Lecture 8. SLAM with known correspondences

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1 Simultaneous Localization (x_t) and Mapping (m)

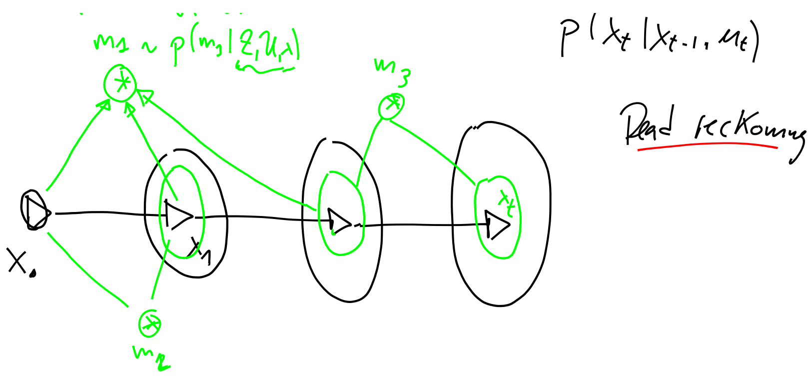
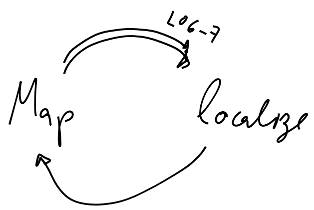


Figure 1: SLAM scheme. Open loop calculation of positions (dead reckoning) results in high uncertain state estimates (black ellipsoids). By taking into account landmark observation, we can reduce the uncertainty on the trajectory estimation (green ellipsoids).

$$\begin{aligned}
\text{Given}: u_{1:t} &= \{u_1, u_2, ..., u_t\} && \text{actions} \\
z_{1:t} &= \{z_1, z_2, ..., z_t\} && \text{obs (L05)}, z = \begin{bmatrix} r \\ \phi \\ s \end{bmatrix} \\
\text{Calculate}: x_{0:t} &= \{x_0, x_1, x_2, ..., x_t\} && \text{trajectory} \\
m &= \left\{ \begin{bmatrix} m_{1,x} \\ m_{1,y} \end{bmatrix}, \begin{bmatrix} m_{2,x} \\ m_{2,y} \end{bmatrix}, ..., \begin{bmatrix} m_{J,x} \\ m_{J,y} \end{bmatrix} \right\} && \text{map of landmarks}
\end{aligned}$$

- The association landmark-observation is not always known
- Incorrect DA can ruin the map



The chicken and the egg problem!
Initial approach: Use KF with known correspondences. Next lecture 9 we will cover unknown DA.

2 EKF SLAM with known correspondences

$$p(x_t, m | z_{1:t}, u_{1:t}, c_t)$$

where correspondences c_t denote the pair landmark m_j and observation z^i such that $\{c_t^i = j\} = c_t$.

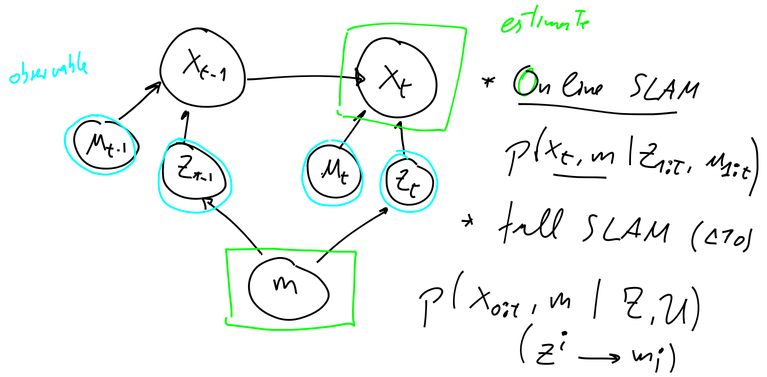


Figure 2: Bayes Network representing the SLAM problem. Variables to estimate are x and m.

Augmented state:

$$\begin{aligned} y_t &= \begin{bmatrix} x_t & m_1 & m_2 & \dots & m_N \end{bmatrix}^T \\ &= \begin{bmatrix} \underbrace{x\ y\ \theta}_{\text{2D Pose}} & \underbrace{m_{1,x}\ m_{1,y}}_{\text{landmark positions}} & \dots & m_{N,x} & m_{N,y} \end{bmatrix}^T \end{aligned}$$

$$y_t \sim \mathcal{N}(\mu_t, \Sigma_t) = \mathcal{N}\left(\begin{bmatrix} \mu_t^x \\ \mu_t^m \end{bmatrix}, \begin{bmatrix} \Sigma_x & \Sigma_{x,m} \\ \Sigma_{m,x} & \Sigma_m \end{bmatrix} \right)$$

$$\left(\begin{array}{c|c} \Sigma_x & \Sigma_{x,m} \\ \hline \Sigma_{m,x} & \Sigma_m \end{array} \right)_{(2N+3)\times(2N+3)}$$

Algorithm 1 EKF (L07) SLAM (ProbRob 314)

1: $\bar{\mu}_t = g(\mu_{t-1}, u_t)$ ▷ Prediction
2: $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$
3: $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q)^{-1}$ ▷ Multiple observations can be corrected
4: $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$ ▷ Sequential vs Batch
5: $\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t$

2.1 Prediction

$$y_t = \begin{bmatrix} x & m_1 & m_2 & \dots & m_N \end{bmatrix}^T$$

$g(y_{t-1}, u_t) \rightarrow$ transition function (Odometry model (PS3), Kinematic model (ProbRob), etc.

$$g(y_{t-1}, u_t) = \begin{bmatrix} g_x(x_{t-1}, u_t) \\ m_1 \\ m_2 \\ \vdots \\ m_N \end{bmatrix}_{3+2N} + \varepsilon_t = \text{(odometry model e.g.)}$$

$$= y_{t-1} + \begin{bmatrix} \delta_{tr} \cdot cos(\theta_{t-1} + \delta_{rot_1}) \\ \delta_{tr} \cdot sin(\theta_{t-1} + \delta_{rot_1}) \\ \delta_{rot_1} + \delta_{rot_2} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

I. $\bar{\mu}_t = g(\mu_{t-1}, u_t)$
II. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$

$$G_t = \frac{\partial g(y_{t-1}, u_t)}{\partial y_{t-1}}\Big|_{\mu_{t-1}} = \begin{bmatrix} \frac{\partial g_x}{\partial x} & \frac{\partial g_x}{\partial m_1} & \cdots & \frac{\partial g_x}{\partial m_N} \\ \frac{\partial m_1}{\partial x} & \frac{\partial m_1}{\partial m_1} & \cdots & \frac{\partial m_1}{\partial m_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial m_N}{\partial x} & \frac{\partial m_N}{\partial m_1} & \cdots & \frac{\partial m_N}{\partial m_N} \end{bmatrix} = \begin{bmatrix} G^x_{t_{3\times 3}} & 0 & \cdots & 0 \\ 0 & I_{2\times 2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & I \end{bmatrix}$$

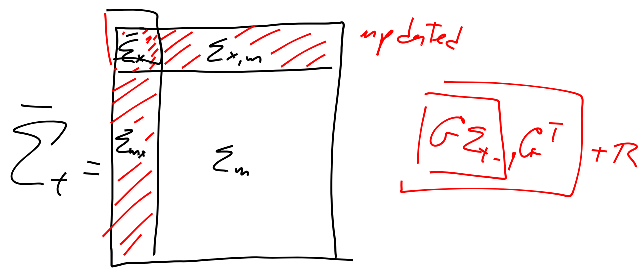
$$= \begin{bmatrix} G_t^x & 0 \\ 0 & I_{2N\times 2N} \end{bmatrix}$$

$$R_t = \begin{bmatrix} R_t^x & 0 \\ 0 & 0_{2N\times 2N} \end{bmatrix} = \begin{bmatrix} V_t^x M_t^x (V_t^x)^T & 0 \\ 0 & 0 \end{bmatrix}$$

Robot noise in state space or action space. Landmark do not propagate $\Rightarrow$ no noise!

state space	action state
$g(y_{t-1}, u_t) + \varepsilon_t$	$g(y_{t-1}, u_t + \varepsilon_t')$
$\varepsilon_t \sim \mathcal{N}(o, R_t^x)$	$\varepsilon_t' \sim \mathcal{N}(0, M_t^x)$

The block matrix Σ_m remains unaltered from time $t-1$ and t in the propagation.



Recall covariance for noise on transition in the action space:

$$M_t^x = \begin{bmatrix} \alpha_1 \delta_{rot_1}^2 + \alpha_2 \delta_{tr}^2 & 0 & 0 \\ 0 & \alpha_3 \delta_{tr}^2 + \alpha_4(\delta_{rot_1}^2 + \delta_{rot_2}^2) & 0 \\ 0 & 0 & \alpha_1 \delta_{rot_2}^2 + \alpha_2 \delta_{tr}^2 \end{bmatrix}$$

2.2 Correction: adding new landmarks

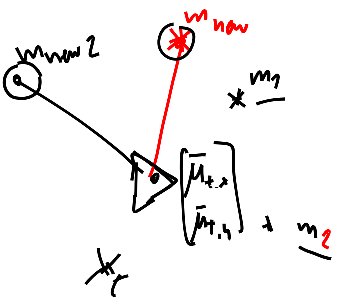
Data association: $\{c_t^i = j \Rightarrow z_i \to m_j\}$
New landmark is observed. We must initialize it.

$$\begin{aligned} z_t = h(y_t, j) + \eta_t &= \text{(L06 2D model)} \\ &= \begin{bmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \end{bmatrix} + \eta_t, \ \phi \in [-\pi, \pi) \end{aligned}$$

Inverse observation model (1 landmark)

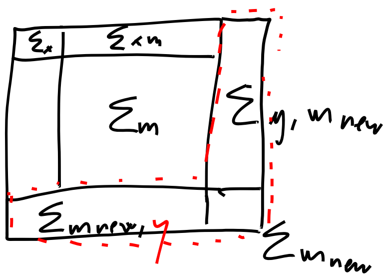
$$m_{j_{new}} = h^{-1}(z_t, \bar{y}_t)|_{\bar{\mu}_t} = \begin{bmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{bmatrix} + r_t \begin{bmatrix} cos(\phi_t + \bar{\mu}_{t,\theta}) \\ sin(\phi_t + \bar{\mu}_{t,\theta}) \end{bmatrix}$$

New landmark is expected w.r.t. robot $m_{new} = h^{-1}(z, y)$



$$\bar{y}_t = \begin{bmatrix} \bar{y}_{t(old)} \\ m_{new} \end{bmatrix} \text{ We have augmented the state vector}$$

Q: and $\bar{\Sigma}_t$? What is the new augmented covariance?



h^{-1} non-linear $\Rightarrow$ Covariance projection

$$h^{-1}(z_t, y_t) \approx h^{-1}(z_t, \mu_t) + L(y_t - \mu_t) + W(z_t - \hat{z}_t)$$

$$\begin{aligned} L = \frac{\partial h^{-1}}{\partial y_t}\Big|_{\bar{\mu}_t} &= \left(\frac{\partial h^{-1}}{\partial m_j} = 0\right) \\ &= \frac{\partial h^{-1}}{\partial x_t}\Big|_{\bar{\mu}_t^x} = \begin{bmatrix} 1 & 0 & -r_t sin(\phi_t + \bar{\mu}_{t,\theta}) \\ 0 & 1 & r_t cos(\phi_t + \bar{\mu}_{t,\theta}) \end{bmatrix} \end{aligned}$$

$$W = \frac{\partial h^{-1}}{\partial z}\Big|_{\bar{\mu}_t} = \begin{bmatrix} cos(\phi_t + \bar{\mu}_{t,\theta}) & -r_t sin(\phi_t + \bar{\mu}_{t,\theta}) \\ sin(\phi_t + \bar{\mu}_{t,\theta}) & r_t cos(\phi_t + \bar{\mu}_{t,\theta}) \end{bmatrix}$$

After linearizing the new landmark inverse model h^{-1}, we calculate all the covariances by the covariance propagation technique.

$$\begin{aligned} \Sigma_{m_{new}} &= E\{(m_{new} - \mu_{new})(m_{new} - \mu_{new})^T\} \\ &= E\{(L\Delta x_t + W\eta_t)(L\Delta x_t + W\eta_t)^T\} \\ &= L\Sigma_x L^T + WQW^T,\ \eta_t \sim \mathcal{N}(0, Q) \end{aligned}$$

$$\begin{aligned} \Sigma_{y,m_{new}} &= E\{(y_t - \mu_t)(m_{new} - \mu_{new})^T\} \\ &= E\{\Delta y(L\Delta x + W\eta_t)^T\} \\ &= (\text{landmarks uncorrelated with noise}) = \begin{bmatrix} \Sigma_x L^T \\ \Sigma_{m,x} L^T \end{bmatrix} \end{aligned}$$

$$\Sigma_{m_{new},y} = L \cdot \begin{bmatrix} \Sigma_x \\ \Sigma_{m,x} \end{bmatrix}^T = L \cdot \begin{bmatrix} \Sigma_x & \Sigma_{m,x} \end{bmatrix}$$

$$\bar{\Sigma}_t = \left[\begin{array}{cc|c} \Sigma_x & \Sigma_{x,m} & \Sigma_x L^T \\ \Sigma_{m,x} & \Sigma_m & \Sigma_{m,x} \cdot L^T \\ \hline L\Sigma_x & L \cdot \Sigma_{x,m} & \Sigma_{m_{new}} \end{array}\right] \qquad \Sigma_{m_{new}} = L\Sigma_x L^T + WQW^T$$

2.3 Correction: conditioning by observations z_t

$$z_t^i = h(y_t, c_t^i) + \eta_t = \begin{bmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \operatorname{atan2}(m_{j,y} - y, m_{j,x} - x) - \theta \end{bmatrix},\ c_t^i = j$$

$$h(y_t, c_t) \approx h(\bar{\mu}_t, c_t) + H_t \Delta y_t$$

$$\begin{aligned} H_t = \frac{\partial h(y_t, c_t^i)}{\partial y_t} &= \begin{bmatrix} \frac{\partial h}{\partial x_t} & \frac{\partial h}{\partial m_1} & \dots & \frac{\partial h}{\partial m_j} & \dots & \frac{\partial h}{\partial m_N} \end{bmatrix} \\ &= \begin{bmatrix} \frac{-(m_{j,x}-x)}{\sqrt{q}} & \frac{-(m_{j,y}-y)}{\sqrt{q}} & 0 & 0 & \dots & \frac{m_{j,x}-x}{\sqrt{q}} & \frac{m_{j,y}-y}{\sqrt{q}} & 0 & \dots & 0 \\ \frac{m_{j,y}-y}{q} & \frac{-(m_{j,x}-x)}{q} & -1 & 0 & \dots & \frac{-(m_{j,y}-y)}{q} & \frac{m_{j,x}-x}{q} & 0 & \dots & 0 \end{bmatrix} \end{aligned}$$

Where $q = (m_{j,x} - x)^2 + (m_{j,y} - y)^2$

$$H_t = \begin{bmatrix} H^x & 0 & \dots & H^j & 0 & \dots & 0 \end{bmatrix}$$

Correction for 1 observation

3 Summary

Online EKF-SLAM

$$p(x_t, m|Z, U, c_t)$$

$$g(y_t, u_t) = \begin{bmatrix} g^x \\ m_1 \\ \vdots \\ m_N \end{bmatrix}, z_t = h(y_t, c_t)$$

$$H_t = \begin{bmatrix} H^x & 0 & \dots & H^j & 0 & \dots & 0 \end{bmatrix}$$

$$\overline{m}_{new} = h^{-1}(y_t, z_t)\Big|_{\bar{\mu}_t}, \mu_t = \begin{bmatrix} \mu_t \\ \mu_{m_{new}} \end{bmatrix}$$

$$\bar{\Sigma}_t = \left[\begin{array}{c|c|c} \Sigma_x & \Sigma_{x,m} & \Sigma_x L^T \\ \hline \Sigma_{m,x} & \Sigma_m & \Sigma_{m,x} \cdot L^T \\ \hline L \cdot \Sigma_x & L \cdot \Sigma_{x,m} & L\Sigma_x L^T + WQW^T \end{array}\right]$$