Probability and Statistics 1 cat2

Bsc. mathematics and computer science (Jomo Kenyatta University of Agriculture and Technology)

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

STA 2100 PROBABILITY AND STATISTICS I

CAT 2 **Time:** 1.25 Hours

Instructions: answer all questions

Question one

a) Two events E and F satisfy $P(E) = 0.35$, $P(F) = 0.45$ and $P(E \cap F') = 0.15$. Illustrate the above information in a fully completed two way table hence or otherwise find $P(E|F)$ and $P(F|E')$. (4 marks)

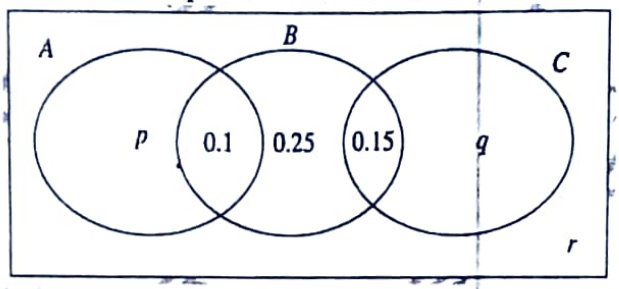
b) In a village, 70% of the families have dogs and 45% have dogs and cats. 18% of the families in this village have neither a dog nor a cat.

(i) Illustrate this information in a fully completed Venn diagram. (2 marks)

(ii) Find the percentage of families that do not have a cat. (1 mark)

(iii) Determine the probability that a family picked at random will own dogs or cats, but not both. (1 mark)

c) The Venn diagram below shows the probabilities associated with three events A , B and C .



Some of the probabilities in the Venn diagram are given in terms of the constants p, q and r.

(i) Given that the events A and B are independent, calculate the value of p. (2 marks)

(ii) Given further that $P(B|C) = 0.75$, find the value of q and the value of r. (3 marks)

(iii) Determine $P(A \cup C|B)$. (2 marks)

d) On a randomly chosen day, the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively.

(i) Draw a tree diagram to represent this information (2 marks)

(ii) Find the probability that on a randomly chosen day Bill travels on foot and is late. (1 mark)

(iii)Given that Bill is late, find the probability that he did not travel on foot. (2 marks)

Question two

a) In a factory, machines A, B and C are all producing metal rods of the same length. Machine A produces 35% of the rods, machine B produces 25% and the rest are produced by machine C. Of their production of rods, A, B and C produces 3%, 6% and 5% defective rods respectively. If a finished rod is randomly selected;

(i) Use Bayes' theorem to find the probability that this rod is defective (3 marks)

(ii) and found to be defective, what are the chances that it was produced by machine C (1 mark)

b) A company manufacturing energy saving light bulbs claims the mean lifetime of a bulb is 8,000 hours. It is known from past quality assurance procedures that the probability of any particular

light bulb having a lifetime of less than 5,000 hours is 0.1. a random sample of 30 light bulbs is taken what is the probability that; (i) exactly 3 light bilb (ii) at most 3 light bilbs (iii) more than 3 light bulbs last less than 5,000 hours? (Hint: use binomial formula) (6 marks)

c) The probability mass function of a random variable X is given below:

x	1	2	3	4	5	6	7
$P(X = x)$	K	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Determine the value of the constant k hence find $P(X \leq 6)$ and $P(3 < X \leq 6)$ (4 marks)

d) Patrick, a sales agent at ISUZU motor dealers, has established that on a randomly selected day he could sell 0, 1, 2 or 3 vehicles. The probability distribution of his daily sales N is given by $P(N = n) = \begin{cases} a(3-n), & n = 0, 1, 2 \\ b & \text{for } n = 3 \end{cases}$ where a and b are constants. He is paid a daily retainer of sh1100 and a commission of sh1500 for every sale he made. If his average daily wage is sh3500, determine; (i) E(N) hence find the values of a and b (ii) the standard deviation of his daily earnings. (6 marks)