Perception in Robotics
Term 3, 2021. L3. Extra task

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This problem consists of a single task worth of 1% in addition to your course grade. The problem is not compulsory: you still can obtain the maximum grade for the course even without doing it.

Submission Instructions

Your assignment must be submitted by 23:59pm on **February 11, Thursday**. You are to upload your assignment directly to Canvas as **one** attachment: a pdf-file typeset in Latex with the full derivation and your **commentaries** that explain the derivation. The solution without any commentary would not be graded.

Task 1: Marginalization of the information form of a Gaussian PDF (1 point)

Apart from the standard parametrization of a multivariate gaussian PDF there exist another parametrization that is called canonical or information parameterization:

$$p(x_a, x_b) = \alpha \exp\left\{ -\frac{1}{2}\left[x-\mu\right]^\intercal \Sigma^{-1}\left[x-\mu\right] \right\}$$

Lets define new variables $\Lambda = \Sigma^{-1}$ and $\eta = \Sigma^{-1}\mu$ then we can rewrite the parametrization above in the following way:

$$p(x_a, x_b) = \alpha \exp\left\{ -\frac{1}{2}\left[x-\Lambda^{-1}\eta\right]^\intercal \Lambda \left[x-\Lambda^{-1}\eta\right] \right\}$$

As you will see by completing the exercise, this parameterization allows to perform certain operations with Gaussians faster. This parameterization will play a major role in the methods that will be covered closer to the end of the course (Information filter, sqrt-SAM).

If we expand the exponent then we will obtain the following:

$$p(x_a, x_b) = \alpha \exp\left\{ -\frac{1}{2}\begin{bmatrix} x_a \\ x_b \end{bmatrix}^\intercal \begin{bmatrix} \Lambda_a & \Lambda_{ab} \\ \Lambda_{ba} & \Lambda_b \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} + \begin{bmatrix} \eta_a \\ \eta_b \end{bmatrix}^\intercal \begin{bmatrix} x_a \\ x_b \end{bmatrix} - \frac{1}{2}\begin{bmatrix} \eta_a \\ \eta_b \end{bmatrix}^\intercal \begin{bmatrix} \Lambda_a & \Lambda_{ab} \\ \Lambda_{ba} & \Lambda_b \end{bmatrix}^{-1} \begin{bmatrix} \eta_a \\ \eta_b \end{bmatrix} \right\}$$

Then we can derive equations for conditioning of the information form:

Since we will search for the solution Δ in the form shown in (1) then we can directly tell that $\Lambda_{a|b} = \Lambda_a$. This is due to the fact that the gathering of **all 2nd order terms** with x_a contains only Λ_a in between them.

$$\begin{aligned}\Delta &= -\frac{1}{2}x_a^\intercal \Lambda_{a|b} x_a + \eta_{a|b}^\intercal x_a - \frac{1}{2}\eta_{a|b}^\intercal \Lambda_{a|b}^{-1}\eta_{a|b} + const \\ &= -\frac{1}{2}x_a^\intercal \Lambda_a x_a + \eta_{a|b}^\intercal x_a - \frac{1}{2}\eta_{a|b}^\intercal \Lambda_a^{-1}\eta_{a|b} + const\end{aligned} \tag{1}$$

By gathering **all 1st order terms** with x_a we get that:

$$\eta_{a|b}^\intercal x_a = -\frac{1}{2}2x_b^\intercal \Lambda_{ba} x_a + \eta_a^\intercal x_a = (\eta_a - \Lambda_{ab}x_b)^\intercal x_a \tag{2}$$

Hence:

$$\eta_{a|b}^c = \eta_a - \Lambda_{ab}x_b$$
$$\Lambda_{a|b}^c = \Lambda_a$$

Find **marginal** $p(x_a)$ PDFs of information parameterization by completing the square in the same manner as was explained in the lecture.

Tip: To marginalize PDF use the fact that $p(x_a) = \int p(x_a, x_b)dx_b = \int p(x_b|x_a)p(x_a)dx_b = p(x_a)\int p(x_b|x_a)dx_b$