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Physics for engineers - Lecture notes 1-10

Mechatronics engineering (Jomo Kenyatta University of Agriculture and Technology)

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SPH 110: FUNDAMENTALS OF PHYSICS I

1.0 UNITS AND DIMENSIONS

1.1 Introduction

Measurable quantities in physics are assigned units of measurements.

Quantities are divided into 2 namely:-

1) Basic / fundamental quantities
2) Derived quantities

1.2 Basic /Fundamental quantities

They don't depend on other quantities. These quantities are used to fully describe other physical quantities. They include:-

Basic Quantity	***S. I. Unit***	***Symbol***
Length	Metre	*m*
Mass	Kilogramme	*kg*
Time	second	*s*
Amount of substance	mole	*mol*
Electric current	Ampere	*A*
Thermodynamic temp.	Kelvin	*K*
Luminous intensity	candela	*cd*

1.3 Derived quantities

They are described in terms of basic or fundamental quantities e.g. volume, area, pressure, density etc.

Metre: It's the distance between two points. The standard of a metre is marked on a bar of platinum (90%) – Iridium (10%) alloy kept at 0^{o}c.

Second: It's the duration of 9, 192, 631, 770 periods of certain microwave radiation emitted by the ceasium atom. The atomic clock is the most accurate and other clocks (secondary) are set compared to it.

Kilogramme: The standard mass is the platinum. Iridium cylinder whose mass is exactly one kilogramme

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Note: The physical quantities, time, mass and length are fundamental quantities we use in our study of mechanics.

1.4 Dimension and dimension Analysis

Dimension: It is a physical property described by the words time, length or mass. (This property is the same no matter what units it is expressed.

Dimension:	***Symbol***
Length:	L
Time:	T
Mass:	M

Dimension Analysis:

It is a technique of establishing the validity of a solution to a problem, a unit or an equation by checking for dimensional consistency.

Dimensional units must have the following properties:-

1) For addition and subtraction, quantities must have the same dimensional units.
2) For division and multiplication they may have different units
3) For equations to hold they must have the same dimensional units on both sides.

Note: Constants and angles have their dimensional units as 1 e.g π, Cos θ, Sin θ, Tan θ, 1, 2…., ½ , $^4/_3$, exp, *ln,* log, etc.

Examples

1. Define and give the dimensional units for the following:-

   a) Speed: It is the rate of change of displacement
   Dimension of velocity = $\frac{L}{T} = LT^{-1}$ (S.I unit is metre per second)

   b) Acceleration: It is the rate of change of velocity
   Dimensions of acceleration = $\frac{LT^{-1}}{T}$ = LT^{-2}

   Density: It is the mass per unit volume
   Dimensions of density $\frac{\mathrm{M}}{L^3} = ML^{-3}$ (S.I unit kg/m^3)

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c) Dimension of energy ; Energy = Force x distance

$$E = Ma.S$$

$$E = MLT^{-2}.L$$

$$E = ML^2T^{-2}$$

2. Prove if the following dimensions are correct.

a) $T = 2\pi\sqrt{\frac{l}{g}}$, $2\pi = 1$

$T = \sqrt{\frac{l}{g}}$, $T = \sqrt{\left(\frac{L}{LT^{-2}}\right)} = T$

Both sides have dimensions of time.

b) $V = \pi r^2 h$ $\pi = 1$
$V = r^2 h = L^2. L = L^3$

Both sides have dimensions of volume

c) $V = u + at$
$V = u + at = LT^{-1} + LT^{-2} \times T = LT^{-1} + LT^{-1} = LT^{-1}$
Both sides have dimensions of velocity.

d) $E = mc^2$ (C is speed of light)

$E = M(LT^{-1})^2 = ML^2T^{-2}$

Both sides have dimensions of energy

e) $d \sin\theta = n\lambda$, n and $\sin\theta$ are dimensionless
$L = L$.
Both have dimensions of length.

3. Find the units of constants below

a) $F = -kx$ where k is spring constant

$$k = -\frac{F}{x} = -\frac{Ma}{L} = -\frac{MT^{-2}}{L} = -ML^{-1}T^{-2}$$

b) $N(t) = N \exp(-kt)$ exponential is dimensionless
$kt = 1$

$kT = 1$

$k = \frac{1}{T}$

$k = T^{-1}$

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4. The expression for kinetic energy is K.E = ½ mv^2 and potential energy is P.E = *mgh*. Show that both expressions have the same dimensions, hence can be subtracted or added from each other.

$$\tfrac{1}{2} mv^2 = mgh$$
$$M(LT^{-1})^2 = MLT^{-2} \times L$$
$$ML^2T^{-2} = ML^2T^{-2}$$

5. The period *T* of a pendulum is given by the dimension equation $T = km^x l^y g^z$, where *m* is mass of the bob, *l* is the length of the string, *g* is acceleration due to gravity and *k*, *x, y, z,* are constants. Calculate the values of *x, y,* and *z.*

$T = M^x L^y (LT^{-2})^z = M^x L^{y+z} T^{-2z}$
$-2z = 1, z = -{}^1/_2$

$y + z = 0, y = {}^1/_2$
$x = 0.$

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2.0 VECTORS

2.1 Introduction

If a sack of flour has a mass of 10kg, that mass is not dependent on where the flour, whether it at rest in a storeroom on land or in motion on a ship in sea. The above statement describes only the magnitude / size (10kg) but not the position. This shows that mass is a *scalar* quantity.

For a quantity like velocity it is quite different. To a passenger in Mombasa desiring to go to Nairobi city on a bus moving at 20m/s, it obviously makes a big difference whether the bus is moving towards Nairobi city or Malindi town. Here both direction and size/magnitude are vitally important. Such a quantity like velocity is a *vector* quantity.

2.2 Scalar and Vector quantities

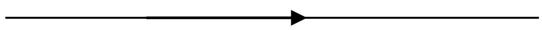
Scalar quantity: It is a physical quantity that has no direction and it is completely specified by its magnitude / size alone, e.g. mass, energy, time, etc.

Vector quantity: It is a physical quantity that is completely specified only when both its magnitude / size and direction are given, e.g. velocity, displacement, force, momentum, acceleration etc.

2.3 Representing vectors

A vector quantity is represented in many ways.

Pictorial representation: A vector is represented by a directed line segment (arrow). Where length of the line is the size/magnitude while the arrow shows direction.



Symbol representation:
Vector ***A*** can be represented as in:-

$\vec{A}$ - Arrow on top

A - wavy line below

A - Bold face

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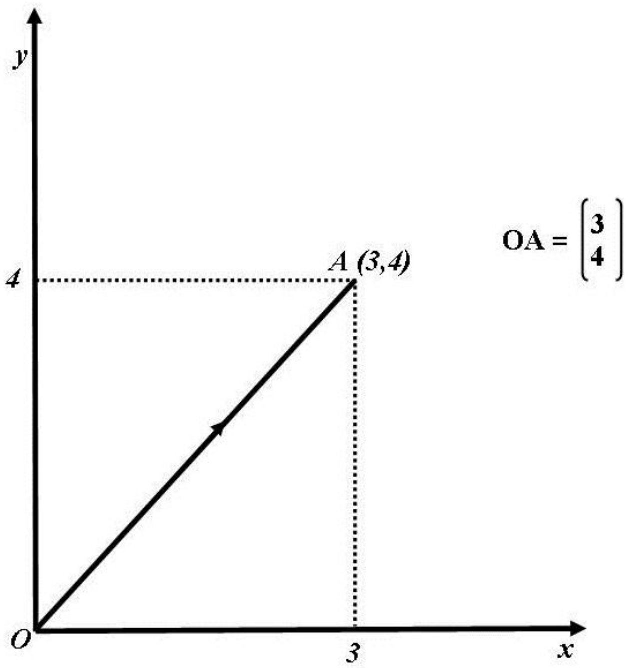
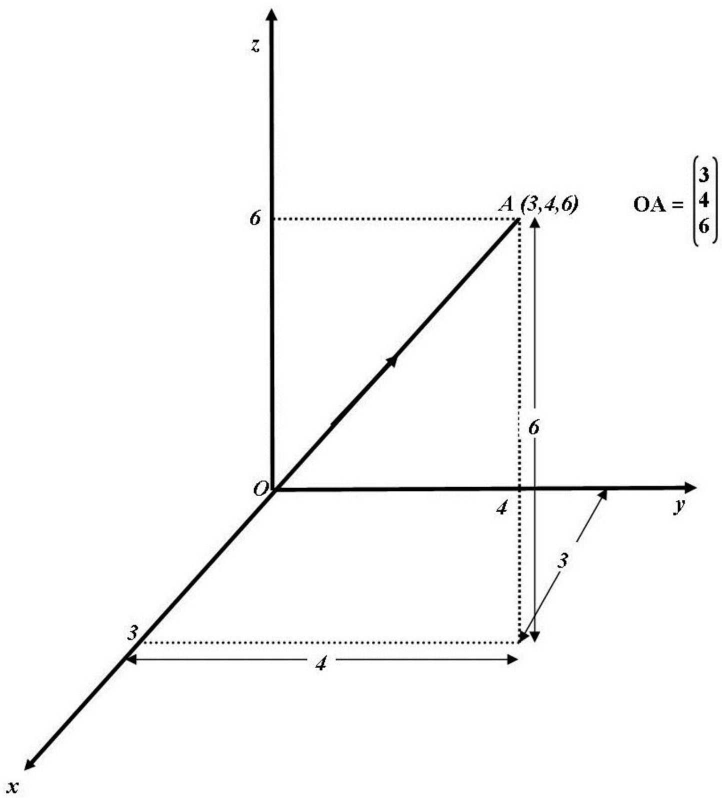
Vectors can be analyzed when represented on a coordinate system.

i) Cartesian or rectangular co-ordinate. *(xy* plane/2 dimension)



ii) *xyz* plane (3 dimension)



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Vectors can also be represented in terms of **i, j** and **k.**

i.e **OA** = $\begin{bmatrix}3\\4\end{bmatrix}$; **OA** = $3\mathbf{i} + 4\mathbf{j}$

OA = $\begin{bmatrix}3\\4\\6\end{bmatrix}$; **OA** = $3\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$

Position vector: It is a vector drawn from the origin of some coordinate system to a point in space to indicate position of object with respect to origin, i.e

OA = $\begin{bmatrix}3\\4\end{bmatrix}$ or **OA** = $\begin{bmatrix}3\\4\\6\end{bmatrix}$

Displacement vector: It is a directed line segment (arrow) whose length indicates the magnitude of the displacement and whose direction is the direction of displacement.

2.4 Operation on vectors

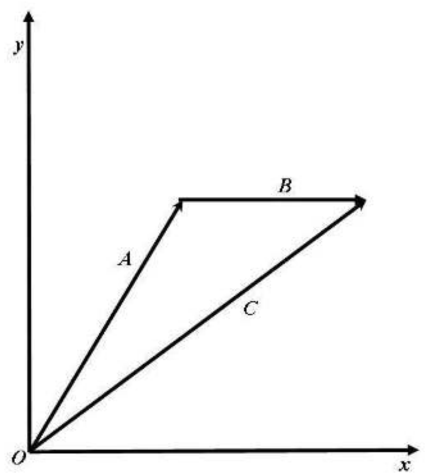
2.4.1 Vector addition

In addition it means two vectors are added to get another vector, i.e

A + **B** = **C**

There are two ways of doing this:-

Triangle method: If **A** and **B** are drawn to scale with tail of **B** at the tip of **A**, then **C** is a vector from the tail of **A** to the tip of **B.**



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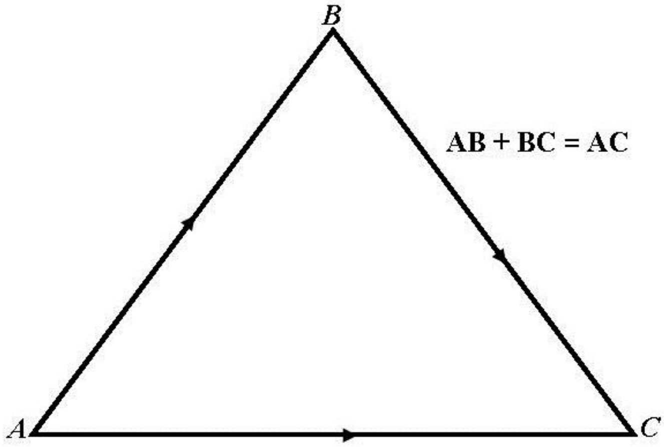
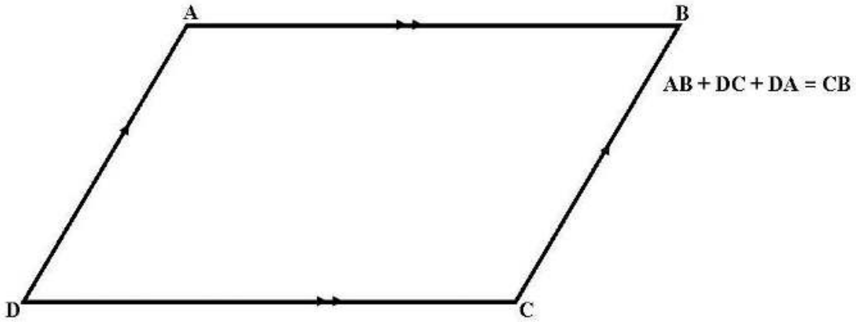
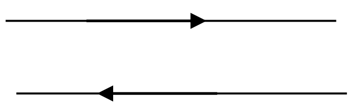
Tip – to – tip Method (polygon): It is an extension of the triangle method to two or more than two vectors.





2.4.2: <u>Vector subtraction</u>

The negative of a vector of equal magnitude but different direction.



$\mathbf{A} = -\mathbf{B}$

Vector subtraction is vector addition of opposite vectors.

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

<u>Example:</u>

1. Given that $\mathbf{A} = 5\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = 2\mathbf{i} - 4\mathbf{j}$

   Find: a) $\mathbf{A} + \mathbf{B}$ b) $\mathbf{A} - \mathbf{B}$

   a) $\mathbf{A} + \mathbf{B} = 7i - j$ b) $\mathbf{A} - \mathbf{B} = 3i - 7j$

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2.4.3: Multiplication of Vectors

We have two ways of a vector multiplication

- Dot / scalar product
- Cross/ vector product

Dot / Scalar product

Means the result is a scalar.

If there are two vectors **A** and **B** then the dot product of the two vectors is defined as

$\mathbf{A.B} = |\mathbf{A}||\mathbf{B}| \cos\theta$

Where θ is the angle between the two vectors.

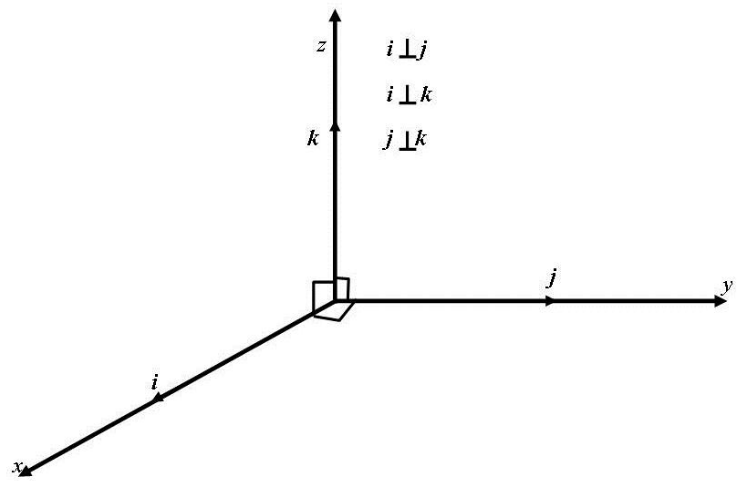
Note: dot product commute, i.e

$\mathbf{A}.\mathbf{B} = \mathbf{B}.\mathbf{A}$

If the vectors are perpendicular to each other then the angle between them is 90^o.

$$\mathbf{A.B} = |\mathbf{A}||\mathbf{B}| \cos 90^o = 0$$

If we express in terms of **i**, **j** and **k** then



$\mathbf{k.j} = \mathbf{i.k} = 0$, $\mathbf{k.j} = \mathbf{j.k} = 0$ **and** $\mathbf{i.j} = \mathbf{j.i} = 0$

Also $\mathbf{i.i} = \mathbf{j.j} = \mathbf{k.k} = 1$

Consider vectors $\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{B} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then,

$$\begin{aligned}
\mathbf{A}.\mathbf{B} &= (a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k})\,(b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) \\
&= a_1b_1\mathbf{i.i} + a_1b_2\mathbf{i.j} + a_1b_3\mathbf{i.k} + a_2b_1\mathbf{j.i} + \\
&\quad a_2b_2\mathbf{j.j} + a_2b_3\mathbf{j.k} + a_3b_1\mathbf{k.i} + a_3b_2\mathbf{k.j} + \\
&\quad a_3b_3\mathbf{k.k} \\
&= a_1b_1 + 0 + 0 + 0 + a_2b_2 + 0 + 0 + 0 + a_3b_3 \\
&= a_1b_1 + a_2b_2 + a_3b_3
\end{aligned}$$

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Example

1) Find **A**.**B** and **B**.**A** given that

$\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{B} = -\mathbf{j} - 2\mathbf{j} + \mathbf{k}$

$\mathbf{A.B} = (2x - 1) + (3 \text{ x } -2) + (4 \text{ x } 1) = {}^{-}4$

$\mathbf{B.A} = ({}^{-}1 \text{ x } 2) + ({}^{-}2 \text{ x } 3) + (1 \text{ x } 4) = {}^{-}4$

2) Given that, $|\mathbf{A}| = \sqrt{14}$, $|\mathbf{B}| = \sqrt{16}$ and the angle between **A** and **B** is 30°, find **A**.**B**

$\mathbf{A}.\ \mathbf{B} = |\mathbf{A}||\mathbf{B}| \text{ Cos } \theta = \sqrt{14} \text{ x } \sqrt{16} \text{ Cos } 30^0 = 12.9514$

Cross Product

Given that two vectors **A** and **B** the cross product of **A** and **B** is defined as

$\mathbf{A} \text{ x } \mathbf{B} = |\mathbf{A}||\mathbf{B}| \text{ Sin } \theta$

Consider two vectors

$\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{B} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$

Represent in matrix form $\mathbf{A} \text{ x } \mathbf{B} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$\mathbf{A} \text{ x } \mathbf{B} = \mathbf{i}\,[(a_2\, b_3) - (b_2\, a_3)] + \mathbf{j}\,[(b_1\, a_3) - (a_1\, b_3)] + \mathbf{k}\,[(a_1 b_2) - (b_1 a_2)]$

Example.

Given that $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{B} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

Find **A** x **B**

$\mathbf{A} \text{ x } \mathbf{B} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 3 & -1 \\ -1 & 1 & 2 \end{vmatrix}$

$\mathbf{A} \text{ x } \mathbf{B} = \mathbf{i}\,[(3 \text{ x } 2) - (1 \text{ x } -1)] + \mathbf{j}\,[(-1 \text{ x } -1) - (2 \text{ x } 2)] + \mathbf{k}\,[(2 \text{ x } 1) - (3 \text{ x } -1)]$

$= 7\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$

2.4.4: Multiplication with scalars

Consider vectors **A, B** and scalar S then

$S\,(\mathbf{A} + \mathbf{B}) = S\mathbf{A} + S\mathbf{B}$

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2.4.5: Magnitude and Direction of a vector

Given that $\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, then

$|\mathbf{A}| = \sqrt{[(a_1)^2 + (a_2)^2 + (a_3)^2]}$

Example:

1. Given that $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, find $|\mathbf{A}|$

   $|A| = \sqrt{[(2)^2 + (3)^2 + (4)^2]} = 5.39$ units

2. If $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$ and $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{B} = 5\mathbf{j} + 6\mathbf{j} + 7\mathbf{k}$. What is $\mathbf{C}$, $|\mathbf{C}|$ and angle between $\mathbf{C}$ and x axis.

   $\mathbf{C} = -\mathbf{A} - \mathbf{B} = -7\mathbf{i} - 9\mathbf{j} - 11\mathbf{k}$

   $|\mathbf{C}| = \sqrt{[(-7)^2 + (-9)^2 + (-11)^2]} = 15.84$ units

   $\theta = \text{Tan}(-9/-7) = 52.13^o$

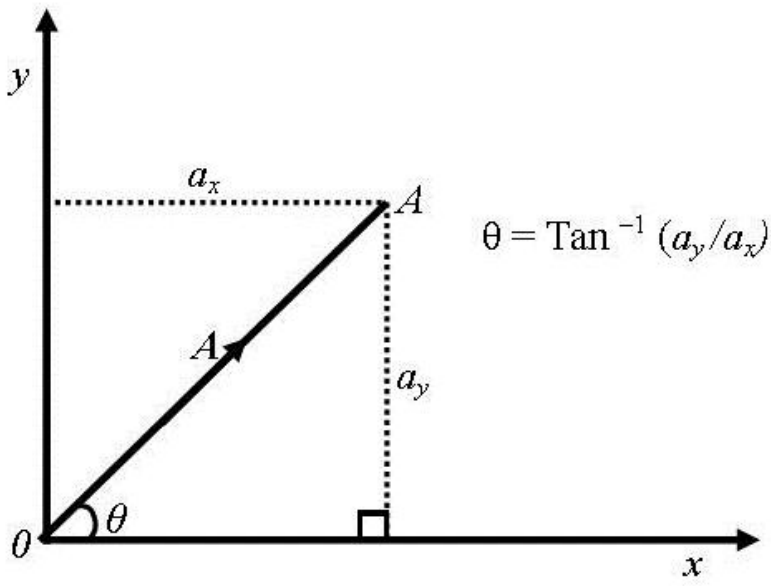
2.4.6: Angle between vectors

We find angles between vectors by using the dot product. This is because dot product gives the result of a scalar.

$\mathbf{A.B} = |\mathbf{A}||\mathbf{B}| \text{ Cos } \theta$, $\theta = \text{Cos}^{-1}\left(\frac{\boldsymbol{A.B}}{|\boldsymbol{A}||\boldsymbol{B}|}\right)$

2.4.7. Angle between vector and axes

Consider vector **A** as shown:-



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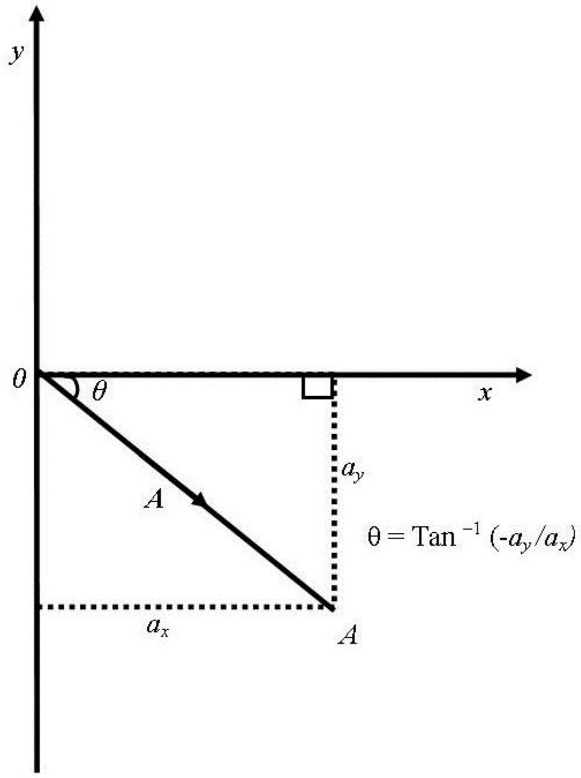


Note: The negative sign shows the angle is below x – axis

<u>Example:</u>

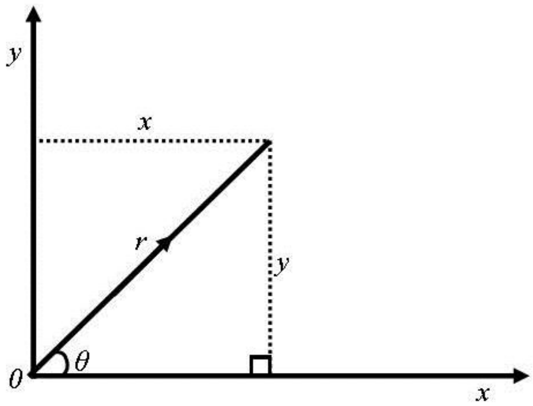
Given that **OA** = $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and **OB** = $\begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$, find the angle between them.

$|\mathbf{A}| = \sqrt{[(2)^2 + (3)^2 + (1)^2]} = \sqrt{14}$, $|\mathbf{B}| = \sqrt{[(5)^2 + (-2)^2 + (2)^2]} = \sqrt{33}$,

$\theta = \text{Cos}^{-1}\left(\frac{\boldsymbol{A.B}}{|\boldsymbol{A}||\boldsymbol{B}|}\right) = \text{Cos}^{-1}\left(\frac{\mathbf{6}}{\sqrt{(\mathbf{14 \text{ x } 33})}}\right) = 73.79^0$

Magnitude and Direction in Two Dimension

The rectangular co-ordinates (x,y) and polar co-ordinates (r,θ) are related by $x = r \text{ Cos } \theta$, $y = r \, Sin \, \theta$, $r = \sqrt{x^2 + y^2}$ and Tan $\theta = y/x$



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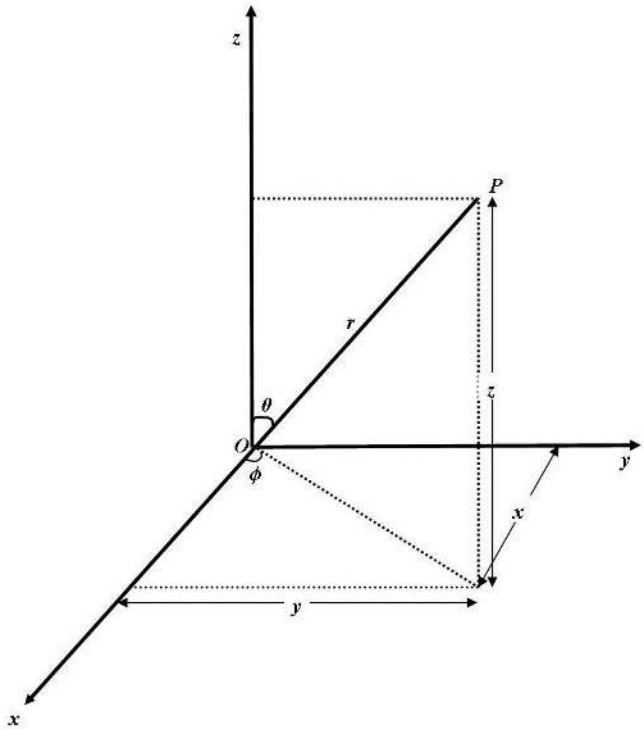
Three dimension (x,y,z) co-ordinate and spherical co-ordinate.
The rectangular co-ordinate (x, y, z) and spherical co-ordinates (r, θ, ϕ) are related by:
$x = r \, Sin\, \theta \, Cos\phi$, $y = r \, Sin\, \theta \, Sin\phi$, $z = r\, Cos\, \theta$, $r = \sqrt{x^2 + y^2 + z^2}$, $Tan\, \theta = \sqrt{\frac{x^2 + y^2}{Z^2}}$
and $Tan\, \phi = y/x$



Examples.

1) Find the magnitude and direction of the following vectors.
   a) **A** = $5\boldsymbol{i} + 3\boldsymbol{j}$ b) **B** = $10\boldsymbol{i} - 7\boldsymbol{j}$ c) **C** = $-2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$

   Solution

   a) $|\mathbf{A}| = r = \sqrt{5^2 + 3^2} = 5.83$
      $\theta = \text{Cos}^{-1}\left(\frac{5}{5.83}\right) = 30.96^0$

   b) $|\mathbf{B}| = r = \sqrt{10^2 + -7^2} = 12.21$

      $\theta = \text{Cos}^{-1}\left(\frac{10}{12.21}\right) = 35.02^0$

   c) $|\mathbf{C}| = r = \sqrt{-2^2 + -3^2 + 4^2} = 5.39$

      $\theta = \text{Tan}^{-1}\left(\frac{\sqrt{-2^2 + -3^2}}{\sqrt{4}}\right) = 42.03^0$
      $\phi = \text{Tan}^{-1}\left(\frac{-3}{-2}\right) = 56.31^0$

2) The rectangular components of the vectors which lie in $x - y$ plane have their magnitudes and directions given below. Find the x and y components of the vectors.
   a) $r = 10$ and $\theta = 30^0$ b) $r = 7$ and $\theta = 60^0$

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Solution

a) $x = r \text{ Cos } \theta = 10 \text{ Cos } 30^0 = 8.66$, $y = r \text{ Sin } \theta = 10 \text{ Sin } 30^0 = 5$

b) $x = r \text{ Cos } \theta = 7 \text{ Cos } 60^0 = 3.5$, $y = r \text{ Sin } \theta = 7 \text{ Sin } 60^0 = 6.06$

3. a) Find the magnitude and direction of the resultant vector **A** = 5**i** + 3**j** and **B** = 2**i** – 4**j**

Solution

$\boldsymbol{R} = \boldsymbol{A} + \boldsymbol{B} = 7\boldsymbol{i} - \boldsymbol{j}$

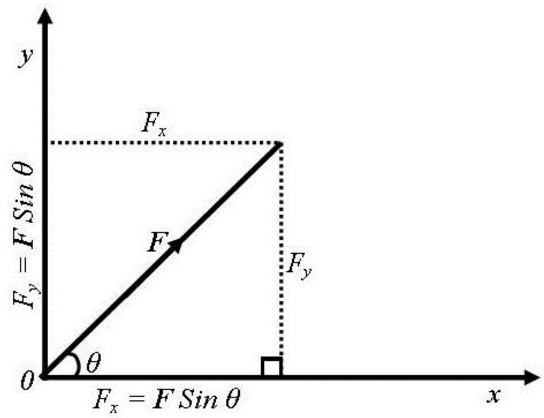
$|\boldsymbol{R}| = \sqrt{(7^2 + -1^2)} = 7.07$

$Tan\ \theta = \frac{y}{x} = \frac{-1}{7}$

$\theta = -8.13^0$

2.4.8 Resolution of Vectors

A component of a vector is the effective part of a vector in that direction.
Consider a Force **F** pulling in the direction as shown.



X component of **F** is **F** Cos θ
Y component of **F** is **F** Sin θ

Example

Consider two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ pulling as shown below. Find the X and Y components of the forces given that

$|\mathbf{F_1}| = 2.88$ and $|\mathbf{F_2}| = 3.44$

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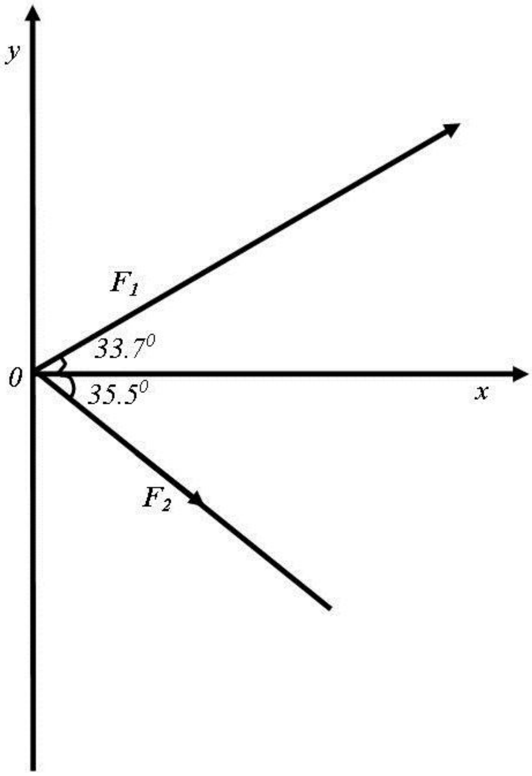

$F_{1x} = |\mathbf{F}_1|$ Cos 33.7° = 2.88 Cos 33.7° = 2.40, F_{1y} = $|\mathbf{F}_1|$ Sin 33.7° = 2.88 Sin 33.7° = 1.60

$\mathbf{F}_1 = 2.4\mathbf{i} + 1.6\mathbf{j}$
Similarly
F_{2x} = $|\mathbf{F}_2|$ Cos 35.5° = 3.44 Cos 35.5° = 2.80, $F_{2y} = |\mathbf{F}_2|$ Sin (-35.5)
= 3.44 Sin (-35.5°) = 2.00

$\mathbf{F}_2 = 2.80\mathbf{i} - 2.00\mathbf{j}$

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3.0: FORCE

Introduction

Force is defined as pull or push in on a body, it is a vector quantity it is measured in Newtons. A Newton is the force that gives a mass of *1kg* an acceleration of $1m/s^2$

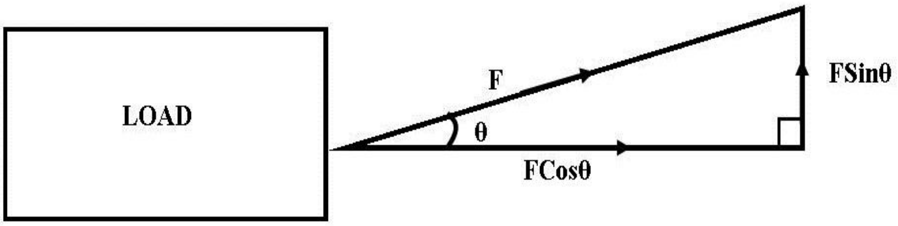
Task:

(1) Give five effects of force.
(2) Name and explain at least 10 different types of force.

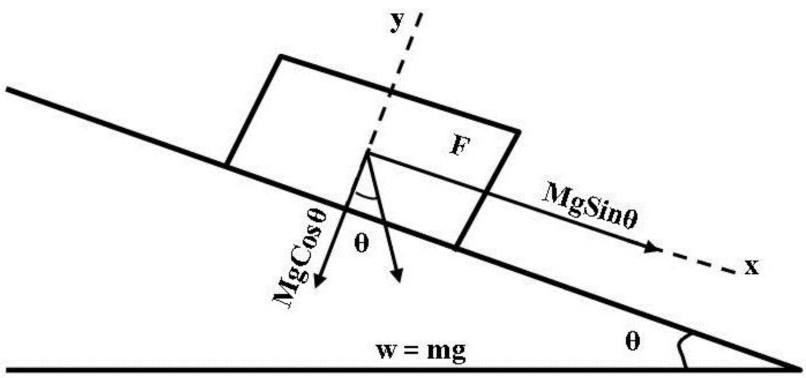
3.1: Resolution of forces

Forces is a vector quantity which can also be expressed in x and y on rectangular coordinates

Consider a force F pulling a load along a surface at an angle θ



The horizontal component of the force is $F \cos \theta$ while the vertical component is $F \sin \theta$. Also consider a load sliding along an inclined plane at a constant acceleration



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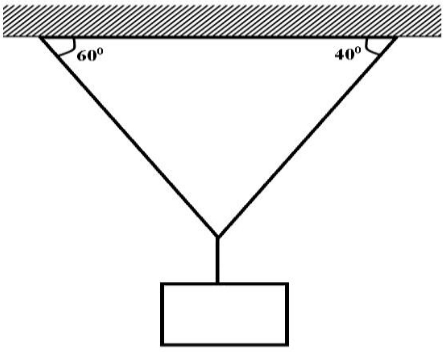
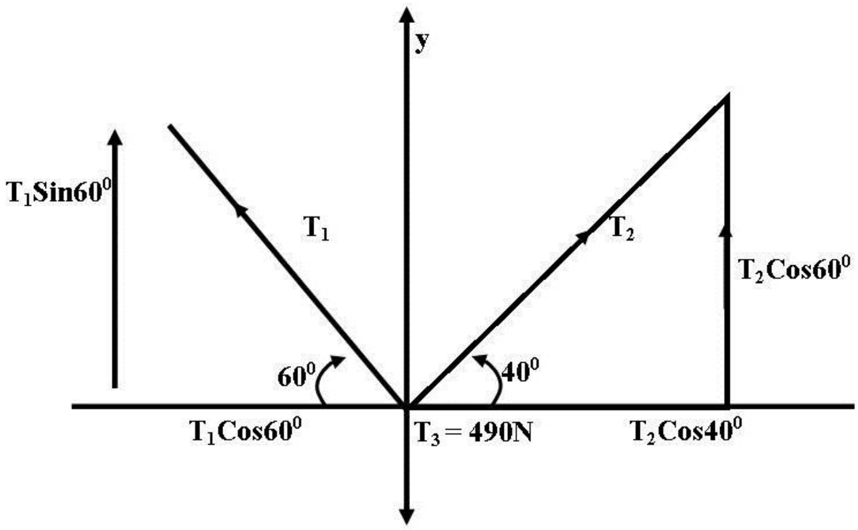
Example

Find the tension in each cord if the weight of suspended object is *490N*



Solution



$\sum F_y = 0$

$T_3 - 490 = 0$

$\sum F_x = 0$

$T_2\ Cos40^0 - T_1\ Cos\ 60^0 = 0$

$T_2 = 0.653T_1$

$\sum Fy = 0,\ T_2Sin40^0 + T_1\ Sin\ 60^0 - 490N = 0$

$(0.653T_1)\ Sin\ 40^0 + T_1\ Sin\ 60^0 = 490$

$T_1 = 381N$

$T_2 = 0.653\ (381) = 249N$

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3.2: Friction Force

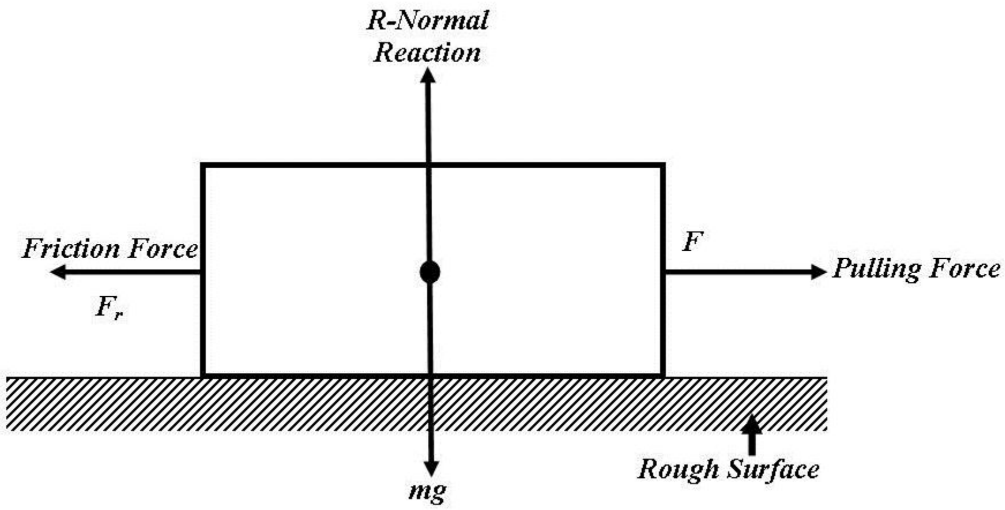
Friction force is a force that opposes relative motion between two surfaces.

Friction is an example of dissipative or resistive force this force convert mechanical energy (kinetic energy) to other energies e.g. sound and heat by resisting motion of bodies. Other examples are air resistance and viscosity of fluids.

Task:

1. Friction is a nuisance, explain?
2. Friction is vital explain?
3. How can you reduce friction?

For a body resting on a rough surface, when external force (F) is applied on it (pushing /pulling)



Then this force has to overcome friction (F_r) before the body moves. The body is moved by a net force (F_{net}) i.e.

$F_{net} = F - F_r$

Frictional force F_r depends on the normal reaction ($R = mg$)

$F_r \propto R$

$F_r = \mu R$

Where μ is the coefficient of friction, μ depends on the nature of two surfaces that are in relative motion.

Static friction:- This is the frictional force exerted by one surface on another when there is no relative motion of the two surfaces

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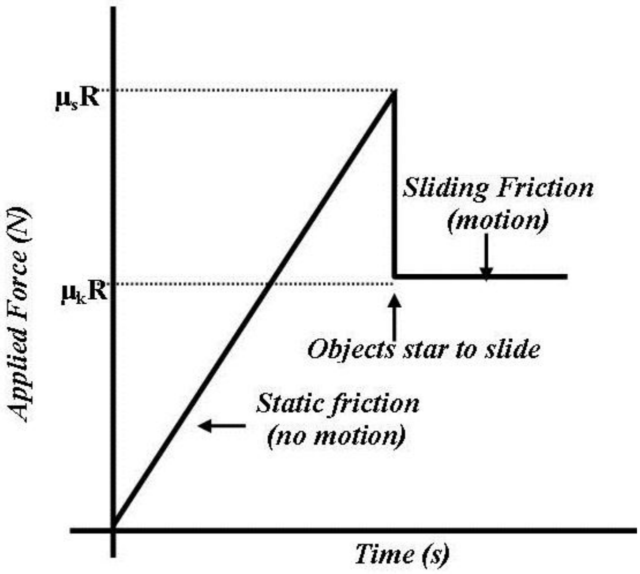
Sliding (Kinetic friction):- The frictional force exerted by one surface on another when one surface slides over another surface.

$F_{sliding/kinetic} = F_k = \mu_k R$ - μ_k - coefficient of kinetic/sliding friction

$F_{static} = F_s \leq \mu_s R$ – μ_s coefficient of static friction



Task:

(1) State the Laws of friction

Examples

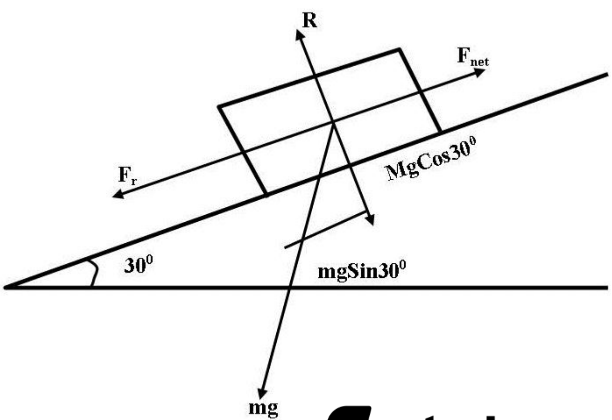
1. A block of wood of mass *20kg* requires a horizontal force of *50N* to pull it with a uniform velocity along a horizontal surface. Calculate the coefficient of friction between the block and the surface.

   $R = mg = 20 \times 10 = 200N,$

   $F_r = \mu R$

   $\mu = {}^{50}/_{200} = 0.25$

2. A mass of *5kg* is placed on a plane inclined at an angle of 30^0 to be horizontal. Calculate the force required to pull the mass up the plane at uniform velocity if $\mu = 0.5$



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$R = mgCos30^0 = 50Cos30^0 = 43.3N$

$F_r = \mu R = \mu mgCos\theta = 0.5 \times 50Cos30^0 = 21.65N$

There are two forces opposing motion, F_r and mgSinθ

$F_{net} = F – (F_r + mgSin\theta)$

We have uniform motions, net force is zero

$F = F_r + mgSin\theta$

$F = 21.65 + 50Sin30^0 = 46.65N$

3.3: <u>Newton Laws</u>

First Law (law of inertia):- every body continues to be in state of rest or to move with uniform velocity unless a resultant force acts on it.

Implication of the 1st law – causes inertia

Inertia:- is the property of an object that resists change of motion

Second law:– The rate of change of momentum is directly proportional to the change causing it (resultant force) and takes place in the direction of force.

Momentum:- – is defined as the product of mass of a body and its velocity.

$\boldsymbol{p} = m\boldsymbol{v}$……………………………..(1)

Consider a force **F**, acting on a body of mass, m for a time t, causing a change in velocity from $\boldsymbol{u}$ to $\boldsymbol{v}$, then:-

$\Delta\mathbf{p} = m\boldsymbol{v} - m\boldsymbol{u}$……………………… (2)

From 2nd law $\frac{\Delta \boldsymbol{P}}{\Delta t} = \frac{m\boldsymbol{v} - m\boldsymbol{u}}{\Delta t}$

$\boldsymbol{F} \propto \frac{\Delta \boldsymbol{P}}{\Delta t}$

$\boldsymbol{F} = K\frac{\Delta \boldsymbol{P}}{\Delta t}$ *however, K = 1*

$\boldsymbol{F} = K\frac{\Delta \boldsymbol{P}}{\Delta t} = \frac{m\boldsymbol{v} - m\boldsymbol{u}}{\Delta t} = \frac{m(\boldsymbol{v} - \boldsymbol{u})}{\Delta t}$

But $\boldsymbol{a} = \frac{(\boldsymbol{v} - \boldsymbol{u})}{\Delta t}$

Therefore $\boldsymbol{F} = m\boldsymbol{a}$………………………….. (3)

Task: Kinetic energy, $K = ½mv^2$ and momentum, $p = mv$, show that, $K = \frac{p^2}{2m}$

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Examples

1) A car of mass *1200kg* travelling at *45m/s* is brought to rest in 9 seconds. Calculate the average retardation of the car and the average force applied by the brakes.

$a = \frac{v-u}{t} = \frac{0-45}{9} = -5ms^{-2}$

$F = ma = (1200 \times -5)N = -6000N$

2) A truck weighs $1.0 \times 1.0^5 N$ and free to move. What force will give it an acceleration of $1.5ms^{-2}$

$F = ma = \frac{1.0 \times 10^5}{10} \times 1.5N = 1.5 \times 10^4 N$

Third Law: For Every Action, There is An Equal and Opposite Reaction.

Note: A body moving in a straight line has linear momentum.

3.4: Impulse – momentum theorem

States that the change in momentum of a body is equal to the sum of the force acting on the body with respect to time.

$m\boldsymbol{v}-m\boldsymbol{u} = \int \boldsymbol{F} dt$

Impulse change the velocity of a body of mass m from $\boldsymbol{u}$ to $\boldsymbol{v}$.

3.4.1: Conservation of linear momentum

Momentum of an isolated system is always conserved,

An isolated system is one that has zero interaction with its environment e.g, pressure, temperature. e.t.c. In practice it is easy to isolate a system.

Consider two bodies of mass m_1 and m_2 moving in the same direction on a smooth horizontal surface at velocities $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ respectively.

Initial momentum will be $\boldsymbol{p}_1 = m_1\boldsymbol{u}_1$ and $\boldsymbol{p}_2 = m_2\boldsymbol{u}_2$

Total momentum before collision $\mathbf{p} = \boldsymbol{p}_1 + \boldsymbol{p}_2 = m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2$

After collision, their velocities are $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ respectively.

Total momentum after collision, $\boldsymbol{p} = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$

By conservation of linear momentum therefore,

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Total initial momentum = Total final momentum

i.e, $m_1\boldsymbol{u}_1 + m_2\,\boldsymbol{u}_2 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$ (isolated system)

3.4.2: Elastic Collision.

It's a case where bodies separate after collision

3.4.3: Inelastic Collision

Colliding bodies coalesce/combine after collision. During perfect elastic collision, the momentum $\boldsymbol{p}$ of the bodies is conserved and also the kinetic energy possessed by the body is conserved.

i.e, $\frac{1}{2}m_1\boldsymbol{u}_1^2 + \frac{1}{2}m_2\,\boldsymbol{u}_2^2 = \frac{1}{2}m_1\boldsymbol{v}_1^2 + \frac{1}{2}m_2\boldsymbol{v}_2^2$.

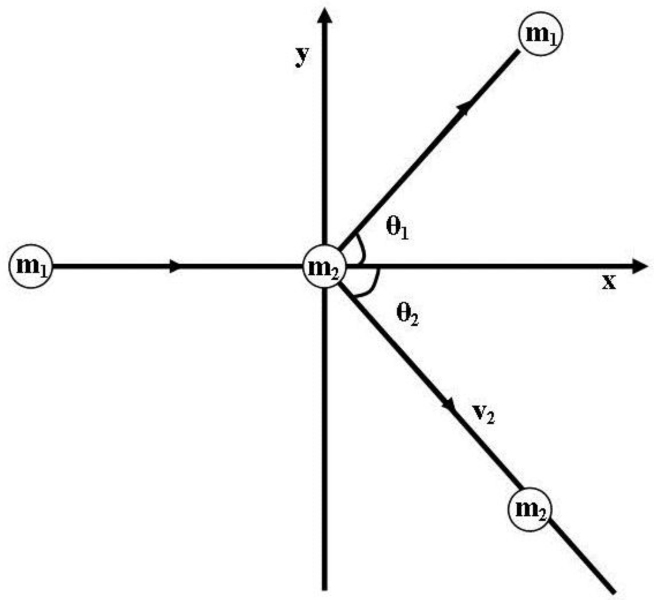
The bodies combine and move with a common final velocity v

i.e, $\frac{1}{2}m_1\boldsymbol{u}_1^2 + \frac{1}{2}m_2\,\boldsymbol{u}_2^2 = \frac{1}{2}(m_1 + m_2)\boldsymbol{v}^2$

During imperfect elastic collision, momentum is conserved but energy is not conserved.

3.4.4 Collision in Two Dimensions.

In this kind of collision the bodies collide at angles. This is represented in a rectangular coordinate.



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The law that controls the above scattering starter:-

"If no external force acts on a system, the components of the total linear momentum of the system along any chosen axis are the same after a collision as before the collision."

On resolution:-

X - Components: $m_1u_1 + 0 = m_1v_1Cos\theta_1 + m_2v_2Cos\theta_2$

Y - Components: $0 + 0 = m_1\, v_1\, sin\theta_1 - m_2\, v_2Sin\theta_2$

Or

$m_1\, v_1\, Sin\theta_1 = m_2\, v_2\, Sin\theta_2$

If the collision is elastic then kinetic energy is conserved.

$\frac{1}{2}m_1\boldsymbol{u}_1^2 + 0 = \frac{1}{2}m_1\boldsymbol{v}_1^2 + \frac{1}{2}m_2\boldsymbol{v}_2^2.$

3.4.5 Co-efficient of resolution, *e*

Is the ratio of the velocity of separation to the velocity of approach.

$e = \frac{v_2 - v_1}{u_2 - u_1}$

$e > 1$ - explosive / super elastic collision

e $= 1$ - elastic collision

e < 1 - inelastic collision

$e = 0$ –completely inelastic.

Examples

1) An object A of mass $2kg$ is moving with velocity of $3ms^{-1}$ and collides head-on with an object B of mass $1kg$ moving in the opposite direction with a velocity of $4ms^{-1}$. After collision both objects stick so that they move with a common velocity v, calculate v.

Solution.

Total momentum before collision is equal to total momentum after collision.

$m_Av_A - m_B\, v_B = (m_A + m_B)v$

$(3 \times 2) - (1\times 4) = (2 + 1)\, v$

$v = {}^2/_3 = 0.67\, ms^{-1}$

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2) A bullet of mass *20g*, traveling horizontally at $100ms^{-1}$ embeds itself in the centre of the block of wood of mass *1kg* which is suspended by a light vertical string *1m* long. Calculate maximum inclination to the string to the vertical. (take $g = 9.8ms^{-2}$)

Solution.

Using the principle of conservation of momentum then

$M_1 u_1 + m_2 u_2 = (m_1 + m_2) v$

$v = 1.96ms^{-1}$

From the law of conservation of energy then

$\frac{1}{2} mv^2 = mg h$

$v^2 = gh$

$h = l - l \cos\theta$

$h = l(1 - \cos\theta)$

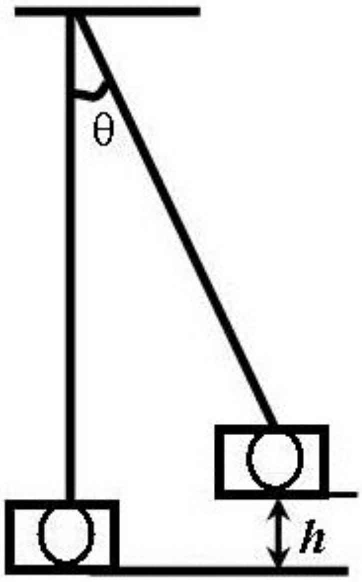
$v^2 = 2gl(1 - \cos\theta)$

$(1.96)^2 = 2 \times 9.8 \times 1 (1 - \cos\theta)$

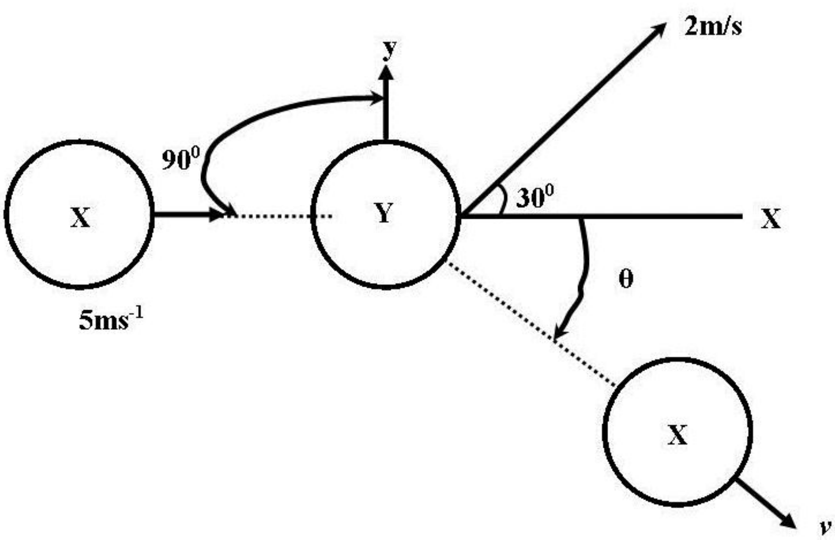
$1 - \cos\theta = \frac{1.96^2}{19.6} - 1$

$\theta = \cos^{-1}(0.8038)$

$\theta = 37^0$



3) A pool ball *x* of mass *0.3kg* moving with velocity $5\ ms^{-1}$, hits a stationary ball *y* of mass *0.4kg*. *Y* moves off with a velocity of $2ms^{-1}$ at 30^0 to the initial direction of *x*. Find the velocity of *x* and its direction after hitting *y*



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In initial direction of x, from conservation of momentum

$0.3v\ Cos\theta + 0.4 \times 2\ Cos\ 30^0 = 0.3 \times 5$

$0.3v\ Cos\theta = 1.5 - 0.8\ Cos\ 30^0 = 0.8$………………………………….. (1)

Along y, 90^0 to initial x direction, initial momentum = 0, so in this direction,

$0.4 \times 2\ Sin30^0 - 0.3 \times v\ Sin\theta = 0$

$0.3\ v\ Sin\theta = 0.4$ ………………………………………………………….……… (2)

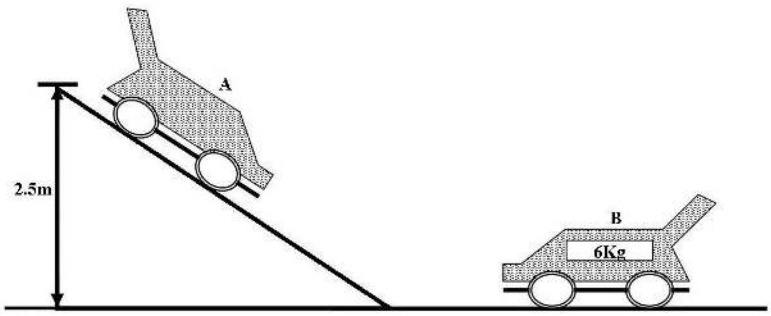
Dividing equation (2) by (1)

$\frac{Sin\theta}{Cos\theta} = Tan\theta = \frac{0.4}{0.8} = 0.5$

$\theta = 27^0$

From (2) $v = \frac{0.4}{0.3\ Sin\ 27^0} = 3ms^{-1}$

4) A trolley of mass 8kg is held at rest on a smooth inclined plane as shown. When released, it moves down through a vertical height of 2.5m while accelerating. It then collides with a second trolley of mass 6kg which is at rest on a smooth horizontal plane.



After collision, the two trolleys coalesce and move forward. Calculate:

a. The velocity of A just before collision.
b. The common velocity of the two trolleys after the collision.
c. The kinetic energy just before and after the collision. Account for the difference in kinetic energy.

Solution:

a) From the principle of conservation of energy

$\frac{1}{2}mv^2 = mgh$, $v = \sqrt{2gh}$

$v = \sqrt{2 \times 10 \times 2.5} = 7.07m/s$

b) $m_1u_1 + m_2u_2 = (m_1 + m_1)v$

$8 \times 7.07 + 0 = (8 + 6)v$

$v = 4.04m/.$

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c) $Ke = \frac{1}{2} m_1 u_1^2 = \frac{1}{2} \times 8 \times (7.07)^2 = 200J$

$Ke = \frac{1}{2} (m_1 + m_2) v^2 = \frac{1}{2} \times (8 + 6)(4.04)^2 = 114.25J$

3.4.6 Variation of weight in a lift

Weight of a body inside a lift increases when the lift accelerates upwards and decreases when the lift accelerates downwards.

Suppose a person is standing on a weighing balance the gravitation force acts vertically downwards. A weight balance exerts reaction force (R) in the upward direction hence the net force on the person is $F = mg - R$

But the person is stationary, hence the net force acting on the person is considered to be zero, if the lift is moving at a constant velocity $(a = 0)$

$Mg - R = 0$

$R = mg$

The weighing balance reaches the actual weight of person

When the lift moves up at acceleration, a then;

$F = R - mg = ma$

$R = mg + ma$

$R = m(g+a)$

Hence the weighing balance reads slightly higher value than actual weight of the person. Thus person feels his/her weight has increased slightly above the normal value.

When a lift is moving downwards.

$F = mg - R$

$Ma = mg - R$

$R = mg - ma$

$R = m(g-a)$

The weight of the body will be slightly less than the weight in normal state.

Example

1) A man of mass $75kg$ stands on a weighing machine in a lift. Determine the readings in the weighing machine whom the lift moves.
   a) Upward with acceleration if $2ms^{-2}$
   b) Downward with acceleration of $2ms^{-2}$

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Solution.

a) $R = m(g+a) = 75\ (10 + 2) = 900\ N$

b) $R = m(g-a) = 75\ (10 - 2) = 600\ N$

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4.0: RECTILINEAR MOTION

Introduction

Motion: is continuous change of position of a body.

We have three types of motion namely:

1) Rectilinear (linear) motion
2) Circular motion
3) Vibrational motion

Rectilinear (linear) motion: This is motion on a straight line.

Mechanics: This is the study of motion of objects and the causes of that motion.

Kinematics: This is the study of motion without considering its causes.

Dynamics: This is the study of motion considering its causes.

4.1: Linear motion

It is the study of motion on a straight line.

4.1.1: Definition of terms

Distance (D): It is the measure of length between two points, it is a scalar quantity and it is measured in metres.

Displacement (S): It is the measure of length between two points in a specified direction, it is a vector quantity and it is measured in meters.

Speed (s): It is the rate of change of distance, it is a scalar quantity and it is measured in metres per second, *(m/s or* ms^{-1}*).*

Velocity (v): It is the rate of change of displacement, it is a vector quantity and it is measured in meters per second, *(m/s or* ms^{-1}*).*

$$v = \frac{dS}{dt}$$

Acceleration *(a):* It is the rate of change of velocity, it is a vector quantity measured in meters per second squared, (m/s^2 *or* ms^{-2})

$$a = \frac{dv}{dt} = \frac{d^2v}{dt^2}$$

Average velocity *(v_{av}):* This is the ratio of average velocity to time interval.

$$v_{av} = \frac{v+u}{2}$$

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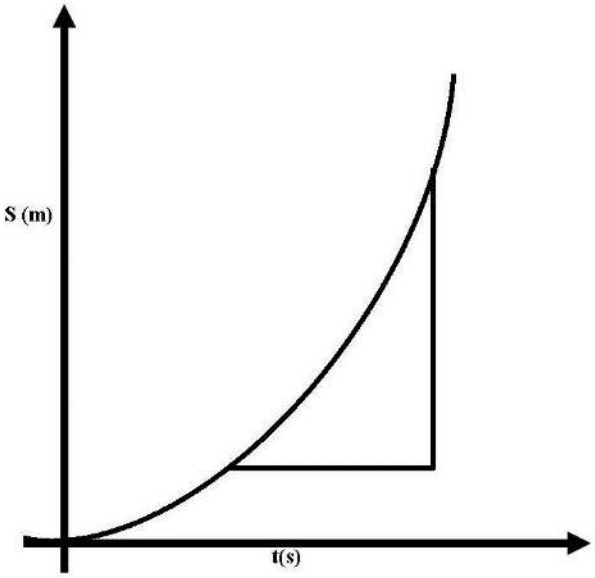
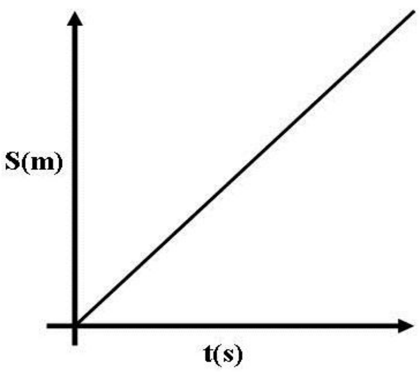
<u>Displacement - time graph.</u>

(i)



The gradient at any given instant of this graph for a body that is changing position represents the instantaneous velocity.

(ii)



If the displacement – time graph is a straight line then the body is undergoing uniform velocity.

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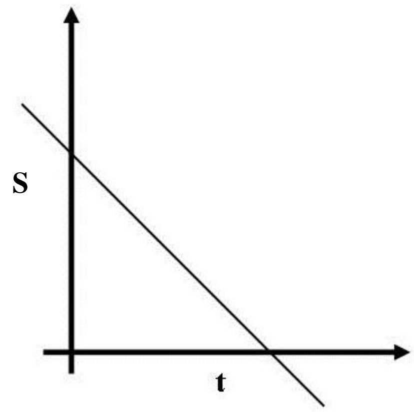
(iii)

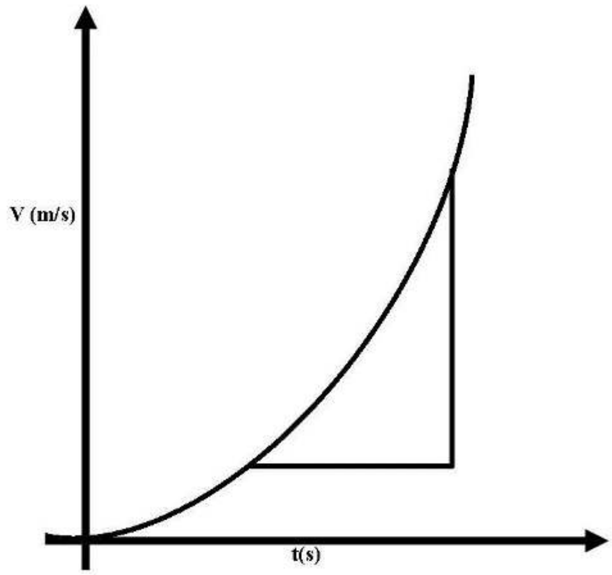


Negative gradient means the body is moving in the opposite direction to the original direction.

Note: The area under the graph has no meaning

Velocity – time graphs

(i)



The gradient at any instant of this graph for a body that is changing position represents the instantaneous velocity

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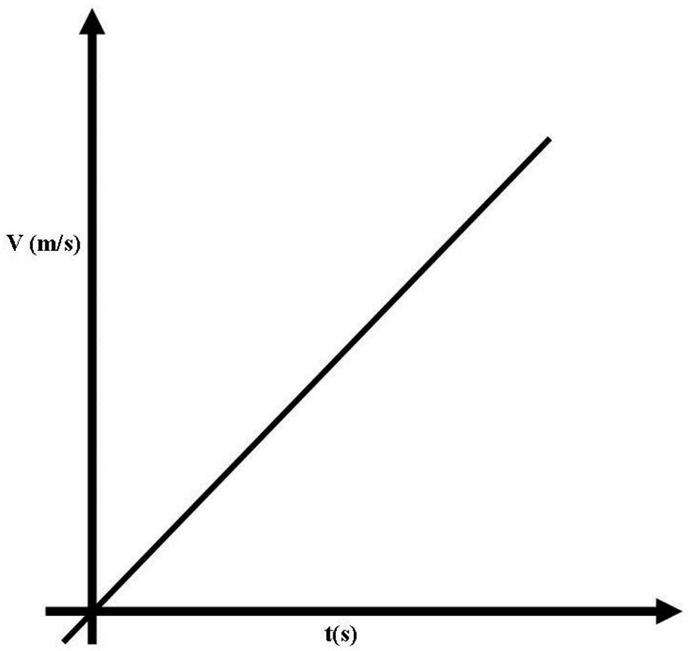
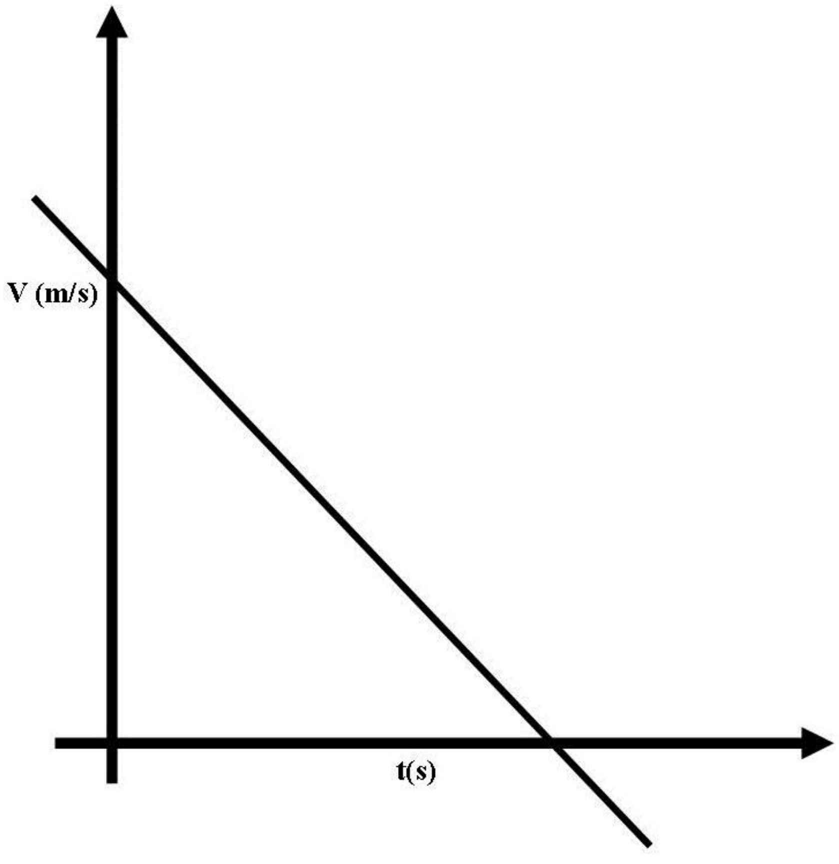
(ii)



If the velocity – graph is a straight line then the body undergoing uniform acceleration.

(iii)



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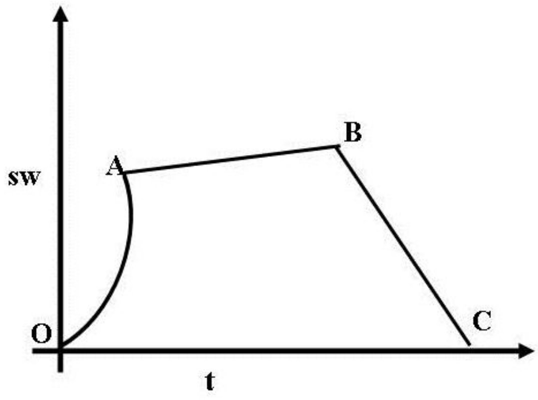
Negative gradient shows that the body is decelerating or retarding. The area under the graph is the displacement.

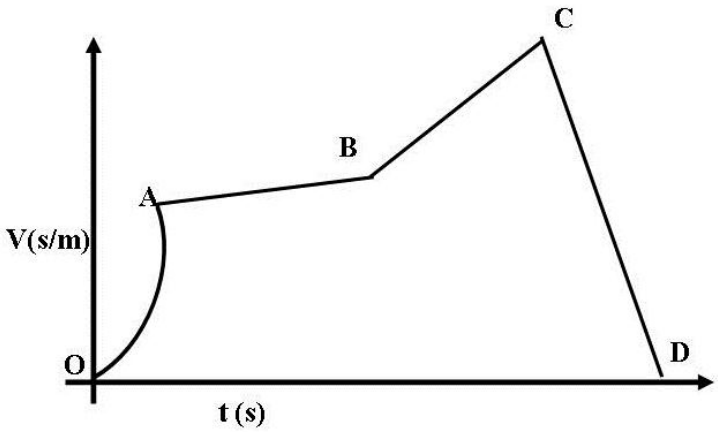
Task:

Describe the motion in the graphs below.

(i)



(ii)



4.2: Equations of linear motion

4.2.1: First equation of linear motion

Consider a body starting with an initial velocity u and it increases or (accelerates) to a final velocity v in time t then the:-

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first equation:

$a = \frac{v-u}{t}$

$\boxed{v=u+at}$

4.2.2: Second equation of a linear motion

Suppose a body accelerates with uniform acceleration *a*, in a time *t* and reaches a velocity *v*. The distance *S* traveled by the object in time *t* is given by

S = average velocity *x* time

Where average velocity =, $v_{av} = \frac{v+u}{2}$

$s = (\frac{v+u}{2})\ x\ t$

$s = (\frac{u+at+u}{2})t$

$s = \left(\frac{2u}{2}\right) + \left(\frac{at}{2}\right)t$

$s = ut + \frac{at^2}{2}$

4.2.3: Third Equation of linear motion

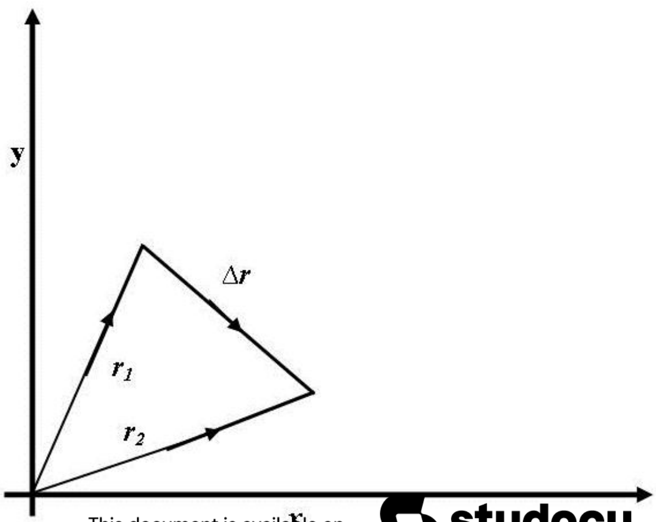
Also $t = \left(\frac{v-u}{a}\right)$ from 1st equation

$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right) = \left(\frac{v^2-u^2}{2a}\right)$

$v^2 = u^2 + 2as$

4.3: Velocity vectors for motion in a plane and acceleration

4.3.1: Velocity vectors



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$\Delta \boldsymbol{r}$ is the displacement vector i.e the vector difference between position vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ and

$$\Delta \boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$$

If an object undergoes a displacement $\Delta \boldsymbol{r}$ in time Δt then its average velocity vector (v) is defined as

$$v = \frac{\Delta r}{\Delta t} = \frac{r_2 - r_1}{t_2 - t_1}$$

The bar means average value of velocity.

Instantaneous velocity in vector notation.

$$\boldsymbol{v} = \frac{Lim}{\Delta t \to 0} \frac{\Delta \boldsymbol{r}}{\Delta t}$$

The limit of the average velocity over a time interval that approaches zero but always includes the desired instant of time.

4.3.2: Acceleration vectors

Suppose that at time $t,$ the velocity is specified by vector $\boldsymbol{v}_1$ where as at sometime later t_2, the velocity vector has a new value $\boldsymbol{v}_2$ then we define acceleration vector as the change of velocity of an object during a given time interval divided by that time interval

$$\bar{a} = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{t_2 - t_1} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

Instantaneous acceleration

$$\bar{a} = \frac{Lim}{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t}$$

The acceleration at a particular instant of time, the limit of the average acceleration over a time interval that approaches zero but always includes the desired instant time.

The displacement vector $\Delta \boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$ could be replaced by two vector displacements along X and Y axes of the rectangular coordinate system. These components are the magnitude $\Delta \boldsymbol{x}$ and $\Delta \boldsymbol{y}$**.** The scalar magnitudes of Δx and Δy are called rectangular components of $\Delta \boldsymbol{r}$. The same approach is taken by velocity and accelerations A velocity $\boldsymbol{v}$ in x-y plane can be replaced by, one along x axis and another along y axis of magnitude v_x and v_y respectively.

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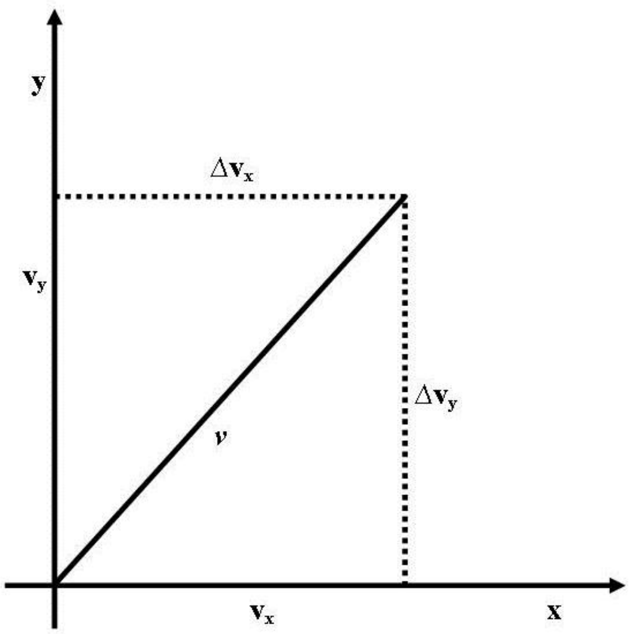

The vector equations introduced for displacement, velocity and acceleration can be summarized in table below.

Physical quantity	Vector notation	Equations for Rectangular components	
		X - axis	y - axis
Displacement	$\Delta \boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$	$\Delta x = x_2 - x_1$	$\Delta y = y_2 - y_1$
Average velocity	$\boldsymbol{v} = \frac{\Delta \boldsymbol{r}}{\Delta t}$	$\boldsymbol{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_2 - x}{t_2 - t}$	$\boldsymbol{v}_y = \frac{\Delta y}{\Delta t} = \frac{y}{t}$
Instantaneous velocity	$\boldsymbol{v} = \frac{Lim}{\Delta t \rightarrow}$	$v_x = \frac{Lim}{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$	$v_y = \frac{Lim}{\Delta t \rightarrow 0} \frac{\Delta y}{\Delta x}$
Average velocity	$\bar{a} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{t_2 - t_1}$	$\bar{\boldsymbol{a}}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}$	$\bar{\boldsymbol{a}}_y = \frac{\Delta v_y}{\Delta t} = \frac{v_{2y} - v_{1y}}{t_2 - t_1}$
Instantaneous acceleration	$\bar{a} = \frac{Lim}{\Delta t \rightarrow 0} \frac{\Delta \boldsymbol{v}}{\Delta t}$	$a_x = \frac{Lim}{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t}$	$a_y = \frac{Lim}{\Delta t \rightarrow 0} \frac{\Delta v_y}{\Delta t}$

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4.3.3: <u>Uniformly accelerated motion on a straight line</u>

Uniformly accelerated motion is motion on a straight line with an acceleration of constant magnitude. We modify the notation we have used so far i.e $t_1 = 0$ *and* $t_2 = t$. The initial position be x_0 and initial velocity be v_0 while final position be x and final velocity be v. If we consider motion in x direction.

$v = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t-0} = \frac{x - x_0}{t}$ ………………… (1)

On solving we have

$x=x_0+vt$…………………………….…….(2)

The average velocity

$v = \frac{v_{0+v}}{2}$…………………………………. (3)

Substituting equation (3) into (2)

$x = x_0 + \left(\frac{v_{0+v}}{2}\right) t$ … … … … ….…………….(4)

The constant acceleration is given by

$a = \frac{\Delta v}{\Delta t} = \frac{v-v_0}{t-0}$…………………………….….(5)

Therefore

$v = v_0 + at$……………………….…….. (6)

If we substitute equation (4) into 6)

$x = x_0 + \left(\frac{v_0+v_0+\text{at}}{2}\right) t = x_0 + v_0 + \frac{at^2}{2}$………(7)

This is a perfect general equation of a body moving in a straight line with uniform acceleration a and initial position x_0 and initial velocity v_0 *at* $t = 0$. We can also develop an equation governing motion but time does not explicitly appear. We use equations and (1) in (4).

I .e $x = x_0 + v\left(\frac{v-v_0}{a}\right)$i.e. $t=\left(\frac{v-v_0}{a}\right)$

From (6) $t=\left(\frac{v-v_0}{a}\right)$

Substituting equation (8) into (4)

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right)$$

$x - x_0 = \left(\frac{v-v_0}{2}\right)\left(\frac{v-v_0}{a}\right) = \frac{v^2 - v_0^2}{2a}$

$v^2=v_0^2+2a\,(x-x_0)$

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The equation of uniformly accelerated motion in one dimension.

$$x = x_0 + vt$$

$$x = x_0 + v_o t + \frac{at^2}{2}$$

$$v^2 = v_0^2 + 2a\,(x - x_0)$$

$$v = v_0 + at$$

$$v = \frac{v_0 + v}{2}$$

Example

1) A force of *140N* acts on a body rest, the body moves a distance in *10* seconds. Calculate:
   a) Acceleration of the body
   b) The distance moved by the body
   c) The velocity of the body

Solution

a) $F_x = ma_{x,} \Rightarrow 140 = (32.5kg)a$
   $a_x = 4.31ms^{-2}$

b) $x = x_0 + v_{0x}t + ½\, a_x t^2$
   $x = ½\,(4.31)^1\,(10.0)^2 = 216m$

c) $V_x = v_{0x} + a_x t$
   $Vx = O + (4.31)\,(10)$
   $= 43.1\ m/s$

4.4: Motion Under Gravity

We have three types of motion under gravity, namely:

1) Free fall
2) Vertical projection
3) Horizontal projection

4.4.1: Vertical Projection

The Earth pulls all bodies towards its centre. A pull of gravity produces a constant acceleration on a body moving vertically upwards. This is called gravitational acceleration *(g)*. Equations of linear motion can be modified by replacing *a* with –*g*, since the body is moving away from the centre of the Earth and *S* is replaced by *H*.

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$v = u - gt$……………………………………….(1)

$H = ut - {}^{1}/_{2}gt^2$……………………………………… (2)

$v^2 = u^2 - 2gH$ ……………………………….(3)

Time taken to reach maximum height (t)

From equation (1)

$v = u\text{-}gt$…………………………………………… *(4)*

At maximum height $v = 0$, the body becomes momentarily at rest, equation (4) becomes

$u = gt, \Rightarrow t = {}^{u}/_{g}$………………………………. *(5)*

Time of flight (T)

Time taken by body to reach maximum height and back to point of projection.

$T=2t = 2{}^{u}/_{g}$…………………………………………….. *(6)*

Maximum Height Reached (H_{max})

At maximum height $v = 0$ using equation (3)

$v^2 = u^2 - 2gh$

$2g\, H_{max} = u^2$

$H_{max} = \frac{u^2}{2g}$………………………………………………………(7)

4.4.2: Free Fall

In free fall or downward motion in equations of linear motion, a is replaced by $+g$ while S is replaced by H

$v=u+gt$………………*(1)*

$H=ut + ½\, gt^2$………..*(2)*

$v^2 = u^2 + 2gH$………….*(3)*

4.4.3: Horizontal projection

This occurs when a body is projected horizontally from a height, h. The body has not continued moving but lands after having same horizontal displacement (range) R.

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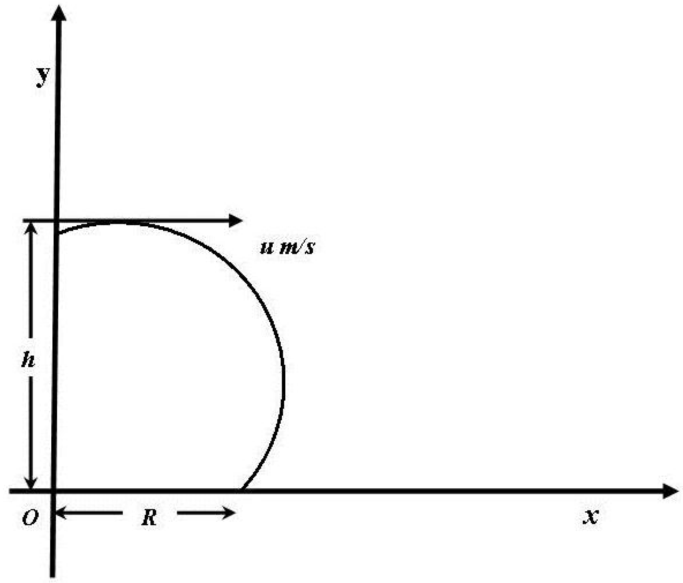


We consider the vertical and horizontal motions from *0* independently consider vertical motion from 0, the vertical velocity $u=0$ from.

$h=ut+{}^1/_2\,gt^2$

$h = {}^1/_2gt^2$…………………………………………………(1)

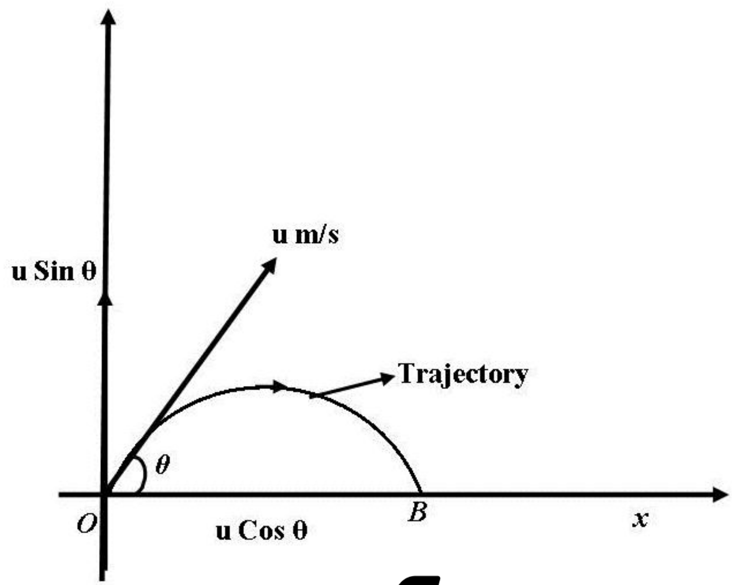
For horizontal projection acceleration is zero.

From $S = ut + {}^1/2at^2$ $a = 0$ *and* $S = R$

$R = ut$……………………………………………(2)

4.4.4: <u>Projectile Motion</u>

Consider a body thrown with a velocity u at an angle θ to the horizontal. We consider the vertical and horizontal motion separately in motion of this kind and use components.



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Vertical motion component $y = u \sin\theta$ *and* $a = -g$, When the project reaches the ground at B, the vertical distance covered will be zero from

$S = ut + {}^{1}/{}_{2}at^{2}$

$O = u \sin\theta\, t - \frac{1}{2} gt^{2}$

$t = \frac{2u \sin\theta}{g}$……………………………………………….....(1)

Where t, is the time of flight.

Horizontal motion since g acts vertically, it has no component in a horizontal direction.

From $S = ut + {}^{1}/{}_{2}at^{2}$, $a=0$ and $S=R$

$R = OB = ut = u\cos\theta \times \frac{2u \sin\theta}{g} = \frac{2u^2 \sin\theta \cos\theta}{g}$

From $2 \sin\theta \cos\theta = \sin 2\theta$ – Trigonometric identity

Therefore $R = \frac{u^2(\sin 2\theta)}{g}$

Note: For a given velocity of projection, the range is maximum when $\sin 2\theta = 1$, *i.e* the range has a maximum value At $\theta = 45^0$ which is $R = \frac{u^2}{g}$

Trajectory

The horizontal component of motion (x) is given by $u_x = u \cos\theta$.

At any given time the horizontal component is given by horizontal velocity x time.

$x = u_x t = ut \cos\theta$………………………………………… (1)

Vertical Displacement, y

$y = u_y t - {}^{1}/{}_{2}gt^{2}$…………………………………………….. (2)

But $u_y = u \sin\theta$…………………………………………… (3)

From (1), making t the subject then

$t = \frac{x}{u \cos\theta}$ ………………………………………………… (4)

Substituting (3) and (4) into (2), we get

$y = u \sin\theta \frac{x}{u \cos\theta} - {}^{1}/{}_{2}g \left(\frac{x}{u \cos\theta}\right)^2$

For $\frac{\sin\theta}{\cos\theta} = \tan\theta$ – Trigonometric identity

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Therefore, $y = x \tan \theta - \frac{1}{2}gx^2 \text{ u} \cos^2 \theta$

Compare $y = ax - bx^2$, by taking $Tan\ \theta$ and $\frac{1}{2}gu\ Cos^2\theta$ as constants a and b respectively.

This is an equation of a parabola described.

Example:

1) A golfer strikes a golf ball on level ground, the ball leaves the ground with initial velocity of *30.0m/s* at an angle of 45^0 above the horizontal. Calculate the ball's:
   a) Horizontal and vertical components of initial velocity
   b) Time taken to reach maximum height
   c) Maximum height kicked
   d) Range of the ball on the ground
   e) Velocity with which it strikes

Solution.

a) $v_x = u\ Cos\theta = 30\ Cos\ 45^0 = 25.98m/s$, $v_y = u\ Sin\theta = 30\ Sin\ 45^0 = 15ms$

b) $t = \frac{u\ Sin\theta}{g} = \frac{30\ Sin\ 45^0}{9.8} = 2.16\ sec.$

c) $y = u\ Sin\theta.t - \frac{1}{2}gt^2 = 30\ Sin\ 45^0 \times 2.16 - \frac{1}{2} \times 9.8 \times (2.16)^2 = 22.9m$

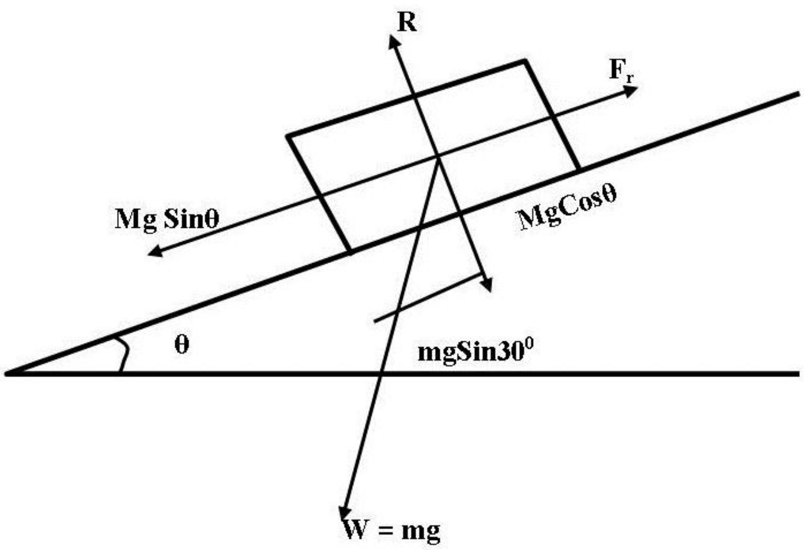
d) $R = \frac{u^2}{g} = \frac{30^2}{9.8} = 91.8\ m$

e) $v = \sqrt{v_x{}^2 + v_y{}^2}$ But $v_x = u\ Cos\theta = 21.2m/s$, & $v_y = u\ Sin\theta - gt = -21.2\ m/s$

$v = \sqrt{(21.2)^2 + (-21.2)^2} = 30.0\ m/s$

4.4.5: Motion of an inclined plane

Body moving down a rough inclined plane.



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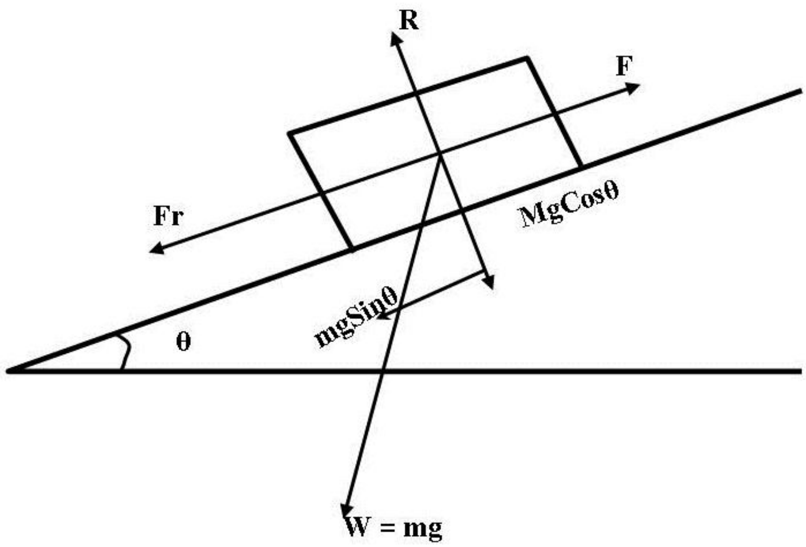
$Fr = mg \, Sin \, \theta$………………………………………… (1)

$R = mg \, Cos \, \theta$………………………………………… (2)

$\mu = \frac{Fr}{R} = \frac{mg \sin \theta}{mg \, Cos \, \theta}$ Using $\frac{Sin \, \theta}{Cos \, \theta} = tan \, \theta$ - Trigonometric identity

Then $\mu = tan \, \theta$ ……………..(3)

Body moving up a rough inclined plane



Substituting equation (1) into (2) for R

$F - mg \, Sin \, \theta - \mu mgCos\theta = 0$, $F = mg \, Sin \, \theta + \mu mgCos\theta$

$F = mg \, (Sin\theta + \mu \, Cos\theta)$ …………………………………………………….(3)

Examples

1) A man pulls a sledge of mass $16kg$ up an inclined plane of slope 22^0 to the horizontal. If he uses an effort of $1400N$ and coefficient of friction of the slope is 0.130. What extra load of wood can he place on the sledge? (Use $g = 9.8ms^{-2}$).

   Solution
   $m = m_{sledge} + m_{wood}$
   $F = mg \, (Sin\theta + \mu \, Cos\theta)$
   $1400N = m \times 9.8 \, (Sin \, 22^0 + 0.13 \, Cos \, 22^0)$
   $m = 289kg$
   $m_{wood} = 289kg - 16kg = 273kg$.

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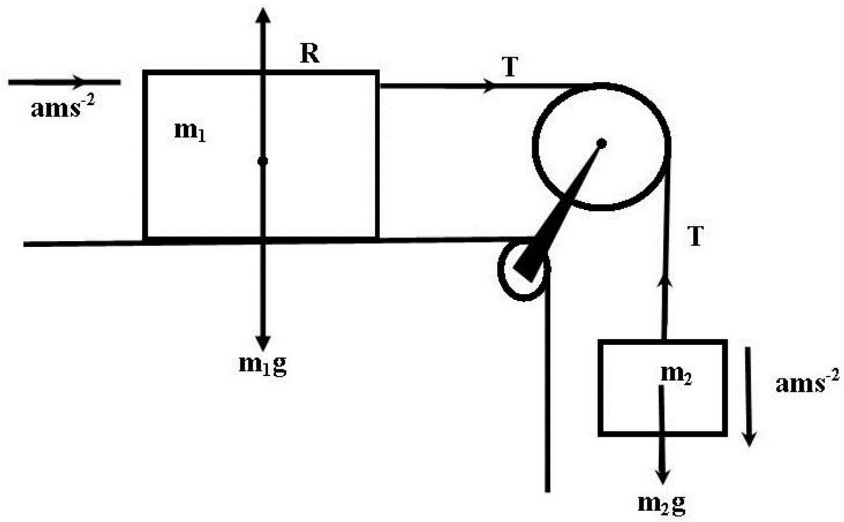
4.5: Motion of connected bodies

Case I: One body on a horizontal plane and other hanging freely

The string is inextensible, the pulley and the horizontal surface are frictionless.



For $m_2 > m_1$ and ignoring friction forces, the equations of motion are:

$T = m_1a$……………………………………………………… (1)

$m_2g - T = m_2a$…………………………………………… (2)

Adding (1) and (2)

$m_2g = m_1a = m_2a = a(m_1+m_2)$

Therefore $a = \frac{m_2 g}{m_1 + m_2}$ ……………………………….. (3)

From Equation (1)

$T = m_1a = m_1\left[\frac{m_2 g}{(m_1 + m_2)}\right] = \frac{m_1 m_2 g}{m_1 + m_2}$ …………………..(4)

Suppose the horizontal surface is rough with coefficient of friction, μ. Then

$T = m_2g - m_2a$ …………………………………… (1)

$R - m_1g = 0 \Rightarrow R = m_1g$

$T - F_r = m_1a$, But $F_r = \mu R$

$T - \mu R = m_1a$, But $R = m_1g$

$T - \mu\, m_1g = m_1a$……………………………… (2)

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Subtracting (2) from (1)

$T - m_2g = -m_2a$

$\underline{T - \mu m_1g = m_1a}$

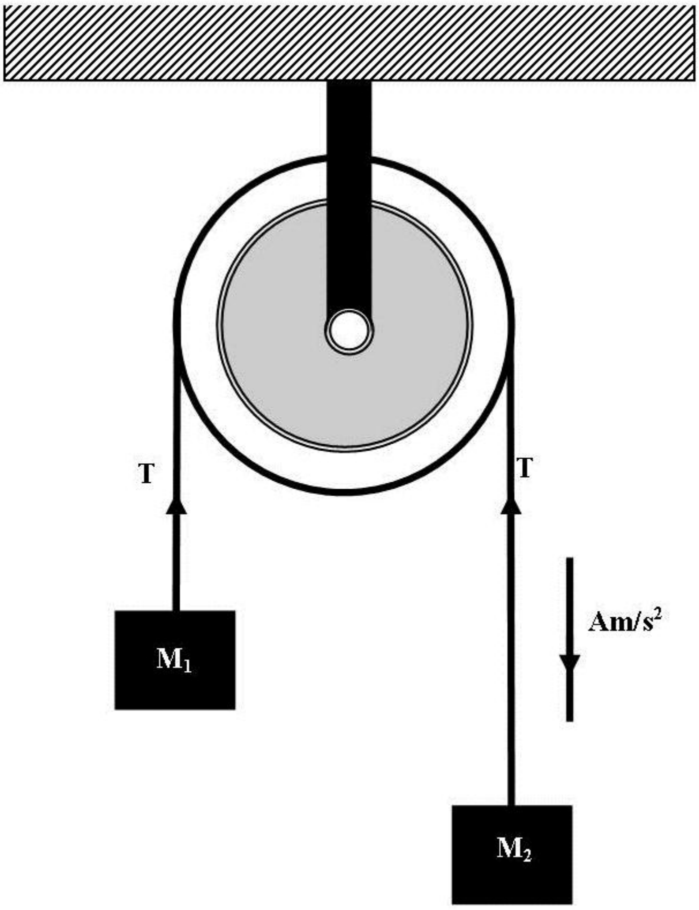
$\mu m_1g - m_2g = -m_2a - m_1a$ Solving for a we get

$a = \frac{(m_2 - \mu m_1)g}{(m_2 + m_1)}$ ………………………………………….…. (3)

Substituting (3) into (2) for a we get

$T = \frac{(m_1 m_2 g)(1 + \mu)}{(m_2 + m_1)}$……………………………………….…… (4)

Case II: Two bodies having freely and supported by a frictionless pulley.



$T - m_1g = m_1a$…………………………….……………. (1)

$m_2 g - T = m_2a$…………………………………….………. (2)

Adding (1) and (2)

$m_2g - m_1g = m_1a + m_2a$ and solving for a

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$(m_2 - m_1)\,g = a\,(m_1 + m_2)$

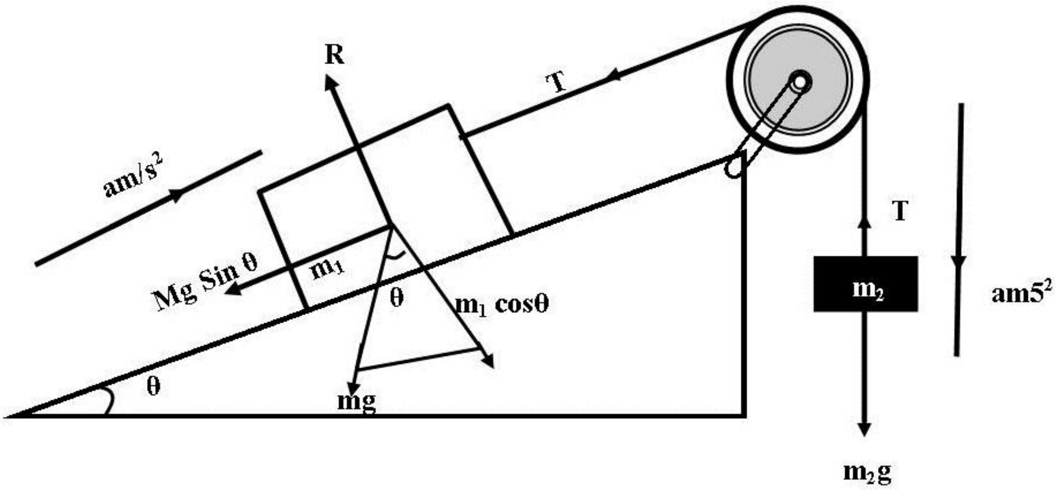
$a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$………………………………….. (3)

Substituting (3) into (1) and solving for *T*

$T = m_1 \frac{(m_2 - m_1)g}{m_1 + m_2} + m_1 g$

$T = \frac{(2m_1 m_2)g}{m_1 + m_2}$……………………………………(4)

Case III: One body lying on an inclined plane and the other hanging freely.



For $m_2 > m_1$

$T - m_1\, g \sin\theta = m_1\, a$……………………………………….…… *(1)*

$M_2\, g - T = m_2\, a$…………….…………………………………… *(2)*

Adding (1) and (2)

$m_2 g - m_1 \sin\theta = a\,(m_1 + m_2)$ Solving for *a* we get

$a = \frac{(m_2 - m_1 \text{ Sin } \theta)g}{m_1 + m_2}$……………………………………….………*(3)*

Substituting (3) into (1) and solving for *T*, we get

$T = m_1\left[\frac{(m_2 + m_1 Sin\theta)g}{(m_1 + m_2)}\right] + m_1 g \sin\theta$

$T = \frac{m_1 m_2\, g(1 + Sin\,\theta)}{m_1 + m_2}$……………………………………… (4)

Suppose the inclined plane is rough with coefficient of friction, μ, then

$T - m_1 g Sin\theta - F_r = m_1 a,$ But $R = m_1 g\ Cos\ \theta$ and $F_r = \mu R$

$T - m_1 g Sin\theta - \mu\, m_1 g\ Cos\ \theta = m_1 a,$……………………………….. *(5)*

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Subtracting (5) from (2)

$T - m_2g = -m_2a$

$\underline{T - m_1gSin\theta - \mu\, m_1g\, Cos\, \theta = m_1a}$

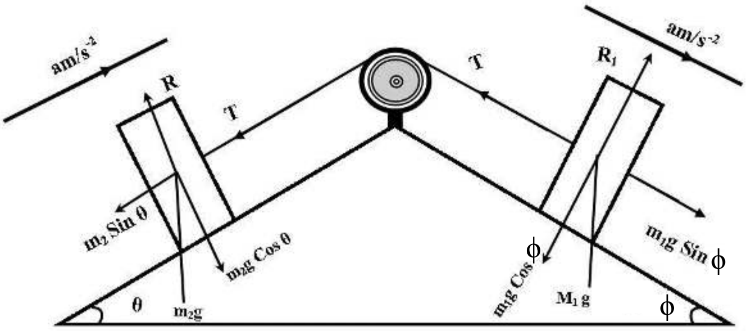
$m_1gSin\theta - m_2g + \mu\, m_1g\, Cos\, \theta = -m_2a - m_1a$ Solving for a we get

$a = \frac{(m_2 - m_1\, Sin\,\theta - \mu m_1\, Cos\,\theta)g}{m_2 + m_1}$ ……………………………………….. *(6)*

Substituting (6) into (1) and solving for T, we get

$T = \frac{(1 + Sin\,\theta + \mu\, Cos\,\theta) m_1 m_2 g}{m_2 + m_1}$ ………………………………….. *(7)*

Case IV: Two Bodies on two inclined planes for $m_1 > m_2$



We have

$T - m_2g\, Sin\,\theta = m_2a$……………………………………………………….. *(1)*

$m_1g\, Sin\,\phi - T = m_1a$……………………………………………………… *(2)*

Adding (1) and (2)

$m_1g\,$ Sin ϕ – m_2g Sin θ = $(m_1+m_2)a$

$a = \frac{(m_1\, \text{Sin}\,\phi - m_2\, \text{Sin}\,\theta)g}{m_1 + m_2}$ ……………………………………………..….(3)

From (1)

$T = m_2\, a + m_2\, g\, Sin\,\theta$…………………………………………………………(4)

Substituting (4) into (3)

$T = m_2\left[\frac{(m_1\, \text{Sin}\,\phi - m_2\, \text{Sin}\,\theta)g}{m_1 + m_2}\right] + m_2\, g\, Sin\,\theta$

$T = \frac{m_1\, m_2\, g\, (\text{Sin}\,\phi + \text{Sin}\,\theta)}{m_1 + m_2}$………………………………………………………. *(5)*

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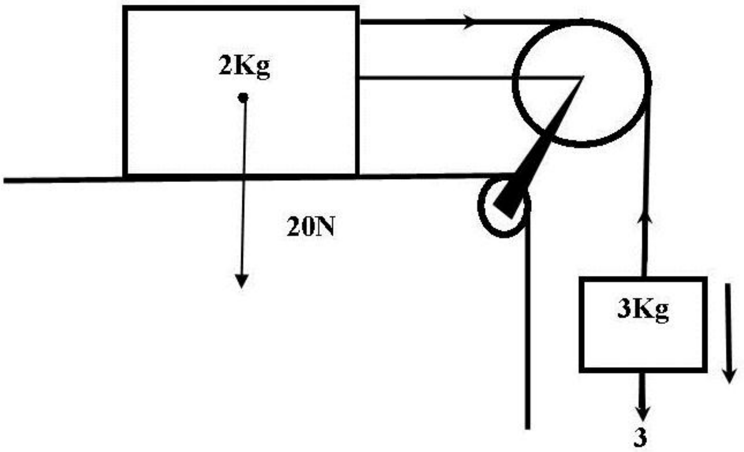
Examples.

1) A wooden block of mass *2kg* is pulled along a horizontal surface by light inextensible spring attached over frictionless pulley to a mass of *3kg* hanging vertically. Calculate the acceleration of the system and tension in the string.

Solution:



30 – T = 3a ………………………………………………………….. (1)

T = 2a……………………………………………………………….(2)

Adding (1) and (2)

3 0 =5a

a= 6m/s²

T = 2x6 = 12N or a = $\frac{3x10}{2+3}$*=6m/s²*

=2x6=12N

2) A light inextensible ship passes over a smooth fixed pulley. A mass of 0.3kg is attached to one of the spring while a mass of 0.2kg is attached to the other end. The system is held at rest then released so that the 0.3kg is descends. Calculate the acceleration of the two masses and the term in the spring (g = 10m/s^2).

Solution.

$a = \frac{(m_2-m_1)g}{m_2+m_1} = \frac{0.1 \ x \ 10}{0.3+0.2} = 2 \ m/s^2$

$T = \frac{2m_2m_1g}{m_2+m_1} = \frac{2 \ x \ 0.2 \ x \ 0.3 \ x \ 10}{0.3+0.2} = 2.4N$

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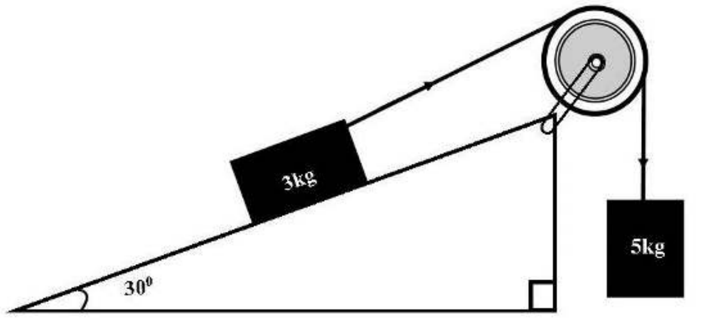
3) A mass of 3kg has on earth inclined plane and connected by a light inextensible string passing over a frictionless pulley at the top of the inclined plane to a mass of 5kg as shown.



Calculate the acceleration and tension of the system. Take $g = 10m/s^2$

Solution.

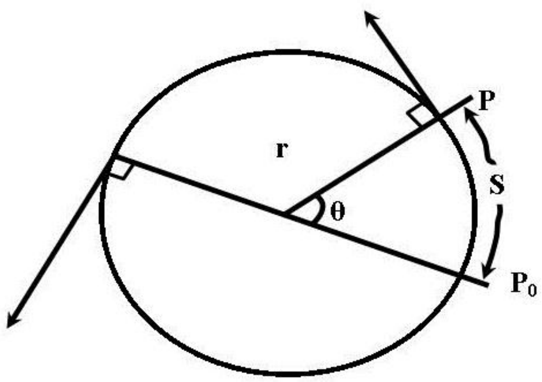
$a = \frac{(m_2 - m_1 \, Sin\,\theta)g}{m_2 + m_1} = \frac{(5-3\,Sin\,30)10}{5+3} = 4.38\text{m/s}^2$

$T = \frac{m_1 m_2 g(1 \;\; + Sin\,\theta)}{m_2 + m_1} = \frac{5\,x\,3\,x\,10\,(1 - Sin\,30)}{5+3} = 28.125N$

4.6: Uniform Circular Motion

Introduction

When a particle moves on a circular part with a constant speed, then its motion is said to be uniform circular motion in a plane. The magnitude of speed remains constant but direction changes.



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Angular Displacement (θ)

Suppose a particle covers a distance ΔS along the circular path in the time interval $\Delta t = t_2 - t_1$

It revolves through an angle $\Delta\theta = \theta_2 - \theta_1$

During the interval, the angle of revolution Δθ is called angular displacement of the particle if r is radius of the circle then angular displacement $\Delta\theta = \frac{\Delta s}{r}$

Angle = $\frac{Arc-length}{Radius}$ *SI* unit is the radian.

If the arc-length = Radius of Circle = 1 radian

Angular Velocity (ω)

Angular velocity of a particle is given by the displacement per unit time

$\omega = \frac{Anguar\ desplacement}{Time\ take}$

$\omega = \frac{\Delta\theta}{\Delta t} \rightarrow \omega = \text{Lim}_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ SI unit radian per second.

If T is the time taken for one revolution

then, $\omega = \frac{2\pi}{T}$, *But* $f = \frac{1}{T}$

$\omega = 2\pi f$

Relationship between Angular Velocity and linear velocity

The particle covers an arc-length Δs in time Δt, hence,

Angular displacement $\Delta\theta = \frac{\Delta s}{r}$, dividing through by Δt

$\frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{r\Delta t}$, $\text{Lim}_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{1}{r}(\text{Lim}_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t}) = \frac{v}{r}$

Therefore $v = r\,\omega$ which is instantaneous linear velocity.

Centripetal Acceleration (a).

A particle is uniform circular motion changes direction hence velocity changes thus the body accelerates. The direction of acceleration is towards the centre thus centripetal petal acceleration.

$\frac{\Delta s}{r} = \frac{\Delta v}{v} \rightarrow \Delta v = \frac{v}{r}\Delta s$, dividing on both sides by Δt we have

$\frac{\Delta v}{\Delta t} = \frac{v\Delta S}{r\Delta t}$, if Δt is infitesmally small $_{\Delta t \rightarrow 0}$, then.

$\text{Lim}_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{v}{r}(\text{Lim}\ \Delta t \rightarrow 0 \frac{\Delta S}{\Delta t}) = \frac{v}{r}\ x\ v = \frac{v2}{r}$

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a = $\frac{v2}{r}$ But $v = r\omega$

$a = \omega^2 r$

Centripetal force (F_c)

From Newton's second law, acceleration is always produced by a force whose direction is the same as that of the acceleration. A body performing circular motion is acted upon by a force, which is always directed towards the centre of the circle.

$F_c = F = ma = m\frac{v2}{r} = m\omega^2 r$

Examples of center petal forces

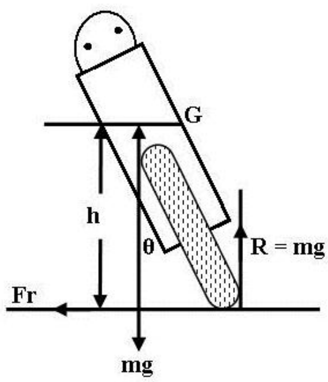
1) A car taking a turn requires a centripetal force. This is provided by the frictional force between the tyres and the road.

2) When a stone is tied on the end of a string is whirled on a circular path, the centripetal force provided by the tension in the string created by drawing the string inward.

Cases of Circular Motion.

Case I: Motion of a bicycle rider round circular track.

When a person on bicycle rides around a circular racing track, the necessary centripetal force $\frac{mv2}{r}$ is actually provided by the frictional force F_r at the ground. F_r has amoment about the c.o.g, *G* equals to F.h which tends to turn the rider outwards.



When the rider leans inwards as shown, this is counter balanced by the moment *R.a* about *G* which is equal to *mg.a* thus provided no slipping occurs.

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$F.h = mg.a$

$^a/_h = \tan\theta = \frac{F}{mg}$, where θ *is* the angle of inclination to the vertical, now,

$F = \frac{mv^2}{r} = mg \tan\theta$

$Tan\theta = \frac{v^2}{rg}$

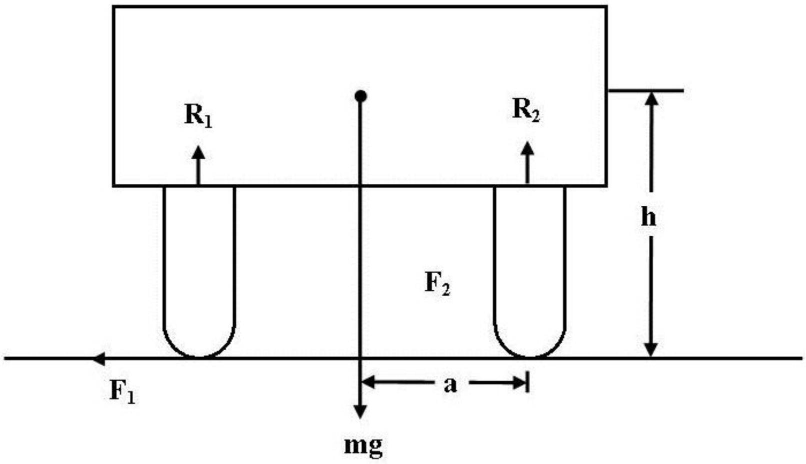
When F is greater than the limiting friction, skidding occurs.

Case II: Motion of a Car Around a Circular Track

Suppose a car is moving with velocity v around a horizontal circular track of radius r, and let R_1 and R_2 be respective normal re actions at the wheels A, B and F_1, F_2 the corresponding frictional forces. Then for circular motion we have $F_1 + F_2 = mv^2$ and vertically $R_1 + R_2 = mg$, also taking moments at G, $(F_1 + F_2)\, h + R_1a - R_2a = 0$, where $2a$ is the distance between the wheels.

G is midway between the wheels and h is the height of G above the ground. From the three equations we find,

$R_2 = {}^1/_2m\ (g + \frac{v^2\, h}{ra})$ and vertically, $R_1 = {}^1/_2m\ (g - \frac{v^2\, h}{ra})$. If $v^2 = \frac{arg}{h_1}$, $R_1 = 0$ and the car turning left is about to overturn outwards. R_1 will be positive if $v^2 < \frac{arg}{h}$



Case III: Motion of a Car Around A Banked Track.

Centripetal force is required for a car to go round a bend on a level surface. The force is provided by the frictional force exerted on the tyres by the road. A suitable banked road removes the need to rely on friction. This provided, no slipping occurs.

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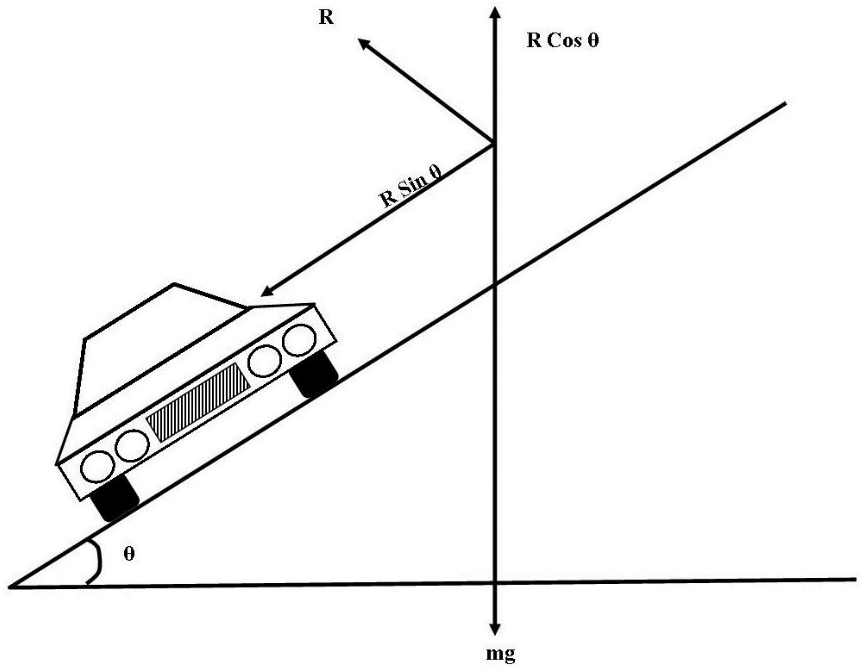
The normal reaction *R,* of the road on the car acquires a horizontal component, *R Sin θ* as a result of banking. Using Newton's 2nd law of motion, *F = ma,* we have, *R Sin θ = ma*

$R \sin\theta = \frac{mv^2}{r}$ ……………………………………………….………. *(1)*



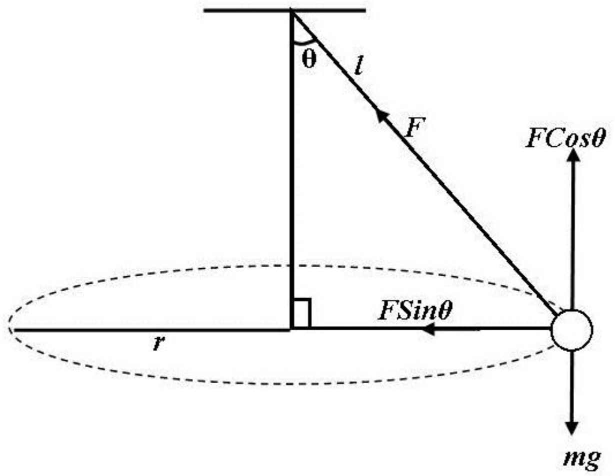
There is no vertical acceleration hence *R Cos θ = mg* ………………………….. (2)

Dividing (1) by (2)

$Tan\ \theta = \frac{v^2}{rg}$, The angle of banking θ is given by $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Case IV: ***<u>Conical Pendulum.</u>***

If a pendulum bob moves in such a way that the string sweeps out a cone, then the bob describes a horizontal circle.



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There are two forces acting on the bob, its weight *(mg)* and tension in string *(F)* centripetal force is provided by the horizontal component of the tension.

F Sin θ, hence from Newton's 2nd law

$F = ma = \frac{mv^2}{r}$

$F\ Sin\theta = \frac{mv^2}{r}$ ………………………………………………... (1)

Since there is no vertical acceleration,

$F\ Cos\ \theta = mg$ …………………………………………………………….. *(2)*

Dividing (1) by (2)

$\frac{FSin\theta}{FCos\theta} = \frac{mv^2/r}{mg}$, $Tan\ \theta = \frac{\text{Sin}\ \theta}{Cos\ \theta}$, Trigonometric identity.

$Tan\ \theta = \frac{v^2}{rg}$

Also $v = \omega r$

$Tan\theta = \frac{\omega^2 r^2}{rg} = \frac{\omega^2 r}{g}$

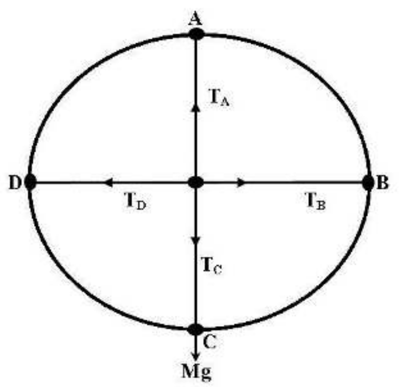
Also $r = l\ Sin\ \theta$, *hence* $Tan\ \theta = \frac{\omega^2 r}{g} = \frac{\omega^2 l\ Sin\ \theta}{g}$

$\frac{Sin\theta}{Cos\theta} = \frac{\omega^2\ l\ Sin\ \theta}{g}$

Therefore $\omega = \sqrt{\frac{g}{l\ Cos\ \theta}}$, Using equation $T = \frac{2\pi}{\omega}$

$T = 2\pi\ \sqrt{\frac{l\ Cos\ \theta}{g}}$

Case V: ***<u>Motion on a Vertical Circle.</u>***



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The string experiences the highest tension when at *C*

$T_c = \frac{mv^2}{r} + mg$ ……………………………………………………. (1)

The string has the lowest tension at *A*

$T_A = \frac{mv^2}{r} - mg$ ……………………………………………………. (2)

The tension at B and D is equal to centripetal force

$T_B = T_D = \frac{mv^2}{r}$……………………………………………………. (3)

Summary

1) $\Delta\theta = \frac{\Delta s}{r}$
2) $\omega = \frac{2\pi}{T} = 2\pi f = {}^{v}/_{r}$
3) $v = r\omega$

4) $a = \frac{v2}{r}$
5) $a = \omega^2 r$
6) $F = m\frac{v2}{r} = m\omega^2 r$
7) $Tan\ \theta = \frac{v^2}{rg}$
8) $R_2 = {}^{1}/_{2}m\ (g + \frac{v^2\ h}{ra})$
9) $R_1 = {}^{1}/_{2}m\ (g - \frac{v^2\ h}{ra})$.
10) $T = 2\pi = \frac{\sqrt{lCos\theta}}{g}$.
11) $T_{bottom} = \frac{mv^2}{r} + mg$.
12) $T_{top} = \frac{mv^2}{r} - mg$.

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5.0: SIMPLE HAMORNIC MOTION

Simple Harmonic motion (S.H.M) is defined as a periodic motion in which the restore force is proportional to the displacement and in the opposite direction $F = -kx$ or

S.H.M is motion change of a particle when directed towards a fixed point and its directly proportional to distance from fixed point.

Fixed point is equilibrium position or rest position.

Periodic motion is motion that repeats itself after a fixed time interval.

Period (T) is the time during which spherical system completes one full cycle of motion.

Frequency (f) is the number of complete cycles per second $T = {}^{1}/_{f}$

Displacement (*x*) is the distance of a moving object from its equilibrium position at any instant of time.

Amplitude (x_0) is the maximum value of displacement.

A system vibrates /oscillates if it satisfies the following conditions;

1. Should be able to store *P.E* (should have elastic properties)
2. Should have some inertia which enables it to possess *K.E.*

The motion of a body is described by the equation of the form

$\frac{d^2x}{dt^2} = -\omega^2 x$ where $\frac{d^2x}{dt^2} = a$ where a is acceleration of the body.

The negative sign indicates that the acceleration is always directed towards the equilibrium position.

For a body moving in a circular path, the linear displacement x is displaced by an angular displacement θ and linear acceleration:

$\frac{d^2x}{dt^2}$ is replaced by $\frac{d^2\theta}{dt^2}$

$\frac{d^2x}{dt^2} = \frac{d^2\theta}{dt^2} = -\omega^2\theta$

Period $T = \frac{2\pi}{\omega}$, where ω is angular velocity.

N/B: Period is independent of amplitude

Frequency (f), $f = \frac{1}{T} = \frac{\omega}{2\pi}$

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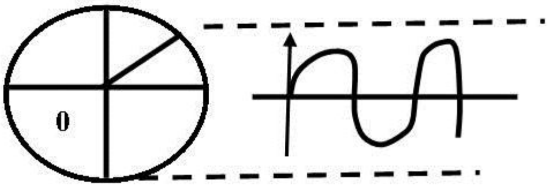
Phase

It expresses position and direction of motion of the vibrating particle at any instant.
Displacement is given by $y = y_0 \, Sin\,(\omega t + \phi)$

ϕ – Phase angle



Example of S.H.M

1. Simple pendulum
2. Mass on a helical spring
3. Vibration of the prongs of the tuning fork.
4. Liquid in a *U* –tube
5. Oscillation of atoms in a molecule

5.1: Relationship between S.H.M and circular motion

Case I: Consider a small ball rotating at constant speed in a vertical circle of radius *r*. we are interested in the *x* component of this circular motion. To find this, we might imagine a large spotlight shining down on the rotating ball from the top of the page and projecting on the *x* axis a shadow of the ball. As the ball rotates through 360^0 or 2π radians, its shadow goes from $+x_0$ to $-x_0$ and back again to $+x_0$ as shown below. If we plot the position of the ball's shadow as a function of angle θ swept out by the rotating ball, we find that *x* executes a perfect cosine curve. This has to be the case because the cosine of θ is the ratio of the adjacent side *(x)* to the hypotenuse *r*. Since *r* is constant and numerically equal to x_0, as the ball rotates its shadow along *x* axis is always proportional to Cosine θ hence,

Cos θ = $\frac{x}{r}$ or $x = r\,Cos\,\theta = x_0\,Cos\,\theta$,

But $\theta = \omega t = 2\pi ft$. Hence

$x = x_0\,Cos\,\theta = x_0\,Cos\,\omega t = x_0\,Cos\,2\pi ft.$

The period of the projected motion along the *x* axis is the same as the period of the circular motion and both complete one cycle in the same time, i.e.

$T = \frac{1}{f} = \frac{2\pi}{\omega}$

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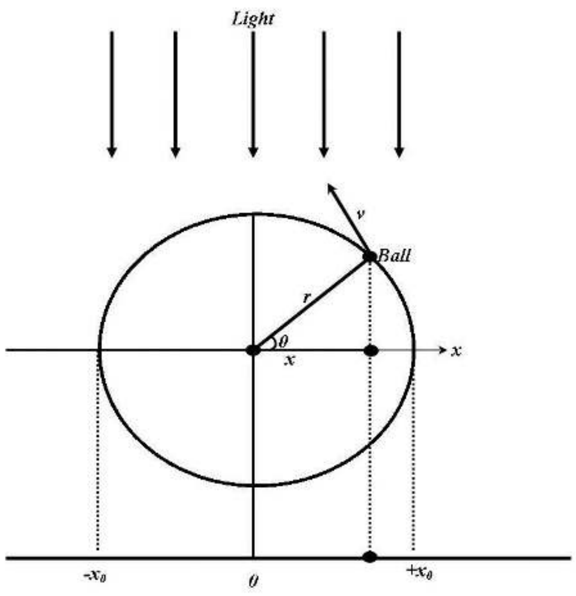
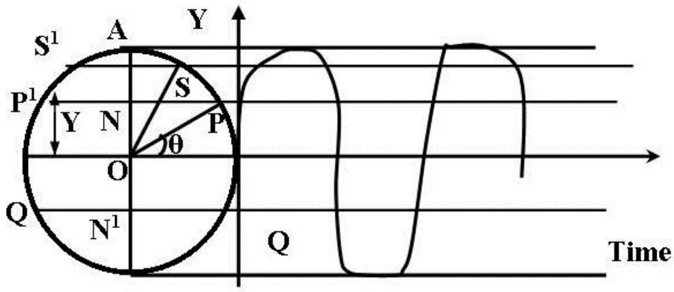


Case II: Consider a particle moving with a uniform speed along a circumference of a circle as shown below.



When particle is at point *P*, the foot of the perpendicular drawn from *P* to the diameter AA^1 of the circle is at point *N*. As the particle moves around the circle, the foot *N* moves on a straight line to and fro about point *O*. The motion of *N* can be projected as shown above. The to and fro motion of *N* along AOA^1 is called S.H.M

Note: The period and hence frequency of rotation of *P* is equal to the period and frequency of oscillation of *N* which is $\frac{2\pi}{\omega}$

Displacement Equation of S.H.M.

At *P*, $\omega = \frac{\theta}{t}$, the displacement of the foot of *N* from *O* is $ON = Y = OP \sin\theta$

But *OP* which is the radius of circular path

$y = r \sin\theta,\ \theta = \omega t$

$y = r \sin\omega t,\ y = S$

$S = r \sin\omega t$ …………………………………………………….(1)

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<u>Velocity Equation for S.H.M.</u>

$v = \frac{dS}{dt}$, $\frac{d(r\ Sin\omega t)}{dt}$ $= r\omega\ Cos\ \omega t$ Recall $\frac{d(Sin\ ax)}{dx}$ $= a\ Cos\ ax$ Standard derivative

$v = r\omega\ Cos\ \omega t$……………………………………………………. (2)

But $NP = r\ Cos\ \theta = r\ Cos\ \omega t = \sqrt{r^2 - y^2}$

$v = \omega\sqrt{r^2 - y^2}$……………………………………………………… (3)

$at\ y = O,\ v = \pm\omega r,$ (velocity in circular path)

At $y = r,\ v = \omega$ x 0 Hence $v = 0$

<u>Acceleration Equation in S. H. M.</u>

$a = \frac{dv}{dt} = \frac{d^2S}{dt^2}$

$a = r\omega \frac{d(Cos\omega t)}{dt} = -r\omega^2\ Sin\omega t,$ Recall $\frac{d(Cos\ ax)}{dx}$ $= -a\ Sin\ ax$ Standard derivative.

Since $y = r\ Sin\ \omega t,$ Then

$a = -\omega^2 y$…………………………………….. (4)

Using (4), acceleration is proportional to displacement and in opposite direction.

At $y = 0,\ a = 0$ and

At $y = r,\ a = \omega^2 r$

5.2: <u>Determination of *g* Using a Simple pendulum</u>

Suppose the mass m is displaced slightly through an angle θ and l is the length of the string at A, the forces acting on the mass m are

i) Weight, mg
ii) Tension in the string, $T = mg\ Cos\ \theta$

Components of weight along x – direction = $mg\ sin\ \theta$ and is the force that tends to restore the bob to the equilibrium point O
Thus the restoring force is
$F = -\ mg\ Sin\ \theta$

The negative sign is due to the fact that the force is opposite to the angular displacement.

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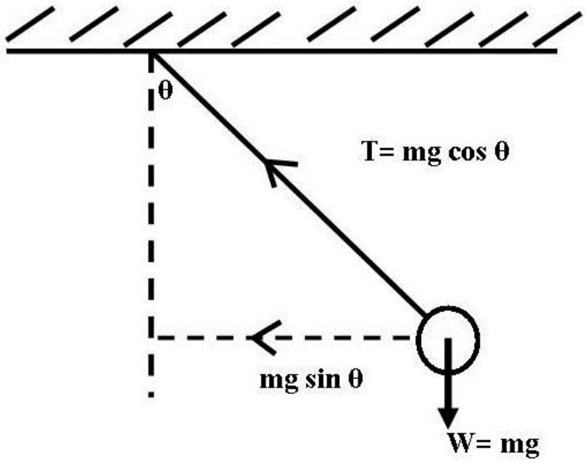

For smaller angles of θ, where θ is in radians $Sin\ \theta \simeq \theta$ = arc length/radius = r/l.
Hence $F = -mg\ Sin\ \theta = -mg\ r/l$

If a is the acceleration of m towards O, then by Newton's second law motion.
$F = ma = -mg\frac{r}{l},\ a = -g\frac{r}{l}$ also $a = -\omega^2 y$ where
$\omega^2 y = g\ {}^r/_l$ for $y = r$, $\omega^2 = \frac{g}{l}$

Hence $\omega^2 = \sqrt{\frac{g}{l}}$ But $T = \frac{2\pi}{\omega}$; $w = \frac{2\pi}{T}$
$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}, \frac{4\pi^2}{T^2} = \frac{g}{l}$, $T^2 = \frac{4\pi^2}{g}\ x\ l$

$T = 2\pi\sqrt{\frac{l}{g}}$, Equation for period of a pendulum in S. H. M.

By measuring l and determining corresponding values of T at small angles of displacement, g can be evaluated by using $T = 2\pi\sqrt{\frac{l}{g}}$. A graph of T^2 against l has the gradient given by

Gradient/slope = $\frac{4\pi^2}{g}$. Hence g = $\frac{4\pi^2}{S}$, where s is slope/gradient.

Note: If we have a spring of mass m_s with a mass of mass m suspended on it then the body oscillates and the period of the system is given by

$$T = 2\pi\left[\frac{(m_s + m)}{K}\right]^{1/2}$$

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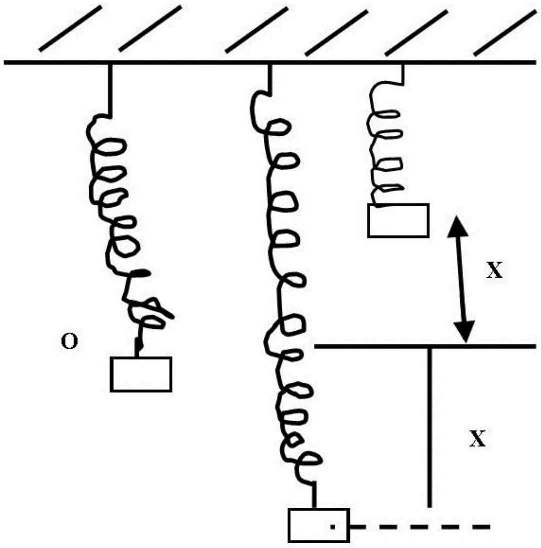
5.3: **Motion of a body suspended on a spring.**

Oscillating system –spring and mass.



When the spring is stretched through x and released then it oscillates as shown above. Suppose the extension x of the spring is stretching proportional to F in spring (Hookes law). F acts in opposite direction to x so $F = -kx$ where F is force constant force per unit extension called spring stiffness. If m is the mass of the body then acceleration a is given by $F = ma$

$ma = -kx$

$a = \frac{-k}{m}x$ But $a = -\omega^2 x$

$$a = \frac{-k}{m}x = -\omega^2 x$$

$\frac{k}{m} = \omega^2$ But $\omega = \frac{2\pi}{T}$

$\frac{k}{m} = \frac{4\pi^2}{T^2}$, $T^2 = \frac{4\pi^2 m}{k}$, $T = 2\pi\sqrt{\frac{m}{k}}$,

$T = 2\pi\sqrt{\frac{m}{k}}$ Equation for period of oscillating spring in S. H. M.

Also $k = \frac{mg}{e}$, Therefore $T = 2\pi\sqrt{\frac{e}{g}}$

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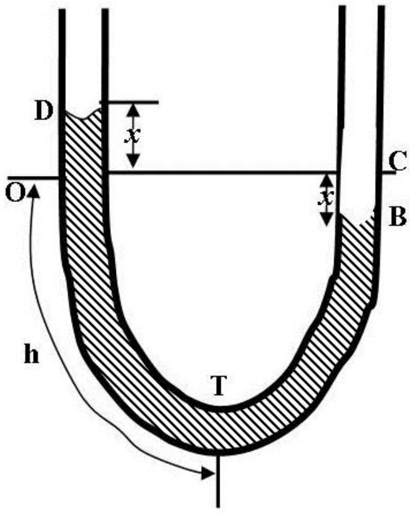
5.4: <u>Oscillations of a liquid in a U-tube</u>

If the liquid on one side of a U-tube *T* is depressed by blowing gently down that side, the levels of the liquid will oscillate for short time about their respective initial positions *O,* before finally coming to rest.



At some instant, suppose that the level of the liquid on the left side of *T* is at *D,* at a height *x* above its original (undisturbed) position *O*. The level *B* of the liquid on the other side is then at a depth *x* below its original position *C*. So excess pressure on the whole liquid is

= excess height x liquid density x g

$= 2x\rho g$

Since pressure = force per unit area

Force on liquid = pressure x area of cross section of the tube

$= 2x\rho g \times A$

Mass of a liquid in U-tube =Volume of liquid x density

$= 2h A\rho$ where $2h$ is total length of liquid at T

From $F= ma$ where a is acceleration towards O or C then

$-2x\rho gA=2hA\rho a$

$a= -\frac{gx}{h}= -\omega^2 x$

$\frac{g}{h}= \omega^2$ for $\omega = \frac{2\pi}{T}$

Then $T^2=4\pi^2\frac{h}{g}$

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$T = 2\pi\sqrt{\frac{h}{g}}$ Equation for period of oscillating liquid in U-Tube in S. H. M.

5.5: <u>Conservation of mechanical energy in S.H.M</u>

If E is the total energy of S.H.M, the we have

$E = K.E + P.E$ is constant.

Consider a mass m executing S.H.M

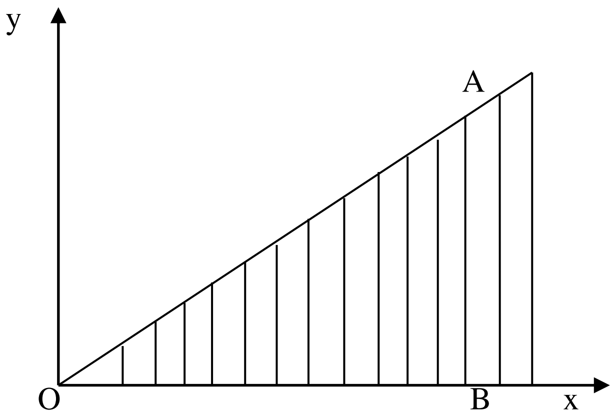
$K.E = {}^1/_2\, mv^2 = {}^1/_2 m\omega^2 \left[\sqrt{(r^2 - y^2)}\,\right]^2$

$= {}^1/_2\, m\omega^2(r^2 - y^2)$…………………………..… *(1)*

P.E for S.H.M.

$a = \frac{d^2y}{dt^2} = -\omega^2 y$……………………………………..… (2)

Drawing graphs of x v_sy, the area under curve is equal to *P.E* of *m*



Area = $½\,(AB \times OB) = {}^1/_2\, Fy = ½\, m\omega^2 y^2$

Total energy = ${}^1/_2 m\omega^2 y^2 + m\omega^2(r^2 - y^2) = {}^1/_2 m\omega^2 r^2$

Total energy, E is independent of time

When $y = r$, E is the form of *P.E*

When y = o, E is the form of *K. E*

The variation of *P.E* with *K.E* with displacement is shown below,

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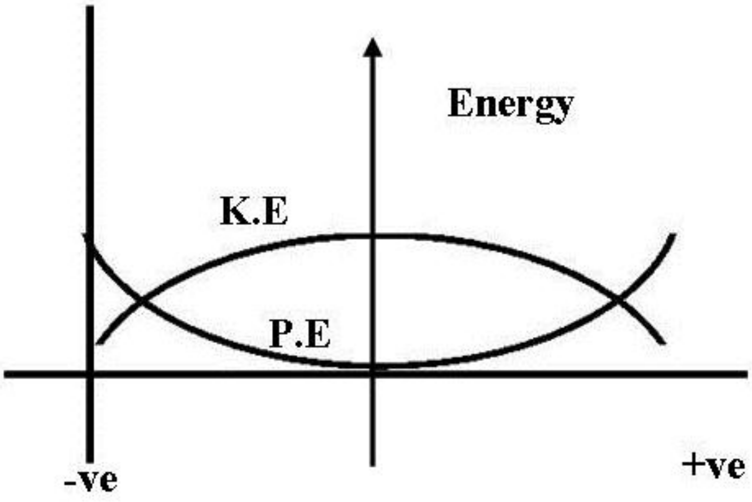

Summary

1. $a = -\omega^2 x$ the minus sign shows that a is always directed towards a fixed point when at distance x from centre of oscillation.
2. $T = 2\pi/\omega$, $\omega = 2\pi f$
3. $v = \omega\sqrt{(r^2 - y^2)}$
4. $v_{max} = \pm\omega r$
5. $a_{max} = -\omega^2 r$
6. $K.E_{max} = \frac{1}{2}mr^2\omega^2$
7. $T = 2\pi\sqrt{\frac{h}{g}}$
8. $T = 2\pi\sqrt{\frac{m}{k}}$ Also $k = \frac{mg}{e}$, Therefore $T = 2\pi\sqrt{\frac{e}{g}}$
9. $T = 2\pi\sqrt{\frac{l}{g}}$,

Examples

1.) A steel strip, clamped at one end vibrates with a frequency of 20 Hz and an amplitude of 5mm at the end where a small mass of $2g$ is positioned. Find:-

a) Velocity of the end when passing through the zero position.
b) The acceleration at the maximum displacement.
c) The maximum kinetic and potential energies of the mass

Solution

a) Let $y = r\ \text{Sin}\omega t$ where r = amplitude
$V = \omega\sqrt{(r^2 - y^2)}$
at zero position $y = 0$
$\omega = 2\pi f = 2\pi \times 20 = 40\pi$
$r = 0.005m,\ V_{max} = \omega r = 40\pi \times 0.005 = 0.628m/s$

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b) $a = -\omega^2 y$ at maximum displacement.
$a = (40\pi)^2 \times 0.005 = 79m/s^2$

c) $K.\ E_{max} = \frac{1}{2}mv^2_{max} = \frac{1}{2}(2 \times 10^{-3}) \times (0.628)^2 = 3.9 \times 10^{-4}J$
Max. $P.\ E$ at $V = 0 = max\ K.E = 3.9 \times 10^{-4}J$

2.) A mass vibrates through an amplitude of *2.0* cm in *S.H.M* with a period of *1.0 seconds*. What distance is moved from the centre of the centre of oscillation in *0.4 seconds?*

Solution

Let $y = r\ \mathrm{Sin}\omega t$ where $r = 2cm$ and $\omega = 2\pi/T = 2\pi/1 = 2\pi$
$y = 2\mathrm{Sin}\ (2\pi \times 0.4) = 2.\mathrm{Sin}\ 144^0 = 1.2$cm

3.) A vertical spring fixed at one end has a mass of *2kg* attached at the other end. The spring constant $k = 5N/mm$, Calculate:
a) Spring extension
b) Energy in spring

Solution

a) $F = kx,\ x = {}^F/_k = {}^{20}/_5 = 4mm$
b) $E = \frac{1}{2}kx^2 = \frac{1}{2} \times 5 \times 10^3 \times (4 \times 10^{-3})^2 = 0.04J$

4. A small mass of *0.2 kg* is attached on helical Spring and produces an extension of *0.015m.* The mass is now pulled down *10mm* and set into vertical oscillation of amplitude *10mm.* What is the:-
a) Period is oscillation?
b) Maximum kinetic energy of the mass?
c) Potential energy of the spring when the mass is *5mm* below the centre of oscillation (g = 9.8m/s^2)

Solution

a) $T = 2\pi\sqrt{\frac{m}{k}}$, $k = mg/e = (0.2 \times 9.8)/0.015$

$T = 2\pi\sqrt{\frac{0.2 \times 0.015}{0.2 \times 9.8}} = 0.25\ s$

b) $K.E_{max} = \frac{1}{2}mv^2_{max}, v_{max} = r\omega$ and r is amplitude.

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{9.8}{0.015}}$

$K.E_{max} = \frac{1}{2} \times 0.2 \times (\sqrt{\frac{9.8}{0.015}})^2 = 6.5 \times 10^{-3}J)$

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c) The potential energy of the spring is given by $\frac{1}{2}kx^2$ where k is force constant and x is extension from its original length. The centre of oscillation is *15mm* below the unstretched length, so *5mm* below the centre of oscillation, $x = 20mm$.
$k = mg/e = (0.2 \times 9.8)/0.015$

$P.E = \frac{1}{2}kx^2 = \frac{1}{2}[(0.2 \times 9.8)/0.015] \times 0.02 = 2.6 \times 10^{-2}$J

5. A simple pendulum has a bob of mass 0.3kg and length of 1m. The bob is drawn aside though an angle of 6^0 and released from rest. Calculate:-
   a) Maximum velocity of the bob.
   b) Maximum acceleration.
   c) Maximum tension in the string.

   Solution

   a) $T = 2\pi\sqrt{\frac{l}{g}} = \frac{2\pi}{\omega}$, $\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{1}}$

   $r = l\ Sin\theta = 1\ Sin\ 6^0 = 0.10m$

   $v_{max} = \omega r = \sqrt{\frac{9.8}{1}} \times 0.10 = 0.31m/s$

   b) $a_{max} = \omega^2 r = (\sqrt{\frac{9.8}{1}})^2 \times 0.10 = 0.98m/s^2$

   c) Maximum tension, $T = \frac{mv^2}{r} + mg$

   $T = \frac{0.3 \times (0.31)^2}{1} + 0.3 \times 9.8 = 2.97N$

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6.0: MECHANICAL PROPERTIES OF MATTER

6.1: Elasticity:

It is the ability of materials to increase in length or size when force is applied on them and rejoin their original length when force is withdrawn.

6.2: Tensile Stress (σ):

It's defined as the force per unit area

$$\text{Tensile Stress } (\sigma) = \frac{Force\ (F)}{Cross\ section\ Area\ (A)}$$

$$\sigma = {}^{F}/_{A}$$

6.3: Tensile strain (α)

It is the extension per unit length

$$Tensile\ Strain\ (\alpha) = \frac{Change\ in\ Length\ (e)}{Original\ length\ (l)}$$

$$\alpha = {}^{e}/_{l}$$

6.4: Young's Modulus (γ/E)

Young modules is the ratio of tensile stress to tensile strain

$$\text{Young's Modulus } (\gamma/E) = \frac{Tensile\ Stress}{Tensile\ Strain}$$

$$\gamma/E = \frac{F/A}{e/l} = \frac{Fl}{Ae}$$

6.5: Energy Stored in a Wire

Work Done = Force *x* Distance

= Average Force *x* Extension

= ½ *Fe* (Energy stored in a wire)

Since $F = \gamma \frac{Ae}{l}$ $from\ \gamma = \frac{Fl}{Ae}$

$$E = \frac{1}{2}\gamma\frac{EAe^2}{l} = \frac{1}{2}\gamma\frac{Ae^2}{l}$$

Area under graph of F v_s e gives the energy stored in the wire.

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6.6: Energy Per Unit Volume of Wire

Energy Stored = ½ *Fe*

Energy Per Unit Volume = $\frac{Energy\ Stored\ in\ Wire}{Volume\ of\ Wire}$

$$= \frac{1/2Fe}{Al} = \frac{Fe}{2Al}$$

$$= \frac{1}{2}\frac{F}{A}x\frac{e}{l}$$

$$\frac{1}{2}\ stress\ x\ strain$$

Summary

Stress (σ) = $\frac{Force}{Cross\ Sectional\ Area} = \frac{F}{A}$

Strain $(\alpha) = \frac{e}{l}$

Young Modulus (γ) = $\frac{Tensile\ Stress}{Tensile\ Strain} = \frac{F/A}{e/l} = \frac{Fl}{Ae}$

Energy $= \frac{1}{2}\frac{EAe^2}{l}$

Energy per unit volume = ½ Stress *x* Strain

Example

A *4m* long copper wire of cross-sectional area $1.2cm^2$ stretched by a force of $4.8\ x\ 10^3N$. If the young's modulus ϒ = $1.2\ x\ 10^{11}\ N/M^2$, *calculate:*

a) Tensile stress
b) Tensile strain
c) Increase in length of wire

Solution

a) Tensile stress

$\frac{F}{A} = \frac{4.8\ x 10^3}{1.2\ x\ 10^{-4}} = 4\ x\ 10^7\ N/m^2$

b) Tensile strain = $\frac{Stress}{Young's\ Modulus} = \frac{4\ x10^7}{1.2\ x\ 10^{11}\ N/m^2} = 3.3\ x\ 10^{-4}$

c) Extension = Strain *x* Original Length

$= 3.3\ x\ 10^{-4}\ x\ 4 = 13.2\ x\ 10^{-4}$

$= 1.32\ x10^{-3}m$

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6.7: Surface Tension

This is the property of liquid that makes it behave as if it had a stretched membrane for a surface.

A molecule in the interior of a liquid is subjected to weak attractive forces exerted on it by other liquid molecules. Since these forces have in all directions, they cancel out and there is no net unbalanced force on the molecule. To remove this molecule from the interior of the liquid, however energy for must be supplied to overcome these attractive forces. Molecules on the surface of the liquid are pulled back towards the interior of the liquid, since the molecules attracting them are in the interior of the liquid.

A surface molecule is surrounded by only about half as many molecules as one in the interior. And so only about half as much energy is needed to remove a molecule from the surface as from the interior. As a consequence, a molecule in the surface layer has higher potential energy than a molecule in the interior of the liquid, and so increasing the surface layer increases the potential energy of the liquid.

The liquid tends to minimize if its potential energy as a result. As the surface of a liquid behaves like a stretched membrane and produces a surface tension in the liquid.

Surface tension explains the spherical shape of liquid droplets, floating of object on surface liquids.

Surface tension (γ) on the surface film of a liquid is defined as the ratio of the molecular force (F) parallel to the surface of the liquid to the length (L) of the surface film across which this force acts.

$\gamma = \frac{F}{l}$ S.I. Unit is *N/m*

Task: State and explain two factors that affect surface tension

Example

The surface tension of soap solutions is $2 \times 10^{-2} N/m$. How much work will be done in making soap bubble of diameter *2cm* by blowing?

Solution.

Wok done: = Surface tension *x* area

$= 2 \times 10^{-2} \times 2.512 \times 10^{-3}$

$= 5.02 \times 10^{-5} J$

6.8: Fluid Flow

Fluids include both liquids and gases.

Steady flow/streamline flow/uniform flow.

The flow of fluid is orderly. All the particles that pass a given point follow the same path at the same speed. In streamline flow, the streamlines coincide with the lines of flow.

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Turbulent/Disorderly/Disturbed flow

It is a disorderly flow, particles pass a given point with varying speeds.

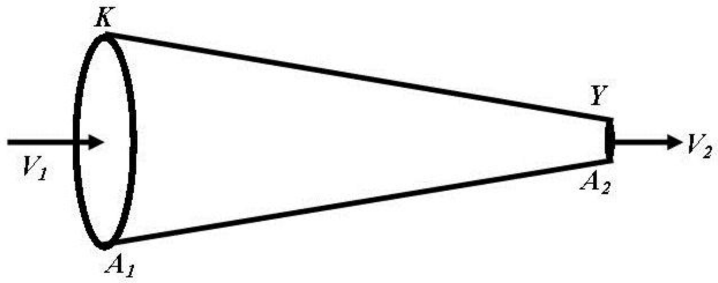
Ideal liquid

It is the one that has;

1. Zero compressibility i.e volume and density are constant on pressing/compressing.
2. Zero velocity i.e non- viscous exhibit zero friction.

Principle of continuity

It states that;- '*when an incompressible and non – viscous liquid flow in streamline motion through a tube of non- uniform cross-section, then the product of the area of cross – section and the velocity of flow is the same at every point in the tube'*.



Consider a liquid flowing in the streamlined motion through a non-uniform tube *X Y*. Let cross – sectional areas *X* and *Y* be A_1 and A_2 respectively and velocities of fluid be v_1 and v_1 respectively.

Let ρ be the density of the liquid, the liquid entering *X* covers a distance of *v,* per second.
Thus the volume of the liquid entering at end *X* per second is $A_1 \times v_1$.
Mass of liquid entering point *X* per second is ρ x $A_1 \times v_1$
Similarly mass of liquid coming out of *Y* per second is ρ x $A_2 \times v_2$
But the liquid which enters at one end must leave at the other end.
Hence both masses are equal i.e
$\rho \text{ x } A_1 \times v_1 = \rho \text{ x } A_2 \times v_2$

$A_1 \times v_1 = A_2 \times v_2$
$A \times v$ = constant which is the equation of continuity.
The velocity of the liquid is smaller as the wider part of the tube and lager in the narrow part.

Energy of flowing liquid

There are 3 types of energies stored in a flowing liquid

i. **Pressure energy**
If ρ is the pressure on an area *A* of a liquid and the liquid moves through a distance *l* due to this pressure, then pressure energy of the liquid can be determined as follows:

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Work done = *pressure* x *Area* x *distance* $=\rho Al$

Presure energy per unit Volume is,

$P = \frac{\rho Al}{Al} = \rho$

ii. Kinetic Energy

It is energy possessed by a flowing liquid per unit Volume. If a liquid of mass m and Volume V is flowing with velocity v, then

$K.E = \frac{1}{2}mv^2$ *Kinetic per unit Volume is*

K.E. per unit volume $= \frac{1}{2}mv^2\ V$ *or* $\frac{1}{2}\rho v^2$ Since $\rho = \frac{m}{V}$

iii. Potential energy

If a liquid of mass m is at a height h from the surface of the earth, then its potential energy is

$P.E = mgh$

P.E per unit volume of the liquid $= \frac{mgh}{V} = \rho gh$ Since $\rho = \frac{m}{V}$

Bernoullis theorem

When an incompressible and non – viscous fluid flows in a streamlined motion from one place to another then at every point of its path the total energy per unit volume is constant, i.e

$P + \frac{1}{2}\rho v^2 + \rho gh$ = constant

Bernoullis theorem is one way the principle of conservation of energy for a flowing fluid.

Work done per second on the liquid entering the tube at X is

force x distance = $P_1A_1V_1$ (Since P x A = Force)

Similarly work done per second against $F = P_2 \times A_2$ by the fluid leaving tube at Y is

$P_2A_2V_2$

Net work done on the liquid = $P_1A_1V_1 = P_2A_2V_2$

But $A_1V_1 = A_2V_2 = \frac{m}{\rho}$, where m is the mass of the flowing fluid and ρ is the density of fluid.

Net work done on the liquid $= (P_1 - P_2)\frac{m}{\rho}$

The kinetic energy of the fluid entering at X per second is $\frac{1}{2}m_1v_1^2$ and that of fluid leaning at Y per second is as $\frac{1}{2}m_2v_2^2$. Therefore increase in kinetic energy of the fluid is $\frac{1}{2}m\,(v_2^2 - v_1^2)$ for $m_1 = m_2$

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The potential energy of the liquid at X P. E. = mgh_1, and that at Y P. E = mgh_2

Decrease in P.E of fluid = $mg\ (h_1-h_2)$.

Net increase in energy due to the net work done on fluid, hence

Net work done = The net increase in energy

$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}m\ (v_2^2-v_1^2) - mg\ (h_1-h_2)$. At horizontal level, $h_1 = h_2 = 0$.

Therefore, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$, hence,

$P_1 + \frac{1}{2}\rho v_1^2$ =constant

Pressure difference $P_1 - P_2 = \frac{1}{2}\rho\ (v_2^2-v_1^2)$ applying equation of continuity.

$A_1 \times v_1 = A_2 \times v_2,\ v_2^2 = \left(\frac{A_1}{A_2}\right)^2 v_1^2$

$P_1 - P_2 = \frac{1}{2}\rho\ (\left(\frac{A_1}{A_2}\right)^2 v_1^2-v_1^2) = \frac{1}{2}\rho v_1^2(\left(\frac{A_1}{A_2}\right)^2-1)$

Examples

1) The pressure difference between two points along a horizontal pipe, through which water is flowing is *1.4cm* of mercury. Due to non-uniform cross section, the speed of flow of water at the point of greater cross section is *60cm/sec*. Calculate the speed at the other part:

$P_1 + ½\rho v_1^2 = P_2 + ½\rho v_2 \quad V_2^2 = \frac{2(P_1-P_2)}{\rho} + v_1^2 \quad = (0.014 \times 3600 \times 10)\ N/m)^2$

$= \sqrt{\frac{2(0.014 \times 13600 \times 10)}{13600} + (0.6)^2} = 2ms^{-1}$

2) Air flows over the upper surface of the wings of an aeroplane at a speed of *100m* per second and past the lower wings at *90m* per second, calculate:
   a) Pressure difference on the wings.
   b) Lift force on the aeroplane if it has a total area of $10m^2$.
      (Take density of air as $1.3kg/m^3$)

Solution.

a) $\Delta P = P_1 - P_2 = \frac{1}{2}\rho\ (v_2^2-v_1^2) = \frac{1}{2} \times 1.3\ (100^2 - 90^2) = 1235N/m$

b) Lift force = Pressure Difference x Total Area of wings.
   Lift force $(F) = 1235 \times 10 = 12350N$

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6.9: Viscosity

Viscosity is the property of the fluid by virtue of which it opposes the relative motion between its adjacent layers.

When a body falls through a liquid or any viscous medium under gravity then the layer of the liquid in immediate contact of the body is carried along with it while the liquid at a large distance from the body is at rest. Thus there is relative motion between layers of the liquid. The viscous force in the liquid opposes relative motion. This force increases with velocity of the body and a stage is reached when it becomes equal to the driving force (resultant force propelling the body through the liquid).

Stoke showed that, *if a small sphere of radius 'r' is moving with terminal velocity, 'v_t' through a perfectly homogenous liquid of infinite extension and having a viscosity, 'η' then the viscous relating force acting in the sphere is given by*.

$$F = 6\pi \eta r v_t \quad - \textit{ Stoke's Law}$$

Consider a sphere of radius r, density ρ falling through a medium of density σ and co-efficient of viscosityη.

When the sphere attains terminal velocity v_t then its subject to three forces, namely:

1) Its weight acting downwards, $W = {}^4/_3\pi r^3 \rho g$.
2) Upthrust due to buoyancy, $U = {}^4/_3\pi r^3 \sigma g$.
3) Viscous force acting upwards, $F = 6\pi \eta r v_t$

The resultant downward force is ${}^4/_3\pi r^3 (\rho - \sigma)g$.
Since the sphere has attained a constant velocity, then resultant driving force must be equal to the retarding viscous force.
$6\pi \eta r v_t = {}^4/_3\pi r^3(\rho - \sigma)g$

$$V_t = {}^2/_9 \frac{r^2(\rho-\sigma)g}{\eta}$$

Thus $v \propto r^2$
Graph of v against r^2 has a gradient/slope $= {}^2/_9\, \frac{(\rho-\sigma)g}{\eta}$

Therefore: $\eta = {}^2/_9\, \frac{(\rho-\sigma)g}{s}$, s – slope/gradient

Summary

$$V_t = {}^2/_9 \frac{r^2(\rho-\sigma)g}{\eta}$$

$$\eta = {}^2/_9\, \frac{(\rho-\sigma)g}{s}$$

Example

1) A drop of water of radius *0.0015m* is falling in air. If the co-efficient of viscosity of air is 1.8×10^{-5} *kg/ms*. What will be the terminal velocity of the drop (take density of water *1000* $kg/m,^3$ neglect density of air).

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Solution

$$V_t = {}^2/_9 \frac{r^2(\rho-\sigma)g}{\eta}$$

$$V_t = 2/9 \frac{(0.0015)^2 \ x \ (9.8 \ x \ 1000)}{1.8 \ x \ 10^{-5}}$$

$$= 272.2 \ m/s$$

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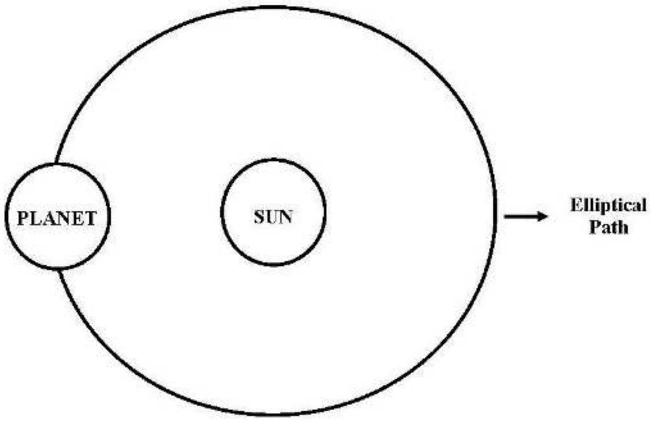
7.0: GRAVITATION

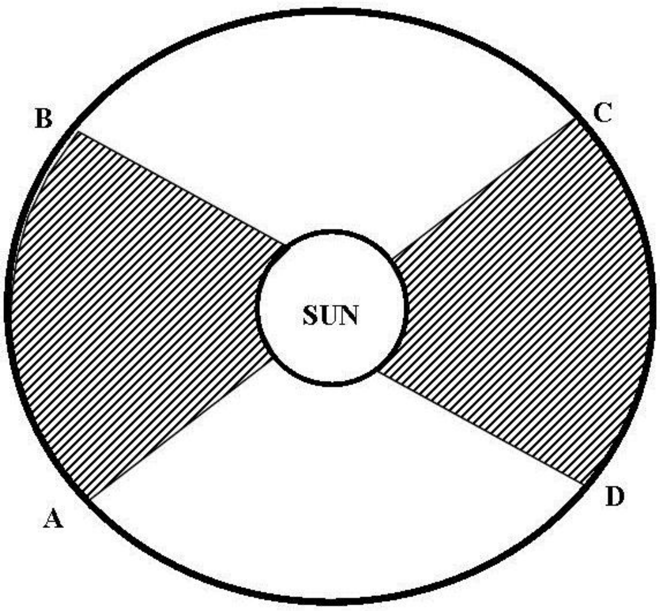
7.1: Kepler's Laws

Keplers laws deal with motion of planets.

Law 1*: The planet describes ellipses about the Sun as one focus.*



Law 2: *The line joining the Sun and the planets sweeps out equal areas in equal times.*



The time taken to move from *A* to *B* is equal to the time taken to move from *A* to *D*, thus the areas *OAB* and *OCD* are equal.

Law 3: *The squares of the periods of revolutions of the planets are proportional to the cubes of their mean distances from the Sun i.e* $T^2 = r^3$.

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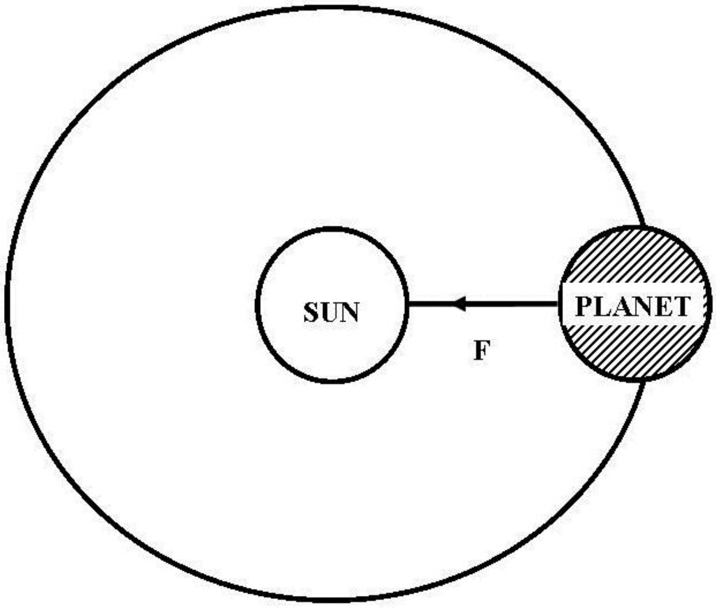
7.2: Newton's Laws of Gravitation.

Newton arrived at his law of gravitation by considering the motion of a planet moving in a circle around the Sun as the centre as shown.



They experience a force of attraction which is proportional to the product of the masses and also inversely proportional to the radius.
The force acting on the planet of mass *m* is:

1) Directly proportional to the product of the mass of the Sun and the planet.
2) Inversely proportional to the square of the disturbance between the Sun and the planet, i.e. $F = \frac{Gm_1 m_2}{r^2}$
   r = distance of separation.
   F = gravitational force of attraction between bodies of masses m_1 and m_2 and whose centres are *r apart.*
   G = Universal gravitational constant *(6.67 x* $10^{-11} Nm^2 kg^{-2}$*)*

To show that Kepler's third law is consistent with $F = \frac{Gm_1 m_2}{r^2}$,
Consider the motion of a planet moving in a circle around the Sun as the centre.

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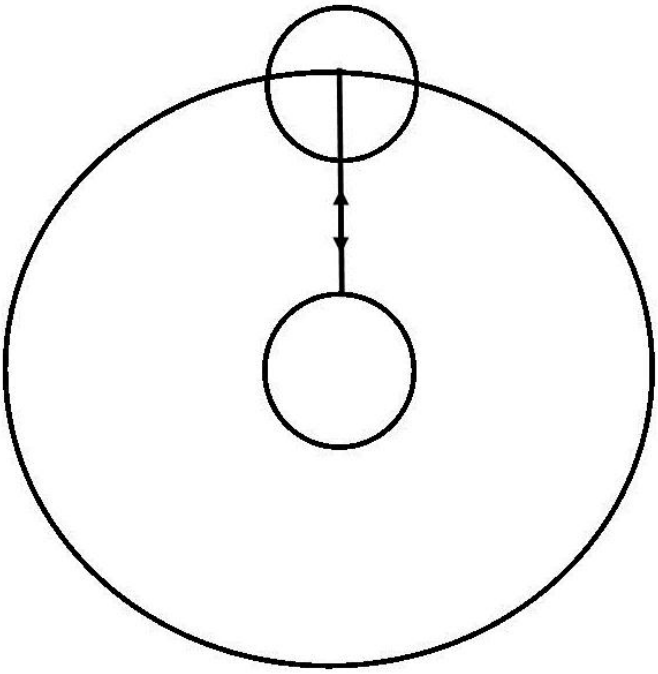

The force acting on the planet of mass $m = m\omega^2 r$ which is provided for by the force of gravity, r – radius of circle, ω angular velocity of motion.
Since $\omega = \frac{2\pi}{T}$, then $F = \frac{mr(2\pi)^2}{T}$, from $F = m\omega^2 r = \frac{4\pi^2 r}{T^2}$.

Also $F = \frac{km}{r^2}$ inverse square law

Therefore $\frac{km}{r^2} = \frac{4m\omega^2 r}{T^2}$, making T^2 the subject.

$T^2 = \frac{4m\omega^2 r^3}{km} = \frac{4\omega^2 r^3}{k}$, hence $T^2 \propto r^3$

Kepler's third law is consistent with Newton's Law of Universal gravitation.

7.3: <u>The Moon's And Motion Around The Earth.</u>

The moon has a period of 27.3 days and the force on it is given by:
$F = m\omega^2 r$, r –radius of orbit, m – mass of the moon.

The force acting on the moon as it revolves around the earth.
$F = \frac{4\pi^2 mr}{T^2}$, since $\omega = \frac{2\pi}{T}$

Consider the moon to be on the surface of the earth, $F = mg$, g = acceleration due to gravity.

Equating the two forces.

$mg = \frac{4\pi^2 mr}{T^2}$, $g = \frac{4\pi^2 r}{T^2}$, where r – radius of earth = $6.4 \times 10^6 m$

$$g = \frac{4 \text{ x } (3.142) \text{ x } 6.4 \text{ x} 10^6}{(27.3 \text{ x} 24 \text{ x } 3600)^{\ 2}} = 9.8\ N/kg$$

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Assuming that the force of attraction varies as the inverse square of the two masses (earth and moon).

$F_1 = \frac{GMm}{R^2}$ and $F_2 = \frac{GMm}{r_E 2}$

$F_1 : F_2 = \frac{F_1}{F_2} = \frac{GMm}{R^2} / \frac{GMn}{r_E 2} = \frac{1}{R^2} / \frac{1}{r_E 2}$

$F_1 : F_2 = \frac{1}{R^2} : \frac{1}{r_E 2}$……………………………..*(1)*

$F_1 = \frac{4\pi^2 \, mR}{T^2}$, $F_{2 = mg}$ - Holds when the moon is on the earth's surface

$\frac{4\pi^2 \, mR}{T^2} : mg$ ……………………………… *(2)*

$\frac{\frac{4\pi^2 \, mR}{T^2}}{mg} = \frac{1}{R^2} / \frac{1}{r_E 2}$

$Mg \times \frac{1}{R^2} = \frac{4\pi^2 \, T^2 mR}{T^2} \times \frac{1}{r_E 2}$

$g = \frac{4\pi^2 \, R^3}{T^2 r_E{}^2}$

7.4: *Mass of the Earth.*

If a body of mass *m* is on the earth's surface, the force acting on the body of its weight, *mg*. This same force is given by the law of universal gravitational.

i.e. $F = \frac{GMM_E}{r^2}$ M =mass of body, M_E =mass of earth, r = radius of earth.

Equating the two forces.

$mg = \frac{GMm_e}{r_e 2}$, $g = \frac{Gm_e}{r_e 2}$

$m_e = \frac{gr_e 2}{G} = \frac{9.8 \, N \, Kg \times (6.4 \times 10^6)^2 m^2}{6.67 \, x \, 10^{-11} \, Nm^2 \, / \, Kg^2}$

$m_e = 6.018 \times 10^{24}$ kg

7.5: Density of Earth.

Density is the ratio of mass to volume of a substance. To get the volume, the earth is spherical.

$V_e = {}^4/_3 \pi r_e^3 = {}^4/_3 \pi \, (6.4 \times 10^6 m)^3 = 1.098 \times 10^{21} m^3$

$\rho_e = \frac{m_e}{V_e} = \frac{6.018 \times 10^{24} \, kg}{1.098 \times 10^{21} m^3} = 5{,}480.54 \text{ kg/m}^3$

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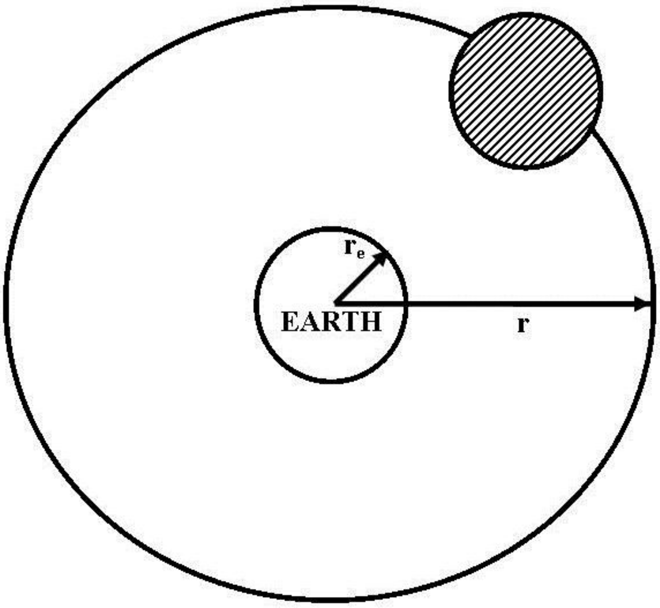
Note: The density of the earth varies with the depth, but it is largely independent of direction.

7.6: Body Outside the Earth ($r>r_e$).

The acceleration due to gravity at the surface of the earth can be determined by re-arranging the equation

$$g = \frac{Gm_e}{r_e 2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

The acceleration due to gravity at a point outside the earth is the value the body could have if the entire mass of the earth was concentrated at the centre.



The gravitational acceleration g^1 at a distance $r > r_e$ is given as

$$g^1 = \frac{GM\quad e}{\mathrm{r}\,2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

Dividing (2) by (1)

$$\frac{g^1}{g} = \frac{Gm_{e\,/\,r^2}}{Gm_e/\,r_e 2};\ \frac{g^1}{g} = \frac{r_e 2}{r^2}$$

$$g^1 = \frac{gr_e 2}{r^2}$$

$g^1 \propto \frac{1}{r^2}$, g^1 is inversely proportional to the square of the radius.

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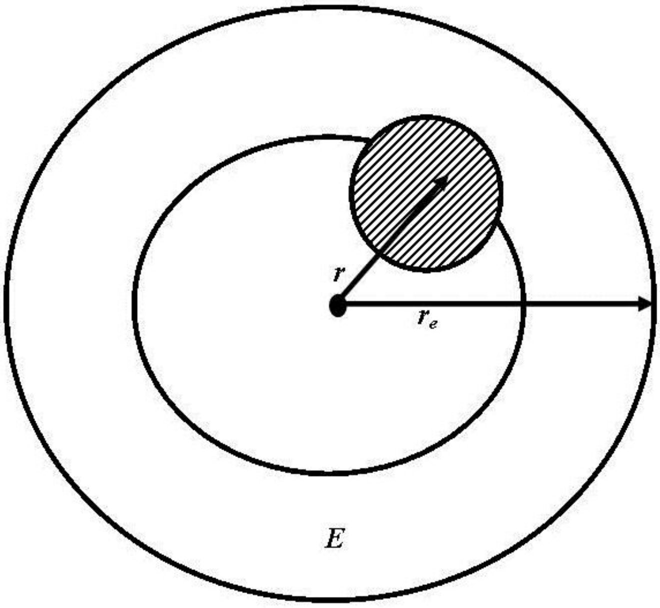
7.7: Body Inside the Earth ($r<r_e$).

The acceleration due to gravity at appoint inside the earth that is at a distance r from the centre of the earth is due to only the sphere of radius r.



The volume of the sphere of radius r of the earth is ${}^{4}/_{3}\pi r_e^{3}$, and if we let the density of the earth to be ρ_e,, then the mass of the sphere

$m_e = {}^{4}/_{3}\pi r_e^{3}\rho_e$

$m = {}^{4}/_{3}\pi r^{3}\rho$

$\frac{m}{m_e} = \frac{r^3}{r_e 3}$

$m = \frac{m_e r^3}{r_e 3}$, Recall $g^1 = \frac{GM}{r^2}$

$g^1 = \frac{G m_e r^3}{r_e 3} \text{x} \frac{1}{r^2}$

$g^1 = \frac{G m_e r}{r_e 3}$, Recall $g = \frac{G m_e}{r_e 2}$

$G m_e = g r_e^2$

$g^1 = \frac{g r_e{}^2 r}{r_e 3}, \; = \frac{rg}{r_e}$

$\frac{g}{r_e}$ = constant

Therefore, $g^1 = kr \Rightarrow g^1 \propto r$, g^1 is directly proportional to the radius

For $r = r_e$ then $g^1 = g$

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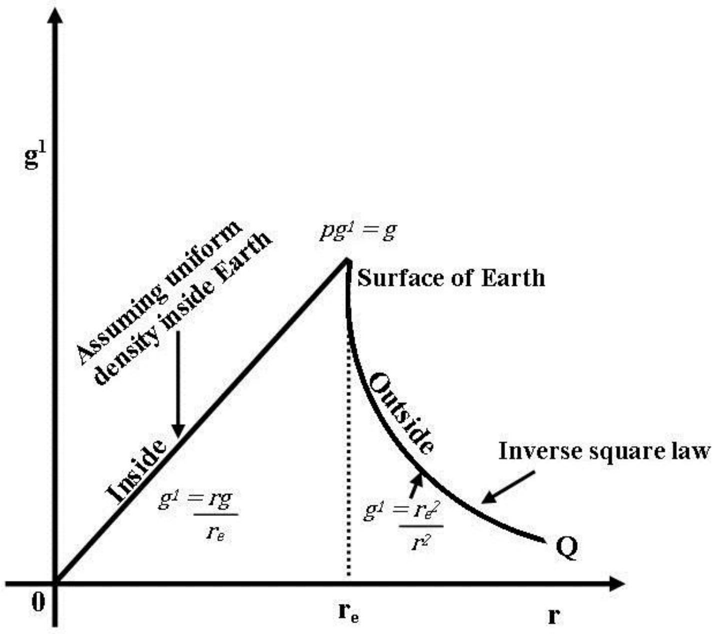

For points outside the Earth, the gravitational force obeys an inverse – square law. So the acceleration of free fall, $g^1 \propto {}^1/_{r}{}^2$ where r is the distance to the centre of the Earth from points P to Q. The maximum value of small g is obtained as the Earth's surface where $r = r_e$.

Inside the Earth point O to P, g^1 is not inversely proportional to square of the distance from the centre, assuming a uniform Earth's density which is not true in practice, theory shows that g^1 varies linearly with distance from the centre of the Earth.

7.8: Force on Astronaut Weightlessness

When a rocket is fired to launch a space craft and astronaut into orbit round the Earth, the initial thrust must be very high owing to the large initial acceleration required. The acceleration, a, is of order $15g$, where g is the gravitational acceleration at the Earth's surface. Suppose R is the reaction of the couch to which the astronaut is initially strapped, then from $F = ma$,

$R - mg = ma$

$R - mg = m\,(15g)$

$R = 16mg$

The force if 16 times the weight of the astronaut; therefore he experiences a large force.

In the orbit, however the state of affairs is different, this time the acceleration of a space craft and astronaut are both g^1 in magnitude where g^1 is the acceleration due to gravity of a particular height of the orbit.

If R^1 is the reaction of the surface of the astronaut then, for circular motion F

$F = mg^1 - R^1 = ma = mg^1$

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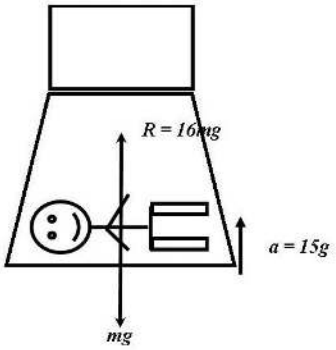
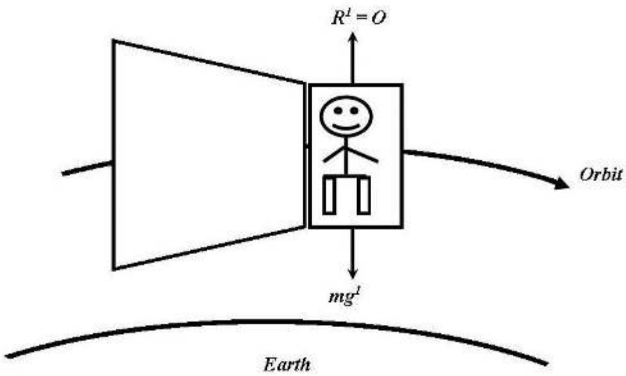
Thus $R^1 = 0$ no reaction on the floor, he experiences weightlessness.





7.9: Satellites

Satellites are kept in orbit by the gravitational attraction of Earth. Consider a satellite of mass m which circles the earth of mass m_e close to its surface.

$$\frac{MV^3}{r_e} = \frac{Gm_e m}{r_e 2} = mg$$

$v^2 = r_e g$ for $r_e = 6.4 \text{ x } 10^6 m$

Then $v = 8 \text{ x } 10^3 \, ms^{-1}$, satellites move at a speed of $8 \text{ x } 10^3 ms^{-1}$ to stay in orbit.

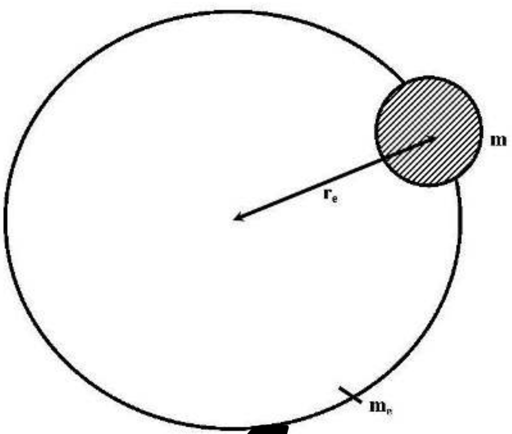
Period in orbit (T) $= \frac{Circumference\ of\ Earth}{Velocity\ of\ Satellite}$

$$= \frac{2\pi \text{ x } 6.4 \text{ x } 10^6 m}{8 \text{ x } 10^3 \text{m}s^{-1}}$$

$= 83.79$ *minutes.*

7.9.1: Escape Velocity

The speed that is enough to put the satellite into space is called escape velocity.



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Mass m experiences a force F which is defined by $F = \frac{Gm_e m}{r_e 2}$ just to be moved from the earth's surface.

The energy needed to propel the body to escape gravity is given by

$KE = {}^1/_2 mv^2$.

The KE is equal to force *(F)* of the body x displacement *(r)*

Therefore, $E = \frac{Gmm_e}{r_e 2} x\ r = \frac{Gmm_e}{r_e} = {}^1/_2 mv^2$

$${}^1/_2 v^2 = \frac{Gm_e}{r_e}$$

$$v = \sqrt{\frac{2Gm_e}{r_e}}$$

$$v = \sqrt{\frac{2x6.67\ x\ 10^{-11} Nm^2/Kg^2 x\ \ 6.018\ x\ 10^{24}}{6.4\ x\ 10^6\ m}}$$

$v = 11\ km/s$

7.9.2: Parking Orbits.

Consider satellite of mass m circling the Earth in the plane of the Equator in orbit concentric with the Earth, suppose the direction of rotation is the same as the Earth and the orbit is at a distance R from the centre Earth, then if v is the speed in orbit then.

$$\frac{mv^2}{R} = \frac{Gm_e m}{R^2}$$

But $Gm_e = gr_e^2$

$$\frac{mv^2}{R} = \frac{mgr_e^{\ \ 2}}{R^2}$$

$$v^2 = \frac{gr_e^{\ \ 2}}{R}$$

If T is the satellite's period in its orbit, then $v = \frac{2\pi R}{T}$

$$\frac{4\pi^2 R^2}{T^2} = \frac{gr_e^{\ \ 2}}{R}$$

$$T^2 = \frac{4\pi^2 R^3}{gr_e^2}$$

If the period of the satellite in its orbit is exactly equal to the period of the Earth as it turns about its axis which is 24 hours, the satellite will stay over the same place on the Earth while the Earth rotates. This is called Parking Orbits.

Since $T = 24$ hrs, then $R = \sqrt[3]{\frac{T^2 gr_e^2}{4\pi^2}}$

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$R = \sqrt[3]{\frac{(24 \text{ x } 36009.8 \text{ x } (6.4 \; x \; 10^6)^2}{4\pi^2}}$

R = 42,400 km

The height above the Earth's surface of the parking orbit.

$R - r_e = 42{,}400 – 6400 = 36{,}000 \; km$

In the orbit, assuming it is circular, the speed of satellite.

$= \frac{2\pi R}{T} = \frac{2\pi \text{ x } 42400}{24 \text{ x } 3600} = 3.2 \text{ kms}^{-1}$

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8.0: THERMAL PHYSICS /HEAT AND THERMODYNAMICS.

It is the study of thermal energy/heat energy. The central concept of thermodynamics is temperature.

Temperature is the degree of hotness or coldness of a body expressed on some scale. Temperature is measured in Kelvin (*K*)

Temperature is measured by a thermometer.

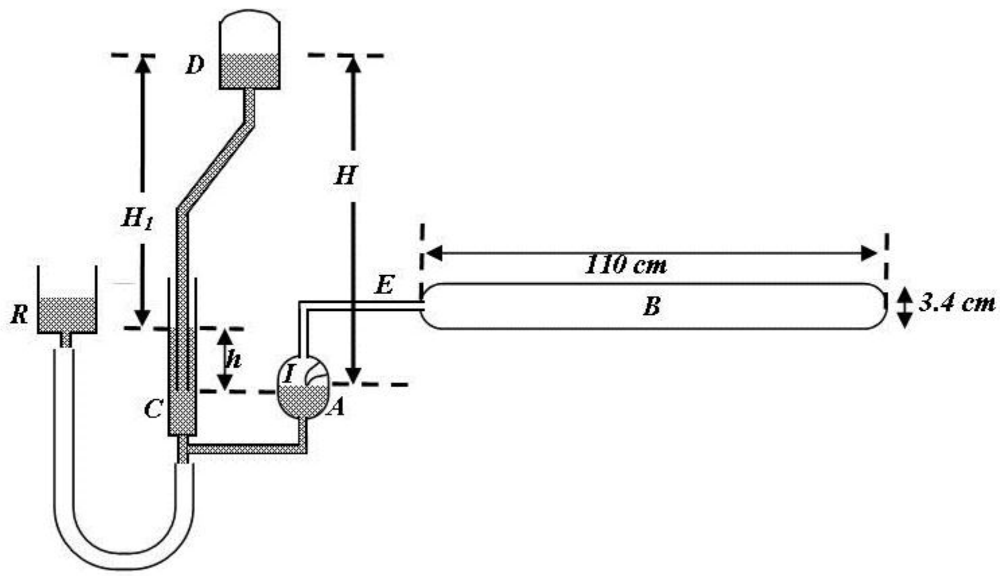
Types of Thermometers.

1) Constant – volume gas thermometers.
2) Resistance thermometers.
3) Mercury – in – glass thermometers.
4) Electric thermometers.
5) Thermocouple thermometers.

Gas Thermometry.

In most accurate work temperatures are measured by gas thermometers, e.g. by the changes in pressure of a gas at constant volume. Gases behave the same at low temperatures. Gas thermometers temperatures are used as standard temperatures. Temperatures measured with other types of thermometers are changed to a gas thermometer temperature. The figure below shows a constant volume hydrogen thermometer. *B* is a bulb of platinum – iridium holding the gas. Volume is defined by index *I* in glass tube *A*. The pressure is adjusted by raising or lowering the mercury reservoir *R*. a barometer *CD* is fitted directly into the pressure measuring system, if H_1 is its height, and *h* the difference in level between the mercury surface *A* and *C*, then the pressure *H* of the hydrogen in *mmHg* is given by

$$H = H_1 + h.$$



Tube *E* is called dead space of thermometer. Its diameter is *0.7mm,* it contains only a fraction of mass of gas. The readings of these, when they are used to measure unknown temperatures, can then be converted into temperatures on ideal gas scale.

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Helium gas is widely used in gas thermometers. The Celcius temperature θ on constant volume gas thermometer scale can be calculated from

$$\theta = \frac{P_\theta - P_0}{P_{100} - P_0} \text{ x } 100°\text{C}$$

P_θ, P_o and P_{100} are respective gas pressures of the unknown temperature θ, the ice point and steam point. Gas thermometers are bulky and difficult to use.

Electric Thermometers.

Electric thermometers are more accurate and less cumbersome. Temperatures on a thermoelectric scale would be calculated as

$$\theta_E = \frac{E_\theta - E_0}{E_{100} - E_0} \text{ x } 100°\text{C} .$$

E_θ, E_o and E_{100} are respective thermoelectric E.M.F.s at the unknown temperature θ, the ice point and the steam point. These thermometers have a temperature range of -250℃ to 1500℃.

Resistance Thermometers.

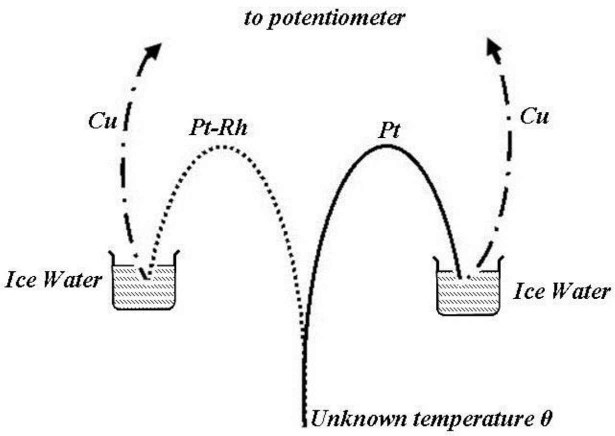
Resistance thermometers are made of platinum. The celecius temperature θ_P on a platinum resistance thermometer scale can be calculated by

$$\theta_P = \frac{R_\theta - R_0}{R_{100} - R_0} \text{ x } 100°\text{C} .$$

where R_θ, R_o and R_{100} are respective resistances at then unknown temperature θ, the ice point and the steam point. They have wide temperature range of -200℃ to 1200℃ used to measure liquid temperatures accurately.

Thermocouple Thermometers.

A thermocouple is a thermometer that has wires made from copper, platinum and platinum rhodium alloy. The wires are joined and dipped in ice.



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Temperature θ measured by this thermocouple is defined by the relation

$E = a + b\theta + c\theta^2$

Where *a, b* and *c* are constants. The values of *a*, *b* and *c* are determined by measuring the value of *E* at gold point *1064.43*℃*,* silver point *960.8*℃ and freezing antimony *630.74*℃.

Examples.

1) The pressure recorded by a constant volume gas thermometer at a *Kelvin* Temperature *T* is 4.0 x $10^4 N/m$. Calculate *T* if the pressure at triple point 273.16*K* is 4.2 x $10^4 N/m$.

   Solution.

   $T = \frac{P_T}{P_{tr}}$ x 273.16K = $\frac{4.8 \text{ x } 10^4}{4.0 \text{ x } 10^4}$ x 273.16K = 312K

2) A platinum wire has a resistance of 2.000Ω, 2.778 Ω and 5.280 Ω at meeting point of ice. Boiling of water and boiling point of sulphur. Calculate the value of B.P of sulphur.

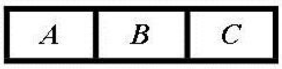
   Solution.

   $\theta_p \frac{R_\theta - R_0}{R_{100} - R_0} = \frac{5.20 - 2.000}{2.778 - 2.000} = 421.6\ ℃$

Zeroth Law

Two bodies are said to be in thermal equilibrium with each other if they have the same temperature.

Zeroth Law: It states that if two bodies A and B are in thermal equilibrium separately with a third body, C then A and B are also in thermal equilibrium with each other.



A Kelvin (1*K*) is defined as $\frac{1}{273.16}$ of the temperature of the triple point of water, i. e solid, liquid and gas.

Triple point is the temperature and pressure at which a substance can exist in all three phases (liquid, liquid and vapour simultaneously). When substances are heated they expand, on cooling they contract.

The linear expansion of substances is characterized by an average expansion coefficient (α) defined by

$$\alpha = \frac{1}{l} \text{ x } \frac{\Delta\iota}{\Delta T} = \frac{\Delta l}{l\Delta T}$$

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Where; l = original length

Δl = change in length

ΔT = Change in temperature

α = linear expansivity

The average volume expansion coefficient β for a homogenous substance

$$\beta = 3\alpha = \frac{3\Delta l}{l\Delta T}$$

An ideal/perfect gas is one that obeys the equation of state.

PV = nRT, where n – number of moles of a gas, V - volume of gas, P - pressure, T – Temperature (K) and R - Universal gas constant (8.31J / mo (K).

A real gas the one that liquefies at low temperature. The absolute temperature is related to the temperature on the Celsius scale by

T(K) = T(^{0}C) + 273.16, where T(^{0}C) is Celcius temperature scale.

Heat flow is a form of energy transfer that takes place as a consequence of a temperature difference only. The internal energy of a substance is a function of its temperature generally with increasing temperature.

A calorie is the amount of heat necessary to raise the temperature of 1 gramme of water 1^0 C.

1 Calorie = 4.186J.

Heat capacity C, of any substance is defined as the amount of heat energy needed to raise the temperature of a substance by 1 degree centigrade or 1 Kelvin.

$$C = \frac{Q}{\Delta T}; \quad Q = C\Delta T$$

Specific heat capacity c, is the amount of heat energy required to change the temperature of a unit mass of a substance by 1 degree centigrade or 1 Kelvin.

$$c = \frac{Q}{m\Delta T}; \quad Q = mc\Delta T$$

The heat that is required to change the phase of a pure substance of mass m is given by

Q = mL, where L is the latent (hidden) heat.

L depends on the nature of phase change and properties of a substance. If the change is solid to liquid then it is known as latent heat of fusion (L_f), and if it is liquid to vapour then it is known as latent heat of vaporization (L_v).

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Specific Heat Capacities of Gases at Constant Volume and Pressure.

When we warm a gas, we may let it expand or not, as we please. If we do not let it expand – if we warm it in a closed vessel – then it does no external work, and all the heat we give it goes to increase its internal energy.

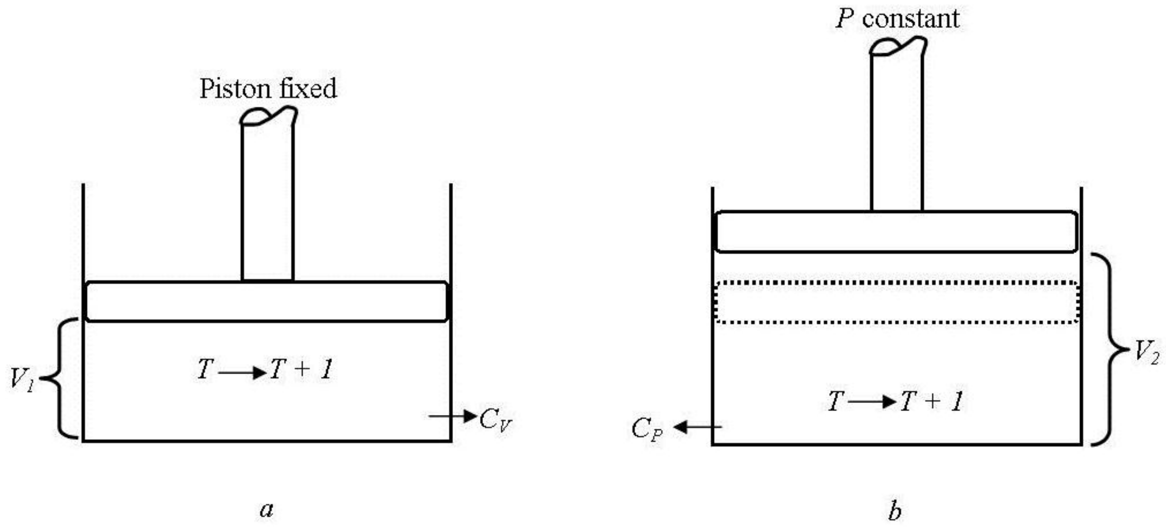
The heat required to warm one mole of a gas through one degree, when its volume is kept constant, is called the molar heat capacity of the gas at constant volume. It is denoted by C_V and generally expressed in $JMol^{-1}K^{-1}$.

We can also warm a gas while keeping its pressure constant and define the corresponding heat capacity:

The molar heat capacity of a gas at constant pressure is the heat required to warm one mole of it by one degree, when its pressure is kept constant. It is denoted by C_P and has same units as C_V.

Molar heat capacities.

The figure below shows how we can find a relationship between the molar heat capacities of a gas. We first consider one mole of the gas warmed through 1*K* at constant volume (*a*). The heat required is C_V *Joules* and goes wholly to increase the internal energy *U*. Then next we consider 1 mole warmed through 1*K* at constant pressure, *(b)*.



It expands from V_1 to V_2 and does an amount of work given by

$W = P$ x Volume change $= P\,(V_2 - V_1)$……………………………….. (1)

Further since the temperature rise of the gas is *1K,* and the internal energy of the gas is independent of the volume, then rise in internal energy is C_V, the molar heat capacity at constant volume, hence from

$\Delta Q = \Delta U + P\Delta V$……………………………………………………. (2)

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The total heat energy required to warm a gas constant pressure is given by:

$C_P = C_V + P(V_2 - V_1)$…………………………………………………………. (3)

Using equation of state for 1 mole.

$PV = RT$ ………………………………………………………………….. (4)

Where T is absolute temperature of the gas and R is the molar gas constant. If T_1 is the absolute temperature before warming and $T_1 + 1$ is the absolute temperature after warming then:

$PV_1 = RT_1$……………………………………………………………… (5)

And $PV_2 = R(T_1 + 1)$…………………………………………………… (6)

Subtracting (5) from (6)

$P(V_2 - V_1) = R$ then equation (3) becomes

$C_P = C_V + R$ or $C_P - C_V = R$…………………………………………. (7)

Also $\gamma = C_P/C_V$ ……………………………………………………….. (8)

Heat transfer in Solids, Liquids and Gases.

Heat is transferred by three fundamentally distinct mechanisms: conduction, convection and radiation.

Conduction: It is the exchange of K.E between colliding molecules. The rate at which heat flows by conduction through a slab of area A is given by

$$\frac{dQ}{dt} = \frac{-kAdT}{dx}$$

Where k is the thermal conductivity and $\frac{\mathrm{dT}}{\mathrm{d}x}$ is the temperature gradient.

$k = \frac{dQ}{dt} / \frac{AdT}{dx}$, $Q = kAt \ \frac{T_2 - T_1}{x}$ Q is heat energy conducted by a conductor, A is surface area of a conducting surface, t is time taken by the conducting process, x is length/distance covered by heat energy and T_1 and T_2 are initial and final temperatures between two conducting bodies.

Example

1) Calculate the quantity of heat conducted through $2m^2$ of a brick wall $12cm$ thick in 1 hour if the temperature on one side is 8^0C and on the other side is 28^0C. (take thermal conductivity of brick = $0.13Wm^{-1}K^{-1}$)

Solution.

Temperature gradient $\left(\frac{\mathrm{dT}}{\mathrm{d}x}\right) = \frac{28-8}{12\ x\ 10^{-2}}$ Km^{-1} and t = 3,600s

$Q = kAt$ x $\frac{\mathrm{dT}}{\mathrm{d}x}$ = 0.13 x 2 x 3,600s x $\frac{28-8}{12\ x\ 10^{-2}}$ Km^{-1}

$= 156{,}000J$

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Convection: It is heat transfer process in which the heated substance moves from one place to another.

Radiation: All bodies radiate and absorb energy in form of electromagnetic waves. A body that is hotter than its surroundings radiates more energy than it absorbs, whereas a body that is cooler than its surroundings absorbs more energy than it radiates. The Sun is one of the surfaces that radiates heat.

Stefan's Radiation Law:

The rate at which energy is radiated by an object at Kelvin temperature, *T* is proportional to the surface area *A*, of the object, to the fourth power of the absolute temperature and to the emissivity (*e)* of the substance emitting the radiation.

$\frac{\Delta Q}{\Delta t} = e\sigma AT^4$, σ is the Stefan-Boltzmann constant = 5.67 x 10^{-8} w/m^2. k^4, OR

$\frac{\Delta Q}{\Delta t} = e\sigma A\ (T - T_0)^4$, $\frac{\Delta Q}{\Delta t}$ = power,

e ranges from 0 – 1

Dark and rough surfaces have *e* closer to 1, while bright/white surfaces have *e* closer to 0.

Another law governing thermal radiation is **The Wien Displacement Law.**

$\lambda_{(max)}$ T = W = 2.898 x 10^{-3} M.K

W – Wiens constant

<u>Example</u>

1) If the radius of the Sun is 6.96 x 10^8m and its emissivity is about 0.93, find the rate at which it radiates heat at 6000K.

   <u>Solution</u>

   $\frac{\Delta Q}{\Delta t} = e\sigma T^4 = 0.93 \text{ x}5.67 \text{ x } 67 \text{ x } 10^{-8}\ 4\pi \text{ x } (6.96 \text{ x } 10^8)^2 \text{ x } (6000)^4$

   $= 4.2 \text{ x } 10^{26}$ W

2) The surface temperature of the sun is about 5000K. If the Sun is a radiator, at what wavelength would its spectrum be peak?

   <u>Solution.</u>

   $\lambda_{(max)} = \frac{2.898\ x\ 10^{-3}\ mK}{5000\text{K}} = 5.79 \text{ x } 10^{-7}$ m

Equation of state for gases:

$\frac{PV}{T}$ = Constant $\left[\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right]$

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8.1: Kinetic Thoery of Gases.

Kinetic theory is used to explain the behaviour of gases in terms of motion of their molecules. The theory postulates that a gas consists of numerous molecules moving at high velocities in random motion, as well as with the walls of the container. This theory is based on the following assumptions:

1) The volume of the molecules is negligible compared to the volume occupied by the gas (they may be considered as points).
2) There are no inter-molecular forces except during collisions.
3) All collisions are perfectly elastic, i.e both kinetic energy and momentum are conserved.
4) Duration of collision is negligible compared to the time in between them.

Summary

1. $\alpha = \frac{1}{l} \text{x} \frac{\Delta\iota}{\Delta T} = \frac{\Delta l}{l\Delta T}$
2. $\beta = 3\alpha = \frac{3\Delta l}{l\Delta T}$
3. $T(K) = T(^0C) + 273.16$
4. $C = \frac{Q}{\Delta T}$; $Q = C\Delta T$
5. $c = \frac{Q}{m\Delta T}$; $Q = mc\Delta T$
6. $Q = mL$
7. $\frac{dQ}{dt} = \frac{-KAdT}{dx}$
8. $\frac{\Delta Q}{\Delta t} = e\sigma AT^4$, $\frac{\Delta Q}{\Delta t} = e\sigma A\ (T - T_0)^4$, $\frac{\Delta Q}{\Delta t}$ = power,
9. $\lambda_{(max)}T = W = 2.898 \text{ x } 10^{-3}$ M.K
10. $\frac{PV}{T}$ = Constant, $\left[\frac{P_{1V_1}}{T_1} + \frac{P_{2V_2}}{T_2}\right]$
11. $PV = nRT$
12. $T = \frac{P_T}{P_{tr}} \text{ x } 273.16K$
13. $\theta = \frac{P_\theta - P_0}{P_{100} - P_0} \text{ x } 100°C$
14. $\theta_E = \frac{E_\theta - E_0}{E_{100} - E_0} \text{ x } 100°C$.
15. $\theta_P = \frac{R_\theta - R_0}{R_{100} - R_0} \text{ x } 100°C$.
16. $E = a + b\theta + c\theta^2$
17. $C_P = C_V + R$ or $C_P - C_V = R$

Examples

1) Convert the following values into Kelvin scale.
   a) 80^0C b) -40^0C

Solution

a) T = 80 + 273.16 = 353.16K
b) T = -40 + 273.16 = 233.16K

2) Calculate the change in length of invar of length 1m when temperature changes from 20^0C to 30^0C. Take linear expansivity of invar as $7.0 \text{ x } 10^{-7}/^0C$.

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Solution

From $\alpha = \frac{\Delta\ell}{\ell\Delta T}$

$\Delta\ell = \alpha\ell\Delta T$

$\Delta\ell = 7.0 \times 10^{-7} \times 1 \times 10 = 7.0 \times 10^{-6}$ m.

3) Suppose *1*kg of a piece of aluminum at *200^0C* is dropped into *5kg* of water at *20^0C*. If no heat is exchanged with the surrounding, what is the final temperature of the water and aluminium? (take specific heat capacities of water and aluminium as *$4200Jkg^{-1}K^{-1}$* and *$924Jkg^{-1}k^{-1}$).*

Solution.

Heat lost by aluminium = Heat gained by water

$m_a c_a \Delta T_a = m_w c_w \Delta T_w$

1 x 924 (200 – T) = 5 x 4200 (T-20)

184800 – 924T = 21000T – 420000

$T = \frac{604800}{21924} = 27.59^0C$

4) Calculate the total amount of heat absorbed when *5kg* of ice is heated from *-20^0C* until it vaporizes at *100^0C*. Take specific heat capacities of ice and water as *$2100\ Jkg^{-1}K^{-1}$* and *$4200\ Jkg^{-1}K^{-1}$* respectively, specific latent heat of fusion of ice as *3.34* x *$10^5J\ Kg^{-1}$* and specific latent heat of vaporization as *2.26* x *$10^6\ Jkg^{-1}$*

Solution.

Q_1 from -20^0C to 0^0C

$Q_1 = 5 \times 2100 \times 20 = 210000J$

Q_2 at 0^0C

$Q_2 = 5 \times 3.34 \times 10^5 = 1{,}670{,}000J$

Q_3 from 0^0C to 100^0C

$Q_3 = 5 \times 4200 \times 100 = 2{,}100{,}000J$

Q_4 at 100^0C

$Q_4 = 5 \times 2.26 \times 10^6 = 11{,}300{,}000J$

$Q = Q_1 + Q_2 + Q_3 + Q_4 = 15.28$ MJ

5) Find the volume of 1 mole of any gas at STP.

Solution.

PV = nRT

$$V = \frac{nRT}{P} + \frac{1\ mole\ x\ 8.31J\ /\ mol.K\ x\ 273}{1.01\ x\ 10^2\ N/m^2}$$

$= 22.\ X\ 10^{-3}m^3$

6) A tank of nitrogen gas at 0^0C and an absolute pressure of 50.0 atm has a volume of $0.100m^3$. What is the mass of nitrogen in the tank?

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Solution

P = 50.0 atm = 50 (1.01 x 10^5 N/m^2) = 50.5 x 10^5 N/m^2

V = 0.100 m^3.   T = 0^0C = 273K

Since PV = nRT, the number of moles of nitrogen present.

$$n = \frac{PV}{RT} = \frac{50.5\ x 10^5\ x\ 0.100}{8.31\ x\ 273} = 2.23\ x\ 10^2\ moles$$

1 mole of nitrogen has a mass of 28g

Therefore mass of nitrogen = 22.23 x 10^{-2} moles $\frac{28g}{1\ mol.}$

= 6.24 kg

7) A steel cylinder of capacity 0.5m^3 contains 30000 pa when temperature is 300K. What will be the pressure of nitrogen if it is allowed to flow into another cylinder of capacity 9.5m^3 with temperature is reduced to 250K?

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \qquad P_2 = \frac{P_1V_1T_2}{V_2T_1} = \frac{30{,}000\ x\ 0.5\ x\ 250}{9.5\ x\ 300} = 1{,}315.79 \text{ pa}$$

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9.0: WAVES AND VIBRATIONS.

Wave motion is a means of transferring energy from one point to another without there being any transfer of matter between the points.

Waves are classified into two categories namely,

1) Mechanical waves e.g water waves, waves in string, sound etc.
2) Electromagnetic waves e.g. light, radio, X – rays etc.

Differences between mechanical and electromagnetic

a) Mechanical waves do need a medium for propagation while electromagnetic waves do not.
b) Mechanical travel at varied speeds depending on their initial energy and the material they are traversing through, while electromagnetic waves move at the speed of light, e.g $c = 3.0 \times 10^8$ m/s.

Differences between oscillations and waves.

1) An oscillation is confined to a body while a wave extends through space.
2) A wave is a means by which energy is transferred from one place to another while an oscillation is a means by which energy can be stored in a in a confined mass.
3) Any oscillation can be resolved into a number of simple harmonic motions of different amplitudes and frequencies while a wave can be resolved into a continuous set of oscillations in the medium.

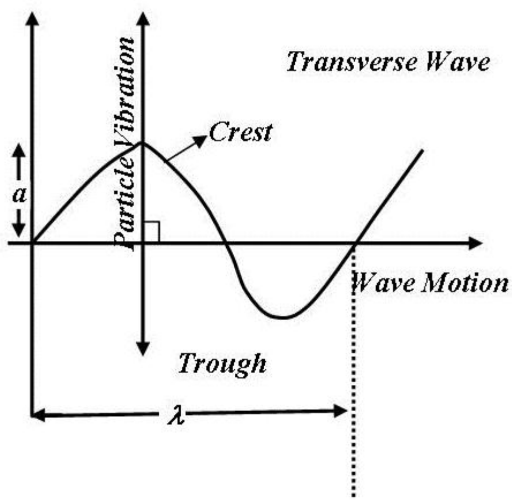
Waves obey the following properties.
Reflection; refraction, diffraction, interference and polarization.

Waves and Their Characterization.

Waves *s* are further are classified into two classes depending on their mode of propagation, i.e.

1) **Transverse waves** – It is a wave in which the particles of the medium move in the direction perpendicular to the direction of wave motion, e.g water waves, waves in a string (rope), all electromagnetic waves etc.

2) **Longitudinal Waves** – are waves in which the particles of the medium move in a direction parallel to the direction of the wave motion e.g sound wave, waves in a slingy spring.



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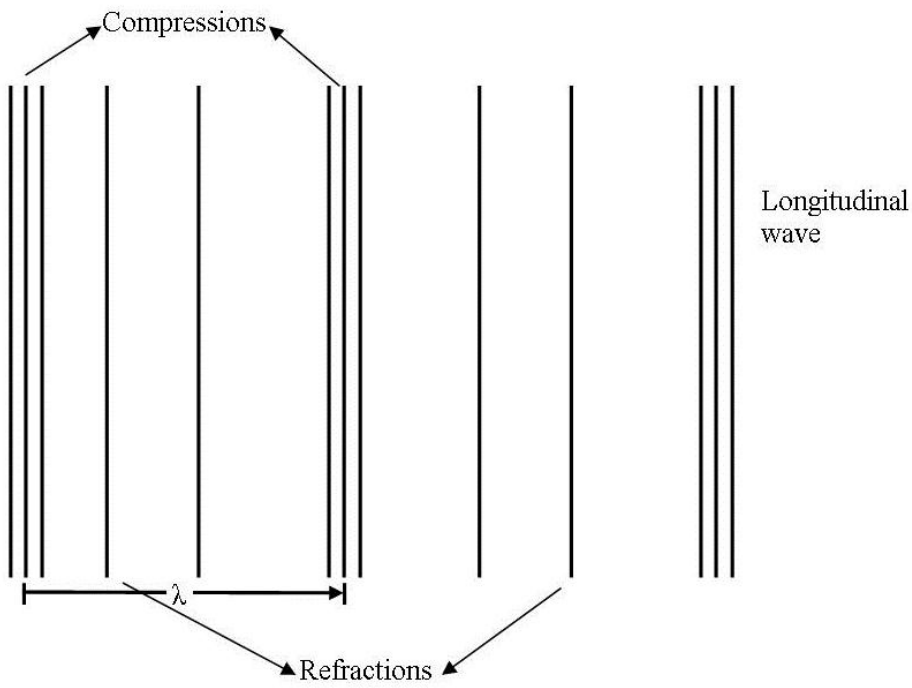

Definition of Terms:

1) **Amplitude (*a*):** It is the maximum displacement from the mean position, measured in metres (*m*).
2) **Wavelength (λ)**: It is the length of a complete cycle of a wave, measured in metres (*m*).
3) **Frequency (*f*):** It is the number of cycles made by a wave in a second, measured in cycles per second (*c/s* or cs^{-1}) where:
   I c/s = **I** Hertz (*Hz*)

4) **Period (*F*):** It is the time taken for wave to make a complete oscillation. It is measured in seconds (*s*).

$$T = {}^{1}/_{f}$$

If T is the period f the frequency and then $v = {}^{\lambda}/_{T}$. But $T = \frac{1}{f} => f = \frac{1}{T}$

5) **Phase Difference.**

Consider the wave below:

An equation can be formed to represent generally the displacement *y,* of a vibrating particle in a medium in which a wave passes. Suppose the wave moves from left to right their amplitude at the origin *0* then vibrates according to the equation

$y = a\ Sin\ \omega t.$ where t is the time and $\omega = 2\pi f$.

A particle *P* at a distance *x* from *0* to the right the phase of the vibration will be different from that at *0.*

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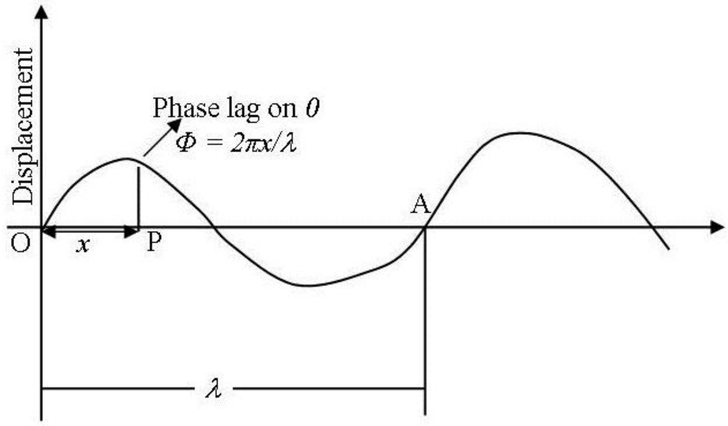

A distance λ from *0* corresponds to a phase difference of 2π. So the phase difference $\emptyset$ at *P* is given by ($^x/_\lambda$) x 2π.

Or $\frac{2\pi x}{\lambda}$ Then the displacement of any particle at a distance *x* from the origin is given by

$y = a\ Sin\ (\omega t - \phi)$,

$y = a\ Sin\ (\omega t - \frac{2\pi x}{\lambda})$, since $\omega = 2\pi f = \frac{2\pi v}{\lambda}$ where *v* is speed of the wave.

Then $y = a\ Sin\ (\frac{2\pi v}{\lambda} t - \frac{2\pi x}{\lambda})$,

$y = a \text{ Sin} \frac{2\pi}{\lambda} (vt - x)$

Also $\omega = \frac{2\pi}{T}$ then, $y = a \text{ Sin } \pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

The negative sign shows the wave is moving from left to right. If a wave travels in opposite direction, then,

$y = a \text{ Sin}\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right)$

Equation of Waves

Waves can be expressed using Sine or Cosine function, i.e. a wave in one dimension can be expressed as $y = A\ Sin \left[\frac{2\pi}{\lambda}\ (x - vt)\right]$

$y = A\ Sin\ (kx - \omega t)$

where *A* – amplitude

ω - ampular frequency

$\omega = \frac{2\pi}{T} = 2\pi f$, *K* =is the wave number

$K = \frac{2\pi}{\lambda}$

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Examples.

1) A generator at one end of a very long string makes a wave governed by the equation,

$y = (6.0\ cm)\ Cos\ \frac{\pi}{2}\ [(2.0\ m^{-1})x + (8.05^{-1})t]$, Calculate the frequency, period, amplitude wavelength and speed of the wave.

Solution

Amplitude = 6.0 cm

$y = (6.0\ cm)\ Cos\ [\pi x m^{-1} + 4\pi t s^{-1}]$

$K = \frac{2\pi}{\lambda} = \pi m^{-1}$

$\frac{2}{\lambda} = 1m^{-1}$, $\lambda = 2m$

$\omega = 2\pi f$, $4\pi s^{-1} = 2\pi f$

$f = 2s^{-1} = T = {}^{1}/_{f}$

$T = {}^{1}/_{2} = 0.5$ seconds

$v = f\lambda = 2 \text{ x } 2 = 4ms^{-1}$

2) Suppose a wave is represented by

$y = a \text{ Sin } (20000\pi t - \frac{\pi x}{0.17})$

Find frequency, period, wavelength, speed and phase difference, given that $x = 0.17$.

Solution.

$y = a \text{ Sin } \frac{2\pi}{\lambda}(vt - x)$

$\frac{2\pi v}{\lambda} = 2000\pi$ and $\frac{2\pi}{\lambda} = \frac{\pi}{0.17}$

$\lambda = 2 \text{ x } 0.17$

$\lambda = 0.34$ metres

$v = 1000\lambda = 1000 \text{ x } 0.34 = 340\ ms^{-1}$

$f = \frac{v}{\lambda} = \frac{340}{0.34} = 1000Hz$

$T = \frac{1}{f} = \frac{1}{1000} = 1 \text{ x} 10^{-3}s$

If the waves are separated by 0.17, then $\phi = \frac{2\pi\ x\ 0.17}{0.17} = 2\pi$:

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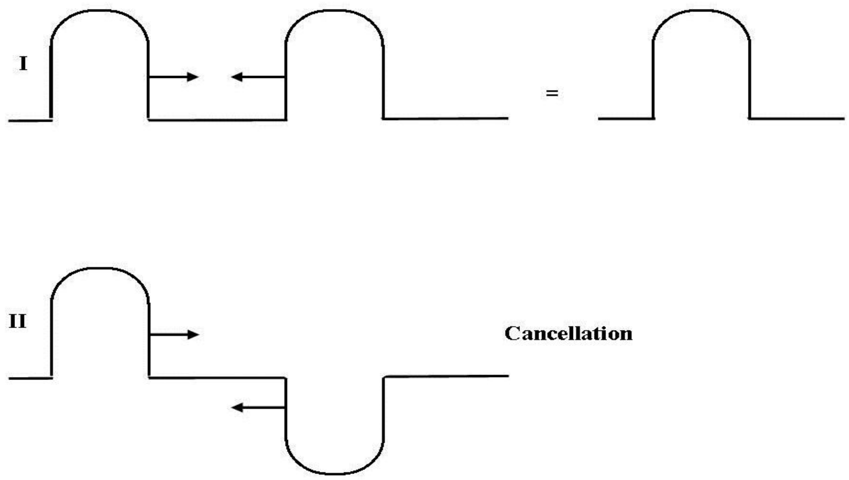
9.2: Principle of Superposition.

When two ends of a string are jerked as shown below, then the waves interfere.



The above observation shows that in I the waves interfere constructively and in II, they interfere destructively. The observation can be explained by applying the principle of superposition which states that, '*whenever two waves travelling in the same medium, the total displacement at any point is the sum of the separate displacements due to the waves at that point.'*

Consider two waves y_1 and y_2 travelling in the same direction and phase difference between them is ϕ.

$y_1(x, t) = a \, Sin\,(kx - \omega t)$ and $y_2(x, t) = a\,Sin\,(kx - \omega t + \phi)$

$y^1(x, t) = y_1(x, t) + y_2(x, t) = a\,Sin\,(kx - \omega t) + a\,Sin\,(kx - \omega t + \phi)$

Using $Sin\,\alpha + Sin\,\beta = 2\,Sin\,½\,(\alpha + \beta)\,Cos\,½\,(\alpha - \beta)$ Trigonometric identity.

$y^1(x, t) = a[2\,Sin\,½\,(kx - \omega t) + (kx - \omega t + \phi)]\,Cos\,½[(kx - \omega t + \phi) - (kx - \omega t)]$

$= a[2\,Sin\,½\,(2\,kx - 2\omega t)\,Cos\,½\phi] = 2a\,Cos\,½\phi\,Sin\,(kx - \omega t + ½\phi)$

The amplitude $= 2a\,Cos\,½\phi$ and phase difference $= ½\phi$ if $\phi = 0$, then the amplitude doubles.

Task: Consider two waves $y_1(x, t) = a\,Sin\,(kx + \omega t)$ and $y_2(x, t) = a\,Sin\,(kx - \omega t)$ travelling in opposite direction, find $y^1(x, t) = y_1(x, t) + y_2(x, t)$

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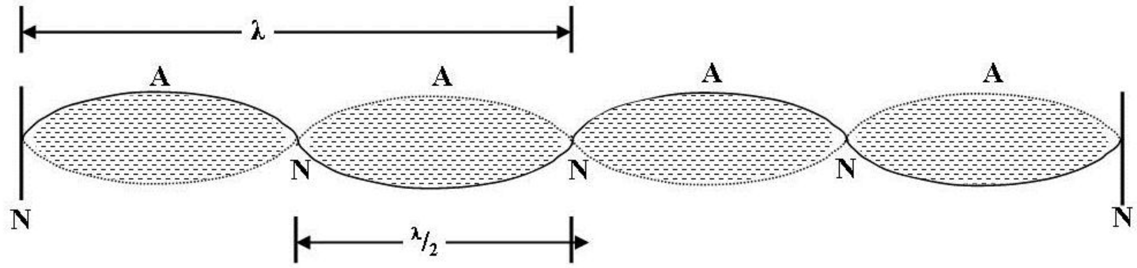
Stationary/Standing Waves.

A stationary (standing) wave is formed when two equal progressive (travelling) waves travelling in opposite directions are superposed on each other.



- Stationary waves have nodes at points of zero displacement and antinodes at points of maximum displacement.
- In a stationary wave, vibrations of particles at points between successive nodes are in phase.
- Between successive nodes, particles have different amplitudes of vibrations.
- The distance between successive nodes or antinodes is $\frac{\lambda}{2}$. The distance between a node and the next antinodes is $\frac{\lambda}{4}$.
  <u>Task:</u> Give differences between a stationary and progressive wave.

9.3: Standing Waves on Strings.

Musical instruments like guitar produce their sound as a result of stretched strings being made to vibrate by plucking them. A transverse wave of a given frequency travels along the string and is reflected back by a fixed end causing the incident and reflected waves to interfere and form a stationary wave. A vibrating string exhibits different stationary waves (modes of vibrations) depending on where it has been plucked. The frequency of the transverse wave depends on the tension (T) and mass per unit length (μ) of the vibrating string. It can be shown that the velocity of the transverse wave along the string is given by:

$v = \sqrt{\frac{T}{m/l}}$ for $\mu = {}^{m}/_{l}$

$v = \sqrt{\frac{T}{\mu}}$

Fundamental Frequency (f_0).

This is the lowest frequency that can be obtained when a musical instruments is played. A stationary wave in its simplest form possible, produces a fundamental note which gives sound its basic pitch.

Overtone: It is a fundamental note accompanied by other notes smaller in amplitude but of higher frequencies than the fundamental frequency, these notes are called overtones.

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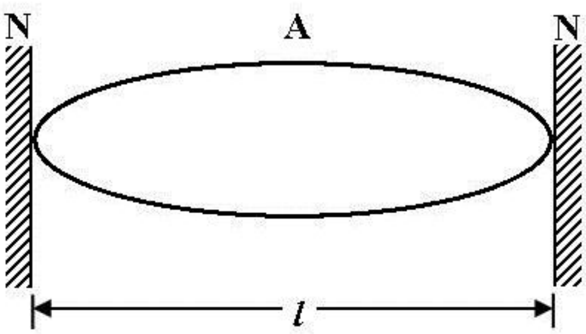
Harmonics: This is the name given to a note whose frequency is a whole number multiple of the fundamental frequency. Frequencies f_0, $2f_0$, $3f_0$ and $4f_0$ are the first, second, third and fourth harmonics respectively.

Modes of Vibration.

Fundamental Frequency (First Harmonics)

The string is plucked in the middle. This produces the simplest possible stationary wave as shown below.



$l = \frac{\lambda}{2}$, $\lambda = 2l$

$f = \frac{v}{\lambda} = \frac{v}{2l}$ for $v = \sqrt{\frac{T}{\mu}}$

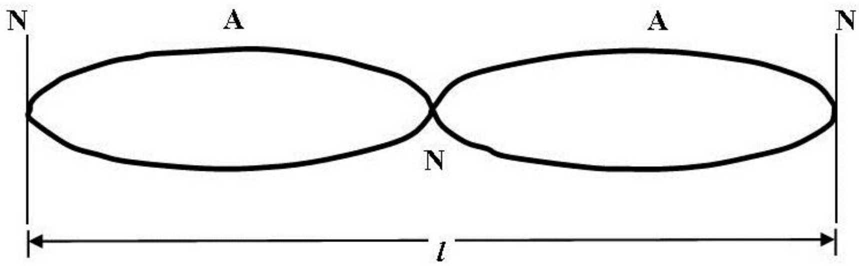
$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

This is the frequency of the fundamental or lowest note obtained from a string.

Therefore $f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

First Overtone (Second Harmonic)

This is obtained by holding the midpoint of the vibrating string and plucking the string at a point a quarter of its length from one end.



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If λ_1 is the wavelength and f_1 the frequency, then

$l = \lambda_1; f_1 = \frac{v}{\lambda_1} = \frac{v}{l}$ …………………………………………….……. (1)

But $f_0 = \frac{v}{2l}$ ……………………………………………………….. (2)

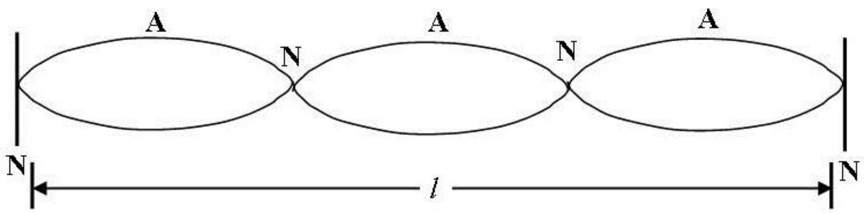
Dividing equation (2) by (1)

$\frac{f_0}{f_1} = \frac{(V/2l)}{(V/l)} = \frac{1}{2}$

$f_1 = 2f_0 = \frac{1}{l}\sqrt{\frac{T}{\mu}}$

Second Overtone (Third Harmonic)

It is obtained by plucking the string in the middle while touching the string one-third from end.



Let wavelength be λ_2 and frequency f_2

$f_2 = \frac{v}{\lambda_2}$, $l = \frac{2}{3}\lambda_2$, $\lambda_2 = \frac{2}{3}l$

$f_2 = \frac{3v}{2l}$…………………………………………………………………. *(1)*

$f_0 = \frac{v}{2l}$………………………………………………………………. *(2)*

Dividing equation (2) by (1).

$\frac{f_0}{f_2} = \frac{(V/2l)}{(3V/2l)} = \frac{1}{3}$

Therefore $f_2 = 3f_0 = \frac{3}{2l}\sqrt{\frac{T}{\mu}}$

It therefore follows that, n^{th} overtone,

$f_n = (n + 1) f_0$

Note: waves in vibrating strings give both odd and even harmonics.

Examples.

1) The length of a stretched string is *0.4m*. Its mass is *1* x 10^{-4}kg. If the tension in the string is *10N*, calculate:
   a) The velocity of the transverse wave in the string.
   b) The fundamental frequency.

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c) The frequency of the first overtone.

Solution

a) $v = \sqrt{\frac{T}{\mu}}$, $\mu = \frac{m}{l} = \frac{1 \text{ x } 10^{-4}\text{Kg}}{0.4m}$ $= 0.00025kg/m^3$

$v = \sqrt{\frac{10N}{0.00025kg/m^3}} = 200m/s$

b) $f_0 = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{200}{2 \text{ x } 0.4} = 250\text{Hz}$

c) $f_1 = 2f_0 = 2 \text{ x } 250\text{Hz} = 500\text{Hz}$

2) A wire of uniform cross-section area has a tension of *20N* and produces a note of *100Hz* when plucked in the middle. Find its diameter if it is *1m* long and has a density of *6000*kg/m^3. State the frequency of the third overtone and the wavelength of the third harmonic.

Solution.

This is the first harmonic

$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$, $v = \sqrt{\frac{T}{\mu}}$,

$v = 2lf_0 = 2 \text{ x } 1 \text{ x } 100 = 200\text{m/s}$

$v^2 = \frac{T}{\mu}$, $\mu = \frac{20}{200 \text{ x } 200} = 0.0005\text{Kg/m}$

$\mu = \frac{m}{l}$, $m = \mu l = 0.0005 \text{ x } 1 = 0.0005Kg$

$V = \frac{m}{\rho} = \frac{5 \text{ x } 10^{-4}}{6000} = 8.33 \text{ x } 10^{-8}m^3$

$V = \pi\left(\frac{d}{2}\right)^2 l = 8.33 \text{ x } 10^{-8}m^3$

$V = \pi\left(\frac{d}{2}\right)^2 \text{ x} 1 = 8.33 \text{ x } 10^{-8}m^3$

$d = \left(\sqrt{\frac{8.33 \text{ x } 10^{-8}m^3}{\pi}}\right) \text{ x } 2 = 3.26 \text{ x } 10^{-4}\text{m}$

Third overtone is fourth harmonic.

$f_3 = 4f_0 = 100 \text{ x } 4 = 400Hz$

Third harmonic

$f_1 = 3f_0 = 3 \text{ x } 100 = 300Hz$

$v = f_1\lambda_1$, $\lambda_1 = \frac{200}{300} = 0.67m$

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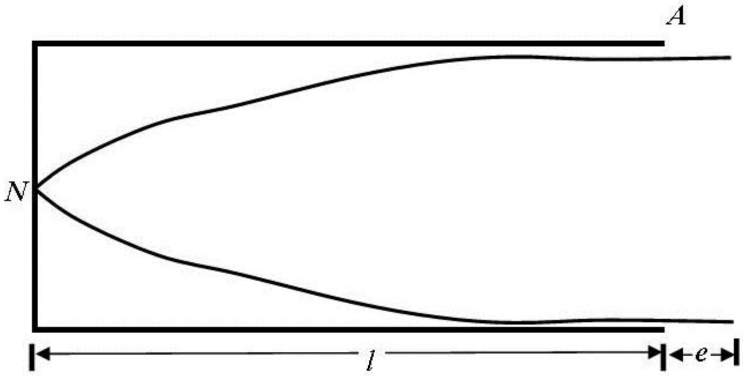
Waves in Pipes (Vibrating Air Collumns)

Closed Pipes.

When air is blown in the pipe trough the open end, the vibration produces a longitudinal wave which travels along the pipe and undergoes reflection at the other end. The reflected wave then interferes with the incidental wave to form a stationary wave.

Fundamental Frequency (First Harmonic)

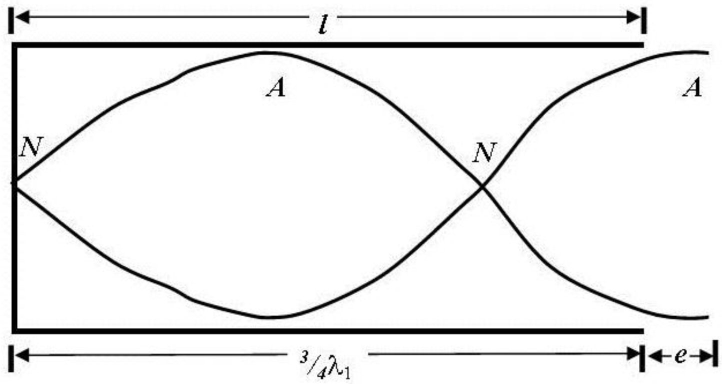


Suppose frequency is f_0, wavelength is λ_0 and e is end correction since the antinode may not be formed exactly at the end of the pipe.

$(l + e) = \left(\frac{1}{4}\right)\lambda_0 \Rightarrow \lambda_0 = 4(l +)e$ e – correction

But $f_0 = \frac{V}{\lambda_0} = \frac{V}{4\,(l+e)}$

First Overtone (Third Harmonic).



$(l + e) = \left(\frac{3}{4}\right)\lambda_1$, $\lambda_1 = \frac{V}{f_1}$

$(l + e) = \left(\frac{3V}{4f_1}\right)$, $\Rightarrow f_1 = \frac{3V}{4(l+e)}$

But $f_0 = \frac{V}{4(l+e)}$, Then $f_1 = 3f_0$, The frequency of the first overtone is thrice the frequency of the fundamental. It is the third harmonic.

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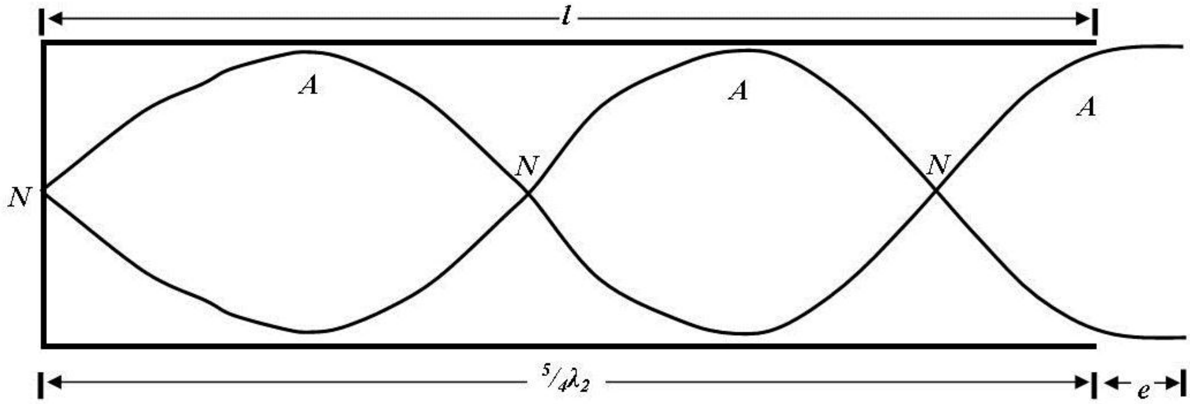
Second Overtone (Fifth Harmonic).



$(l + e) = {}^5/_4\lambda_2$, $\lambda_2 = \frac{4\,(l+e)}{5}$

$\lambda_2 = \frac{V}{f_2}$

$f_2 = \frac{5V}{4\,(l+e)}$, from $f_0 = \frac{V}{4(l+e)}$, then

$f_2 = 5f_0$

The frequency of the second overton is five times the fundamental frequency. It is the fifth harmonic.

It can shown that:

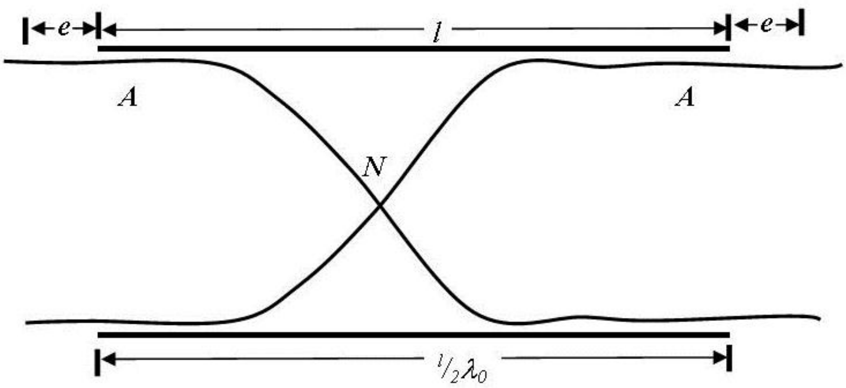
3^{rd} overtone, $f_3 = 7f_0$
4^{th} overtone, $f_4 = 9f_0$
5^{th} overtone, $f_5 = 11f_0$
n^{th} overtone, $f_n = (2n + 1)f_0$

Note: A closed pipe produces only odd Harmonics.

Open Pipe

When air is blown, a wave travels from one end through the pipe and on meeting therefore air at the other end is reflected back.

Fundamental Frequency (First Harmonic).



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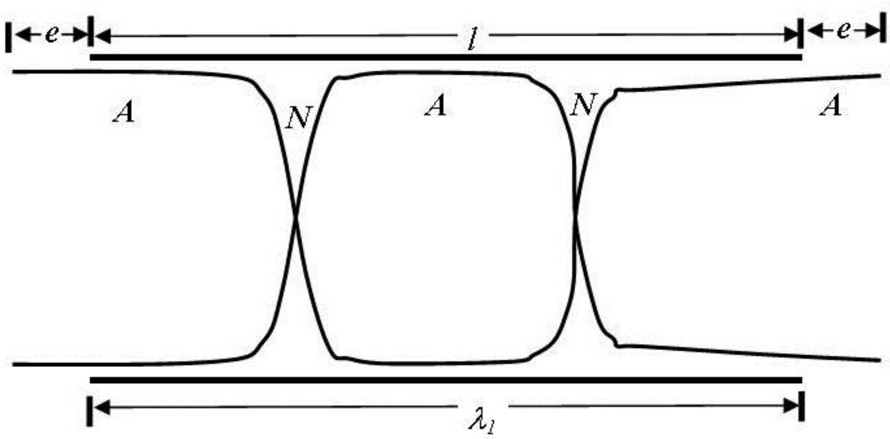
$(l + 2e) = {}^1/_2\lambda_0$, $\lambda_0 = 2(l + 2e)$, But = $\lambda_0 = \frac{V}{f_0}$

$f_0 = \frac{V}{\lambda_0} = \frac{V}{2\,(l + 2e)}$

$f_0 = \frac{V}{2\,(l+2e)}$

First Overtone (second Harmonic)



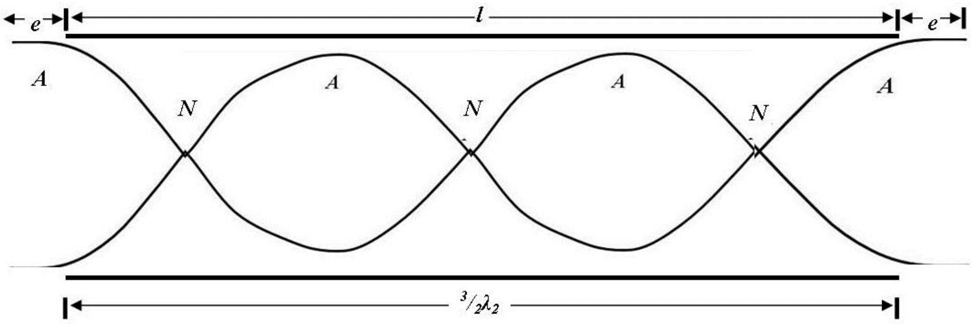
$(l + 2e) = \lambda_1$, $\lambda_1 = \frac{.}{f_1}$

$f_1 = \frac{V}{\lambda_1} = \frac{V}{(l + 2e)}$ From $f_0 = \frac{V}{2(l + 2e)}$

Therefore, $f_1 = 2f_0$

The frequency of the first overtone is twice the fundamental frequency. It is the second harmonic.

Second Overtone (Third Harmonic).



$(l + 2e) = \frac{3}{2}\,\lambda_2$, $\lambda_2 = \frac{V}{f_2}$; $f_2 = \frac{V}{\lambda_2}$

$\lambda_2 = \frac{2\,(l + 2e)}{3}$

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$f_2 = \frac{3V}{2\,(l + 2e)}$ From $f_0 = \frac{V}{2\,(l + 2e)}$ then

$F = f_2 = 3f_0$

It can be shown that in a similar way that overtones of an open pipe are as follows:
3rd overtone, $f_3 = 4f_0$
4th overtone, $f_4 = 5f_0$
5th overtone, $f_5 = 6f_0 \ldots$
nth overtone, $f_n = (n + 1)f_0$

Note: An open pipe produces both even and odd harmonics.

Task: Sound from a stringed instrument and an open pipe is of high quality than from a closed pipe. Explain.

Example.

A closed pipe has a length of 30cm. if the speed of sound is 330 ms^{-1} and the end correction is 1cm, calculate:

a) The fundamental frequency.
b) The wavelength of the second overtone.
c) The next two frequencies after the fundamental frequency.

Solution.

a) For a closed pipe,

$f_0 = \frac{V}{4(l + e)} = \frac{330}{4\,(0.3+0.01)} = 226.1\ Hz$

b) 2nd Overtone, $f_2 = 5f_0 = 5 \times 266.1 = 1331\ Hz$

$\lambda_2 = \frac{V}{f_2} = \frac{330}{1331} = 0.248m$

c) $f_1 = 3f_0 = 3 \times 266.1 = 798.3\ Hz$

$f_2 = 5f_0 = 5 \times 266.1 = 1331\ Hz$

9.5: Speed of Sound in Matter.

Sound is a mechanical wave which needs a medium for propagation. Sound cannot travel through vacuum. Sound travels more rapidly in liquids than in gases and more rapidly in solid metals than in liquids.

If the medium is a fluid, i.e a liquid or a gas, the most important inertial property is the density of the undisturbed fluid, ρ_0, for this is an inertial property independent of volume and it is measured by its bulk modulus, B given by

$B = \frac{-P}{(\Delta V/\,V_0)}$, where P is pressure.

i.e. $P = \frac{F}{A}$, The negative sign arises because an increase in pressure produces a decrease in volume.

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V new volume while V_0 is original volume. Bulk modulus is a measure of how difficult it is to compress materials.

For speed of sound in a fluid:
$v = \left(\frac{B}{\rho_0}\right)^{½}$, speed of sound in solids, we replace the Bulk modulus by Young's modulus, γ
$V = \left(\frac{\gamma}{\rho_0}\right)^{½}$,

The speed of longitudinal waves in fluids depends directly on square root of Bulk modulus and inversely on the square root of equilibrium density of the fluid.

The velocity of sound varies little with temperature for liquids or solids. In gases however, the dependence is considerate. It can be shown that the speed of sound v_1 at a temeperature T_1 is related to speed of sound v_2 at a temperature T_2 by the equation.

$$\frac{v_1}{v_2} = \left(\frac{T_1}{T_2}\right)^{½}$$

The speed of sound in a gas is proportional to the square root of the absolute (Kelvin) temperature.

$$\frac{V_1}{V_2} = \left(\frac{T_1}{T_2}\right)^{½} = \left(\frac{\mathrm{B}/\rho_{01}}{\mathrm{B}/\rho_{02}}\right)^{½}$$

Example.
The density of hydrogen iodide gas is 5.79 kgm^{-3} at STP. What is the speed of sound in hydrogen iodide at 50^0C. assume that the speed of sound in air is 331 m/s and Bulk Modulus for ideal gases is the same (density of air 1.29 kg/m^3).

Solution.
$v_{HI} = \left(\frac{\rho_{0\,\mathrm{air}}}{\rho_{0\,\mathrm{HI}}}\right)^{½} v_{air} = \left(\frac{1.29\ \mathrm{kg}/m^3}{5.79\ \mathrm{kg}/m^3}\right) = 331$ m/s

$v_{HI} = 156\ m/s$

$$\frac{v_{50}}{v_0} = \left(\frac{323}{273}\right)^{½}$$

$v_{50} = \left(\frac{323}{273}\right)^{½}$ x 156 = 170 m/s

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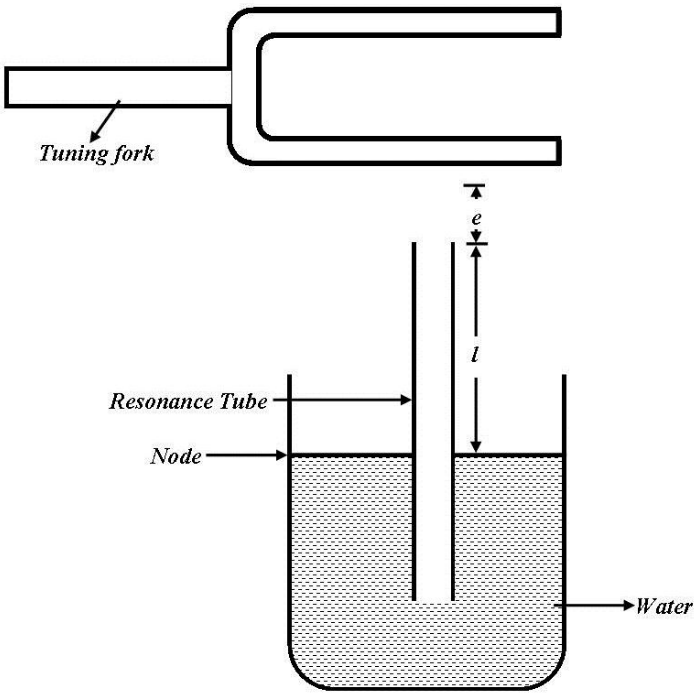
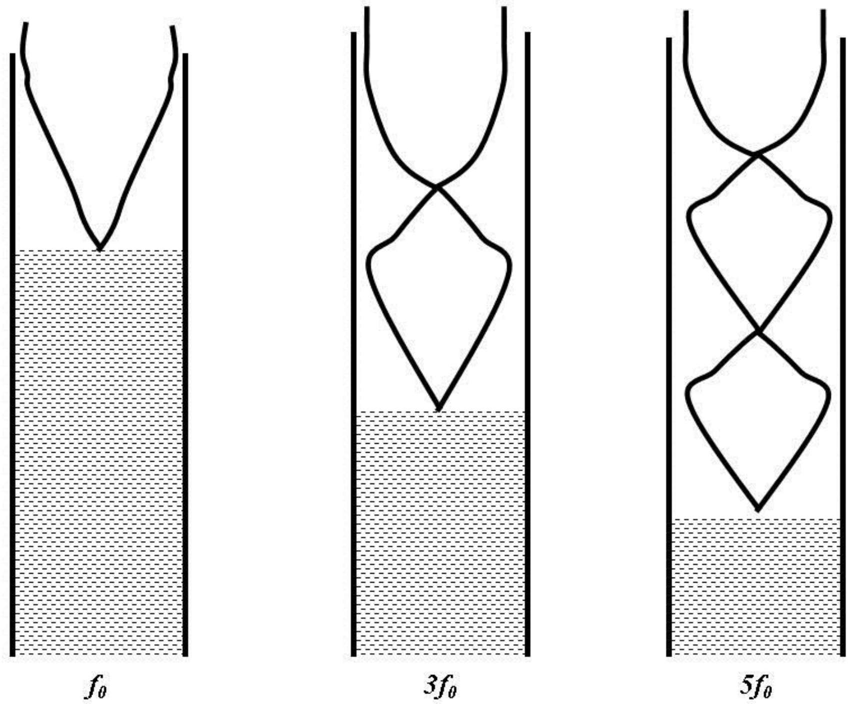
Determination of velocity of sound using resonance tube and tuning forks of known frequencies.



Strike a tuning fork on a hard surface and let it vibrate near the open end of the resonance tube in which the level of water is at its mouth. Raise the resonance tube out of the water to increase the length l until the loudest note is heard. Different readings of l are further obtained when the resonance tube is raised further as shown below:



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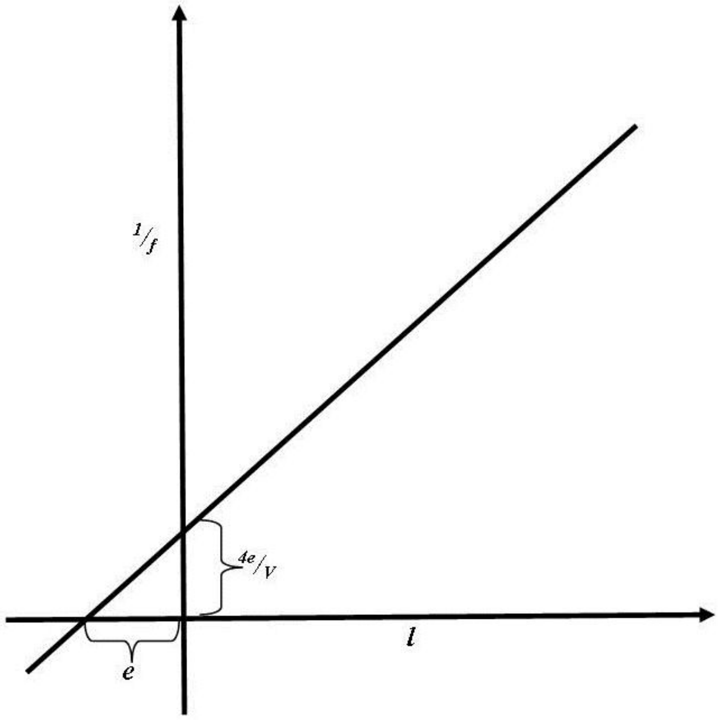
If a graph of $^1/_f$ against l is plotted, then:



From $l + e = \frac{\lambda}{4}$, $l + e = \frac{V}{4f}$

$\frac{1}{f} = \frac{4l}{V} + \frac{4e}{V}$

Slope = $\frac{4}{V}$

Example.

When sounding a tuning fork of frequency 528 Hz is placed over a column of air dipped in water, resonance is obtained at length of 14.8cm and 45.0cm respectively of the air column. Find:

a) Velocity of the sound in air.
b) The end connection of the pipe.

Solution.

a) From $l + e = \frac{\lambda}{4}$, and $l + e = \frac{3}{4}\lambda$ then

$0.148 + e = \frac{\lambda}{4}$……………………………………………………(1)

$0.45 + e = \frac{3}{4}\lambda$ …………………………………………… (2)

Subtracting (1) from (2)

$0.302 = \frac{\lambda}{2}$, $\lambda = 0.604$m

$v = f\lambda = 0.604 \times 528 = 319\ ms^{-1}$

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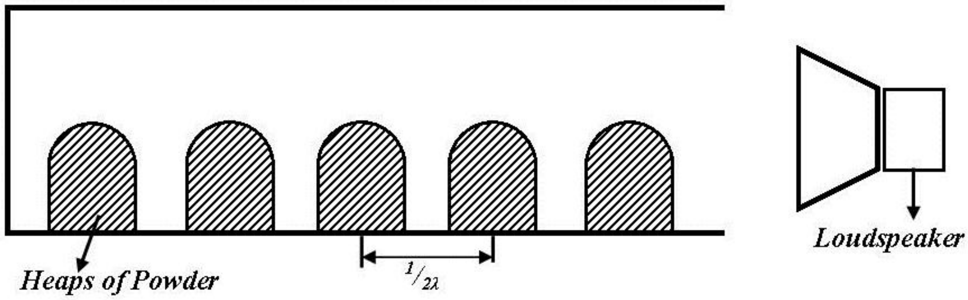
b) Using (1)

$e = (\frac{\lambda}{4} - 0.148) = (\frac{0.604}{4} - 0.148) = 0.003\text{m}.$

Estimating Speed of Sound Using Kundts' Apparatus



When powder is placed inside the tube and a loud speaker of known frequency sounded near its open end for some time, the powder forms into heaps at regular positions along the tube. The heaps are arranged due to constructive interference between the incoming sound wave from the speaker and the reflected waves. The distance between two successive heaps is

$$x = {}^1/_2\lambda.$$

Example

Using Kundts' apparatus, an oscillator is set at 3KHz. The dust settles in the tube such that the distance between the adjacent heaps is 5.75 cm. Calculate:

a) Speed of sound in air.
b) The new distance between heaps of dust when frequency is set at 5KHz.

Solution.

a) $x = {}^1/_2\lambda$

$\lambda = 2x$

$\lambda = 2 \times 5.75 \times 10^{-2} = 0.115\text{m}$

$V = f\lambda = 3000 \times 0.115 = 345\text{ms}^{-1}$

b) $\lambda = \frac{V}{f} = \frac{345}{5 \times 10^3} = 0.069\text{m}$

$X = {}^1/_2\lambda = {}^1/_2 \times 0.069 = 0.0345\text{m}.$

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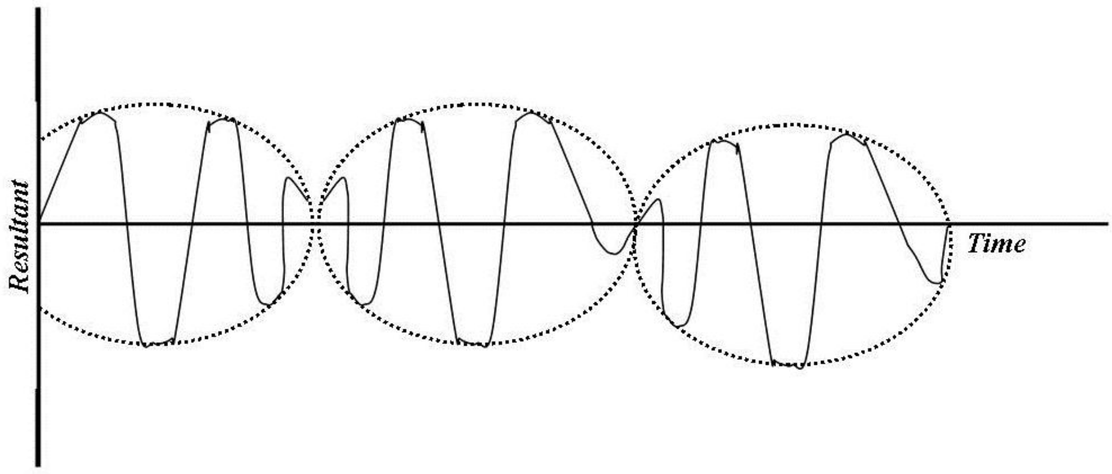
9.6: Beats.

If two notes of nearly equal frequency are sounded together, a periodic rise and fall in loudness can be heard. This phenomenon is known as *Beats*.

When the two waves travel in the same direction they interfere and produce a resultant wave shown below.



Beat Frequency

It is the number of times the sound reaches maximum intensity per second. If the frequencies of the two sources are f_1 and f_2 then in time t the following equations hold if $f_1 > f_2$.

$f_1 t - f_2 t = 1$... (1)

$f_1 - f_2 = 1/t$... (2)

it means one beat has been made in t seconds so that $1/t$ is the number of beats per second or beat frequency (f).

$f_1 - f_2 = f$... (3)

Therefore $f = 1/t$.

Note: The phenomenon of beats can be used to measure unknown frequency of a note using equation (3).

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Examples.

1) Two tuning forks *x* and *y* produce three beats per second when sounding. If *x* has a frequency of 428Hz. What is the possible frequencies of *y?*

Solution.
$f_y = f_x - f$ or $f_y = f_x + f$.
$f_y = 428 + 3$ or $428 - 3 = 431Hz$ or 425Hz

2) Two frequencies of 260Hz and 256Hz are sounded together:
   a) How many maxima are heard per second
   b) Find beat period

Solution.
a) $f = 260 - 256 = 4Hz$ 4 maxima are heard per second.
b) Beat period, $T = 1/f = ¼ = 0.25s$.

9.7: Intensity of Sound

Intensity is defined as the amount of energy transported per unit second per unit area normal to the direction of wave propagation.

$$I = \frac{E/t}{A} = \frac{E}{t} = P$$

$I = \frac{P}{A}$. Dimensions of intensity are watts/area (W/m^2).

The power is distributed uniformly per unit spherical area, i.e.

$I = \frac{P}{4\pi r^2}$, i.e $I = \propto \frac{I}{r^2}$

Sound waves spreading out from a small source are weakened according to inverse square law.

Note: *If on the other hand, the energy is radiated uniformly into a hemisphere, e.g, a sound source on the ground, the power will be distributed over a hemispherical area* $2\pi r^2$ *and te intensity will be given by:*

$I = \frac{P}{2\pi r^2}$

Because of enormous range of intensities to which the ear is sensitive, and because the psychological sensation of loudness varies with intensity not directly but more nearly logarithmically, a logarithmic scale is used to describe the intensity level of sound wave. The intensity level β measured in decibels (dB) is dfined by $\beta = 10 \log \frac{I}{I_0}$, where I is the intensity level corresponding to the level β and I_0 is inference level which we take to be threshold of hearing $I_0 = 1 \times 10^{-12}$W/m^2

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Examples.

1) A dog barking delivers about 1mW of power. If this power is uniformly distributed over a hemispherical area, what is sound level intensity at a distance of 5m.
(Take $I_0 = 1 \times 10^{-12}$ W/m^2).

Solution.

$I = \frac{P}{A}$, $A = 2\pi r^2 = 2\pi(5m)^2 = 157m^2$

$I = \frac{1 \times 10^{-3}\, w}{157}$, $= 6.37 \times 10^{-6}$ w/m^2

$\beta = 10 \log \frac{I}{I_0} = 10 \log \frac{6.37 \times 10^{-6}}{1 \times 10^{-12}}$

$= 10 (\log 6.37 + \log 10^6)$

$= 10 (0.8 + 6) = 68$dB

2) At a cocktail party, 38 people were speaking equally loudly. If only one person was talking, the sound level would be 72 decibels. Find the sound level when all 38 people were talking. (Take $I_0 = 1 \times 10^{-12}$ W/m^2).

Solution.

$\beta = 10 \log \frac{I}{I_0}$, $72 = 10 \log \frac{I}{1 \times 10^{-12}}$

$72 = 10 \log I + 10 \log (1 \times 10^{-12}) = 10 \text{ Log } I + 120, - 4.8 = \text{ Log } I.$

Taking anti-logs on both sides, you get $I = 1.58 \times 10^{-5}$. Intensity for 38 people will be; $38 \times 1.58 \times 10^{-5} = 6.023 \times 10^{-4}$.

$\beta = 10 \log \frac{I}{I_0} = 10 \log \frac{6.023 \times 10^{-4}}{1 \times 10^{-12}} = 87.8$ dB.

9.8: Doppler's Effect

When a vehicle travelling at high speed and sounding its horn passes a pedestrian, the pitch as heard by the pedestrian drops sharply as the vehicle passes. When the source of the sound is moving towards an observer or observer towards the source the pitch of sound heard is higher than actual pitch. However, when the sound source moves away from the observer or the observer moves away from the source the pitch is lowered.

When there is a relative motion between the source of sound and the observer, the pitch of the sound heard is not the same as that when both the observer and the source of sound are stationary. If the pitch of the sound heard by the

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observer is higher than the source, this would indicate that the distance between the observer and the source is decreasing. Similarly, if the pitch of the sound heard by the observer is lower than that of the source, it would indicate that the distance between the observer and the source is increasing.

"The apparent change in pitch produced by the relative motion between the source and the observer is called Doppler effect".

The frequency of the sound emitted by the source remains unchanged as does that vocally of sound in the transmitting medium.

Case I: ***Source moving towards a stationary observer.***

Suppose a stationary source emits sound waves of frequency f and velocity v, then f waves are produced per second. If the source now moves towards a stationary observe with velocity v_s, then the distance covered by sound will be the product of velocity and time.

$(v - v_s) \times 1 = v - v_s$

Wavelength of waves as observed by observer is given by:

$$\lambda^1 = \frac{v - v_s}{f}$$

Apparent frequency, $f^1 = \frac{Velocity\ of\ sound}{Apparent\ wavelength}$

$$f^1 = \frac{v}{\lambda^1} = \frac{v}{v - v_s / f} = f\frac{v}{v - v_s}$$

Since $v > (v - v_s)$ then the apparent frequency f^1 is higher than the frequency of the source.

Case II: ***Source moving away from a stationary observer with constant velocity, v_s.***

The distance occupied by f waves in a second is $v + v_s$ apparent wavelength λ^1 heard by observer.

$$\lambda^1 = \frac{v + v_s}{f} \qquad \text{for } v = f\lambda$$

$$f^1 = \frac{v}{\lambda^1} = \frac{v}{v + v_s}$$

Case III: ***Source stationery and the observer moving along with constant velocity, v_0.***

The wavelength is unchanged in this case. The relative velocity between the observer and sound $v - v_0$

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$V = f\lambda$

Apparent frequency wave, $f^1 = \frac{Relative\ Velocity}{Wavelength}$

$f^1 = \frac{v - v_0}{\lambda}$ from $\lambda = \frac{v}{f}$

$f^1 = \frac{(v - v_0)f}{v}$

Since $v - v_0 < v$ then $f > f^1$

Case IV: ***Observing Moving towards a stationary source with a constant, v_0.***

The relative velocity between observer and sound source is $v + v_0$

$v = f\lambda$

Apparent frequency, $f^1 = \frac{Relative\ Velocity}{Wavelength}$

$= \frac{v + v_0}{\lambda}$, $\lambda = \frac{v}{f}$, *therefore*

$f^1 = \frac{(v + v_0)f}{v}$

Since $(v + v_0) > v$ then $f^1 > f$

Application of Doppler Effect

1) Radar speed traps: used by traffic police officers to measure speed of vehicles.
2) Measurement of high temperatures, e.g. in nuclear reactors.
3) Used in astronomy to measure speed of rotating sun, starts, earth.

Examples.

1) The frequency of a car horn is 400Hz. What frequency is observed if the car moves towards a stationary receiver at a velocity of 30m/s. (take velocity of sound in air as 340 m/s).

Solution.

$\lambda^1 = \frac{v - v_s}{f} = \frac{340 - 30}{400} = 0.775$ m

$f^1 = f\frac{v}{v - v_s} = \frac{400 \text{ x } 340}{340 - 30} = 438.71$Hz

2) A horn of a stationary car has a frequency of 400Hz, what frequency is observed by an observer moving towards the car at 30m/s (take speed of sound as 340 m/s).

$f^1 = \frac{(v + v_0)}{v} f = \frac{340 + 30}{340}$ x 400

= 435.29 Hz.

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9.9: Ultrasonics/Ultrasound.

The frequency range of the human ear is 20 – 20,000Hz. This range is known as audiofrequaency. Below 20Hz sound is known as infrasonic. Sound with frequency above 20,000Hz is known as ultrasonic. Ultrasonic waves are generated by using quartz crystals as sources. Such crystals are piezoelectric, which means that if an alternating electric voltage is applied to them, they vibrate mechanically and generate sound waves at the same frequency as applied voltage. They have very short wavelength.

Application of Ultrasonic Waves.

1) Echo sounding technique to determine the depth of seas and oceans. Ultrasound is used because it penetrates deepest and it is reflected easily.
2) Detecting flaws and cracks in metal castings.
3) In medicine, it used to study and monitor development of foetus because they are safe compared to X-rays.

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