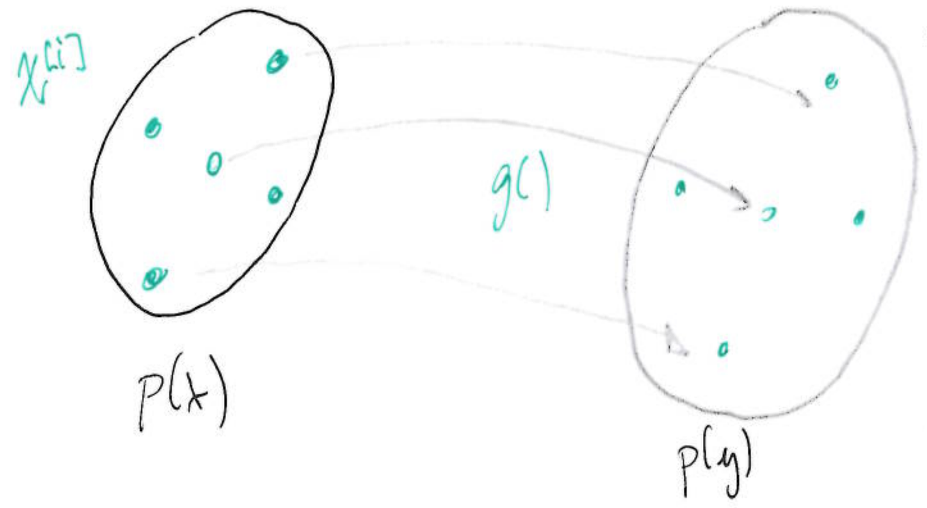
* Unscented transformation



$$y^{[i]} = g(\chi^{[i]}, u_t)$$

We transform a set of sigma points instead of a 1st Taylor exp.

○ Choosing the sigma points

$$\chi^{[0]} = \mu$$

ith column vector

dim (x)

$$\chi^{[i]} = \mu + \left(\sqrt{n+\lambda}\ \sqrt{\Sigma_x}\right)_i \quad , \quad i = 1, \dots, n$$

$$\chi^{[i]} = \mu - \left(\sqrt{n+\lambda} \cdot \sqrt{\Sigma_x}\right)_i \qquad i = n+1, \dots, 2n$$

recall $\quad \Sigma = L \cdot L^T$ (Cholesky) $\Rightarrow \sqrt{\Sigma} = L$

$$\gamma = \sqrt{n+\lambda} = K \qquad \text{(radius)}$$

○ Sigma weights

mean $\quad w_m^{[0]} = \dfrac{\lambda}{n+\lambda}$

covariance $\quad w_c^{[0]} = \dfrac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta)$

Parameter (for reference)

$\beta = 2$
$\alpha \in [0, 1]$
$\lambda = \alpha^2 (n+K) - n$
$K \geq 0$

$$w_m^{[i]} = w_c^{[i]} = \frac{1}{2(n+\lambda)} \qquad \forall i = 1, \dots, 2n$$

7.5

Unscented Kalman Filter

Inputs $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ (ProbRob 70)

I
1: $\mathcal{X}_{t-1} = \{ \mu_{t-1}, \mu_{t-1} + \gamma\sqrt{\Sigma_{t-1}}, \dots \}$ (Build 2n+1 Sigma p.)

2: $\bar{\mathcal{X}}^*_t = g(u_t, \mathcal{X}_{t-1})$

3: $\bar{\mu}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{\mathcal{X}}_t^{*[i]}$

II
4: $\bar{\Sigma}_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{X}}_t^{*[i]} - \bar{\mu}_t)(\bar{\mathcal{X}}_t^{*[i]} - \bar{\mu}_t)^T + R_t$ (pi2 pi)

5: $\bar{\mathcal{X}}_t = \{$ create sigma points from $\mathcal{N}(\bar{\mu}_t, \bar{\Sigma}_t) \}$

6: $\bar{Z}_t = h(\bar{\mathcal{X}}_t)$

7: $\hat{z}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{Z}_t^{[i]}$ $(\hat{z} = H_t \cdot \bar{x}_t)$ on EKF

8: $S_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{Z}_t^{[i]} - \hat{z}_t)(\bar{Z}_t^{[i]} - \hat{z}_t)^T + Q_t$ $(S = H\Sigma H^T + Q)$

9: $\bar{\Sigma}_t^{x,z} = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{X}}_t^{[i]} - \bar{\mu}_t)(\bar{Z}_t^{[i]} - \hat{z}_t)^T$ (Crosscovarian $\bar{\Sigma}_t \cdot H_t$)

III 10: $K_t = \bar{\Sigma}_t^{x,z} \cdot S_t^{-1}$

IV 11: $\mu_t = \bar{\mu}_t + K_t(z_t - \hat{z}_t)$

V 12: $\Sigma_t = \bar{\Sigma}_t - K_t S_t K_t^T$

$$\Sigma_t = \bar{\Sigma}_t - K_t S_t (S_t^{-1})^T (\Sigma_t^{x,z})^T$$
$$= \bar{\Sigma}_t - K_t (\bar{\Sigma}_t H_t)^T$$
$$= \bar{\Sigma}_t - K_t H_t \bar{\Sigma}_t$$

EKF

UKF summary:

- Highly efficient, same complexity as EKF + extra constant.
- Better linearization (Jacobian vs sigma points)
- Derivative free
- Still not optimal

* Gaussian Canonical parametrization

$$\mathcal{N}(\mu,\Sigma) = \eta \cdot \exp\left\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right\}$$

$$= \eta \cdot \exp\left\{-\frac{1}{2} x^T \Sigma^{-1} x^T + x^T \Sigma \mu \underbrace{- \frac{1}{2} \mu^T \Sigma^{-1} \mu}_{\text{cst.}}\right\}$$

$$= \eta' \cdot \exp\left\{-\frac{1}{2} x^T \Sigma^{-1} x + x^T \Sigma^{-1} \mu\right\}$$

$\Lambda = \Sigma^{-1}$ information matrix (L03)

$\xi = \Sigma^{-1} \cdot \mu$ information vector

$$= \eta' \cdot \boxed{\exp\left\{-\frac{1}{2} x^T \Lambda x + x^T \xi\right\} = \mathcal{N}^{-1}(\xi, \Lambda)}$$

Canonical form: simplied and elegant form.
We can do the same derivations to obtain the KF equivalent (Information filter) for linear Gaussians and the Extended IF for linearized Gaussian systems.

* Information Filter

Inputs: $\xi_{t-1}, \Lambda_{t-1}, u_t, z_t$ (Prob Rob 73)

1: $\bar{\Lambda}_t = (A_t \Lambda_{t-1}^{-1} A_t^T + R_t)^{-1}$

2: $\bar{\xi}_t = \bar{\Lambda}_t (A_t \cdot \Lambda_{t-1}^{-1} \xi_{t-1} + B_t \cdot u_t)$

prediction (marginalization)

3: $\Lambda_t = C_t^T Q_t^{-1} C_t + \bar{\Lambda}_t$

4: $\xi_t = C_t^T Q_t^{-1} z_t + \bar{\xi}_t$

correction (conditioning) (easier)

return ξ_t, Λ_t

There exists a duality between marginalizing a Gaussian and conditioning a canonical form, both are easy.

Next lecture: localization Prob Rob Ch 7