

## Lecture 09: Data Association

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### 1 Euclidean nearest neighbours

The data association problem consist of finding for each observation  $z^i$  to which landmark  $m_j$  it corresponds. In other words:

$$c_t = \{c_t^i\}, (z^i \rightarrow m_j)$$

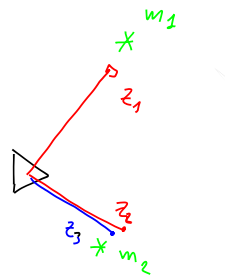


Figure 1: Euclidean nearest neighbours

$$c_t^i = \arg \min_j \|m_j - z_t^i\|_2$$

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#### Algorithm 1 Euclidean nearest neighbour

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1: for  $i = 1 : K$  do
2:   for  $j = 1 : N$  do
3:      $c_t^i = \min(\|m_j - z_t^i\|_2)$ 
4:   end for
5: end for

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Pros:

- Match each  $z^i$  to closest  $m_j$
- Easy and Fast  $\mathcal{O}(K \cdot N)$

Cons: Greedy Data association.

### 2 Mahalanobis nearest neighbour

$$d_{ij}^2 = \|m_j - z^i\|_{\Sigma}^2 = (m_j - z^i)^T \Sigma^{-1} (m_j - z^i)$$

MH captures uncertainty

Pros:

- More robust to noise

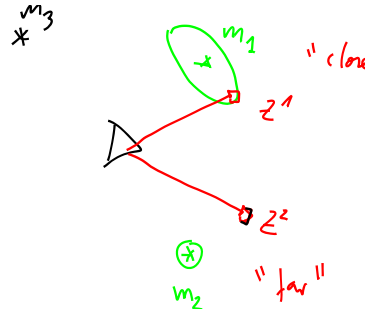


Figure 2: Mahalanobis nearest neighbours. The term “far” and “close” denote uncertainty for MH distances.

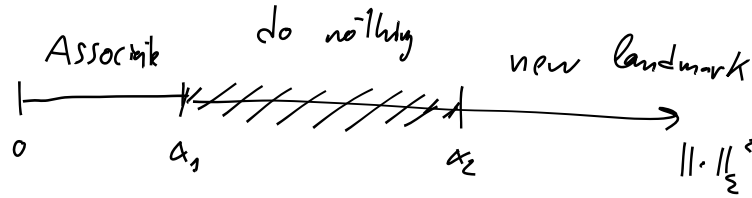


Figure 3: Decision Making for new observations. New landmarks might need conformation over several observations.

- Easy and fast  $\mathcal{O}(K \cdot N)$

Cons: Still greedy.

## 3 Maximum Likelihood (ML) Data Association

From the Bayes Filter we should calculate the distribution of correspondences:

$$\begin{aligned} p(x_{0:t}, c_{1:t} | Z, U) &= (\text{product rule}) \\ &= p(c_{1:t} | x_{0:t}, Z, U) p(x_{0:t} | Z, U) \end{aligned}$$

Very complicated! Sequence of all correspondences should be re-calculated completely for all observations.

### 3.1 Assumption I: Solve DA incrementally

$p(c_t | z_t, y_t)$ . The history of correspondences  $c_{1:t}$  only depends on the last correspondence. Assume previous correspondences were correct.

$$p(c_t | z_t, y_t) = p(c_t | z_t, y_t, c_{1:t-1})$$

$$p(c_t | z_t, y_t) = \frac{p(z_t | c_t, y_t) p(c_t | y_t)}{p(z_t | y_t)} \propto p(z_t | c_t, y_t)$$

Posterior  $\propto$  likelihood for a given  $c_t$ .

$$c_t^* = \arg \max_{c_t} \{p(z_t | c_t, y_t)\} \text{ ML estimator}$$

$$\begin{aligned}
 p(z_t | c_t, y_t) &= p(z_t^1 | z_t^{2:t}, y_t, c_t) \\
 &\quad \cdot p(z_t^2 | z_t^{3:t}, y_t, c_t) \\
 &\quad \vdots \\
 &\quad \cdot p(z_t^K | y_t, c_t)
 \end{aligned}$$

## 3.2 Assumption II: Independence

$$\begin{aligned}
 p(z_t | c_t, y_t) &\simeq \prod_{i=1}^K p(z_t^i | c_t^i, y_t) \\
 c_t^* &= \arg \max_{c_t} \prod_{i=1}^K p(z_t^i | c_t^i, y_t),
 \end{aligned}$$

where  $c_t = \{c_t^1 = m_{j_1}, c_t^2 = m_{j_2}, \dots, c_t^i = m_{j_i}, \dots\}$   
 Since  $c_t$  are independent:

$$\max_{c_t} \left( \prod_{i=1}^K p(z_t^i | c_t^i, y_t) \right) = \prod_{i=1}^K \max_{c_t^i} p(z_t^i | c_t^i, y_t)$$

Evaluate  $c_t^i$  individually!

$$z_t^i \sim \mathcal{N}(z_t^i; h(\bar{\mu}_t, c^i), H_t^i \bar{\Sigma}_t (H_t^i)^T + Q_t)$$

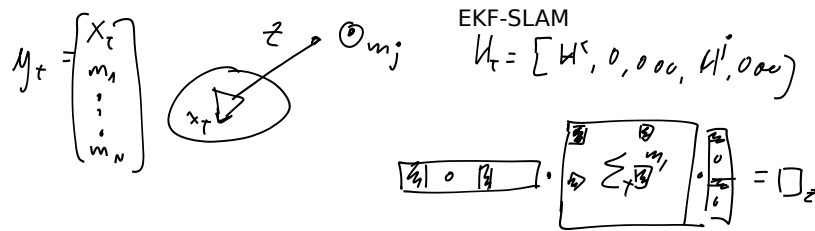


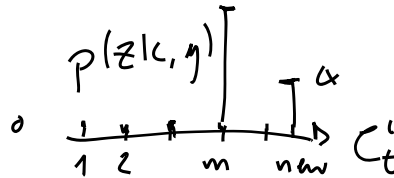
Figure 4: The Jacobian used to propagate the covariances of observations is the same as explained in the EKF-SLAM lecture.

New landmarks (ProbRob 322) under this method would not be detected. Is for that reason that the hypothesis of being a new landmark should be considered:

$$p(z_t^i | c_t^i = \text{new}, y_t) = \alpha$$

where  $\alpha$  is a threshold value, hard to tune in practice.

We create a landmark only if distance to all other landmarks is higher than  $\alpha$ .



## 4 Joint DA

Efficient way to express DA: tree representation as seen in Fig. 5.

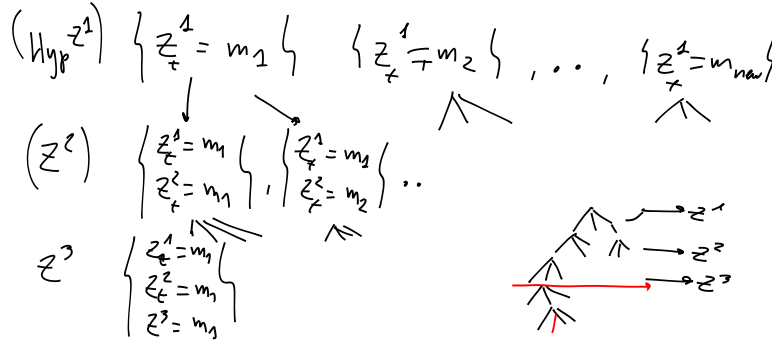


Figure 5: Joint Data Association. A tree structure captures all possible combinations of landmark-observation pairs.

### 4.1 Individual compatibility test

$\chi$  squared test ( $\alpha$  confidence level: 90, 95, 99%)

$$d_{11}^2 = \|m_1 - z^1\|_{\Sigma}^2 < \chi_{\alpha}^2$$

$$d_{12}^2 < \chi_{\alpha}^2$$

$$d_{13}^2 = \|m_1 - z^3\|_{\Sigma}^2 < \chi_{\alpha}^2$$

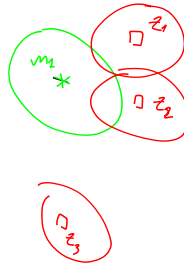


Figure 6: Individual compatibility

Fast filter to reject clearly incorrect hypothesis.

### 4.2 Joint Compatibility (JC) [1]

Content provides more accurate solutions.

Set of hypothesis  $\rightarrow$  size exponential

$$\mathcal{H}_l = \{z_1 = m_1, z_2 = m_3, z_3 = m_{new}\}$$

We will evaluate joint candidates based on their joint compatibility (JC)

$$\text{if } d_{\mathcal{H}_l}^2 < \chi_{\alpha}^2$$

For Individual compatibility (IC):

$$f_{ij}(y, z) = \frac{z^i - h(y, j)}{4}$$

For Joint compatibility:

$$f_{\mathcal{H}_l}(y, z) = \begin{bmatrix} f_{1,j_1}(y, z) \\ f_{2,j_2}(y, z) \\ \vdots \\ f_{l,j_l}(y, z) \end{bmatrix} = [f_{\mathcal{H}_{l-1}}(y, z) f_{l,j_l}(y, z)]$$

$$\mathcal{H} = \underbrace{\{c^1, c^2, \dots, c^K\}}_{\mathcal{H}_1}$$

$$\underbrace{\hspace{10em}}_{\mathcal{H}_2}$$

$$\underbrace{\hspace{10em}}_{\mathcal{H}_l}$$

If we rewrite  $f_{\mathcal{H}_l}(y, z)$  incrementally:

$$f_{\mathcal{H}_l}(y, z) = \begin{bmatrix} f_{\mathcal{H}_{l-1}}(y, z) \\ f_{l,j_l}(y, z) \end{bmatrix} \simeq f_{\mathcal{H}_l}(\bar{y}, z) + G_{\mathcal{H}_l}(y - \bar{y}) + H_{\mathcal{H}_l}(z - \bar{z})$$

Covariance of the joint innovation

$$C_{\mathcal{H}_l^i} = H_{\mathcal{H}_l^i} S H_{\mathcal{H}_l^i}^T + G_{\mathcal{H}_l^i} \Sigma G_{\mathcal{H}_l^i}^T$$

$$d_{\mathcal{H}_l^i}^2 = f(\bar{y}, z)^T \cdot C_{\mathcal{H}_l^i}^{-1} \cdot f(\bar{y}, z) < \chi_{d,\alpha}^2, d = \dim(\mathcal{H}_l^i)$$

This test is carried out incrementally.

## 4.3 JC branch and Bound

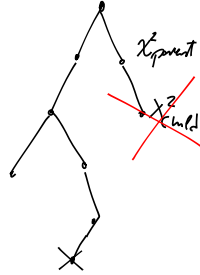


Figure 7: JCBB

Knowing that evaluating branched with the JC test allows us to discard branches without evaluating, since

$$\chi_{parent}^2 < \chi_{child}^2$$

$C_{H_l^i}$  captures cross-correlations of the observations while ML doesn't (assumed independence). More accurate and probabilistically more complete.

Slow (not exponential but it is on the worst case).

## References

- [1] Neira, José, and Juan D. Tardós. "Data association in stochastic mapping using the joint compatibility test." IEEE Transactions on robotics and automation 17.6 (2001): 890-897.

## 5 Summary

$c^i = \arg \min_j \|m_j - z^i\|_z$  – Euclidean Nearest Neighbor

$c^i = \arg \min_j \|m_j - z^i\|_{\Sigma_j}$  – Mahalanobis Nearest Neighbor

$c_t^* = \arg \max_{c_t} \{p(z_t | c_t, y_t)\}$  – Maximum Likelihood

$z_t = \{z^1, z^2, \dots, z^k\}$

Assumption:  $z$  independent

$\max(\prod p(\cdot)) = \prod (\max_{c^i} p(\cdot))$  – is equivalent to Mahalanobis Nearest Neighbor

### 5.1 JCBB: Joint Compatibility Branch and Bound

Evaluates hypothesis recursively and eliminates branches.  $d_{\mathcal{H}_l^i}^z < \chi_{d_i \alpha}^z$ , where  $\alpha$  – confidence level,  $d = \dim(f_{\mathcal{H}_l^i})$  – dimensionality.