Lecture 12. Incremental SAM and Pose SLAM

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1 Incremental square root factorization (iSAM Kaess'2008

As new observations are available, update A, R without recalculating everything.

1.1 QR factorization incrementally

$$A = QR \Rightarrow R = Q^T A$$

New observation:

$$\begin{bmatrix} Q^T & \\ & 1 \end{bmatrix} \cdot \begin{bmatrix} A \\ W^T \end{bmatrix} = \begin{bmatrix} R \\ W^T \end{bmatrix}, \begin{bmatrix} d \\ \gamma \end{bmatrix}_{(\text{new})} \text{(according update the vector } d\text{)}, \; b = \begin{bmatrix} d \\ e \end{bmatrix}$$

$$\begin{array}{ll} \text{Odometry :} & W^T = [\quad \ldots \quad G_i^{i-1} - I \qquad\qquad] \\ \text{Observation :} & W^T = [\quad H^i \quad \ldots \qquad\qquad J^i \quad] \text{ (sparse)} \end{array}$$

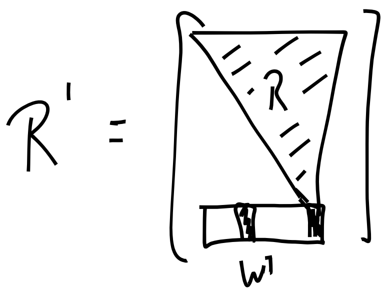

Figure 1: R matrix after adding a new observation.

Objective: make R' upper triangularly again

1.2 Givens rotations

$$\Phi = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \qquad \text{such as} \qquad \Phi \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} r & s \\ 0 & t \end{bmatrix}$$

A sequence of given rotations produce the QR decomposition.

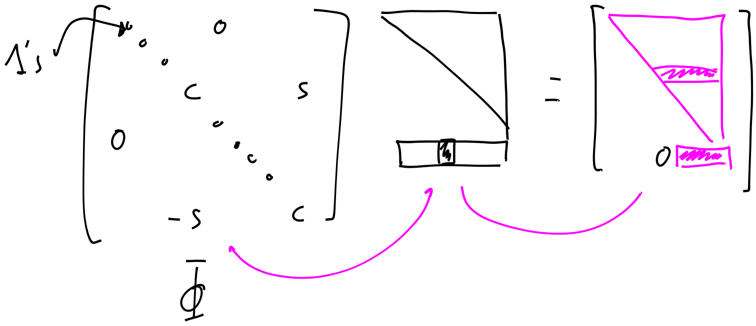

Figure 2: QR decomposition: a single givens rotation adds one zero.

We keep applying below the diagonal until we get an upper triangular matrix.

2 iSAM algorithm

1: New information W^T, update $\begin{bmatrix} R \\ W^T \end{bmatrix}$;

2: Givens rotations until R' upper triangular update d'

3: Solve $R'\delta = d'$

2.1 Discussion

- It is only possible for some time, eventually we need recalculate R
- The ordering is important for next incremental poses. May produce fill-ins → iSAM2 (Kaess 2011) and the Bayes tree graph

3 Data Association in SAM

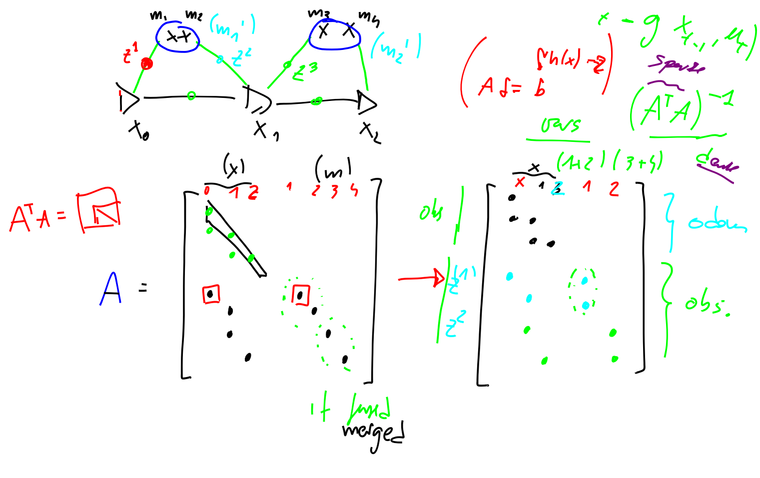

Figure 3: The adjacency matrix allows for manipulation of associations by changing the graph structure.

Grouping landmarks, undoing wrong correspondences etc. implies a change on A (EKF "must" filter correct c_t). Grouping landmarks by a likelihood test + greedy but true table.

$$\Delta_{j,k} = \begin{bmatrix} m_k - m_j \\ m_j - m_k \end{bmatrix} \qquad p(a,b)$$

$$\|\Delta_{j,k}\|^2_{\Sigma_{j,k}} < \chi^2_{d,\alpha} \qquad \begin{pmatrix} d & = 4 \\ \alpha & = \text{confidence interval} \end{pmatrix}$$

Other alternatives might work. Open problem.

Example:

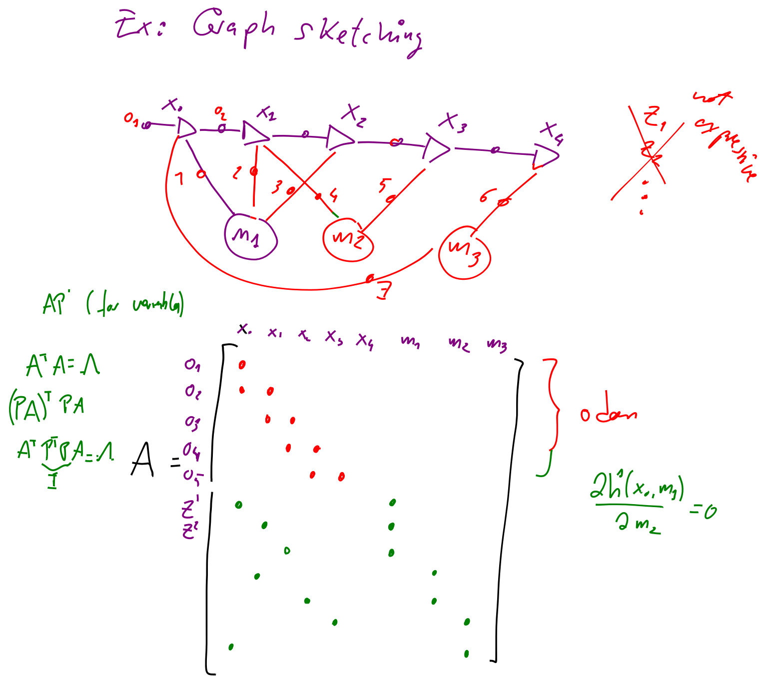

Figure 4: Graph example of the adjacency matrix.

For the data association we need the posterior covariance Σ_t and from there marginalize everything except j, k landmarks in order to calculate Mahalanobis distance or ML (L09).

4 Covariance in $\sqrt{\text{SAM}}$

$\Sigma = \Lambda^{-1}$ (Λ is sparse but inversion is not efficient $\rightarrow$ dense)

Idea: No need to invert Λ, we have R

$$\begin{aligned} \Lambda = A^T A = R^T R \quad &= \Sigma^{-1} \\ &\Downarrow \\ R^T R \Sigma \quad &= I \end{aligned}$$

$$\begin{cases} R^T \cdot Y & = I \\ R \cdot \Sigma & = Y \end{cases} \qquad \text{2 back-substitution, now of a matrix (set of vectors)}$$

4.1 Landmarks elimination

Eliminating m_1 will add more factors to substitute the previous factors to m_1.

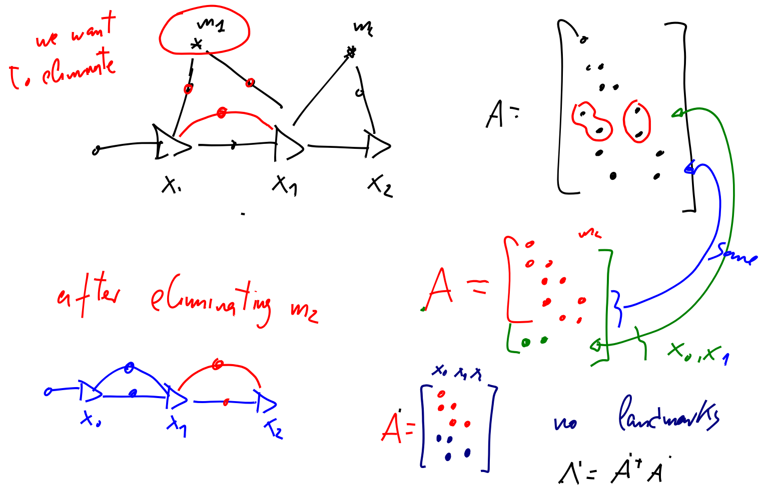

Figure 5: Example of landmark elimination in the graph and adjacency matrix.

All landmarks have been eliminated, but the new factors have appeared to express the equivalent graph.

5 Relation to the Schur complement

$$\Lambda = A^T A = \begin{bmatrix} \Lambda_x & \Lambda_{xm} \\ \Lambda_{mx} & \Lambda_m \end{bmatrix}, \delta = \begin{bmatrix} \delta_x \\ \delta_m \end{bmatrix}, A^T b = \begin{bmatrix} b_x \\ b_m \end{bmatrix}$$

$$(\Lambda_x - \Lambda_{xm}\Lambda_m^{-1}\Lambda_{mx})\delta_x = b_x - \Lambda_{xm}\Lambda_m^{-1}$$

The Schur complement is equivalent to eliminate (marginalize) all landmarks in the information matrix. These new information blocks are the result of the marginalization of landmarks, and they maintain the same relations as in the original problem.

	marginalize	condition
Σ	✓	Schur complement
Λ	Schur complement	✓

6 Pose SLAM

Only poses are estimated.

Example: 2D

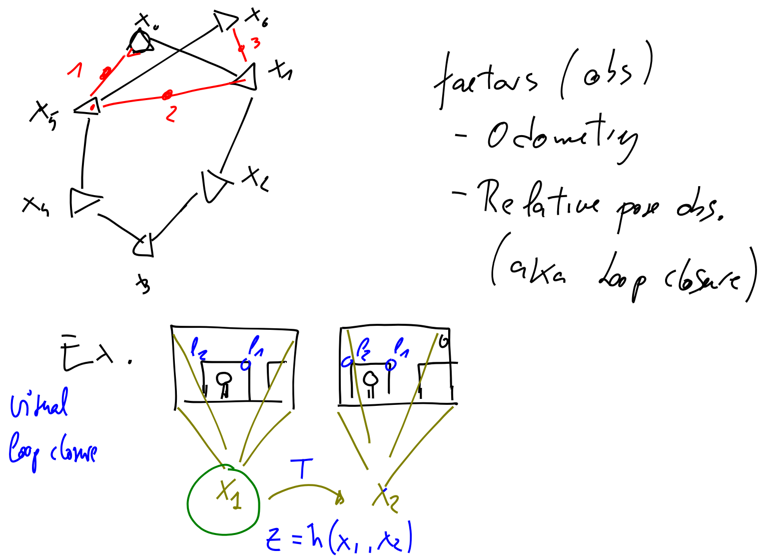

Figure 6: Poses are observed from different poses, for instance, from x_5 we observe the initial pose x_0. In the bottom part there is an example for visual loop closure, relating a pair of poses from visual information.

6.1 Observations in 2D poses

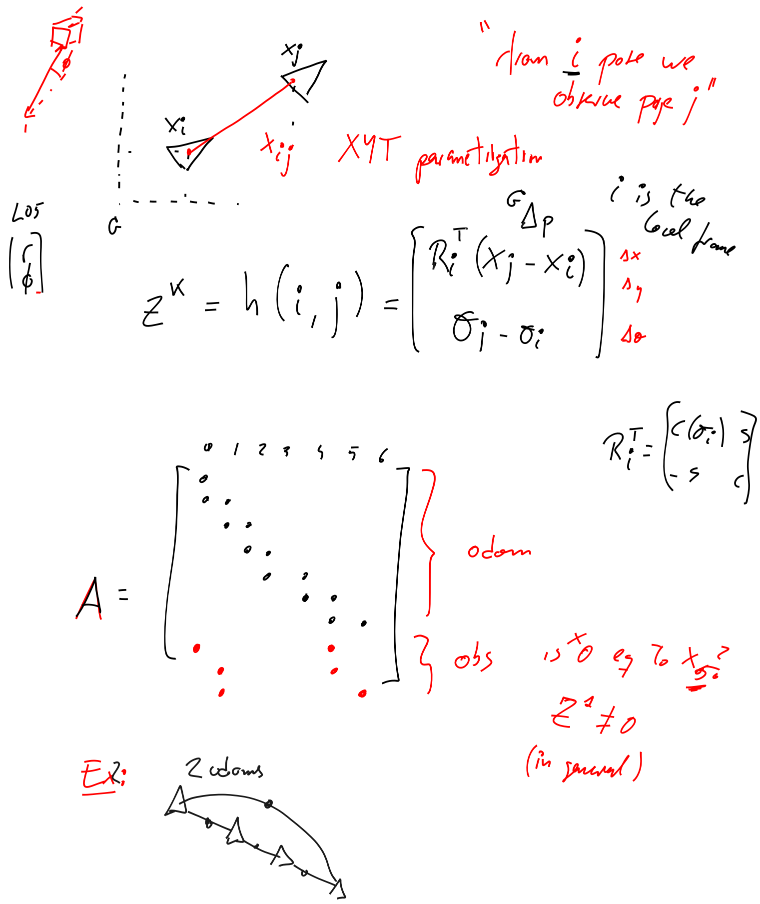

Figure 7: Observations in the adjacency matrix indicate relation between poses.

$$2DPoseJacobian \qquad H_k^i = \begin{bmatrix} R_i^T & 0 \\ 0 & 1 \end{bmatrix} \qquad H_k^i = \begin{bmatrix} -R_i^T & -s\Delta x + c\Delta x \\ & -c\Delta x - s\Delta x \\ 0 & -1 \end{bmatrix}$$

1. Pose SLAM obtained after marginalizing landmarks (not practical)
2. Virtual observation between poses → Registration problem (L14)