Formulas - calculus2.

Engineering Mechanics (New Jersey Institute of Technology)

MATH 112 Formulas

This list may not be completely comprehensive but will give students a good idea of the formulas they may encounter on exams.

Don't forget to look over the MATH 111 Calculus 1 Review Sheet. Students should come in knowing those formulas as well

Chapter 6

- **Volumes Using Cross Sections:** $V = \int_a^b A(x)\,dx$
  - **Rotation about a horizontal axis:** $V = \int_a^b \pi\left[R(x)\right]^2 dx$
  - **Rotation about a vertical axis:** $V = \int_c^d \pi\left[R(y)\right]^2 dy$
  - **Washers about a horizontal axis:** $V = \int_a^b \pi\left(\left[R(x)\right]^2 - \left[r(x)\right]^2\right) dx$
  - **Washers about a vertical axis:** $V = \int_c^d \pi\left(\left[R(y)\right]^2 - \left[r(y)\right]^2\right) dy$

- **Volume using Cylindrical Shells:**
  - **Rotation about a vertical axis:** $V = \int_a^b 2\pi r(x)h(x)\,dx$
  - **Rotation about a horizontal axis** $V = \int_c^d 2\pi r(y)h(y)\,dy$

- **Arc Length:** $L = \int_a^b \sqrt{1 + \left[f'(x)\right]^2}\,dx$ $\quad L = \int_c^d \sqrt{1 + \left[g'(y)\right]^2}\,dy$

- **Surface Area:**
  - **Rotation about a horizontal axis:** $S = \int_a^b 2\pi f(x)\sqrt{1 + \left[f'(x)\right]^2}\,dx$
  - **Rotation about a vertical axis:** $S = \int_c^d 2\pi g(y)\sqrt{1 + \left[g'(y)\right]^2}\,dy$

- **Work:**
  - **Force/weight:** $F = ma$
  - **Work:** F constant: $W = Fd$; F variable: $W = \int_a^b F(x)\,dx$
  - **Hooke's Law:** $F = kx$

Chapter 7

- $\sinh x = \dfrac{e^x - e^{-x}}{2}$ $\qquad \cosh x = \dfrac{e^x + e^{-x}}{2}$
- $\tanh x = \dfrac{\sinh x}{\cosh x}$ $\qquad \coth x = \dfrac{\cosh x}{\sinh x}$ $\quad \operatorname{csch} x = \dfrac{1}{\sinh x}$ $\quad \operatorname{sech} x = \dfrac{1}{\cosh x}$

- **Pythagorean Identities**
  - $\cosh^2 x - \sinh^2 x = 1$; $\qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$; $\qquad \coth^2 x - 1 = \operatorname{csch}^2 x$

- **Double Angle Identities**
  - $\sinh(2x) = 2\sinh x \cosh x$; $\qquad \cosh(2x) = \cosh^2 x + \sinh^2 x$

- **Power Reducing Formulas**
  - $\cosh^2 x = \dfrac{\cosh(2x) + 1}{2}$; $\qquad \sinh^2 x = \dfrac{\cosh(2x) - 1}{2}$

- **Derivatives of Hyperbolic Trig Functions**
  - $\dfrac{d}{dx} \sinh x = \cosh x$
  - $\dfrac{d}{dx} \cosh x = \sinh x$
  - $\dfrac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$
  - $\dfrac{d}{dx} \tanh x = \operatorname{sech}^2 x$
  - $\dfrac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$
  - $\dfrac{d}{dx} \coth x = -\operatorname{csch}^2 x$

Chapter 8

- **Integration by Parts:** $\int u\,dv = uv - \int v\,du$

- **Trig Substitution**
  - $(a^2 + x^2)^{m/n}$: $\quad x = a\tan\theta$, $\quad a^2 + x^2 = a^2\sec^2\theta$
  - $(a^2 - x^2)^{m/n}$: $\quad x = a\sin\theta$, $\quad a^2 - x^2 = a^2\cos^2\theta$
  - $(x^2 - a^2)^{m/n}$: $\quad x = a\sec\theta$, $\quad x^2 - a^2 = a^2\tan^2\theta$
- **Trapezoid Rule:** $T = \dfrac{1}{2}\left[y_0 + 2y_1 + \ldots + 2y_{n-1} + y_n\right]\Delta x$
  - **Area Estimation Error:** $|E_T| \le \dfrac{M(b-a)^3}{12n^2}$ $\quad$ (M upper bound for $|f''|$)
- **Simpson's Rule:** $S = \dfrac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right)$
  - **Area Estimation Error:** $|E_S| \le \dfrac{M(b-a)^5}{180n^4}$ $\quad$ (M upper bound for $|f^{(4)}|$)

$\int^{\infty} dx$ (converges for $p > 1$

•

1 x^p diverges for $p \le 1$

Chapter 10

- **Limits of Common Sequences**
  - $\lim_{n\to\infty} \frac{\ln n}{n} = 0$
  - $\lim_{n\to\infty} n^{1/n} = 1$
  - $\lim_{n\to\infty} x^{1/n} = 1$ for $x > 0$
  - $\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$ for any x
  - $\lim_{n\to\infty} \frac{x^n}{n!} = 0$ for any x
- **Geometric Series:** $S_n = \frac{a(1-r^n)}{1-r}$ If $|r| < 1, \quad \lim_{n\to\infty} S_n = \frac{a}{1-r}$
- **Error with Integral Test:** $\int_{n+1}^{\infty} f(x)\,dx \le R_n \le \int_n^{\infty} f(x)\,dx$
  - $S_n + \int_{n+1}^{\infty} f(x)\,dx \quad \le \quad S \quad \le \quad S_n + \int_n^{\infty} f(x)\,dx$
- **Error with Alternating Series:** $|R_n| < a_{n+1}$
- **Taylor Series Coefficients:** $a_n = \frac{f^{(n)}(a)}{n!}$
- **Common Taylor Series**
  - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $-1 < x < 1$
  - $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
  - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
  - $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
  - $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$
  - $\ln(x+1) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$ or $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$
- **Taylor's Formula:** $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$

  where $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$ for some c between a and x
- **Remainder Estimation Theorem:** $|R_n(x)| \le M \frac{|x-a|^{n+1}}{(n+1)!}$ where M is an upper bound for $f^{(n+1)}(c)$ for all c between a and x inclusive

The Binomial Series

For $-1 < x < 1$, $\quad (1+x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k,$ where we define

•
$$\binom{m}{1} = m, \quad \binom{m}{2} = \frac{m(m-1)}{2!}, \quad \binom{m}{k} = \frac{m(m-1)(m-2)\cdots(m-k+1)}{k!}$$