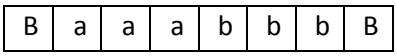
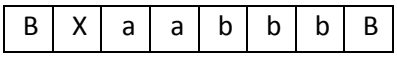
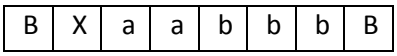
First, the tape symbols are



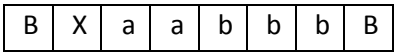
™(q_0, a) → (q_1, Xaabbb, R)



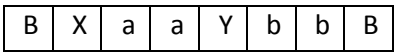
™(q_1, a) → (q_1, Xaabbb, R)



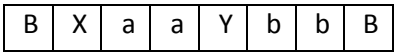
™(q_1, a) → (q_1, Xaabbb, R)



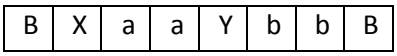
™(q_1, b) → (q_2, XaaYbb, L)



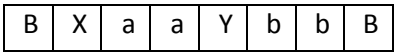
™(q_2, a) → (q_2, XaaYbb, L)



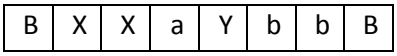
™(q_2, a) → (q_2, XaaYbb, L)



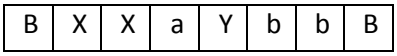
™(q_2, X) → (q_0, XaaYbb, R)



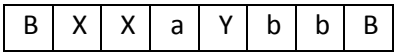
™(q_0, a) → (q_1, XXaYbb, R)



™(q_1, a) → (q_1, XXaYbb, R)



™(q_1, Y) → (q_1, XXaYbb, R)



™(q_1, b) → (q_2, XXaYYb, L)

B	X	X	a	Y	Y	b	B

™(q_2, Y) → (q_2, XXaYYb, L)

B	X	X	a	Y	Y	b	B

™(q_2, a) → (q_2, XXaYYb, L)

B	X	X	a	Y	Y	b	B

™(q_2, X) → (q_0, XXaYYb, R)

B	X	X	a	Y	Y	b	B

™(q_0, a) → (q_1, XXXYYb, R)

B	X	X	X	Y	Y	b	B

™(q_1, Y) → (q_1, XXXYYb, R)

B	X	X	X	Y	Y	b	B

™(q_1, Y) → (q_1, XXXYYb, R)

B	X	X	X	Y	Y	b	B

™(q_1, b) → (q_2, XXXYYY, L)

B	X	X	X	Y	Y	Y	B

™(q_2, Y) → (q_2, XXXYYY, L)

B	X	X	X	Y	Y	Y	B

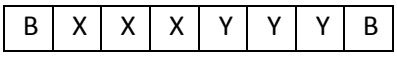
™(q_2, Y) → (q_0, XXXYYY, R)

B	X	X	X	Y	Y	Y	B

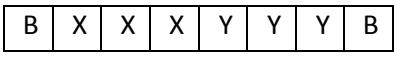
™(q_0, Y) → (q_3, XXXYYY, R)

B	X	X	X	Y	Y	Y	B

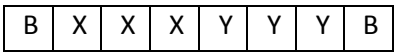
$\delta_M(q_3, Y) \rightarrow (q_3, XXXYYY, R)$



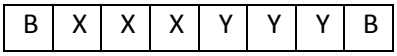
$\delta_M(q_3, Y) \rightarrow (q_3, XXXYYY, R)$



$\delta_M(q_3, Y) \rightarrow (q_3, XXXYYY, R)$



$\delta_M(q_3, B) \rightarrow (q_3, XXXYYY, H)$



Exampel 8.3 Design a TM to accept the language

$$L = \{WW_R, \text{ where } W \in (a, b)^+\}$$

Show an ID for the string 'abaaaaba' with tape symbols.

Solution: W^R is the reverse of W. If W starts with 'a' or 'b', W^R ends with 'a' or 'b', respectively. If W ends with 'a' or 'b', W_R starts with 'a' or 'b', respectively. The TM can be designed as follows.

If the string W starts with 'a', upon traversal, that 'a' is replaced by X with a state change from q_0 to q_1, and the head moves to one right. The transitional function is

$$\delta_M(q_0, a) \rightarrow (q_1, X, R)$$

W^R ends with 'a' if W starts with 'a'. The machine needs to search the end 'a' of W^R. Before that, the machine needs to traverse the end symbols of W and the beginning symbols of W^R. The transitional functions are

$$\delta_M(q_1, a) \rightarrow (q_1, a, R)$$

$$\delta_M(q_1, b) \rightarrow (q_1, b, R)$$

After the end symbol of W^R, there exists the blank symbol B. In the traversal process, if the machine gets a B, it traverses back to the left side and gets the end symbol of W^R. The transitional functions are

$$\delta_M(q_1, B) \rightarrow (q_2, B, L)$$

$$\delta_M(q_2, a) \rightarrow (q_3, X, L)$$

Now, the machine needs to search for the second symbol of W. Before that, it has to traverse the beginning symbols of W^R and the end symbols of W. The transitional functions are

$$\delta_M(q_3, a) \rightarrow (q_3, a, L)$$

$$\delta_M(q_3, b) \rightarrow (q_3, b, L)$$

When the machine gets the rightmost X, it recognizes that the next symbol of W exists after that 'X'. The transitional function is

$$\delta_M(q_3, X) \rightarrow (q_0, X, R)$$

If the string W starts with ‘b’, the transitions are the same as the previous one but with some states changed. The transitional functions are

$$™(q_0, a) \rightarrow (q_4, Y, R)$$

$$™(q_4, a) \rightarrow (q_4, a, R)$$

$$™(q_4, b) \rightarrow (q_4, b, R)$$

$$™(q_4, B) \rightarrow (q_5, B, L)$$

$$™(q_5, b) \rightarrow (q_6, Y, L)$$

$$™(q_6, a) \rightarrow (q_6, a, L)$$

$$™(q_6, b) \rightarrow (q_6, b, L)$$

$$™(q_6, Y) \rightarrow (q_0, Y, R)$$

After the fi rst traversal, i.e., from the second traversal onwards, the machine need not traverse up to the end of W^R. In state q_1 (W starts with ‘a’) or q_4 (W starts with ‘b’), if the machine gets X or Y, it traverses back to the left to point the rightmost ‘a’ or ‘b’. The transitional functions are

$$™(q_1, X) \rightarrow (q_2, X, L)$$

$$™(q_1, Y) \rightarrow (q_2, Y, L)$$

$$™(q_4, X) \rightarrow (q_5, X, L)$$

$$™(q_4, Y) \rightarrow (q_5, Y, L)$$

When all the symbols of W and W^R are traversed, the machine gets an X or Y in the state q_0. The machine halts if in state q_0 it gets an X or Y. The transitional functions are

$$™(q_0, X) \rightarrow (q_f, X, H)$$

$$™(q_0, Y) \rightarrow (q_f, Y, H)$$

The transitional functions can be given in a tabular format as follows.

state	a	B	B	X	Y
q_0	(q_1, X, R)	(q_4, Y, R)	–	(q_f, X, H)	(q_f, Y, H)
q_1	(q_1, a, R)	(q_1, b, R)	(q_2, B, L)	(q_2, X, L)	(q_2, Y, L)
q_2	(q_3, X, L)	–	–	–	–
q_3	(q_3, a, L)	(q_3, b, L)		(q_0, X, R)	–
q_4	(q_4, a, R)	(q_4, b, R)	(q_5, B, L)	(q_5, X, L)	(q_5, Y, L)
q_5	–	(q_6, Y, L)	–	–	–
q_6	(q_6, a, L)	(q_6, b, L)	–	–	(q_0, Y, R)