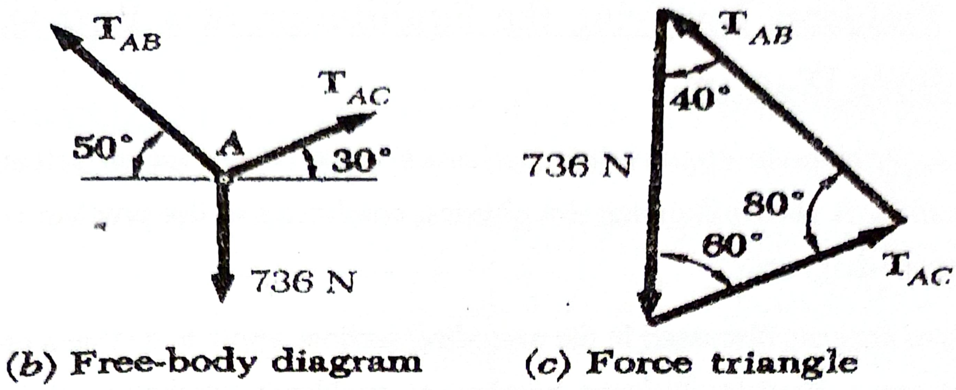


Fig. 2.26

In order to solve this problem, a free-body diagram must be drawn, showing a particle in equilibrium. Since we are interested in the rope tensions, the free-body diagram should include at least one of these tensions and, if possible, both tensions. Point A is seen to be a good free body for this problem. The free-body diagram of point A is shown in Fig. 2.26b. It shows point A and the forces exerted on A by the vertical cable and the two ropes. The force exerted by the cable is directed downward and is equal to the weight W of the crate. Recalling Eq. (1.4), we write

$$W = mg = (75\ kg)(9.81\ m/s^2) = 736\ N$$

and indicate this value in the free-body diagram. The forces exerted by the two ropes are not known. Since they are respectively equal in magnitude to the tension in rope AB and rope AC, we denote them by T_{AB} and T_{AC} and draw them away from A in the directions shown in the space diagram. No other detail is included in the free-body diagram.

Since point A is in equilibrium, the three forces acting on it must form a closed triangle when drawn in tip-to-tail fashion. This force triangle has been drawn in Fig. 2.26c. The values T and T of the tension in the ropes may be found graphically if the triangle is drawn to scale, or they may be found by trigonometry. If the latter method of solution is chosen, we use the law of sines and write

$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{AC}}{\sin 40^\circ} = \frac{736\,N}{\sin 80^\circ}$$

When a particle is in equilibrium under three forces, the problem may always be solved by drawing a force triangle. When a particle is in equilibrium under more than three forces, the problem may be solved graphically by drawing a force polygon.

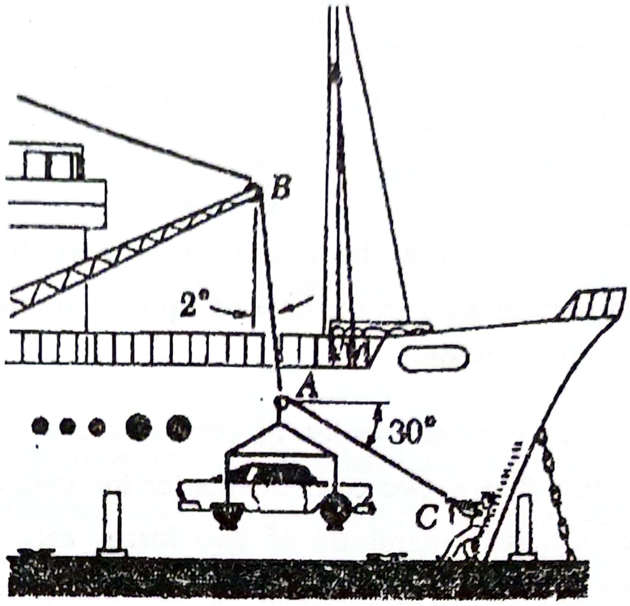
$$\sum F_x = 0 \qquad \sum F_y = 0$$

These equations may be solved for no more than two unknowns; similarly, the force triangle used in the case of equilibrium under three forces may be solved for two unknowns.

The more common types of problems are those where the two unknowns represent (1) the two components (or the magnitude and direction) of a single force, (2) the magnitude of two forces each of known direction. Problems involving the determination of the maximum or minimum value of the magnitude of a force are also encountered.

EXAMPLE 2.11

In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A rope is tied to the cable at A and pulled in order to center the automobile over its intended position. The angle between the cable and the vertical is 2°, while the angle between the rope and the horizontal is 30°. What is the tension in the rope?



Solution.

Point A is chosen as a free body, and the complete free-body diagram is drawn. T_{AB} is the tension in the cable AB, and T_{AC} is the tension in the rope.

Equilibrium Condition. Since only three forces act on the free body, we draw a force triangle to express that it is in equilibrium. Using the law of sines, we write

$$\frac{T_{AB}}{\sin 120^\circ} = \frac{T_{AC}}{\sin 2^\circ} = \frac{3500\ \text{lb}}{\sin 58^\circ}$$

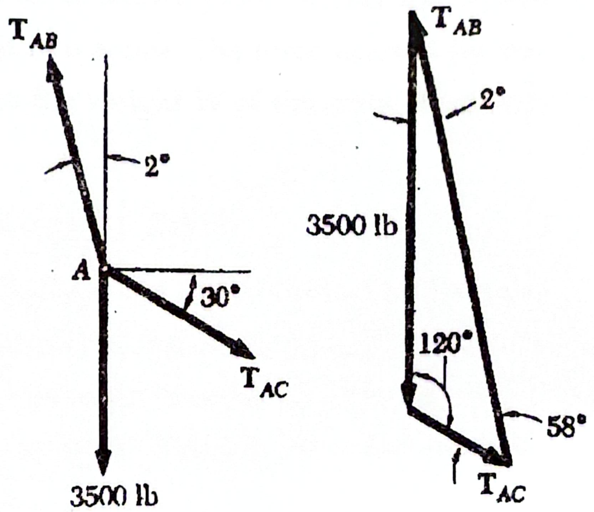


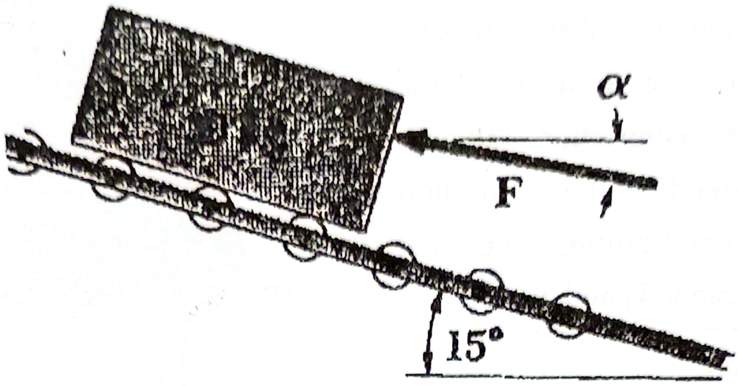
Fig. 2.27

With a calculator, we first compute and store the value of the last quotient. Multiplying this value successively by sin 120° and sin 2°, we obtain

$T_{AB} = 3570$ lb $\qquad T_{AC} = 144$ lb

EXAMPLE 2.12

Determine the magnitude and direction of the smallest force F which will maintain the package shown in equilibrium. Note that the force exerted by the rollers on the package is perpendicular to the incline.



Solution.

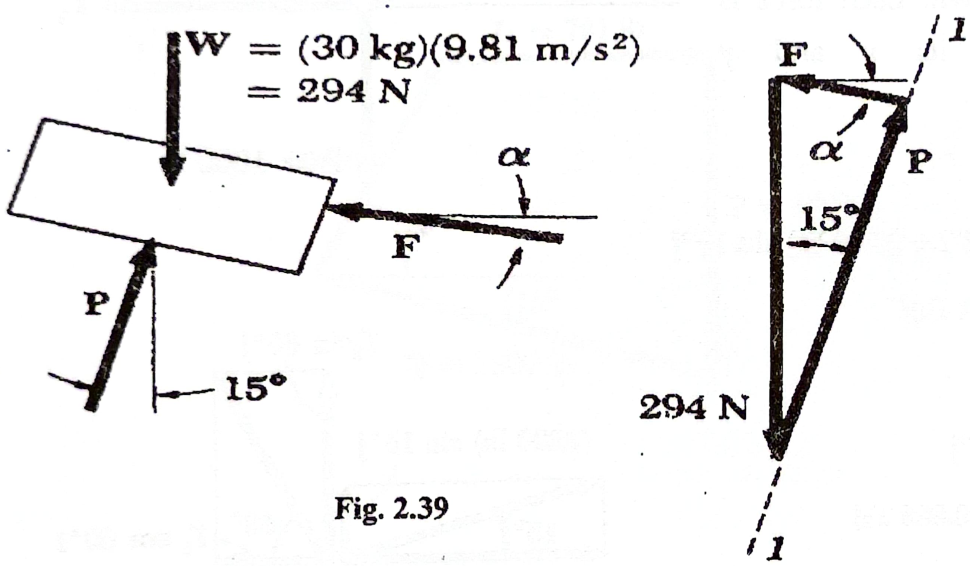


Fig. 2.39

We choose the package as a free body, assuming that it may be treated as a particle. We draw the corresponding free-body diagram.

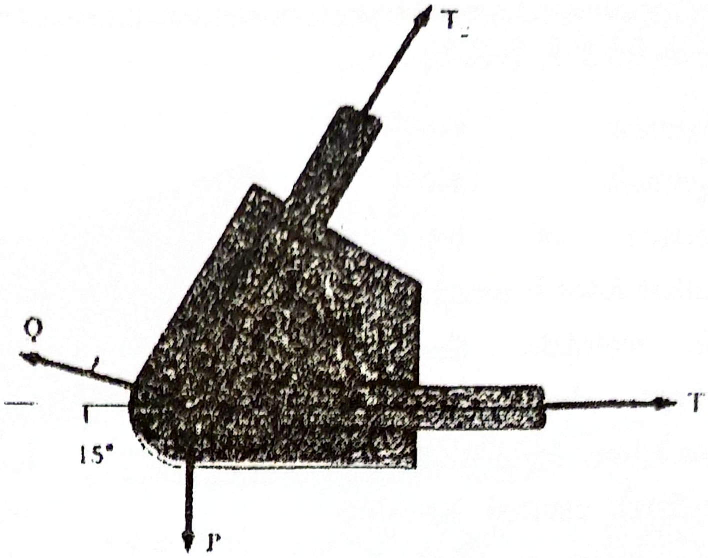
Equilibrium Condition. Since only three forces act on the free body, we draw a force triangle to express that it is in equilibrium. Line 1,1 represents the known direction of P. In order to obtain the mini mum value of the force F, we choose the direction of F perpendicular to that of P. From the geometry of the triangle obtained, we find

F = (294 N) sin 15° = 76.1 N $\alpha = 15°$

F=76.1N. ,15°

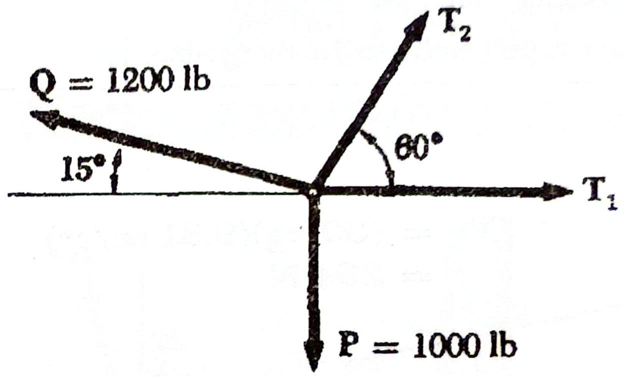
EXAMPLE 2.13

Two forces P and Q of magnitude P = 1000 lb and Q = 1200 lb are applied to the aircraft connection shown. Knowing that the connection is in equilibrium, determine the tensions T_1 and T_2.



Solution.

The connection is considered a particle and taken as a free body. It is acted upon by four forces directed as shown. Each force is resolved into its x and y components.



$$P = -(1000)j$$

$$Q = -(1200)\cos 15°i + (1200\ lb)\sin 15°j$$

$$= -(1159)i + (311\ lb)j$$

$$T_1 = T_1 i$$

$$T_2 = T_2 \cos 60°j$$

$$= 0.500\ T_2 i\ 0.866\ T_2 j$$

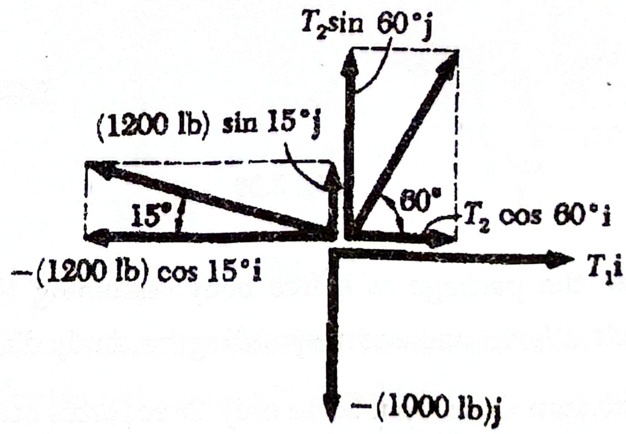


Fig. 2.29

Equilibrium Condition. Since the connection is in equilibrium, the resultant of the forces must be zero. Thus

$$R = P + Q + T_1 + T_2 = 0$$

Substituting for P. Q, T_1 and T_2 the expressions obtained above, and factoring the unit vectors i and j, we have

$$(-1159) + T_1 + 0.500\ T_2) + (-1000 + 311 + 0.866\ T_2)j = 0$$

This equation will be satisfied if, and only if, the coefficients of i and j are equal to zero. We thus obtain the following two equilibrium equations, which express, respectively, that the sum of the x components and the sum of the y components of the given forces must be zero.

$(\sum F_x = 0:)$ $\quad - 1159 + T_1 + 0.500\, T_2 = 0$

$(\sum F_y = 0:)$ $\quad - 1000 + 311 + 0.866\, T_2 = 0$

Solving these equations, we find

$T_1 = 761$ lb $\qquad T_2 = 766$lb

In drawing the free-body diagram, we assumed a sense for each known force. A positive sign in the answer indicates that the assumed sense is correct. The complete force polygon may be drawn to check the results.