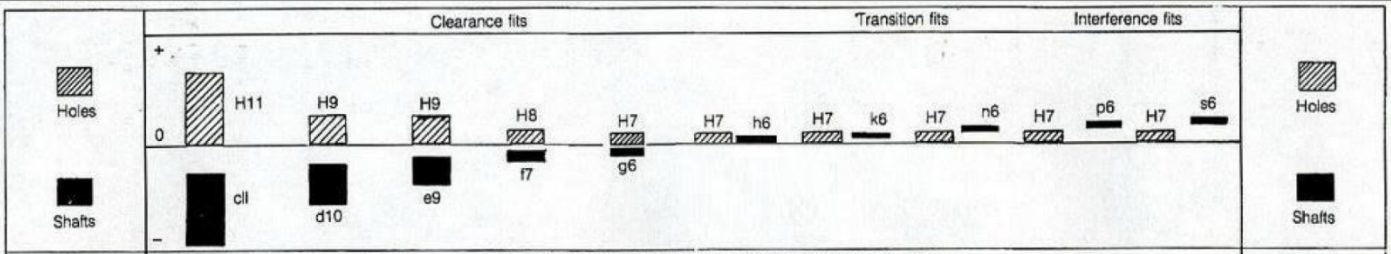
CAD Formula Sheet

Computer Aided Design (University of Ontario Institute of Technology)

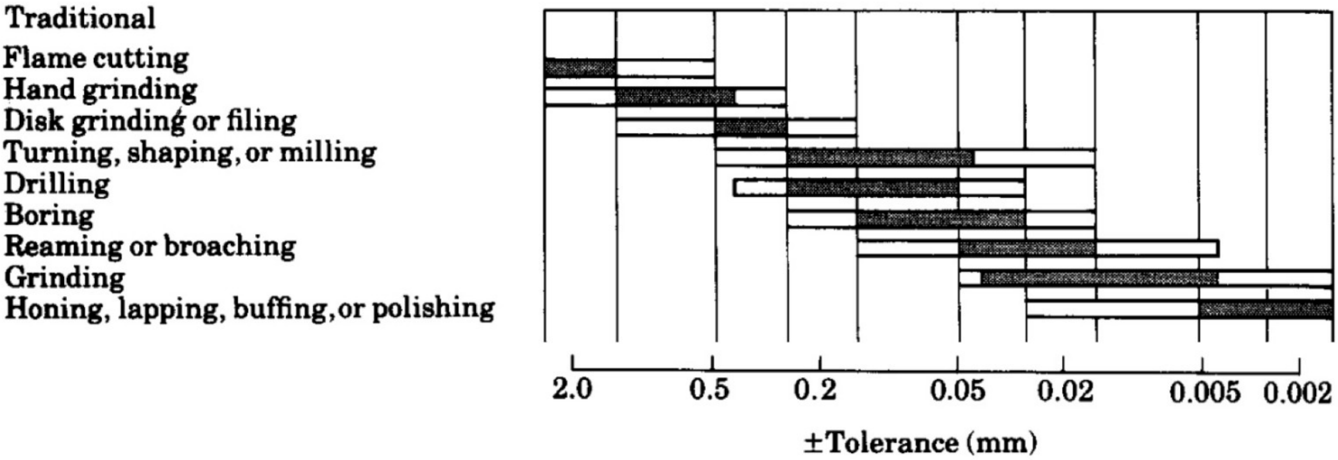
Equations and Supporting Information

Types of Fits



Upper and lower deviations for tolerance class (Values μm)

Basic size (mm) Above	Up to and incl.	H11 +	c11 −	H9 +	d10 −	H9 +	e9 −	H8 +	f7 −	H7 +	g6 −	H7 +	h6 +	H7 +	k6 +	H7 +	n6 +	H7 +	p6 +	H7 +	s6 +	Basic size (mm) Above	Up to and incl.
0	3	60 / 0	60 / 120	25 / 0	20 / 60	25 / 0	14 / 39	14 / 0	6 / 16	10 / 0	2 / 8	10 / 0	6 / 0	10 / 0	6 / 0	10 / 0	10 / 4	10 / 0	12 / 6	10 / 0	20 / 14	0	3
3	6	75 / 0	70 / 145	30 / 0	30 / 78	30 / 0	20 / 50	18 / 0	10 / 22	12 / 0	4 / 12	15 / 0	9 / 0	15 / 0	10 / 1	15 / 0	19 / 10	15 / 0	24 / 15	15 / 0	32 / 23	3	6
6	10	90 / 0	80 / 170	36 / 0	40 / 98	36 / 0	25 / 61	22 / 0	13 / 28	15 / 0	5 / 14	18 / 0	11 / 0	18 / 0	12 / 1	18 / 0	23 / 12	18 / 0	29 / 18	18 / 0	39 / 28	6	10
10	18	110 / 0	95 / 205	43 / 0	50 / 120	43 / 0	32 / 75	27 / 0	16 / 34	18 / 0	6 / 17	21 / 0	13 / 0	21 / 0	15 / 2	21 / 0	28 / 15	21 / 0	35 / 22	21 / 0	48 / 35	10	18
18	30	130 / 0	110 / 240																			18	30
30	40	160 / 0	120 / 280	62 / 0	80 / 180	62 / 0	50 / 112	39 / 0	25 / 50	25 / 0	9 / 25	25 / 0	16 / 0	25 / 0	18 / 2	25 / 0	33 / 17	25 / 0	42 / 26	25 / 0	59 / 43	30	40
40	50	160 / 0	130 / 290																			40	50
50	65	190 / 0	140 / 330	74 / 0	100 / 220	74 / 0	60 / 134	46 / 0	30 / 60	30 / 0	10 / 29	30 / 0	19 / 0	30 / 0	21 / 2	30 / 0	39 / 20	30 / 0	51 / 32	30 / 0	72 / 53	50	65
65	80	190 / 0	150 / 340																	30 / 0	78 / 59	65	80



Transformation Matrices

$$[R_Z] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [S] = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix} \quad [M] = \begin{bmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$$

$$[x^* \quad y^* \quad z^* \quad 1]^T = \begin{bmatrix} 1 & 0 & 0 & x_d \\ 0 & 1 & 0 & y_d \\ 0 & 0 & 1 & z_d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Parametric Curve Equations

$$\boldsymbol{P} = \boldsymbol{P_1} + u(\boldsymbol{P_2} - \boldsymbol{P_1})$$
$$0 \leq u \leq 1$$

$$\left.\begin{aligned} x &= x_c + R\cos u \\ y &= y_c + R\sin u \\ z &= z_c \end{aligned}\right\} \quad 0 \leq u \leq 2\pi$$

$$\mathbf{P}(u) = \left(2u^3 - 3u^2 + 1\right)\ \mathbf{P}_0 + \left(-2u^3 + 3u^2\right)\ \mathbf{P}_1 + \left(u^3 - 2u^2 + u\right)\ \mathbf{P'}_0 + \left(u^3 - u^2\right)\ \mathbf{P'}_1$$

$$\mathbf{P}(u) = \mathbf{P}_0(1-u)^n + \mathbf{P}_1 C(n,1)u(1-u)^{n-1} + \mathbf{P}_2 C(n,2)u^2(1-u)^{n-2} + \ldots + \mathbf{P}_{n-1} C(n,n-1)u^{n-1}(1-u) + \mathbf{P}_n u^n$$

$$B_{i,n}(u) = C(n,i)u^i(1-u)^{n-i}$$

Good luck with your assignment!

Equations Sheet

Bezier Curves

$$\mathbf{P}(u) = \mathbf{P}_0(1-u)^n + \mathbf{P}_1 C(n,1)u(1-u)^{n-1} + \mathbf{P}_2 C(n,2)u^2(1-u)^{n-2} + \ldots + \mathbf{P}_{n-1} C(n,n-1)u^{n-1}(1-u) + \mathbf{P}_n u^n$$

$$B_{i,n}(u) = C(n,i)u^i(1-u)^{n-i}$$

where $C(n,i) = \dfrac{n!}{i!(n-i)!}$

NURBS Curves

$$N_{i,1} = \begin{cases} 1 & u_i < u < u_{i+1} \\ 0 & elsewhere \end{cases}$$

$$N_{i,k} = \frac{(u-u_i)N_{i,k-1}}{(u_{i+k-1}-u_i)} + \frac{(u_{i+k}-u)N_{i+1,k-1}}{(u_{i+k}-u_{i+1})}$$

$$R_{i,k}(u) = \frac{w_i N_{i,k}(u)}{\sum_{i=0}^{n} w_i N_{i,k}(u)}$$

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{P}_i R_{i,k}(u) \qquad (0 \le u \le u_{\max})$$

Hermite Bicubic Surfaces

$$\mathbf{P}(u,v) = \mathbf{U}^T[M_H][B][M_H]^T\mathbf{V}$$

$$[B] = \left[\begin{array}{cc:cc} \mathbf{P}_{00} & \mathbf{P}_{01} & \mathbf{P}_{v00} & \mathbf{P}_{v01} \\ \mathbf{P}_{10} & \mathbf{P}_{11} & \mathbf{P}_{v10} & \mathbf{P}_{v11} \\ \hdashline \mathbf{P}_{u00} & \mathbf{P}_{u01} & \mathbf{P}_{uv00} & \mathbf{P}_{uv01} \\ \mathbf{P}_{u10} & \mathbf{P}_{u11} & \mathbf{P}_{uv10} & \mathbf{P}_{uv11} \end{array}\right] = \left[\begin{array}{c:c} [P] & [P_v] \\ \hdashline [P_u] & [P_{uv}] \end{array}\right]$$

$$[M_H] = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

Bezier Surfaces

$$\boldsymbol{P(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} P_{ij} B_{i,n}(u) B_{j,m}(v)}$$
$$\boldsymbol{0 \le u \le 1, 0 \le v \le 1}$$

$$\begin{aligned} \mathbf{P}(u,v) &= \sum_{i=0}^{n} B_{i,n}(u)[\mathbf{P}_{i0}B_{0,m}(v) + \mathbf{P}_{i1}B_{1,m}(v) + \ldots + \mathbf{P}_{im}B_{m,m}(v)] \\ &= B_{0,n}(u)[\mathbf{P}_{00}B_{0,m}(v) + \mathbf{P}_{01}B_{1,m}(v) + \ldots + \mathbf{P}_{0m}B_{m,m}(v)] \\ &\quad + B_{1,n}(u)[\mathbf{P}_{10}B_{0,m}(v) + \mathbf{P}_{11}B_{1,m}(v) + \ldots + \mathbf{P}_{1m}B_{m,m}(v)] \\ &\quad + \ldots \\ &\quad + B_{n,n}(u)[\mathbf{P}_{n0}B_{0,m}(v) + \mathbf{P}_{n1}B_{1,m}(v) + \ldots + \mathbf{P}_{nm}B_{m,m}(v)] \end{aligned}$$

Finite Element Analysis

Element stiffness matrix

$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{21} \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$KU = R \qquad U = \begin{bmatrix} u_1 \\ \vdots \\ u_8 \end{bmatrix} \quad , \quad \boldsymbol{R} = \begin{bmatrix} R_1 \\ \vdots \\ R_8 \end{bmatrix}$$

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