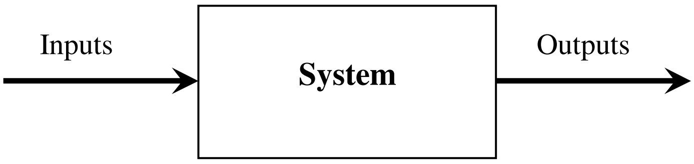
2. Introduction to Systems

2.1 What is a system?

- As stated in the first section, a **system** is a collection of components acting together to perform a specific task – it is anything with inputs and outputs.



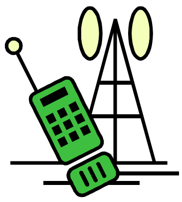
- A **component** is a single functioning unit of the system.

- Systems are not always physical – they can extend to more abstract phenomena, for example: economical, transportation, population, biological, etc.

- Example systems include:

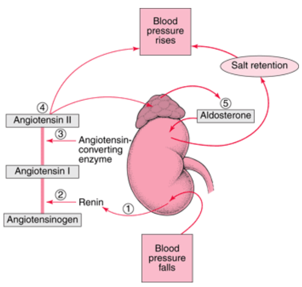
Toaster

Stereo



Mobile Phone

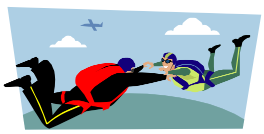
Car



Blood Pressure Regulation

Tipping

The Human Brain



Sky Diving

- Most systems are sub-systems of other larger systems. For example:
  - a car is a system with many subsystems including brakes, steering, suspension, etc.
  - electricity supply grid - transmission lines, transformers, consumer appliances, etc.
  - human body - breathing, blood pressure, heart rate, sleeping etc.

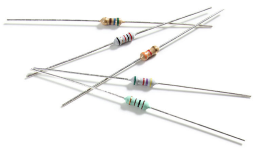
2.2 Dynamical and static systems

- A **dynamical system** is where the current output depends on past inputs. As such, a dynamical system is also referred to as a system with memory.

- On the other hand, a **static system** is where the current output depends only on the current input. It cannot depend on past values and, as such, these systems are referred to as systems with no memory.

- In a static system, the output remains constant if the input does not change. In a dynamical system the output can change with time even though the input is no longer changing.

- Examples of dynamical systems include:
  - steering a car
  - accelerating a car
  - aircraft altitude/ship steering autopilot
  - domestic central heating
  - the currency exchange rate
  - blood pressure control
  - the weather
  - the human body
  - a cup of tea !



- *Can you give some examples of systems that are not dynamical (i.e. static systems)?*

- The world is full of dynamical systems. In fact, the world itself is a dynamical system!

2.3 Categorizing systems

- Systems can be categorized as:
  - linear or nonlinear
  - time-invariant or time-varying
  - continuous or discrete

2.3.1 Linear v Nonlinear

- **Linear** systems possess two important properties, namely *superposition* and *homogeneity*. If these properties are not withheld, then the system is said to be **nonlinear**.

- *The principle of superposition* - this states that the response produced by a combination of different inputs is the sum of the individual responses produced by each of the different inputs acting alone.

- In other words, the response of a multi-input linear system can be determined by dealing with each input in turn and then adding the results.

- *Homogeneity* (or *scaling*) – this states that if an input $u(t)$ produces and output $y(t)$ then, for a linear system, an input $ku(t)$ will produce an output $ky(t)$, where k is a scalar.

- We can easily demonstrate these properties with some trivial examples.

- Consider the following function with output y and inputs u_1 and u_2.

$$y(t) = 3u_1(t) + 5u_2(t)$$

- If, at an arbitrary point in time, $u_1(t) = 2$ and $u_2(t) = 3$, then:

$$y(t) = 3(2) + 5(3) = 21$$

- Now, let's obtain the output $y(t)$ by considering each input separately, as follows:

  First, let $u_2(t) = 0$, then $y_1(t) = 3(2) + 5(0) = 6$

  Now, let $u_1(t) = 0$, then $y_2(t) = 3(0) + 5(3) = 15$

  Overall, $y(t) = y1(t) + y2(t) = 6 + 15 = 21$ … as above

- Hence, this very simple system obeys the principle of superposition.

- If we multiply both inputs by a factor of 2 (for example) then:

$$y(t) = 3(4) + 5(6) = 42 \qquad = \qquad 2 \text{ x } 21$$

- Thus, the new output is also multiplied (i.e. scaled) by a factor of 2. Hence the system possesses the homogeneity property and thus this system is linear.

- Now consider another very simple system:

$$y(t) = 3u_1(t) + 5u_2(t) + 4$$

- Taking an arbitrary point in time, where $u_1(t) = 2$ and $u_2(t) = 3$, then:

$$y(t) = 3(2) + 5(3) + 4 = 25$$

- Once again, let's obtain the output $y(t)$ by considering each input separately, as follows:

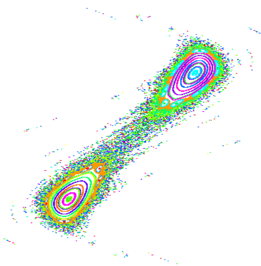
  First, let $u_2(t) = 0$, then $y_1(t) = 3(2) + 5(0) + 4 = 10$

  Now, let $u_1(t) = 0$, then $y_2(t) = 3(0) + 5(3)+4 = 19$

  Overall, $y(t) = y1(t) + y2(t) = 10 + 19 = 29$ … different to above

- Hence, this system is not linear as it does not obey the principle of superposition.

- ***Ex2.1 Is the following system linear or nonlinear?***



$$y(t) = 3u_1(t)^2$$

- Take the arbitrary value, $u_1(t) = 2$ then:

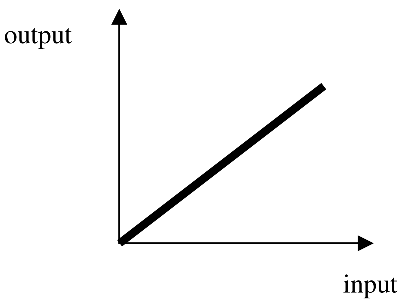
$$y(t) = 3(2)^2 = 3(4) = 12$$

- Now, let's scale the input by a factor of 2, i.e. $2 \times u_1(t)$, then:

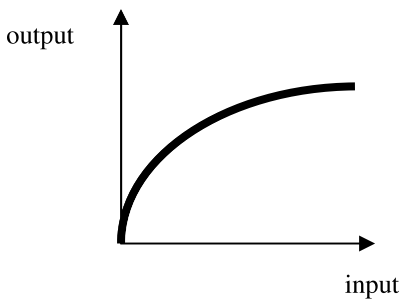
$$y(t) = 3(4)^2 = 3(16) = 48 \qquad \neq \qquad 2 \times 12$$

- Hence, this system is nonlinear, as it does not possess the homogeneity property.

- Graphically the difference between linear and nonlinear systems can be illustrated as follows:

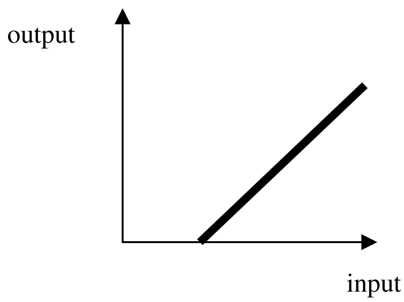


Linear system



Nonlinear system

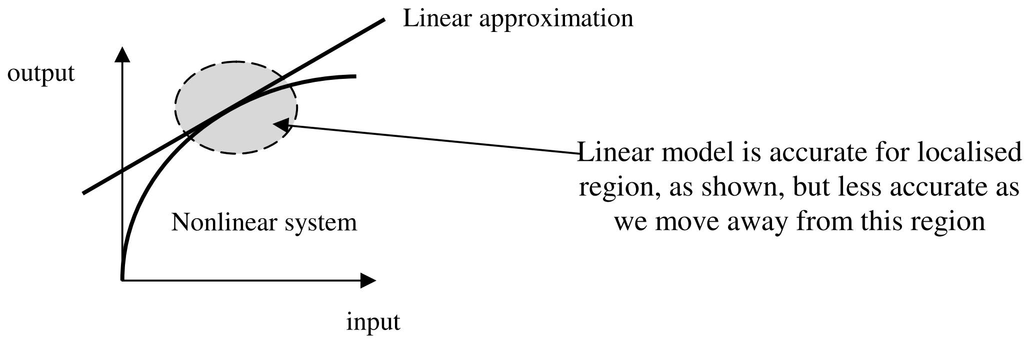
- *Question – is the following representative of a system that is linear or nonlinear? Why?*



- In general, analyzing nonlinear systems can be complicated and can require difficult mathematical calculations.

- It is, therefore, common practice to *linearize* such systems about specific operating points. Thus, simpler, linear mathematical techniques can now be applied to the linear approximation of the nonlinear system.

- Graphically, we can demonstrate the concept of linearization as follows:



- You will study linearization in more detail in EE211 System Dynamics.

2.3.2 Time-invariant v Time-varying

- The output of a **time-invariant** system does not explicitly depend on *time* whereas the output of a **time-varying** system does.

- For example, consider the following systems with output y and input u:

System 1: $y(t) = 3u(t)$

and

System 2: $y(t) = sin(t)\, u(t)$

- Here, system 1 is time-invariant but system 2 is time-varying as it depends explicitly on the time parameter t.

- Note, the input function (or signal) $u(t)$ clearly depends on time, as it is a function of time.

- However, it is the relationship between the input $u(t)$ and the output $y(t)$, i.e. the system, that we are referring to and the question becomes does this contain a parameter directly affected by time?

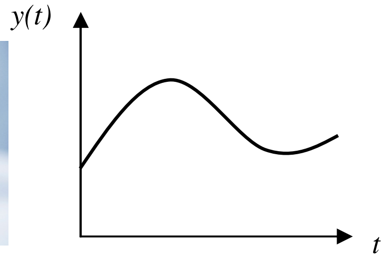
- In system 1, the system is simply a constant value, i.e. 3, and independent of time. In system 2, the system is $sin(t)$ and clearly dependent on time and hence time-varying.

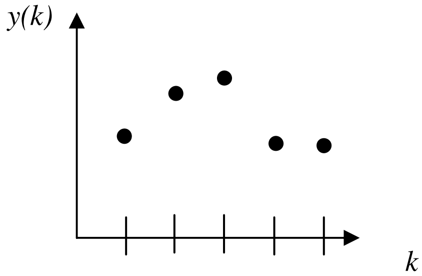
- A resistor is a time-invariant system (recall Ohm's Law) while an aircraft can be regarded as a time-varying system, as it's dynamics are effective by parameters of the system that change with time, such as weight change due to fuel consumption.

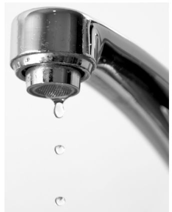
2.3.3 Continuous v Discrete

- **Continuous-time** systems involve signals that are continuous in time while **discrete-time** systems involve variables that only change at discrete instants of time.

- This concept is simply captured in the following illustration:





Continuous-time output

Discrete-time output

- ***In this module, we will focus on continuous- time, linear time-invariant (LTI) system and analysis.***

2.4 Mathematical modelling (*an overview*)

- In order to design a controller for a system, we must first model the system, so that we can analyse its response and predict its performance.

- The model of the system is typically based on a mathematical description of the dynamical characteristics of the system.

- This mathematical description is referred to as a **mathematical model.**

- For many practical systems, useful mathematical models are generally described in terms of **differential equations**, for continuous-time systems, or **difference equations**, for discrete-time systems.

- For example, a sky diver falling under gravity g and encountering a resistance proportional to his velocity v can be modelled by the following first order *differential equation*, where k is a constant:

$$\frac{dv}{dt} + kv = g$$

- In contrast, given that a county's population in year k is $x(k)$ and that there is a constant birth rate b and a constant death rate d, then the country's population growth can be modelled by the *difference equation*:

$$x(k+1) = x(k) + bx(k) - dx(k)$$

- While these equations can be solved in their current form, it is easier to work with them in an alternative format know as the **transfer function** format.

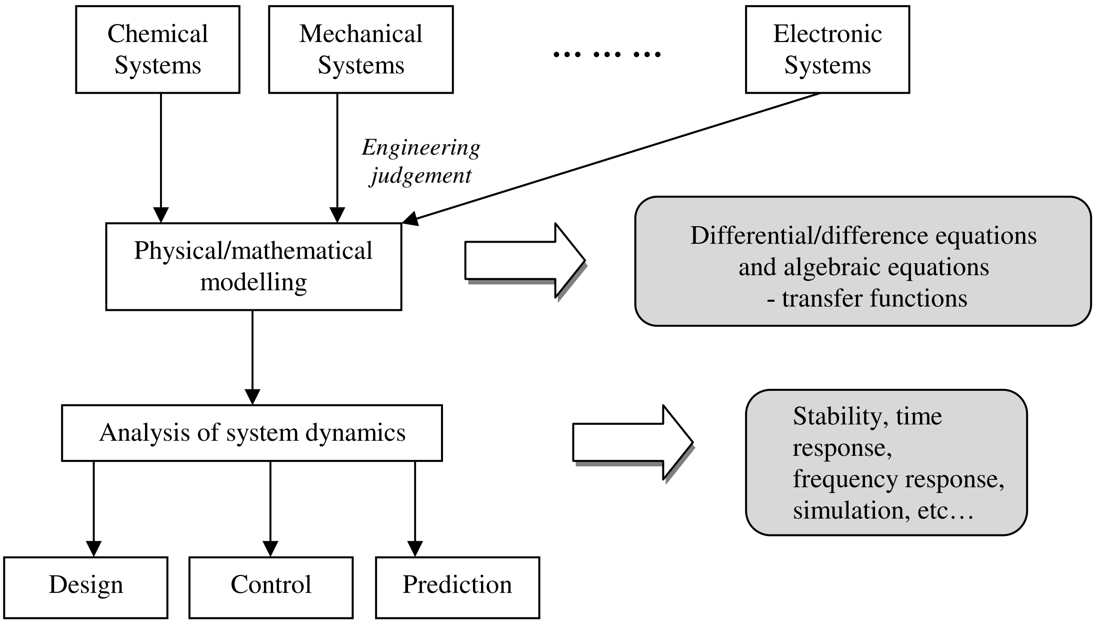
- **Laplace transforms** are used to convert differential equations into algebraic form, while **Z-transforms** are used to convert difference equations.

- ***Later in this module, we will introduce and examine the transfer function format for continuous-time systems and, consequently, introduce and briefly examine Laplace transforms.***

- *Laplace Transforms are also covered in more detail in the mathematic modules EE112 and (more significantly) EE206.*

- *Z-transforms will not be covered in this module but will be studied in detail in other modules later in the programme, including EE206, EE211 and EE213.*

2.5 The big picture



- System dynamics involves developing a mathematical model of a dynamical system with a view to analyzing the system's performance.

- This, in turn, can lead to better design, better control and better prediction of the system.

- Ultimately, the success of all these depends on the validity of the underlying mathematical model. In other words, the validity of a prediction depends largely on how accurately the mathematical model captures the behaviour of the actual system.

- This concept of modelling, analysis and design/control/prediction is summarized in the 'big picture' diagram given previously.

- ***In terms of this module, we will be focus primarily on some basic modelling, with a key emphasis on the transfer function representation for continuous-time systems.***