

First-Class Functions in Racket

Design of Programming Languages

First-Class Values

A value is **first-class** if it satisfies all of these properties:

- It can be named by a variable
- It can be passed as an argument to a function;
- It can be returned as the result of a function;
- It can be stored as an element in a data structure (e.g., a list);
- It can be created in any context.

Examples from Racket: numbers, boolean, strings, characters, lists, ... and **functions!**

Functions can be Named

```
(define dbl (λ (x) (* 2 x)))  
  
(define avg (λ (a b) (/ (+ a b) 2)))  
  
(define pow  
  (λ (base expt)  
    (if (= expt 0)  
        1 ⇒* 3  
        (* base (pow base (- expt 1))))))
```

Recall syntactic sugar:

```
(define (dbl x) (* 2 x))  
  
(define (avg a b) (/ (+ a b) 2))  
  
(define (pow base expt) ...)
```

Functions can be Passed as Arguments

```
(define app-3-5 (λ (f) (f 3 5)))
```

```
(define sub2 (λ (x y) (- x y)))
```

```
({app-3-5} sub2)
```

```
⇒ ((λ (f) (f 3 5)) {sub2}) [varref]
```

```
⇒ {((λ (f) (f 3 5)) (λ (x y) (- x y)))} [varref]
```

```
⇒ {((λ (x y) (- x y)) 3 5)} [function call]
```

```
⇒ {(- 3 5)} [function call]
```

```
⇒ -2 [subtraction]
```



More Functions-as-Arguments

What are the values of the following?

```
(app-3-5 avg)
```

```
(app-3-5 pow)
```

```
(app-3-5 ( $\lambda$  (a b) a) )
```

```
(app-3-5 +)
```



Functions can be Returned as Results from Other Functions

```
(define make-linear-function  
  (λ (a b) ; a and b are numbers  
    (λ (x) (+ (* a x) b))))
```

```
(define 4x+7 (make-linear-function 4 7))
```

```
(4x+7 0)
```

```
(4x+7 1)
```

```
(4x+7 2)
```

```
(make-linear-function 6 1)
```

```
((make-linear-function 6 1) 2)
```

```
((app-3-5 make-linear-function) 2)
```

More Functions-as-Returned-Values



```
(define flip2  
  (λ (binop)  
    (λ (x y) (binop y x))))
```

```
((flip2 sub2) 4 7)
```

```
(app-3-5 (flip2 sub2))
```

```
((flip2 pow) 2 3))
```

```
(app-3-5 (flip2 pow))
```

```
(define g ((flip2 make-linear-function) 4 7))
```

```
(list (g 0) (g 1) (g 2))
```

```
((app-3-5 (flip2 make-linear-function)) 2)
```



Functions can be Stored in Lists

```
(define funs (list sub2 avg pow app-3-5  
                  make-linear-function flip2))
```

```
((first funs) 4 7)
```

```
((fourth funs) (third funs))
```

```
((fourth funs) ((sixth funs) (third funs)))
```

```
((fourth funs) (fifth funs) 2)
```

```
((fourth funs) ((sixth funs) (fifth funs)) 2)
```


Functions can be Created in Any Context

- In some languages (e.g., C) functions can be defined only at top-level. One function cannot be declared inside of another.
- Racket functions like `make-linear-function` and `flip2` depend crucially on the ability to create one function inside of another function.

Python Functions are First-Class!

```
def sub2 (x,y):  
    return x - y
```

```
def app_3_5 (f):  
    return f(3,5)
```

```
def make_linear_function(a, b):  
    return lambda x: a*x + b
```

```
def flip2 (binop):  
    return lambda x,y: binop(y,x)
```

```
In [2]: app_3_5(sub2)
```

```
Out[2]: -2
```

```
In [3]: app_3_5(flip2(sub2))
```

```
Out[3]: 2
```

```
In [4]: app_3_5(make_linear_function)(2)
```

```
Out[4]: 11
```

```
In [5]: app_3_5(flip2(make_linear_function))(2)
```

```
Out[5]: 13
```

JavaScript Functions are First-Class!

```
function sub2 (x,y) {  
  { return x-y; }  
}
```

```
function app_3_5 (f)  
{ return f(3,5); }
```

```
function make_linear_function(a,b) {  
  return function(x) {return a*x + b;};  
}
```

```
function flip2(binop) {  
  return function(x,y)  
    { return binop(y,x); }  
}
```

```
> app_3_5(sub2)
```

```
< -2
```

```
> app_3_5(flip2(sub2))
```

```
< 2
```

```
> app_3_5(make_linear_function)(2)
```

```
< 11
```

```
> app_3_5(flip2(make_linear_function))(2)
```

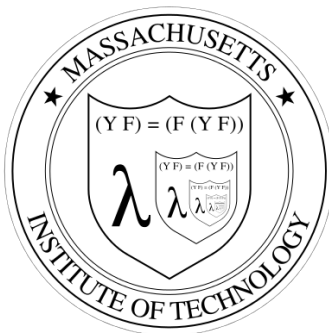
```
< 13
```

Summary (and Preview!)

*Data and procedures and the values they amass,
Higher-order functions to combine and mix and match,
Objects with their local state, the messages they pass,
A property, a package, a control point for a catch —
In the Lambda Order they are all first-class.*

*One Thing to name them all, One Thing to define them,
One Thing to place them in environments and bind them,
In the Lambda Order they are all first-class.*

Abstract for the *Revised4 Report on the Algorithmic Language Scheme (R4RS)*, MIT Artificial Intelligence Lab Memo 848b, November 1991



Emblem for the Grand Recursive Order
of the Knights of the Lambda Calculus



More Functions-as-Arguments Solutions

What are the values of the following?

`(app-3-5 avg)` \Rightarrow^* **4**

`(app-3-5 pow)` \Rightarrow^* **243 ; 3^5**

`(app-3-5 (λ (a b) a))` \Rightarrow^* **3**

`(app-3-5 +)` \Rightarrow^* **8**



Functions can be Returned as Results from Other Functions Solutions

```
(define make-linear-function  
  (λ (a b) ; a and b are numbers  
    (λ (x) (+ (* a x) b))))
```

make-linear-function \mapsto (λ (a b) (λ (x) (+ (* a x) b))))

```
(define 4x+7 (make-linear-function 4 7))
```

4x+7 \mapsto (λ (x) (+ (* 4 x) 7)))

; Note: This illustrates that functions are data structures! **make-linear-function**
; returns something similar to a Java object that “remembers” instance vars a and b!

(4x+7 0) \Rightarrow^* 7

(4x+7 1) \Rightarrow^* 11

(4x+7 2) \Rightarrow^* 15

(make-linear-function 6 1) \Rightarrow^* (λ (x) (+ (* 6 x) 1)))

((make-linear-function 6 1) 2) \Rightarrow^* 13

((app-3-5 make-linear-function) 2) \Rightarrow^* 11

More Functions-as-Returned-Values Solutions



```
(define flip2
  (λ (binop)
    (λ (x y) (binop y x))))
```

```
flip2 ↦ (λ (binop) (λ (x y) (binop x y)) )
```

```
((flip2 sub2) 4 7) ⇒* 3
```

```
(app-3-5 (flip2 sub2)) ⇒* 2
```

```
((flip2 pow) 2 3)) ⇒* 9 ; 3^2
```

```
(app-3-5 (flip2 pow)) ⇒* 125 ; 5^3
```

```
(define g ((flip2 make-linear-function) 4 7))
```

```
g ↦ (λ (x) (+ (* 7 x) 4))
```

```
(list (g 0) (g 1) (g 2)) ⇒* '(4 11 18)
```

```
((app-3-5 (flip2 make-linear-function)) 2) ⇒* 13
```

Functions can be Stored in Lists Solutions



```
(define funs (list sub2 avg pow app-3-5  
                  make-linear-function flip2))
```

; funs is a list of 6 functions. In Racket, the printed representation of this list is:

```
' (#<procedure:sub2> #<procedure:avg>  
  #<procedure:pow> #<procedure:app-3-5>  
  #<procedure:make-linear-function> #<procedure:flip2>)
```

```
((first funs) 4 7)  $\Rightarrow^*$  -3
```

```
((fourth funs) (third funs))  $\Rightarrow^*$  243 ; 35
```

```
((fourth funs) ((sixth funs) (third funs)))  $\Rightarrow^*$  125 ; 53
```

```
((fourth funs) (fifth funs)) 2)  $\Rightarrow^*$  11
```

```
((fourth funs) ((sixth funs) (fifth funs))) 2)  $\Rightarrow^*$  13
```