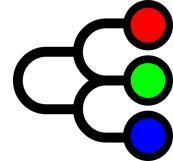


Combinatorial Programming with Functions

slides.com/jod/manyworlds-modref

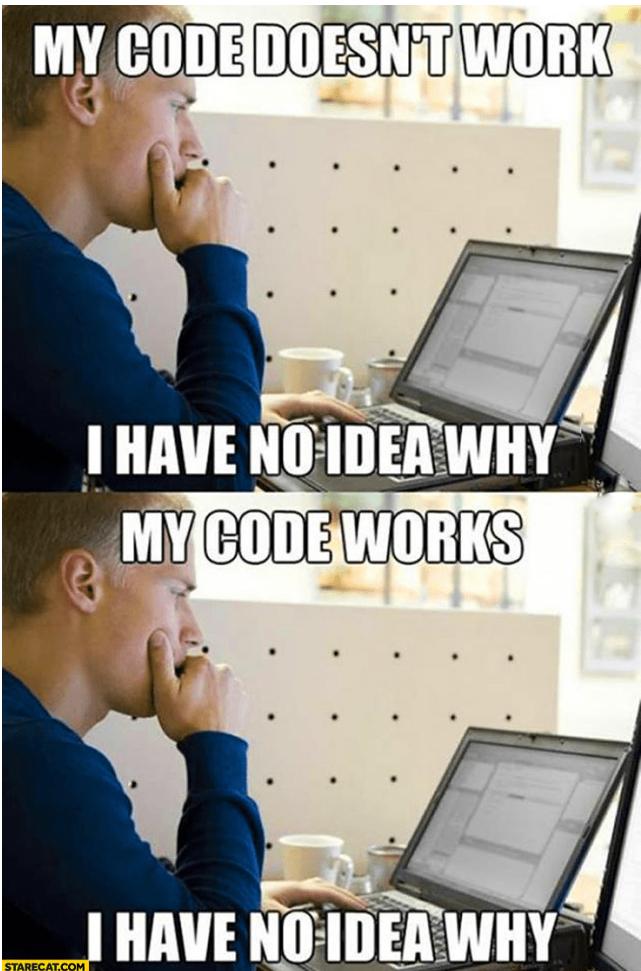
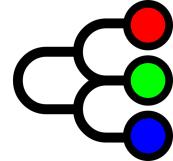
We were all n00bs once...



STARECAT.COM

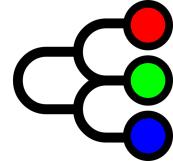
NF
S_

...with a problem to solve



- a tournament roster for your sports club
- a seat arrangement for a wedding
- a dice odds calculation for a game
- a class schedule for a school
- ...

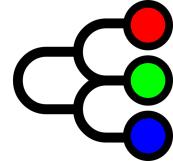
...with a problem to solve



- a tournament roster for your sports club
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- ...

You were good at high-school math, regularly use spreadsheet formulas.

...with a problem to solve

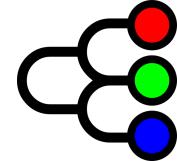


- a tournament roster for your sports club
- a seat arrangement for a wedding
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- a class schedule for a school
- ...

You were good at high-school math, regularly use spreadsheet formulas.

So you know about
arithmetic formulas and
function application.

...with a problem to solve



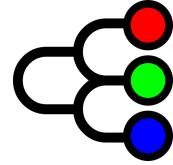
- a tournament roster for your sports club
- a seat arrangement for a wedding
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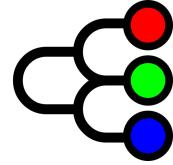
How would you go about this?

ManyWorlds



A friendly, small, simple (??) combinatorial programming language

ManyWorlds



A friendly, small, simple (??) combinatorial programming language

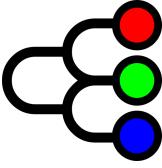
Basic building blocks:

values

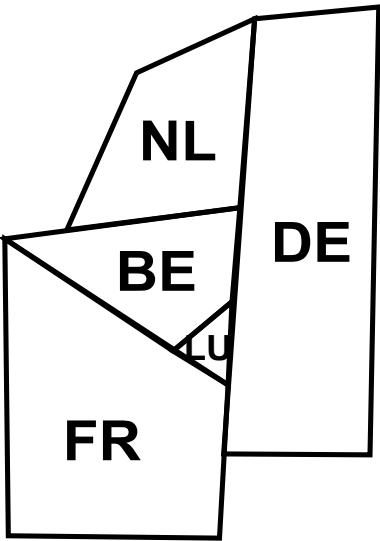
`bool, int, string`

total functions

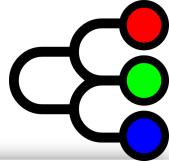
`type* -> type`



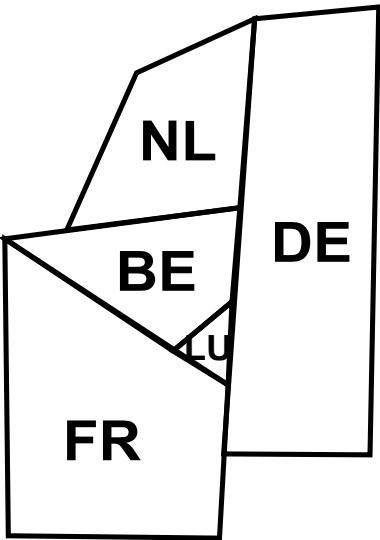
Map coloring



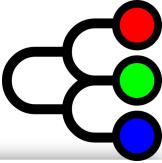
NF
S_



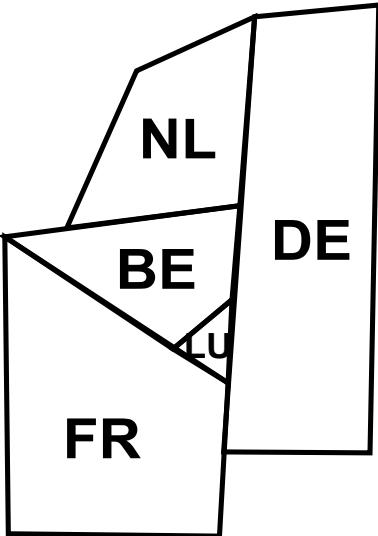
Map coloring



```
1 declare color: string -> {"r", "g", "b", "y"}.
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3 color("NL") != color("BE"). color("NL") != color("DE").
4 color("BE") != color("LU"). color("BE") != color("DE").
5 color("BE") != color("FR"). color("FR") != color("LU").
6 color("FR") != color("DE"). color("LU") != color("DE").
```



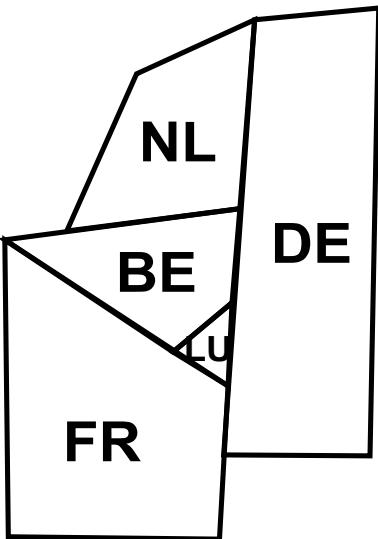
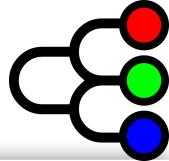
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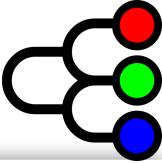
- **User functions:** uninterpreted functions with a total co-domain [*variable(s)* (*arrays*)]

Map coloring

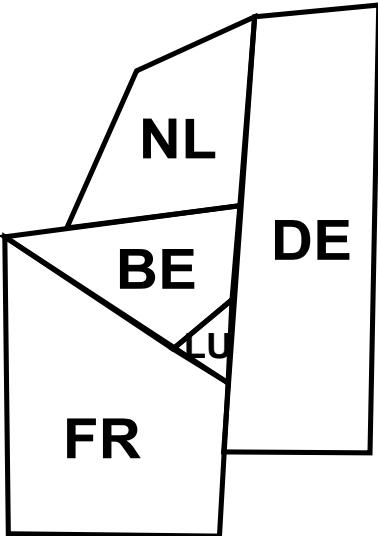


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- **User functions:** uninterpreted functions with a total co-domain [*variable(s)* (*arrays*)]
- **Expressions:** (nested) function applications

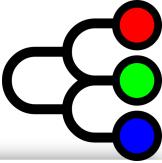


Map coloring

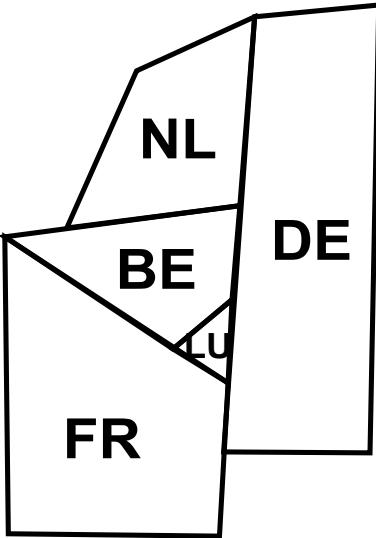


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- **Constraints:** expressions that must be true

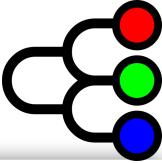


Map coloring

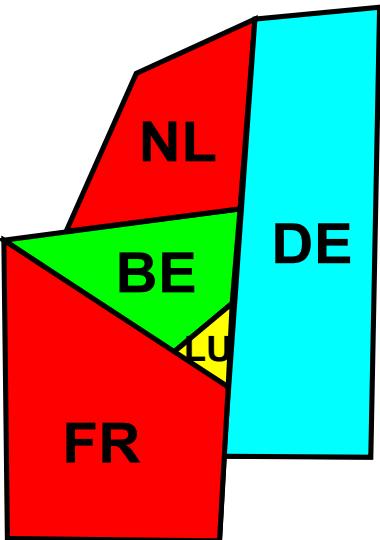


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- **User functions:** uninterpreted functions with a total co-domain [*variable(s)* (*arrays*)]
- **Expressions:** (nested) function applications
- **Constraints:** expressions that must be true
- **Worlds:** user function interpretations that make all constraints true [*solutions*]



Map coloring



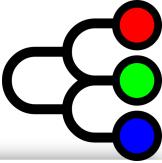
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```

FINDING...

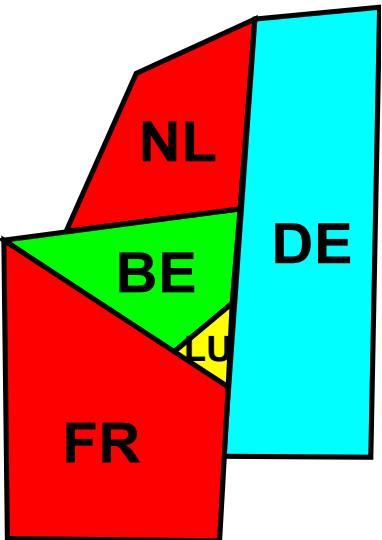
FOUND WORLD

```
define color as {("BE", "g"), ("DE", "b"), ("FR", "r"),
("LU", "y"), ("NL", "r")} default unknown.
```

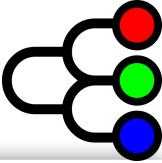
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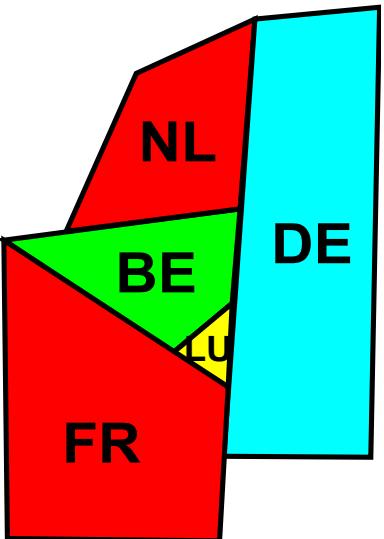
Map coloring bis



```
1 declare color: string -> {"r", "g", "b", "y"}.
2
3 // declare border: string, string -> bool.
4 decdef border as {("NL", "BE"), ("NL", "DE"),
5 ("BE", "LU"), ("BE", "DE"), ("BE", "FR"),
6 ("FR", "LU"), ("FR", "DE"), ("LU", "DE")}.
7
8 all [ color(x)!=color(y) for x,y where border(x,y) ].
```



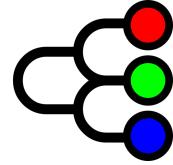
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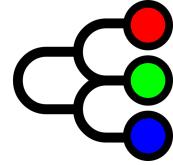
"Fold-Map-Filter"
expression

FMF expressions



```
all [ color(x)!=color(y) for x,y where border(x,y) ].
```

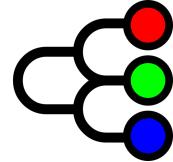
FMF expressions



```
all [ color(x)!=color(y) for x,y where border(x,y) ].
```

Filter: select all x, y where
 $\text{border}(x, y)$ holds

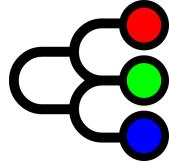
FMF expressions



```
all [ color(x)!=color(y) for x,y where border(x,y) ].
```

Map: map those x, y to
 $\text{color}(x) \neq \text{color}(y)$

Filter: select all x, y where
 $\text{border}(x, y)$ holds



FMF expressions

Fold: reduce those

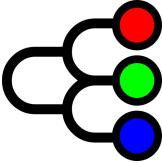
`color(x) != color(y)`

to true iff all are true

```
all [ color(x) != color(y) for x,y where border(x,y) ].
```

Map: map those x, y to
`color(x) != color(y)`

Filter: select all x, y where
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FMF expressions

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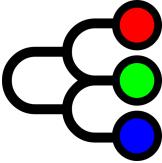
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```

Map: map those x, y to
`color(x) != color(y)`

Filter: select all x, y where
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all
any
none
count
sum
product

min
max
distinct
same
odd
even



FMF expressions

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`color(x) != color(y)`

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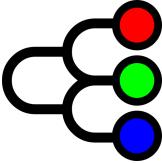
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- generalize quantification / aggregates



FMF expressions

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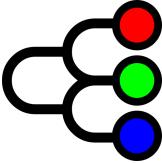
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- generalize quantification / aggregates
- take over role of "joker" global constraints - e.g., `alldiff_except_0`



FMF expressions

Fold: reduce those

`color(x) != color(y)`

to true iff all are true

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all [ color(x) != color(y) for x,y where border(x,y) ].
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Map: map those x, y to
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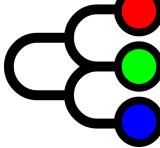
Filter: select all x, y where
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all
any
none
count
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- generalize quantification / aggregates
- take over role of "joker" global constraints - e.g., `alldiff_except_0`
- nestable

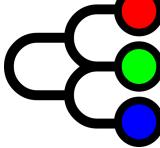
More language extensions



Done

- builtin functions and operators
- syntactic sugar
- arbitrary precision integers
- user types
- intensional definitions
- python definitions

More language extensions



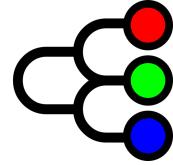
Done

- builtin functions and operators
- syntactic sugar
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- python definitions

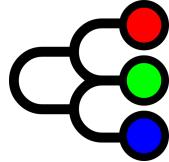
To do

- pseudo-rational division
- tuple values
- recursive definitions
- ...

Inferences & objectives



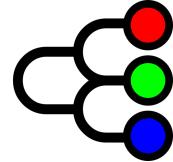
Inferences & objectives



inferences

find
count
intersect

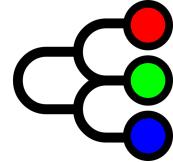
Inferences & objectives



objectives

	-	@maximize	@minimize	@mode
inferences				
find				
count				
intersect				

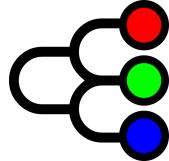
Inferences & objectives



objectives

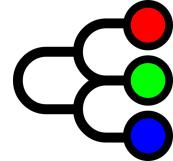
inferences	-	@maximize	@minimize	@mode
find	✓	✓	✓	✓
count	✓	✓	✓	✓
intersect	✓	✓	✓	✓

Count + @mode example



```
1 // we have 5 dice
2 decdef die as {"d" 1 .. 5}.
3
4 // with 1 to 6 dots
5 decdef dots as {1 .. 6}.
6
7 // rolling assigns a number of dots to each die
8 declare roll: die -> dots.
9
10 // the sum of the dice rolls must be 14
11 sum[ roll(x) for x where die(x) ] = 14.
12
13 // at most two dice can have rolled the same dots
14 all[
15   count[ roll(x) = y for x where die(x) ] <= 2
16 for y where dots(y)].|
17
18 // show the most common value ('mode') of the highest die
19 // this also yields statistics on the highest die
20 @mode max[ roll(x) for x where die(x) ].
```

Count + @mode example



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1 // we have 5 dice
2 decdef die as {"d" 1 .. 5}.
3
4 // with 1 to 6 dots
5 decdef dots as {1 .. 6}.
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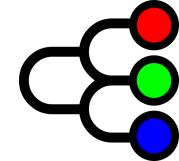
PARSING...
#seconds 0.000000

COMPILING...
#vars 61 #constraints 124 #seconds 0.001000
7776 candidate(s) exist

CALCULATING DISTRIBUTION OF WORLDS...
#seconds 0.060000
450 world(s) exist
-> 5.787037% of candidates

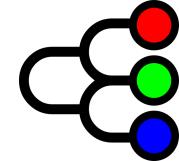
5: 210 (mode objective fixed to this)
6: 150
4: 90
mean: 5.133333 (77/15)
median: 5

Development support



- online IDE
- simple syntax highlighting
- helpful error/warning messages
- **debugging** support

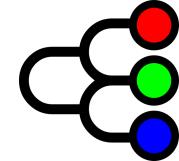
Development support



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(3-valued) **expression evaluation**

Development support

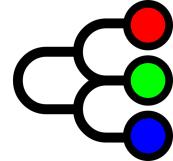


- online IDE
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(3-valued) **expression evaluation**

```
declare drinksAlcohol: -> bool.  
declare age: -> {0 .. 150}.  
  
age() >= 18 implies drinksAlcohol().  
  
define drinksAlcohol() as true.  
define age() as 0.
```

Development support



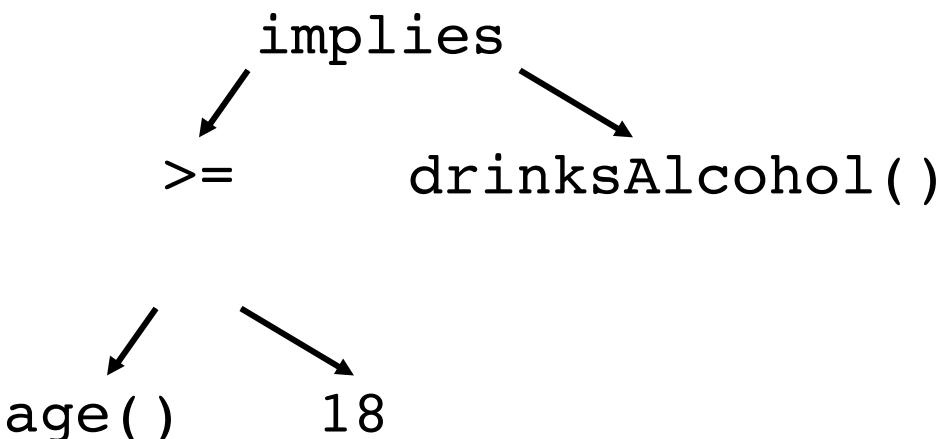
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(3-valued) expression evaluation

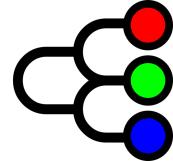
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declare drinksAlcohol: -> bool.  
declare age: -> {0 .. 150}.
```

```
age() >= 18 implies drinksAlcohol().
```

```
define drinksAlcohol() as true.  
define age() as 0.
```



Development support



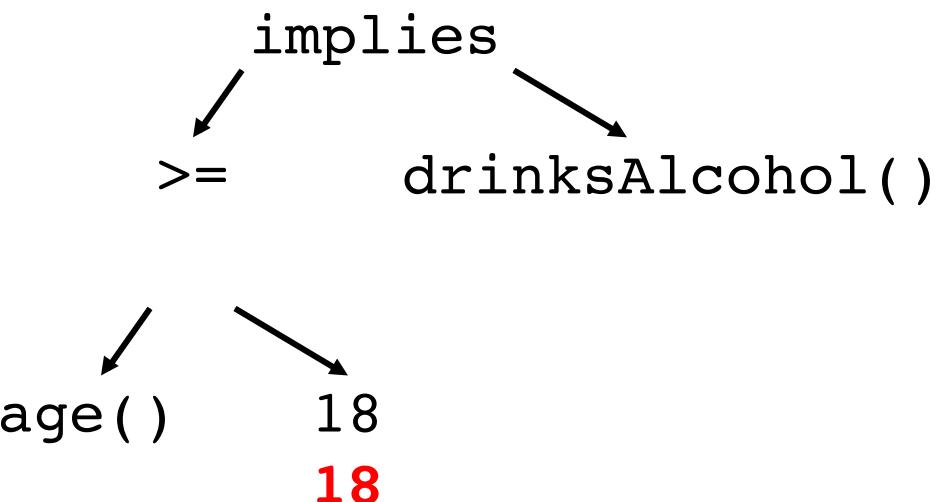
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(3-valued) expression evaluation

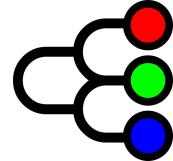
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declare drinksAlcohol: -> bool.  
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age() >= 18 implies drinksAlcohol().
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```



Development support



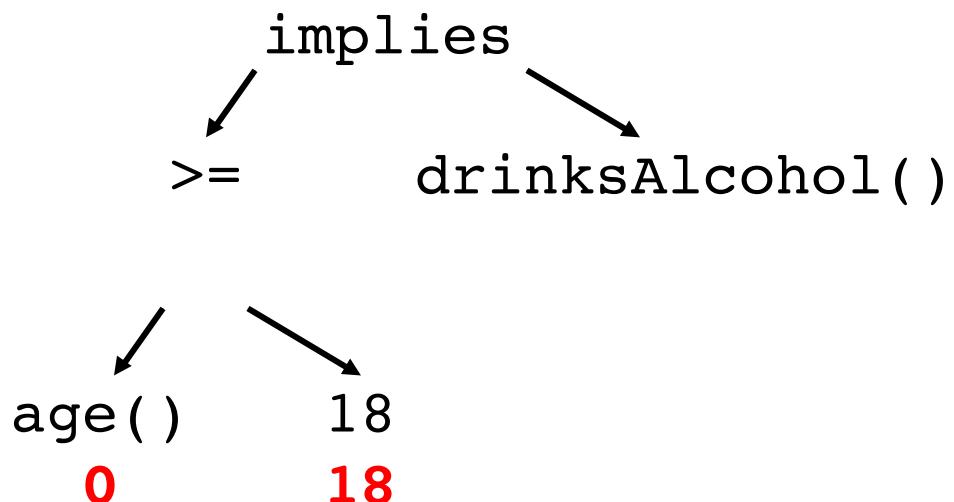
- online IDE
- simple syntax highlighting
- helpful error/warning messages
- **debugging** support

(3-valued) expression evaluation

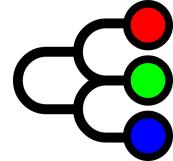
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declare drinksAlcohol: -> bool.  
declare age: -> {0 .. 150}.
```

```
age() >= 18 implies drinksAlcohol().
```

```
define drinksAlcohol() as true.  
define age() as 0.
```



Development support



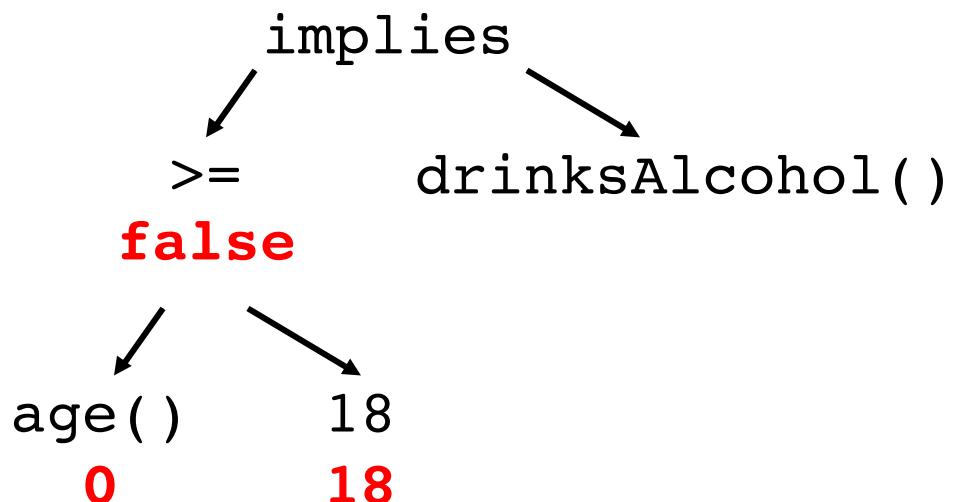
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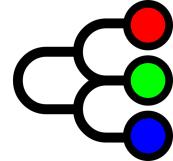
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Development support



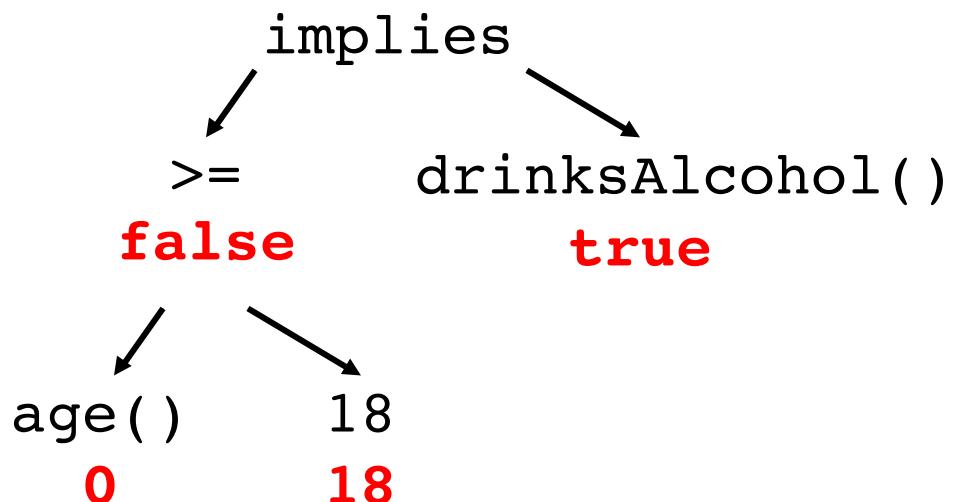
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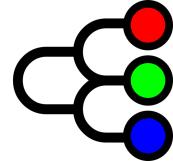
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Development support



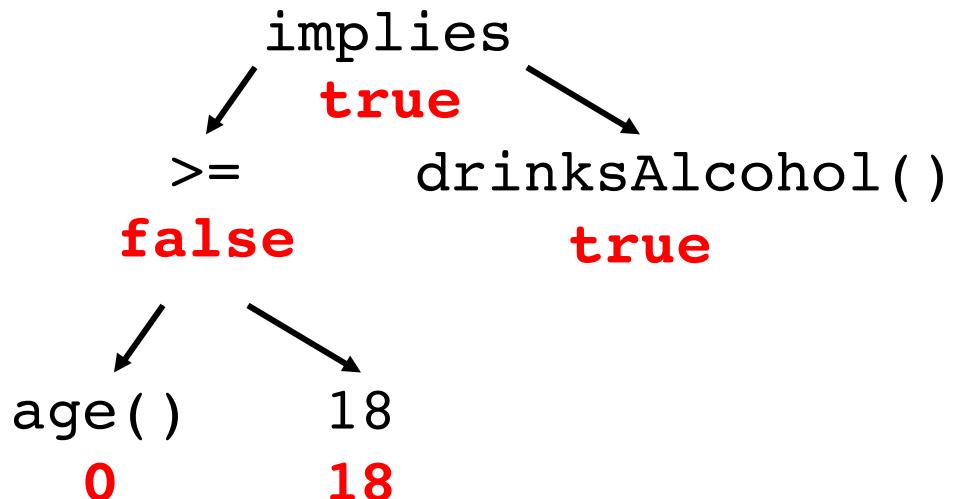
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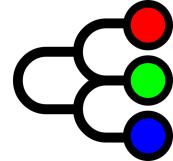
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Development support



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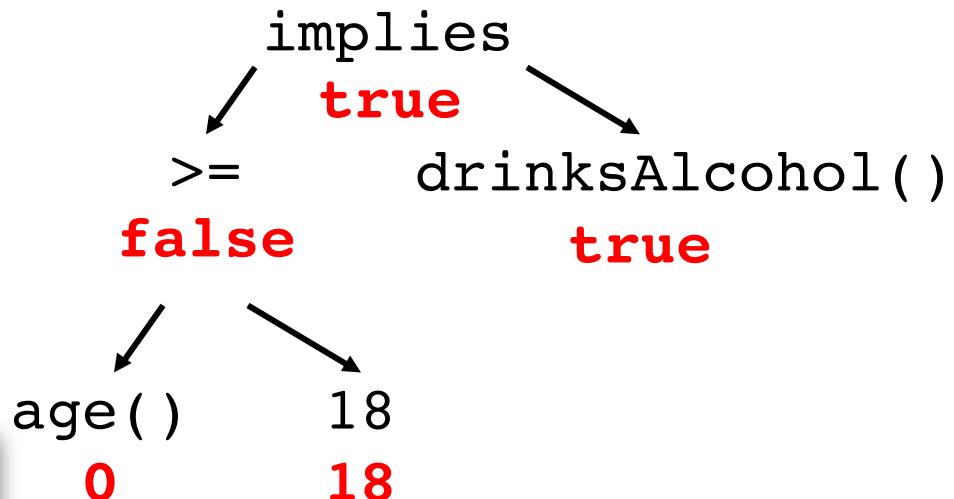
(3-valued) expression evaluation

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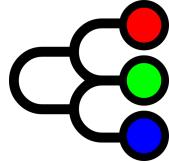
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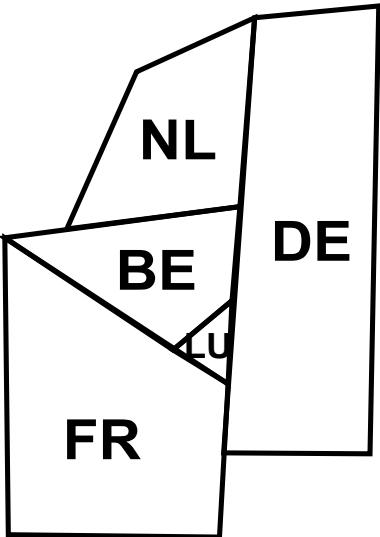
```
•      •      age() [0]  
•      >= [false]  
•      •      18  
implies [true]  
•      drinksAlcohol() [true].
```



Development support

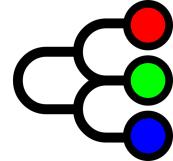


detailed UNSAT explanation

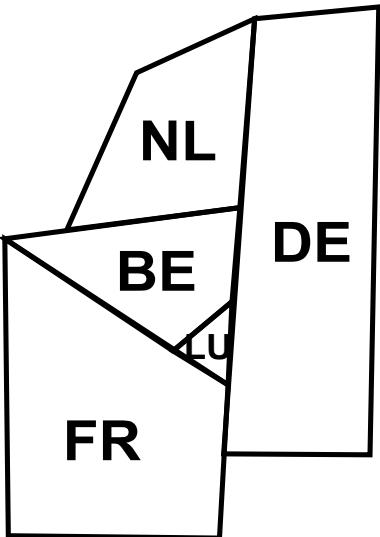


```
1 declare color: string -> {"r", "g", "b", "y"}.
2
3 decdef border as {("NL", "BE"), ("NL", "DE"),
4 ("BE", "LU"), ("BE", "DE"), ("BE", "FR"),
5 ("FR", "LU"), ("FR", "DE"), ("LU", "DE")}.
6
7 all [
8   color(x)!=color(y)
9 for x,y where border(x,y) ].
```

Development support

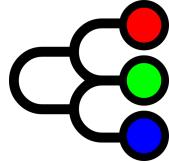


detailed UNSAT explanation

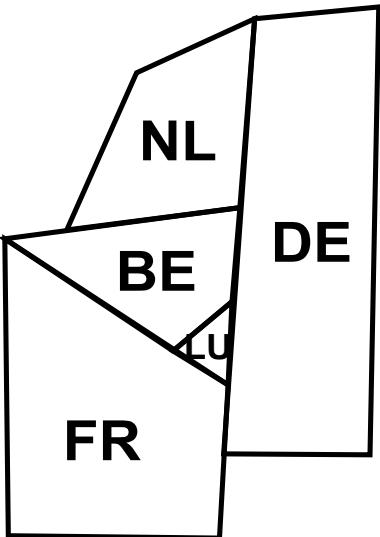


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Development support



detailed UNSAT explanation



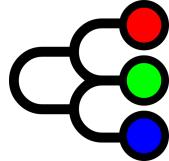
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FOUND UNSATISFIABILITY

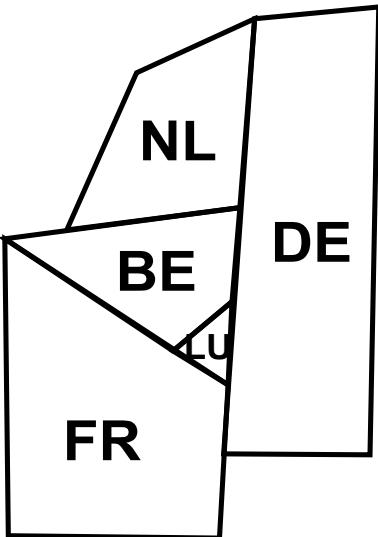
BLOCKERS

Line 7: all[not color(x)=color(y) for x,y where border(x,y)]

Development support



detailed UNSAT explanation



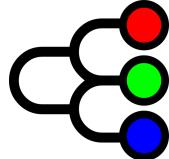
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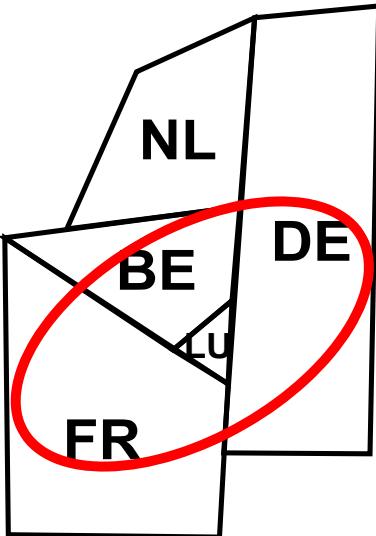
DETAILED BLOCKERS

Line 7: not color("BE")=color("DE")
Line 7: not color("BE")=color("FR")
Line 7: not color("BE")=color("LU")
Line 7: not color("DE")=color("FR")
Line 7: not color("DE")=color("LU")
Line 7: not color("FR")=color("LU")

Development support



detailed UNSAT explanation

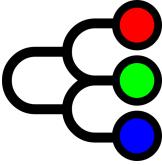


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8   color(x)!=color(y)
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DETAILED BLOCKERS

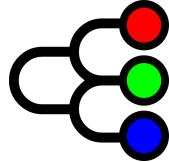
```
Line 7: not color("BE")=color("DE")
Line 7: not color("BE")=color("FR")
Line 7: not color("BE")=color("LU")
Line 7: not color("DE")=color("FR")
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Performance?

Backend solver: Exact

<https://gitlab.com/nonfiction-software/exact>



Performance?

Backend solver: Exact

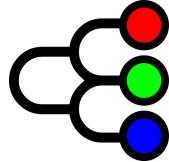
<https://gitlab.com/nonfiction-software/exact>

On-call rostering

Instance	MiniZinc (Gecode)		MiniZinc (OR-Tools)		ManyWorlds (Exact)	
	Objective	Time	Objective	Time	Objective	Time
2s-200d	65	25 000+	64	6204.30	64	0.04
4s-100d	58	25 000+	61	25 000+	2	0.24
10s-100d-C	47	25 000+	47	1.29	47	0.06
20s-100d-B	59	25 000+	58	25 000+	16	0.43
30s-400d-A	293	25 000+	299	25 000+	2	11.12

■ **Table 1** Objective values and runtimes (in seconds) for three different approaches to solve the on-call rostering problem. Entries in bold denote that optimality was proven.

<https://github.com/MiniZinc/mzn-challenge/blob/develop/2018/on-call-rostering/oc-roster.mzn>



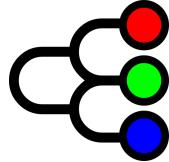
Where to get

Running

- Website with examples & online editor:
manyworlds.site
- Source code (should compile on Linux)
gitlab.com/nonfiction-software/manyworlds
- Dockerfile and Docker image

Learning

- The [ModRef paper](#)
- These slides: slides.com/jod/manyworlds-modref
- Mailing list: groups.google.com/g/manyworlds-lang
- Subreddit: reddit.com/r/manyworlds



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