The Pros of `cons`: Pairs and Lists in Racket

Design of Programming Languages

Modified Slides of Lyn Turbak

Racket Values

- booleans: `#t, #f`
- numbers:
  - integers: `42, 0, -273`
  - rationals: `2/3, -251/17`
  - floating point (including scientific notation):
    `98.6, -6.125, 3.141592653589793, 6.023e23`
  - complex: `3+2i, 17-23i, 4.5-1.4142i`

  Note: some are *exact*, the rest are *inexact*. See docs.
- strings: `"cat"`, `"CS251"`, `"αβγ"`,
  `"To be\nor not\nto be"`
- characters: `#\a`, `#\A`, `#\5`, `#\space`, `#\tab`, `#\newline`
- anonymous functions: `(lambda (a b) (+ a (* b c)))`

What about compound data?

`cons` Glues Two Values into a Pair

A new kind of value:

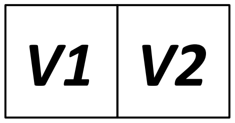
- pairs (a.k.a. `cons` cells): `(cons` ***V1 V2*** `)`
  e.g.,
  - `(cons 17 42)`
  - `(cons 3.14159 #t)`
  - `(cons "CS251" (λ (x) (* 2 x))`
  - `(cons (cons 3 4.5) (cons #f #\a))`

In Racket,
type Command-\
to get λ char

- Can glue any number of values into a `cons` tree!

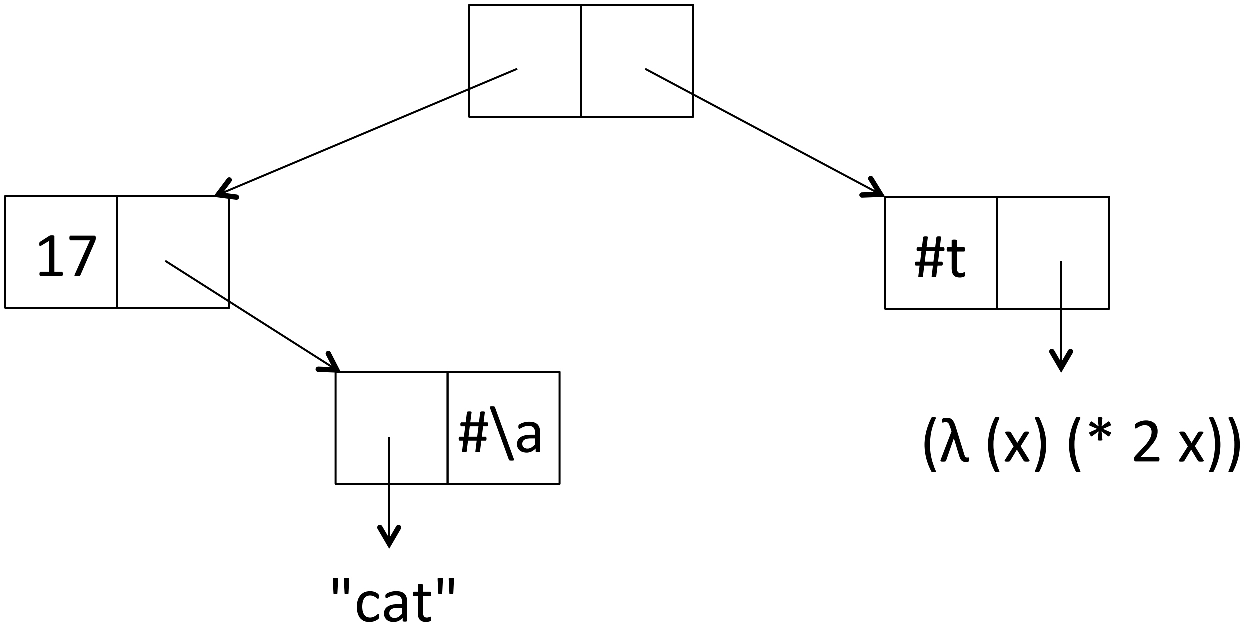
Box-and-pointer diagrams for `cons` trees

`(cons` ***V1 V2*****`)`**



Convention: put “small” values (numbers, booleans, characters) inside a box, and draw a pointers to “large” values (functions, strings, pairs) outside a box.

```
(cons (cons 17 (cons "cat" #\a))
      (cons #t (λ (x) (* 2 x))))
```



Evaluation Rules for `cons`

Big step semantics:

$$\frac{\begin{array}{l} E1 \downarrow V1 \\ E2 \downarrow V2 \end{array}}{\texttt{(cons } E1\ \ E2\texttt{)} \downarrow \texttt{(cons } V1\ \ V2\texttt{)}}\ [\text{cons}]$$

Small-step semantics:

`cons` has no special evaluation rules. Its two operands are evaluated left-to-right until a value **`(cons`** ***V1 V2*****`)`** is reached:

`(cons` ***E1 E2*****`)`**

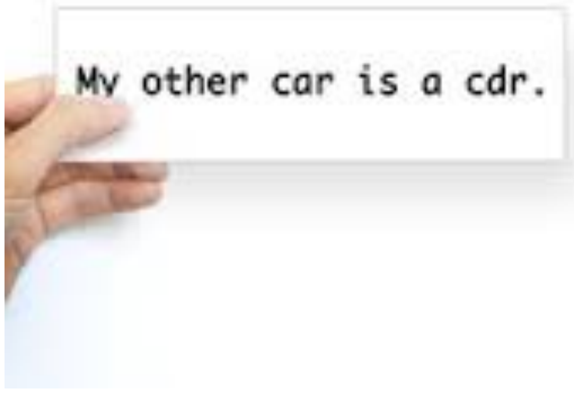
$\Rightarrow^*$ **`(cons`** ***V1 E2*****`)`** ; first evaluate ***E1*** to ***V1*** step-by-step

$\Rightarrow^*$ **`(cons`** ***V1 V2*****`)`** ; then evaluate ***E2*** to ***V2*** step-by-step

cons evaluation example

```
(cons (cons {(+ 1 2)} (< 3 4))
      (cons (> 5 6) (* 7 8)))
⇒ (cons (cons 3 {(< 3 4)})
        (cons (> 5 6) (* 7 8)))
⇒ (cons (cons 3 #t) (cons {(> 5 6)} (* 7 8)))
⇒ (cons (cons 3 #t) (cons #f {(* 7 8)}))
⇒ (cons (cons 3 #t) (cons #f 56))
```

`car` and `cdr`



- `car` extracts the left value of a pair

  `(car (cons 7 4))` ⇒ `7`

- `cdr` extract the right value of a pair

  `(cdr (cons 7 4))` ⇒ `4`

Why these names?

- `car` from “contents of address register”
- `cdr` from “contents of decrement register”

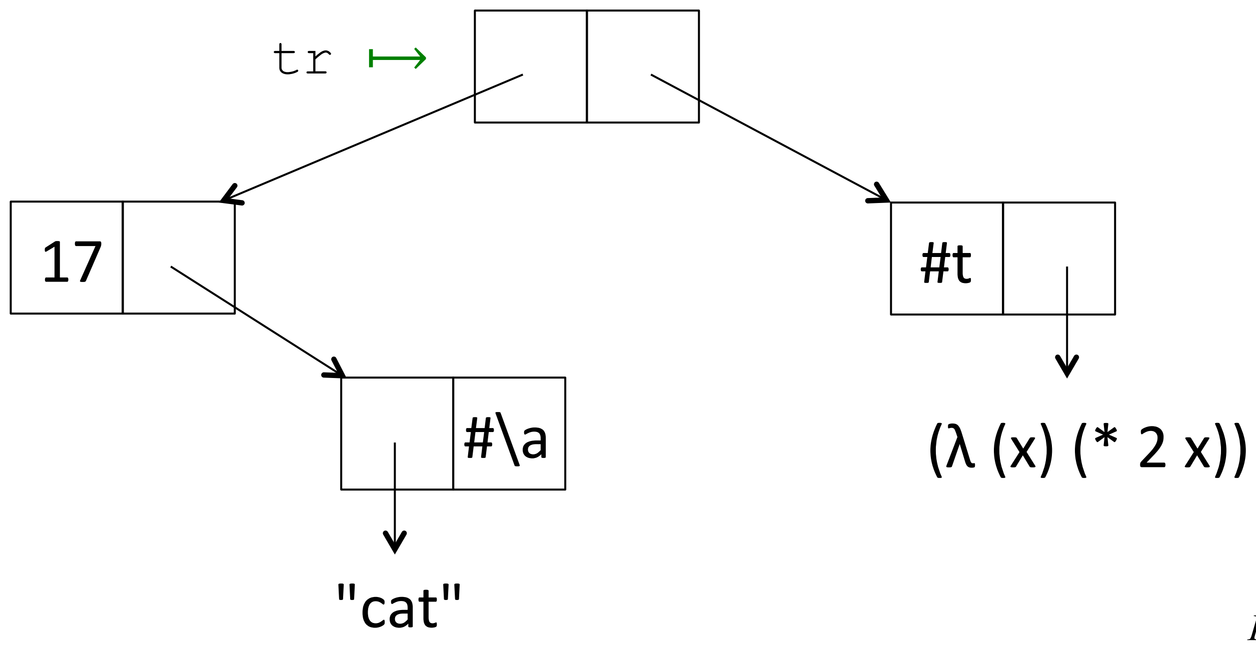
Practice with `car` and `cdr`

it's *your* turn

Write expressions using `car`, `cdr`, and `tr` that extract the five leaves of this tree:

```
(define tr (cons (cons 17 (cons "cat" #\a))
                 (cons #t (λ (x) (* 2 x)))))
```

```
tr ⟼ (cons (cons 17 (cons "cat" #\a))
           (cons #t (λ (x) (* 2 x)))), …
```



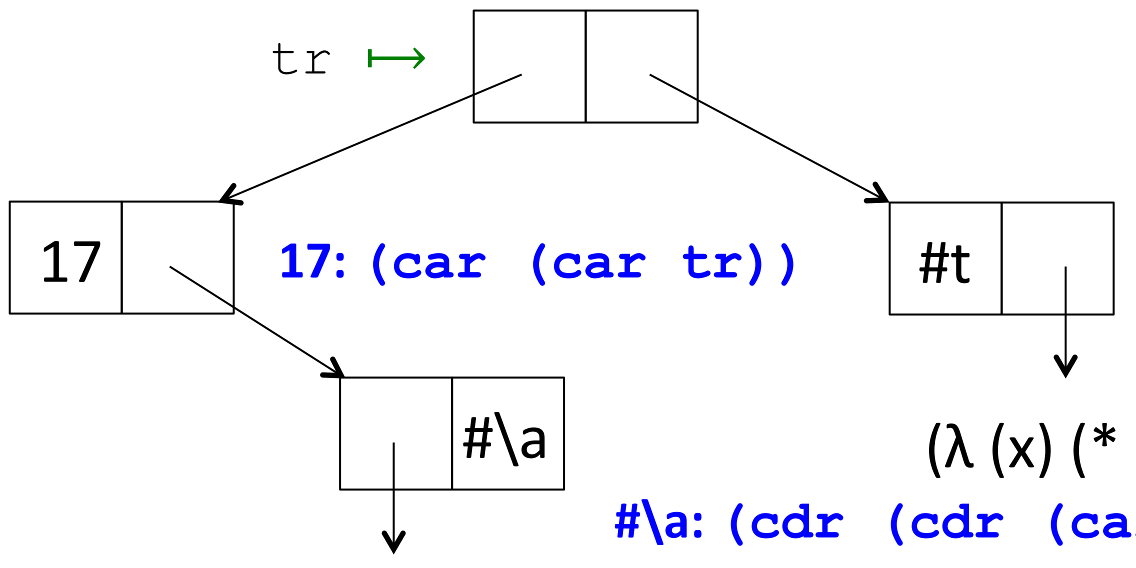
Practice with `car` and `cdr` **Solutions**

it's *your* turn

Write expressions using `car`, `cdr`, and `tr` that extract the five leaves of this tree:

```
(define tr (cons (cons 17 (cons "cat" #\a))
                 (cons #t (λ (x) (* 2 x)))))
```

```
tr ⟼ (cons (cons 17 (cons "cat" #\a))
           (cons #t (λ (x) (* 2 x)))), …
```



17: `(car (car tr))`

#t: `(car (cdr tr))`

(λ (x) (* 2 x)) : **`(cdr (cdr tr))`**

#\a: `(cdr (cdr (car tr)))`

"cat" : **`(car (cdr (car tr)))`**

`cadr` and friends

- `(caar` ***E*** `)` means `(car (car` ***E*** `))`
- `(cadr` ***E*** `)` means `(car (cdr` ***E*** `))`
- `(cdar` ***E*** `)` means `(cdr (car` ***E*** `))`
- `(cddr` ***E*** `)` means `(cdr (cdr` ***E*** `))`
- `(caaar` ***E*** `)` means `(car (car (car` ***E*** `)))`

⋮

- `(cddddr` ***E*** `)` means `(cdr (cdr (cdr (cdr` ***E*** `))))`

Any sequence of up to four `a`s and `d`s between `c`...`r` is supported.

Evaluation Rules for `car` and `cdr`

Big-step semantics:

$$\frac{E \downarrow (\texttt{cons}\ V1\ V2)}{(\texttt{car}\ E) \downarrow V1}\ [\text{car}] \qquad \frac{E \downarrow (\texttt{cons}\ V1\ V2)}{(\texttt{cdr}\ E) \downarrow V2}\ [\text{cdr}]$$

Small-step semantics:

$$(\texttt{car}\ (\texttt{cons}\ V1\ V2)) \Rightarrow V1\ \ [\text{car}]$$

$$(\texttt{cdr}\ (\texttt{cons}\ V1\ V2)) \Rightarrow V2\ \ [\text{cdr}]$$

Semantics Puzzle

According to the rules on the previous page, what is the result of evaluating this expression?

```
    (car (cons (+ 2 3) (* 4 #t)))
```

Note: there are two ``natural" answers. Racket gives one, but there are languages that give the other one!

Semantics Puzzle **Solutions**

According to the rules on the previous page, what is the result of evaluating this expression?

```
(car (cons (+ 2 3) (* 4 #t)))
```

Answer:

```
(car (cons {(+ 2 3)} (* 4 #t)))
⇒ (car (cons 5 (* 4 #t)))
```

Stuck at `(* 4 #t)`

Note: there are two ``natural” answers. Racket gives one, but there are languages that give the other one!

Side note: In so-called lazy languages like Haskell, (cons E1 E2) is a value (even if E1 and E2 aren't values) and car and cdr work as follows:

(car (cons *E1* *E2*))
⇒ ***E1*** [lazy-car]

(cdr (cons *E1* *E2*))
⇒ ***E2*** [aazy-cdr]

```
{(car (cons (+ 2 3) (* 4 #t)))}
⇒ {(+ 2 3)} [lazy-car]
⇒ 5 [addition]
```

Printed Representations in Racket Interpreter

```
> (lambda (x) (* x 2))
#<procedure>

> (cons (+ 1 2) (* 3 4))
'(3 . 12)

> (cons (cons 5 6) (cons 7 8))
'((5 . 6) 7 . 8)

> (cons 1 (cons 2 (cons 3 4)))
'(1 2 3 . 4)
```

What’s going on here?

Display Notation, Print Notation and Dotted Pairs

- The **display notation** for `(cons` ***V1 V2***`)` is `(`***DN1*** `.` ***DN2***`)`, where ***DN1*** and ***DN2*** are the display notations for ***V1*** and ***V2***
- In display notation, a dot "eats" a paren pair that follows it directly:

```
((5 . 6) . (7 . 8))
```

    becomes `((5 . 6) 7 . 8)`

```
(1 . (2 . (3 . 4)))
```

    becomes `(1 . (2 3 . 4))`

    becomes `(1 2 3 . 4)`

  Why? Because we'll see this makes lists print prettily.
- The **print notation** for pairs adds a single quote mark before the display notation. (We'll say more about quotation later.)

`display` vs. `print` in Racket

```
> (display (cons 1 (cons 2 null)))
(1 2)

> (display (cons (cons 5 6) (cons 7 8)))
((5 . 6) 7 . 8)

> (display (cons 1 (cons 2 (cons 3 4))))
(1 2 3 . 4)
```

```
> (print (cons 1 (cons 2 null)))
'(1 2)

> (print (cons (cons 5 6) (cons 7 8)))
'((5 . 6) 7 . 8)

> (print (cons 1 (cons 2 (cons 3 4))))
'(1 2 3 . 4)
```

Racket interpreter uses print (quoted) notation

```
> (cons 1 (cons 2 null))
'(1 2)

> (cons (cons 5 6) (cons 7 8))
'((5 . 6) 7 . 8)

> (cons 1 (cons 2 (cons 3 4)))
'(1 2 3 . 4)
```

Why? Because, as we'll see later, quoted values evaluate to themselves, and so are an easy way to specify a compound data value. Without the quote, the parentheses would indicate function calls and would generate errors.

```
> '(1 2)
'(1 2)

> '((5 . 6) 7 . 8)
'((5 . 6) 7 . 8)

> '(1 2 3 . 4)
'(1 2 3 . 4)
```

```
> (1 2)
application: not a procedure;
 expected a procedure that can be
applied to arguments
  given: 1
  arguments...:
```

Functions Can Take and Return Pairs

it's your turn

```
 (define (swap-pair pair)
     (cons (cdr pair) (car pair)))

(define (sort-pair pair)
    (if (< (car pair) (cdr pair))
        pair
        (swap-pair pair)))
```

What are the values of these expressions?

- `(swap-pair (cons 1 2))`
- `(sort-pair (cons 4 7))`
- `(sort-pair (cons 8 5))`

Functions Can Take and Return Pairs **Solutions**

it's your turn

```
 (define (swap-pair pair)
     (cons (cdr pair) (car pair)))

(define (sort-pair pair)
    (if (< (car pair) (cdr pair))
        pair
        (swap-pair pair)))
```

What are the values of these expressions?

- `(swap-pair (cons 1 2))` ⇒* **`'(2 . 1)`**
- `(sort-pair (cons 4 7))` ⇒* **`'(4 . 7)`**
- `(sort-pair (cons 8 5))` ⇒* **`'(5 . 8)`**

Lists

In Racket, a **list** is just a recursive pattern of pairs.

A list is either

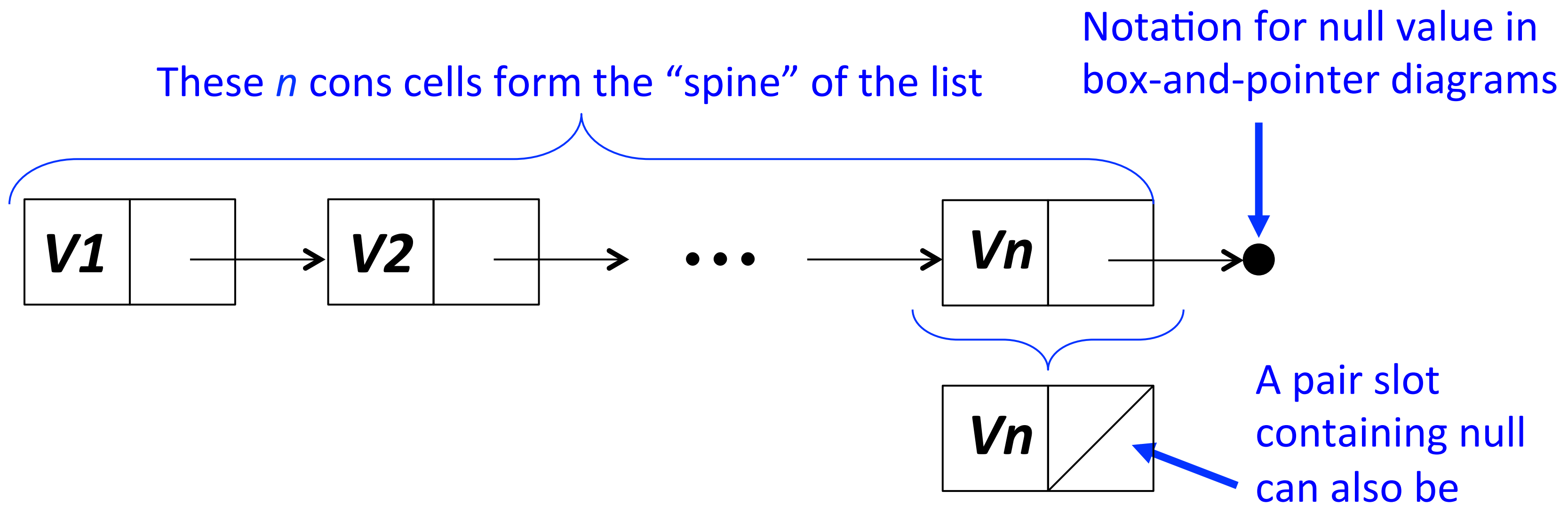
- The empty list `null`, a new value whose display notation is ()
- A nonempty list `(cons` ***Vfirst Vrest***`)` whose
  - first element is ***Vfirst***
  - and the rest of whose elements are the sublist ***Vrest***

E.g., a list of the 3 numbers 7, 2, 4 is written

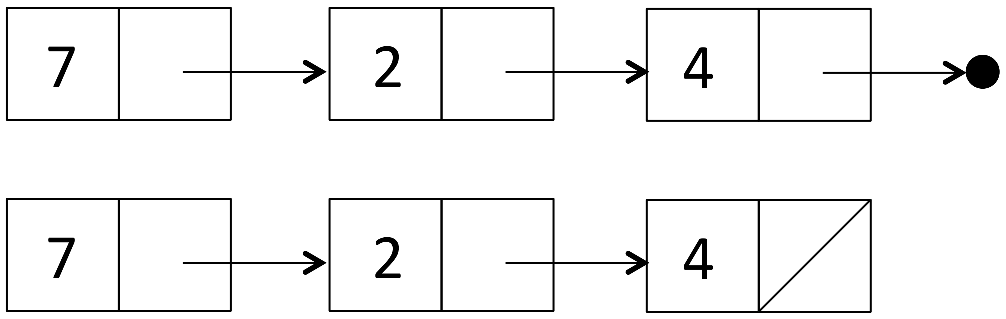
```
(cons 7 (cons 2 (cons 4 null)))
```

Box-and-pointer notation for lists

A list of *n* values is drawn like this:


These *n* cons cells form the "spine" of the list
Notation for null value in box-and-pointer diagrams
V1
V2
• • •
Vn
Vn
A pair slot containing null can also be with a slash through the slot

For example:



`list` sugar

Treat `list` as syntactic sugar:*

- `(list)` desugars to `null`
- `(list` ***E1*** `…)` desugars to `(cons` ***E1*** `(list …))`

For example:

```
(list (+ 1 2) (* 3 4) (< 5 6))
```

desugars to `(cons (+ 1 2) (list (* 3 4) (< 5 6)))`

desugars to `(cons (+ 1 2) (cons (* 3 4) (list (< 5 6))))`

desugars to `(cons (+ 1 2) (cons (* 3 4) (cons (< 5 6) (list))))`

desugars to `(cons (+ 1 2) (cons (* 3 4) (cons (< 5 6) null)))`

* This is a white lie, but we can pretend it's true for now

Display Notation for Lists

The “dot eats parens” rule makes lists display nicely:

`(list 7 2 4)`

desugars to `(cons 7 (cons 2 (cons 4 null))))`

displays as (before rule) `(7 . (2 . (4 . ())))`

displays as (after rule) `(7 2 4)`

prints as `'(7 2 4)`

In Racket:

```
> (cons 7 (cons 2 (cons 4 null)))
'(7 2 4)

> (list 7 2 4)
'(7 2 4)
```

`list` and small-step evaluation

In small-step derivations, it's helpful to both desugar and resugar with `list`:

```
(list (+ 1 2) (* 3 4) (< 5 6))
desugars to (cons {(+ 1 2)} (cons (* 3 4)
                                  (cons (< 5 6) null)))
⇒ (cons 3 (cons {(* 3 4)} (cons (< 5 6) null)))
⇒ (cons 3 (cons 12 (cons {(< 5 6)} null)))
⇒ (cons 3 (cons 12 (cons #t null)))
resugars to (list 3 12 #t)
```

Heck, let's just informally write this as:

```
(list {(+ 1 2)} (* 3 4) (< 5 6))
⇒ (list 3 {(* 3 4)} (< 5 6))
⇒ (list 3 12 {(< 5 6)})
⇒ (list 3 12 #t)
```

`first`, `rest`, and friends

- `first` returns the first element of a list:

  ```
  (first (list 7 2 4)) ⇒ 7
  ```

  (`first` is almost a synonym for `car`, but requires its argument to be a list)

- `rest` returns the sublist of a list containing every element but the first:

  ```
  (rest (list 7 2 4)) ⇒ (list 2 4)
  ```

  (`rest` is almost a synonym for `cdr`, but requires its argument to be a list)

- Also have `second`, `third`, …, `ninth`, `tenth`

- Stylistically, `first`, `rest`, `second`, `third` preferred over `car`, `cdr`, `cadr`, `caddr` because emphasizes that argument is expected to be a list.

`first`, `rest`, and friends examples

```
> (define L '(10 20 (30 40 50 60)))

> (first L)
10

> (second L)
20

> (third L)
'(30 40 50 60)

> (fourth (third L))
60

> (rest (third L))
'(40 50 60)
```

```
> (fourth L)
fourth: list contains too few elements
  list: '(10 20 (30 40 50 60))

> (first '(1 2 3 . 4))
first: contract violation
  expected: (and/c list? (not/c empty?))
  given: '(1 2 3 . 4)
```

`length`

`length` returns the number of top-level elements in a list:

```
> (length (list 7 2 4))
3

> (length '((17 19) (23) () (111 230 235 251 301)))
4

> (length '())
0

> (length '(()))
1

> (length '(1 2 3 . 4))
length: contract violation
  expected: list?
  given: '(1 2 3 . 4)
```

List exercise

```
(define LOL
  (list (list 17 19)
        (list 23 42 57)
        (list 110 (list 111 230 235 251 301) 304 342)))
```

- What is the printed representation of LOL?
- Give expressions involving LOL that return the following values:
  - 19
  - 23
  - 57
  - 251
  - `'(235 251 301)`
- What is the value of

```
(+ (length LOL)
   (length (third LOL))
   (length (second (third LOL))))?
```

List exercise Solutions

it's your turn

```
(define LOL
  (list (list 17 19)
        (list 23 42 57)
        (list 110 (list 111 230 235 251 301) 304 342)))
```

- What is the printed representation of LOL?

  `'((17 19) (23 42 57) (110 (11 230 235 251 301) 304 342))`
- Give expressions involving LOL that return the following values:
  - 19 : `(second (first LOL))`
  - 23 : `(first (second LOL))`
  - 57 : `(third (second LOL))`
  - 251 : `(fourth (second (third LOL)))`
  - `'(235 251 301)` : `(rest (rest (second (third LOL))))`
- What is the value of

```
(+ (length LOL) ; ⇒* 3
   (length (third LOL)) ; ⇒* 4
   (length (second (third LOL))) ; ⇒* 5
   ) ; ⇒* 12
```

append

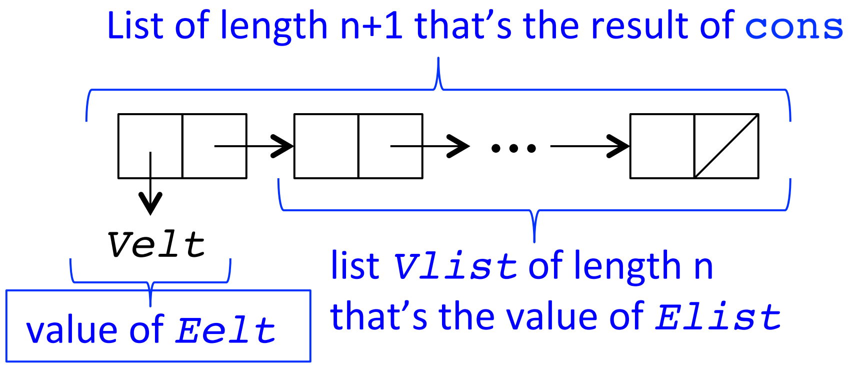
`append` takes any number of lists and returns a list that combines all of the top-level elements of its argument lists.

```
> (append '(17 19) '(23 42 57))
'(17 19 23 42 57)

> (append '(17 19) '(23 42 57) '(111) '() '(230 235 251 301))
'(17 19 23 42 57 111 230 235 251 301)

> (append '((0 1) 2 (3 (4 5))) '(() (6 (7 8) 9)))
'((0 1) 2 (3 (4 5)) () (6 (7 8) 9))

> (append '(0 1) 2 '(3 (4 5)))
append: contract violation
  expected: list?
  given: 2
```

`cons` vs. `list` vs. `append`

`cons`, `list`, and `append` are the three most common ways to build lists. They are very different! Since you will use them extensively in both Racket and Standard ML, it's important to master them now!

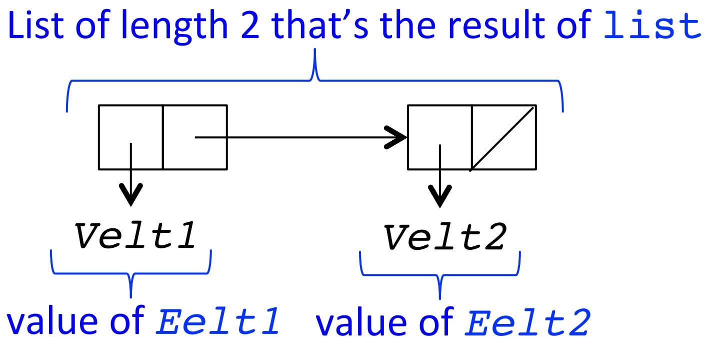
In the context of lists, `(cons` ***`Eelt Elist`***`)` creates one new cons-cell and returns a list whose length is 1 more then the length of its 2nd argument (assumed to be a list here).



List of length n+1 that's the result of cons
Velt
value of Eelt
list Vlist of length n that's the value of Elist

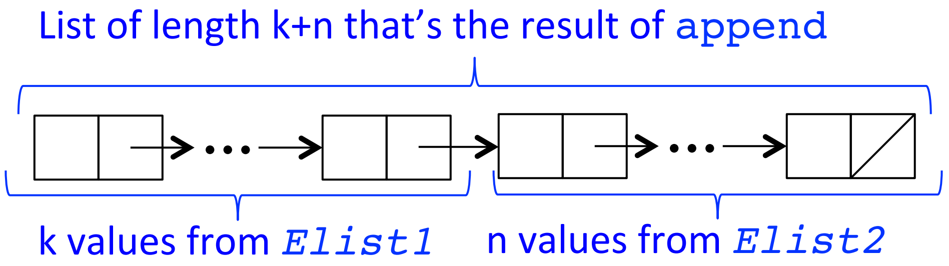
`(list` ***`Eelt1 Eelt2`***`)` creates a list of length 2 using two new cons-cells.

`(list` ***`Eelt1`*** `…` ***`Eeltn`***`)` creates a list of length n



`(append Elist1 Elist2)` only makes sense if `Elist1` and `Elist2` denote lists. It returns a list whose length is the sum of the length of the two lists.

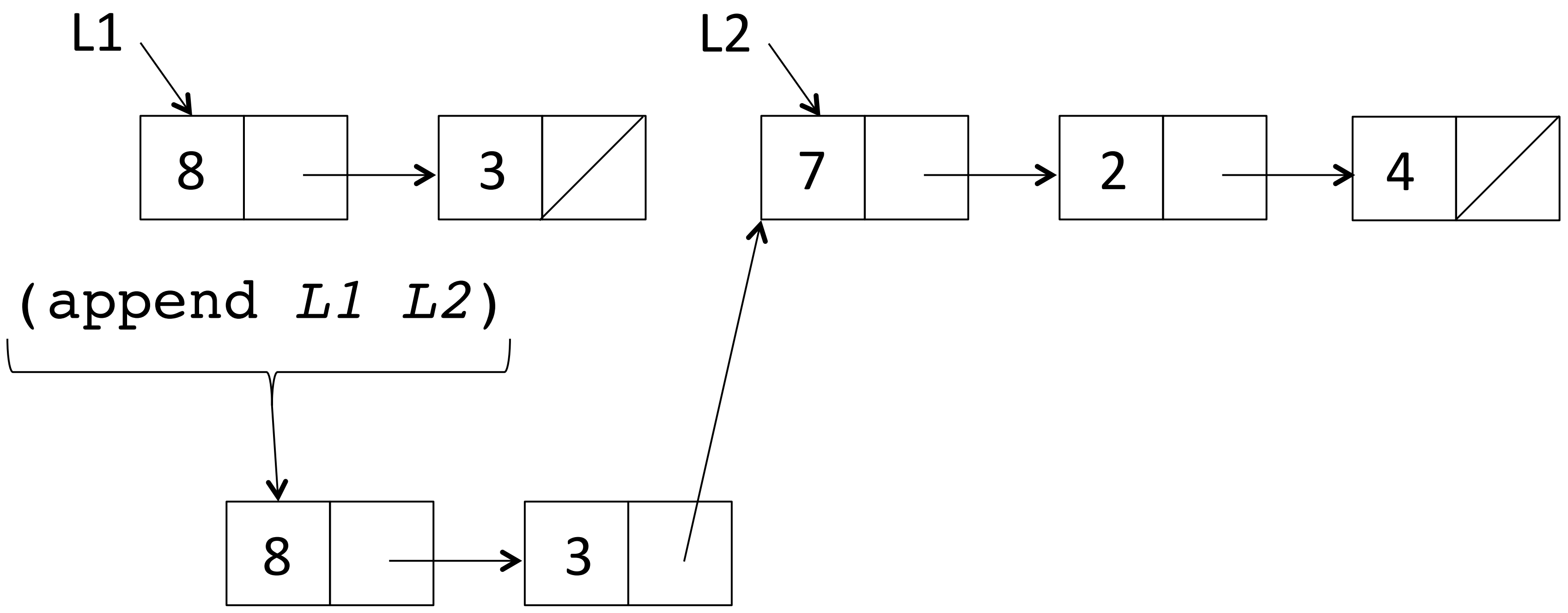
`append` can be applied to *any* number of lists.



`append` and sharing

Given two lists *L1* and *L2*, `(append L1 L2)` copies the list structure of L1 but shares the list structure of L2.

For example:



- This fact important when reasoning about number of cons-cells created by a program.
- We'll see why it's true in the next lecture, when we see how `append` is implemented
- Given more than two lists, `append` copies all but the last and only shares the last.

`cons` vs. `list` vs. `append` exercise

it's *your* turn

Suppose you are given:

```
(define L1 '(7 2 4))
(define L2 '(8 3 5))
```

For each of the following three lists:

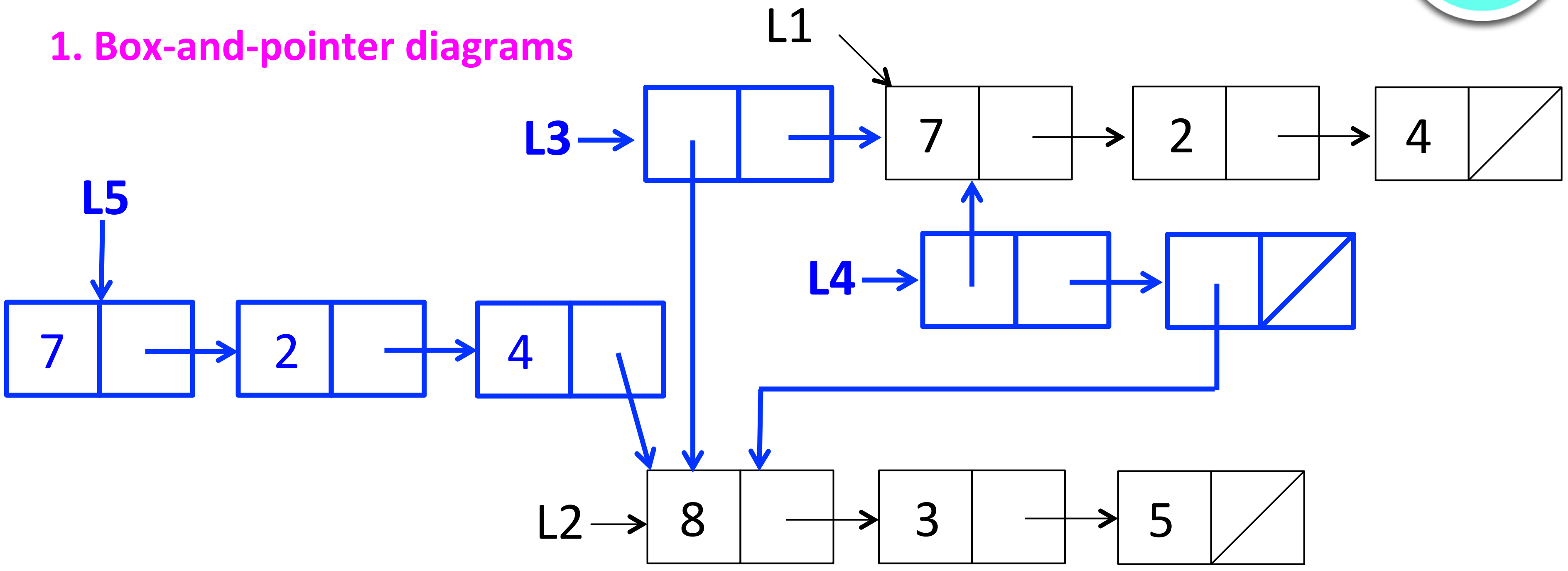
1. Draw the box-and-pointer structure for its value
2. Indicate the number of conses created for its value
3. Write the quoted notation for its value
4. Determine the length of its value

```
(define L3 (cons L1 L2))

(define L4 (list L1 L2))

(define L5 (append L1 L2))
```

`cons` vs. `list` vs. `append` Solutions

it's your turn

1. Box-and-pointer diagrams



List	Definition	2. # Conses	3. Quoted Notation	4. Length
`L3`	`(cons L1 L2)`	1	`'((8 3 5) 7 2 4)`	4
`L4`	`(list L1 L2)`	2	`'((8 3 5) (7 2 4))`	2
`L5`	`(append L1 L2)`	3	`'(8 3 5 7 2 4)`	6

Use `(cons` *`Eval Elist`*`)` rather than `(append (list` *`Eval`*`)` *`Elist`*`)`

Although `(cons` *`Eval Elist`*`)` and `(append (list` *`Eval`*`)` *`Elist`*`)` return equivalent lists, the former is preferred stylistically over the latter (because the former creates only one cons-cell, but the latter creates two).

For example, use this:

```
> (cons (* 6 7) '(17 23 57))
'(42 17 23 57)
```

Rather than this:

```
> (append (list (* 6 7)) '(17 23 57))
'(42 17 23 57)
```