"Group - 39"

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Answer to the Question no: 31

Let us now determine the distribution;

$P(x=0) = \binom{3}{0} \times \left(\frac{1}{8}\right)^0 \times \left(\frac{7}{8}\right)^3 \approx 0.670$

$P(x=1) = \binom{3}{1} \times \left(\frac{1}{8}\right)^1 \times \left(\frac{7}{8}\right)^2 \approx 0.287$

$P(x=2) = \binom{3}{2} \times \left(\frac{1}{8}\right)^2 \times \left(\frac{7}{8}\right)^1 \approx 0.041$

$P(x=3) = \binom{3}{3} \times \left(\frac{1}{8}\right)^3 \times \left(\frac{7}{8}\right)^0 \approx 0.002$

$E[x] = (0\times0.670)+(1\times0.287)+(2\times0.041)+(3\times0.002)$
≈ 0.4

Answer to the Question no: 32

Here, each throw is an independent event, and the player either scores (1 point) with a probability of ~~0.7~~ 0.70 or doesn't score (0 point) with a probability of 0.30

The expected value for one throw is

$E(\text{one throw}) = (1\cdot0.70)+(0\cdot0.30) = 0.70$

$\therefore E(\text{total}) = 0.70\cdot2 = 1.4$

— x —

Answer to the Question no: 33

Mean: $\mu = np$

Variance: $\sigma^2 = np(1-p)$

we are given;

$\mu = 2$, so $np = 2$ (Equation 1)

$\sigma^2 = 4/3$, so $np(1-p) = 4/3$ (Equation 2)

Step 1: substitute $np = 2$

From equation 1, we have $np = 2$

$2(1-p) = 4/3$

Step: 2;

Solve for $1-p$.

Divide both sides by 2

$1-p = \frac{4/3}{2} = 4/3 \div 1/2 = 4/6$
$= 2/3$

Step: 3; Solve for p

Subtract $1-p = 2/3$ from 1;

$p = 1 - 2/3 = 1/3$

Step: 4; $p = 1/3$ back into Equation 1 ($np = 2$)

$n \cdot 1/3 = 2$

Multiply both sides by 3

$n = 2\cdot3 = 6$

Step: 5;

$n=6$ and $p = 1/3$

$\sigma = np(1-p) = 6\cdot1/3\cdot(1-1/3) = 4/3$

The variance matches the give value, and the mean is

$\mu = np = 6\cdot1/3 = 2$

Both condition are satisfied, the values are $n=6$ and $p = 1/3$ $\boxed{\text{Answer}}$

Answer to the Question no: 34

Given that, $n = 4$ (number of throws)

$p = 1/6$ (probability of success)

The possible values of x are 0. 1, 2. 3. 4.

Binomial distribution is;

$$P(x=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where, $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Calculating the probabilities

$P(X=0)$; $P(X=0) = \binom{4}{0}\left(\frac{1}{6}\right)^0\left(5/6\right)^4 = 625/1296$

$P(X=1)$; $P(\cancel{X}=1) = \binom{4}{1}(1/6)^1(5/6)^3 = 125/324$

$P(X=2)$: $P(X=2) = \binom{4}{2}(1/6)^2(5/6)^2 = 25/216$

$P(X=3)$; $P(X=3) = \binom{4}{3}(1/6)^3(5/6)^3 = 5/324$

$P(X=4)$; $P(X=4) = \binom{4}{4}(1/6)^4(5/6)^4 = 1/1296$

The distribution of X, the number of "6"s in 4 throws, is:

$$X \sim Bin(4; 1/6)$$

— x —

Answer to the Question no: 35

Given that,

Binomial distribution;

$X \sim Bin(n, p) = Bin(2, 0.8)$

$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

$P(X=0)$: $P(X=0) = \binom{2}{0} (0.8)^0 (0.2)^2 = 0.04$

$P(X=1)$: $P(X=1) = \binom{2}{1} (0.8)^1 (0.2)^1 = 0.32$

$P(X=2)$: $P(X=2) = \binom{2}{2} (0.8)^2 (0.2)^2 = 0.64$

Expected value:

$E(X) = np$, with $n = 2$ and $p = 0.8$)

$= 2 \cdot 0.8 = 1.6$

Standard Error : $\sigma = \sqrt{0.32} \approx 0.566$

—x—

Answer to the question no: 36 :

Given that,

Total balls : 3 black + 3 white = 6

Number of white balls : 3

Number of black balls : 3

Number of draws : 3

The expected value of a hypergeometric random variable is:

$E(x) = n \cdot k/n$

$= 3 \cdot 3/6 = 3 \cdot 1/2 = 1.5$

The expected value of the number of white balls drawn is 1.5

Answer to the Question no: 37

Given that,

Total envelopes : 5 (with values 10 €, 20€, 30€, 40€, 50 €)

Number of envelopes chosen : 2

X : Total amount won

* Total number of possible pairs;

The number of ways to choose 2 envelopes form from 5 is given by the combination formula

$\binom{5}{2} = \frac{5 \cdot 4}{2 \cdot 1} = 10$ (so, there are 10 possible pairs)

By linearity of expectation;

$E(X) = E(X_1 + X_2) = E(X_1) + E(X_2)$

Compute: $E(X_1)$:

$E(X_1) = 10 \cdot 1/5 + 20(1/5) + 30(1/5) + 40(1/5) + 50(1/5)$

$= \frac{10+20+30+40+50}{5}$

$= 150/5 = 30$

Compute: $E(X_2)$

$E(X_2) = E(X_1) = 30$

* Total sum of all amounts:

$= 10+20+30+40+50$

$= 150$

* population mean

$= 150/5 = 30$

Total expected value: 60

Each pair is equally likely with probability 1/10. Compute the expected value:

$E(X) = \Sigma$ (sum · probability)

$= (30 \cdot 1/10) + (40 \cdot 1/10) + (50 \cdot 1/10) + (60 \cdot 1/10) + (70 \cdot 1/10) + (80 \cdot 1/10) + (90 \cdot 1/10) = 600/10 = 60$ Ans (The expected value of the total amount, is won)

Answer to the Question no; 38 :

probability of winning $1000€ = \frac{1}{1000}$

net win $= 100 - 1 = 99 €$

probability of winning $50€ = \frac{1}{100}$

net win $= 50 - 1 = 49 €$

probability of winning $20€ = \frac{3}{200}$

net win $= 20 - 1 = 19 €$

probability of no winning $= \frac{1000 - 1 - 10 - 15}{1000} = \frac{974}{1000}$

Net win $= -1$

$$E[x] = \frac{99 \cdot 1}{1000} + \frac{49 \cdot 1}{100} + \frac{19 \cdot 3}{200} - \frac{974}{1000} \approx -0.1 €$$

Answer to the Question no: 39 :

P HH means 1 Head and P_{TT} means 2 Tail.

$P_{HH} = \frac{1}{2}, P_{HH'} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P_{TT} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\quad P_{HT'} = 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$,

$$E[win] = (P_{HH} \times 20) + (P_{HH'} \times 40) + (P_{TT} \times (-100))$$

$$= (\frac{1}{2} \times 20) + (\frac{1}{4} \times 40) + (\frac{1}{4} \times (-100))$$

$$= -5$$

Answer to the Question no: 40;

Given that,

Six $1€$ coins

Four $2€$ coins

Two $0.05€$ coins

Total number of coins $= 6+4+2 = 12$

- $X = 0.50€ = P(X=0.50) = \frac{\text{Number of } 0.50€ \text{ coins}}{\text{Total Coins}}$

$= 2/12 = 1/6$

- $X = 1€ = P(X=1) = \frac{6}{12} = 1/2$
- $X = 2€ = P(X=2) = 4/12 = 1/3$

The distribution of the value of the coin picked is;

$P(X=0.50) = 1/6 \approx 0.1667$

$P(X=1) = 1/2 = 0.5$

$P(X=2) = 1/3 \approx 0.33$

(Ans)

CS CamScanner

Answer to the Question no: 41

Given that,

* Each trial involves tossing 6 fair coins
* The number of heads in each trial can range 0 to 6
* There are 90 trials, and the number of heads in each trial is recorded

** Tally the frequencies of each outcome.

$x = 0$: 3 times (at positions 1. 75. 83)

$x = 1$: 9 times (at positions 11, 23, 35, 39, 41. 46. 50, 56. 85)

$x = 2$: 20 times

$x = 3$: 28 times

$x = 4$: 22 times

$x = 5$: 8 times

$x = 6$: 0 times

Total : $3 + 9 + 20 + 28 + 22 + 8 + 0 = 90$, which matches the number of trials

- $P(x = 0) = 3/90 = 1/30 \approx 0.03$

$P(x = 1) = 9/90 = 1/10 \approx 0.1$

$P(x = 2) = 20/90 = 2/9 \approx 0.22$

$P(x = 3) = 28/90 = 14/45 \approx 0.31$

$P(x = 4) = 22/90 = 11/45 \approx 0.24$

$P(x = 5) = 8/90 = 4/45 \approx 0.089$

$P(x = 6) = 0/90 = 0$