1 Assignment for practise

Computer Aided Design (University of Ontario Institute of Technology)

MECE 3030U Computer Aided Design
Assignment 3
Due November 28, 2024

For the handwritten questions in this assignment, please show your calculations using pencil/pen and paper and draw a box around your final answer. Please use only a simple calculator to complete these questions (as you would for an exam). MATLAB can be used to verify your answers but not to demonstrate your ability to solve the assignment questions.

For NX-based questions, please include a .zip file that includes all the files required to open your parts, assembly, motion analysis and finite element analysis.

Please upload your assignment (one .zip file containing a single pdf of your handwritten solutions and all the NX files required to validate your work) by the due date. Name the file **LastName_FirstName_StudentNumber.zip**

Question 1

A. (10 marks)

For the control points and weights below, find the parametric equation and the curve's coordinates at u = 0, u = 0.5, u = 1, for a Non-Uniform Rational B-Spline of degree 2:

$P_1 = [-1, -1]^T$, $P_2 = [0, 3]^T$, $P_3 = [3, 0]^T$, $P_4 = [-2, -2]^T$

$w_1 = 1$, $w_2 = 3$, $w_3 = 1$, $w_4 = 1$

B. (10 marks)

Sketch the NURBS curve and label the tangent vectors at the beginning and end of the curve.

Sketch a Bezier Curve that utilizes the same control points.

Describe why the Bezier Curve is a special case of a NURBS curve (<50 words).

$P_0 = [-1,-1]$ $P_1 = [0,3]$ $P_2 = [3,0]$ $P_3 = [2,-2]$

$\omega_1 = 1$ $\omega_2 = 3$ $\omega_3 = 1$ $\omega_4 = 1$

→ Find $\underset{\text{degree}+1}{K = 3}$ $\underset{\text{ctrl points}-1}{n = 3}$ $\underset{\text{knot vector length}}{m = 7}$

→ Write knot vector $u = [\underbrace{0\ 0\ 0}_{K=3}\ \underbrace{\tfrac{1}{2}}_{\Delta u}\ \underbrace{1\ 1\ 1}_{K=3}]$ ($u_0\ u_1\ u_2\ u_3\ u_4\ u_5\ u_6$) $\Delta u = \frac{1}{m-2K+1} = \frac{1}{2}$

→ Find $N_{n,k}$ pyramid

$K = 3$ $N_{0,3}$ $N_{1,3}$ $N_{2,3}$ $N_{3,3}$ ($n=3$, $k=3$)

$K = 2$ $N_{0,2}$ $N_{1,2}$ $N_{2,2}$ $N_{3,2}$ $N_{4,2}$

$K=1$ $N_{0,1}$ $N_{1,1}$ $N_{2,1}$ $N_{3,1}$ $N_{4,1}$ $N_{5,1}$

→ Calculate $N_{i,1} = \begin{cases} 1 & u_i \le u, u_{i+1} \\ 0 & \text{elsewhere} \end{cases}$

$N_{0,1} = 1$ at $u=0$ $N_{1,1} = 1$ at $u=0$ 0 elsewhere

$N_{2,1} = 1$ at u from $\overset{u_2}{0}$ to $\overset{u_3}{0.5}$ $N_{3,1} = 1$ at u from $\overset{u_3}{0.5}$ to $\overset{u_4}{1}$ 0 elsewhere

$N_{4,1} = 1$ at $u=1$ $N_5 = 1$ at $u=1$ 0 elsewhere

→ Calculate $N_{i,k}$

~~k=2~~

$$N_{i,k} = \frac{(u-u_i)N_{i,k-1}}{(u_{i+k-1}-u_i)} + \frac{(u_{i+k}-u)N_{i+1,k-1}}{(u_{i+k}-u_{i+1})}$$

$i=0$ $k=2$ $$N_{0,2} = \frac{(u-u_0)N_{0,1}}{\underbrace{u_1-u_0}_{=0}} + \frac{(u_2-u)N_{1,1}}{\underbrace{u_2-u_1}_{=0}} = 0$$

if denominator = 0 term = 0

$i=1$ $k=2$ $$N_{1,2} = \frac{(u-u_1)N_{1,1}}{\underbrace{u_2-u_1}_{=0}} + \frac{(u_3-u)N_{2,1}}{\underbrace{u_3-u_2}_{=\frac{1}{2}}} = 2(\overset{u_3}{0.5}-u)N_{2,1}$$

$i=2$ $k=2$ $$N_{2,2} = \frac{(u-\overset{0}{u_2})N_{2,1}}{\underbrace{u_3-u_2}_{=\frac{1}{2}}} + \frac{(\overset{1}{u_4}-u)N_{3,1}}{\underbrace{u_4-u_3}_{\frac{1}{2}}} = 2uN_{2,1} + 2(1-u)N_{3,1}$$

$i=3 \quad k=2 \qquad N_{3,2} = \dfrac{(u-u_3)\,N_{3,1}}{\underbrace{u_4-u_3}_{=1/2}} + \dfrac{(u_5-u)\,N_{4,1}}{\underbrace{u_5-u_4}_{=0}} = 2(u-0.5)\,N_{3,1}$

$i=4 \quad k=2 \qquad N_{4,2} = \dfrac{(u-u_4)\,N_{4,1}}{\underbrace{u_5-u_4}_{=0}} + \dfrac{(u_6-u)\,N_{5,1}}{\underbrace{u_6-u_5}_{=0}} = 0$

$i=0 \quad k=3 \qquad N_{0,3} = \dfrac{(u-u_0)\,N_{0,2}}{\underbrace{u_2-u_0}_{=0}} + \dfrac{(u_3-u)\,N_{1,2}}{\underbrace{u_3-u_1}_{=1/2}} = 2(0.5-u)\,N_{1,2}$

$i=1 \quad k=3 \qquad N_{1,3} = \dfrac{(u-\overset{=0}{u_1})\,N_{1,2}}{\underbrace{u_3-u_1}_{=1/2}} + \dfrac{(\overset{=1}{u_4}-u)\,N_{2,2}}{\underbrace{u_4-u_2}_{1}} = 2u\,N_{1,2} + (1-u)\,N_{2,2}$

$i=2 \quad k=3 \qquad N_{2,3} = \dfrac{(u-\overset{=0}{u_2})\,N_{2,2}}{\underbrace{u_4-u_2}_{=1}} + \dfrac{(\overset{=1}{u_5}-u)\,N_{3,2}}{\underbrace{u_5-u_3}_{1/2}} = u\,N_{2,2} + 2(1-u)\,N_{3,2}$

$i=3 \quad k=3 \qquad N_{3,3} = \dfrac{(u-\overset{=1/2}{u_3})\,N_{3,2}}{\underbrace{u_5-u_3}_{=1/2}} + \dfrac{(\overset{=1}{u_6}-u)\,N_{4,2}}{\underbrace{u_6-u_4}_{=0}} = 2(u-0.5)\,N_{3,2}$

Calculate $u = 0.5$

$N_{0,3} = 2(0.5-0.5)\,N_{1,2} = 0$

$N_{1,3} = 2(0.5)\,N_{1,2} + (1-0.5)\,N_{2,2}$
$= 2(0.5)\,2(0.5-0.5)\,N_{2,1} + (0.5)(2u\,N_{2,1} + 2(1-u)N_{3,1})$
$= 0.5(2)(0.5)(1) \cancel{+ 0.5[(2)(0.5)(1)]} + 0.5(2)(0.5)(1)$
$= 0.5 + 0.5 = 1$

$N_{2,3} = 0.5\,N_{2,2} + 2(1-0.5)\,N_{3,2}$
$= 0.5[2u\,N_{2,1} + 2(1-u)\,N_{3,1}] + 2(0.5-0.5)\,N_{3,1}$

$= 0.5(1) + (1) = 1.5$

$N_{3,3} = 2(0.5-0.5)\,N_{32} = 0$

$$P(0.5) = 0 + \frac{\overset{\text{really } P_2}{P_1}\,\overset{\text{really } w_2}{w_1}\,N_{1,3} + P_2 w_2 N_{2,3}}{w_1 N_{1,3} + w_2 N_{2,3}} + 0$$

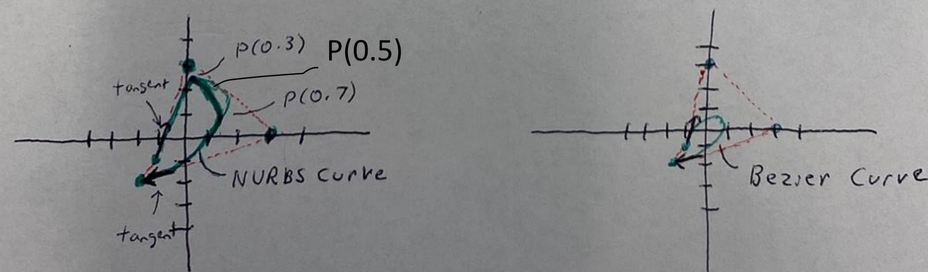
$$= \frac{[0,3]\,3(1) + [3,0]\,1.5(1)}{3 + 1.5}$$

$P(0.5)_x = 1 \qquad P(0.5)_y = 2$

$P(0) = -1,-1 \qquad P(0.5) = 1,2 \qquad P(1) = -2,-2$

check with a sketch

b)



Bezier curve → order = control points = 4 → degree = 3, ∴ Bezier curve follows points less closely than NURBS. Bezier curve has uniform knots, equal(no) weights, so the curve is equally influenced by all control points.

Question 2

A. (10 marks)

A designer selects the following control points to create parametric curves for the front and back of a new car hood. Compute the Bezier Surface equation, P(u,v), that generates the new car hood from the following control points:

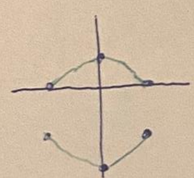
Front of Hood: $P_1 = [-1, -1, 3]^T$, $P_2 = [0, -2, 4]^T$, $P_3 = [1, -1, 3]^T$

Back of Hood: $P_4 = [-1, 0, -1]^T$, $P_5 = [0, 1, 0]^T$, $P_6 = [1, 0, -1]^T$

Same process as Lecture 17 Q2

B. (10 marks)

Describe and sketch which parameters the designer would need to provide to create a similar surface using the Hermite Bicubic Surface approach (<100 words).

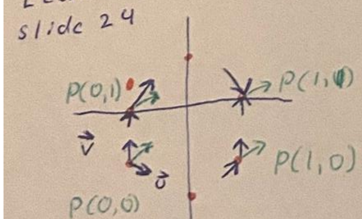


Bezier - give control point for start, end, and some middle points of boundary curves

Hermite - give 8Ontrol points for start and end of boundary curves(corner points = 4)
- give tangent vectors for u, v for each corner point (8 vectors)
- give twist vector for 3D surface curvature (4 vectors)

→ 16 points total

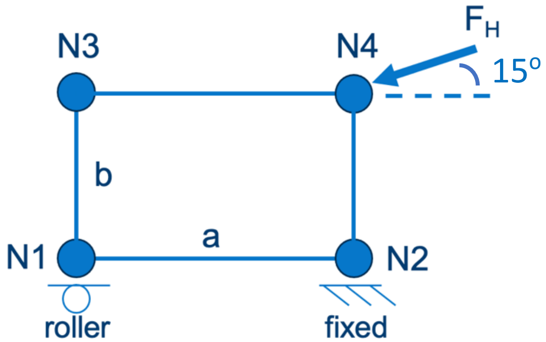
Lecture 12
slide 24



Question 3

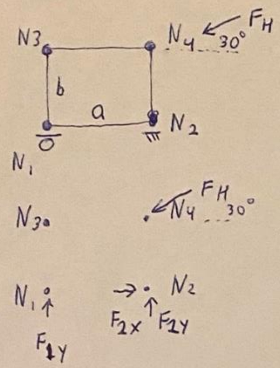
A. (10 marks)

A human loads a truss structure with a force F_H, as shown below. Create the stiffness matrix and create a matrix equation to calculate the reaction forces and node displacements. Assume the elastic modulus is E, the element area is A and element lengths are b and a.



Same as below, change 30 º to 15º angle.

Create stiffness matrix, Rxn forces, node displacements



1) Calculate boundary forces

$\Sigma F_x = 0 \quad F_H \cos 30 = F_{2x}$

$\Sigma F_y = 0 \quad F_H \sin 30 = F_{1y} + F_{2y}$

$F_H\left(\sin 30 - \left(\frac{b}{a}\right)\cos 30\right) = F_{2y}$

$\Sigma M_2 = 0 \quad F_H \overset{\cos}{\cancel{\sin}} 30\ b = F_{1y}\, a$

$F_H (\cos 30)\left(\frac{b}{a}\right) = F_{1y}$

2) Break into elements

$u_{3y} = u_6$ $\quad u_{4y} = u_8$ ~~$F_H \sin 30$~~

$u_{3x} = u_5$ $\quad u_{4x} = u_7$ ~~$F_H \cos 30$~~

roller

$u_{1y} = u_2 = 0$ $\quad u_{2y} = u_4 = 0$ fixed

$u_{1x} = u_1$ $\quad u_{2x} = u_3 = 0$

Use same numbering for reaction forces

R_1 to R_8

$u_2 = 0 \quad u_3 = 0 \quad u_4 = 0$

$u_1, u_5, u_6, u_7, u_8, R_1, R_5, R_6$ unknown (8 eqn)

$F_{1x} = R_1 =$

$F_{1y} = R_2 = F_H \cos 30 \left(\frac{b}{a}\right)$

$F_{2x} = R_3 = F_H \cos 30$

$F_{2y} = R_4 = F_H \left(\sin 30 - \frac{b}{a} \cos 30\right)$

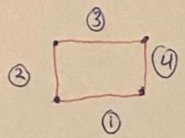
$F_{3x} = R_5 =$

$F_{3y} = R_6 =$

$F_{4x} = R_7 = -F_H \cos 30$

$F_{4y} = R_8 = -F_H \sin 30$

3) Calculate Stiffness matrices



Element 1 Stiffness Matrix

$$\frac{EA}{L_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix}K_{11} & K_{13}\\ K_{31} & K_{33}\end{bmatrix} \Rightarrow \begin{bmatrix}K_{11} & K_{13}\\ K_{31} & K_{33}\end{bmatrix}\begin{bmatrix}u_1\\ u_3\end{bmatrix} = \begin{bmatrix}R_1\\ R_3\end{bmatrix}$$

$\xrightarrow{}u_1$ ● —— a ① —— ● $\xrightarrow{}u_3$

$$K_{11} = \frac{EA}{L_1} \quad K_{13} = -\frac{EA}{L_1} \quad K_{31} = \frac{-EA}{L_1} \quad K_{33} = \frac{EA}{L_1}$$

Similar process for Element 2, 3, 4

$\uparrow u_6$, ② b, $\uparrow u_2$

$$\frac{EA}{L_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix}K_{22} & K_{26}\\ K_{62} & K_{66}\end{bmatrix}$$

$\xrightarrow{}u_5$ ● —— ● $\xrightarrow{}u_7$

$$\frac{EA}{L_3}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{matrix}K_{55} & K_{57}\\ K_{75} & K_{77}\end{matrix}$$

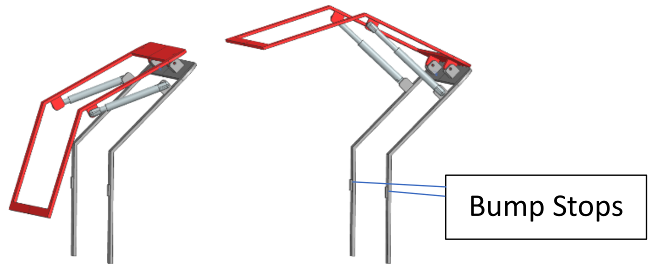
$\uparrow u_8$, $\uparrow u_4$

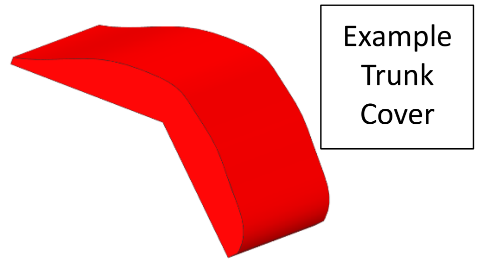
$$\frac{EA}{L_4}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix}K_{44} & K_{48}\\ K_{84} & K_{88}\end{bmatrix}$$

4) Add all k values, Rxn values (R), node displacements (u) into matrix

$$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8 \end{array}
\begin{bmatrix}
\frac{EA}{a} & 0 & -\frac{EA}{a} & 0 & 0 & 0 & 0 & 0\\
0 & \frac{EA}{b} & 0 & 0 & 0 & -\frac{EA}{b} & 0 & 0\\
-\frac{EA}{a} & 0 & \frac{EA}{a} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{EA}{b} & 0 & 0 & 0 & -\frac{EA}{b}\\
0 & 0 & 0 & 0 & \frac{EA}{a} & 0 & -\frac{EA}{a} & 0\\
0 & -\frac{EA}{b} & 0 & 0 & 0 & \frac{EA}{b} & 0 & 0\\
0 & 0 & 0 & 0 & -\frac{EA}{a} & 0 & \frac{EA}{a} & 0\\
0 & 0 & 0 & -\frac{EA}{b} & 0 & 0 & 0 & \frac{EA}{b}
\end{bmatrix}
\begin{bmatrix} u_1\\ 0\\ 0\\ 0\\ u_5\\ u_6\\ u_7\\ u_8 \end{bmatrix}
=
\begin{bmatrix} R_1\\ F_H\cos 30\left(\frac{a}{b}\right)\\ F_H\cos 30\\ F_H\left(\sin 30 - \frac{b}{a}\cos 30\right)\\ R5\\ R6\\ -F_H\cos 30\\ -F_H\sin 30 \end{bmatrix}$$

Question 4 – Solutions are dependent on your trunk frame. Example NX solution attached.
The lift gate for a redesigned sport utility vehicle is being developed based on a previous model:
Hyundai Lift Gate Video: https://www.youtube.com/watch?v=_hbe7WidJqc
The designed components are attached in the .zip file.





Your challenge is to use Siemens NX software to develop a stylish trunk cover using a parametric curve (Studio Spline), and create an assembly, motion analysis, and finite element analysis for the system to determine the system's range of motion and the trunk frame's structural integrity.

A. (20 marks)
Create a stylish trunk cover, example above, that utilizes Studio Spline parametric curves (in addition to standard lines and arcs) and that mounts to the M5 bolt hole locations on the trunk's frame (hint: you can start from a duplicate of the trunk frame part to create your cover).
Create an assembly for the lift gate system shown above and incorporate your trunk cover.
Find a suitable bump stop thickness so the car frame and trunk frame stop when parallel (round to whole numbers). Remodel the car frame part with the correct bump stop thickness.
On your handwritten solutions sheet, state the correct bump stop thickness.

B. (20 marks)
Create a motion analysis showing the trunk opening and closing at 40^{o}/s, using the actuator as the motion driver. On your handwritten solutions sheet, state your method for controlling the actuator's motion.
Create a motion analysis with a force driver for the actuator. Determine the force required to lift your trunk frame and cover. Ensure your analysis has gravity enabled. Assume your trunk frame and trunk cover are made from AISI_Steel_1005. On your handwritten solutions sheet, state the force required to lift your trunk.

C. (10 marks)
Create a finite element analysis for the car frame to analyze the force of the actuators on their ball joints. Determine the maximum stress on the car frame and if the car frame will yield.
Modify the car frame to better distribute loads from the actuator. On your handwritten solutions sheet, state the maximum stress and if your car frame will yield. Explain the geometric changes and feature additions you made to the car frame (<100 words).

Your final assignment for CAD, congrats!
Please reach out for support if desired.
Good luck and enjoy the process,
Aaron

Equations Sheet

Bezier Curves

$$\mathbf{P}(u)=\mathbf{P}_0(1-u)^n+\mathbf{P}_1C(n,1)u(1-u)^{n-1}+\mathbf{P}_2C(n,2)u^2(1-u)^{n-2}$$
$$+\ldots+\mathbf{P}_{n-1}C(n,n-1)u^{n-1}(1-u)+\mathbf{P}_nu^n$$

$$B_{i,n}(u)=C(n,i)u^i(1-u)^{n-i}$$
$$\text{where } C(n,i)=\frac{n!}{i!(n-i)!}$$

NURBS Curves

$$N_{i,1}=\begin{cases}1 & u_i<u<u_{i+1}\\ 0 & elsewhere\end{cases}$$

$$N_{i,k}=\frac{(u-u_i)N_{i,k-1}}{(u_{i+k-1}-u_i)}+\frac{(u_{i+k}-u)N_{i+1,k-1}}{(u_{i+k}-u_{i+1})}$$

$$R_{i,k}(u)=\frac{w_iN_{i,k}(u)}{\sum_{i=0}^{n}w_iN_{i,k}(u)}$$

$$\mathbf{P}(u)=\sum_{i=0}^{n}\mathbf{P}_iR_{i,k}(u)\qquad(0\le u\le u_{\max})$$

Hermite Bicubic Surfaces

$$\mathbf{P}(u,v)=\mathbf{U}^T[M_H][B][M_H]^T\mathbf{V}$$

$$[B]=\left[\begin{array}{cc|cc}\mathbf{P}_{00} & \mathbf{P}_{01} & \mathbf{P}_{v00} & \mathbf{P}_{v01}\\ \mathbf{P}_{10} & \mathbf{P}_{11} & \mathbf{P}_{v10} & \mathbf{P}_{v11}\\ \hline \mathbf{P}_{u00} & \mathbf{P}_{u01} & \mathbf{P}_{uv00} & \mathbf{P}_{uv01}\\ \mathbf{P}_{u10} & \mathbf{P}_{u11} & \mathbf{P}_{uv10} & \mathbf{P}_{uv11}\end{array}\right]=\left[\begin{array}{c|c}[P] & [P_v]\\ \hline [P_u] & [P_{uv}]\end{array}\right]$$

$$[M_H]=\begin{bmatrix}2 & -2 & 1 & 1\\ -3 & 3 & -2 & -1\\ 0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}$$

Bezier Surfaces

$$\boldsymbol{P(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}P_{ij}B_{i,n}(u)B_{j,m}(v)}$$
$$\boldsymbol{0\le u\le 1,0\le v\le 1}$$

$$\begin{aligned}\mathbf{P}(u,v)&=\sum_{i=0}^{n}B_{i,n}(u)\left[\mathbf{P}_{i0}B_{0,m}(v)+\mathbf{P}_{i1}B_{1,m}(v)+\ldots+\mathbf{P}_{im}B_{m,m}(v)\right]\\ &=B_{0,n}(u)\left[\mathbf{P}_{00}B_{0,m}(v)+\mathbf{P}_{01}B_{1,m}(v)+\ldots+\mathbf{P}_{0m}B_{m,m}(v)\right]\\ &\quad+B_{1,n}(u)\left[\mathbf{P}_{10}B_{0,m}(v)+\mathbf{P}_{11}B_{1,m}(v)+\ldots+\mathbf{P}_{1m}B_{m,m}(v)\right]\\ &\quad+\ldots\\ &\quad+B_{n,n}(u)\left[\mathbf{P}_{n0}B_{0,m}(v)+\mathbf{P}_{n1}B_{1,m}(v)+\ldots+\mathbf{P}_{nm}B_{m,m}(v)\right]\end{aligned}$$

Finite Element Analysis

Element stiffness matrix

$$K=\begin{bmatrix}k_{11} & k_{12}\\ k_{21} & k_{21}\end{bmatrix}=\frac{EA}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$KU=R\qquad \boldsymbol{U}=\begin{bmatrix}u_1\\ \vdots\\ u_8\end{bmatrix},\quad \boldsymbol{R}=\begin{bmatrix}R_1\\ \vdots\\ R_8\end{bmatrix}$$