# SPH 111: FUNDAMENTALS OF PHYSICS II

## SEMESTER II  ACADEMIC YEAR: 2024/25


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# Electrostatics (charges at rest)

- **Electrostatics**: is a branch of physics that deals with the phenomena and properties of **stationary or slow moving** electric charges.

***Conductors and Insulators***

- **Conductors** are materials through which charge can move freely; examples include *metals (such as copper in common lamp wire)*, *the human body, and tap water*.
- **Nonconductors**—also called **insulators**—are materials through which charge cannot move freely; examples include *rubber*, *plastic*, *glass*, *and chemically pure water.*
- **Semiconductors** are materials that are intermediate between conductors and insulators; examples include *silicon and germanium in computer chips*.
- **Superconductors** are materials that are perfect conductors, allowing charge to move without any hindrance.

# Properties of the electric charge

There are two types of electric charge: positive and negative.

- Charge is conserved: the principle of charge conservation states that the algebraic sum of all charges in a closed system is conserved.
- The charge $q$ of the proton is given as $+1.6\times10^{-19}$ C and the charge of the electron is $-1.6\times10^{-19}$ C.
- Charge is quantized: the magnitude is always an integer multiple of the basic unit of charge $e$ ($q = ne$)
- Charge is invariant: the charge of a particle is independent of its speed.
- Two stationary, point charges interact via the electrostatic force which is given by Coulomb’s Law.

# Coulomb's Law (electrical force between charged particles)

- The force between any two, stationary point charges, $q_1$ and $q_2$, is:

proportional to the product of their charges and inversely proportional to the square of the distance between them

- The magnitude of the electrostatic force, F, is $F \propto \frac{|q_1||q_2|}{r^2}$

where $r$ is the distance between $q_1$ and $q_2$ $|q_1|$ is the magnitude of the charge $q_1$). $F = k\frac{|q_1||q_2|}{r^2}$ ($k = \frac{1}{4\pi\varepsilon_o} = 9.0 \times 10^9 Nm^2C^{-2}$, $permittivity\ constant, \varepsilon_o = 8.85 \times 10^{-12} C^2/Nm^2$)

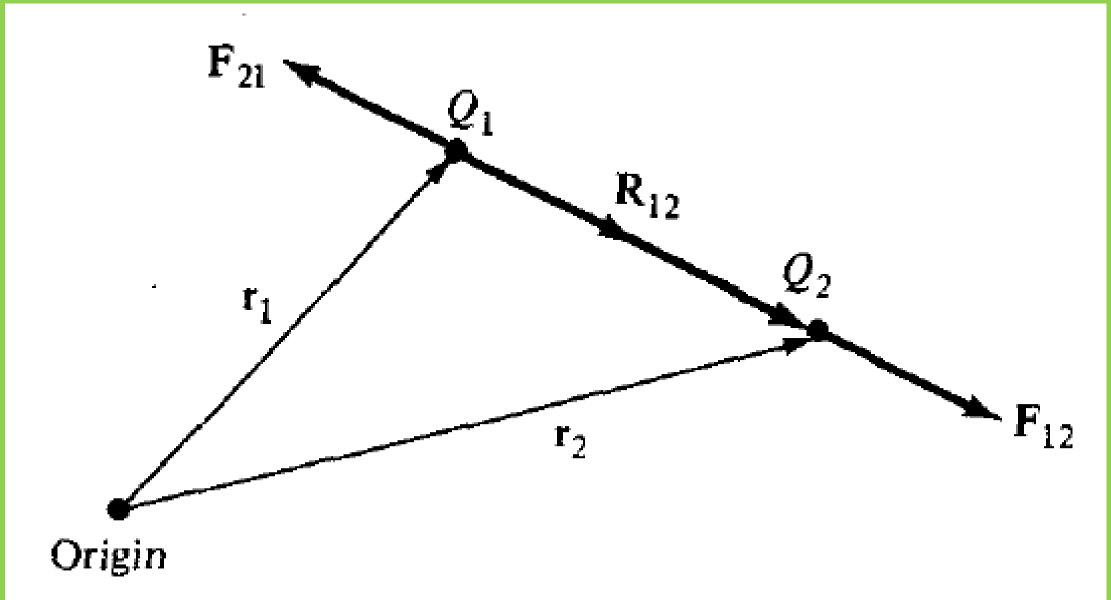
- The electrostatic force is directed along the line joining $q_1$ and $q_2$.
- The force is attractive if the charges are of opposite sign, and repulsive if the charges have the same sign.

# Coulomb's Law cont

If $q_1$ and $q_2$are located at points having position vectors $r_1$ and$r_2$then the Force $F_{12}$ is given by;

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r_{12}^2}\mathbf{a}_{R_{12}}$$

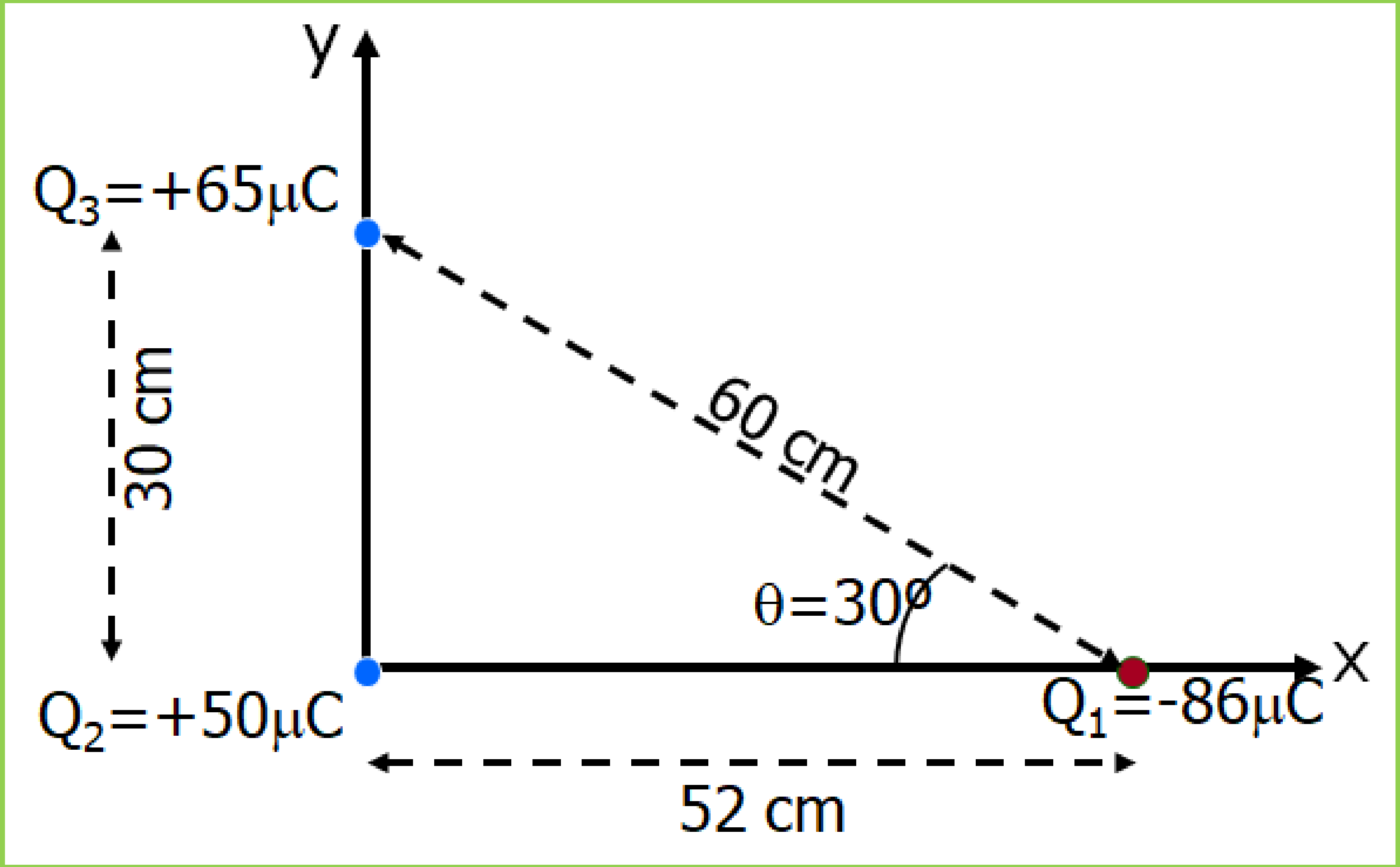
$$\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1$$




The equation is valid for point charges.  If the charged objects are spherical and the charge is uniformly distributed, $r_{12}$ is the distance between the centers of the spheres. *Assignment emtL 3 pg 7*

# Assignment

Calculate the net electrostatic force on charge $Q_3$ due to the charges $Q_1$ and $Q_2$



# Coulomb's Law cont. (Superposition Theorem)

- If there are *n* charged particles, they interact independently in pairs, and the force on any one of them, say particle 1, is given by the vector sum

$$\vec{F}_{1,net} = \vec{F}_{1,2} + \vec{F}_{1,3} + \vec{F}_{1,4} + \vec{F}_{1,5} \ldots \vec{F}_{1,n}$$

- A shell with uniform charge repel or attracts a charge particle that is outside the shell as if all the shell's charge  were concentrated at its centre
- If a charged particle is located inside the a shell of uniform charge there is no net electrostatic force on the particle from the shell.

# **Example 1**(*chap21 & EM01, 2 pdf, Giancoli pg 654*)

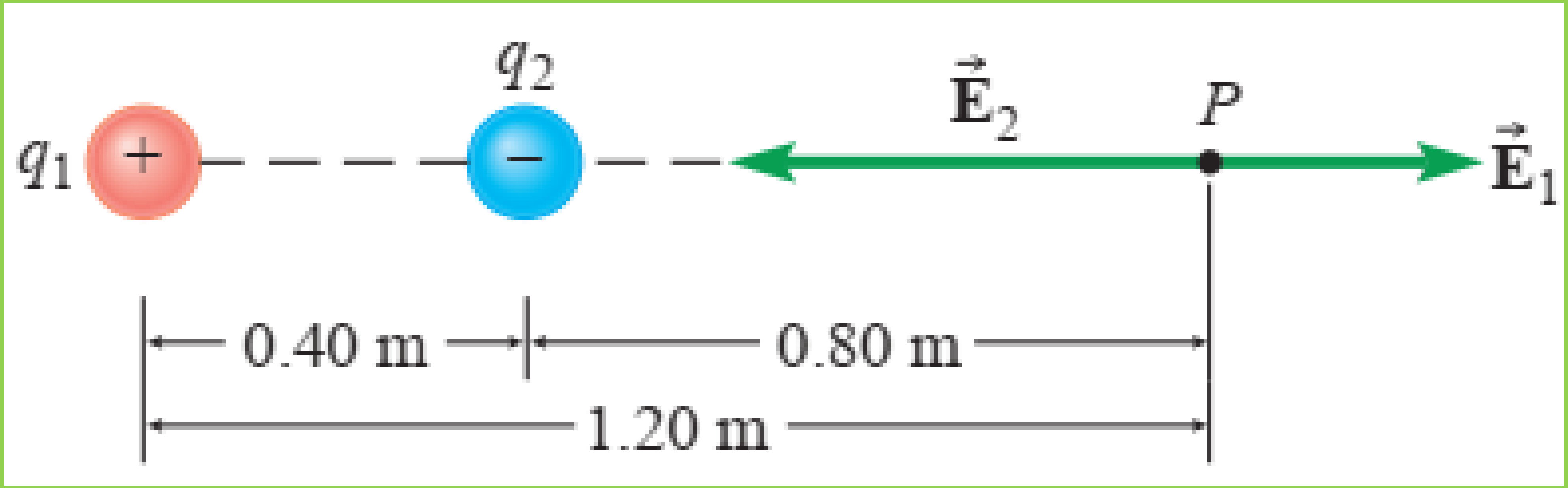
Point charges of 2 μC and -3 μC are at rest 4 cm apart in a vacuum.

Calculate the force on the 2 μC charge.

*Serway pg 713, Halliday pg 641.*

# Example

Two point charges are located on the x-axis. charge $q_1 = +0.6\mu C$ is located at x = 0; charge $q_2 = -0.5\mu C$ is located at $x = 40cm$. Point P is located at x = 120 cm. What is the magnitude and direction of the electric field at point P due to the two charges?



# Solution

$$E_1 = \frac{k|q_1|}{r_1^2} = 8.99 \times 10^9 \frac{N.m^2}{C^2} \times \frac{0.6 \times 10^{-6} C}{(1.20m)^2} = 3.75 \times 10^3 N/C$$

$$E_2 = \frac{k|q_2|}{r_2^2} = 8.99 \times 10^9 \frac{N.m^2}{C^2} \times \frac{0.5 \times 10^{-6} C}{(0.80m)^2} = 7.02 \times 10^3 N/C$$

$E = 7.02 \times 10^3 N/C - 3.75 \times 10^3 N/C = 3.3 \times 10^3 N/C$ *in the x-direction.*

# Electric Fields *Hafez pg 615*

- The electric field ***E*** at a point is defined as the force per unit charge experienced by a stationary charge $q$, which is situated at that point.

$$E = \frac{F}{q}$$

The SI unit for the electric field is $NC^{-1}$

- the charge q is not the charge that creates the field - it is a test charge situated at the point where the electric field is being measured/calculated.
- the charge $q$ must be stationary, since moving charges lead to a magnetic interaction.
- Conceptually, the charge $q$ (sometimes called the test charge $q_o$) is very small to not affect the charge creating the electric field $\boldsymbol{E}$.

# Electric Field cont.

- The electric field is a vector quantity, so it has direction! The direction of the electric field is found using a positive test charge.
- For a positive test charge $q_o$ , the electric field $\boldsymbol{E}$ at a point is given by

$$E = \frac{F}{q_o} = k\frac{q}{r^2} = \frac{q}{4\pi\varepsilon_o r^2}$$

- The electric field, at the point where $q_o$ is situated, points in the same direction as the force $\boldsymbol{F}$.
- If we know the electric field, we can calculate the force on any charge

$$F = qE$$

- The direction of the force depends on the sign of the charge – in the direction of the field for a positive charge, opposite to it for a negative one.

# **Example** (*openstax pg 641, Giancoli pg 654*)

Calculate the strength and direction of the electric field $E$ due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

- We can find the electric field created by a point charge by using the equation $k\frac{q}{r^2}$.

**Solution**

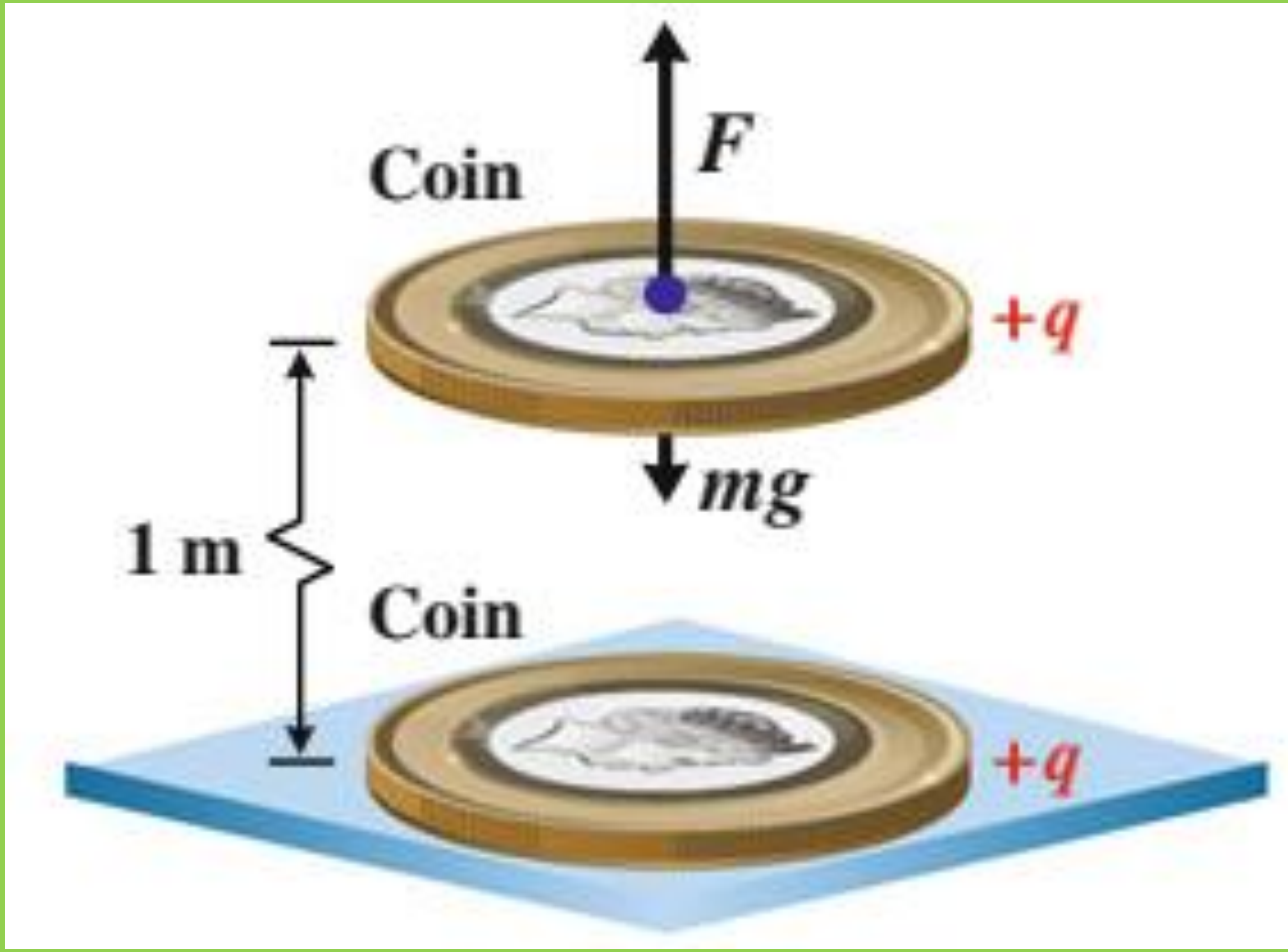
Here $Q =$

Electric field of a single point charge. Calculate the magnitude and direction of the electric field at a point P which is 30 cm to the right of a point charge (*Giancoli pg-657, Example 19.5 OpenStax pg 672*)

# **Example 19.3** ***Hafez pg 625***

Two identical copper coins of mass $m = 2.5g$ contain about $N = 2 \times 10^{22}$ atoms each. A number of electrons $n$ are removed from each coin to acquire a net positive charge $q$. Assume that when we place one of the coins on a table and the second above the first, the second coin stays at rest in air at a distance of $1m$, see Figure . below. (a) Find the value of $q$ that keeps the two coins in that configuration. (b) Find the number of removed electrons $n$ from each coin. (c) Find the fraction of the copper atoms that lost those $n$ electrons in each coin. Assume that each copper atom loses only one electron.

# **Example 19.3** ***Hafez pg 625***



# Example 19.3 *Hafez pg 625*

**Solution:** (a) The upper coin is in equilibrium due to its weight and the electrostatic repulsion between the two charged coins. Therefore:

$mg = k\frac{q\times q}{r^2}$ or $g = \sqrt{\frac{mgr^2}{k}} = \sqrt{\frac{(2.5\times10^{-3}kg)(9.8N/kg)(1m)^2}{9\times10^9 N.m^2/C^2}} = 2.72\times10^{-6}$C

This small charge leads to a measurable force between large bodies.

(b) From the electronic charge $(-e)$ and the total charge $q$ on each coin, we can find the number of removed electrons $n$ as follows: $n = \frac{q}{e} = \frac{2.72\times10^{-6}C}{1.6\times10^{-19}C} = 1.7\times10^{13}$ electrons(Very big number)

(c) The fraction of the copper atoms that loses the $n$ electrons is:

$f = \frac{n}{N} = \frac{1.7\times10^{13}}{2\times10^{22}} = 8.5\times10^{-9}$ (Very small fraction )

# Applications of Coulomb's Law

- Coulomb's law involves the interaction between at least two charged objects, it is often necessary to represent each object.
- Coulomb's law applies to particles and objects that can be modeled as particles, so on hybrid free-body diagrams, represent the objects as dots.
- The distance $r$ that appears in Coulomb's law is the center-to-center distance. When a spherical object is represented as a dot, place the dot at the sphere's center.
- Label information about the charged particles on the diagram.
- Circle the subject. Then use your knowledge of the electrostatic force to draw vectors representing the force exerted by all the other objects on the subject.
- Place the tail of each vector on the subject to indicating the direction.

# Applications of Coulomb's Law cont

- Draw the vector toward a particular object if that object has the opposite sign as the subject (because the force is attractive). Draw the vector away from the object if that object has the same sign as the subject (because the force is repulsive).
- Write the electrostatic force in component form, and then algebraically combine the components taking note about signs.

# Electric Field Lines

- The electric field of a point charge points radially away from a positive charge and towards a negative one.

Electric field lines:

✓Point in the direction of the field vector at every point

✓Start at positive charges or infinity

✓End at negative charges or infinity

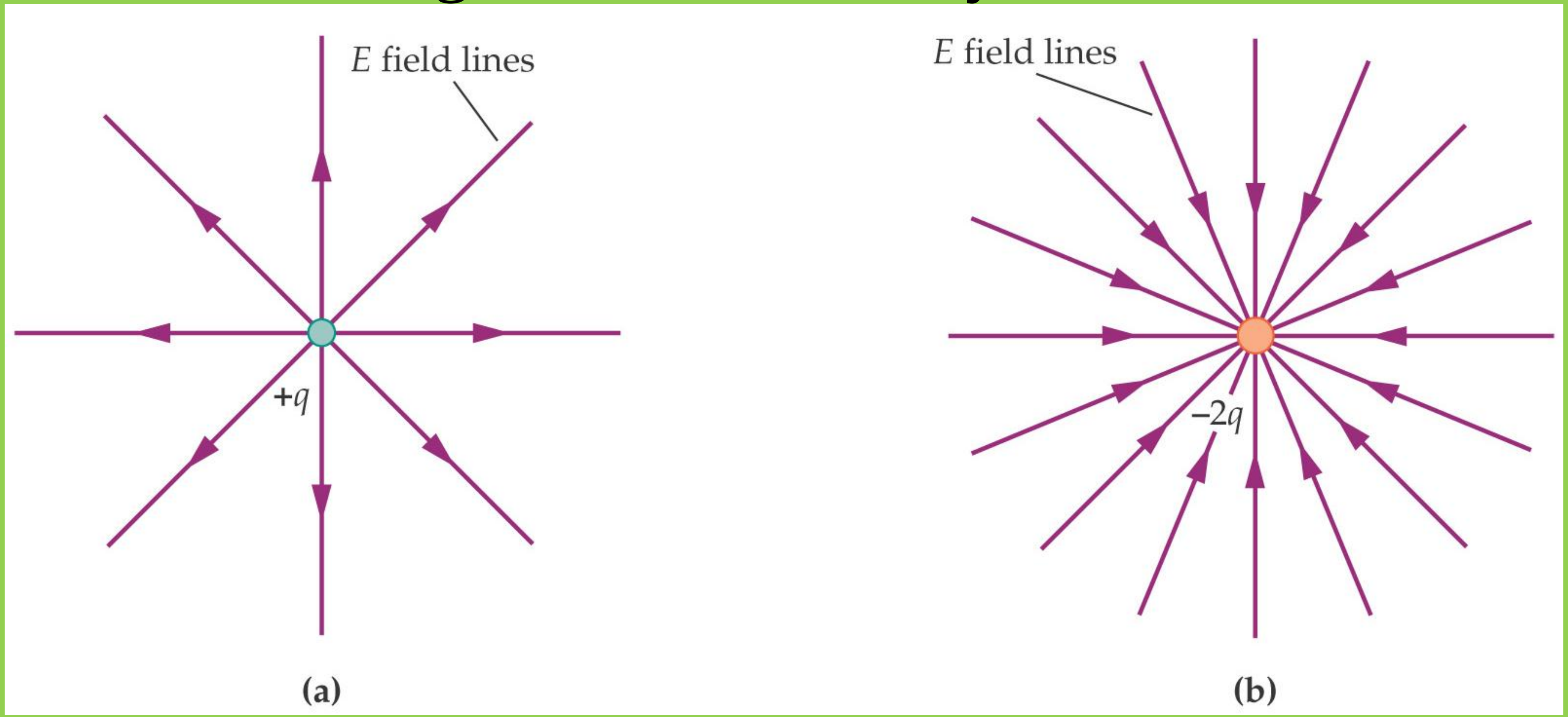
✓Are more dense where the field is stronger

✓Excess charge on a conductor is on the surface

✓ Electric field within a conductor is zero (if charges are static)

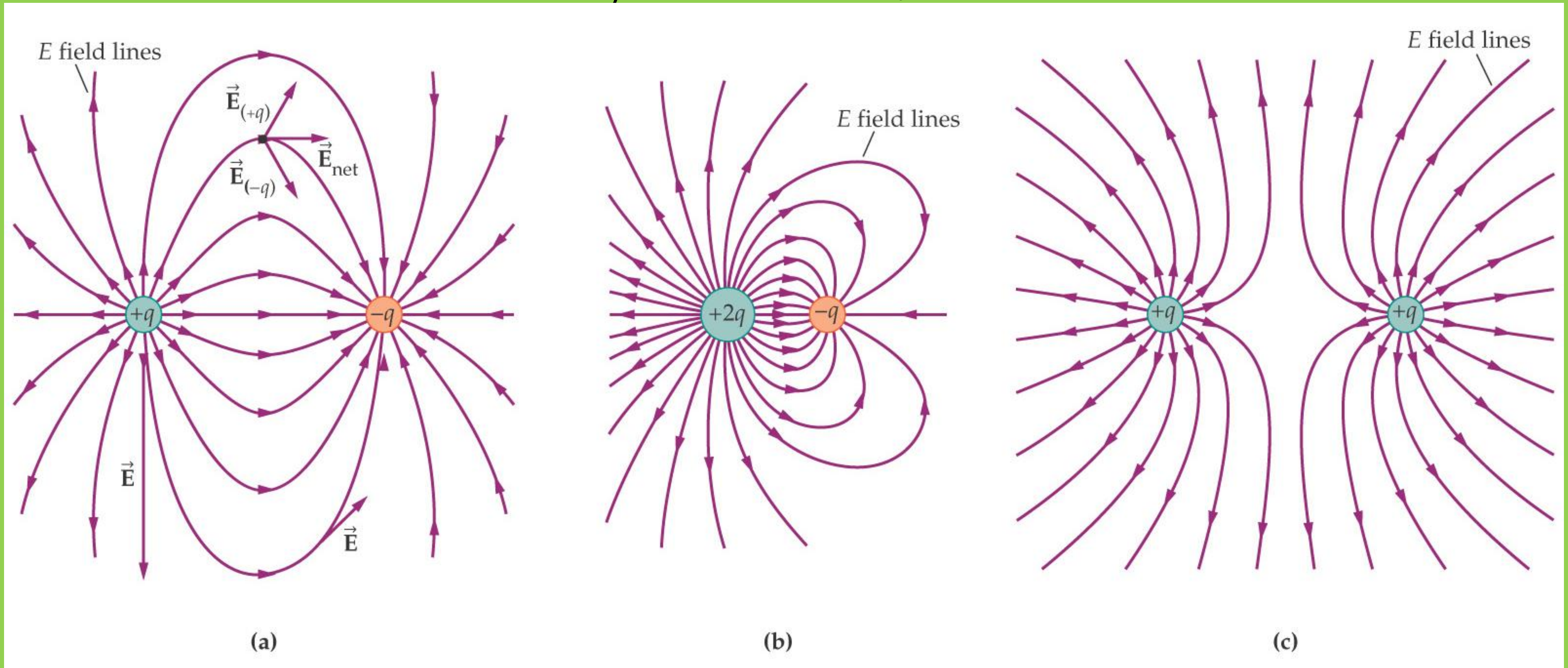
# Electric Field Lines cont.

- The charge on the right is twice the magnitude of the charge on the left (and opposite in sign), so there are twice as many field lines, and they point towards the charge rather than away from it.



# Electric Field Lines cont.

- Combinations of charges. Note that, while the lines are less dense where the field is weaker, the field is not necessarily zero where there are no lines. In fact, there is only one point within the figures below where the field is zero – can you find it?



# Gauss's law

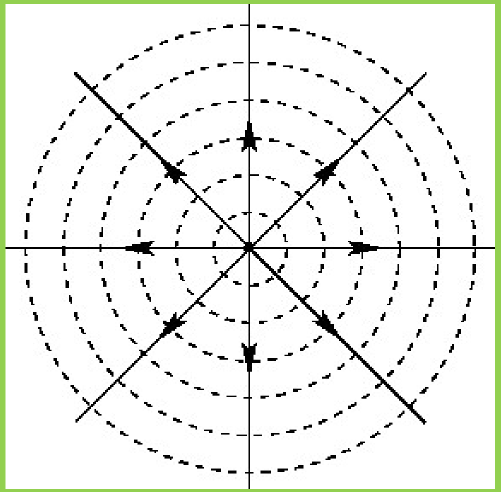
- Electric Flux: measure of the electric field passing through a surface. Imagine the electric field as water flow, and the surface as a net. The flux is how much water passes through the net.
- Gaussian Surface: An imaginary closed surface used to apply Gauss's Law.
- Gauss's law relates the electric flux through a closed surface to the net charge enclosed by that surface. It is mathematically expressed as

$\oint E.dA = \frac{Q_{ecclosed}}{\varepsilon_O}$ where: $\oint$ closed surface $E.dA$ is the **electric flux** through a closed surface, $E$ is the **electric field** at each point on the surface, $dA$ is a small element of the surface area (vector normal to the surface), $Q_{ecclosed}$ is the **net charge enclosed** within the surface,

$\varepsilon_O$ is the **permittivity of free space** $(8.85 \times 10^{-12} \frac{C^2}{N}.m^2)$.

# Gauss's law cont

- Gauss' Law states that the electric flux through any closed surface is proportional to the algebraic sum of all of the charges enclosed in that surface. $\psi \propto Q_{enc} = \frac{1}{\varepsilon_o} Q_{enc}$, where $Q_{enc}$ is the charge enclosed by the surface. Gauss' Law is one of the fundamental laws of electromagnetism.
- Gauss' Law can be used to find the electric field around a charge distribution, for example a point charge.



- The closed surface is taken to be the surface area of a Sphere. By Gauss' Law $\frac{1}{\varepsilon_o} Q_{enc} = E \times 4\pi r^2 \Rightarrow \frac{1}{4\pi\varepsilon_o}\frac{Q_{enc}}{r^2}$
- An inward piercing field is negative flux. An outward Piercing field is positive flux. A skimming field is zero flux.

# Applications of Gauss's Law

- Gauss's law is most useful in cases with **high symmetry** (spherical, cylindrical, and planar charge distributions)
- **Electric Field Due to a Spherical Charge Distribution**: For a point charge $Q$ (or a uniformly charged sphere, outside the sphere)

$$E = k_e \frac{Q}{r^2}, \ (r > R)$$

(Same as Coulomb's law; behaves like a point charge outside.)

- Inside a uniformly charged sphere $(r < R)$. Using Gauss's law, the electric field inside a uniformly charged sphere varies linearly with $r$

$$E = \frac{Q}{4\pi\varepsilon_0 R^3} r$$

# Applications of Gauss's Law cont

- **Electric Field Due to an Infinitely Long Line Charge**: For an infinite line charge with linear charge density $\lambda\ (C/m)$, Gauss's law gives,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where $r$ is the perpendicular distance from the line charge.

**Electric Field Due to an Infinite Plane Sheet of Charge**

For an infinite sheet with surface charge density $\sigma\ \left(\frac{C}{m^2}\right)$, $E = \frac{\sigma}{2\pi\varepsilon_0}$

- The electric field is **constant** and does not depend on distance from the plane.
- Direction: **Away** from a positively charged sheet, **toward** a negatively charged sheet.

# Applications of Gauss's Law cont1

- **Electric Field Between Two Infinite Parallel Plates (Capacitor)** : For two oppositely charged infinite plates with charge densities $+\sigma\ and\ -\sigma:, E = \frac{\sigma}{\varepsilon_0}.$

The field is **uniform** between the plates.

# Advantages of Gauss’s law

- Symmetry: Gauss's Law works best when there's symmetry in the charge distribution, making it easier to choose a Gaussian surface where the electric field is constant.
- Simplification It often simplifies electric field calculations compared to directly using Coulomb's Law, especially for complex charge arrangements.
- Gauss's Law is one of Maxwell's equations, the foundation of classical electromagnetism.
- It applies to any closed surface, but the choice of the Gaussian surface is crucial for making the calculation manageable.

# Magnetic Flux Density ($B$)

- Magnetic flux density, denoted as $B$, represents the strength and direction of a magnetic field at a given point. It is a vector quantity measured in teslas ($T$).
- Magnetic flux density is defined as the amount of magnetic flux ($\Phi$) passing per unit area **perpendicular** to the field:$B = \frac{\Phi}{A}$ where:

$B =$ Magnetic flux density ($T$)

$\Phi =$ Magnetic flux (Wb, Weber)

$A =$ Area through which the flux passes ($m^2$)

# Magnetic Flux Density ($B$) cont

- Magnetic flux $\Phi$ represents the total number of magnetic field lines passing through a given surface: $\Phi = BAcos\theta$ where $\theta$ is the angle between the magnetic field $B$ and the normal to the surface,

- If $B$ is perpendicular to the surface ($\theta = 0^0$), $\Phi = BA$.
- If $B$ is parallel to the surface ($\theta = 90^0$), $\Phi = 0$ (no flux passes through the surface)

**Magnetic flux density and Lorentz Force**

- Magnetic flux density also appears in **Lorentz force**, which is the force a magnetic field exerts on a moving charge:$F = qvB\sin\theta$ where:

$q =$ charge ($C$), $v =$ velocity of the charge ($m/s$), $B =$ magnetic flux density ($T$) and $\theta =$ angle between $v$ and $B$.

# Magnetic Flux Density Around a Straight Wire

- A current-carrying conductor produces a magnetic field around it. The magnetic flux density at a distance $r$ from a long straight wire carrying current $I$ is given by: $B = \frac{\mu_0 I}{2\pi r}$, where, $\mu_0 = 4\pi \times 10^{-7} T.m/A$

(permeability of free space) and $r =$ distance from the wire ($m$).

- Using the **Right-Hand Rule:** Point your thumb in the direction of the current; your curled fingers show the direction of the magnetic field lines.

# Magnetic Flux Density in a Solenoid

- A solenoid is a coil of wire that generates a nearly uniform magnetic field inside. The magnetic flux density inside an **ideal solenoid** is:

$$B = \mu_0 n I$$

$n =$ number of turns per unit length ($turns/m$).,

$I =$ current through the solenoid ($A$).

- If the solenoid contains a **magnetic material** with relative permeability $\mu_r$, then: $B = \mu_0 \mu_r n I$

# Applications of Magnetic Flux Density

- **MRI (Magnetic Resonance Imaging)** Uses strong magnetic fields (up to a few teslas) for medical imaging.
- **Transformers & Motors** Magnetic flux density is crucial in designing electric motors, generators, and transformers.
- **Earth's Magnetic Field** The Earth's magnetic flux density is about 25−65 $\mu T$ (microteslas)

# *Example*

- A solenoid has 500 turns per meter and carries a current of 3A. Calculate the magnetic flux density ($B$) inside the solenoid in:
  (a) Free space ($\mu_r = 1$)
  (b) A medium with a relative permeability $\mu_r = 200$.

*Solution*

The magnetic flux density inside a solenoid is given by $B = \mu_0 \mu_r n I$

(a) $B = (4\pi \times 10^{-7})(1)(500)(3) = 1.88mT$.

(b) $B = (4\pi \times 10^{-7})(200)(500)(3) = 377mT$.

# *Example 2*

- A long, straight wire carries a current of $10A$. Find the magnetic flux density at a distance of 5 cm ($0.05\ m$) from the wire in free space.

*Solution*

The magnetic flux density at a distance $r$ from a long, straight wire is given by $B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7})(10)}{2\pi(0.05)} = 40\mu T$

# Electrostatic Energy

- Electrostatic energy is the potential energy stored in a system of electric charges due to their interactions. It represents the work required to assemble the charge distribution from an infinite separation.
- For point charges The electrostatic energy ($U$) of a system of point charges can be calculated using the formula: $U = \frac{1}{2}\ \Sigma\,(q_i\, V_i)$

where:

$q_i$ is the charge of the $i^{th}$particle

$V_i$ is the electric potential at the location of the i-th particle due to all other charges

# Electrostatic Energy cont

- For continuous charge distributions: The electrostatic energy can be calculated by integrating the product of the charge density and the electric potential over the volume containing the charge: $U = \frac{1}{2}(\rho V)dV$.

where:

$\rho$ is the charge density

$V$ is the electric potential

- Electrostatic energy is stored in the electric field created by the charges. The energy density $(U)$ of the electric field is given by $U = \frac{1}{2}\varepsilon_O E^2$.

where:

$\varepsilon_O$ is the permittivity of free space

# Electrostatic Energy cont1

- The electrostatic potential energy ($U$) of two charges is the work required to bring them from infinity to a separation $r$:

$$U = k\frac{q_1 q_2}{r}$$

Positive $U$ For like charges (repulsion).
Negative $U$ For opposite charges (attraction).

**Electrostatic Potential Energy of a System of Charges**

- For multiple charges, the total electrostatic potential energy is the sum of the energies between all charge pairs $U_{total} \sum_{i<j} k\frac{q_1 q_2}{r_{ij}}$

Where $r_{ij}$the distance between charges $q_i$ and $q_j$.

# Electrostatic Energy cont2

## Energy Stored in a Capacitor

- A capacitor stores electrostatic energy when charged. The energy stored is $U = \frac{1}{2}CV^2$ OR $U = \frac{Q^2}{2C} = \frac{1}{2}$QV

where:

$C$ = capacitance ($F$),

$V$ = voltage ($V$),

$Q$ = charge ($C$).

Energy is stored in the electric field between capacitor plates.

# Electrostatic Energy cont3

**Electrostatic Energy density in an electric field**

- Energy is also stored in an electric field. The energy density(energy per unit volume) in an electric field is $U_E = \frac{1}{2}\varepsilon_O E^2$.

Where,

$U_E =$ energy density$(J/m^3)$

$E =$ electric field strength $(V/m)$

$\varepsilon_O =$ permittivity of free space

This applies to capacitors and dielectrics where energy is distributed through out the field.

# Electrostatic Energy cont4

**Magnetostatic Energy (Electrostatic Energy in Magnetism)**

- In magnetostatics: Magnetic potential energy arises in current-carrying systems. Energy is stored in the magnetic field, similar to how energy is stored in electric fields.
- The energy density in a magnetic field is: $U_B = \frac{1}{2}\frac{B^2}{\mu_0}$, where

$U_B =$ magnetic energy density $(J/m^3)$,

$B =$ magnetic flux density $(T)$,

$\mu_0 =$ permeability of free space $(4\pi \times 10^{-7}0\ H/m)$.

- This shows that energy can be stored both in electric and magnetic fields, forming the basis of electromagnetic waves.

# Applications of Electrostatic and Electrostatic Energy

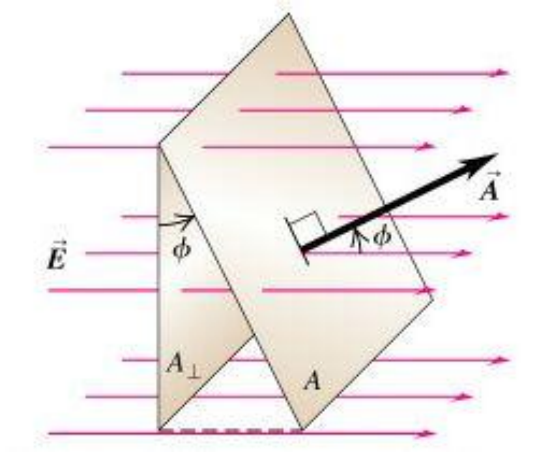
- Capacitors store electrostatic energy in electronic circuits.
- Electrostatic potential energy explains binding energy in atoms.
- High-voltage applications use electrostatic energy storage.
- Magnetic energy storage applies to inductors and transformers.
- Capacitors: Devices that store electrical energy in an electric field.They are used in many electronic circuits.
- Electrostatic painting: A technique that uses an electric field to deposit paint particles onto a surface, providing an even and efficient coating.

# Applications of electrostatic and electrostatic energy cont

- ➢Laser printers and photocopiers: These devices use electrostatic forces to transfer toner particles onto paper, creating images.
- ➢Particle accelerators: Use electric fields to accelerate charged particles to high speeds for research purposes.
- Electrostatic phenomena play a role in various natural events, like lightning and the behavior of dust particles in the atmosphere.
- Controlling and understanding electrostatics is crucial in many technological applications to prevent damage to sensitive equipment or to utilize the forces effectively.

# Electric Flux

If surface area is not perpendicular to the electric field we have to slightly change our definition of the flux




$$\Phi_{E} = E\,A\cos\phi$$

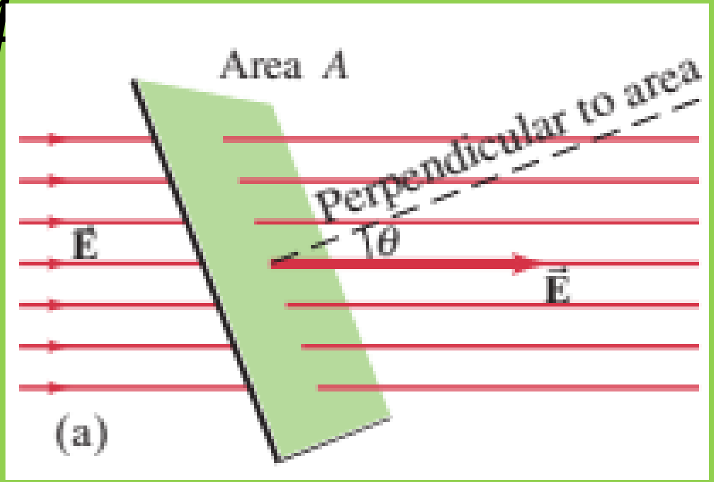
Where$\Phi$is the angle between the field and the unit vector that is perpendicular to the surface

This shows that the relationship between the flux and the electric field and the area vector is just the dot product of two vectors

# **Applications of Gauss's law** *(Giancoli pg 687)*

- ✓To find the electric field *Halliday pg693/3*, *Serway pg 739*, *Giancoli pg683*
- ✓To find the enclosed charge(various charge distributions)-*Serway pg 746*

*Calculate the electric flux through the rectangle shown in Fig. below. The rectangle is 10 cm by 20 cm, the electric field is 200N/C and is uniform at and the angle is 30°. Giancoli pg684*



# Work Done In An Electric Field

- Consider a uniform electric field $E$, but in one dimension. Let the field is directed along the positive $x$-axis. Let a charge move from an initial point $A$ in the electric field to a final point $B$, in the direction of the field.
- The charge moves because of the electrostatic force acting on it (due to the electric field).
- The work done on the charge is equal to the electrostatic force times the displacement $W_{AB} = F\Delta x = qE\left(x_f - x_i\right)$
- Note that $\Delta x$ can be positive or negative, since it is a vector and depends on direction. The charge $q$ can also be positive or negative, depending on what type of charge it is.

# Work Done And The Potential Difference

- When work is done on a charge in moving it from point A to point B
- A and B are at the same potential if $W_{AB} = 0$,
- A and B are at different potentials if $W_{AB} \neq 0$ , i.e. there is a potential difference between the two points.
- The potential difference $\Delta V$ between points $A$ and $B$ in an electrostatic field is defined as the work done per unit charge, that is done on any charge, to move it slowly from $A$ to $B$. $\Delta V = \frac{W_{AB}}{q}$
- The SI unit of potential difference is the Volt, $V$ $(1V = 1\,JC^{-1})$. The potential difference $V$ is the difference in electric potential between points $A$ and $B$ $\Delta V = V_B - V_A$

# Ohm’s Law and Resistance

- Current is the rate of flow of charge $I = \frac{\Delta q}{\Delta t}$
- Current moves along a circuit (and across a component) when there is a potential difference.
- A battery is a source of electrical potential energy, so that a charge can move around a circuit.
- Consider a current moving through a component in a circuit. Electrons collide with the fixed atoms in the component, and lose some of their kinetic energy to the atoms. The vibrational kinetic energy of the atoms increase (and hence the temperature of the component increases).

# Ohm's Law and Resistance

- Ohm's Law says that, for a wide range of potential differences, the potential difference (voltage drop) V across a conductor (component) is proportional to the current $I$ through the conductor. $\Delta V \propto I, \Delta V = IR$.
- R is the resistance. The SI units for voltage, current and resistance are the Volt ($V$), Ampere ($A$) and Ohm ($\Omega$), respectively. The resistance $R$ of a conductor is defined as the ratio of the voltage across the conductor and the current through the conductor. $R = \frac{\Delta V}{I}$
- Where $1\,\Omega = 1VA^{-1}$.

# Resistivity

- It is found experimentally that the resistance $R$ of any wire is directly proportional to its length $l$ and inversely proportional to its cross

-sectional area $A$. $R \propto l,\ R \propto \frac{1}{A}$ or $R = \frac{\rho l}{A}$

Where $\rho$, the constant of proportionality, is called the **resistivity** and depends on the material used.

- Resistivity is property of a *substance*, while resistance is a property of an *object*.
- The resistivity $\rho$ for most materials changes with temperature.

Examples Serway pg839, Halliday pg 782, openstax pg 707.

A silver wire has a length of 5m and a cross sectional radius of 0.002m. Silver has a resistivity of $1.59 \times 10^{-8}\Omega m$. It is connected to a potential difference of 0.006V (i) Calculate the resistance of the wire and the current through the wire.

## ***Solution***

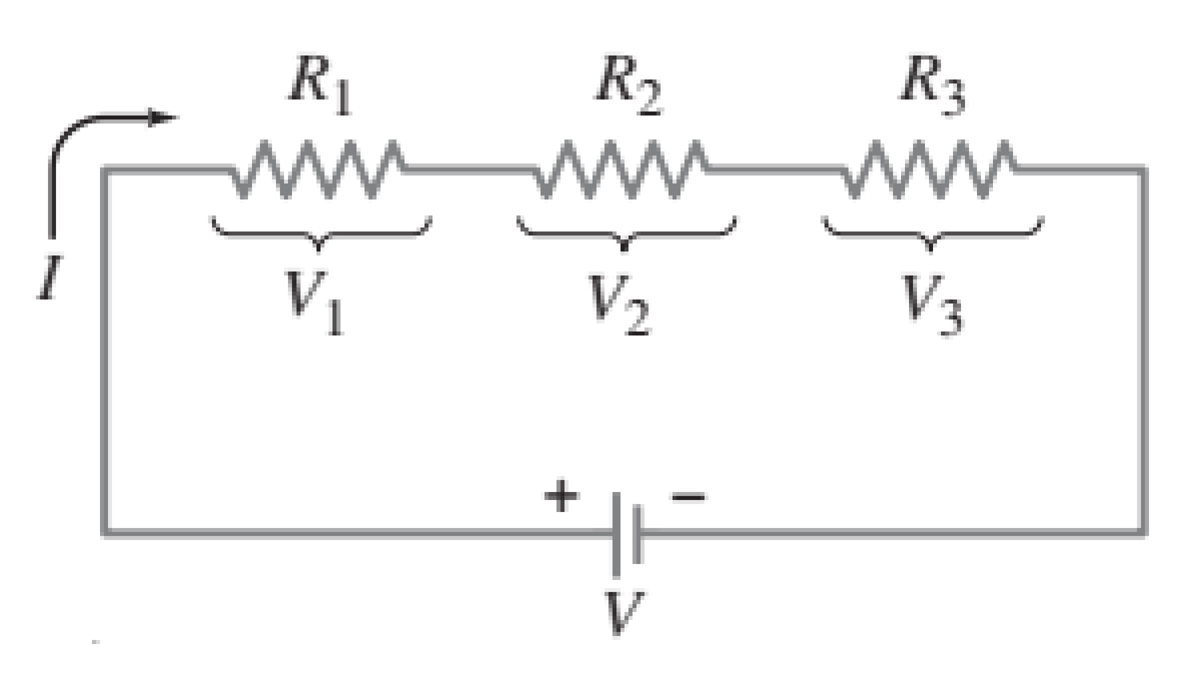
$$R = \frac{\rho l}{A} = \frac{1.59\times10^{-8}\Omega m\times5m}{\pi\times(0.002m)^2} = 6.36 \times 10^{-3}\Omega.$$

$$\Delta V = IR \; or \; I = \frac{V}{R} = \frac{0.006V}{6.36\times10^{-3}\Omega} = 0.9433A.$$

# Resistors in Series and in Parallel

***Series Circuits:*** In a series circuit, the same amount of charge passes through each device. $I_T = I_1 = I_2 = I_3$

The total voltage $V$ is equal to the sum of the voltages across each resistor: $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$



# *Total series resistance*

Due to conservation of charge we can factor out $I$,

$V_{source} = I_{total}(R_1 + R_2 + R_3)$

Since, $V_{source} = I_{total}R_{total}$

$$R_{total} = R_{eq} = R_1 + R_2 + R_3$$

*Examples*

*Openstax pg 727, Halliday pg 810, Serway pg 866, Giancoli pg 789*

https://www.tutorialspoint.com/how-to-calculate-equivalent-resistance-series-and-parallel-circuit-examples

# ***Parallel Circuits:***

In a Parallel circuit, components are connected across the same voltage source, $V_T = V_1 = V_2 = V_3$.

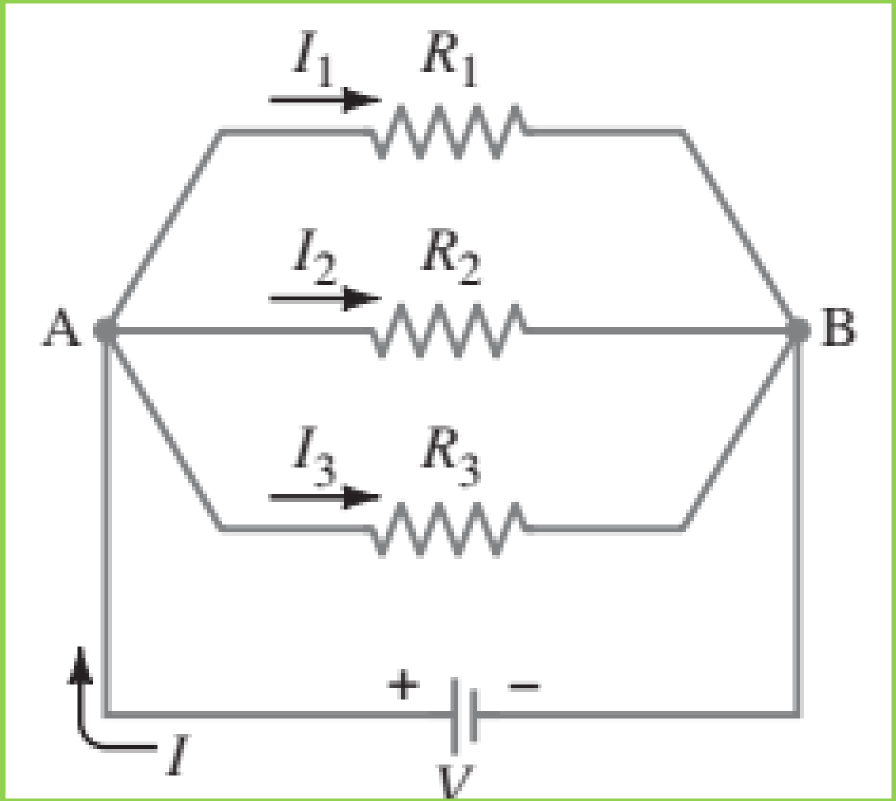
The total current $I_T$ is equal to the sum of the currents across each resistor: $I_T = I_1 + I_2 + I_3 = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$

Since, $I_{source} = I_T = \frac{V_{source}}{R_{total}} = \frac{V_T}{R_{total}}$

$\frac{V_{source}}{R_{total}} = \frac{V_T}{R_1} + \frac{V_T}{R_2} + \frac{V_T}{R_3}$, Or

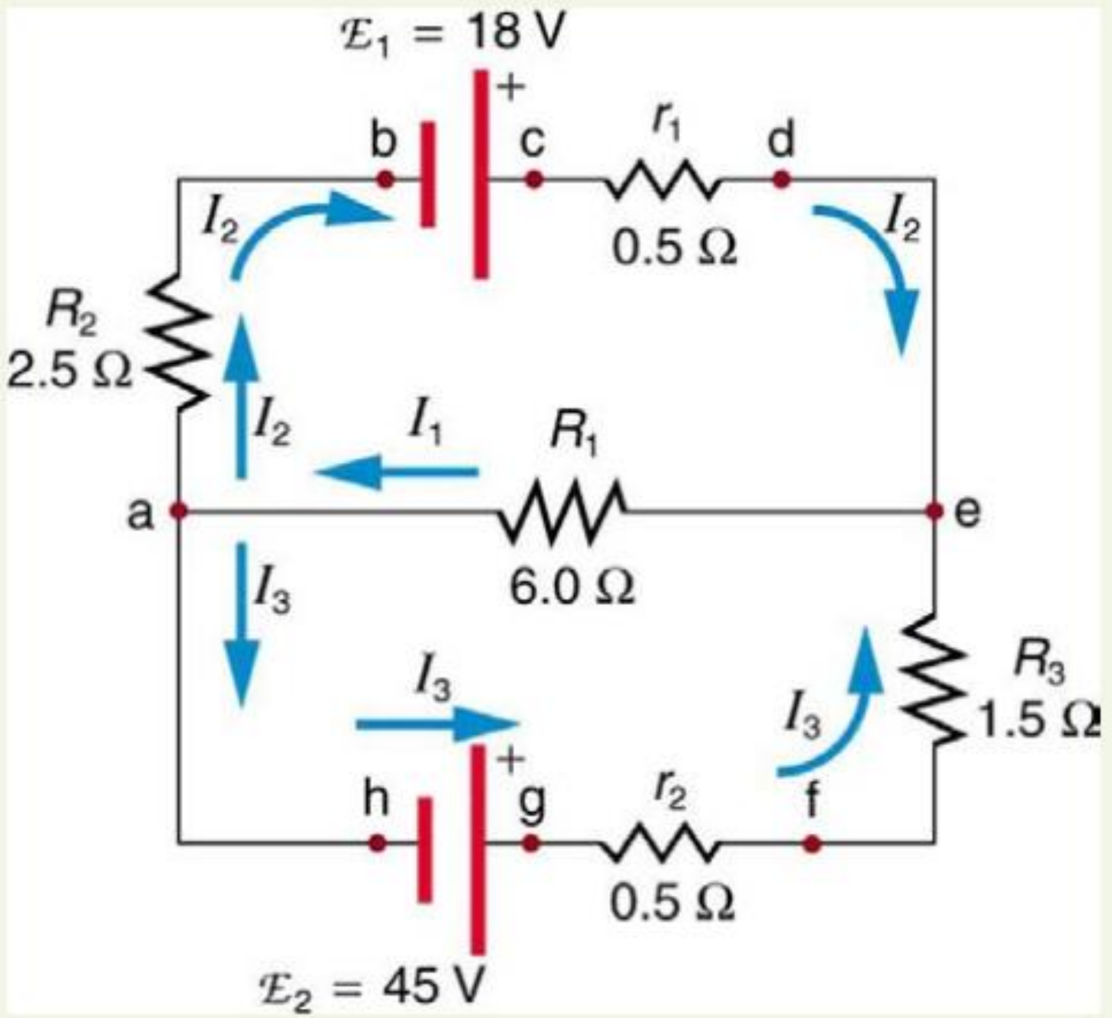
$\frac{1}{R_{eq}} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$, since $\mathrm{I} = \frac{V}{R_{eq}}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



# *Example*

Use Kirchhoff's rules to find the currents $I_1,\ I_2\ and\ I_3$ flowing in the circuit.



# *Solution*

*Three independent equations are needed* $I_1 = I_2 + I_3$*. We consider the loop abcdea which gives*

$$-I_2R_2 + emf_1 - I_2r_1 - I_1R_1 = -I_2(R_2 + r_1) + emf_1 - I_1R_1 = 0.$$

*Substituting values from the circuit diagram for the resistances and emf, and canceling the ampere unit gives*

$-3I_2 + 18 - 6I_1 = 0.$ *Now applying the loop rule to aefgha (or abcdefgha as well) similarly gives*

$$I_1R_1 + I_3R_3 + I_3r_2 - emf_1 = I_1R_1 + I_3(R_3 + r_2) - emf_2 = 0.$$

*Note that the signs are reversed compared with the other loop, because elements are traversed in the opposite direction. Entering the values, we have* $6I_1 + 2I_3 - 45 -= 0.$

# *Solution cont.*

*These three equations are sufficient to solve for the three unknown currents. First, solve the second equation for* $I_2 : I_2 = 6 - 2I_1$.

*Now solve the third equation for* $I_3$. $I_3 = 22.5 - 3I_1$. *Substituting these two new equations into the first one allows us to find a value for* $I_1$. $I_1 = I_2 + I_3 = (6 - 2I_1) + (22.5 - 3I_1) = 28.5 - 5I_1$

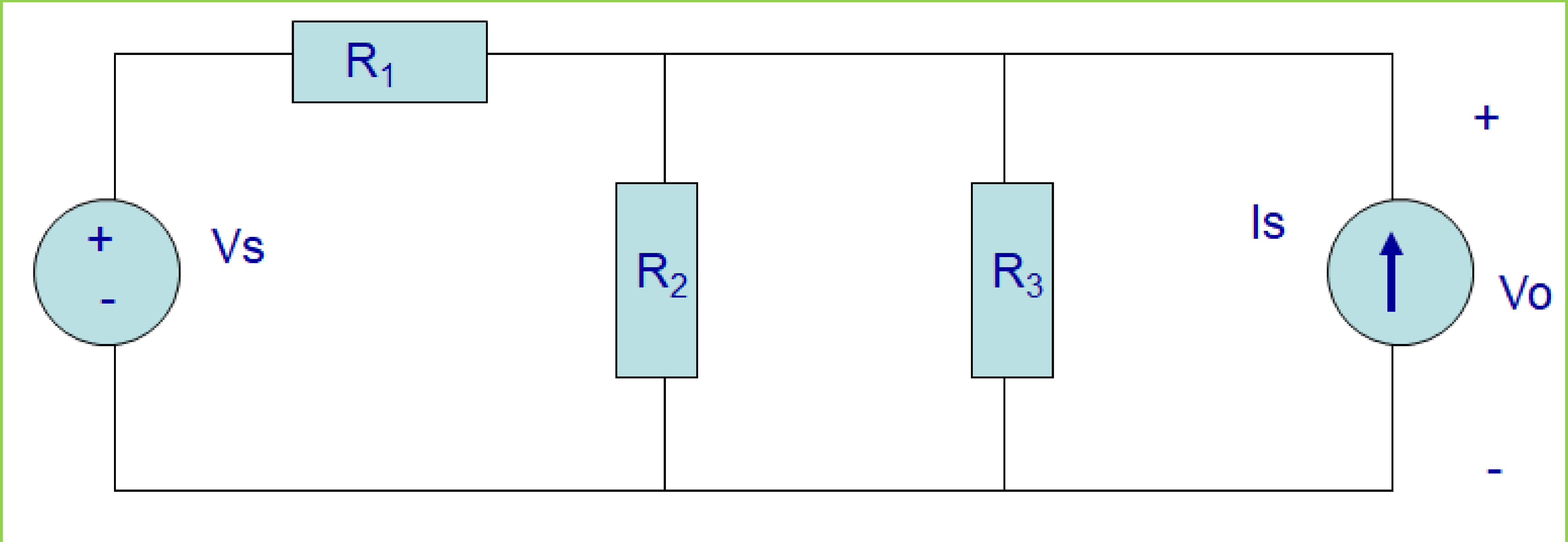
*Combining terms gives* $6I_1 = 28.5,$ *and* $I_1 = 4.75A$. *Substituting this value for* $I_1$ *back into the fourth equation gives* $I_2 = 6 - 2I_1 = 6 - 9.50. I_2 = -3.50A$. *The minus sign means*$I_2$ *flows in the direction opposite to that assumed in the diagram. Finally, substituting the value for* $I_1$ *into the fifth equation gives* $I_3 = 22.5 - 3I_1 = 22.5 - 14.25 = 8.25A$

# Circuit Definitions

- Node – any point where 2 or more circuit elements are connected together
  - Wires usually have negligible resistance
  - Each node has one voltage (w.r.t. ground)
  - Branch – a circuit element between two nodes
  - Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

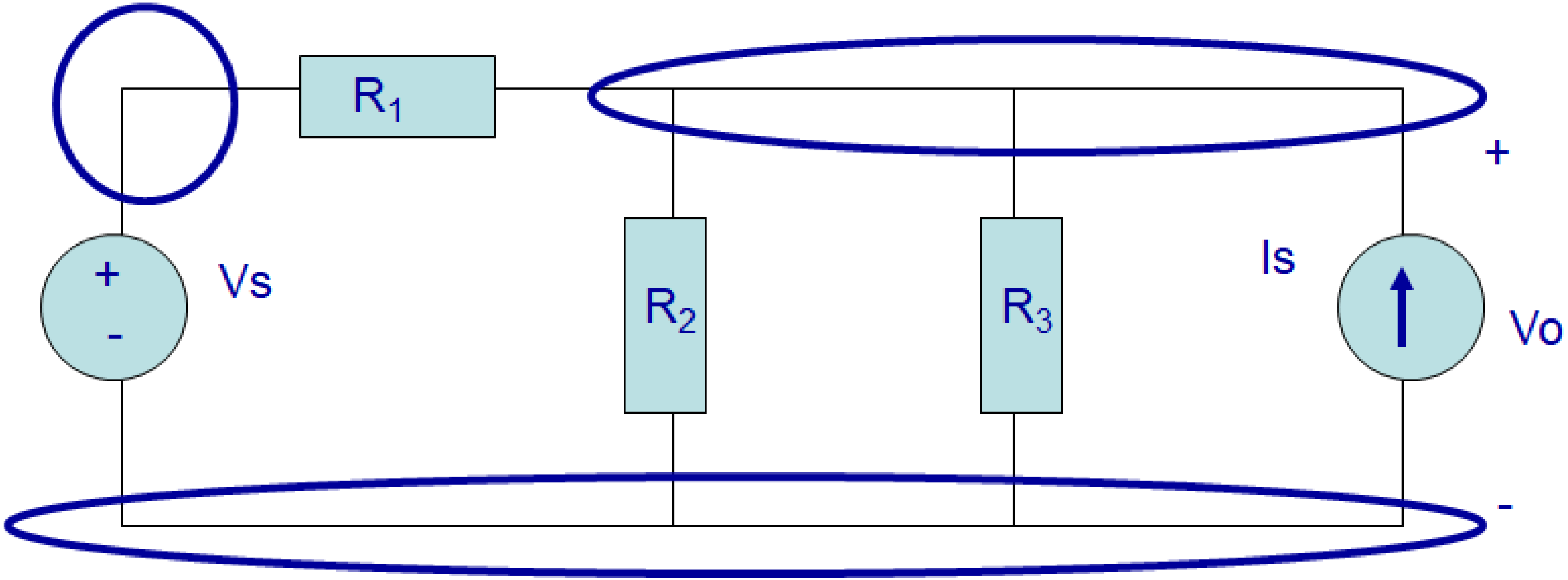
# Example

How many nodes, branches & loops?



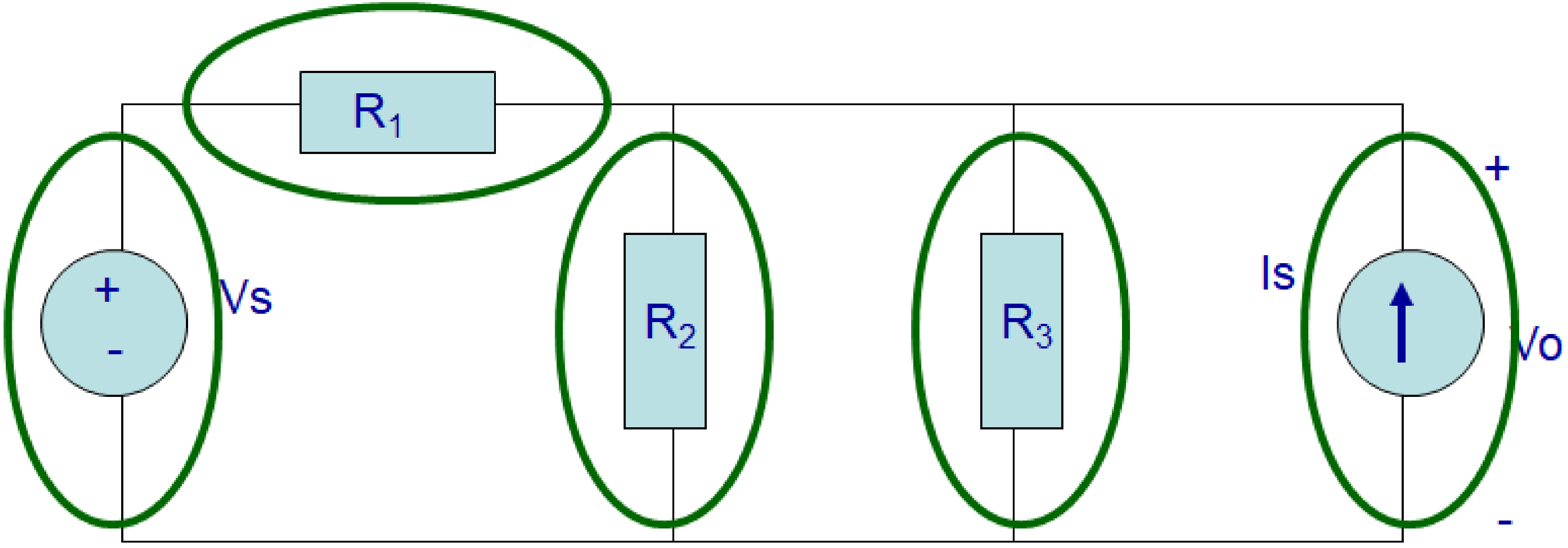
# *Solution*

- Three nodes



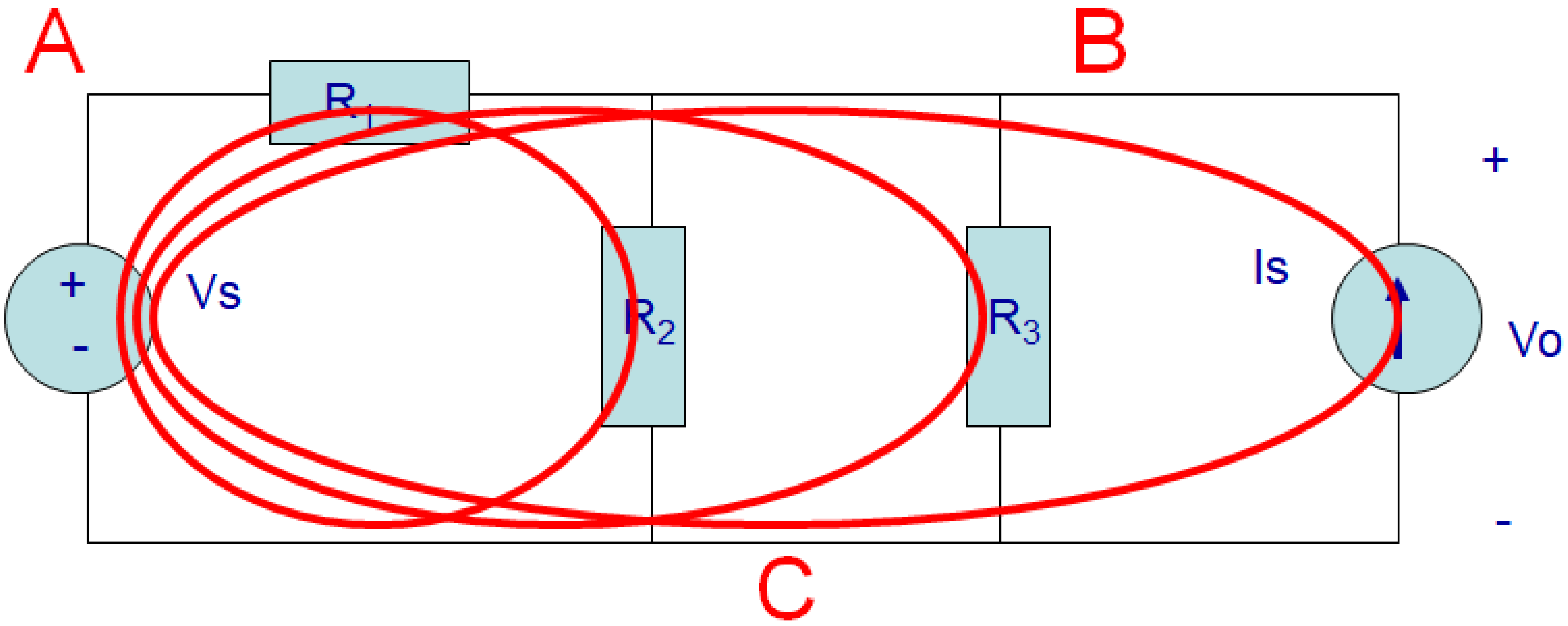
# Solution

- 5 Branches



# *Solution*

- Three Loops, if starting at node A

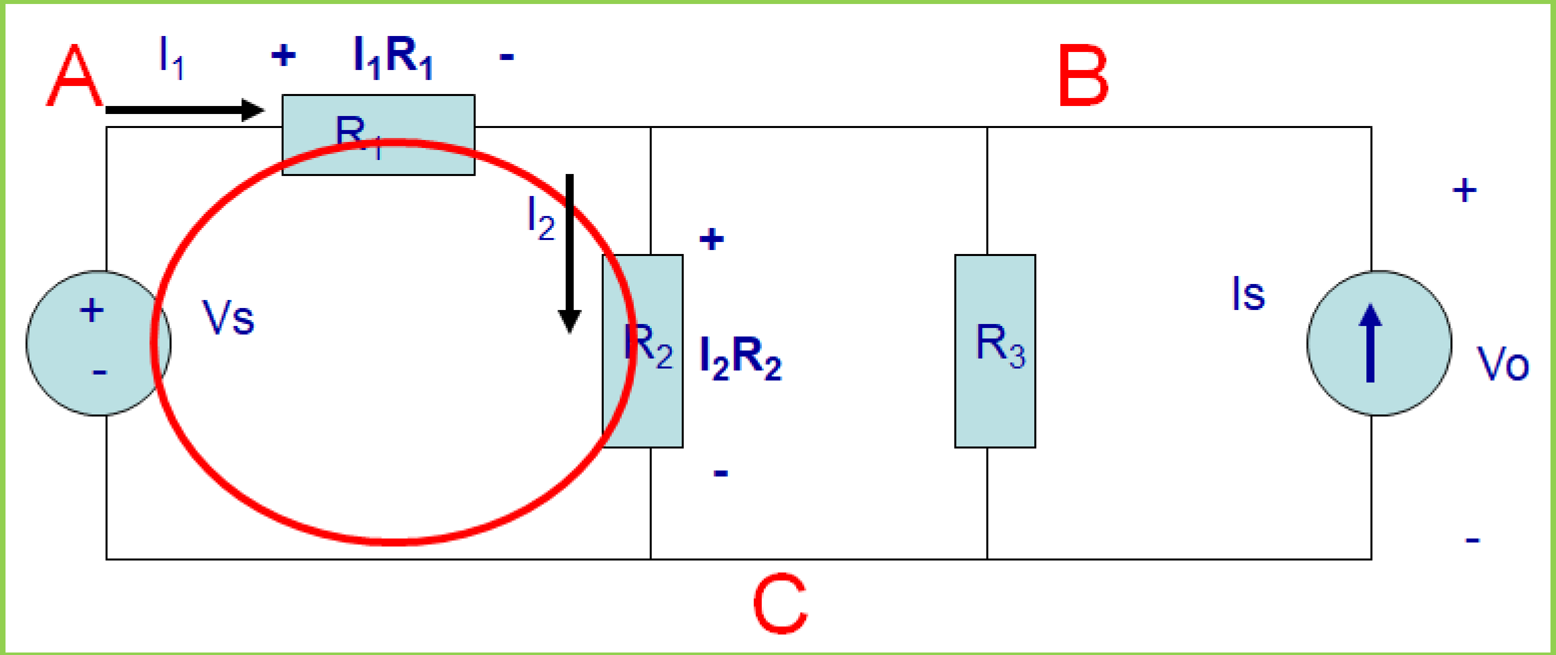


# Kirchoff’s Voltage Law (KVL)

- The algebraic sum of voltages around each loop is zero
- Beginning with one node, add voltages across each branch in the loop (if you encounter a + sign first) and subtract voltages (if you encounter a – sign first)
- Σ voltage drops - Σ voltage rises = 0
- Or  Σ voltage drops = Σ voltage rises

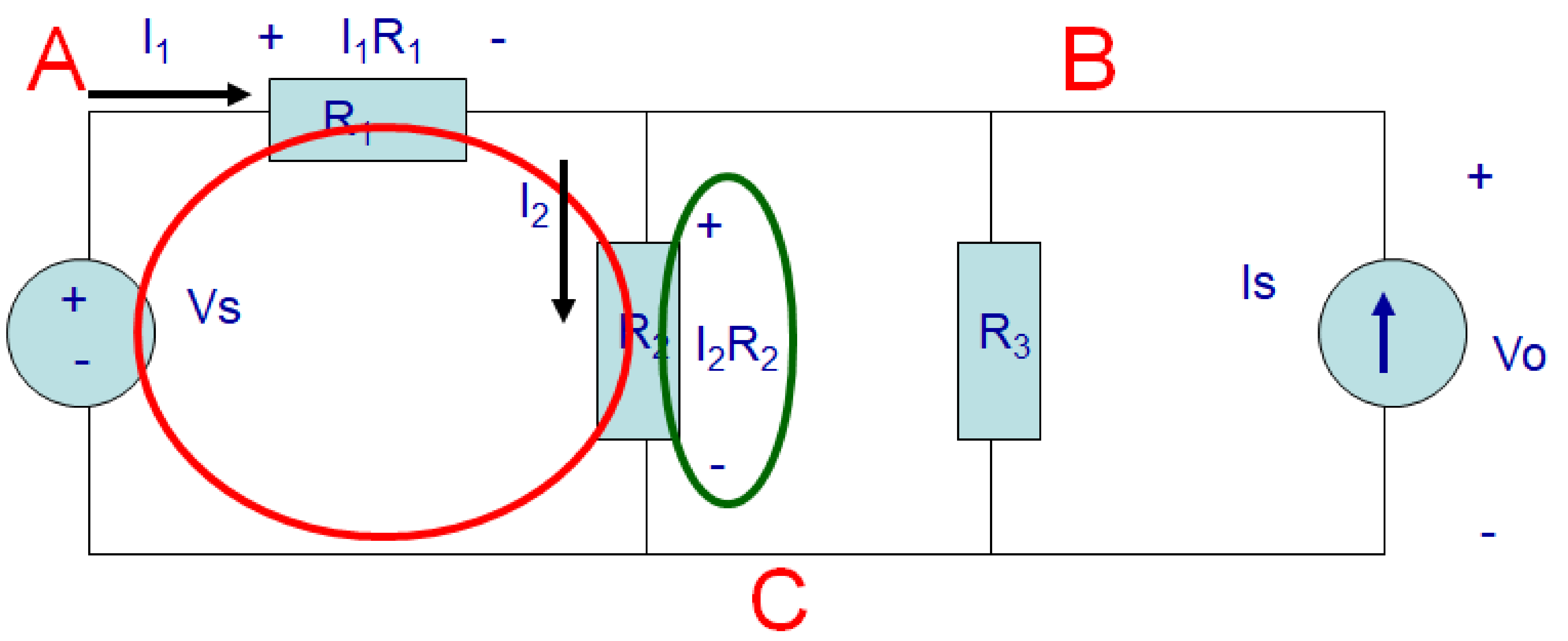
# *Example*

- Kirchoff's Voltage Law around 1st Loop.
- Assign current variables and directions
- Use Ohm's law to assign voltages and polarities consistent with passive devices (current enters at the + side)



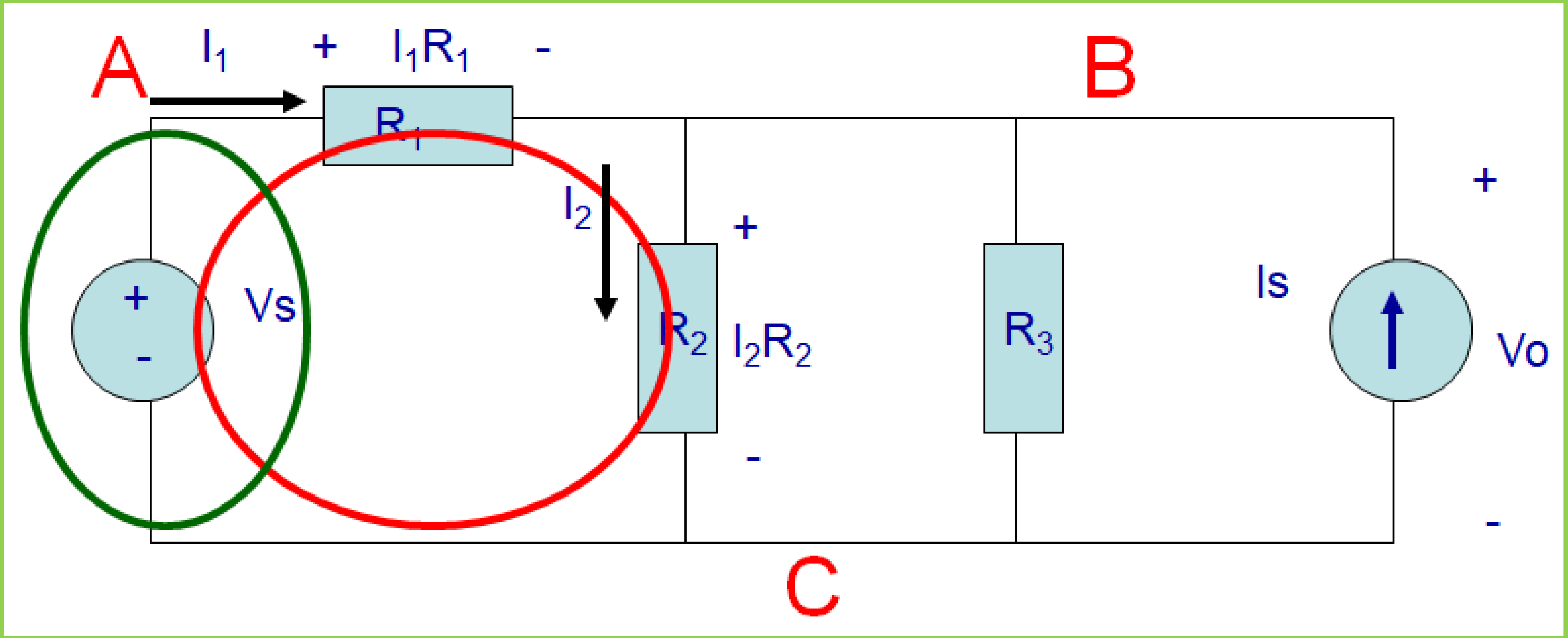
# *Example*

Starting at node A, add the 1st voltage drop: $+ I_1R_1$



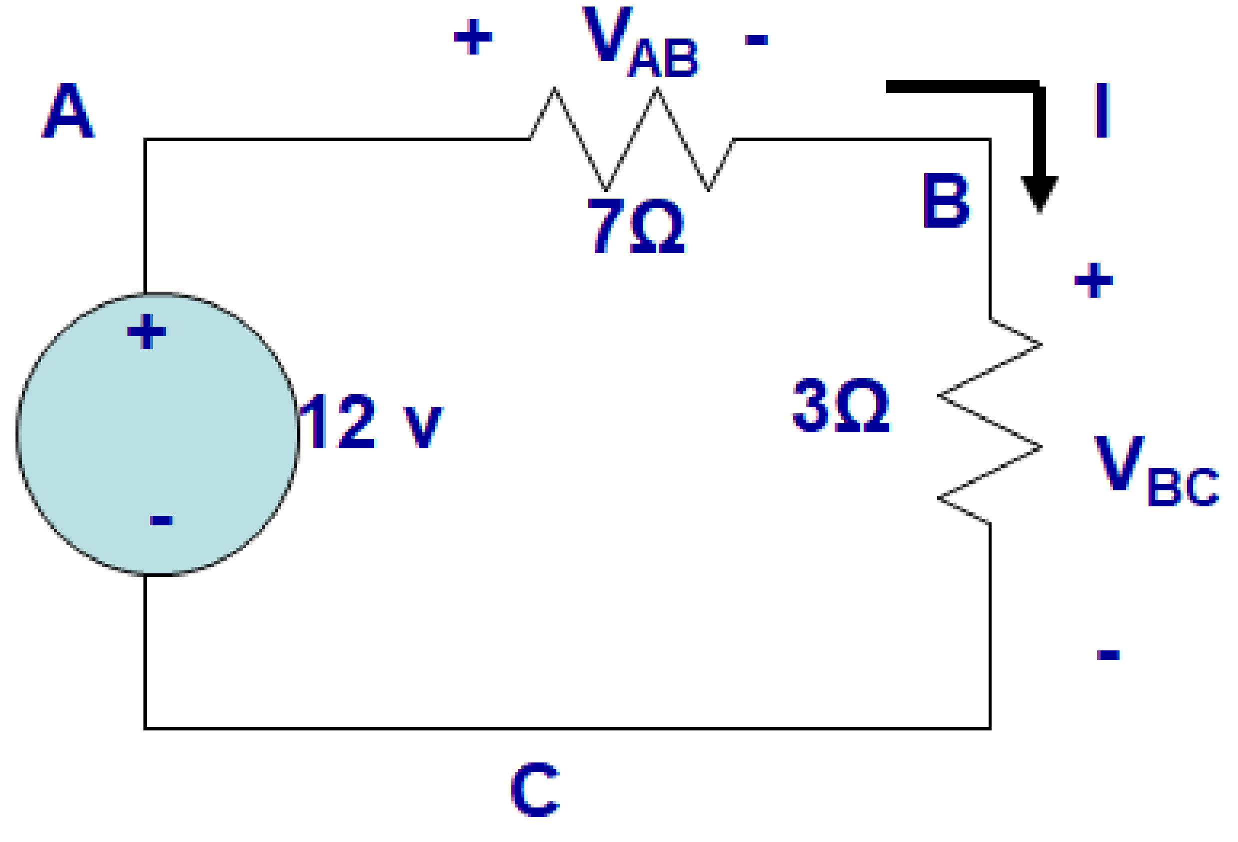
# *Example*

- Subtract the voltage rise from C to A through Vs:+ $I_1R_1 + I_2R_2 - Vs = 0$
- Notice that the sign of each term matches the polarity encountered 1st



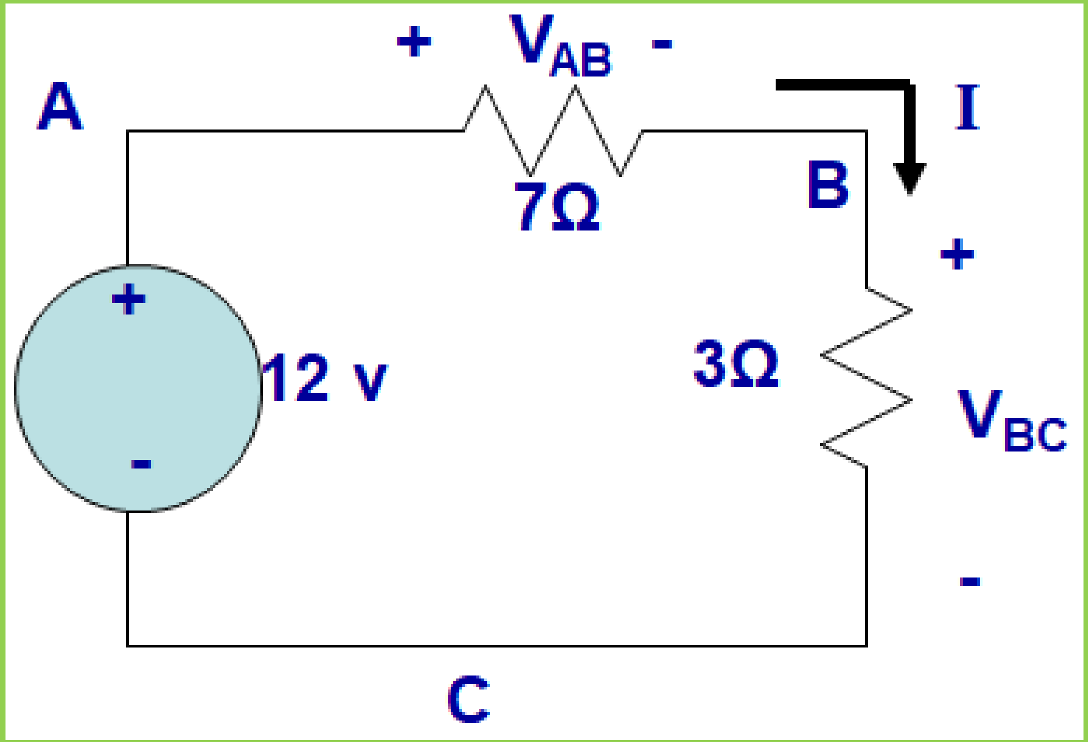
# Circuit Analysis

- When given a circuit with sources and resistors having fixed values, you can use Kirchoff’s two laws and Ohm’s law to determine all branch voltages and currents



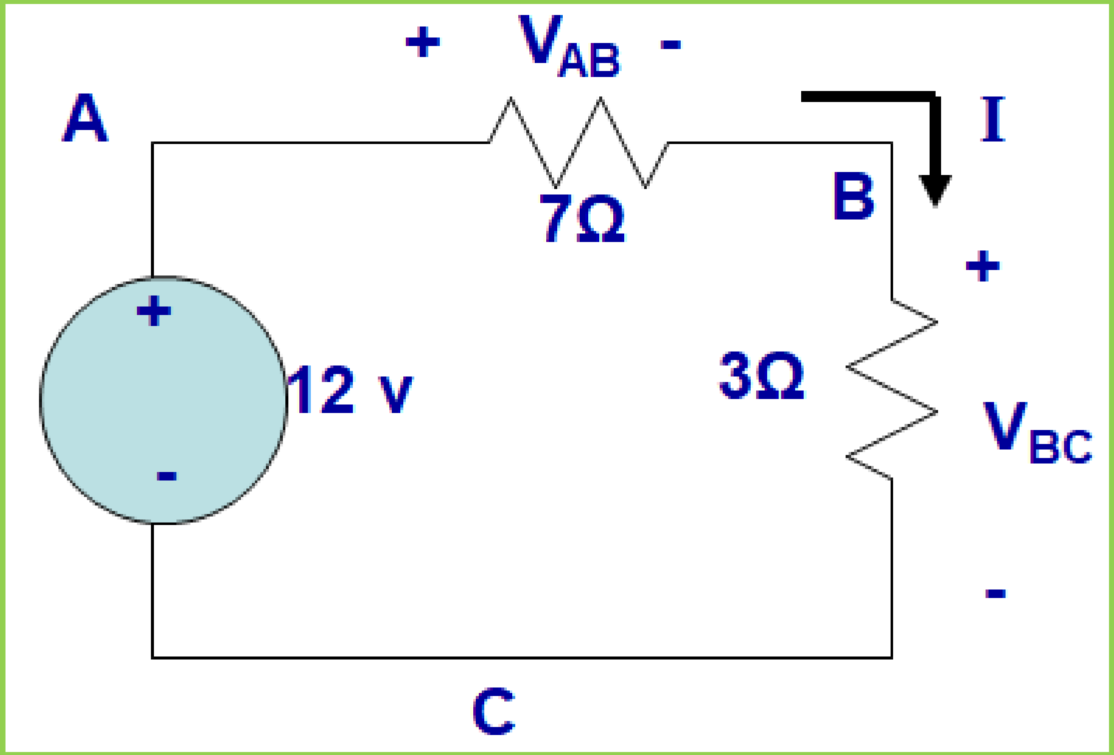
# Circuit Analysis

- By Ohm's law: $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- By KVL: $V_{AB} + V_{BC} - 12\,v = 0$
- Substituting: $I \cdot 7\Omega + I \cdot 3\Omega - 12\,v = 0$
- Solving: $I = 1.2\,A$



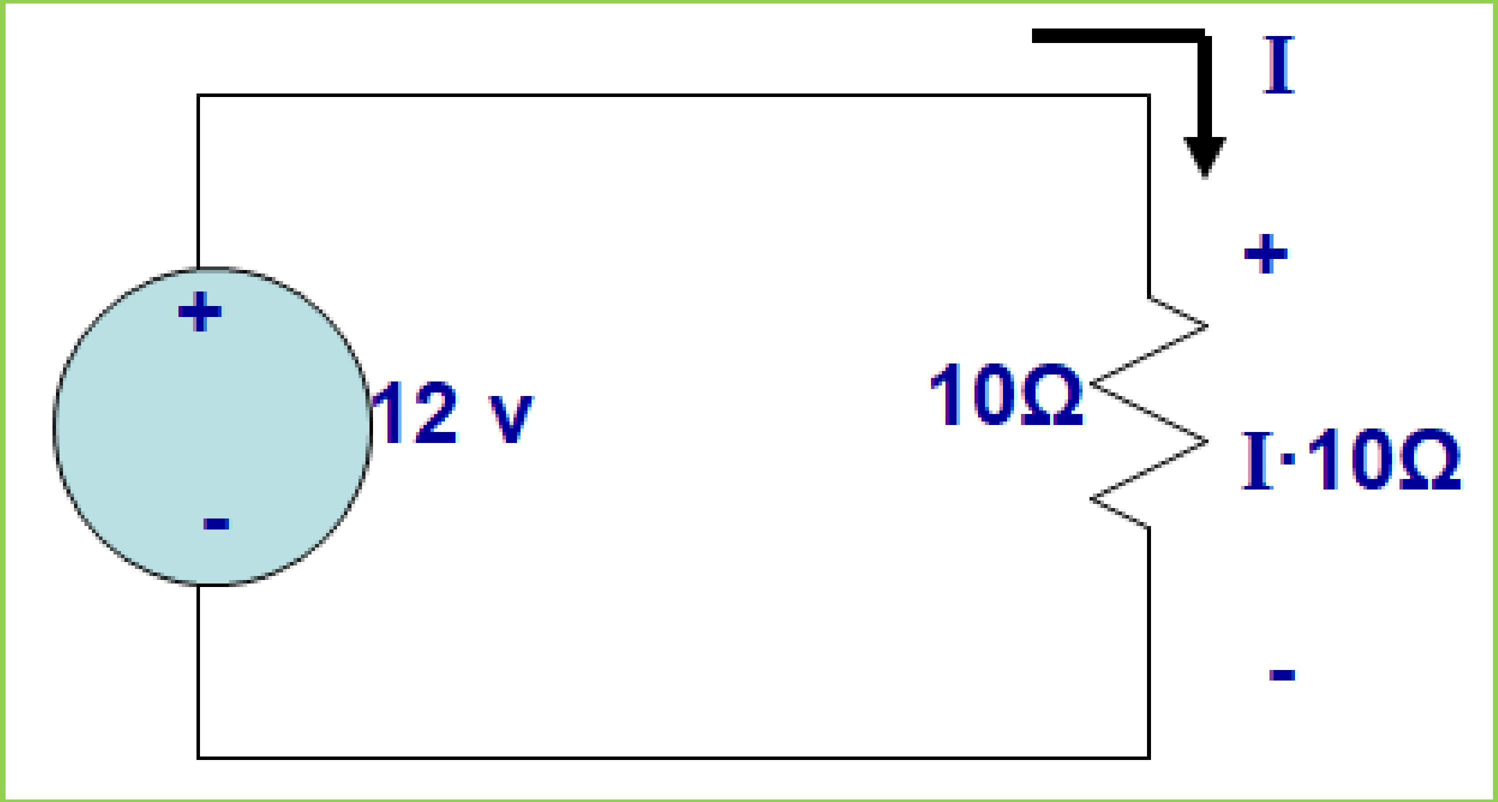
# Circuit Analysis

- Since $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- And $I = 1.2\ A$
- So $V_{AB} = 8.4\ V$ and $V_{BC} = 3.6\ V$



# Series Resistors

- KVL: $+I \cdot 10\Omega - 12\,v = 0,\quad$ So $I = 1.2\,A$
- From the viewpoint of the source, the 7 and 3 ohm resistors in series are equivalent to the 10 ohms



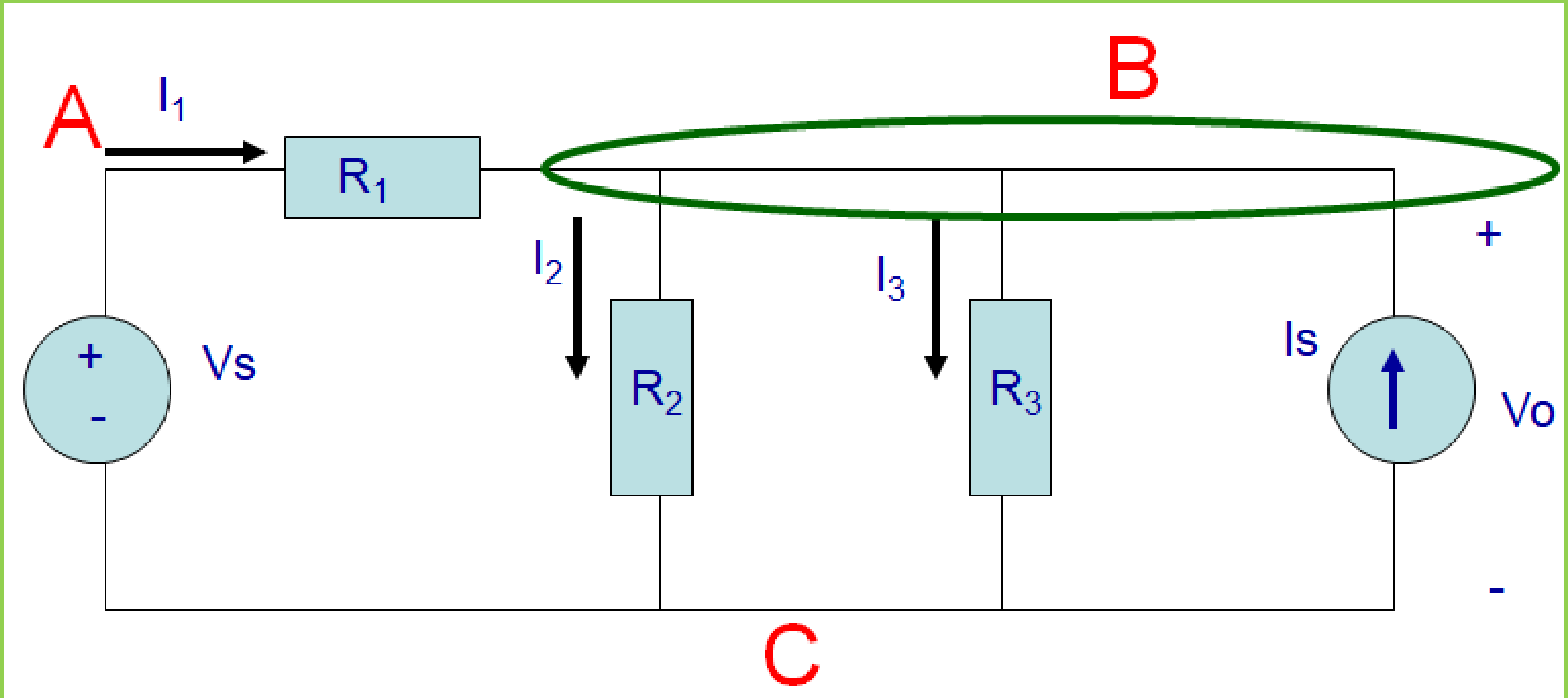
# Kirchoff’s Current Law (KCL)

- The algebraic sum of currents entering a node is zero
- Add each branch current entering the node and subtract each branch current leaving the node
- $\Sigma$ currents in - $\Sigma$ currents out = 0
- Or $\Sigma$ currents in = $\Sigma$ currents out

# *Example*

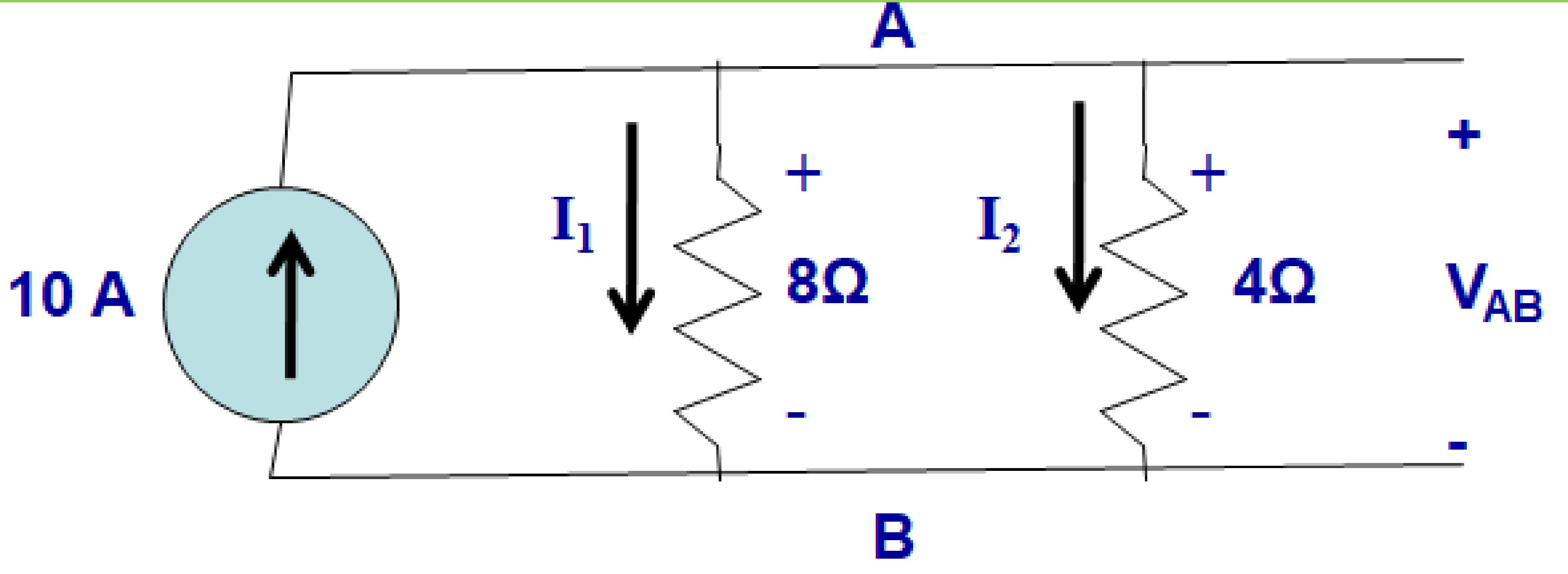
➢Kirchoff's Current Law at B

- Assign current variables and directions
- Add currents in, subtract currents out:  $I_1 - I_2 - I_3 + Is = 0$



# Circuit Analysis

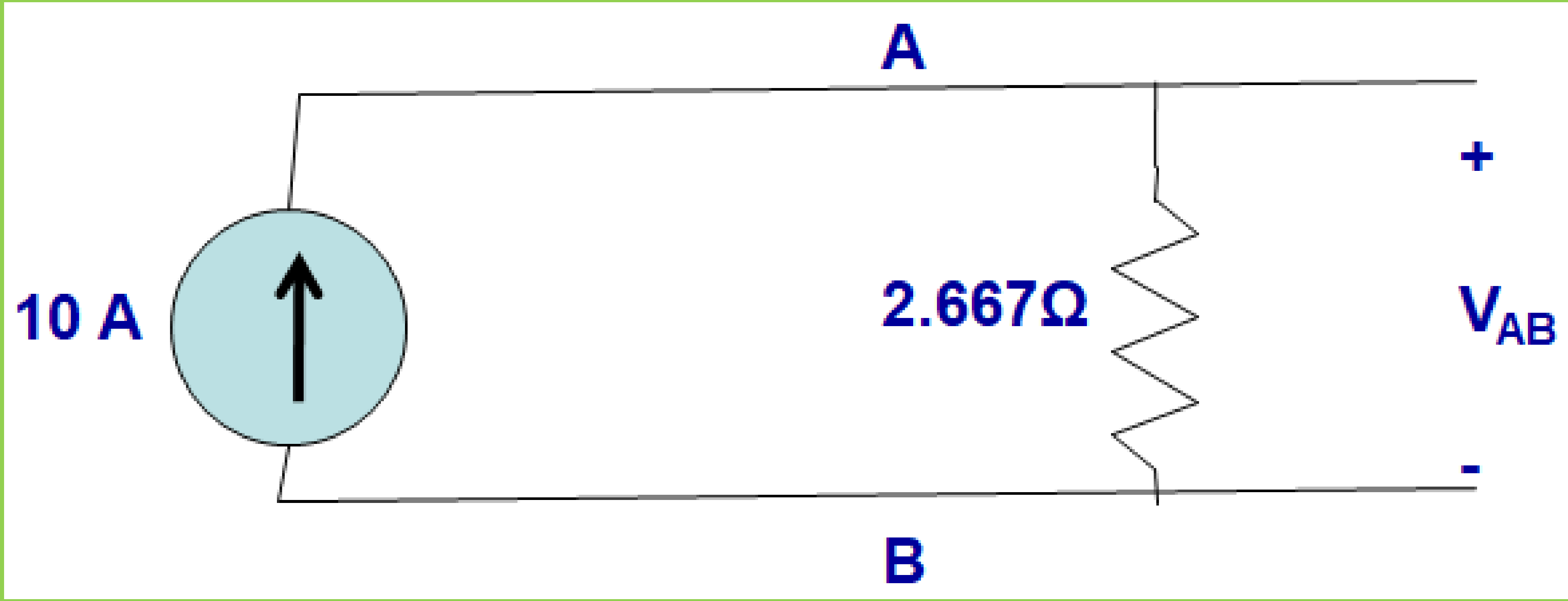
- By KVL: $-I_1 \cdot 8\Omega + I_2 \cdot 4\Omega = 0$, Solving: $I_2 = 2 \cdot I_1$
- By KCL: $10A = I_1 + I_{2,}$ Substituting: $10A = I_1 + 2 \cdot I_1 = 3 \cdot I_1$
- So $I_1 = 3.33\ A$ and $I_2 = 6.67\ A$, And $V_{AB} = 26.33$ volts



# Circuit Analysis

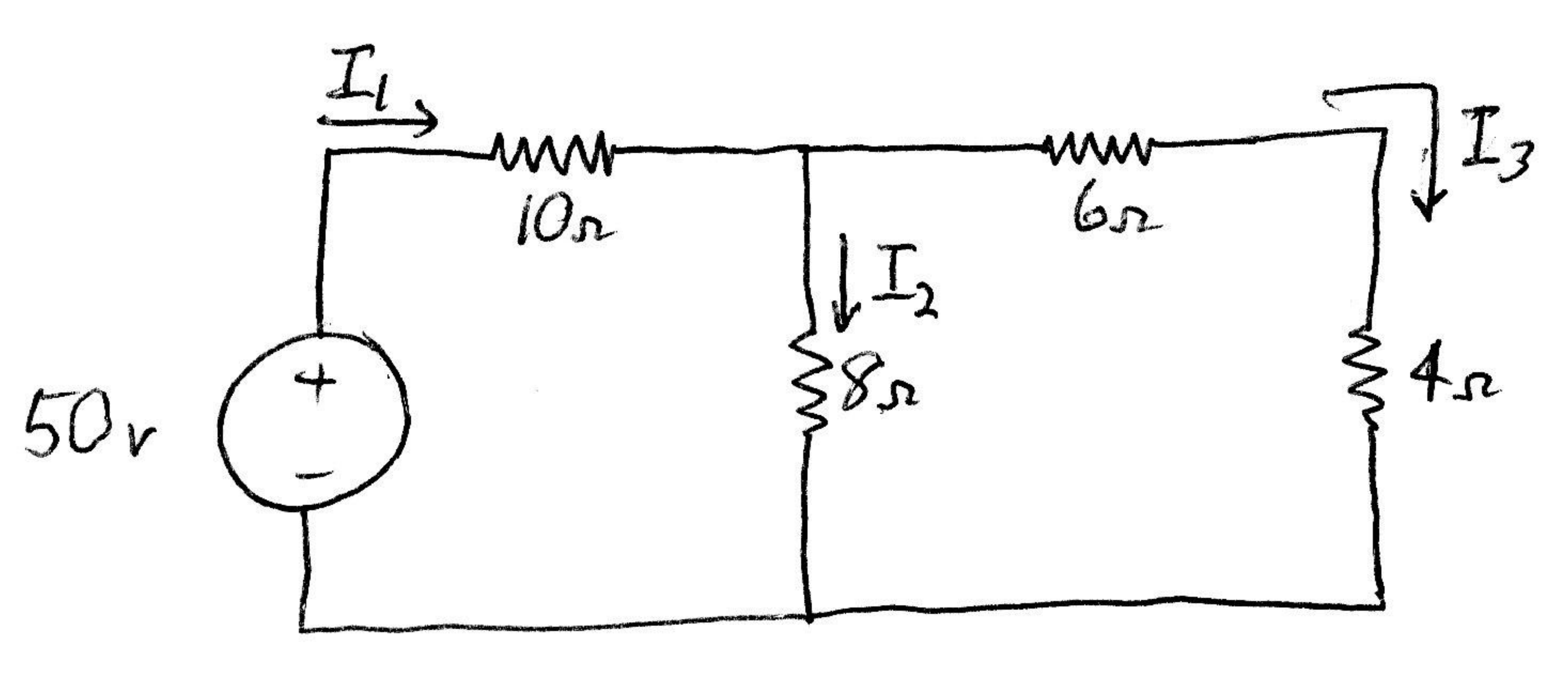
➢By Ohm's Law: $V_{AB} = 10\ A \cdot 2.667\ \Omega$, So $V_{AB} = 26.67$ volts

Replacing two parallel resistors (8 and 4 Ω) by one equivalent one produces the same result from the viewpoint of the rest of the circuit.



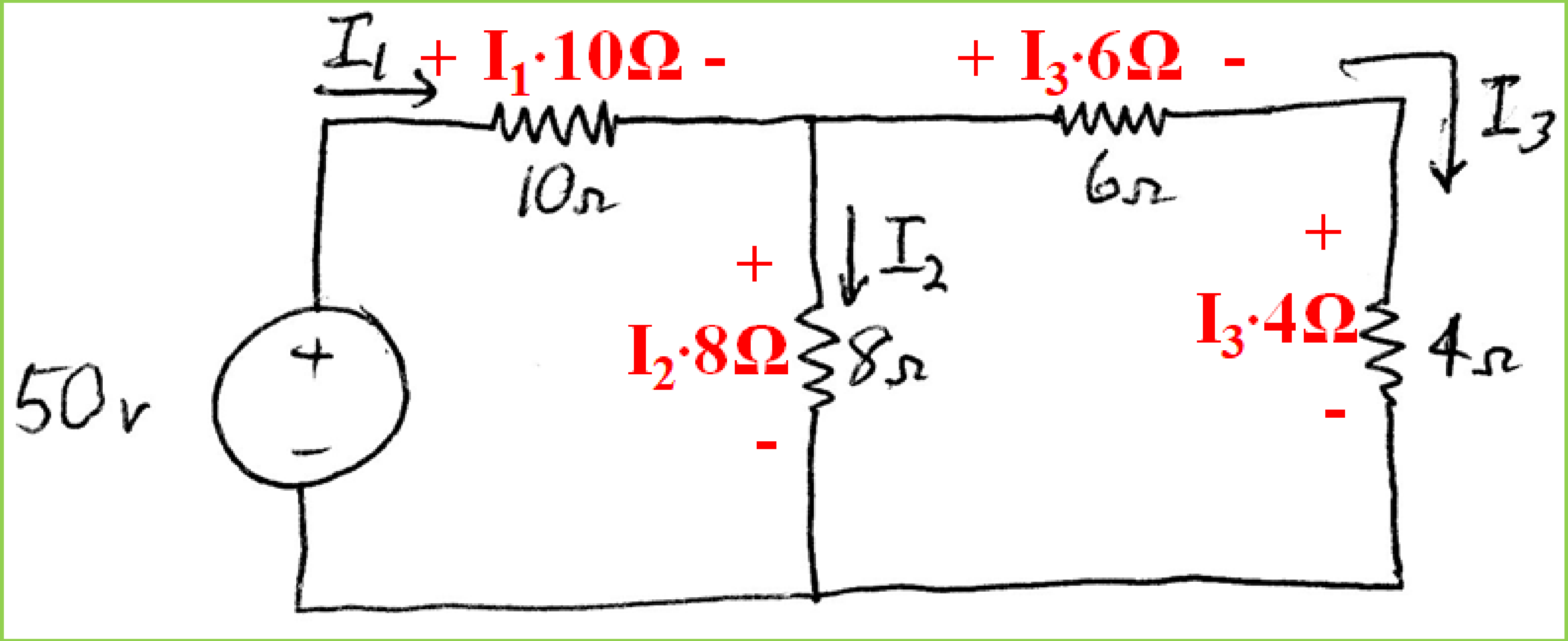
*Example Circuit*

- Solve for the currents through each resistor and the voltages across each resistor



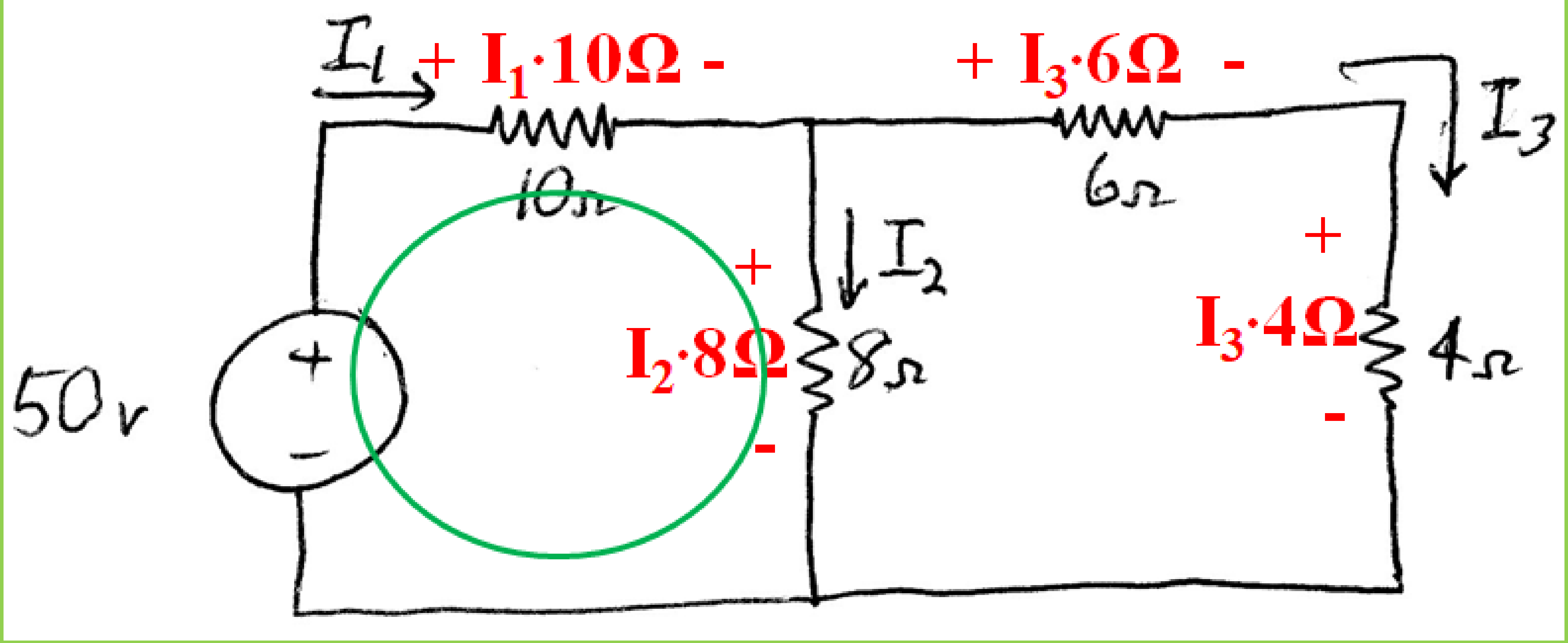
# *Example Circuit*

Using Ohm's law, add polarities and expressions for each resistor voltage



# *Example Circuit*

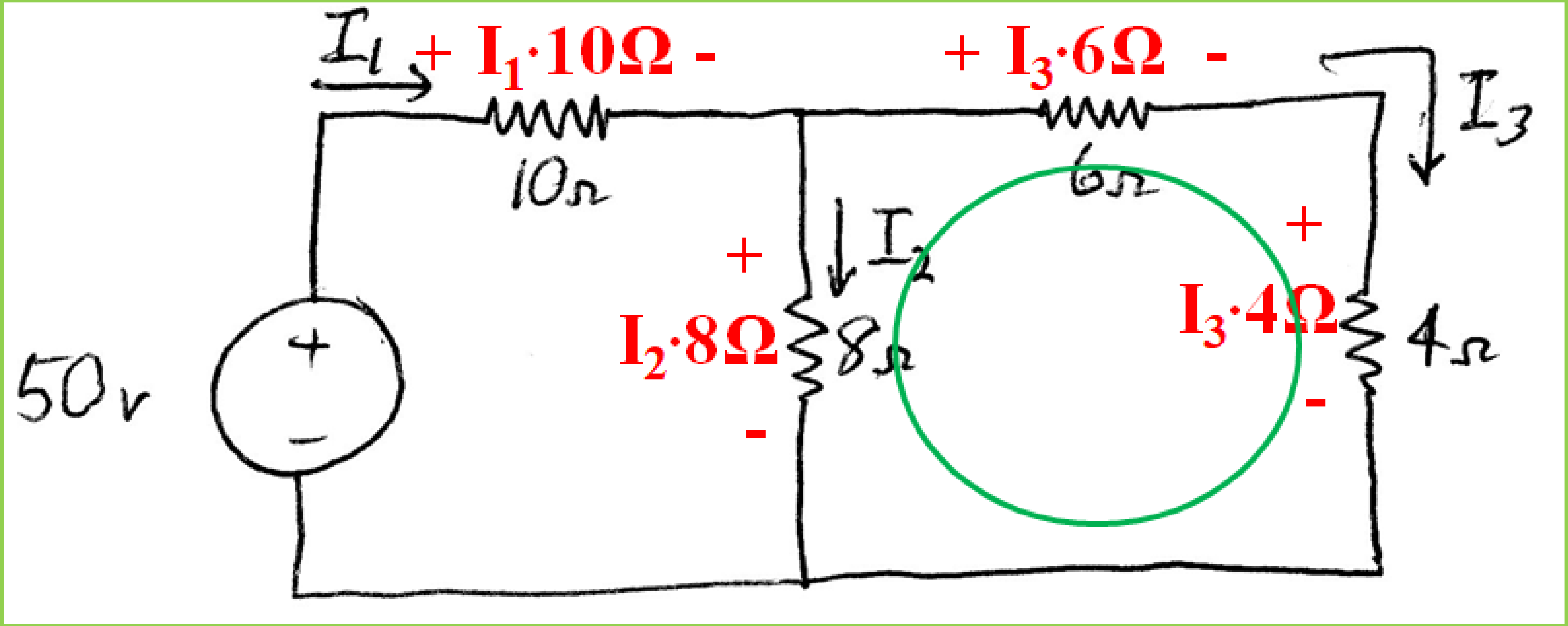
Write 1st Kirchoff's voltage law equation $-50\ v + I_1 \cdot 10\Omega\ + I_2 \cdot 8\Omega =$
$0$



# *Example Circuit*

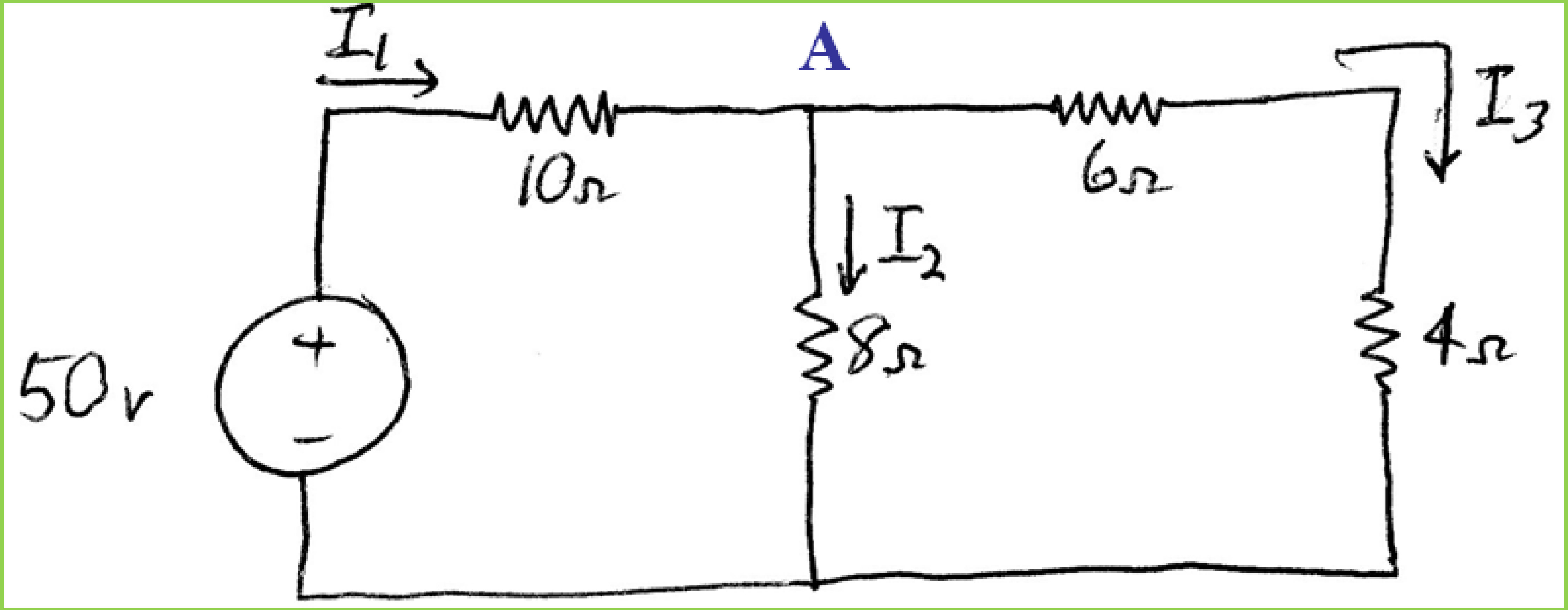
Write 2nd Kirchoff's voltage law equation

$-I_2 \cdot 8\Omega + I_3 \cdot 6\Omega + \ I_3 \cdot 4\Omega = 0$ or $I_2 = I_3 \ \cdot (6 + 4)/8 \ = 1.25 \ \cdot \ I_3$



# *Example Circuit*

Write Kirchoff's current law equation at A $+I_1 - I_2 - I_3 = 0$



# *Example Circuit*

- We now have 3 equations in 3 unknowns, so we can solve for the currents through each resistor, that are used to find the voltage across each resistor
- Since $I_1 - I_2 - I_3 = 0, \quad I_1 = I_2 + I_3$
- Substituting into the 1st KVL equation

$$or\ I_2 \cdot 18\,\Omega + I_3 \cdot 10\,\Omega = 50\,volts$$

# *Example Circuit*

- But from the 2nd KVL equation, $I_2 = 1.25 \cdot I_3$
- Substituting into 1st KVL equation:

$$(1.25 \cdot I_3) \cdot 18\,\Omega + I_3 \cdot 10\,\Omega = 50\, volts$$

Or: $I_3 \cdot 22.5\,\Omega + I_3 \cdot 10\,\Omega = 50\, v$

Or: $I_3 \cdot 32.5\,\Omega = 50\, volts$

Or: $I_3 = 50\, volts/32.5\,\Omega$

Or: $I_3 = 1.538\, amps$

# *Example Circuit*

- Since $I_3 = 1.538\, amps$

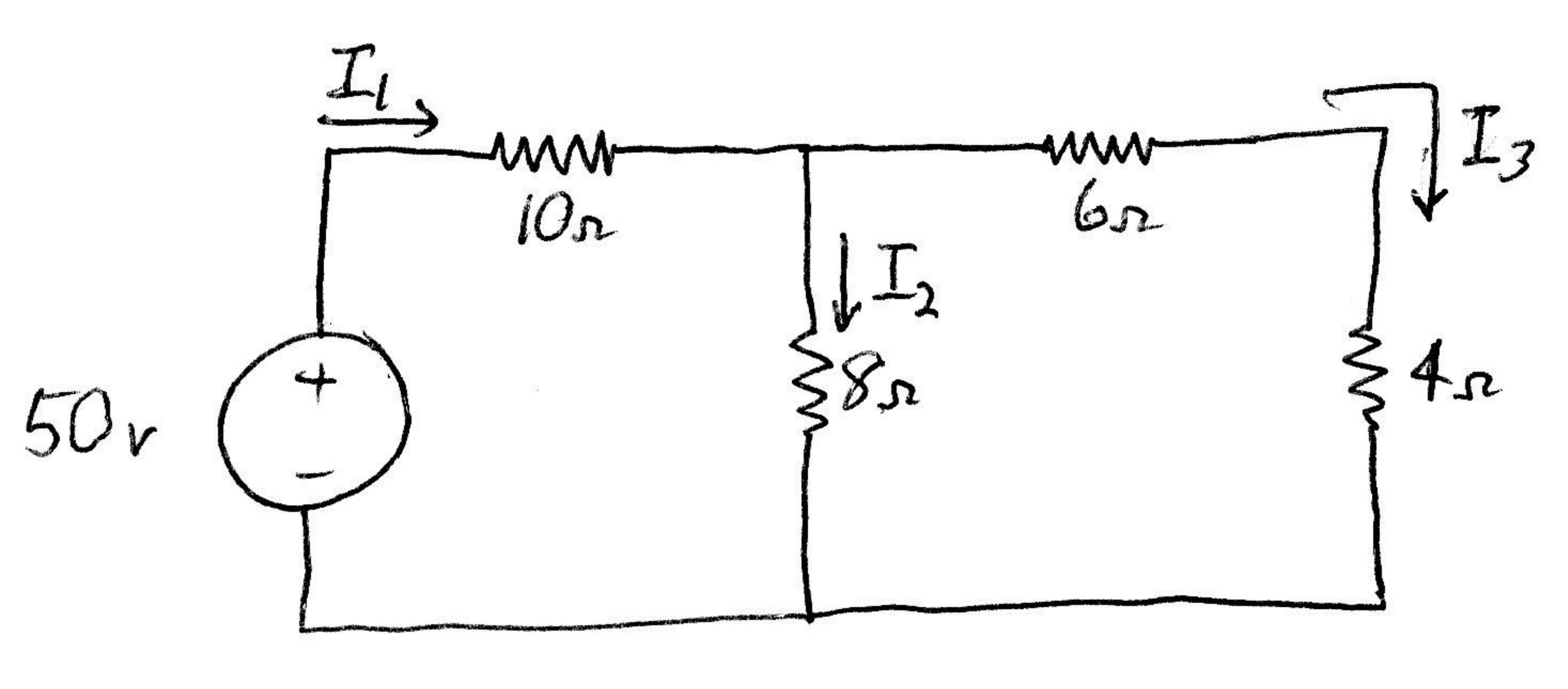
$$I_2 = 1.25 \cdot I_3 = 1.923\, amps$$

- Since $I_1 = I_2 + I_3$, $I_1 = 3.461\, amps$
- The voltages across the resistors:

$$I_1 \cdot 10\Omega = 34.61\, volts$$
$$I_2 \cdot 8\Omega = 15.38\, volts$$
$$I_3 \cdot 6\Omega = 9.23\, volts$$
$$I_3 \cdot 4\Omega = 6.15\, volts$$

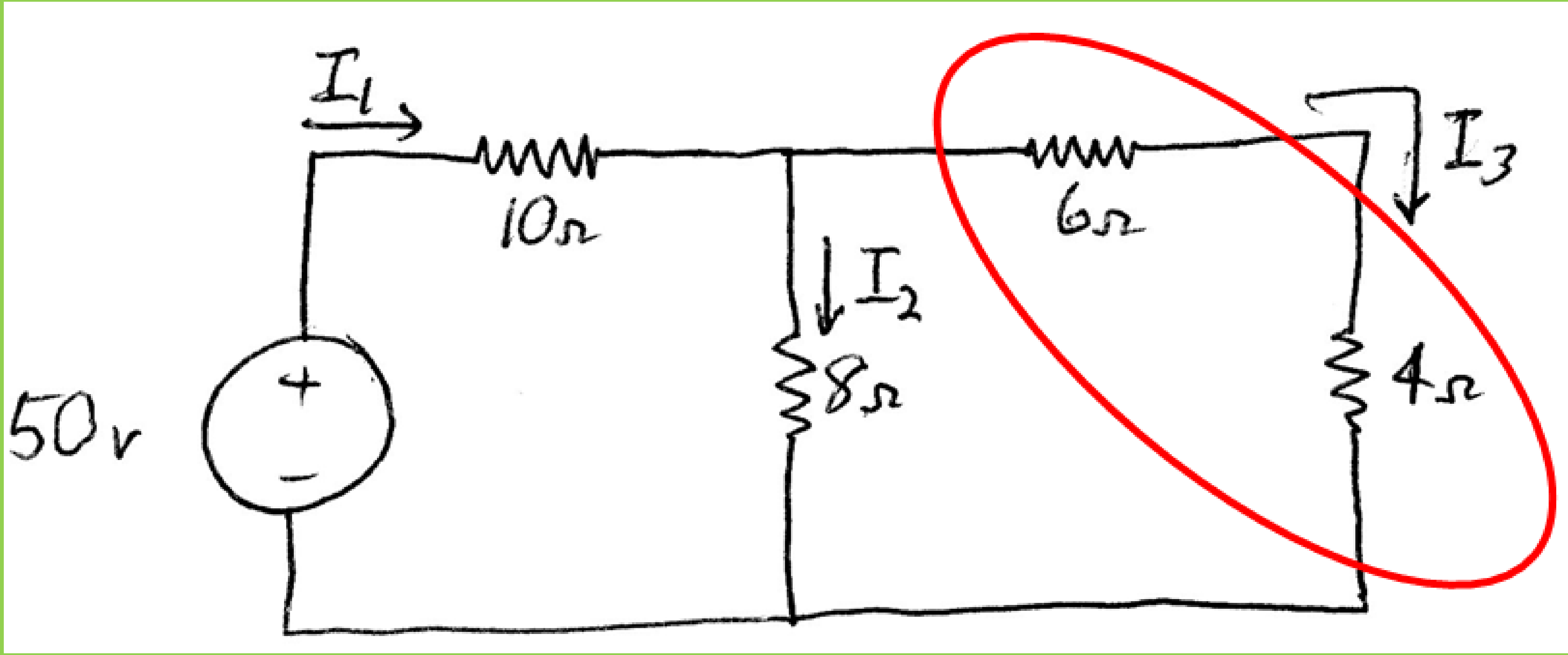
# *Example on resistor combinations*

Solve for the currents through each resistor and the voltages across each resistor using  series and parallel simplification.



# *Example on resistor combinations*

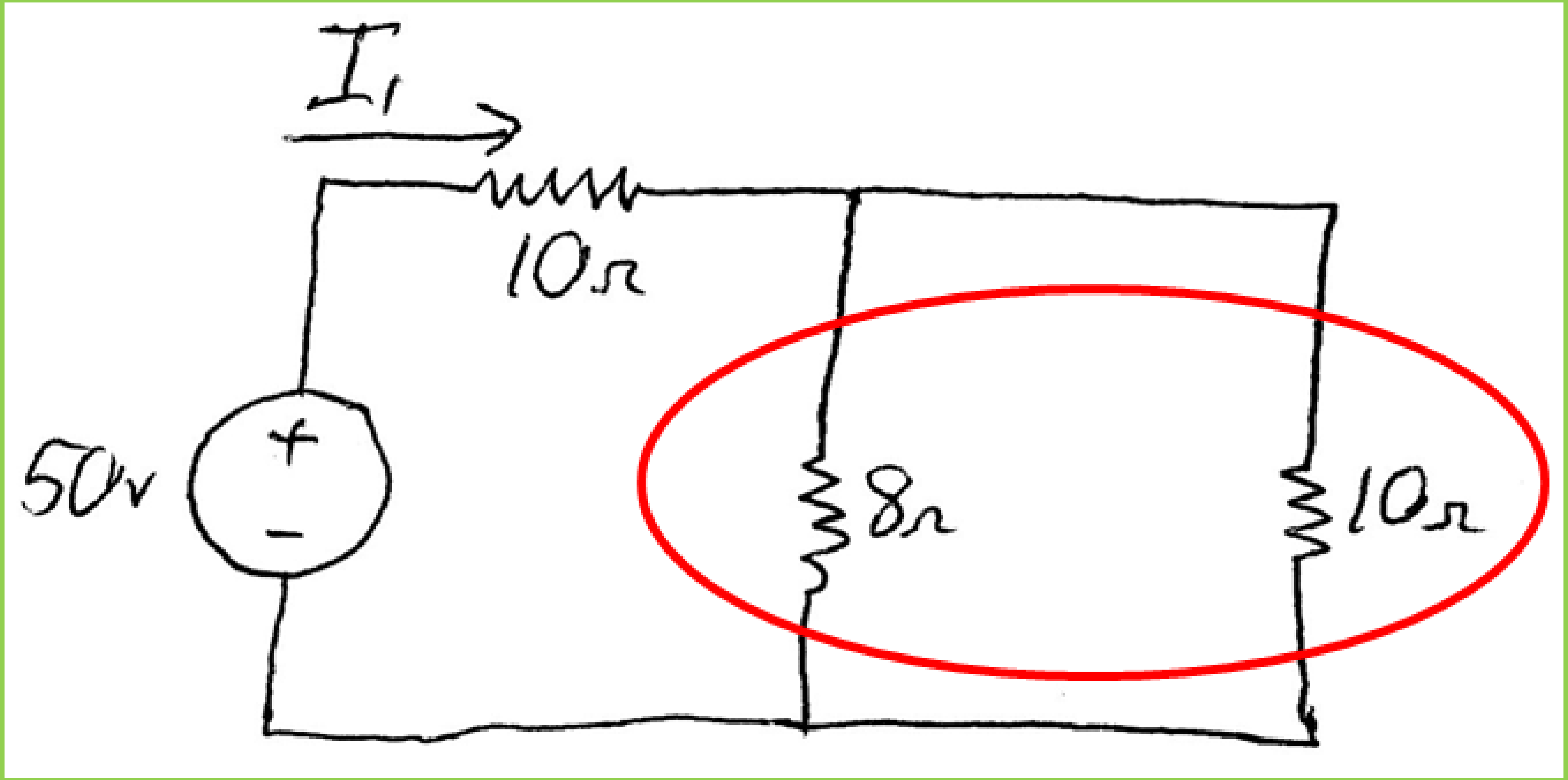
The 6 and 4 ohm resistors are in series, so are combined into $6 + 4 = 10\Omega$



# *Example on resistor combinations*

The 8 and 10 ohm resistors are in parallel, so are combined into

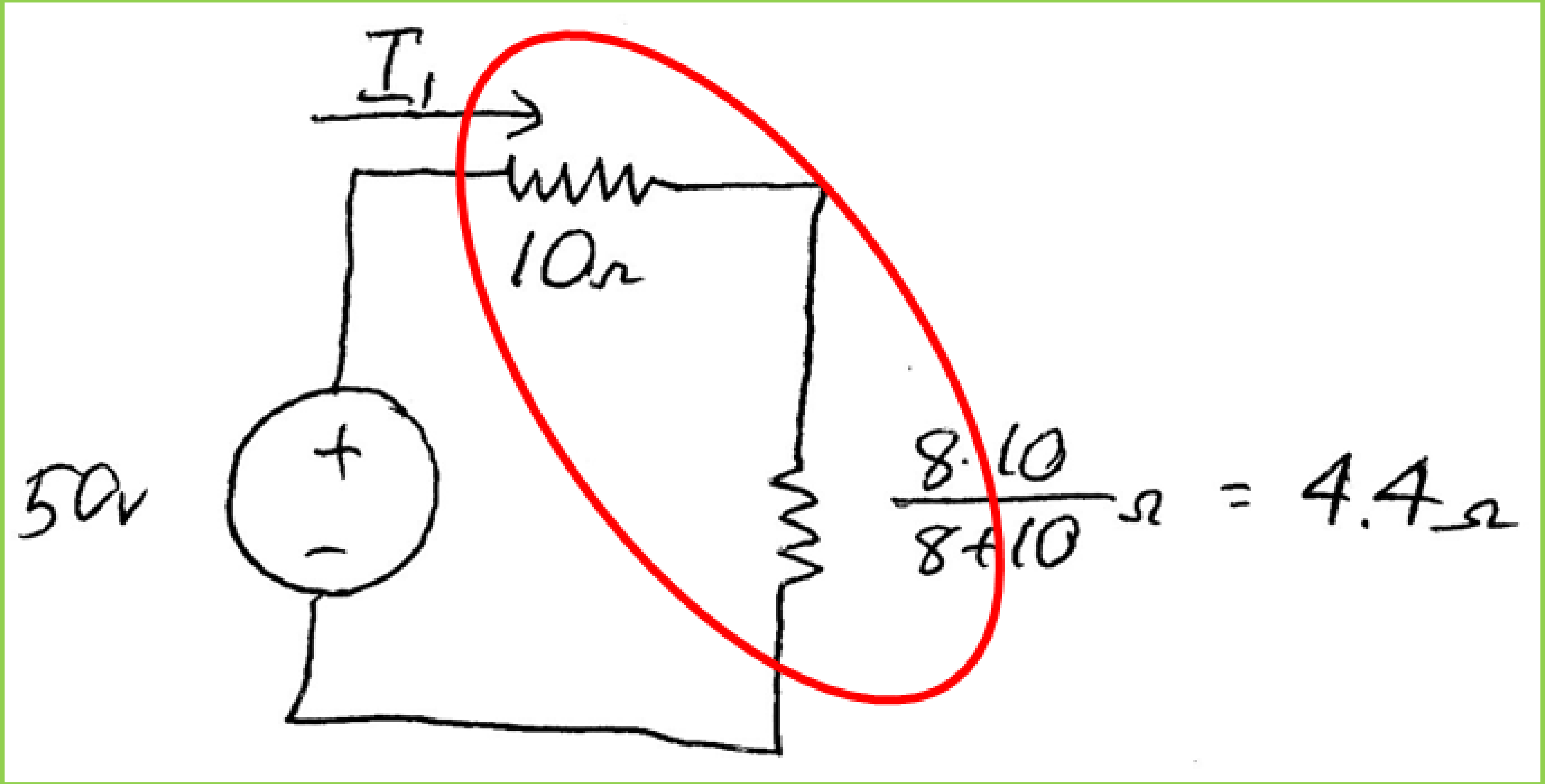
$$8 \cdot 10/(8+10) \ = 14.4\,\Omega$$



# *Example on resistor combinations*

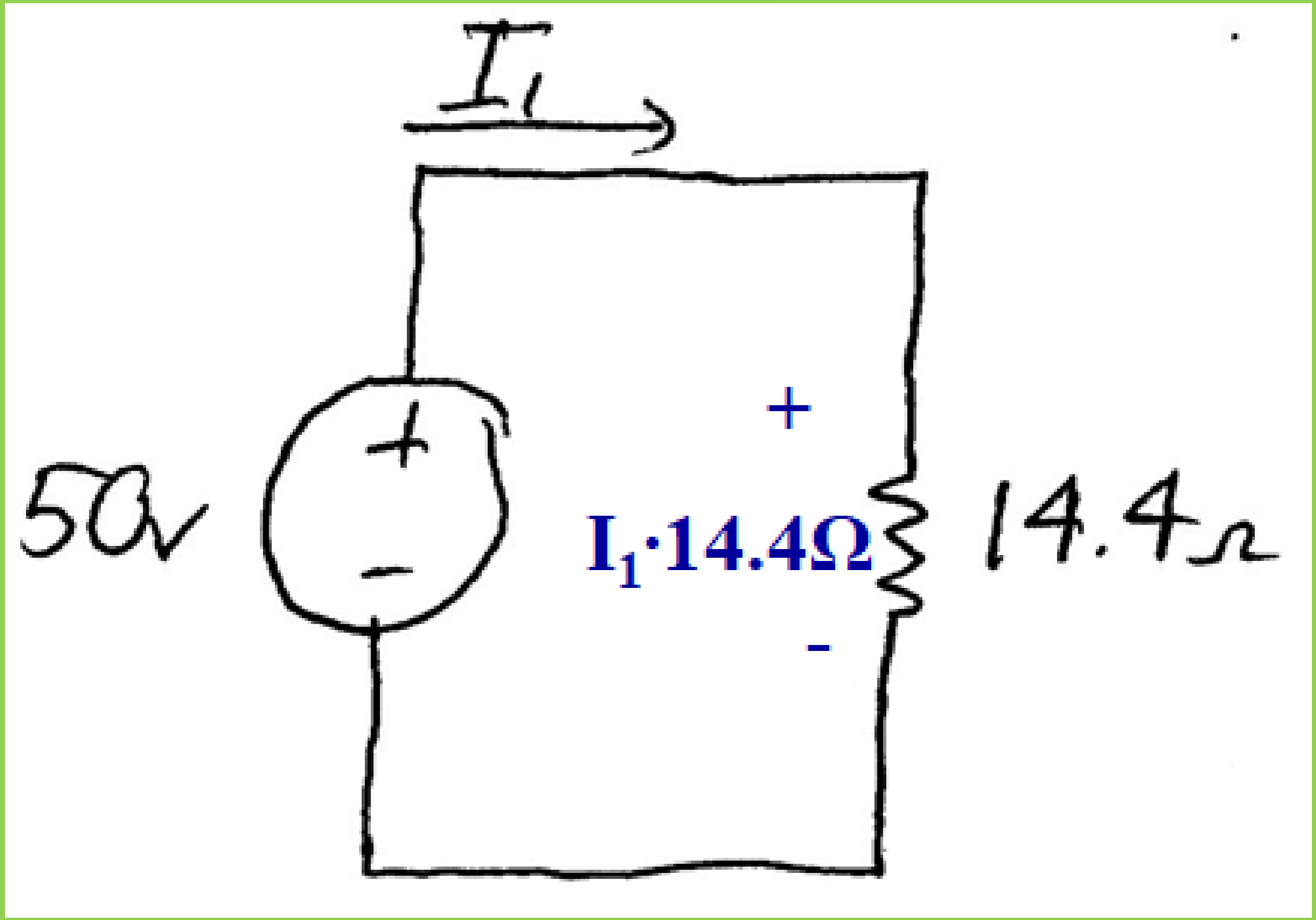
The 10 and 4.4 ohm resistors are in series, so are combined into

$$10 + 4 \ = \ 14.4\Omega$$



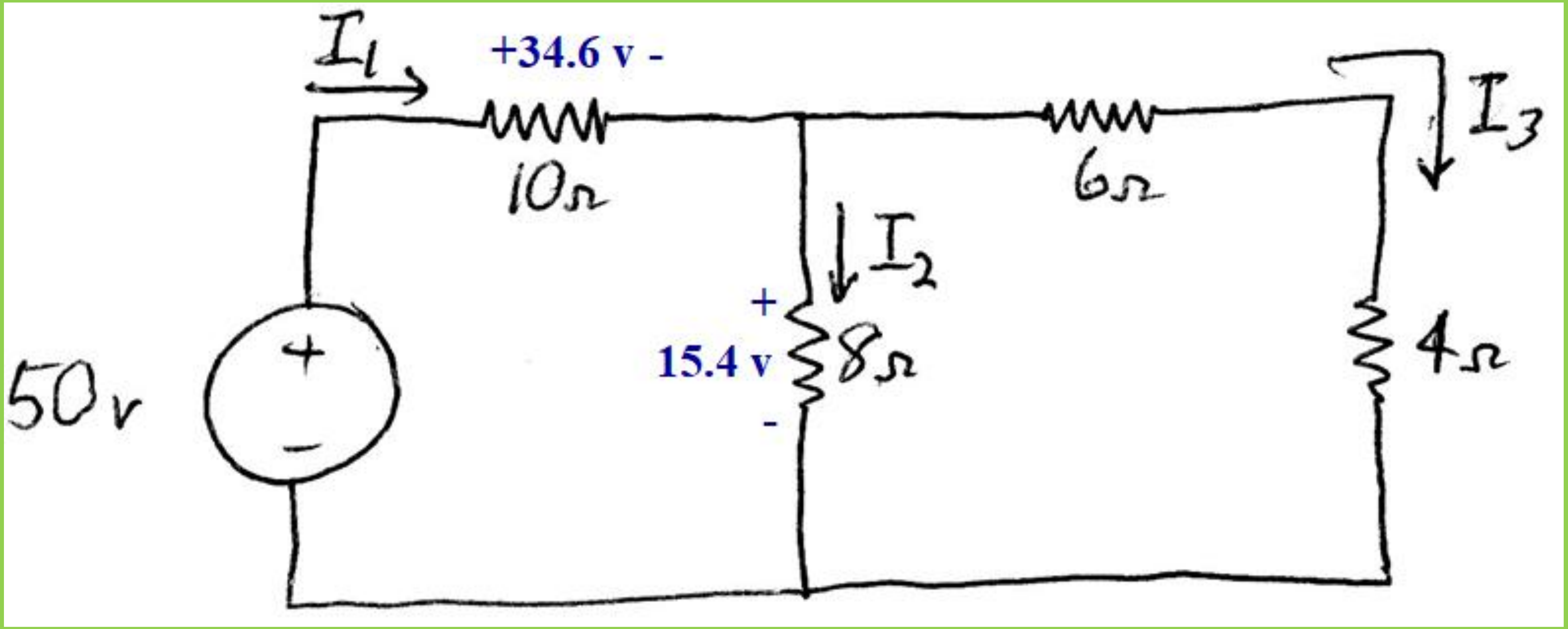
# *Example on resistor combinations*

Writing KVL, $I_1 \cdot 14.4\Omega - 50\ v = 0$ Or $I_1 = 50\ v\ /\ 14.4\Omega = 3.46\ A$



# *Example on resistor combinations*

If $I_1 = 3.46\ A$, then $I_1 \cdot 10\ \Omega = 34.6\ v$ So the voltage across the $8\ \Omega = 15.4\ v$



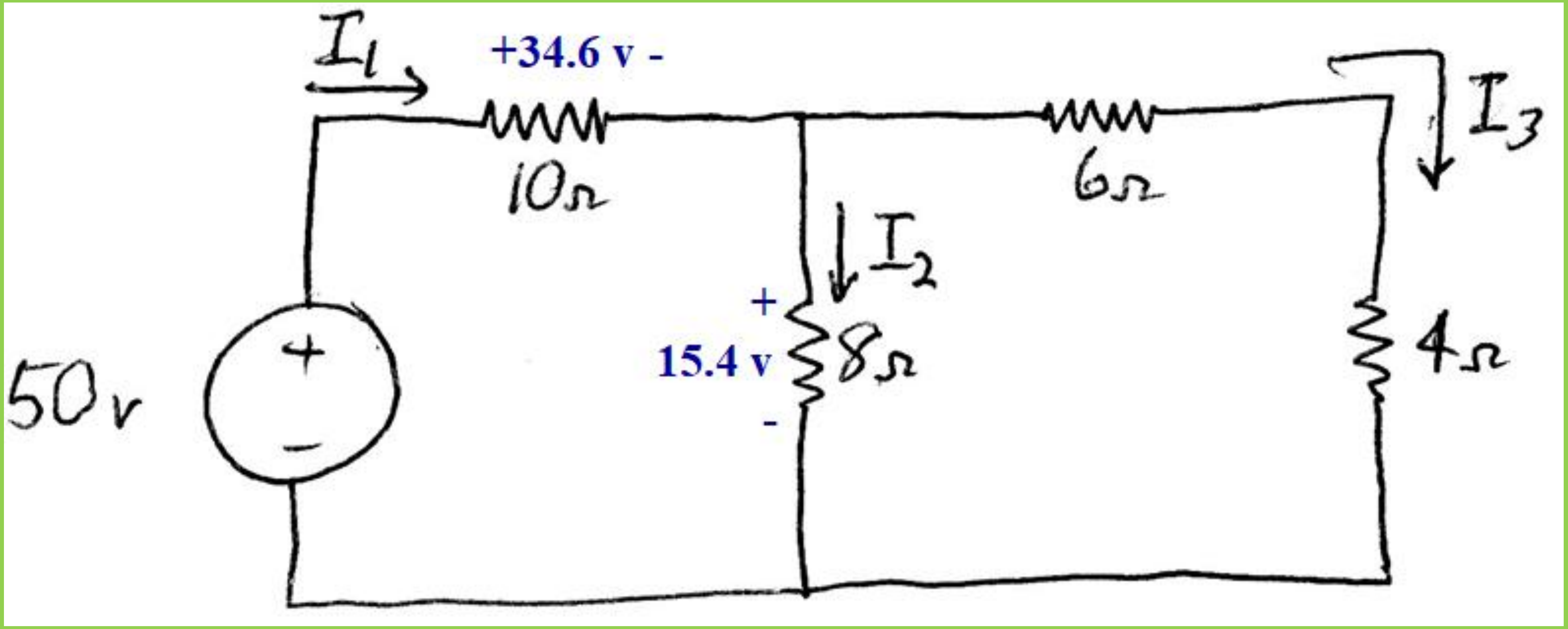
# Summary of electric circuits

1. The amount of current is the same at every place in a series circuit; $I = \frac{q}{t}$.
2. The power provided by the battery ($P = I\Delta V$) is exactly equal to the power dissipated in the resistors ($P = I^2R$)
3. Ohm's Law applies to resistors: $\Delta V = IR$
4. Series circuit: effective $R = R_1 + R_2 + R_3$
5. Parallel circuit: effective $R$ is

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

# *Example on resistor combinations*

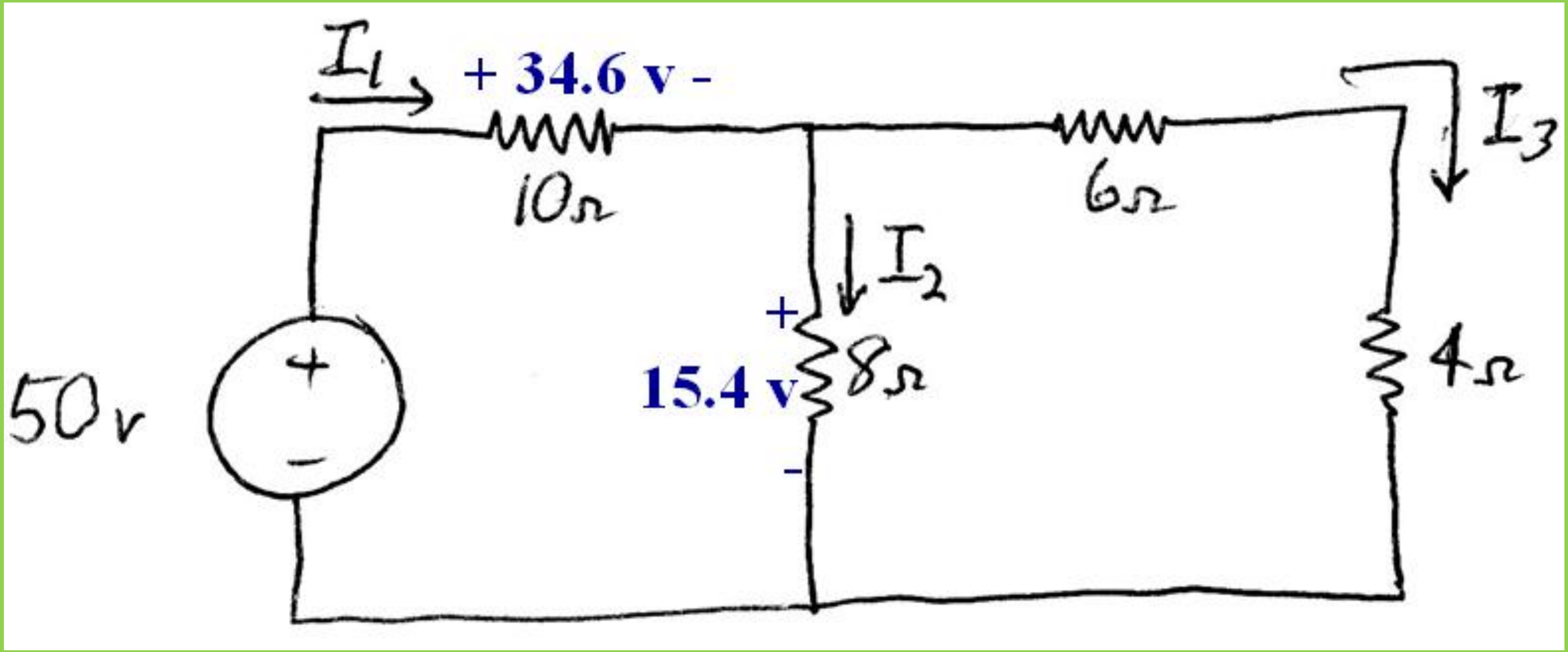
If $I_1 = 3.46\ A$, then $I_1 \cdot 10\ \Omega = 34.6\ v$ So the voltage across the $8\ \Omega = 15.4\ v$



# *Example on resistor combinations*

If $I_2 \cdot 8\ \Omega\ =\ 15.4\ v$, then $I_2\ =\ 15.4/8\ =\ 1.93\ A$

By KCL, $I_1 - I_2 - I_3 = 0,$  so $I_3\ =\ I_1 - I_2\ =\ 1.53\ A$



# Capacitance, Dielectrics, Electric Energy Storage

**Capacitors(*openstax chapter 19 pg 677, Halliday chapter 25  pg743, Serway Part 4 chapter23 , Giancoli chapter24 pg 727)***

- Capacitors are devices that store electric charge.
- The amount of charge $Q$ a *capacitor* can store depends on two major factors the voltage applied and the capacitor's physical characteristics, such as its size. Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - to eliminate sparking in automobile ignition systems
  - energy-storing devices in electronic flashes

**Definition of Capacitance**

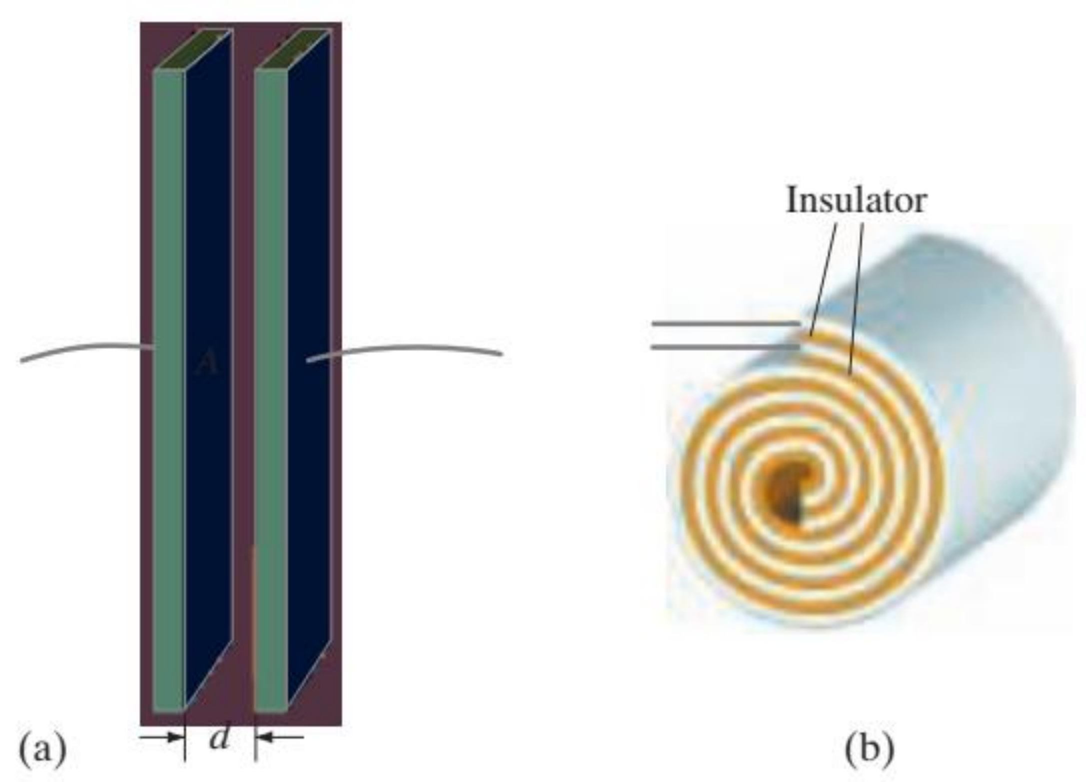
Charge per unit volt i.e.

# Capacitance

- The **capacitance**, *C*, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors. $C = \frac{Q}{V}$, The field is proportional to the charge $E \propto Q$,and since across parallel plates is $V = Ed, V \propto E$ thus $V \propto Q$
- The SI unit of capacitance is the **farad** (F).
- The farad is a large unit, typically you will see microfarads (mF) and picofarads (pF).
- Capacitance will always be a positive quantity
- The capacitance of a given capacitor is constant.
- The capacitance is a measure of the capacitor's ability to store charge .

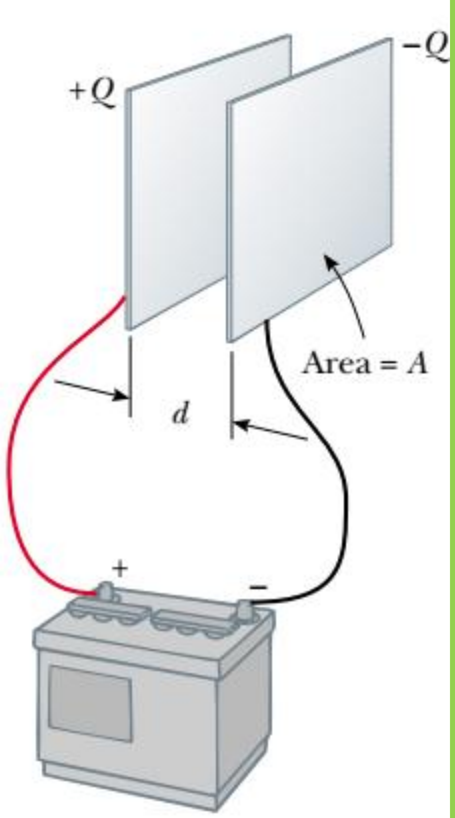
# Capacitor Shapes

- A capacitor consists of two conductors separated by an insulator and Capacitors come in a wide range of sizes and shapes.
- Capacitors: diagrams of (a) parallel plate, (b) cylindrical (rolled up parallel plate).



# Parallel Plate Capacitor

- The parallel plate capacitor shown in *Figure below* has two identical conducting plates, each having a surface area $A$ , separated by a distance $d$ (with no material between the plates). When the capacitor is charged by connecting the plates to the terminals of a battery, the plates carry equal amounts of charge. One plate carries positive charge, and the other carries negative charge.




- It can be shown that for a parallel plate capacitor there are only two factors ( $A$ and $d$ ) that affect its capacitance $C$.
- The capacitance of a parallel plate capacitor in equation form is given by $C = \varepsilon_O \frac{A}{d}$. $A$ is the area of one plate in square meters, and $d$ is the distance between the plates in meters. The constant $\varepsilon_O$ is the permittivity of free space; its numerical value in SI units is $\varepsilon_O$ = 8.85×10$^{-12}$ F/m. The units of $F/m$ are equivalent to $C^2/N \cdot m^2$ .

# Parallel Plate Capacitor cont

- This field applies a force on electrons in the wire just outside of the plates.
- The force causes the electrons to move onto the negative plate.
- This continues until equilibrium is achieved.
    - The plate, the wire and the terminal are all at the same potential.
- At this point, there is no field present in the wire and the movement of the electrons ceases.
- The plate is now negatively charged.
- A similar process occurs at the other plate, electrons moving away from the plate and leaving it positively charged.
- In its final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery.

# Examples

What is the capacitance of a parallel plate capacitor with metal plates, each of area 1.00 $m^2$ , separated by 1.00 mm? (b) What charge is stored in this capacitor if a voltage of $3.00\times10^3$ V is applied to it? *(openstax pg 680)*

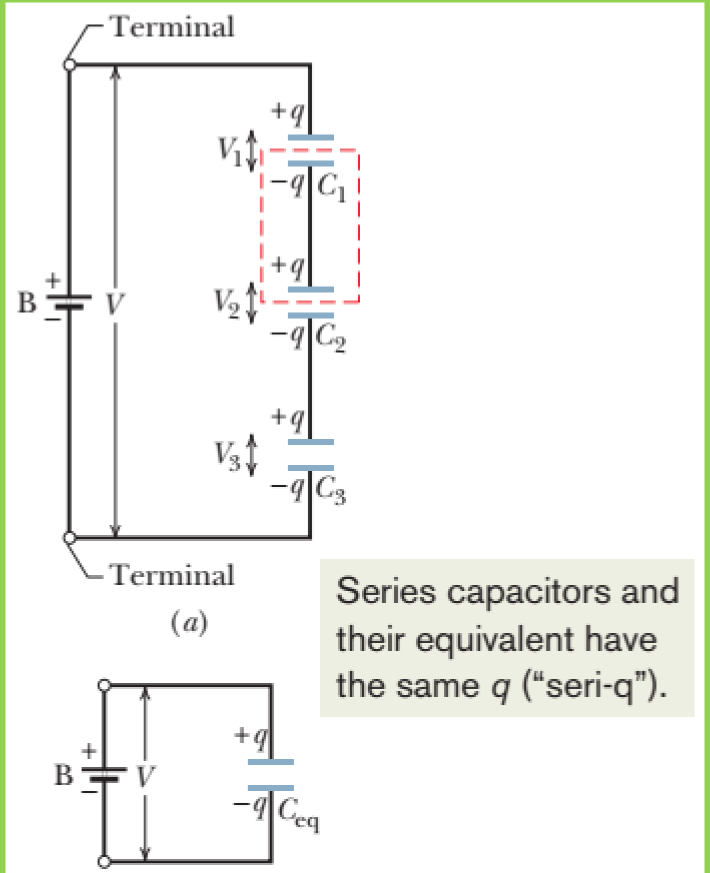
A parallel-plate capacitor with air between the plates has an area $A$ =$2.00 \times 10^{-4} m^2$ and a plate separation $d$ = 1.00 $mm$. Find its capacitance. *(Serway pg 800)*

($a$) Calculate the capacitance of a parallel-plate capacitor whose plates are and are separated by a 1.0-mm air gap. ($b$) What is the charge on each plate if a 12-V battery is connected across the two plates? ($c$) What is the electric field between the plates? ($d$) Estimate the area of the plates needed to achieve a capacitance of 1 F, given the same air gap $d$.

*(Giancoli pg 730)*

# Capacitors in Series and Parallel

- Capacitors can be connected together in various ways. Two common ways are in *series*, or in *parallel (Giancoli pg733, Serway pg 802, Halliday pg 750, openstax 683).*
- Figure below shows three capacitors connected *in series* to battery B.



# Capacitors in Series

- We start with capacitor 3 and work upward to capacitor 1.When the battery is first connected to the series of capacitors, it produces charge $-q$ on the bottom plate of capacitor 3. That charge then repels negative charge from the top plate of capacitor 3 (leaving it with charge $+q$).
- The repelled negative charge moves to the bottom plate of capacitor 2 (giving it charge $-q$). That charge on the bottom plate of capacitor 2 then repels negative charge from the top plate of capacitor 2 (leaving it with charge $+q$) to the bottom plate of capacitor 1 (giving it charge-$q$).
- Finally, the charge on the bottom plate of capacitor 1 helps move negative charge from the top plate of capacitor 1 to the battery, leaving that top plate with charge $+q$.

# Capacitors in Series cont

- Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge *q* and the same *total* potential difference *V* as the actual series capacitors.
- Capacitors connected in series have the same charge to find the potential difference of each actual capacitor

$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{c_2}, V_3 = \frac{q}{C_3}$$

- The total potential difference $V$ due to the battery is the sum

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

- The equivalent capacitance is then $C_{eq} = \frac{q}{V}$ or $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

# Examples on Capacitors in Series and parallel

*(Halliday pg 752, openstax pg685, Serway pg 806, Giancoli pg 734)*

***Exercise***

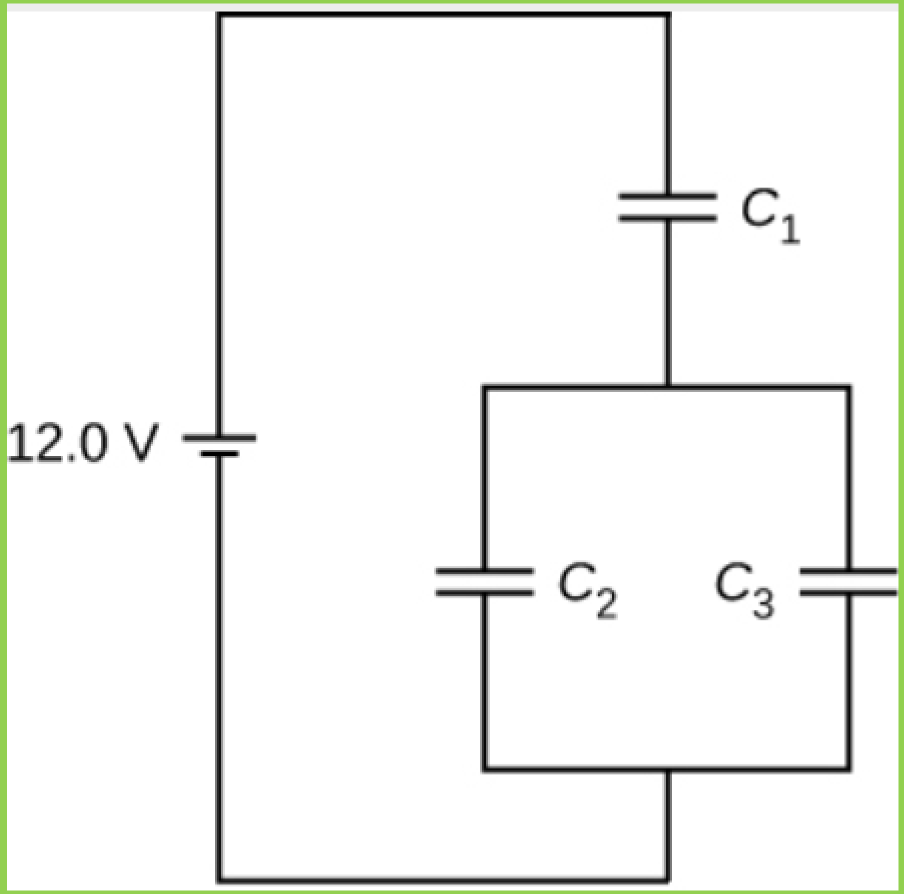
An 8μF capacitor and a 5μF capacitors are connected in parallel and this combination is connected in series with a 2μF capacitor. Then the whole combination is connected to a potential difference of 8V. (i) Calculate the equivalent capacitance of the combination (ii) Calculate the total charge stored by the combination (iii) Calculate the charge across the 2μF capacitor (iv) Calculate the potential difference across the 2μ (v) Calculate the potential difference across the 8μF and 5μF capacitors (vi) Calculate the charge stored by the 8μF and 5μF capacitors.

# Examples on Capacitors in Series and parallel

Calculate the net capacitance $C$ of the capacitor combination shown below when the capacitances $C_1 = 12.0\mu F$, $C_2 = 2.0\mu F$, and $C_3 = 4.0\mu F$. When a $12.0V$ potential difference is maintained across the combination, find the charge and the voltage across each capacitor.

(ii) Calculate also the energy stored in the capacitor network in below when the capacitors are fully charged.

# Examples on Capacitors in Series and parallel cont



# Solution

The equivalent capacitance for is $C_2$ and $C_3$ is $C_{23} = C_2 + C_3 = 2.0\mu F + 4.0\mu F = 6.0\mu F$.The whole three-capacitor combination is equivalent to two capacitors in series
$\frac{1}{C} = \frac{1}{12.0\mu F} + \frac{1}{6.0\mu F} = \frac{1}{4.0\mu F} \Rightarrow C = 4.0\mu F$. the capacitors in series, they have the same charge, i.e.$Q_1 = Q_{23}$. The capacitors $C_2$and $C_3$ share the 12V potential difference $12V = V_1 + V_{23} = \frac{Q_1}{C_1} + \frac{Q_{23}}{C_{23}} = \frac{Q_1}{12.0\mu F} + \frac{Q_1}{6.0\mu F} \Rightarrow$ $Q_1 = 48.0\mu C$, and the potential difference across capacitor1 is $V_1 = \frac{Q_1}{C_1} = \frac{48.0\mu C}{12\mu F} = 4V$.

# Solution cont

Since capacitors and are connected in parallel, they are at the same potential difference$V_2 = V_3 = 12.0V - 4.0V = 8.0V$. Hence, the charges on these two capacitors are, $Q_2 = C_2V_2 = (2.0\mu F)(8.0V) = 16.0\mu C$, $Q_3 = C_3V_3 = (4.0\mu F)(8.0V) = 32.0\mu C$.

(ii) $E_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(12.0\mu F)(4.0V)^2 = 96\mu J$ $E_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(2.0\mu F)(8.0V)^2 = 64\mu J$

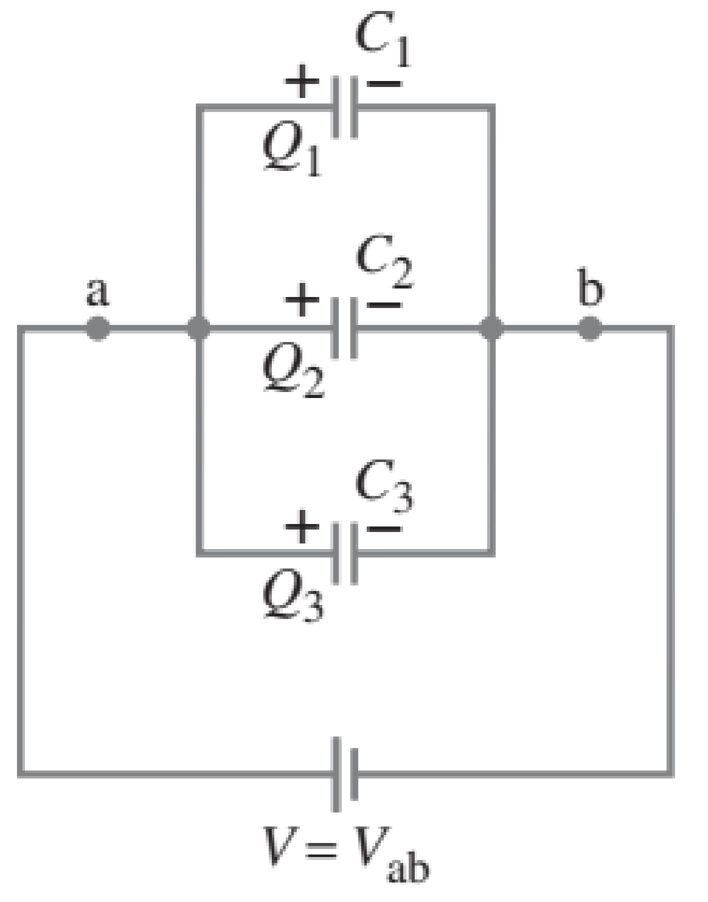
$$E_3 = \frac{1}{2}C_3V_3^2 = \frac{1}{2}(4.0\mu F)(8.0V)^2 = 130\mu J$$

*The total energy stored in this network is* $E_1 + E_2 + E_3 = 96\mu J + 64\mu J + 130\mu J = 290\mu J$.

# Capacitors in Parallel

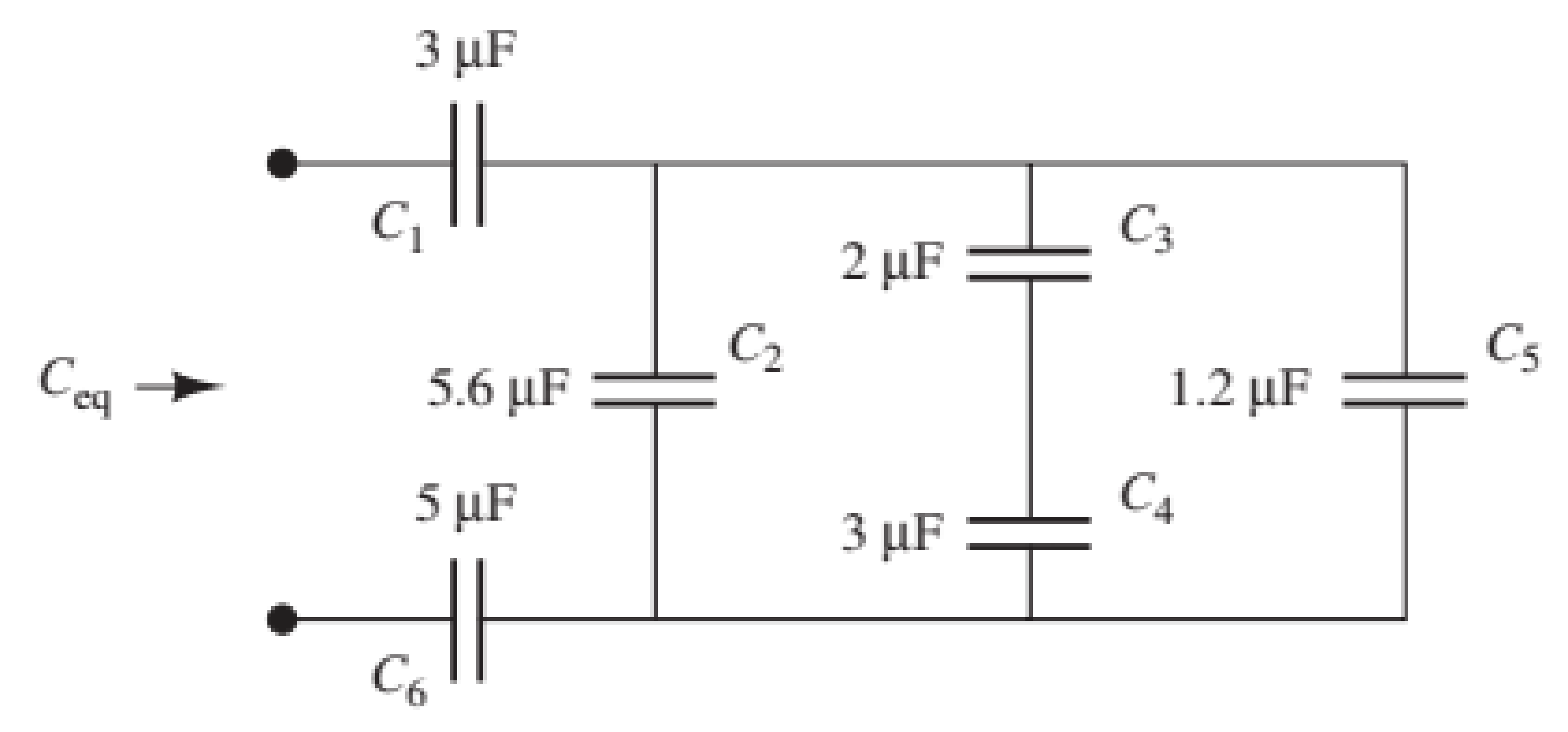
- A circuit containing three capacitors connected in parallel is shown in below. They are in "parallel" because when a battery of voltage $V$ is connected to points $a$ and $b$, this voltage exists across each of the capacitors.
- The individual potential differences across capacitors connected in parallel are the same And are equal to the potential difference applied across the combination.
- The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors. $Q = C_1V + C_2V + C_3V$

$$C_{eq} = C_1 + C_2 + C_3$$

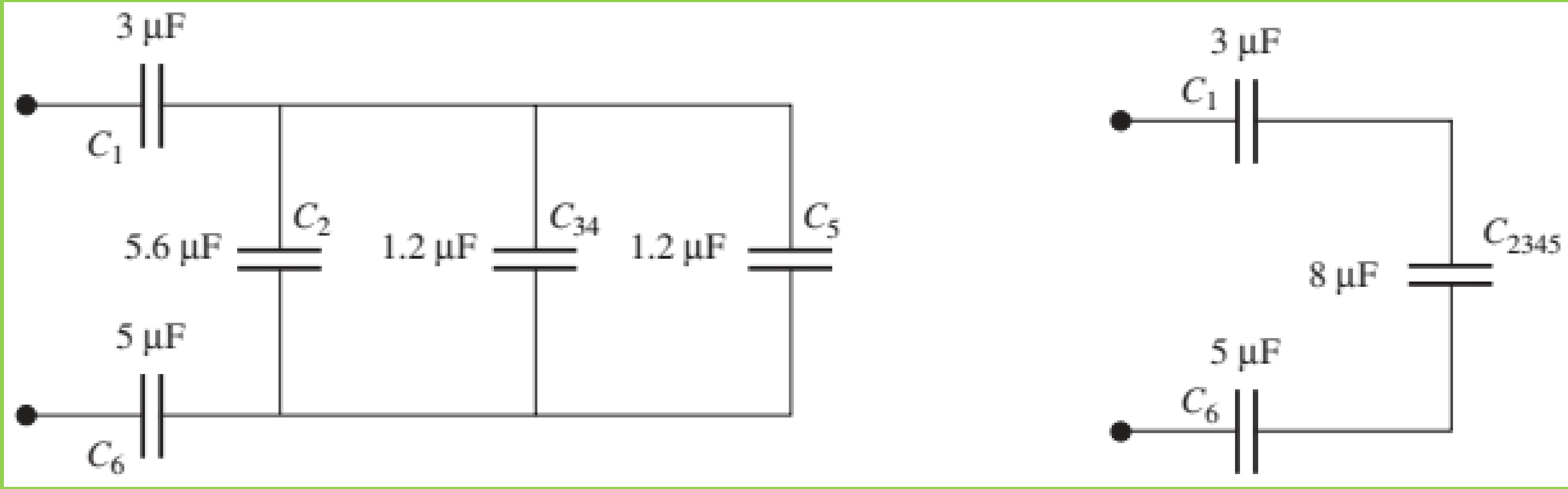


# Worked example

Find the equivalent capacitance of the capacitor network shown in Figure



# *Solution Cont.*



# ***Solution***

$C_3$ and $C_4$ are in series. Combine these capacitors using $\frac{1}{C_3} + \frac{1}{C_4} =$

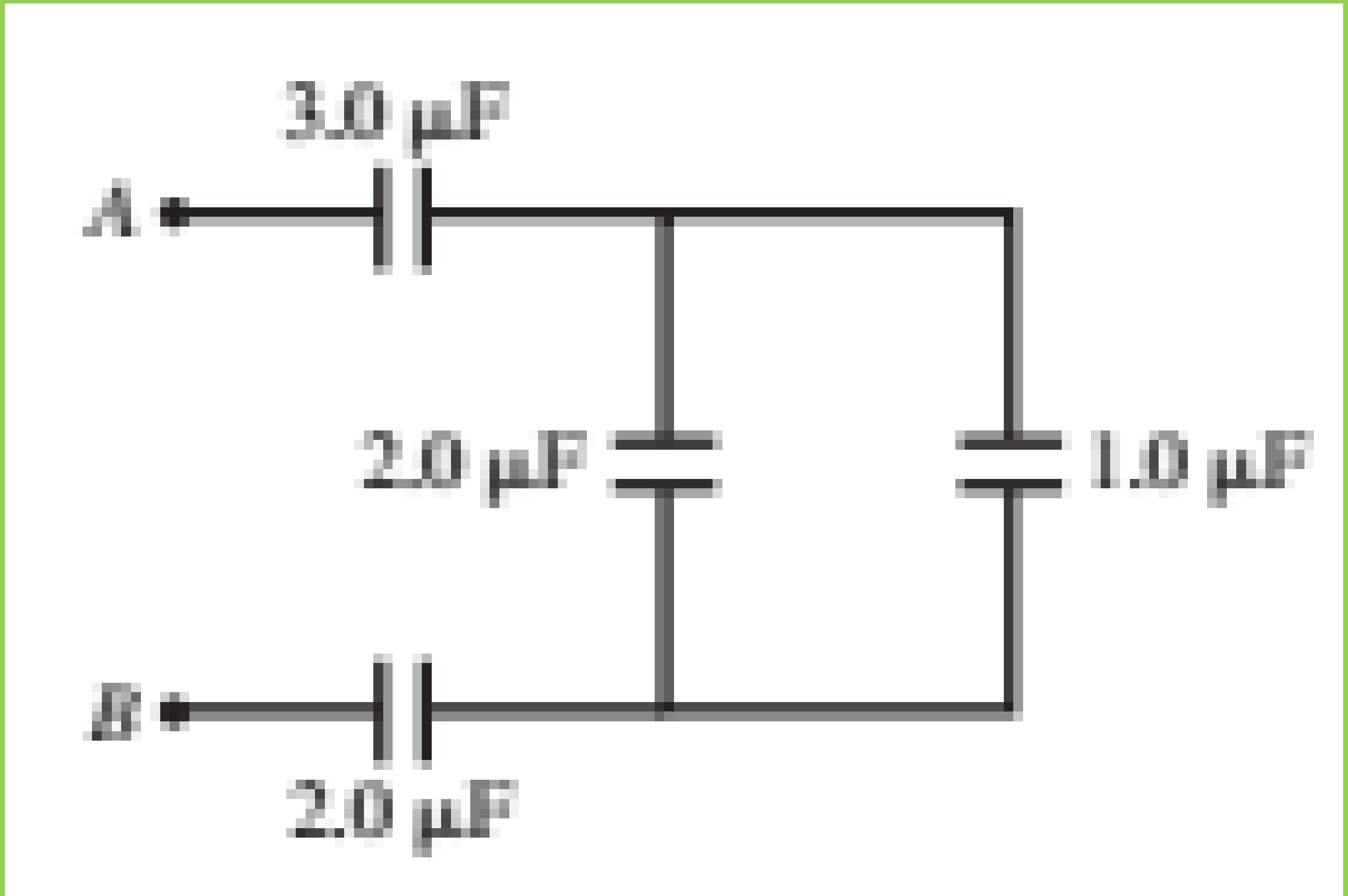
$$C_{34} = 1.2\mu F$$

Now that $C_2$, $C_{34}$, and $C_5$ are in parallel, and their equivalent capacitance corresponds to: $C_{2345} = C_2 + C_{34} + C_5 = 8\mu F$

Finally, find the series combination of the three capacitors, $C_1$, $C_{2345}$, and $C_6$

# *Exercise*

What's the equivalent capacitance measured between $A$ and $B$? Find also the energy stored in the $1\mu F$ capacitor when a $53V$ battery is connected between $A$ and $B$.



# Magnetostatics

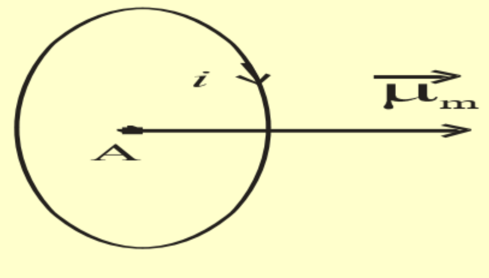
- *Magnetostatics* deals with the behaviour of stationary Magnetic fields.
- Oersterd and Ampere proved experimentally that the current carrying conductor produces a magnetic field around it.
- The origin of Magnetism is linked with current and magnetic quantities are measured in terms of current.

***Magnetic dipole:*** Any two opposite magnetic poles separated by a distance *d* constitute a magnetic dipole.

If $m$ is the magnetic pole strength and $l$ is the length of the magnet, then its dipole moment is given by $\mu_m = m \times l$

If an Electric current of $i$ amperes flows through a circular wire of one turn having an area of cross section $a$ m$^2$, then the magnetic moment is

$\mu_m = i \times a$, Unit: ampere (metre)$^2$

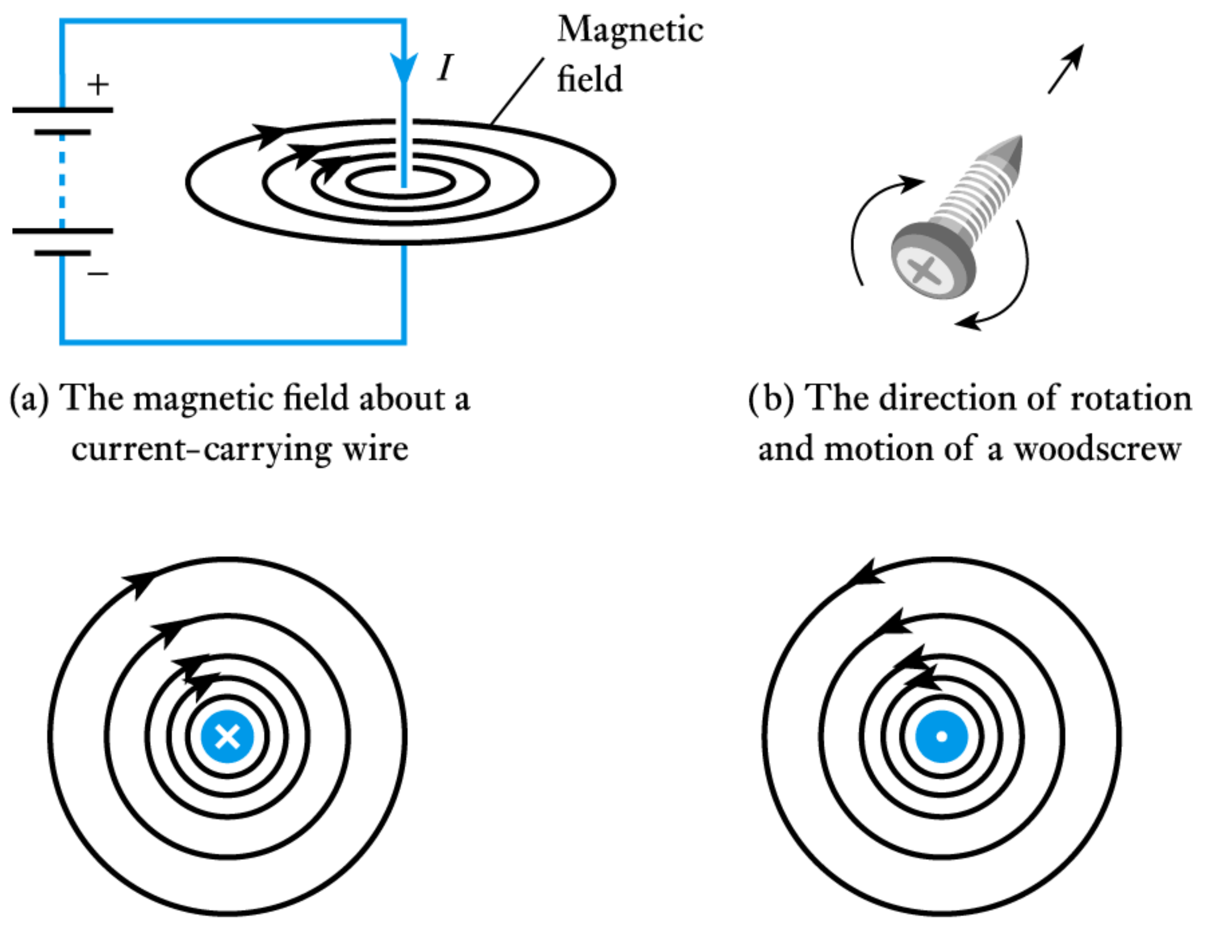



Magnetic moment

# Electromagnetism

***Electromagnetism:*** A wire carrying a current $I$ causes a **magnetomotive force (m.m.f)** $F$

- this produces a **magnetic field**
- $F$ has units of Amperes
- for a single wire $F$ is equal to $I$




(a) The magnetic field about a current-carrying wire

(b) The direction of rotation and motion of a woodscrew

(c) The magnetic field about a current flowing into the page

(d) The magnetic field about a current flowing out of the page

# Magnetic field

- The magnitude of the field is defined by the **magnetic field strength**, ***H*** , where $H = \frac{I}{l}$, where *l* is the length of the magnetic circuit.

***Example:*** A straight wire carries a current of $5\ A$. What is the magnetic field strength $H$ at a distance of $100mm$ from the wire?

Magnetic circuit is circular. $r = 100mm$, so path $= 2\pi r = 0.628m$

$$H = \frac{I}{l} = \frac{5}{0.628} = 7.96A/m$$

The magnetic field produces a **magnetic flux**, $\boldsymbol{\Phi}$ It is defined as the total number of magnetic lines of force passing perpendicular through a given area. It can also be defined as the total number of lines of force emanating from North Pole. Flux has units of weber (Wb)

# **Magnetic flux density**, ***B (or Magnetic Induction)***

- Strength of the flux at a particular location is measured in term of the **magnetic flux density**, ***B***. It is defined as the number of Magnetic Lines of force passing through an unit area of cross section. And it is given by $B = \frac{Magnetic\ Flux}{Unit\ Area} = \frac{\varphi}{A}$ and $B = \frac{F}{m} = \frac{Force\ experienced}{Pole\ strength}$
- Flux density has units of tesla (T) (equivalent to 1 $Wb/m^2$)
- Flux density at a point is determined by the field strength and the material present $B = \mu H$, or $B = \mu_0 \mu_r H$.

where $\mu$ is the permeability of the material, $\mu_r$ is the relative permeability and $\mu_0$ is the permeability of free space

# Magnetic field strength (or) Magnetic field intensity(H)

Magnetic field intensity or magnetic field strength at any point in a magnetic field is e qual to $\frac{1}{\mu}$ times the force per pole strength at that point.

$H = \frac{1}{\mu} \times \left(\frac{F}{m}\right) = \frac{B}{\mu}$ ampere turns/metre, $\mu$ = permeability of the medium**.**

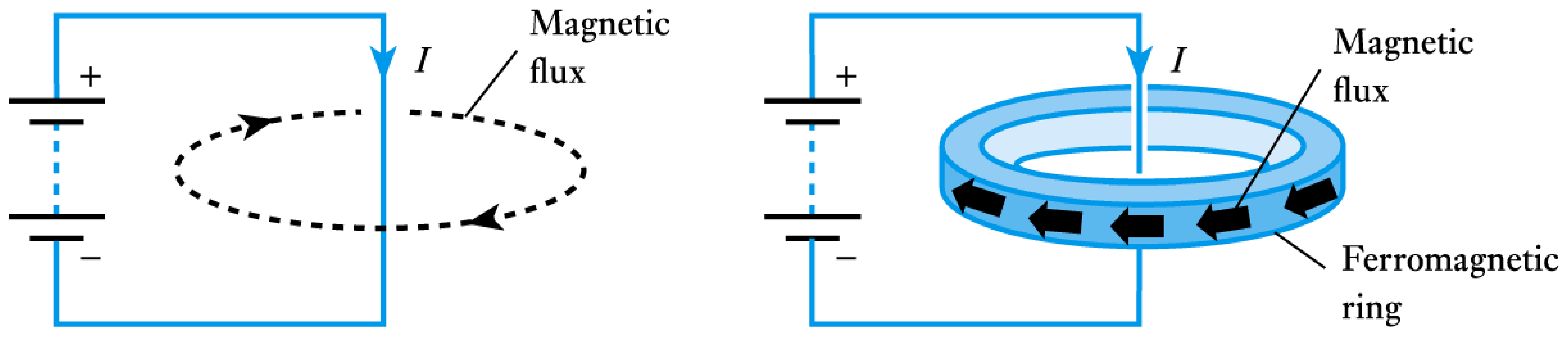
***Magnetization (or) Intensity of Magnetization (M)***

Intensity of magnetization ($M$) is defined as the Magnetic moment per unit Volume. It is expressed in **ampere/metre.**

Intensity of Magnetization measures the magnetization of the magnetized specimen.

# Ferromagnetic material

- Adding a ferromagnetic ring around a wire will increase the flux by several orders of magnitude. since $\mu_r$ for ferromagnetic materials is 1000 or more.

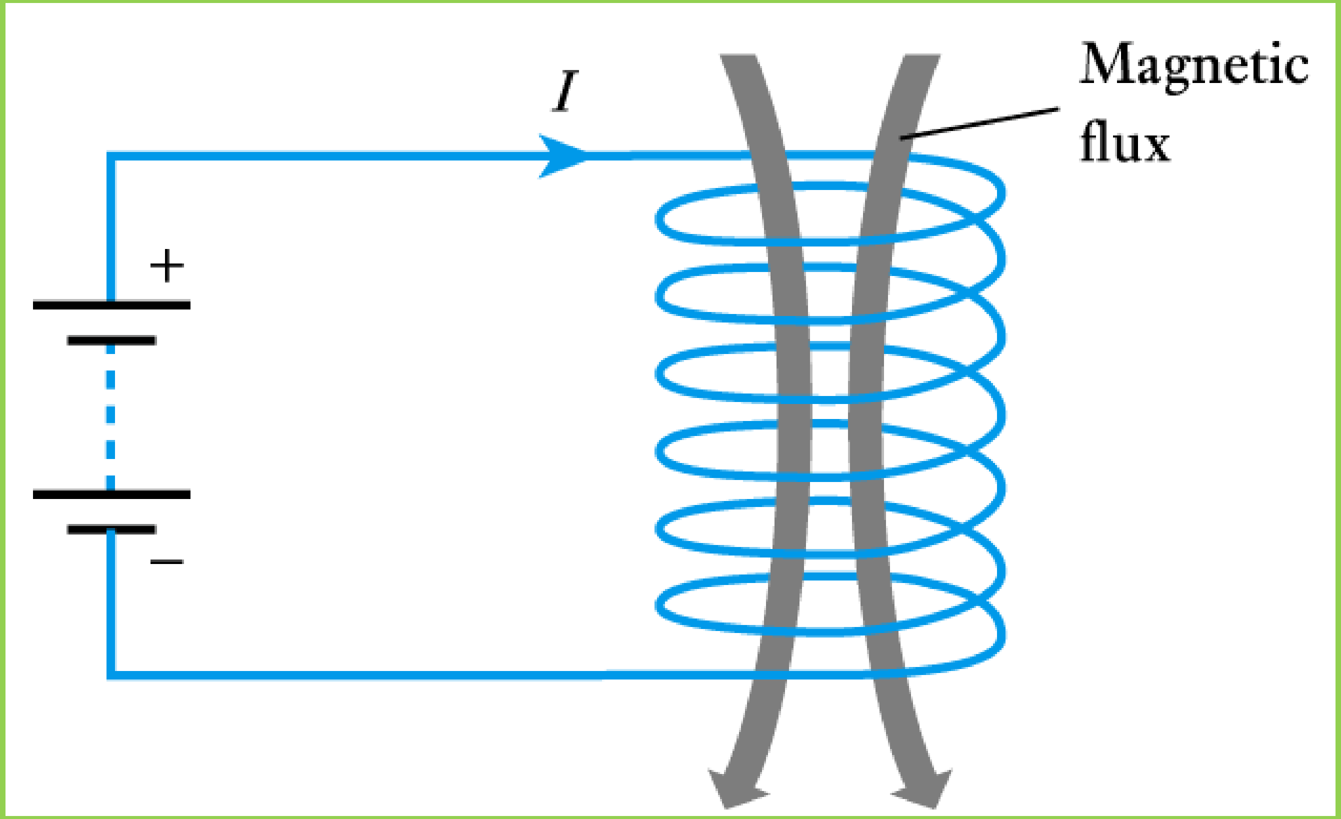



(a) The magnetic flux about a current-carrying wire in air

(b) The effect of adding a ferromagnetic ring

# coil

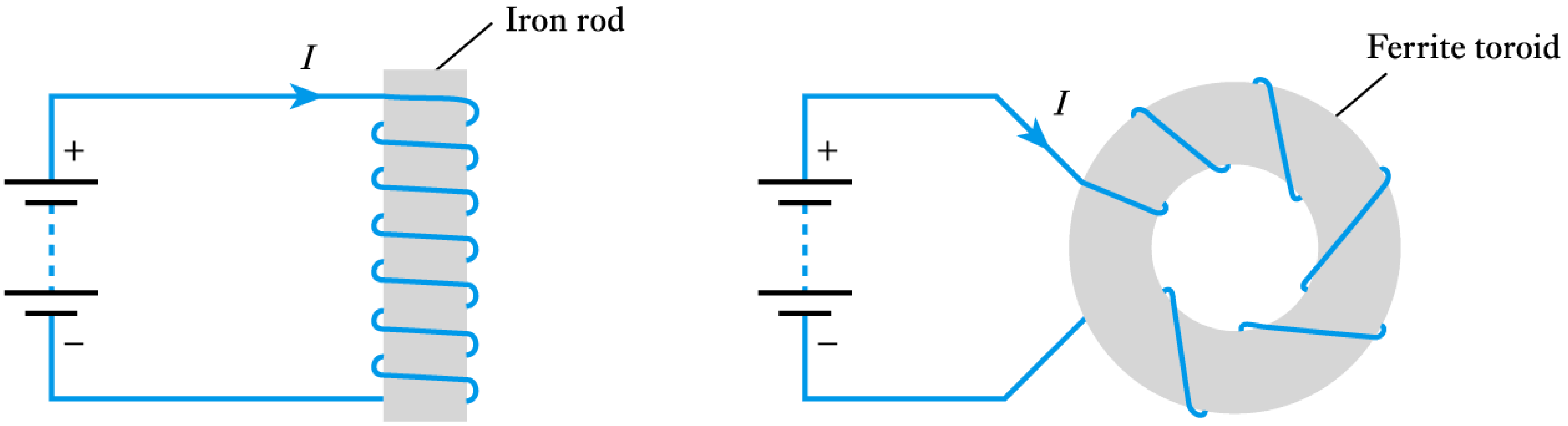
- When a current-carrying wire is formed into a **coil** the magnetic field is concentrated. For a coil of $N$ turns the m.m.f. ($F$) is given by $F = NI$ and the field strength is $H = \frac{NI}{l}$



# Ferromagnetic material cont.

➢The magnetic flux produced is determined by the permeability of the material present

- a ferromagnetic material will increase the flux density




(a) A coil wound on an iron rod to increase the flux density

(b) A coil wound on a ferrite toroid

# Magnetic susceptibility ($\chi$)

- It is defined as the ratio of magnetization produced in a sample to the magnetic field intensity. i.e. magnetization per unit field intensity.
- It is the measure of the ease with which the specimen can be magnetized by the magnetizing force. $\chi = \frac{M}{H}$ No units.

**Magnetic permeability ($\mu$)** : It is the measure of degree at which the lines of force can penetrate through the material.

- It is defined as the ratio of magnetic flux density in the sample to the applied magnetic field intensity. $\mu = \mu_0\mu_r = \frac{B}{H}$

$\mu_0$ = permeability of free space = $4\pi \times 10^{-7} Hm^{-1}$

$\mu_r =$ relative permeability of the medium

# Relative permeability ($\mu_r$)

- It is the ratio of permeability of the medium to the permeability of free space. $\mu_r = \frac{\mu}{\mu_0}$ No units.

## Relation between $\mu_r$ and $\chi$

Total flux density ($B$) in a solid in the presence of magnetic field can be given as $B = \mu_0(H + M)$, Then $\mu_r$ can be related to $\chi$ as $\mu_r = 1 + \chi$.

**Reluctance S:** In a *resistive* circuit, the resistance is a measure of how the circuit opposes the flow of electricity. In a *magnetic* circuit, the **reluctance**, ***S*** is a measure of how the circuit opposes the flow of magnetic flux. In a resistive circuit $R = V/I$, In a magnetic circuit $S = \frac{F}{\Phi}$

The units of reluctance are amperes per weber (A/ Wb)

# Bohr Magneton ($\mu_B$)

- Bohr Magneton is the Magnetic moment produced by one unpaired electron in an atom. It is the fundamental quantum of magnetic moment.

1Bohr Magneton $= \frac{e}{2m} . \frac{h}{2\pi} = \frac{eh}{4\pi m}$ , $1\ \mu_B = 9.27 \times 10^{-24}$ampere metre$^2$

# Moving charges and magnetic fields

- When a charge is placed in an electric field it experiences an electric force provided:
  - The charge must be moving.
  - The velocity of the moving charge must have a component that is perpendicular to the direction of the magnetic field.
- The form of the magnetic force on a moving charged particle is given by the following: $\vec{F} = q_0\vec{v} \times \vec{B}$.

Here $v$ represents the velocity of the particle, $q_0$ is the charge of the particle, and $B$ is the magnetic field.

$$\vec{F} = q_0\vec{v}\vec{B}\sin\theta.$$

The angle θ is the angle between $v$ and $B$.

# *Example*

- A proton in a particle accelerator has a speed of 5.0 x $10^6$ m/s. The proton encounters a magnetic field whose magnitude is 0.40 T and whose direction makes an angle of $30.0^0$ with respect to the proton's velocity. Find the magnitude of the magnetic force on the proton.

- $\vec{F} = q_0\vec{v}\vec{B}\sin\theta = (1.6 \times 10^{-19}C)(5 \times 10^6 m/s)(0.40T)(\sin 3\,0.0^o)$

$$= 1.6 \times 10^{-13} N$$

*Exercise:* A loop of wire has a radius R = 5 cm and current i = 10 A. What is B at the center?

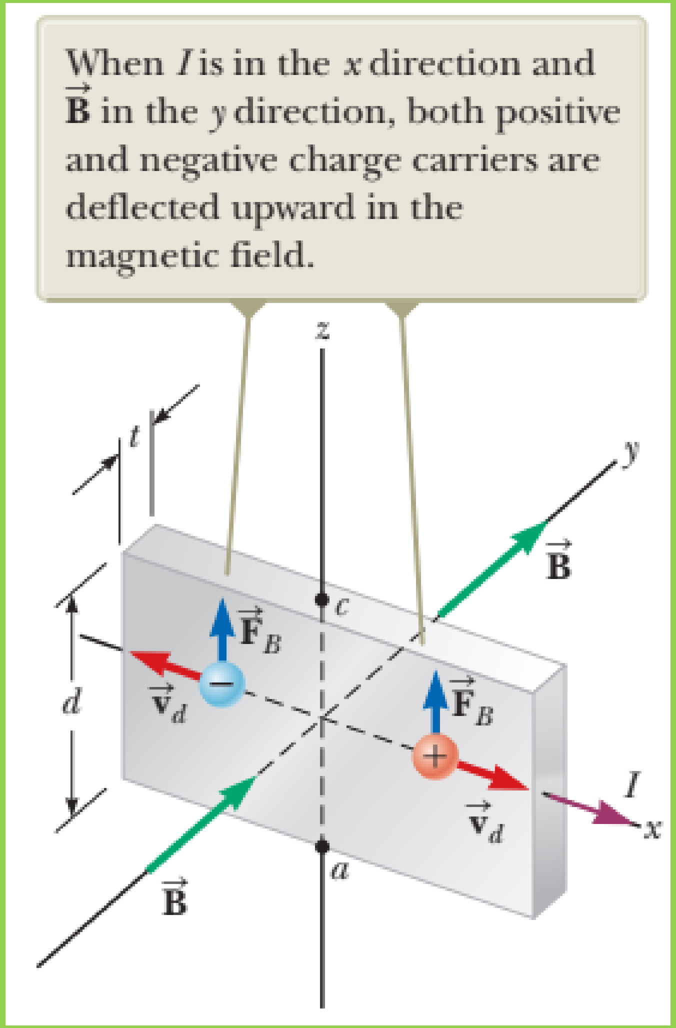
$$B = \frac{\mu_{0I}}{2R} = 4\pi \times 10^{-7}\frac{N}{A^2}\frac{10A}{2(0.05)} = 1.2 \times 10^{-4} T \text{ or } 1.2 \text{ Gauss.}$$

# Hall Effect (serway pg890, Giancolli pg 833, openstax chap 11, Wolfson chap26)

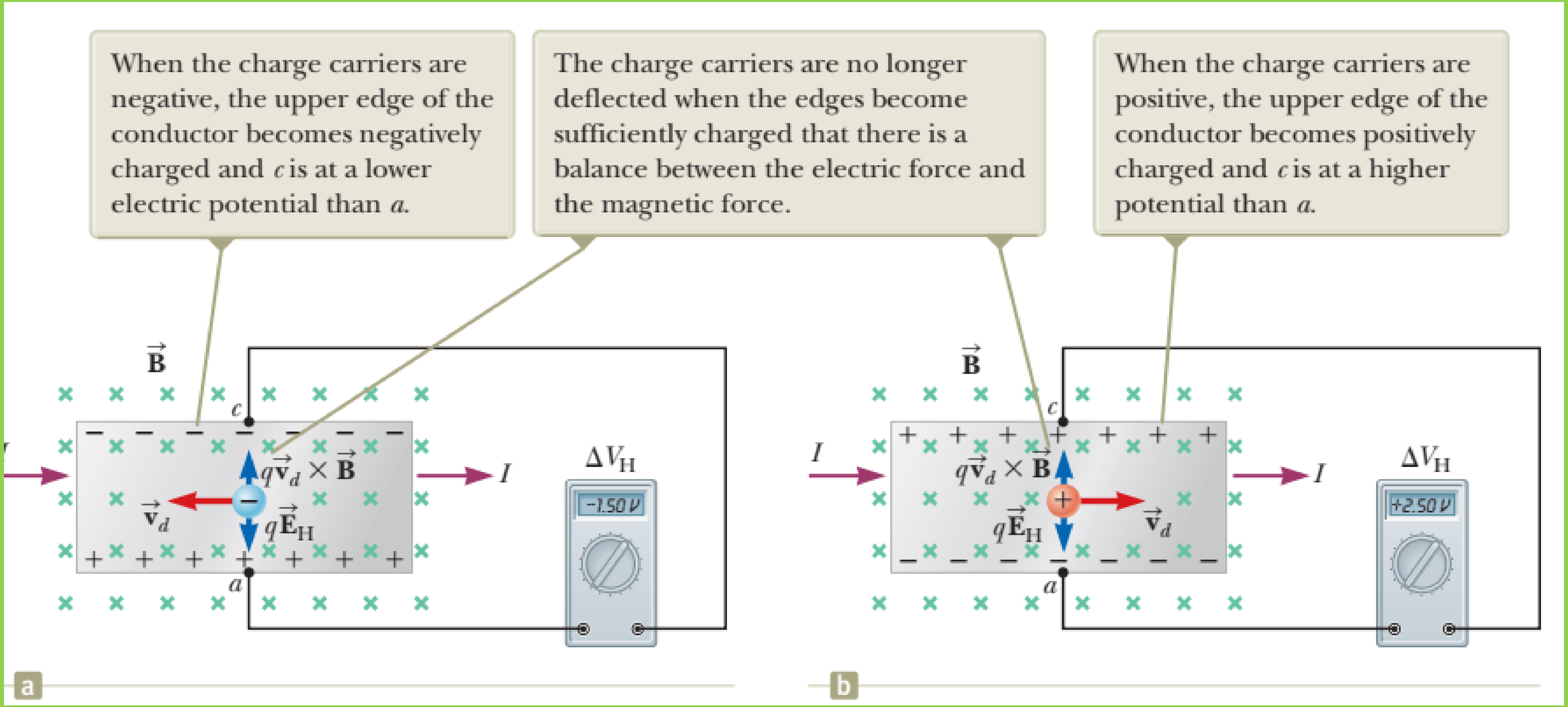
- When a current-carrying conductor or semiconductor like indium arsenide  is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field. This phenomenon, is known as the *Hall effect*.
- The Hall potential difference is directly proportional to the magnetic field strength the *direction* of the electric field and the *sign* of the potential difference depend on the sign of the charge carriers.
- For electrons, the magnitudes of the electric and magnetic forces are $eE$ and $evB$ , respectively, where $v$ is the electron drift speed.
- Hall Effect observation consists of a flat conductor carrying a current $I$ in the $x$ direction as shown in Figure below.
- A uniform magnetic field $\vec{B}$ is applied in the $y$ direction. If the charge carriers are electrons moving in the negative $x$ direction with a drift velocity $\vec{v}_d$, they experience an upward magnetic force $\vec{F}_B = q\vec{v}_d \times \vec{B}$

# Hall Effect cont.1 (Hafez pg843, Andrew pg 411)

- They are deflected upward, and accumulate at the upper edge of the flat conductor, leaving an excess of positive charge at the lower edge (Fig. next slide).
- To observe the Hall effect, a magnetic field is applied to a current-carrying conductor. The Hall voltage is measured between points $a$ and $c$.
- Accumulation of charge at the edges establishes an electric field in the conductor and increases until the electric force on carriers remaining in the bulk of the conductor balances the magnetic force acting on the carriers.



# Hall Effect cont.2



# Hall Effect cont.3

- A sensitive voltmeter connected across the sample as shown in Figure previous slide can measure the potential difference, known as the *Hall voltage* $\Delta V_H$, generated across the conductor.
- If the charge carriers are positive and hence move in the positive $x$ direction (for rightward current) as shown in slide 94 and 95b above , they also experience an upward magnetic force $q\vec{v}_d \times \vec{B}$, which produces a buildup of positive charge on the upper edge and leaves an excess of negative charge on the lower edge.
- The magnetic force exerted on the carriers has magnitude $q\vec{v}_d B$. In equilibrium, this force is balanced by the electric force $q\vec{E}_H$ , where $\vec{E}_H$ is the magnitude of the electric field due to the charge separation (sometimes referred to as the *Hall field*). Therefore, $q\vec{v}_d B= q\vec{E}_H$ or $e\vec{v}_d B= e\vec{E}_H$, $\vec{E}_H = \vec{v}_d B$
- If $d$ is the width of the conductor, the Hall voltage is

# Hall Effect cont.4

$$\Delta V_H = \vec{E}_H d = \vec{v}_d B d$$

- Therefore, the measured Hall voltage gives a value for the drift speed of the charge carriers if $d$ and $B$ are known.
- We can obtain the charge-carrier density $n$ by measuring the current in the sample. we can express the drift speed as $\vec{v}_d = \frac{I}{nqA}$, where $A$ is the cross-sectional area of the conductor. Substituting Equation on non top $\Delta V_H = \frac{IBd}{nqA}$ and since $A = td$, where $t$ is the thickness of the conductor, we can also express the Equation $\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$
- where $R_H = \frac{1}{nq}$ is called the **Hall coefficient.** This relationship shows that a properly calibrated conductor can be used to measure the magnitude of an unknown magnetic field.

# Biot-Savart Law

- Biot -Savart law is used to calculate the magnetic field due to a current carrying conductor.
- According to this law, the magnitude of the magnetic field at any point P due to a small current element $I.dl$ ( $I\ =$ current through the element, $dl\ =$ length of the element) is,

$$dB \propto \frac{Idlsin\,\theta}{r^2}, dB = \frac{\mu_0}{4\pi}.\frac{Idlsin\,\theta}{r^2}$$

In vector notation

$$dB = \frac{\mu_0}{4\pi}.\frac{Idl\times r}{r^3}$$

# Ampere's circuital law

- It states that the line integral of the magnetic field (vector B ) around any closed path or circuit is equal to $\mu_0$(permeability of free space) times the total current ( I ) flowing through the closed circuit

Mathematically,

$$\oint \vec{B}.\vec{d}l = \mu_0 I$$

# Faraday's Law of electromagnetic induction

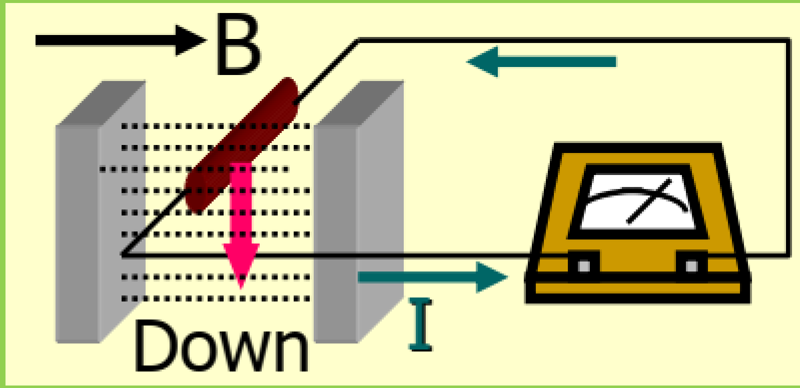
- Michael Faraday found that whenever there is a change in magnetic flux linked with a circuit, an emf is induced resulting a flow of current in the circuit.
- The magnitude of the induced emf is directly proportional to the rate of change of magnetic flux.
- Lenz's rule gives the direction of the induced emf which states that the induced current produced in a circuit always in such a direction that it opposes the change or the cause that produces it.

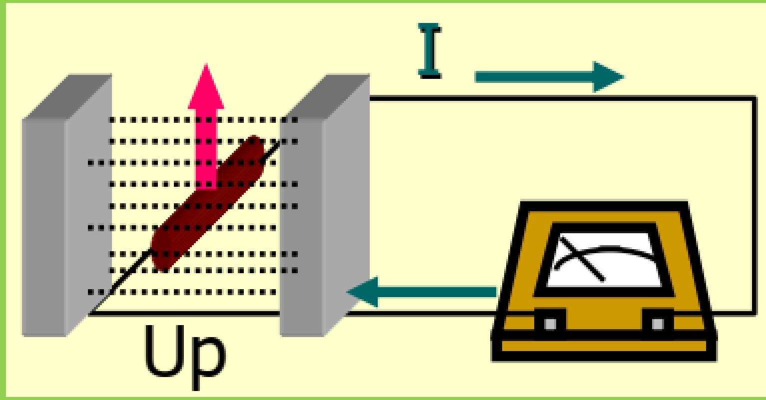
Induced emf $(e) = -\frac{d\varphi}{dt}$

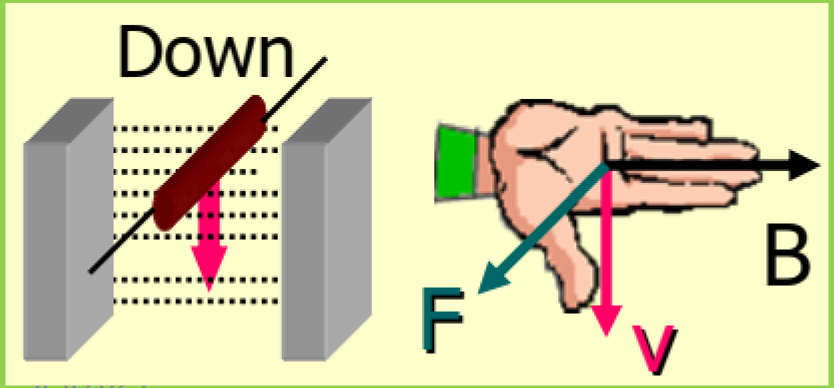
$d\varphi$ is the change magnetic flux linked with a circuit

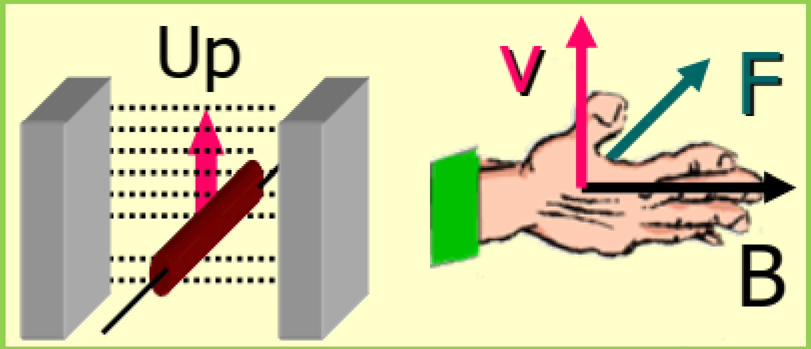
# Induced Current

- When a conductor moves across flux lines, magnetic forces on the free electrons induce an electric current.
- Right-hand force rule shows current outward for down and inward for up motion.



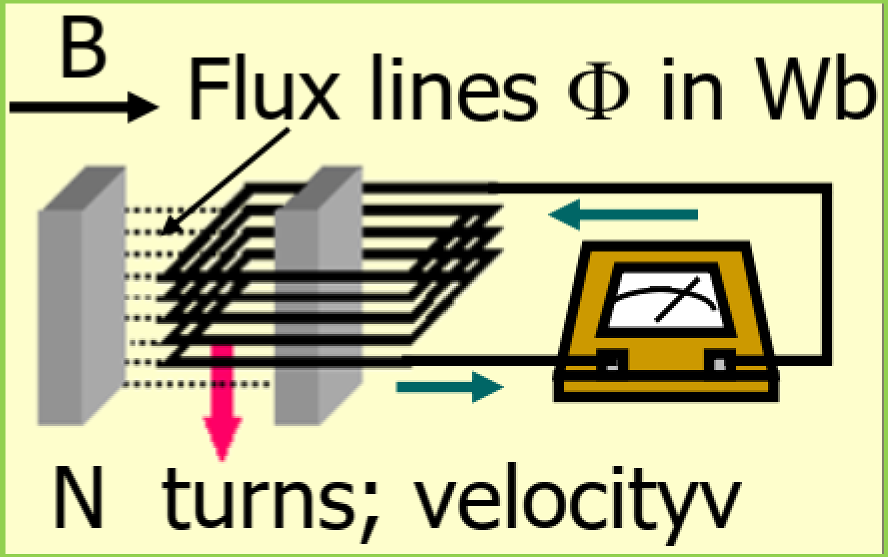



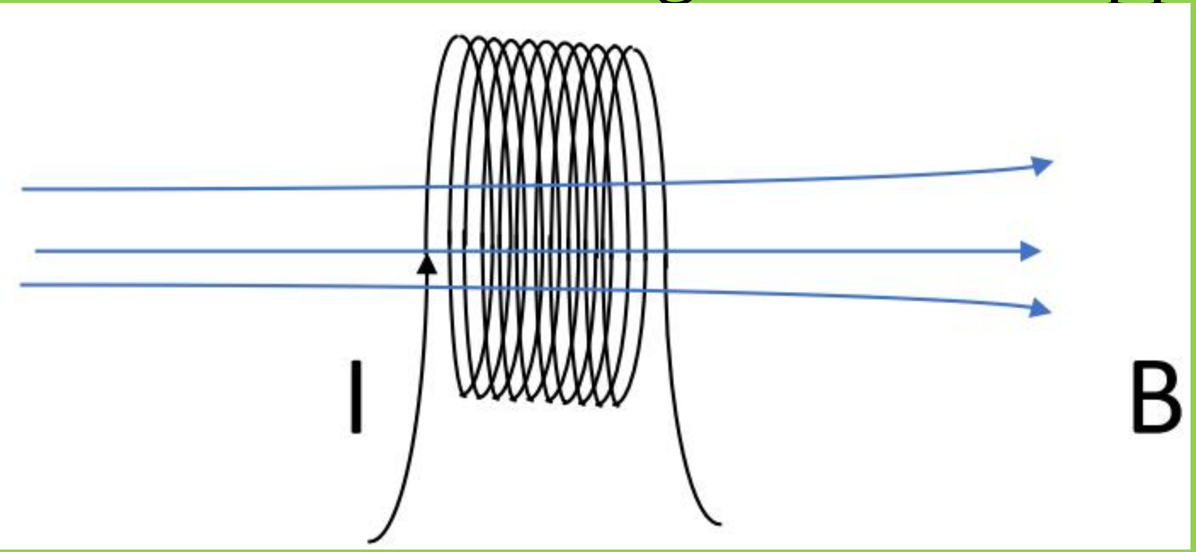
# Induced EMF Observations (Faraday's observations)

- Relative motion induces emf. $\boldsymbol{\varepsilon} = -\boldsymbol{N}\frac{\Delta\boldsymbol{\varphi}}{\Delta\boldsymbol{t}}$ .The negative sign means that **E** opposes its cause.
- Direction of emf depends on direction of motion.
- Emf is proportional to rate at, which lines are cut (v).
- Emf is proportional to the number of turns N



# Self Inductance

- A changing current in a coil can induce an emf in itself.
- If the current is steady, the coil acts like an ordinary piece of wire.
- But if the current changes, $B$ changes and so then does $\Phi_B$, and Faraday tells us there will be an induced emf.
- Lenz's law tells us that the induced emf must be in such a direction as to produce a current which makes a magnetic field opposing the change.



# Implications of Self Inductance

- We define the self inductance of a circuit element (a coil, wire, resistor or whatever) as $L = \frac{\Phi_B}{I}$, From this we have $\Phi_B = \ LI$ and so $\frac{d\Phi_B}{dt} = L\frac{dI}{dt}$

and Faraday's law gives $\varepsilon$ =- $L\frac{dI}{dt}$ .Since this emf opposes changes in the current (in the component) it is often called the "back emf".

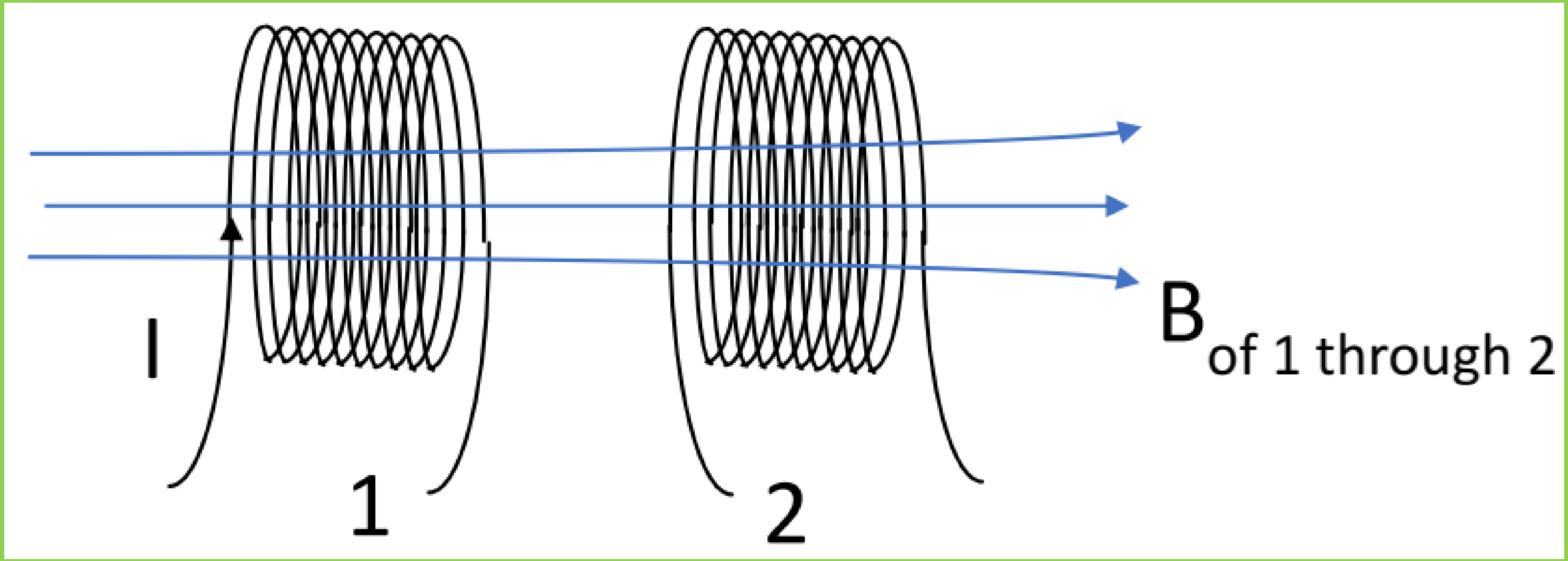
*What is the (self) inductance of a solenoid* $(L = \frac{\Phi_B}{I})$*with area A, length* $d$*, and* $n$ *turns per unit length?*

In the solenoid$B = \mu_0 nI$, so the flux through one turn is:

$\Phi_B = BA = \mu_0 nI$A. The total flux in the solenoid is $(nd)\ \Phi_B$. Therefore $\Phi_B = \mu_0 n^2 IAd$ and so $L = \frac{\Phi_B}{I}$ gives $L = \mu_0 n^2 Ad$.

# Mutual Inductance

- Two coils, 1 & 2, are arranged such that flux from one passes through the other.
- We already know that changing the current in 1 changes the flux (in the other) and so induces an emf in 2.This is mutual inductance.



# Mutual Inductance cont.

- The *mutual inductance*, $M$, tells us how much flux through the second coil, $F_2$ , is caused by a current, $I_1$, through the first: $M = \frac{\Phi_2}{I_1}$ which gives $\Phi_2 = MI_1$ and so $\frac{d\Phi_2}{dt} = M\frac{dI_1}{dt}$ But by Faraday's law:

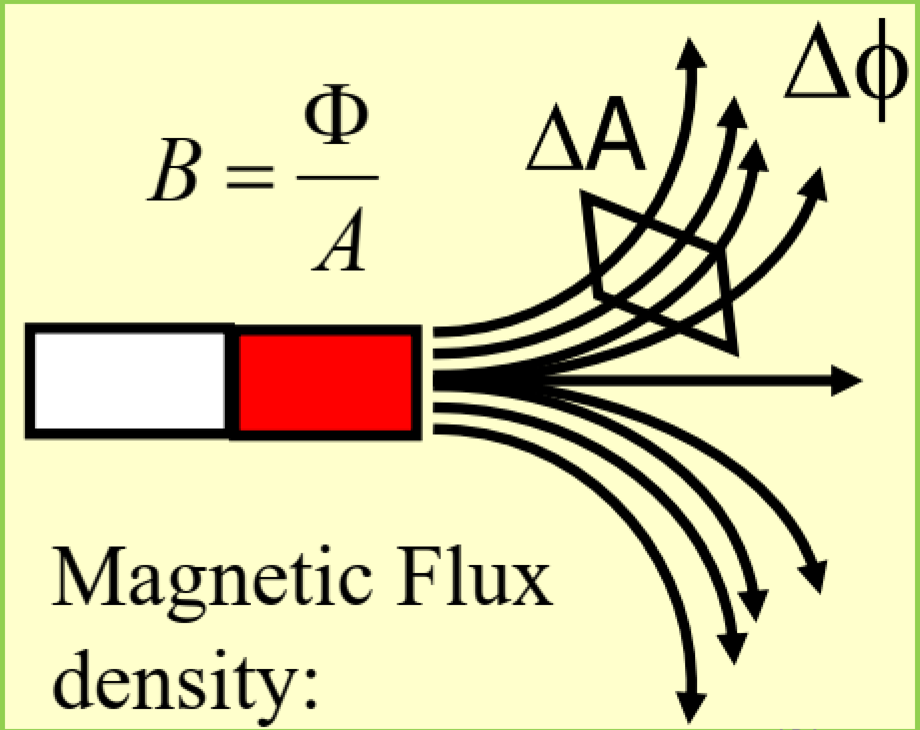
$$\varepsilon_2 = -\frac{d\Phi_2}{dt} = -M\frac{dI_1}{dt}$$

- M arises from the way flux from one coil passes through the other: that is from the geometry and arrangement of the coils.
- Mutual means mutual. Note there is no subscript on M: the effect of 2 on 1 is identical to the effect of 1 on 2.
- Inductance has units: called the "Henry" (H).

1 H = 1 Vs/A

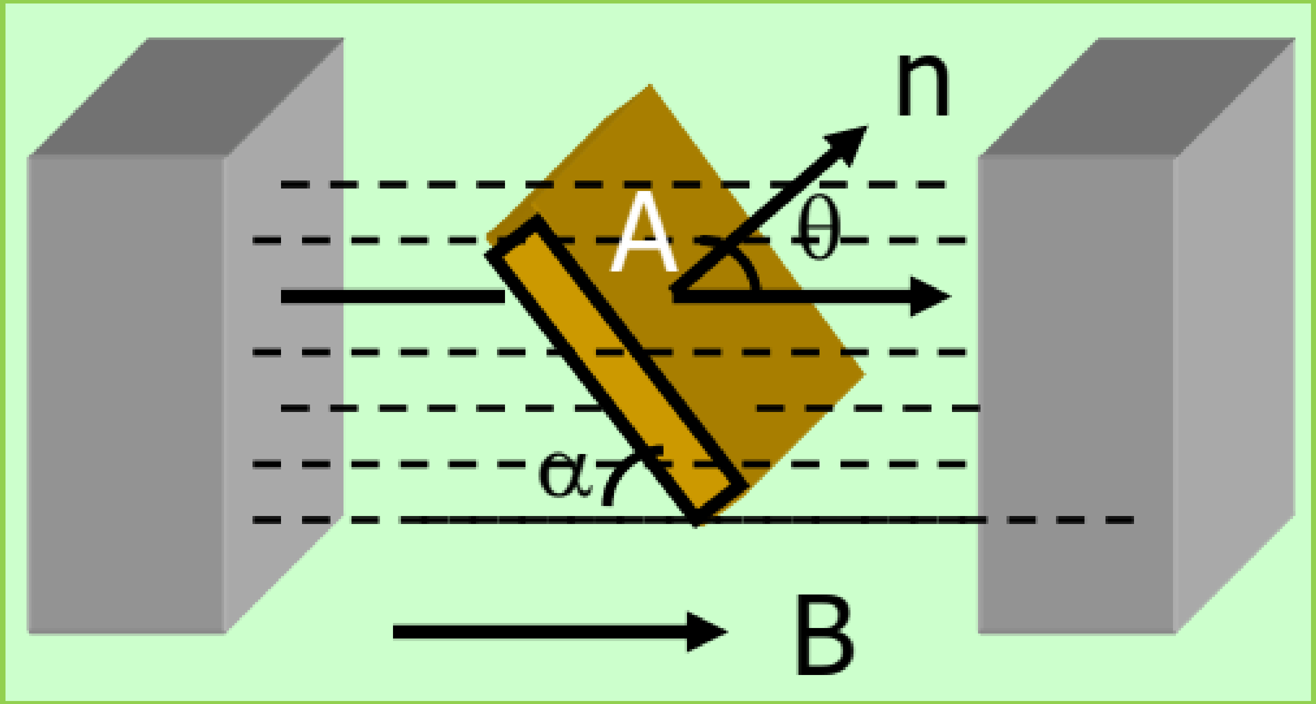
# Magnetic Flux Density

- Magnetic flux lines$\Phi$ are continuous and closed. Direction is that of the $B$ vector at any point.
- When area A is perpendicular to flux $B = \frac{\Phi}{A}$ or $\Phi = BA$
- The unit of flux density is the weber per square meter.



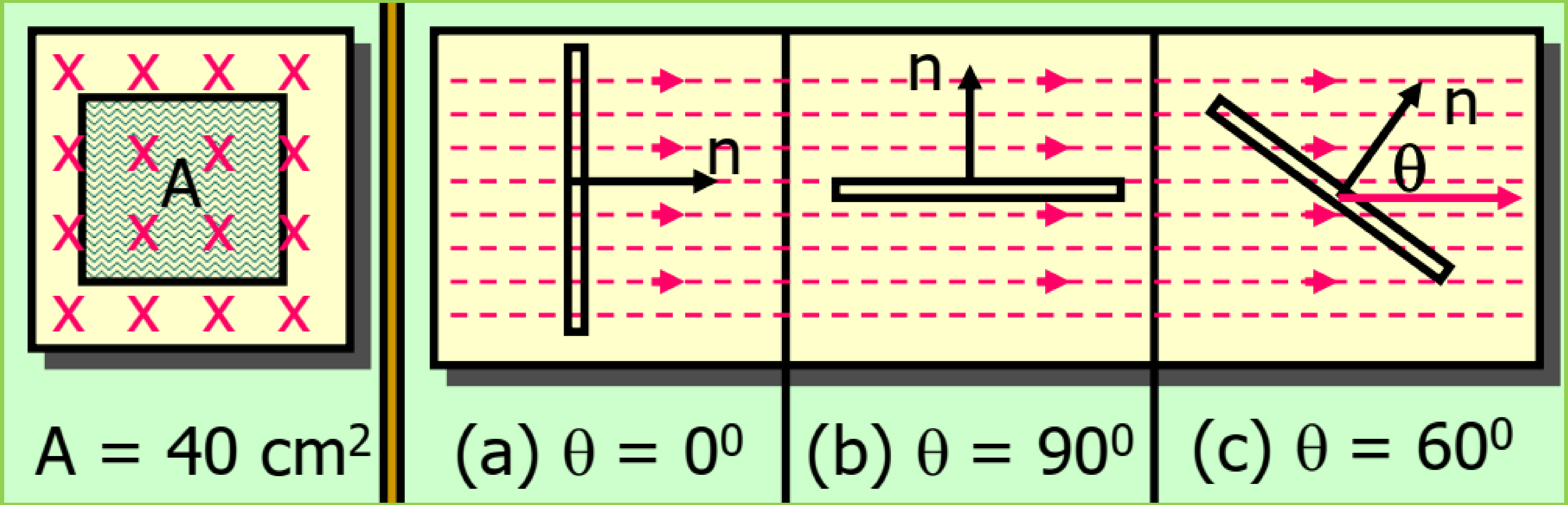
# Flux When Area is Not Perpendicular to Field

- The flux penetrating the area A when the normal vector n makes an angle of $\theta$ with the $B$-field is $\Phi = BA\cos\ \theta$.
- The angle $\theta$ is the complement of the angle $\alpha$ that the plane of the area makes with B field. (Cos $\theta$ = Sin $\alpha$)



# Example

A current loop has an area of $40\ cm^2$ and is placed in a $3 - T$ $B$-field at the given angles. Find the flux $\Phi$ through the loop in each case.



# Solution

(a) $\Phi = BAcos\,0^0 = (3T)(0.004m^2)(1) = 12.0\,mWb.$

(b) $\Phi = BAcos\,90^0 = (3T)(0.004m^2)(0) = 0\,mWb.$

(c) $\Phi = BAcos\,60^0 = (3T)(0.004m^2)(0.5) = 6.0\,mWb.$

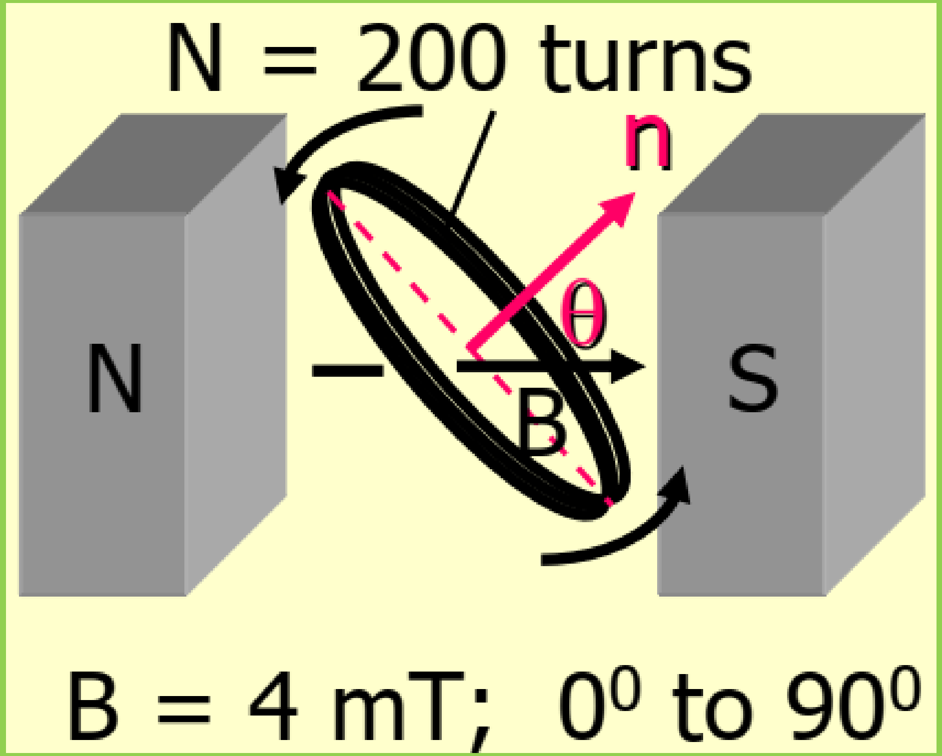
**Application of Faraday's Law**

A change in flux $\Delta\Phi$ can occur by a change in area or by a change in the B-field:

**Example** A coil has $200$ turns of area $30\ cm^2$. It flips from vertical to horizontal position in a time of 0.03 s. What is the induced emf if the constant $B$-field is $4\ mT$?

# Solution

$\Delta A = 30cm^2 - 0 = 30cm^2,$
$\Delta\Phi = B\Delta A = (3mT)(30cm^2),$
$\Delta\Phi = B\Delta A = (0.004T)(0.003cm^2)$
$\Delta\Phi = 1.2 \times 10^{-5}Wb$
$\varepsilon = -N\dfrac{\Delta\varphi}{\Delta t} = -(200)\dfrac{1.2 \times 10^{-5}Wb}{0.03s}$
$\varepsilon = 0.08V$




The negative sign indicates the polarity of the voltage.

The coil of a generator has 400 turns and a cross-sectional area of $4 \times 10^{-3}m^2$ It is rotating in a field of strength 3T with an angular speed of $5rad/s$ . calculate the maximum value of the induced voltage

# Lenz's Law

- An induced current will be in such a direction as to produce a magnetic field that will oppose the motion of the magnetic field that is producing it.

# Electromagnetic waves

- According to Maxwell's modification of Ampere's law, a changing electric field gives rise to a magnetic field.
- It leads to the generation of electromagnetic disturbance comprising of time varying electric and magnetic fields.
- These disturbances can be propagated through space even in the absence of any material medium.
- These disturbances have the properties of a wave and are called electromagnetic waves.

# A.C. circuits

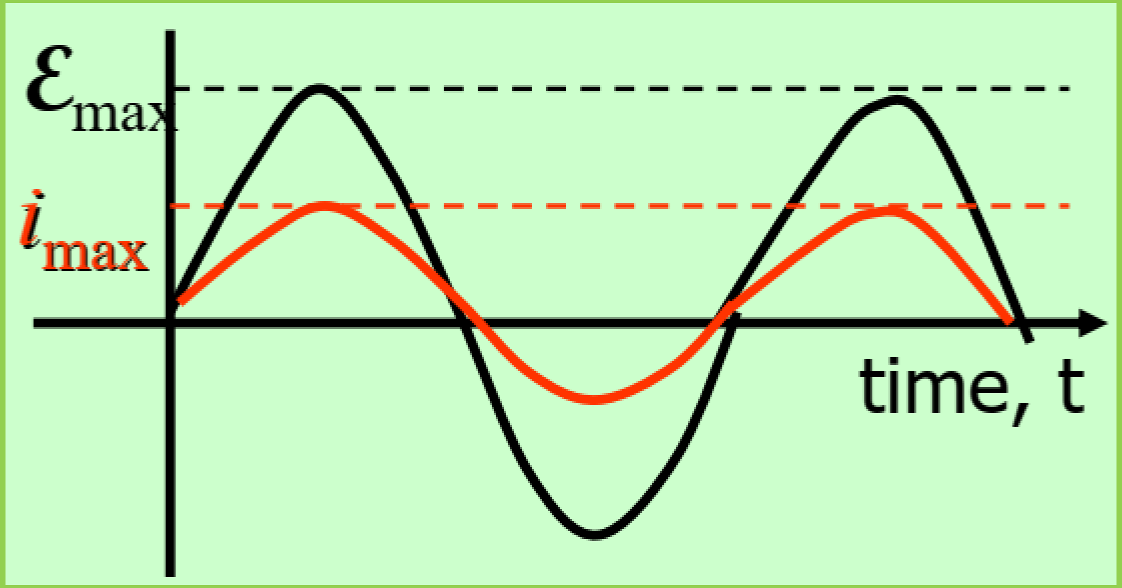
## Alternating Currents

An alternating current such as that produced by a generator has no direction in the sense that direct current has. The magnitudes vary sinusoidally with time as given by

AC-voltage and current

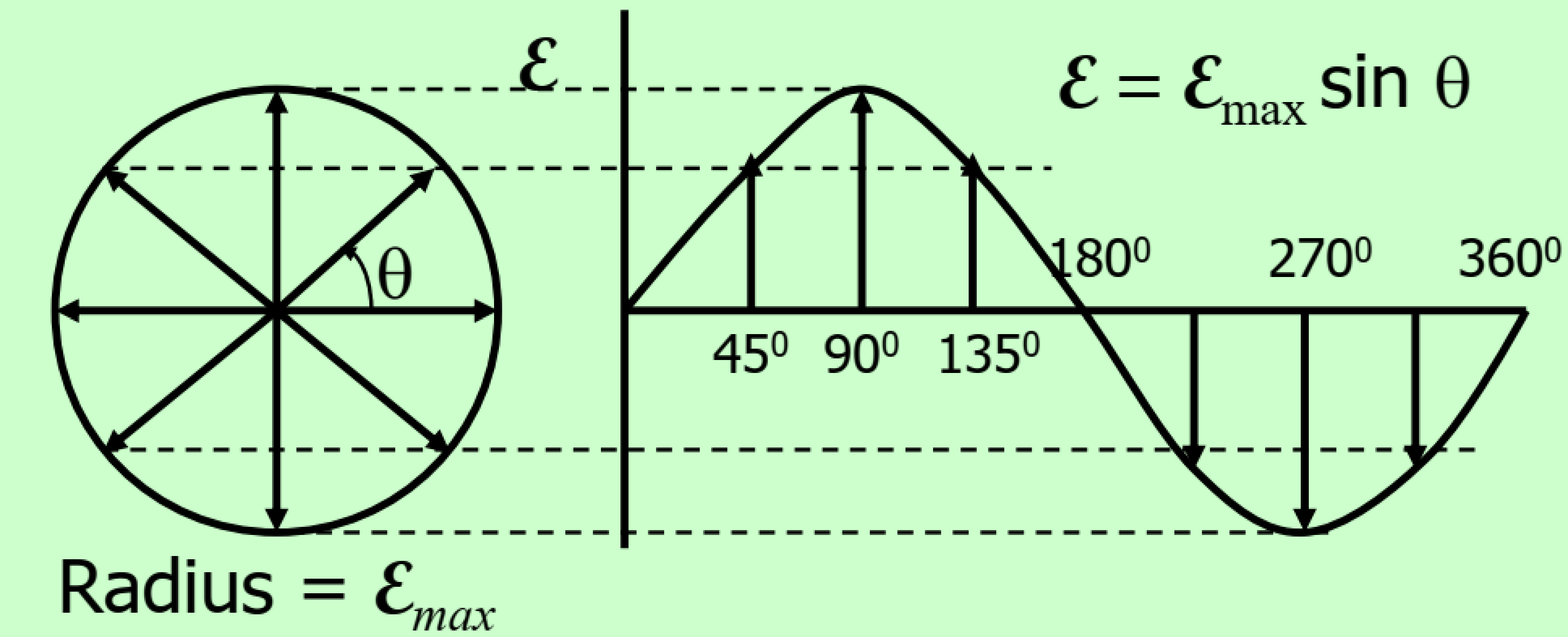
$$\mathcal{E} = \mathcal{E}_{max} \sin \theta$$

$$i = i_{max} \sin \theta$$



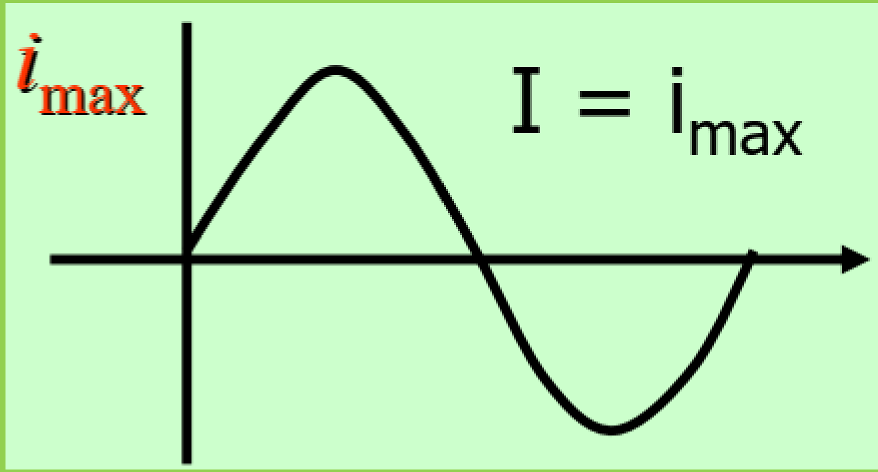
# Rotating Vector Description

- The coordinate of the emf at any instant is the value of $\boldsymbol{\varepsilon_{max}\ sin\ \theta}$ Observe for incremental angles in steps of $45^0$. Same is true for $i$



# Effective AC Current

- The average current in a cycle is zero—half + and half -.




- But energy is expended, regardless of direction. So the "root-mean square" value is useful.

$$I_{rms} = \sqrt{\frac{I^2}{2}} = \frac{I}{0.707}$$

# AC Definitions

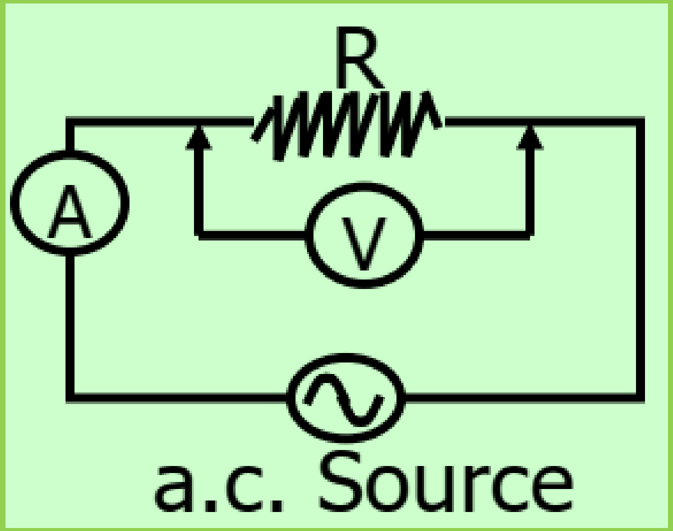
- One effective ampere is that ac current for which the power is the same as for one ampere of dc current.
- Effective current: $i_{eff} = 0.707 i_{max}$
- One effective volt is that ac voltage that gives an effective ampere through a resistance of one ohm. $V_{eff} = 0.707 V_{max}$

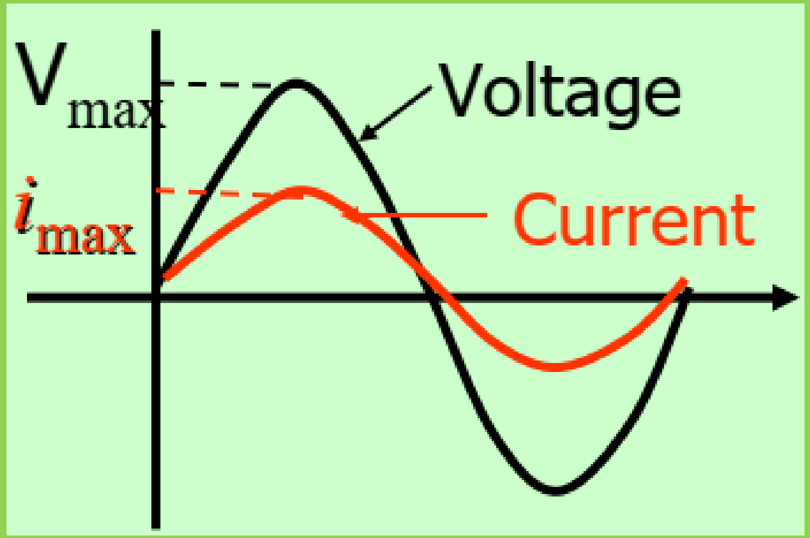
  ***Example 1:*** For a particular device, the house ac voltage is 120-V and the ac current is 10 A. What are their maximum values?

***Solution:*** $i_{max} = \frac{i_{eff}}{0.707} = \frac{10A}{0.707} = 14.14A,\ V_{max} = \frac{V_{eff}}{0.707} = \frac{120V}{0.707} = 170V,$

- The ac voltage actually varies from +170 V to -170 V and the current from 14.1 A to –14.1 A.

# Pure Resistance in AC Circuits

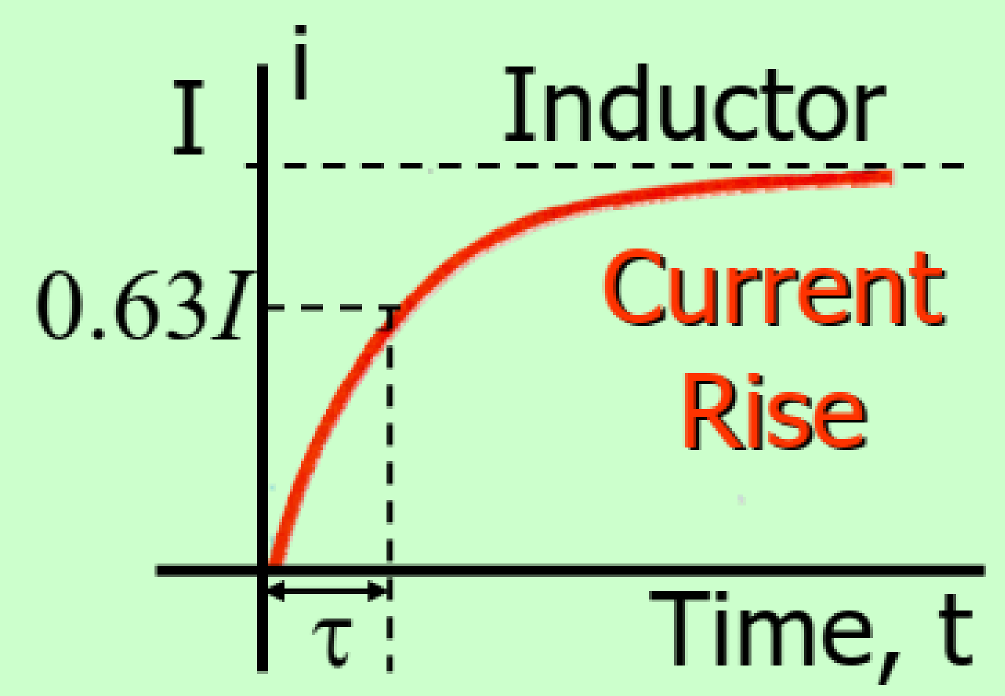


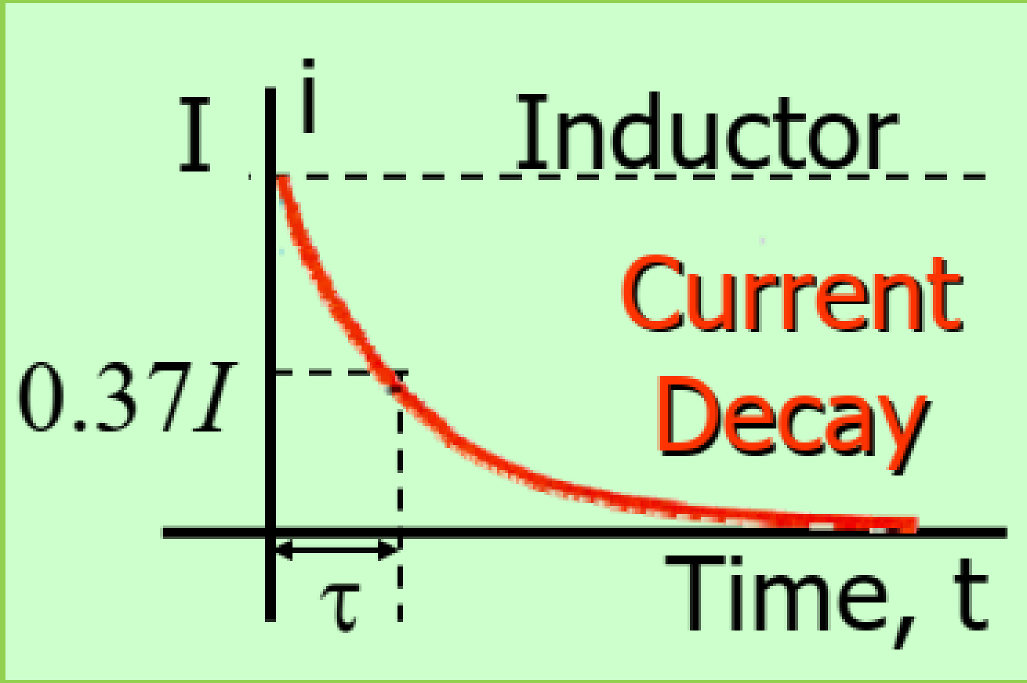




- Voltage and current are in phase, and Ohm's law applies for effective currents and voltages.
- Ohm's law: $V_{eff} = i_{eff}R$

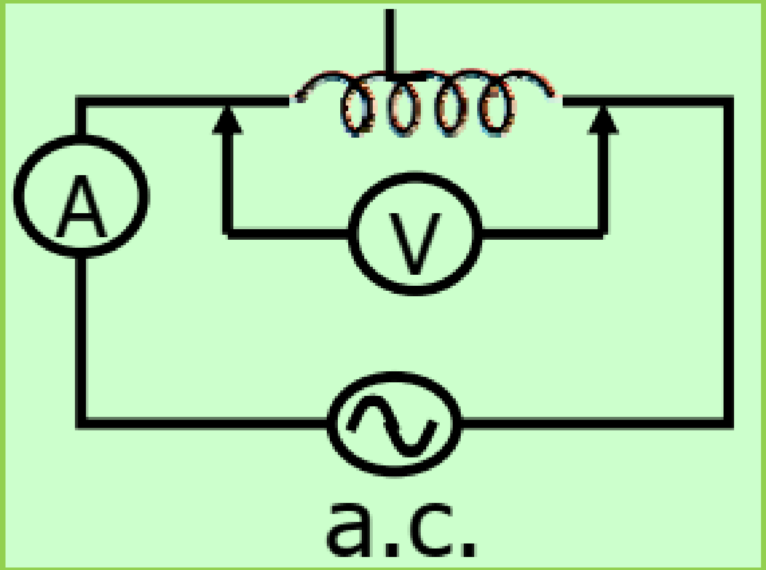
# AC and Inductors

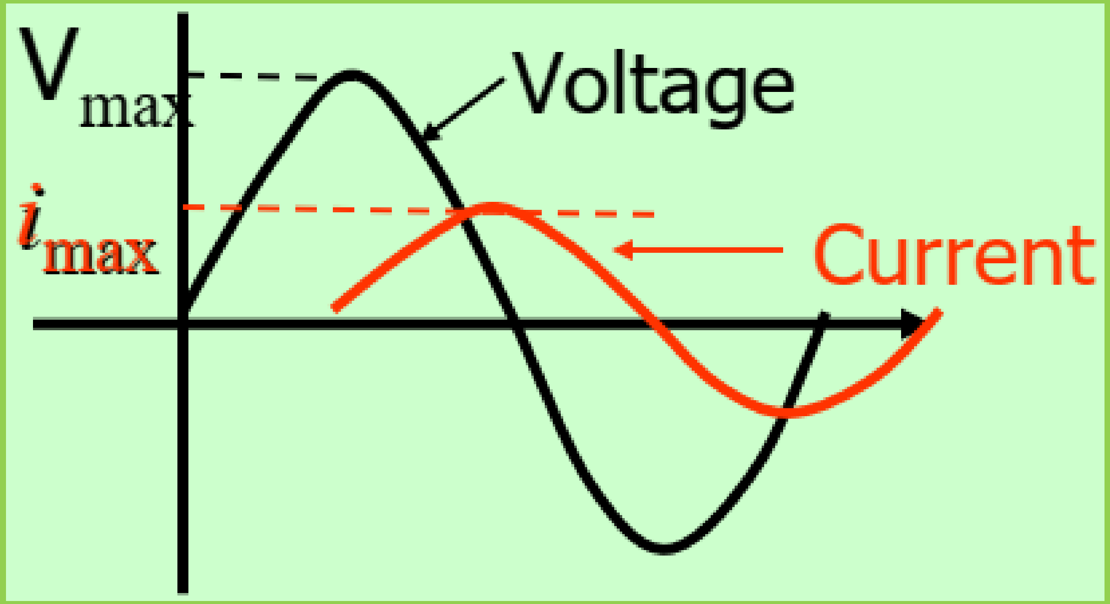
- The voltage V peaks first, causing rapid rise in i current which then peaks as the emf goes to zero. Voltage leads (peaks before) the current by $90^0$. Voltage and current are out of phase.



# A Pure Inductor in AC Circuit

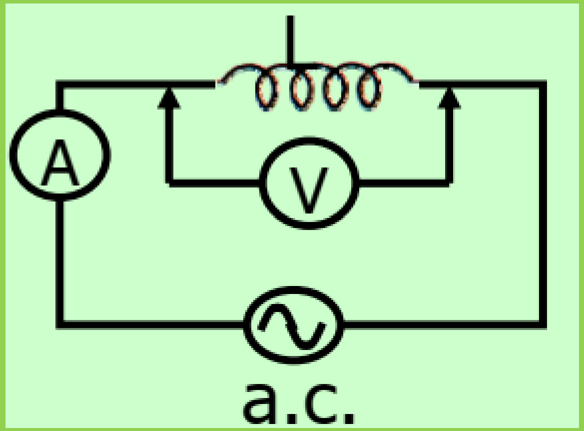



- The voltage peaks $90^0$ before the current peaks. One builds as the other falls and vice versa.
- The reactance may be defined as the nonresistive opposition to the flow of ac current.

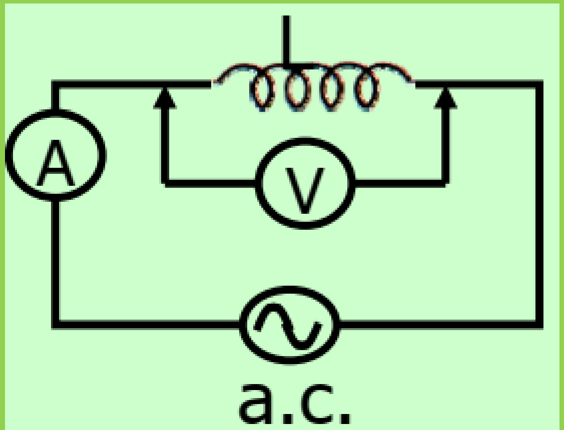
# Inductive Reactance

- The back emf induced by a changing current provides opposition to current, called inductive reactance $X_L$.




- Such losses are temporary, however, since the current changes direction, periodically re-supplying energy so that no net power is lost in one cycle. Inductive reactance $X_L$ is a function of both the inductance and the frequency of the ac current.

# Calculating Inductive Reactance




**Inductive Reactance:**

$X_L = 2\pi f L$ Unit is the $\Omega$

Ohm's law: $V_L = iX_L$

- The voltage reading V in the above circuit at the instant the ac current is i can be found from the inductance in H and the frequency in Hz

$$V_L = i(2\pi f L), \text{ Ohms Law: } V_L = i_{eff} X_L$$

# Example 2:

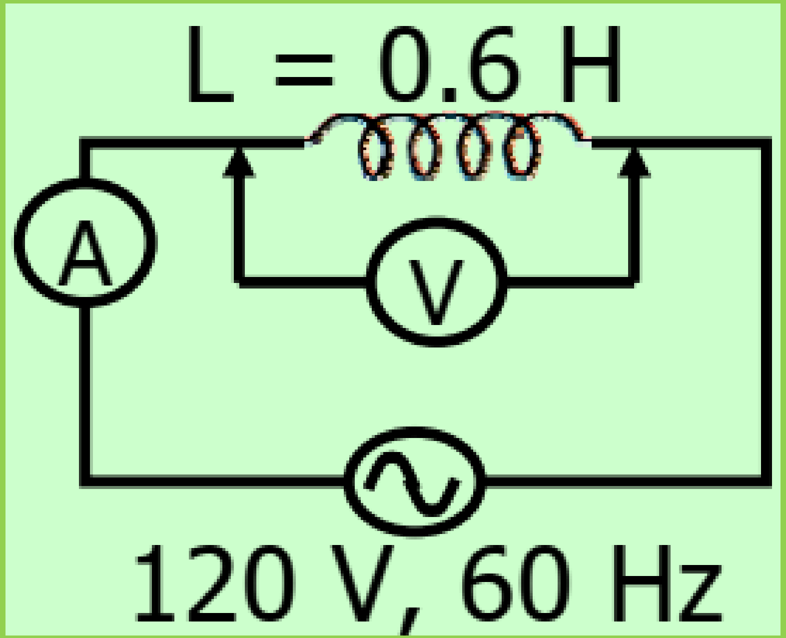
A coil having an inductance of 0.6 H is connected to a 120-V, 60 Hz ac source. Neglecting resistance, what is the effective current through the coil? Reactance

$$X_L = \omega L = 2\pi f L$$

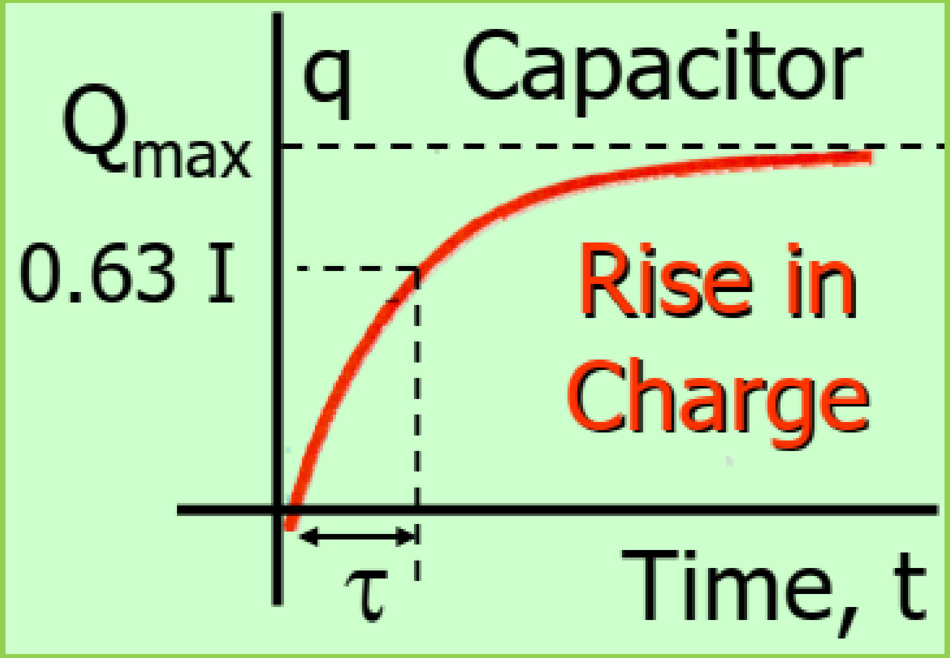
$$X_L = 2\pi(60Hz)(0.6H) = 226\Omega$$

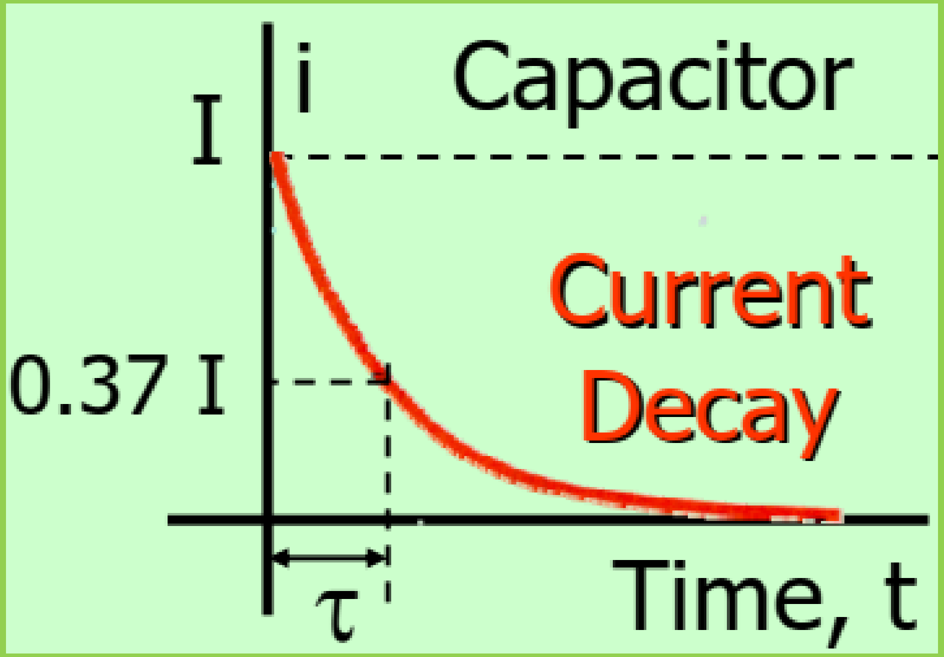
$$i_{eff} = \frac{V_{eff}}{X_L} = \frac{120V}{226\Omega} = 0.531A$$

Show that the peak current is $I_{max} = 0.750A$



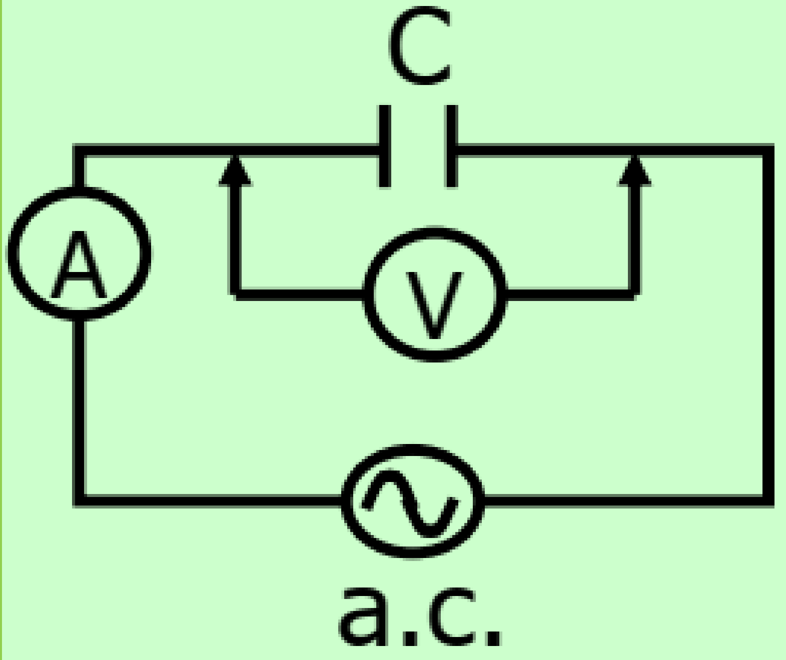
# AC and Capacitance

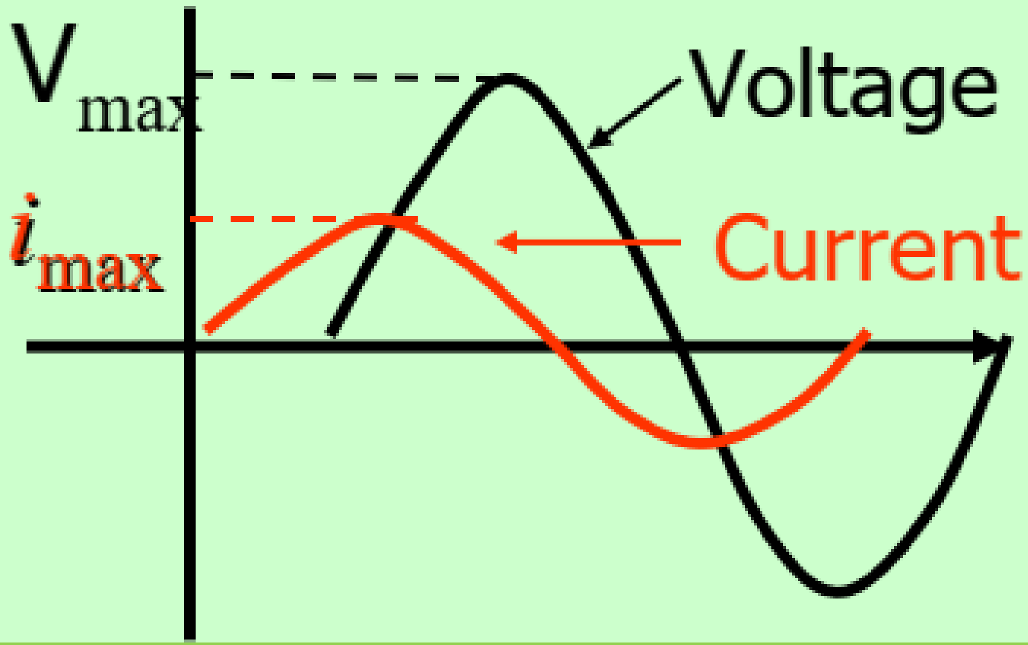



- ➢The voltage V peaks ¼ of a cycle after the current i reaches its maximum. The voltage lags the current. Current i and V out of phase.

# A Pure Capacitor in AC Circuit

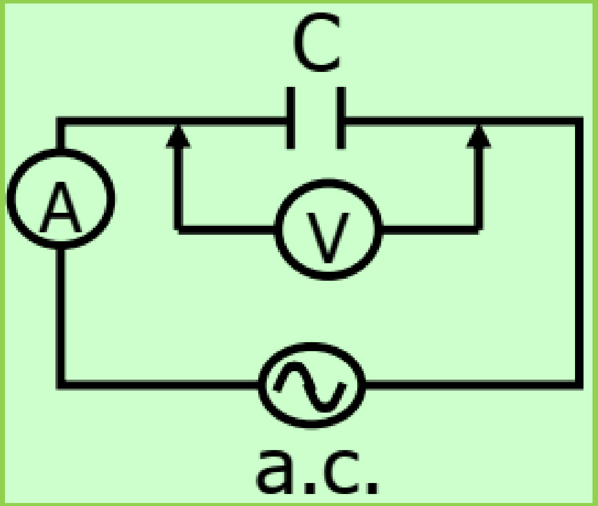



- ➢The voltage peaks $90^0$ after the current peaks. One builds as the other falls and vice versa.
- ➢The diminishing current i builds charge on C which increases the back emf of $V_C$.

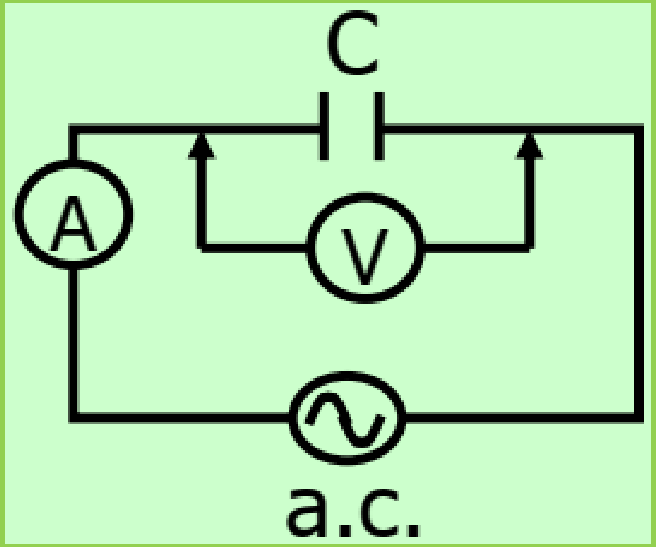
# Capacitive Reactance

- Energy gains and losses are also temporary for capacitors due to the constantly changing ac current.




- No net power is lost in a complete cycle, even though the capacitor does provide non-resistive opposition (reactance) to the flow of ac current.
- Capacitive reactance $X_C$ is affected by both the capacitance and the frequency of the ac current.

# Calculating Inductive Reactance




**Capacitive Reactance:**

$$X_C = \frac{1}{2\pi f C} \quad \text{Unit is the } \Omega$$

Ohm's law: $V_C = iX_C$

- The voltage reading V in the above circuit at the instant the ac current is i can be found from the inductance in F and the frequency in Hz.
- $V_L = \frac{1}{2\pi f L}$ Ohms law: $V_C = i_{eff} X_C$

# Example

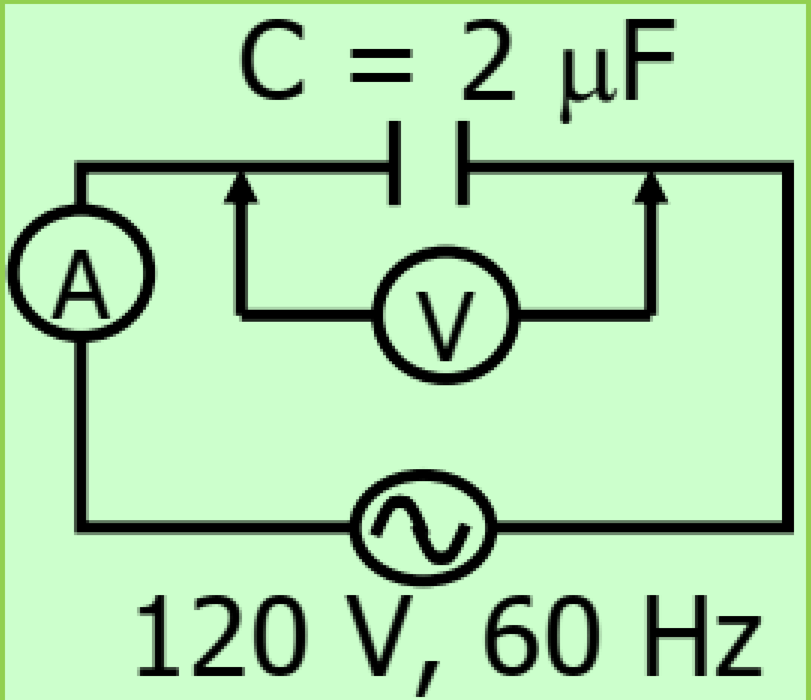
A 2-$\mu F$ capacitor is connected to a 120-V, 60 Hz ac source. Neglecting resistance, what is the effective current through the coil?

Reactance: $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$X_C = \frac{1}{2\pi(60Hz)(2 \times 10^{-6}F)} = 1330\Omega,$$

$$i_{eff} = \frac{V_{eff}}{X_C} = \frac{120V}{1330\Omega} = 90.5mA$$

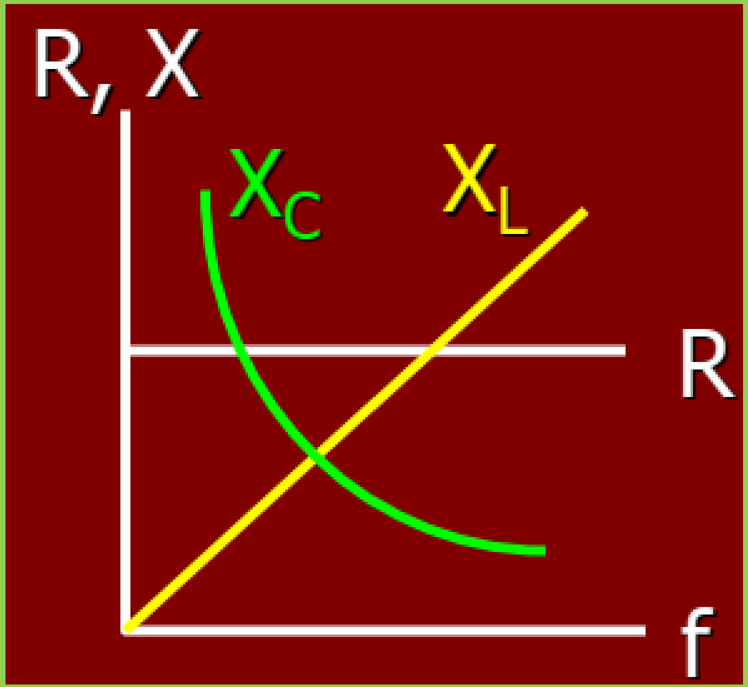
Show that the peak current is $i_{max} = 128mA$




*"ELI" the 'ICE" man* Emf **E** is before current i in inductors L; Emf **E** is after current i in capacitors C.

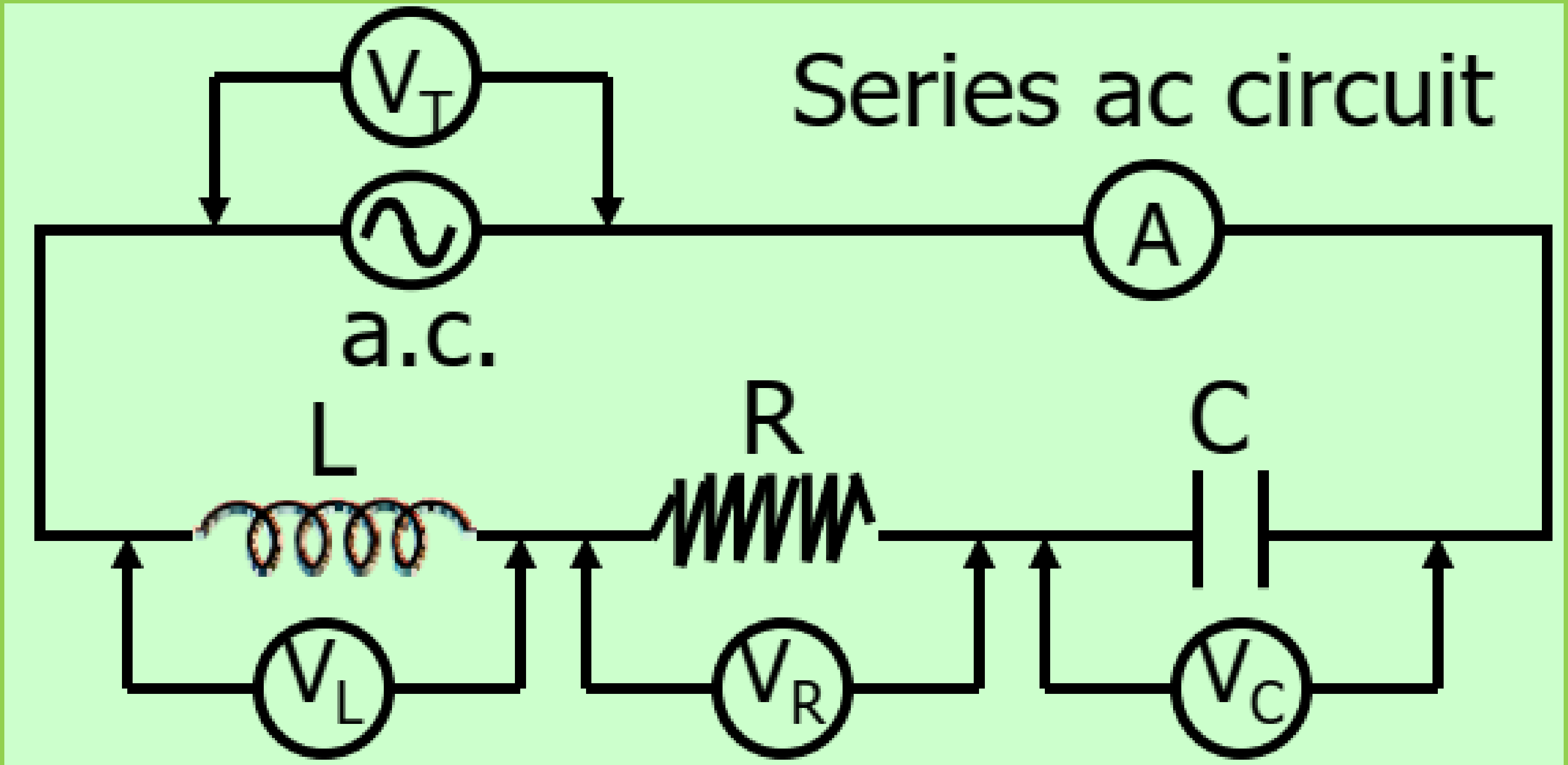
# Frequency and AC Circuits

- Inductive reactance $X_L$ varies directly with frequency as expected since $\varepsilon \propto \frac{\Delta i}{\Delta t}, X_L = 2\pi f L$ and $X_C = \frac{1}{2\pi f C}$
- Capacitive reactance $X_C$ varies inversely with $f$ since rapid ac allows little time for charge to build up on capacitors.



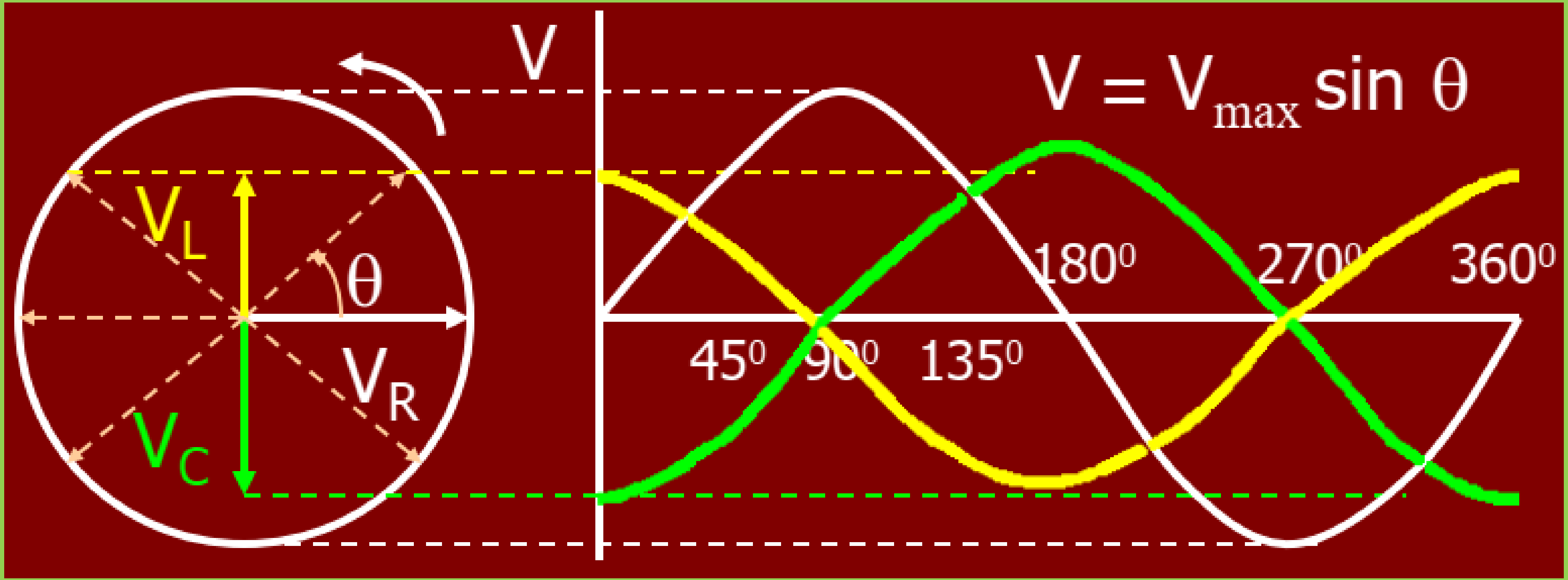
# Series LRC Circuits

- Consider an inductor $L$, a capacitor $C$, and a resistor $R$ all connected in series with an ac source. The instantaneous current and voltages can be measured with meters.



# Phase in a Series AC Circuit

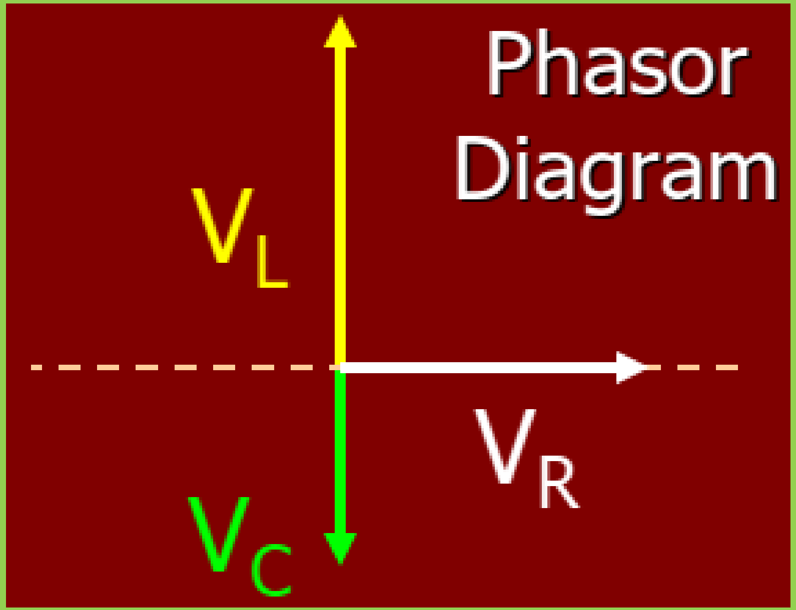
- The voltage leads current in an inductor and lags current in a capacitor. In phase for resistance $R$

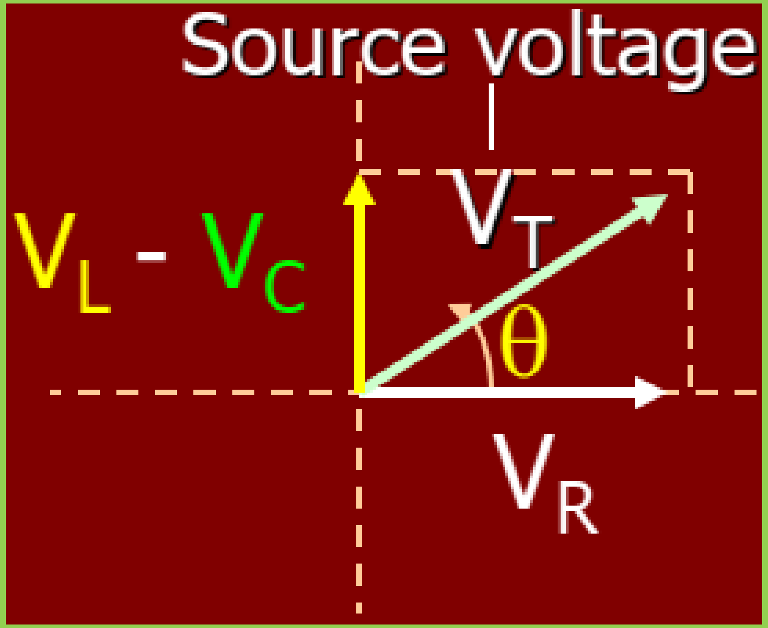



- Rotating phasor diagram generates voltage waves for each element $R$, $L$, and $C$ showing phase relations. Current $i$ is always in phase with $V_R$

# Phasors and Voltage

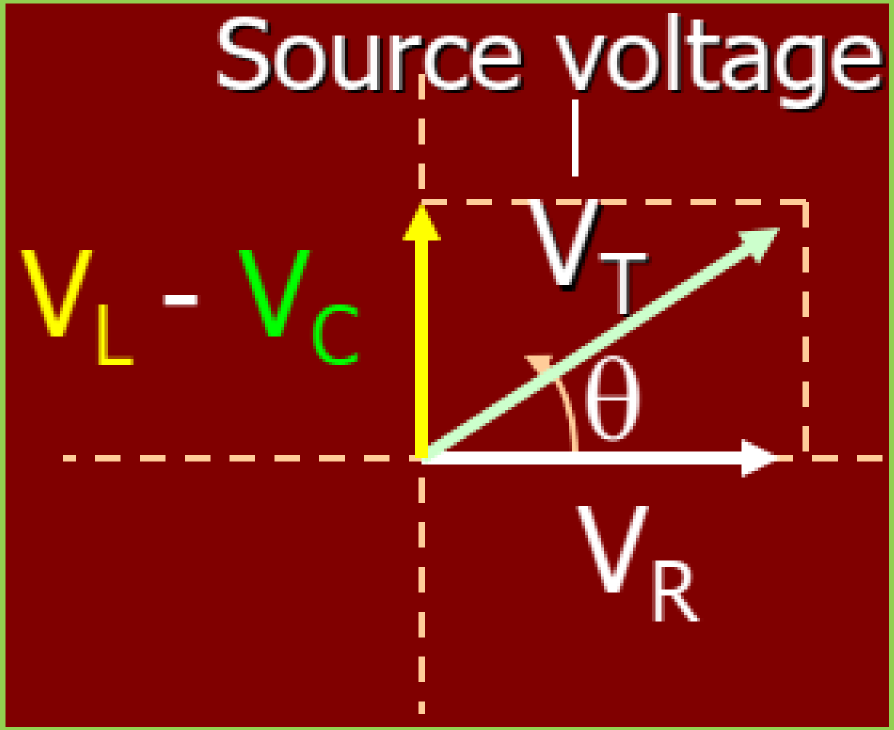
- At time t = 0, suppose we read $V_L, V_R$ and $V_C$ for an ac series circuit. What is the source voltage $V_T$?




- We handle phase differences by finding the vector sum of these readings. $V_T = \sum V_i$. The angle $\theta$ is the phase angle for the ac circuit.

# Calculating Total Source Voltage



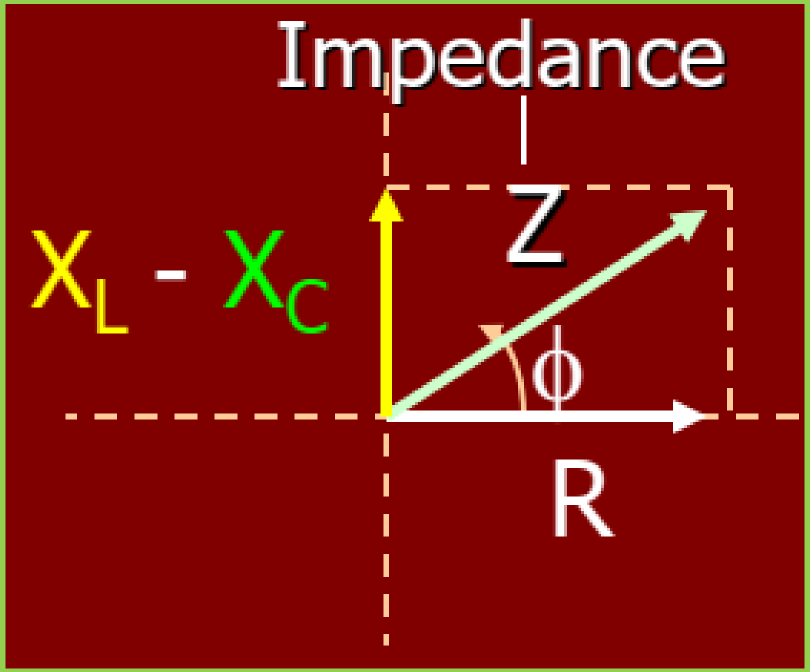

Treating as vectors, we find:

$$V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

Now recall that: $V_R = iR$; $V_L = iX_L$; and $V_C = iV_C$ Substitution into the above voltage equation gives: $V_T = \sqrt[i]{R^2 + (X_L - X_C)^2}$

# Impedance in an AC Circuit




$$V_T = \sqrt[i]{R^2 + (X_L - X_C)^2}$$

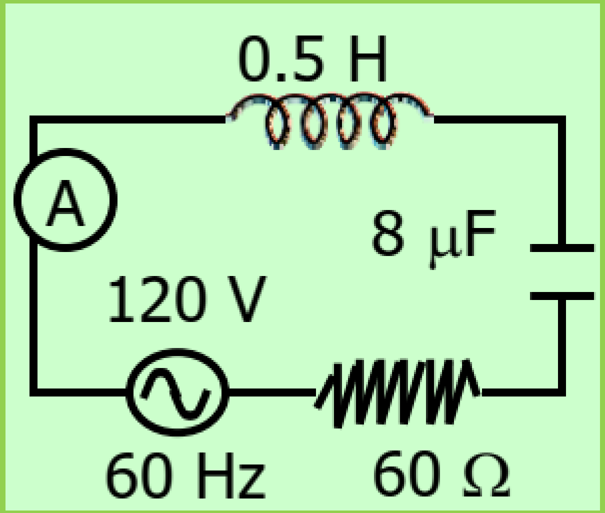
Impedance $Z$ is defined

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- Ohm's law for ac current and impedance: $V_T = iZ$ or $i = \frac{V_T}{Z}$
- The impedance is the combined opposition to ac current consisting of both resistance and reactance

# Example

A 60-Ω resistor, a 0.5 H inductor, and an 8-$\mu$F capacitor are connected in series with a 120-V, 60 Hz ac source. Calculate the impedance for this circuit.




$X_L = 2\pi f L$ and $X_C = \frac{1}{2\pi f C}$

$$X_L = 2\pi(60Hz)(0.6H) = 226\Omega$$

$$X_C = \frac{1}{2\pi(60Hz)(8\times10^{-6}F)} = 332\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(60\Omega\ )^2 + (226\Omega - 332\Omega)^2}$$

Thus the impedance is $Z = 122\Omega$

# Example

Find also the effective current and the phase angle in the previous example. We know values of $X_L, X_C, R$ and $Z$

$i_{eff} = \frac{V_T}{Z} = \frac{120V}{122\Omega} = 0.985A$

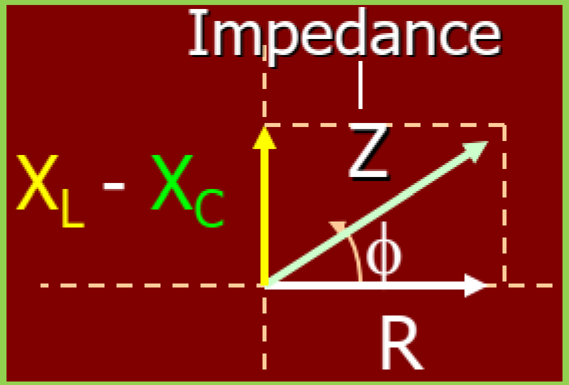
The phase angle, $\phi$. From $X_L - X_C$

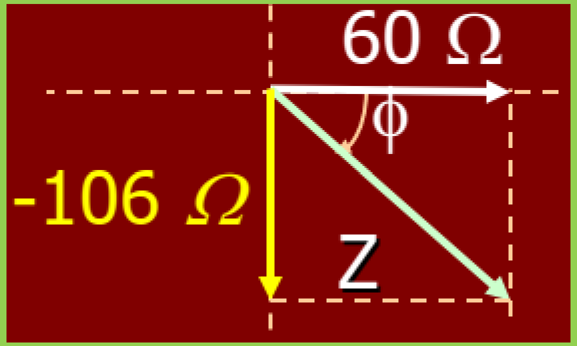
$= 226\Omega - 332\Omega = -106\Omega$

$R$ is 60 $\Omega$, $\tan\phi = \frac{X_L - X_C}{R} = \frac{-106\Omega}{60\Omega}$

$\phi = -60.5^0$

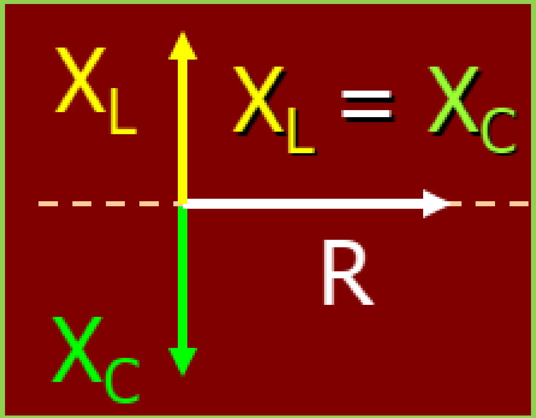
The negative phase angle means that the ac voltage lags the current by $60.5^0$. This is known as a capacitive circuit.



# Resonant Frequency

- Because inductance causes the voltage to lead the current and capacitance causes it to lag the current, they tend to cancel each other out.
- Resonance (Maximum Power) occurs when $X_L = X_C$




$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

$$2\pi f L = \frac{1}{2\pi f C} \text{ i.e.} (4\pi^2 f^2 LC = 1)$$

$$f^2 = \frac{1}{4\pi^2 LC} \; or \; f_r = \frac{1}{2\pi\sqrt{LC}}$$

***Example*** Find the resonant frequency for the previous circuit example: $L = 0.5$ H, C = 8 $\mu$F.

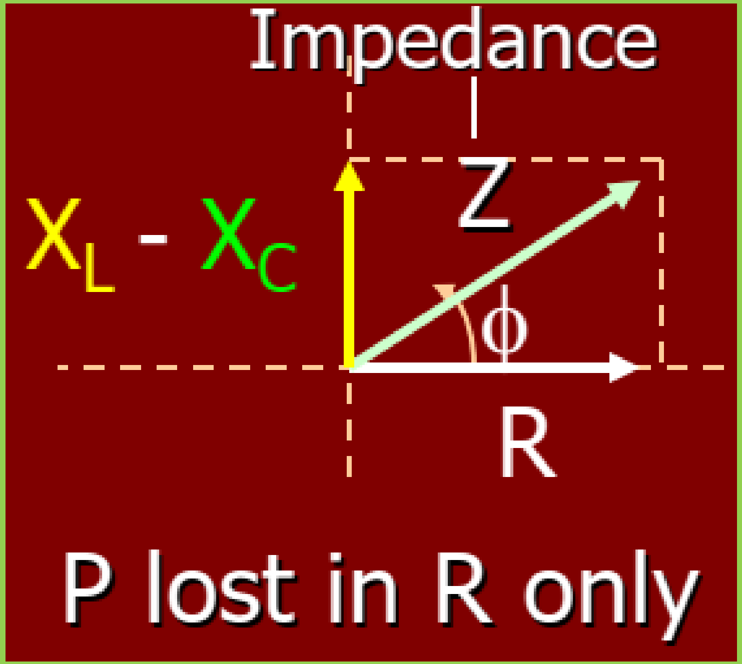
$$f_r = \frac{1}{2\pi\sqrt{(0.5H)(8\times10^{-6}F)}} = 79.6Hz.$$

- At resonance frequency, there is zero reactance (only resistance) and the circuit has phase angle of zero.

# Power in an AC Circuit

No power is consumed by inductance or capacitance. Thus power is a function of the component of the impedance along resistance.

In terms of ac voltage $P = iV cos\phi$, In terms of resistance $R$; $P = i^2R$




The fraction $cos\phi$ is known as the power factor.

# Example

What is the average power loss for the previous example: $V = 120\,V$, $\phi = -60.50, i = 90.5\,A$, and $R = 60\Omega$ .

$P = i^2R = (0.0905)^2(60\Omega)$ =0.491W

The power factor is $\cos 60.5^0$and $\cos\phi = 0.492$ or 49.2%.

- The higher the power factor, the more efficient is the circuit in its use of ac power.

# *Exercise*

A series AC circuit contains the following components: $R = 150\Omega$ , $L = 250\ mH$, $C = 2.0\mu F$, and a source with $\Delta V_{max} = 210\ V$ operating at $50.0\ Hz$. Calculate the (a) inductive reactance, (b) capacitive reactance, (c) impedance, (d) maximum current, and (e) phase angle between current and source voltage.

# Solution

(a) $X_L = \omega L = 2\pi f L = 2\pi(50Hz)(250 \times 10^{-3}H)$ =*78.5Ω*

*(b)* $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(50Hz)(2\times10^{-6}F)}$ =*1.59kΩ*

*(c)* $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{150^2 + (78.5 - 1590)^2} = 1519\Omega.$

*(d)* $i = \frac{V_T}{Z} = \frac{\Delta V_{max}}{Z} = \frac{200V}{1.5\times10^3} = 138mA.$

*(e)* $\phi = tan^{-1} = \left[\frac{X_L - X_C}{R}\right] = \frac{78.5-1590}{150} = tan^{-1}(-10.07) = -84.3^0$

# Low and High pass filters-Introduction

- Filters are circuits that are capable of *passing signals within a band* of frequencies while *rejecting or blocking* signals of frequencies *outside this band*. OR A filter is basically a frequency selective circuit designed to pass specific band of frequencies and block or attenuate input signals outside this band

- Filter can be passive or active filter.

- Passive filters: The circuits built using RC, RL, or RLC circuits.
- Active filters : The circuits that employ one or more op-amps in the design in addition to resistors and capacitors

# Advantages of Active Filters over Passive Filters

- Active filters can be designed to provide required gain, and hence no attenuation as in the case of passive filters (Gain and frequency adjustment flexibility)
- No loading problem, because of high input resistance and low output resistance of op-amp.
- Active Filters are cost effective as a wide variety of economical op-amps are available (Active filters are cheaper than passive filters)

# Applications

- Active filters are mainly used in communication and signal processing circuits.
- They are also employed in a wide range of applications such as entertainment, medical electronics, etc.

# Classification of filters

- **Depending on signal nature:**

 1. Analog filter 2. Digital filter.

- **Depending on components:**

 1. active filter 2. Passive filter.

- **Depending on frequencies(Audio and radio frequency)**

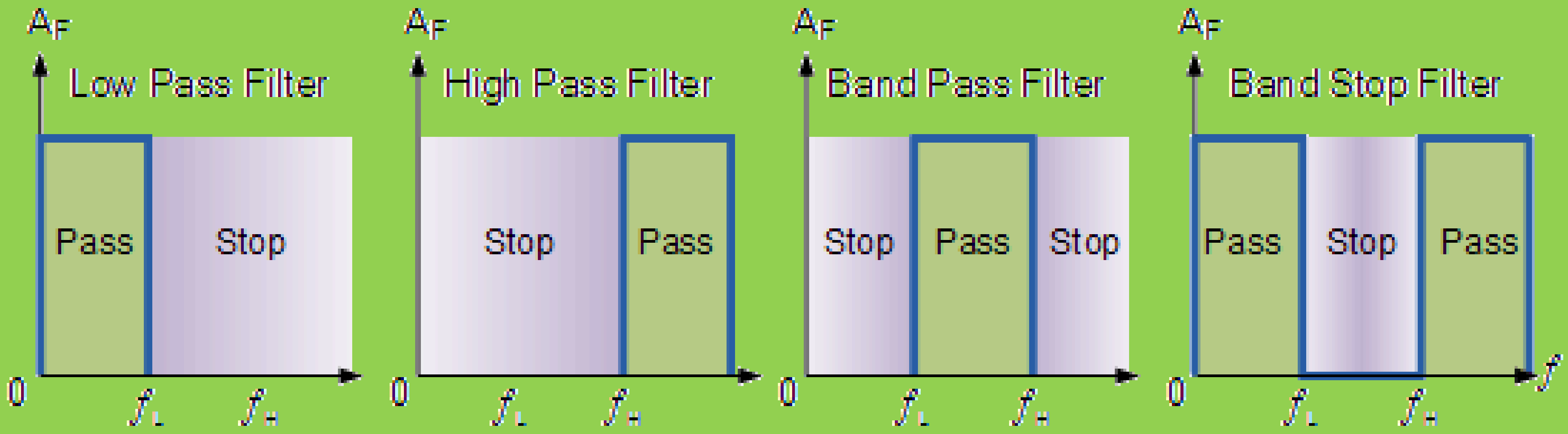
1. low pass filter 2.high pass filter 3.band pass filter 4.band stop

5. All pass filter

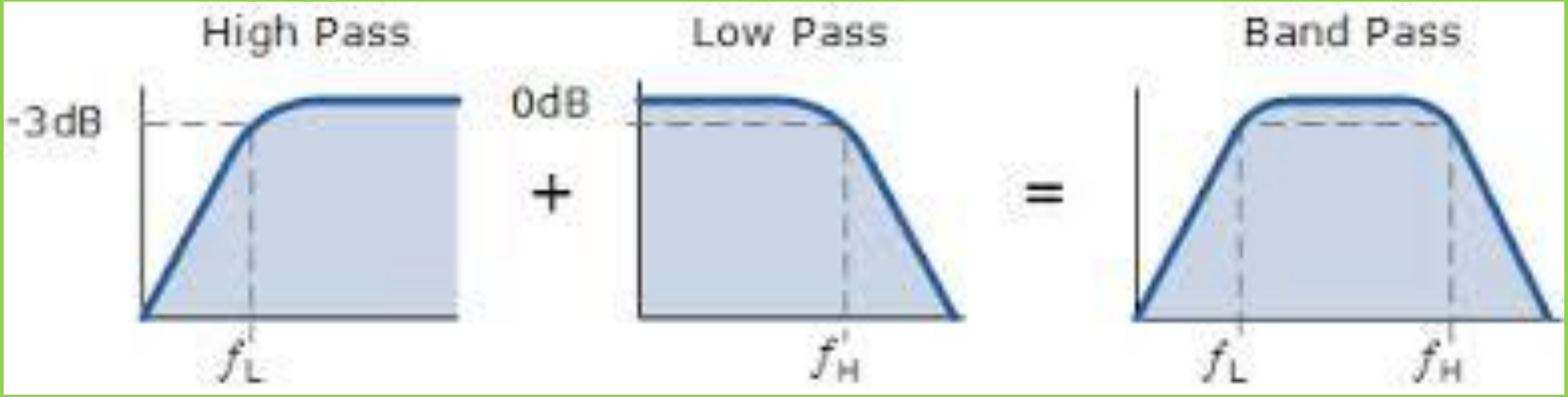
# Active Filters

- There are 4 basic categories of active filters: **Active Filters** contain active components such as operational amplifiers, transistors or FET’s within their circuit design. They draw their power from an external power source and use it to boost or amplify the output signal.
- These are generally  used in communication and signal processing i.e. radio, television etc.
- **Low-pass filters, High-pass filters Band-pass filters, Band-reject filters**
- Each of these filters can be built by using op-amp as the active element combined with RC, RL or RLC circuit as the passive elements.

# Ideal Filter Response Curves



# Actual frequency response



# Terms related to filters

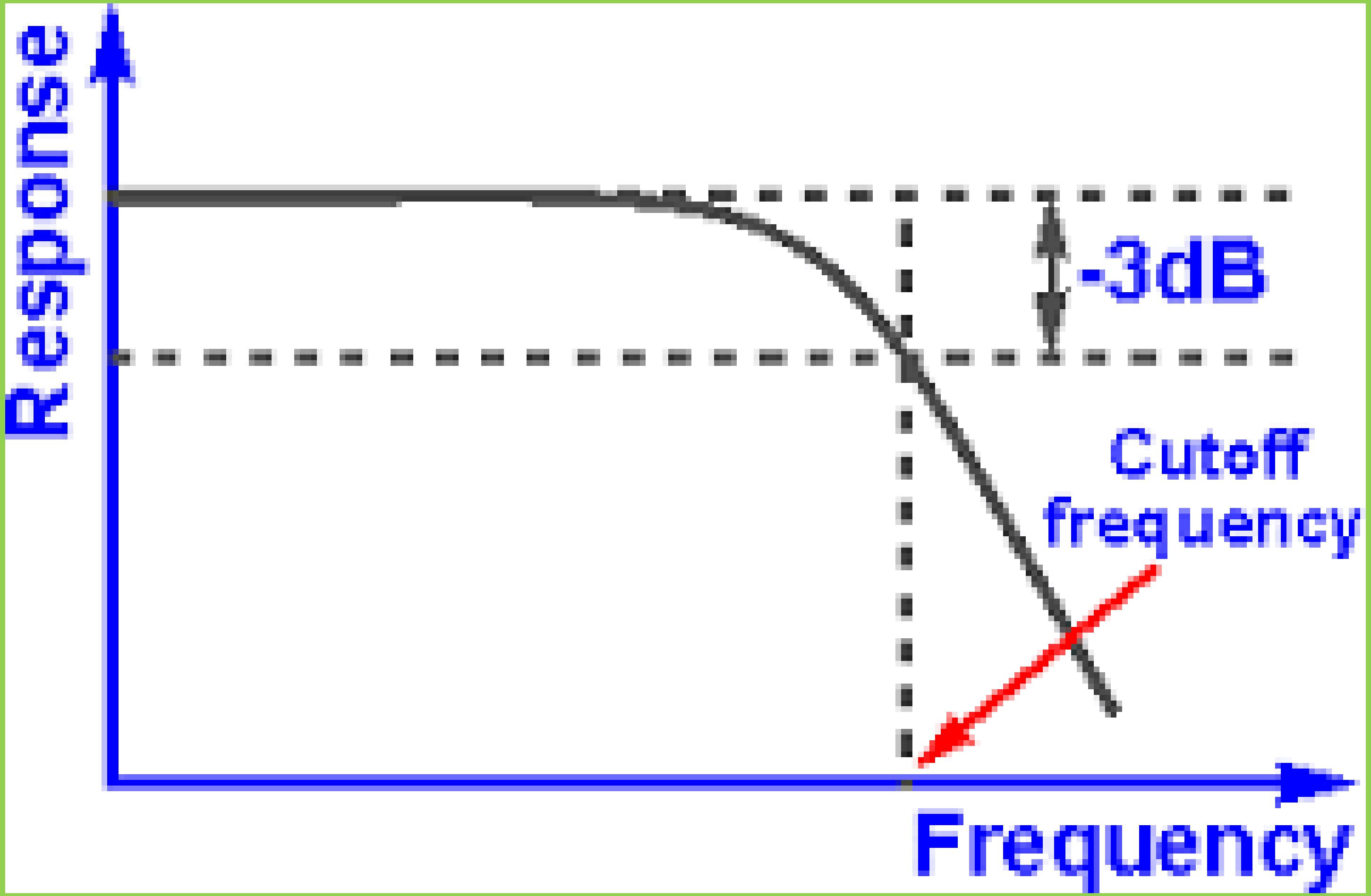
1.**Cut-off frequency-** it is the frequency at which signal strength drops 3dB(i.e. signal power becomes half.)

2. **Pass band-**The range of frequency which is permitted to pass.

3.**Stop band-** The range of frequency which is (not permitted)attenuated to pass.

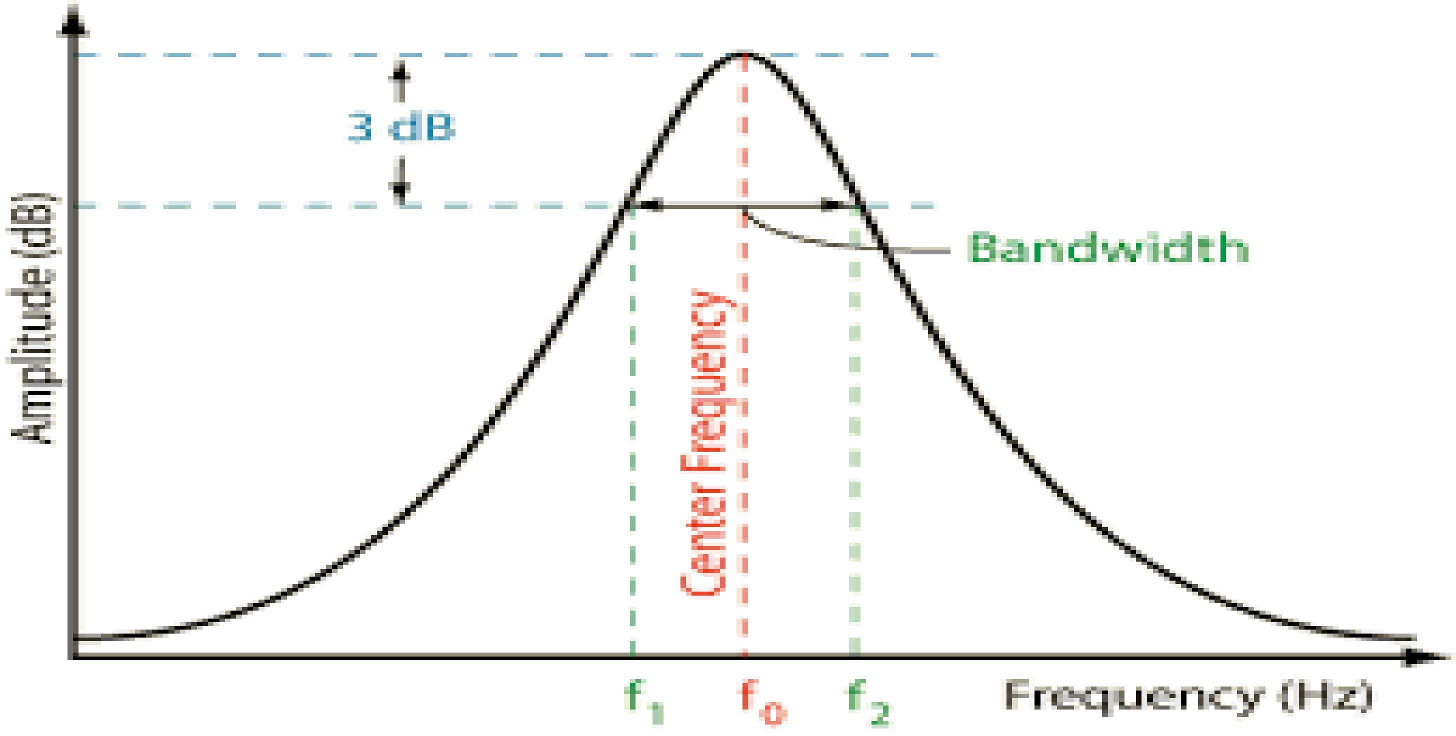
4.**Roll off rate-**The gain of filter falls rapidly in the stop band. The rate at which gain falls off is called the roll off rate.

# Roll rate



# Terms related to filters cont

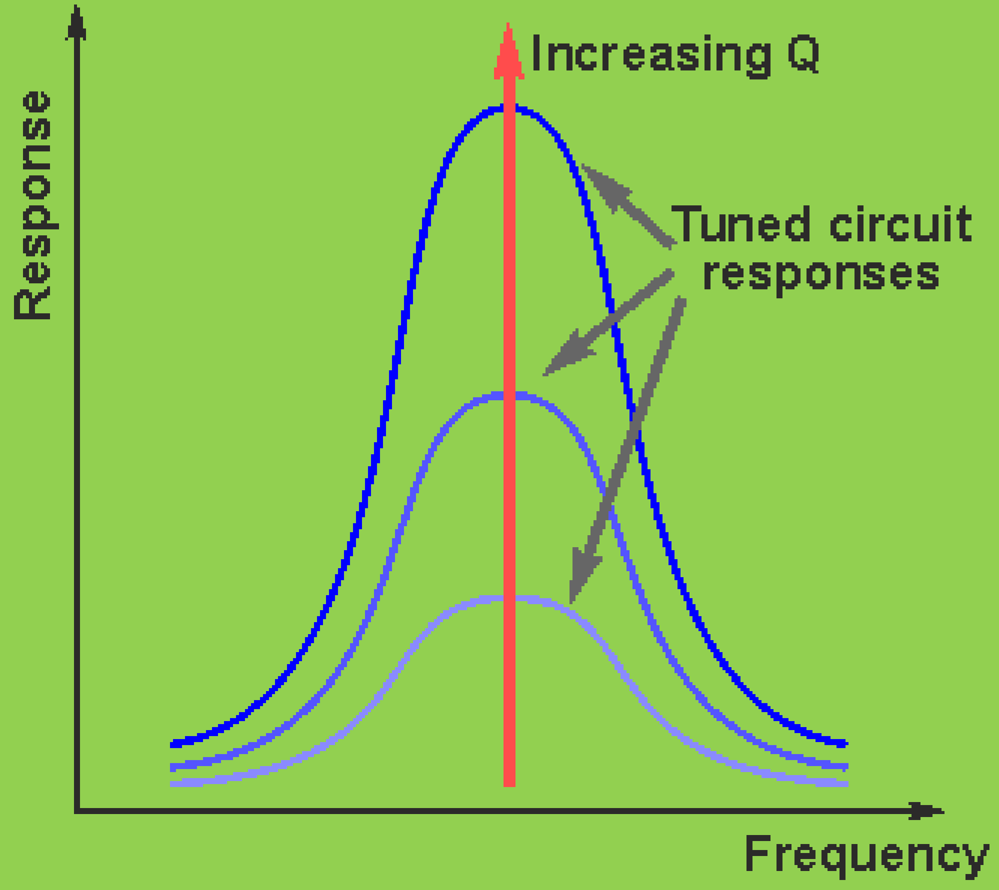
**5. Center frequency-** the **center frequency** of a **filter** or channel is a measure of a **central frequency** between the upper and lower cutoff **frequencies**. OR geometric mean of lower and higher frequencies.



# Terms related to filters cont1

6.**Band width-It** is the difference between the two cut-off frequencies.

7. **Q(quality) factor-it** is the ratio of center frequency to bandwidth of filter. $Q = \frac{f_c}{BW}$



# Terms related to filters cont2

8. **Order of filters-**The filter characteristics is given by order of filter i.e. number of poles.

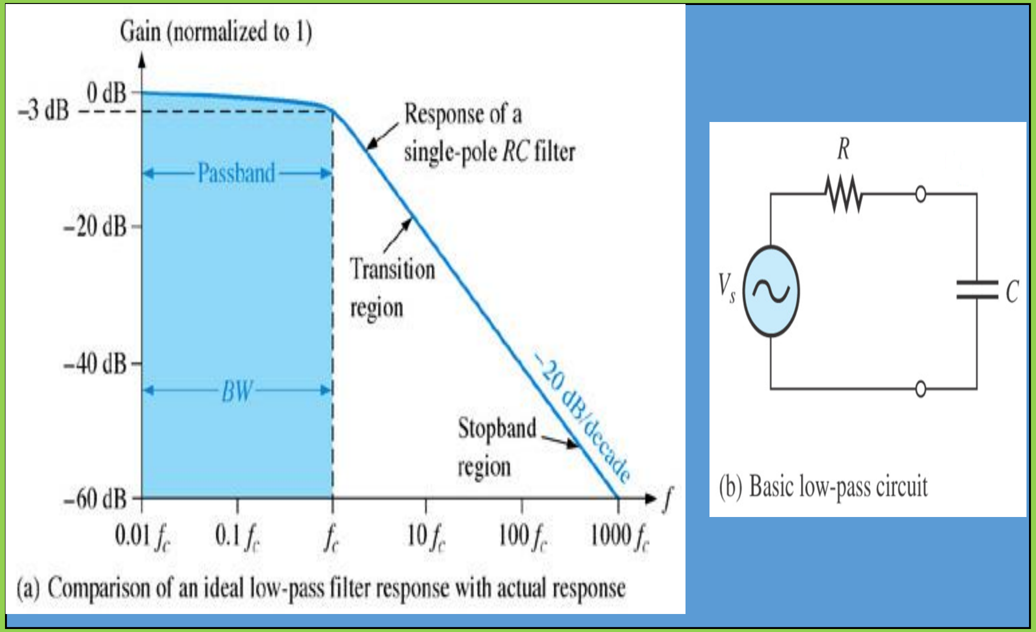
| Filter order | Roll rate |
|---|---|
| 1 | 20 dB/decade |
| 2 | 40 dB/decade |
| 3 | 60 dB/decade |

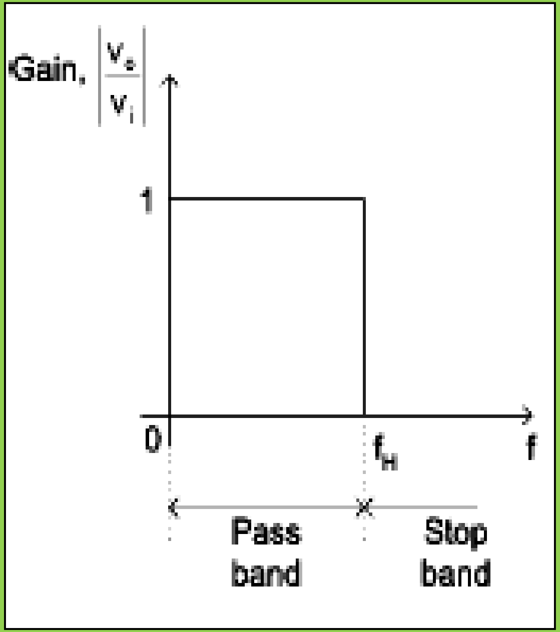
# Comparison between Passive Filter & Active Filter

| Passive Filter | Active Filter |
|---|---|
| R,C or L,C components are used | RC,LC and OP-amp are used. |
| Gain less than 1. | Gain greater than 1. |
| Isolation between input and output absent. | Isolation between input and output present. |
| Source loading can take place. | Source loading can not take place. |
| Frequency  response is not sharp. | Frequency response is sharp |

# Basic Filter responses

**Low Pass Filter Response:** A **low-pass filter** is a filter that passes frequencies from 0Hz to critical frequency, $f_c$ and significantly attenuates all other frequencies. Ideally, the response drops abruptly at the critical frequency, $\boldsymbol{f_H}$




**Actual response** **Ideal response**

# Basic Filter responses cont

**Passband** of a filter is the range of frequencies that are allowed to pass through the filter with minimum attenuation (usually defined as less than -3 dB of attenuation).

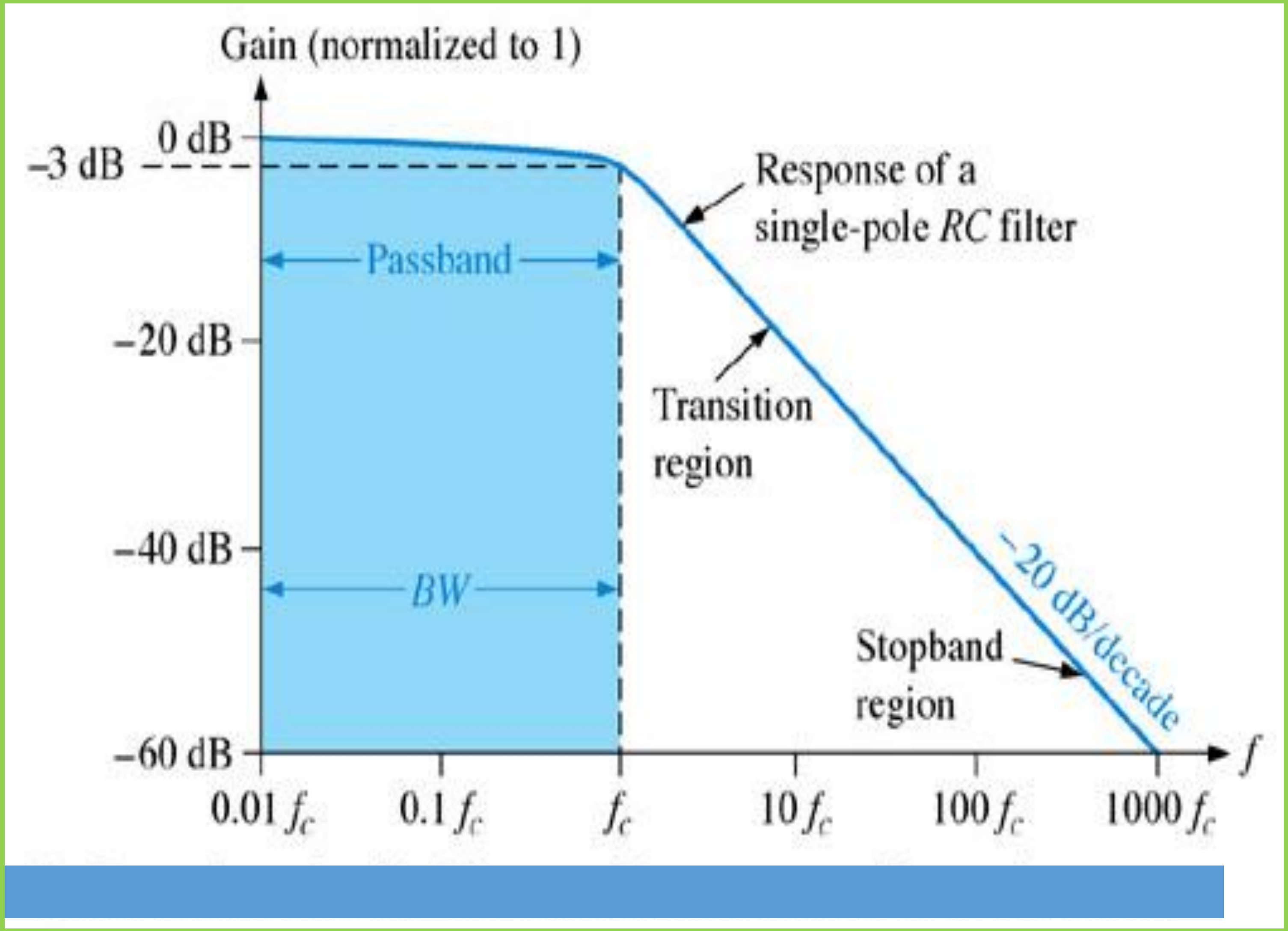
**Transition region** shows the area where the fall-off occurs.

**Stopband** is the range of frequencies that have the most attenuation.

**Critical frequency**, $f_c$, (also called the cutoff frequency) defines the end of the passband and normally specified at the point where the response drops – 3 dB (70.7%) from the passband response.

# Basic Filter responses cont1

*FR curve*

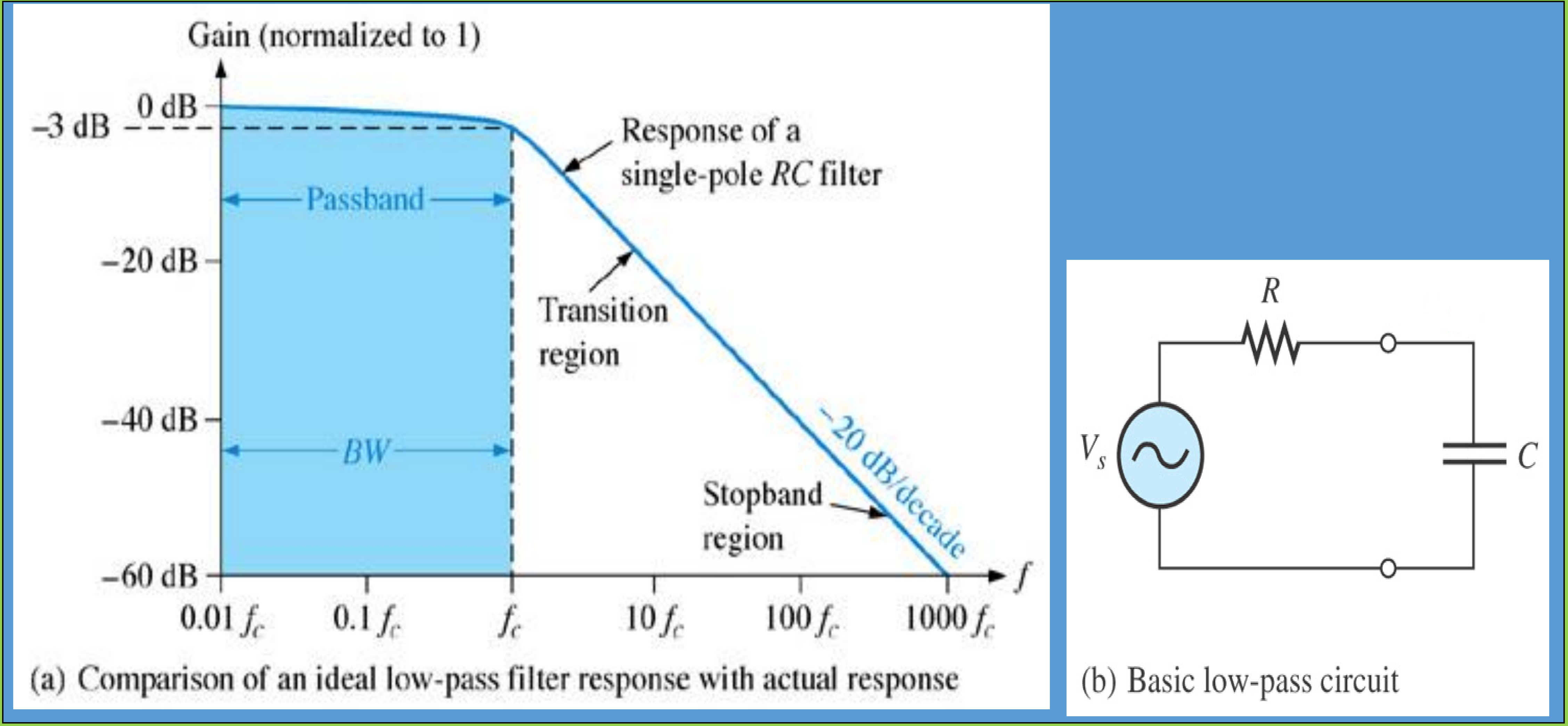


# Basic Filter responses cont2

- At low frequencies, $X_C$ is very high and the capacitor circuit can be considered as open circuit. Under this condition, $V_o = V_{in}$ or $A_V = 1$ (unity).
- At very high frequencies, $X_C$ is very low and the $V_o$ is small as compared with $V_{in}$. Hence the gain falls and drops off gradually as the frequency is increased.

# Basic Filter responses cont3

*FR curve cont*




(a) Comparison of an ideal low-pass filter response with actual response

(b) Basic low-pass circuit

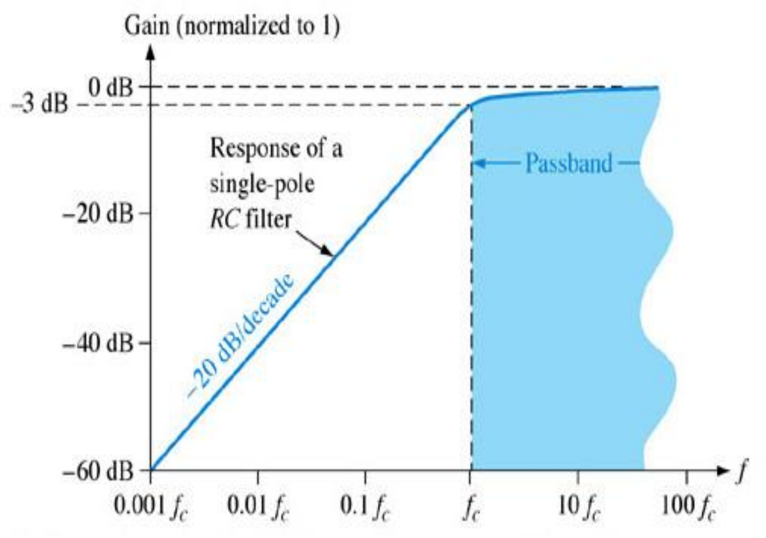
# Basic Filter responses cont4

- The bandwidth of an ideal low-pass filter is equal to $f_c$:$BW = f_c$
- The critical frequency of a low-pass RC filter occurs when $X_C = R$ and can be calculated using the formula below: $f_c = \frac{1}{2\pi RC}$

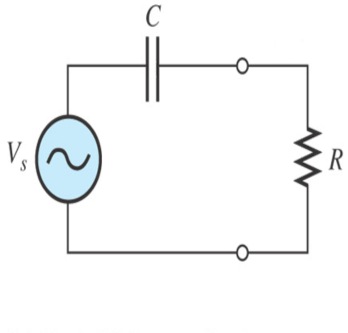
# High-pass filter responses

- A high-pass filter is a filter that significantly attenuates or rejects all frequencies **below** $f c$ and passes all frequencies **above** $f_c$.

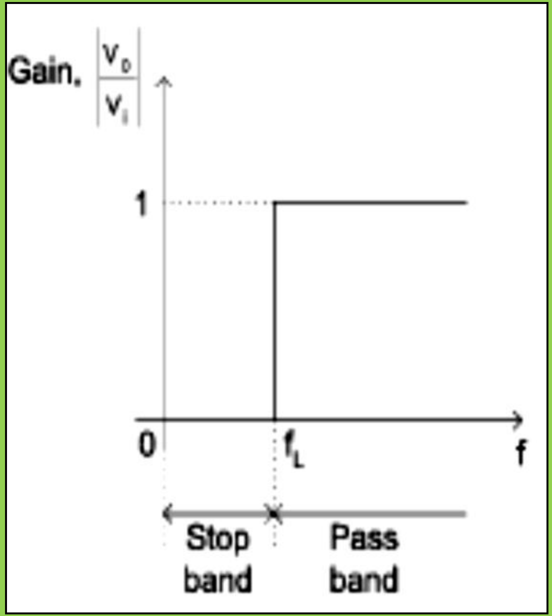
- The passband of a high-pass filter is all frequencies above the critical frequency. Ideally, the response rises abruptly at the critical frequency, $\boldsymbol{f_L}$



(a) Comparison of an ideal high-pass filter response with actual response



(b) Basic high-pass circuit




**Actual response** **Ideal response**

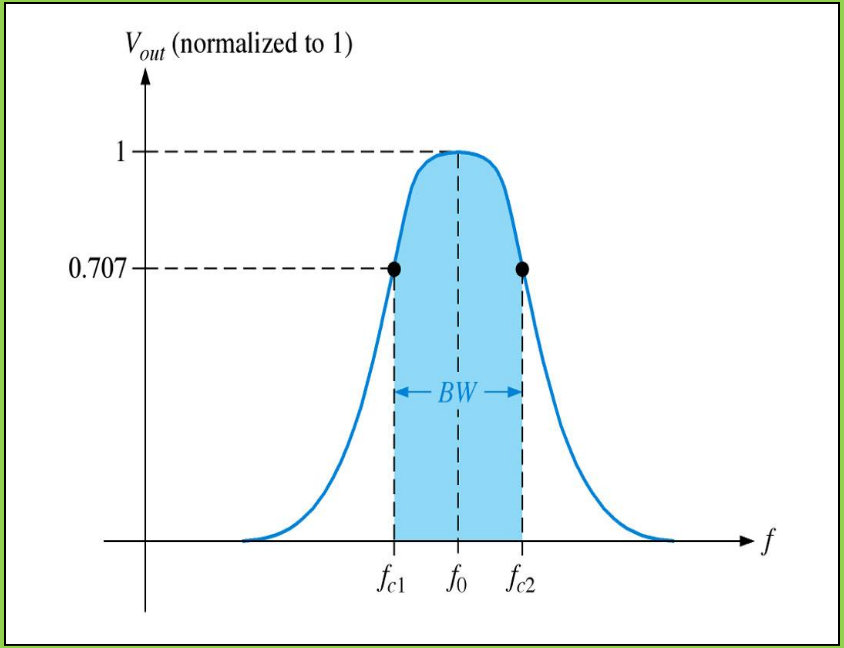
# High-pass filter responses cont

- The critical frequency of a high-pass RC filter occurs when

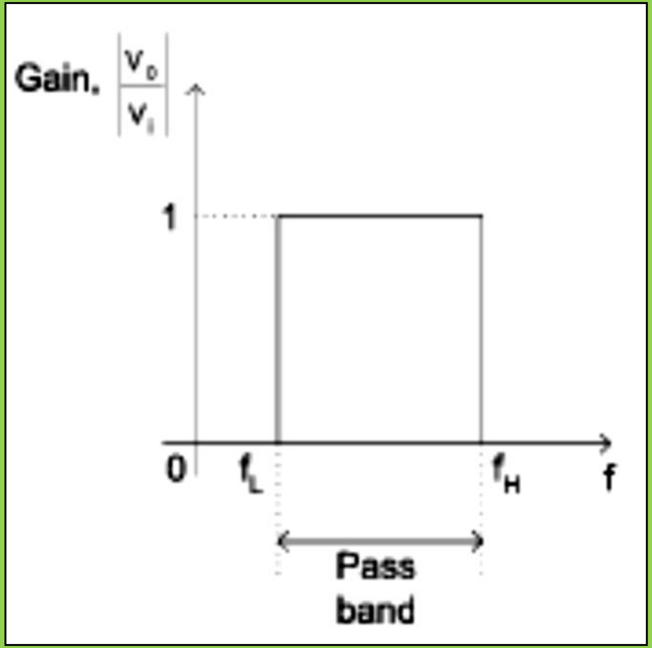
  $X_C = R$ and can be calculated using the formula below: $f_c = \frac{1}{2\pi RC}$

# Band-pass filter responses

- A **band-pass filter** passes all signals lying within a band between a **lower-frequency limit** and **upper-frequency limit** and essentially rejects all other frequencies that are outside this specified band.




**Actual response**

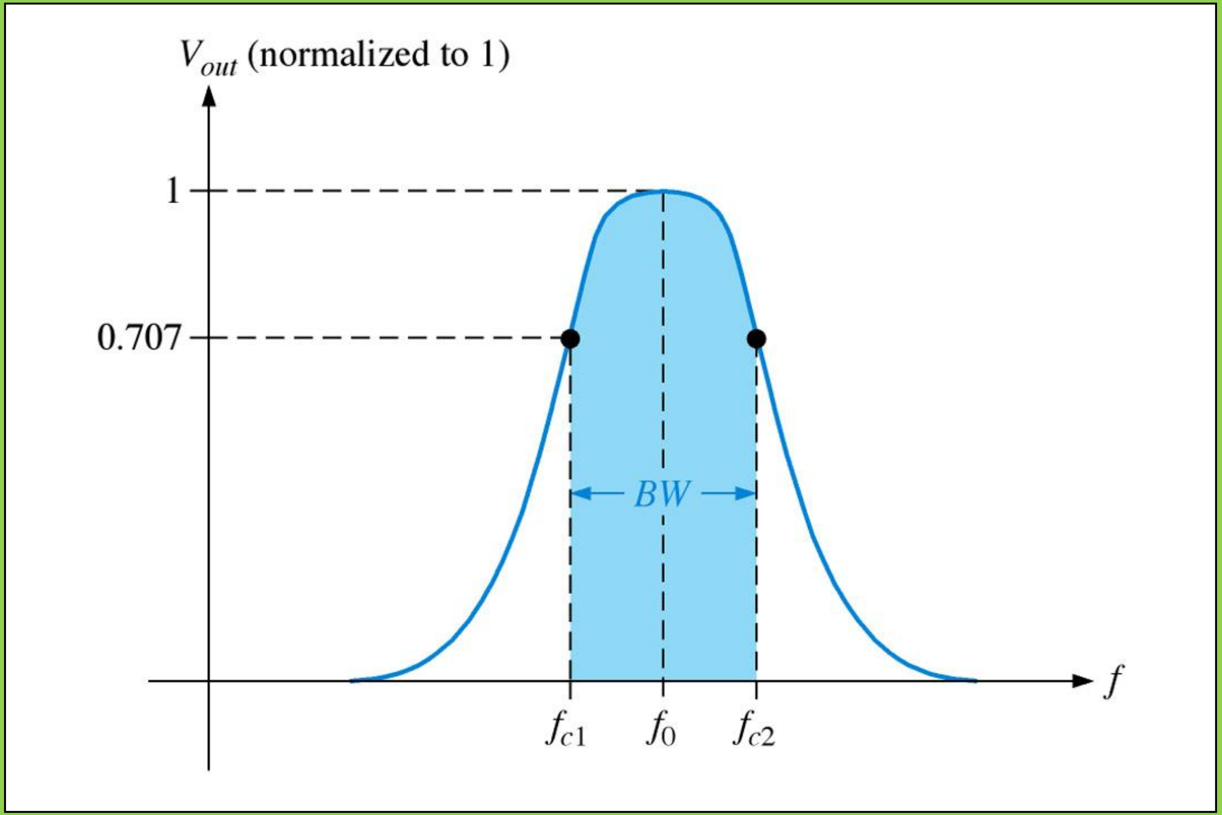



**Ideal response**

# Bandwidth (BW)

The **bandwidth (BW)** is defined as the **difference** between the **upper critical frequency ($f_{c2}$)** and the **lower critical frequency ($f_{c1}$)**

$$BW = f_{c2} - f_{c1}$$



# Band-pass filter responses cont

- The frequency about which the pass band is centered is called the ***center frequency***, $f_o$ and defined as the geometric mean of the critical frequencies.

$$f_0 = \sqrt{f_{c2} \cdot f_{c1}}$$

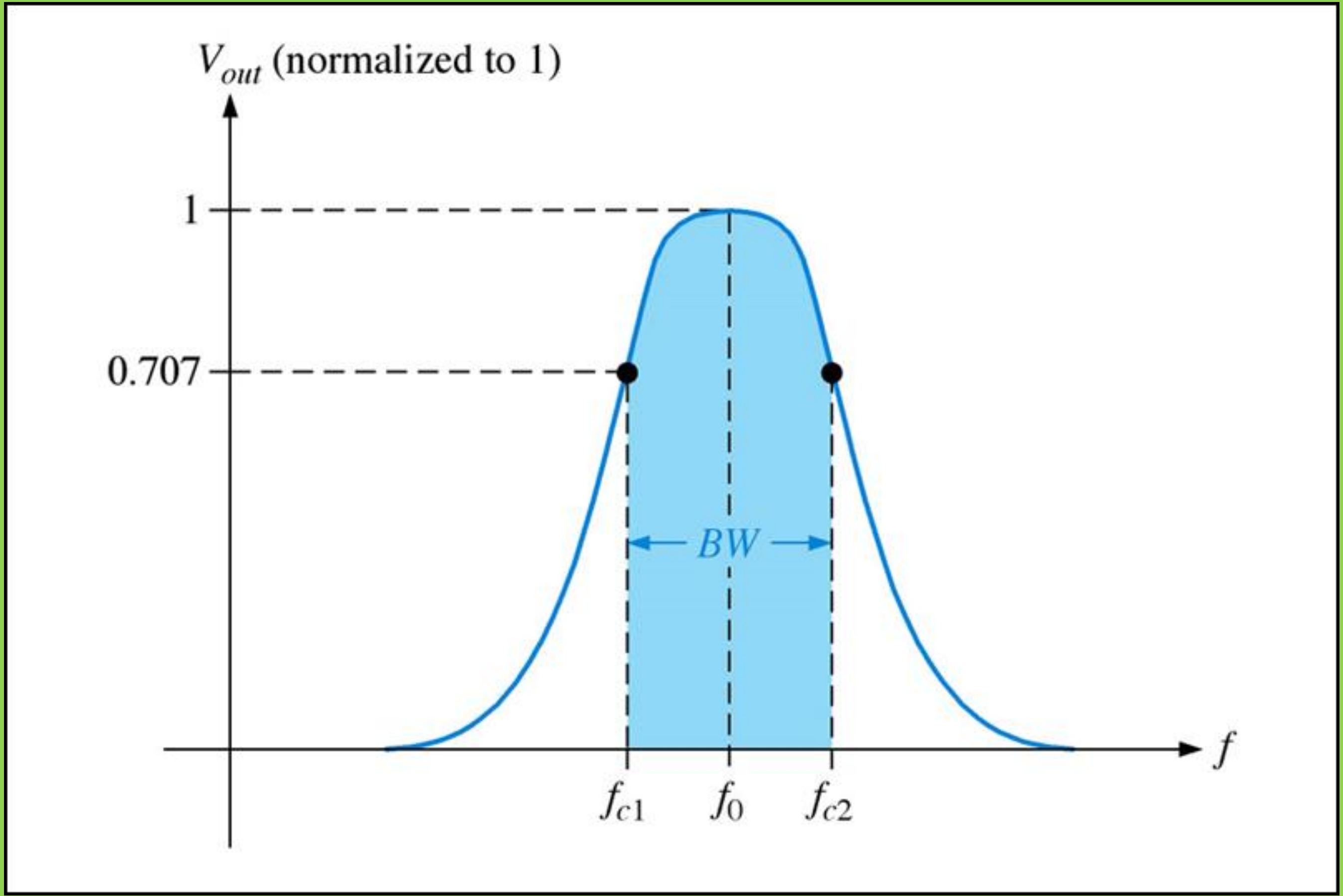
- The ***quality factor (Q)*** of a band-pass filter is the ratio of the center frequency to the bandwidth. $Q = \frac{f_0}{BW}$

- The higher value of Q, the narrower the bandwidth and the better the selectivity for a given value of $f_o$.

- (Q>10) as a narrow-band or (Q<10) as a wide-band

# Band-pass filter responses cont1

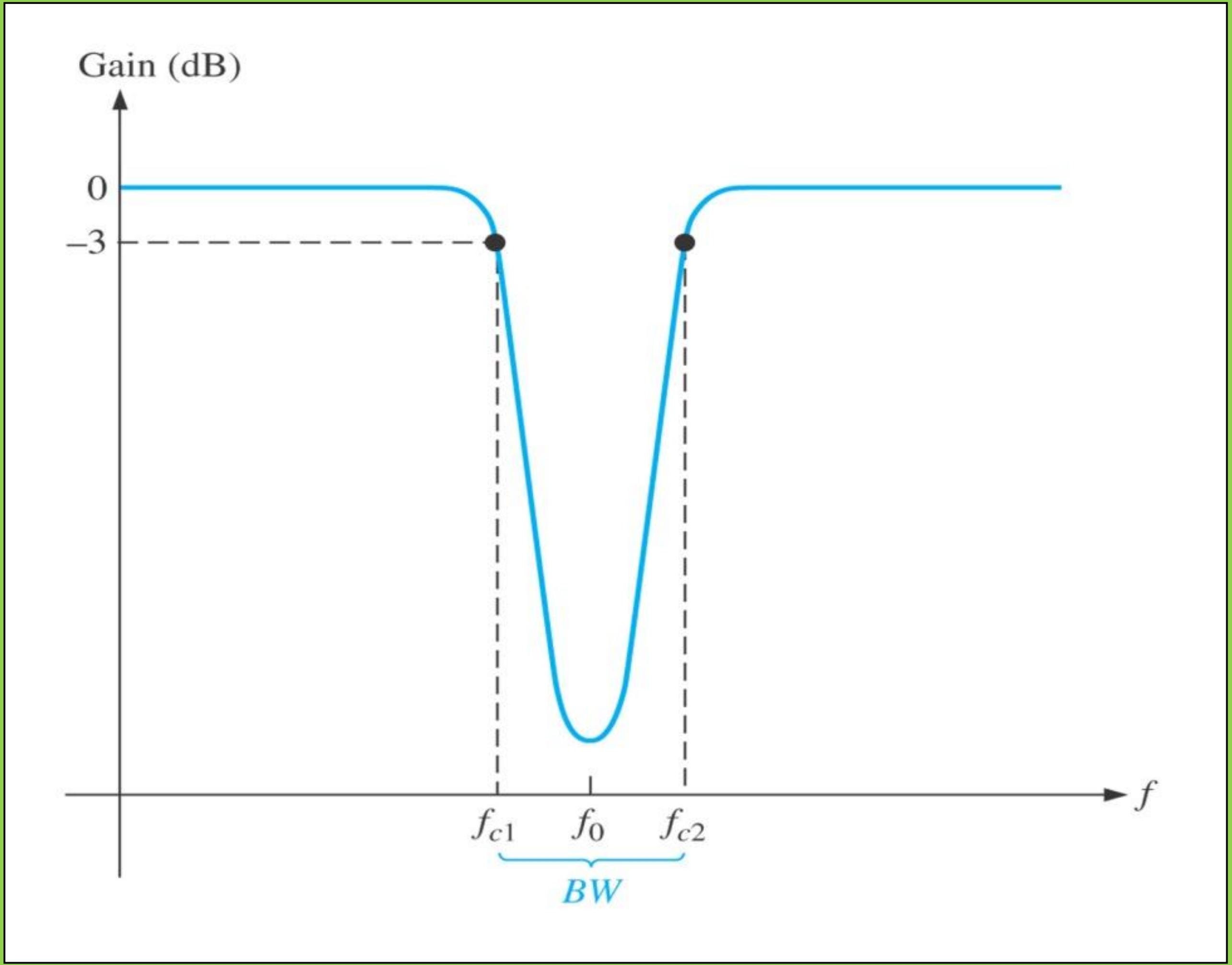
- The quality factor (Q) can also be expressed in terms of the damping factor (DF) of the filter as : $Q = \frac{1}{DF}$



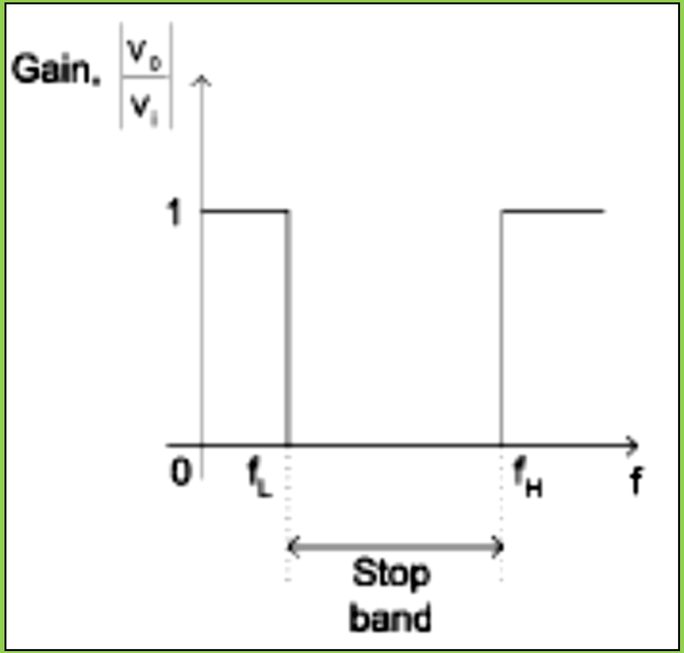
# Band-stop filter responses

- **Band-stop filter** is a filter which its operation is **opposite** to that of the band-pass filter because the frequencies **within** the bandwidth are **rejected**, and the frequencies above $f_{c1}$ and $f_{c2}$ are **passed**.
- For the band-stop filter, the **bandwidth** is a band of frequencies between the 3 dB points, just as in the case of the band-pass filter response.

# Band-stop filter responses cont




Actual response




Ideal response

# Optics (spherical mirrors and Thin Lenses)

## Terms Associated with Spherical Mirrors

- **Spherical Mirror** :A curved mirror formed by a part of a hollow glass sphere with a reflecting surface (created by depositing silver metal) is also referred to as a spherical mirror.
- **Concave Mirror**: A concave mirror is a curved mirror with the reflecting surface on the hollow side (created by depositing silver metal on the outer curved side).
- **Convex Mirror :** A convex mirror is a curved mirror with the reflecting surface on the outerside
- **Centre of Curvature** : The centre of curvature of a curved mirror is defined as the center of the hollow glass sphere of which the curved mirror was (previously) a part.
- **Radius of curvature** The radius of curvature of a curved mirror is defined as the radius of the hollow glass sphere of which the spherical mirror was (previously) a part. Note that any line drawn from the center of curvature C to the mirror surface meets it at right angle and equals

# Terms Associated with Spherical Mirrors con't

- **Principal Axis** :The principal axis of a curved mirror is defined as the imaginary line passing through its pole P and center of curvature C.
- **Focus** :The principal focus is defined as the point on the principal axis where the light rays traveling parallel to the principal axis after reflection actually meet (for a concave mirror) or appear to meet (for a convex mirror). The principal focus is in front of the concave mirror and is behind the convex mirror. The focal length (denoted by FP in the figure) is the distance between the pole P and the principal focus F of a curved mirror. Note that the focal length is half the radius of curvature. Focal Length = Radius of Curvature/2.
- **Pole** : The pole is defined as the geometric center of the curved mirror.

# Ray Diagrams

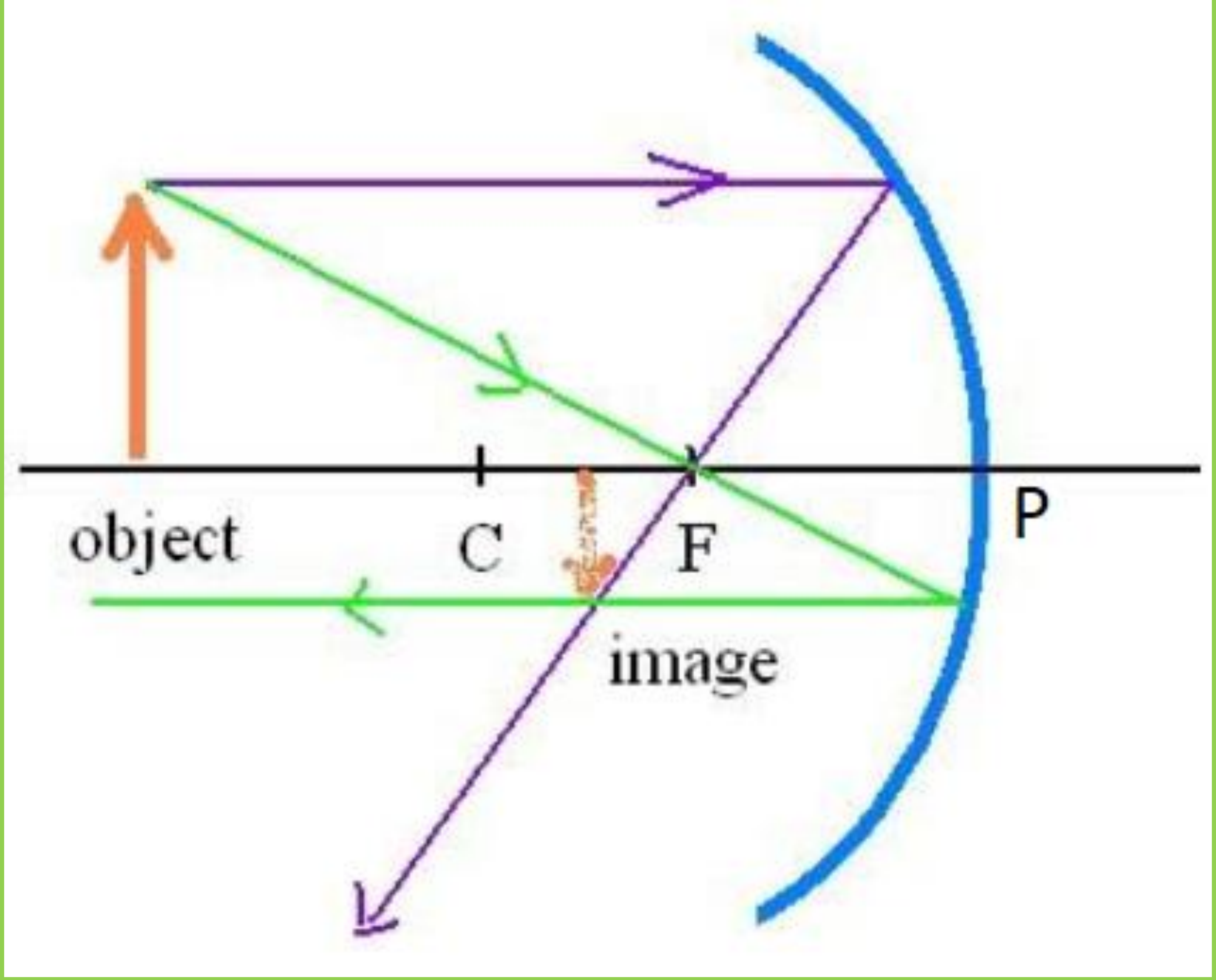
- ***Used for determining location, size, orientation, and type of image***

1. Any light ray traveling parallel to the principal axis is reflected by the curved mirror through the principal focus. It either actually passes (for a concave mirror) or appears to pass (for a convex mirror) through the principal focus

2. Any light ray that passes (for a concave mirror) or appears to pass (for a convex mirror) through the principal focus is reflected by the curved mirror parallel to the principal axis.

3. Any light ray that passes (for a concave mirror) or appears to pass (for a convex mirror) through the center of curvature retraces its initial path after reflection by the curved mirror.

# Reflection for concave mirror

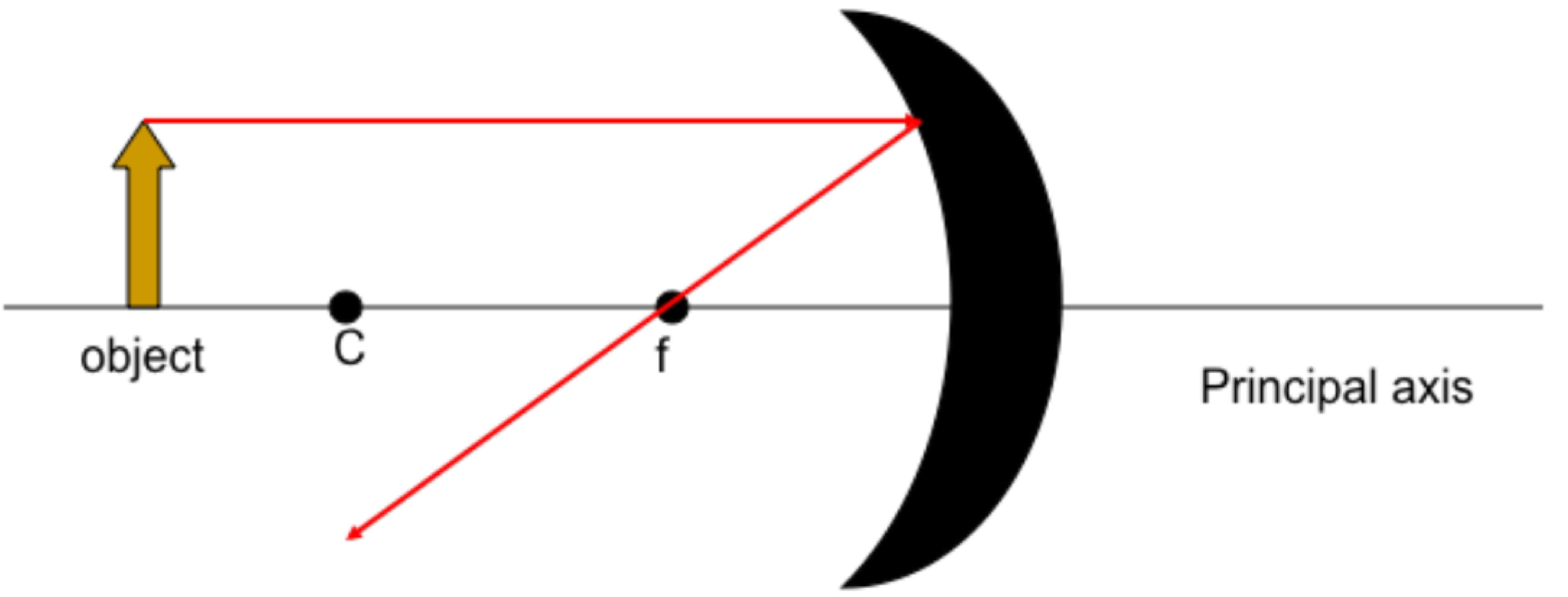


# Sign Convention for Spherical Mirrors

- $f$ is positive for a concave mirror and negative for a convex mirror
- $d_i$ is positive if the image is a real image and located on the object's side of the mirror. Negative if the image is a virtual image and located behind the mirror.
- $h_i$ is positive if the image an upright (and therefore, also virtual). Negative if the image is inverted (and therefore, also real).
- $M$ is positive if the image is enlarged and  Negative if the image is reduced

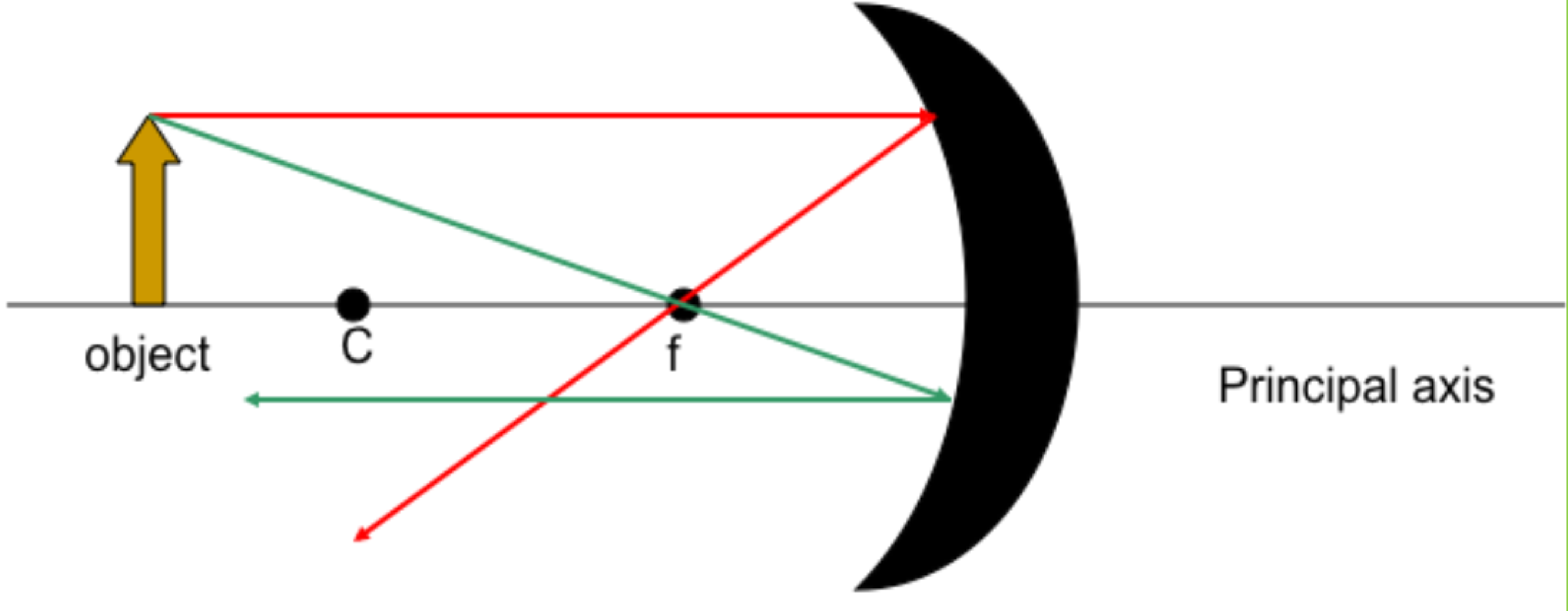
# Ray Diagrams – Concave Mirrors

- Step One: Draw a ray, starting from the top of the object, parallel to the principal axis and then through "*f*" after reflection.



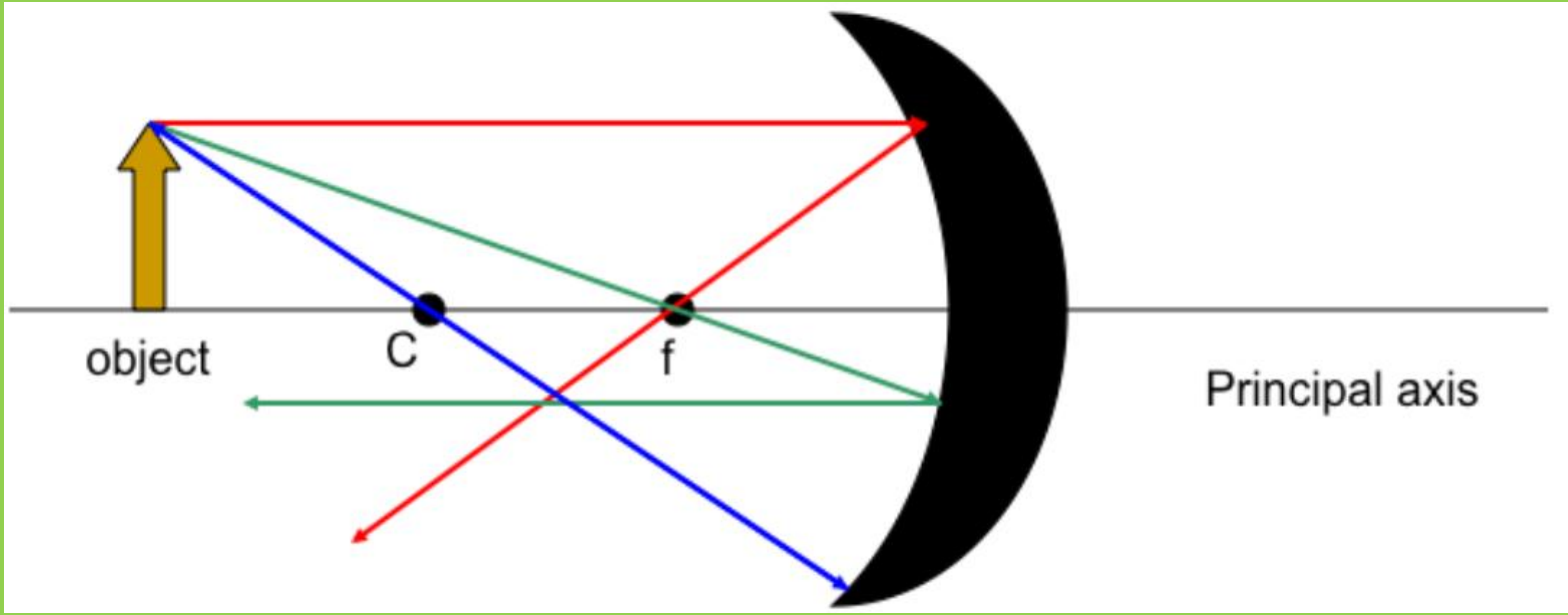
# Ray Diagrams – Concave Mirrors con’t

- Step Two: Draw a ray, starting from the top of the object, through the focal point, then parallel to the principal axis after reflection.



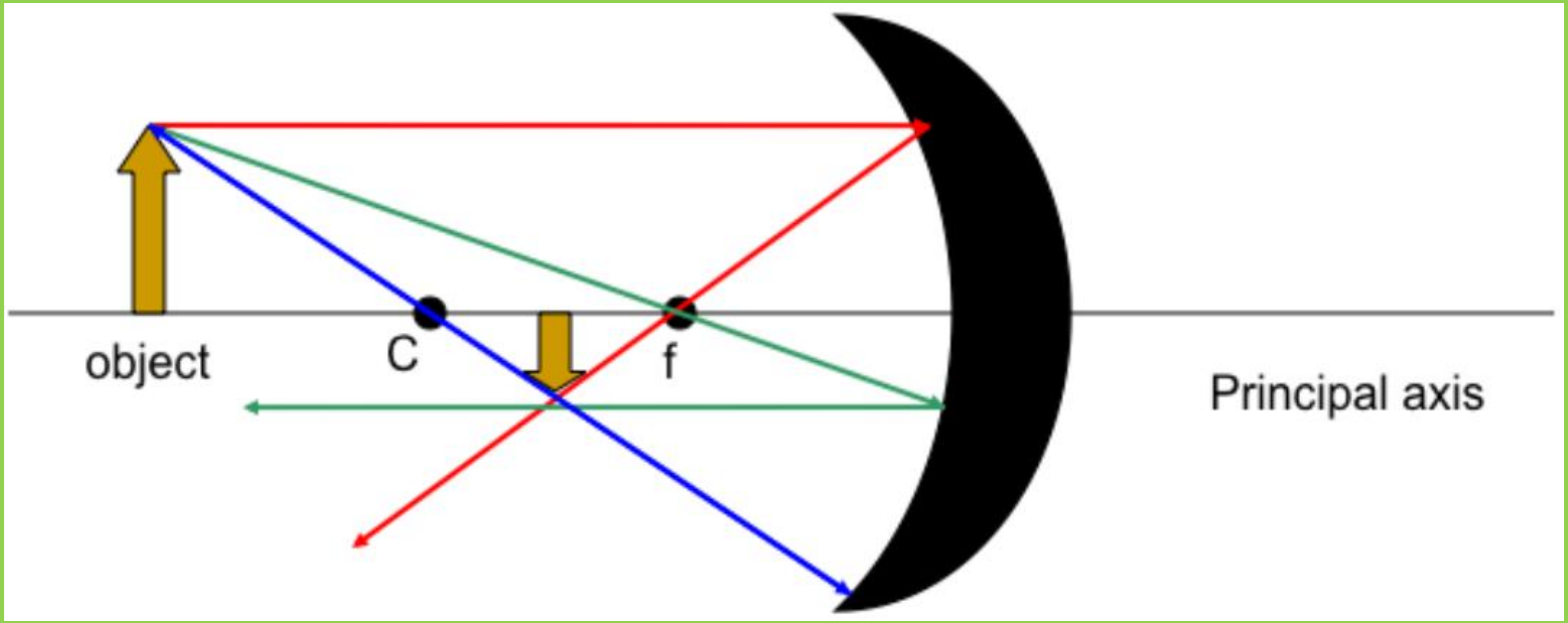
# Ray Diagrams – Concave Mirrors con't

- Step Three: Draw a ray, starting from the top of the object, through C, then back upon itself. *The intersection of these 3 lines is the location of the image (or two).*



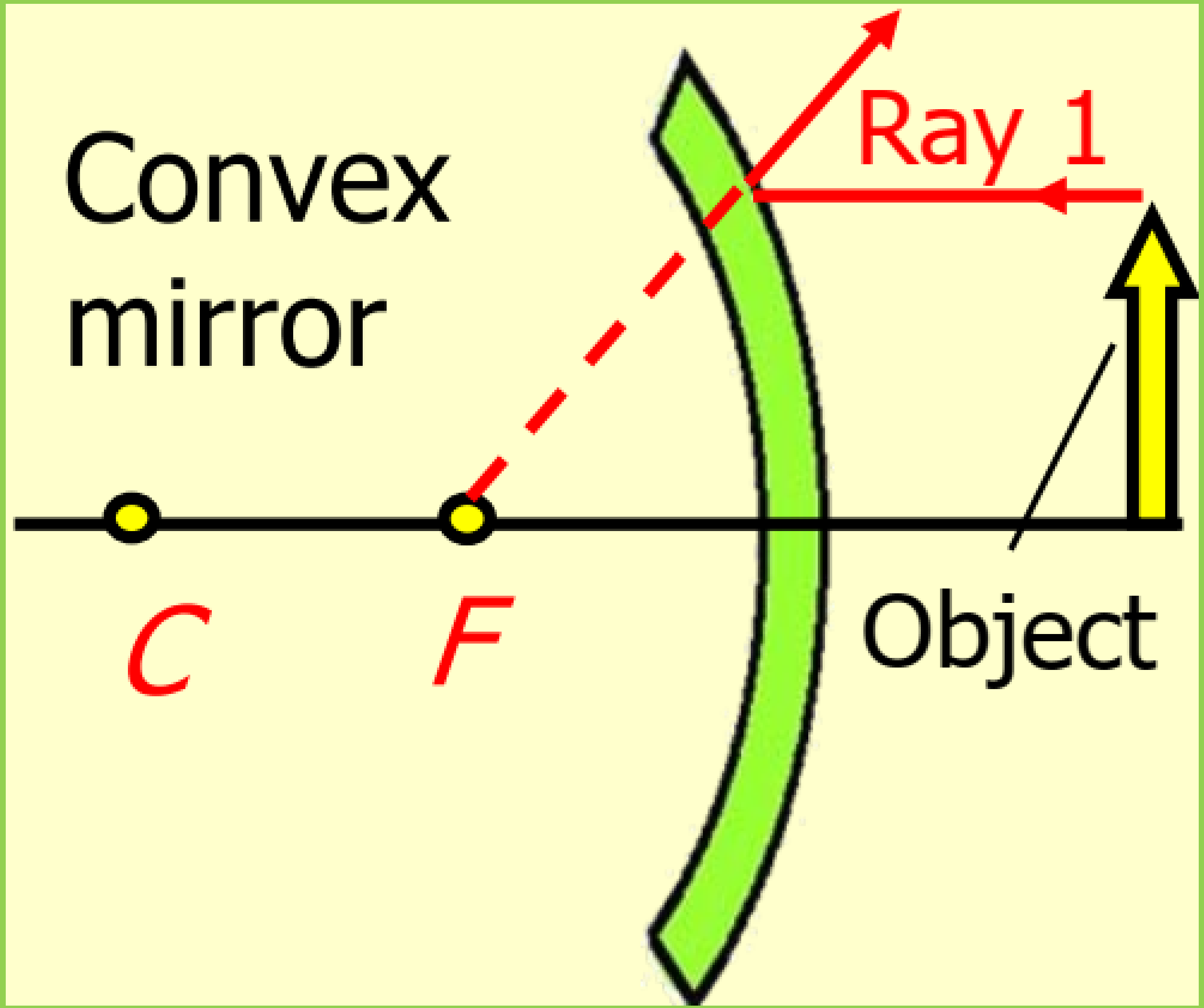
# Ray Diagrams Image Characteristics

After getting the intersection, draw an arrow down from the principal axis to the point of intersection. Then notice: *(a) Image is on the SAME (or opposite) side of the mirror, (b) Image is REDUCED (or enlarged) and (c) Image is INVERTED (or upright)*



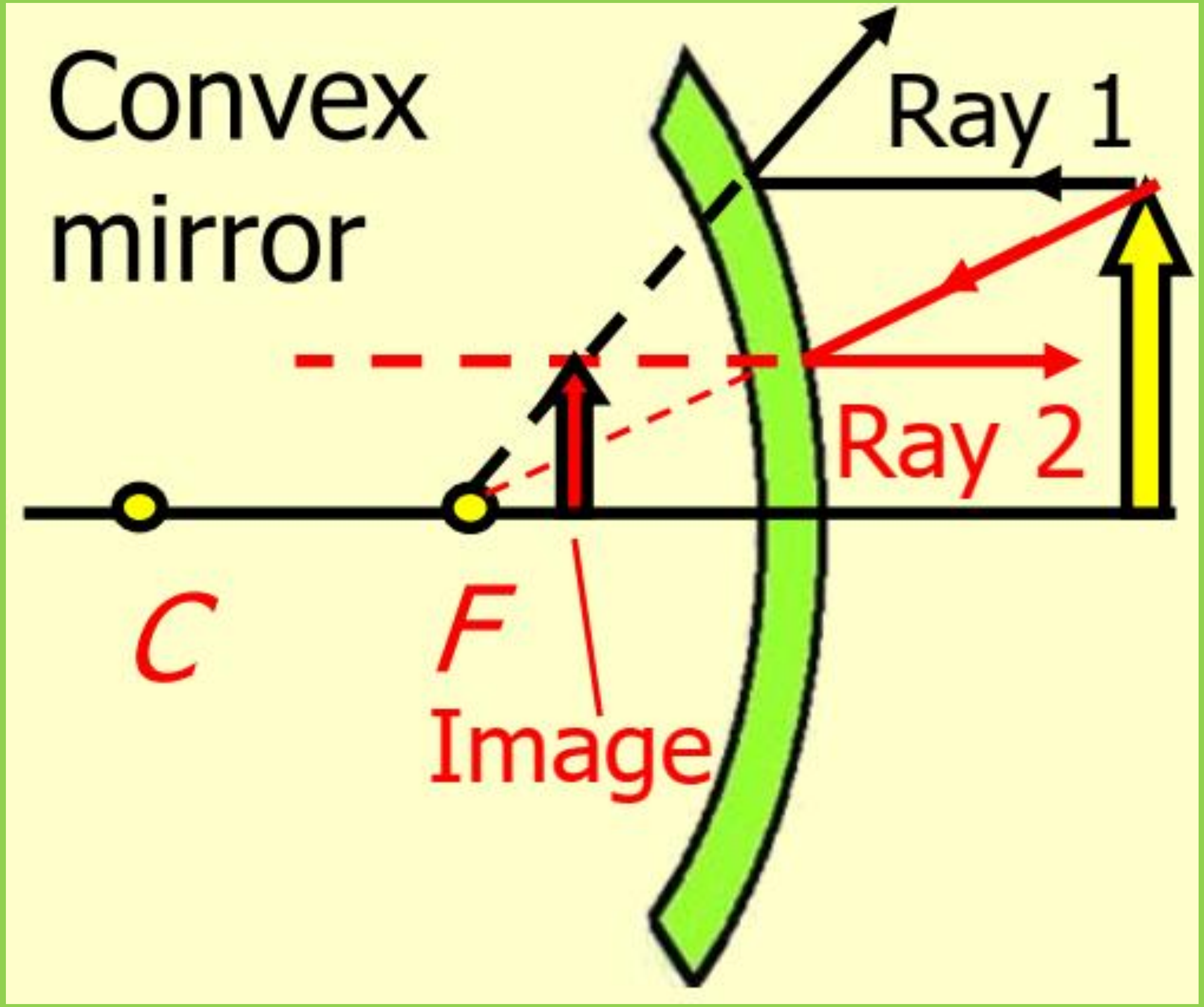
# **Ray Diagrams – Convex Mirrors-** *Image Construction*

Ray 1: A ray parallel to mirror axis appears to come from the focal point of a convex mirror.



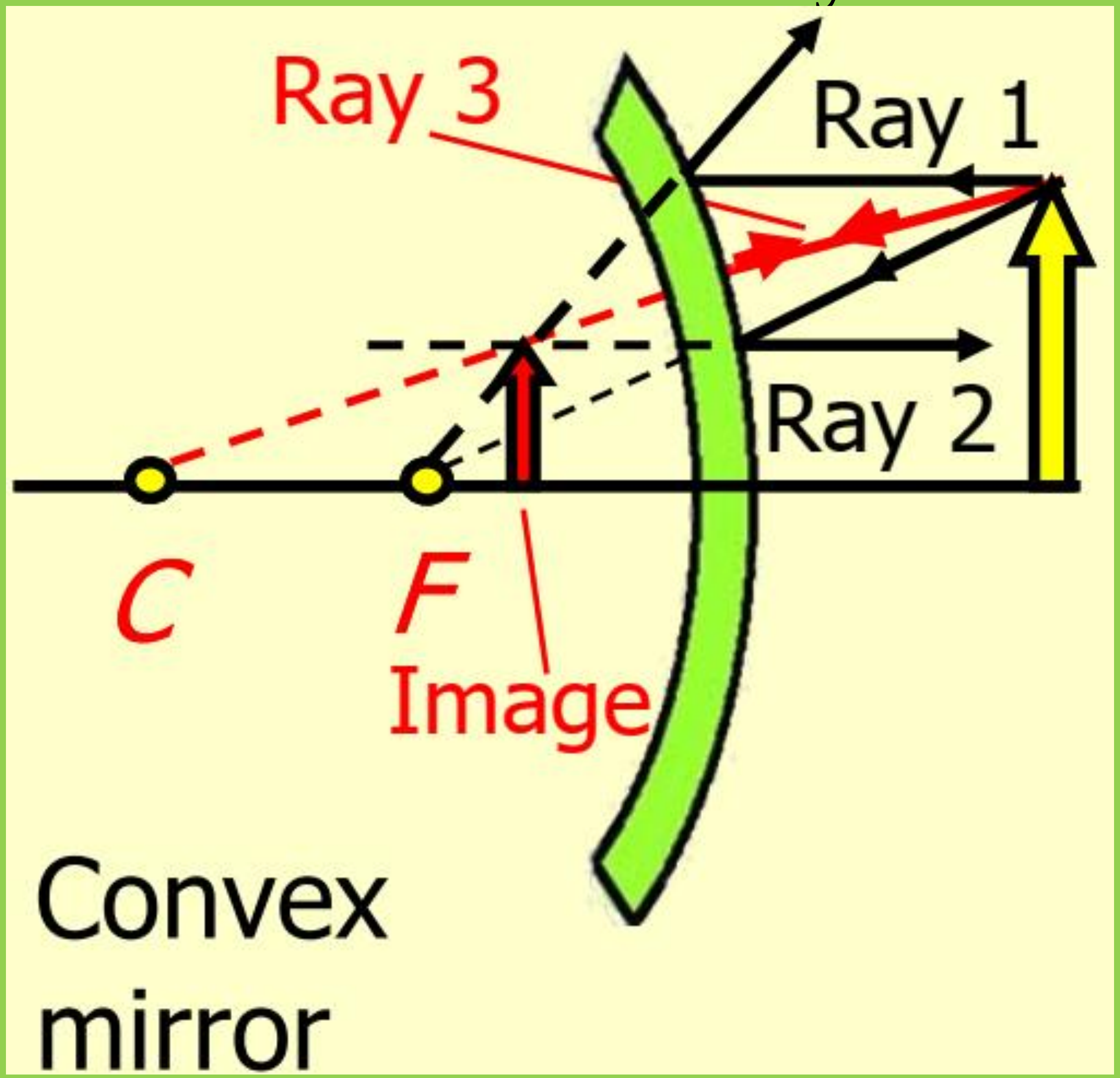
# **Ray Diagrams – Convex Mirrors-** *Image Construction*

Ray 2: A ray proceeding toward the principal focus of a convex mirror is reflected parallel to the mirror axis.



# Ray Diagrams – Convex Mirrors- *Image Construction*

Ray 3: A ray that proceeds along the centre of curvature is always reflected back along its original path.



# Ray Diagrams – Convex Mirrors

## Image Characteristics

*All images are erect, virtual, and diminished. Images get larger as object approaches*

# Optics (mirrors)

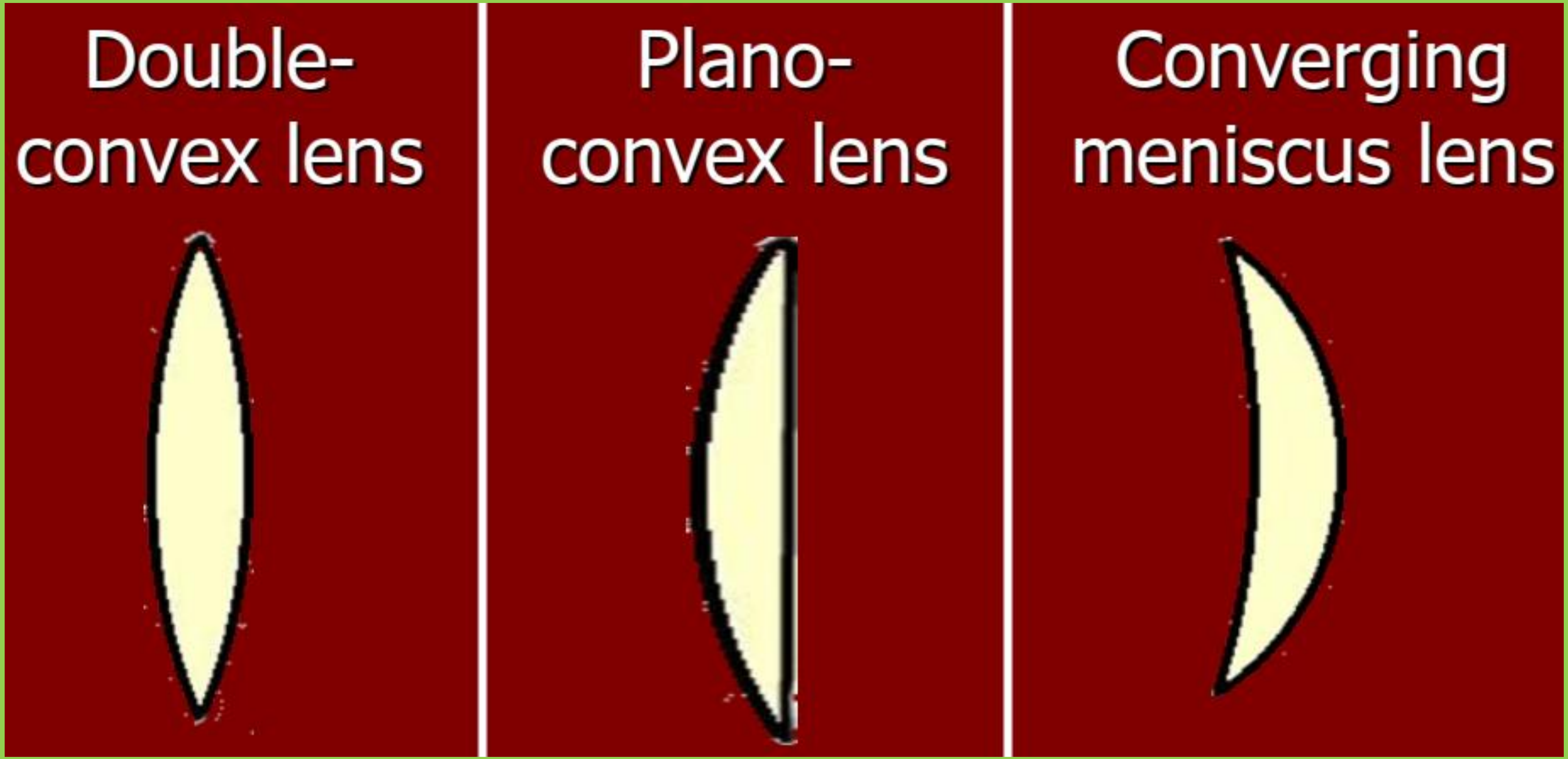
*Giancoli pg 970, Serway pg1127, Halliday pg 1040 &1049 and Openstax pg889.*

*Exercise: 1.* A concave mirror forms an image, on a wall 3.00 m in front of the mirror, of a headlamp filament 10.0 cm in front of the mirror.
(a) What are the radius of curvature and focal length of the mirror?
(b) What is the lateral magnification? What is the image height I the object height is 5.00 mm?

*Exercise: 2.* An external rearview car mirror is convex with a radius of curvature of 16.0 m. Determine the location of the image and its magnification for an object 10.0 m from the mirror.

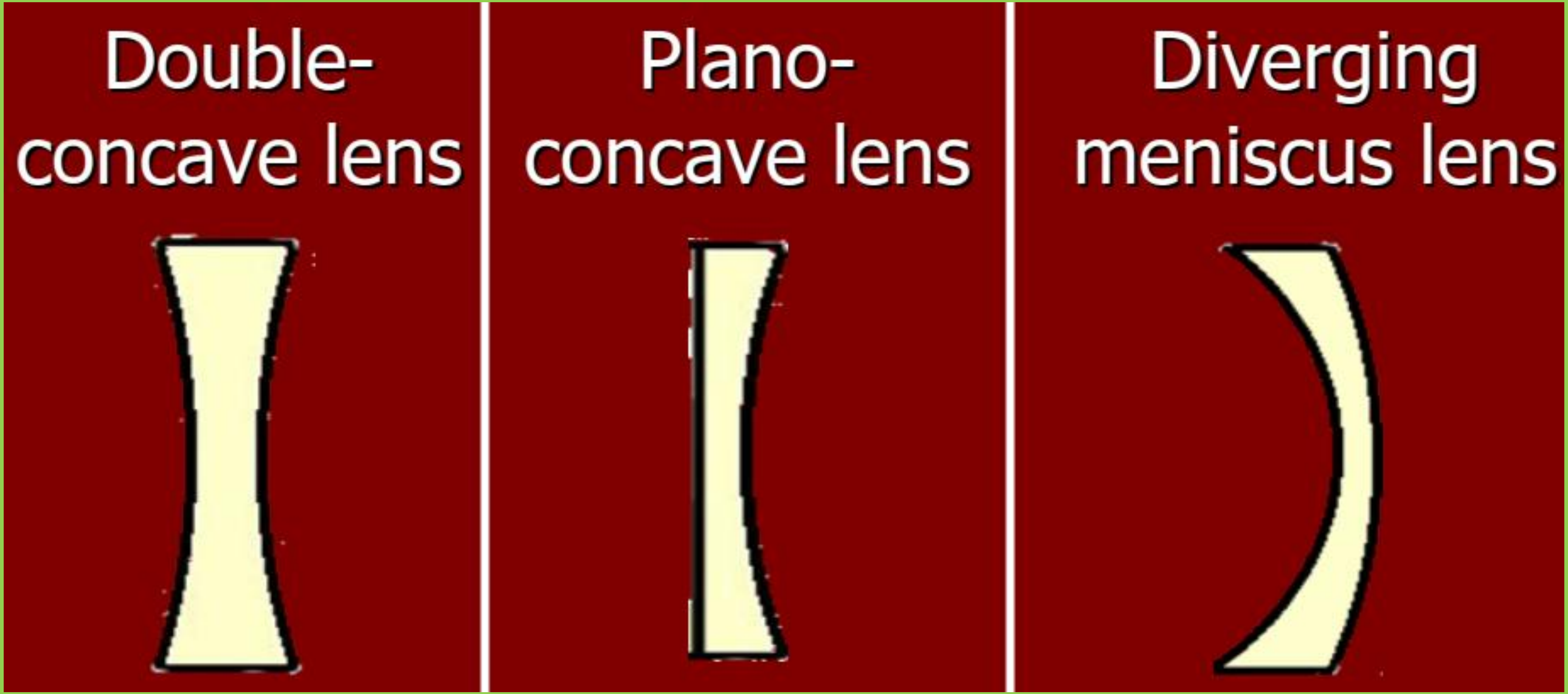
# Thin Lenses (Converging and Diverging Lenses)

**Types of Converging Lenses:** In order for a lens to converge light it must be thicker near the midpoint to allow more bending.

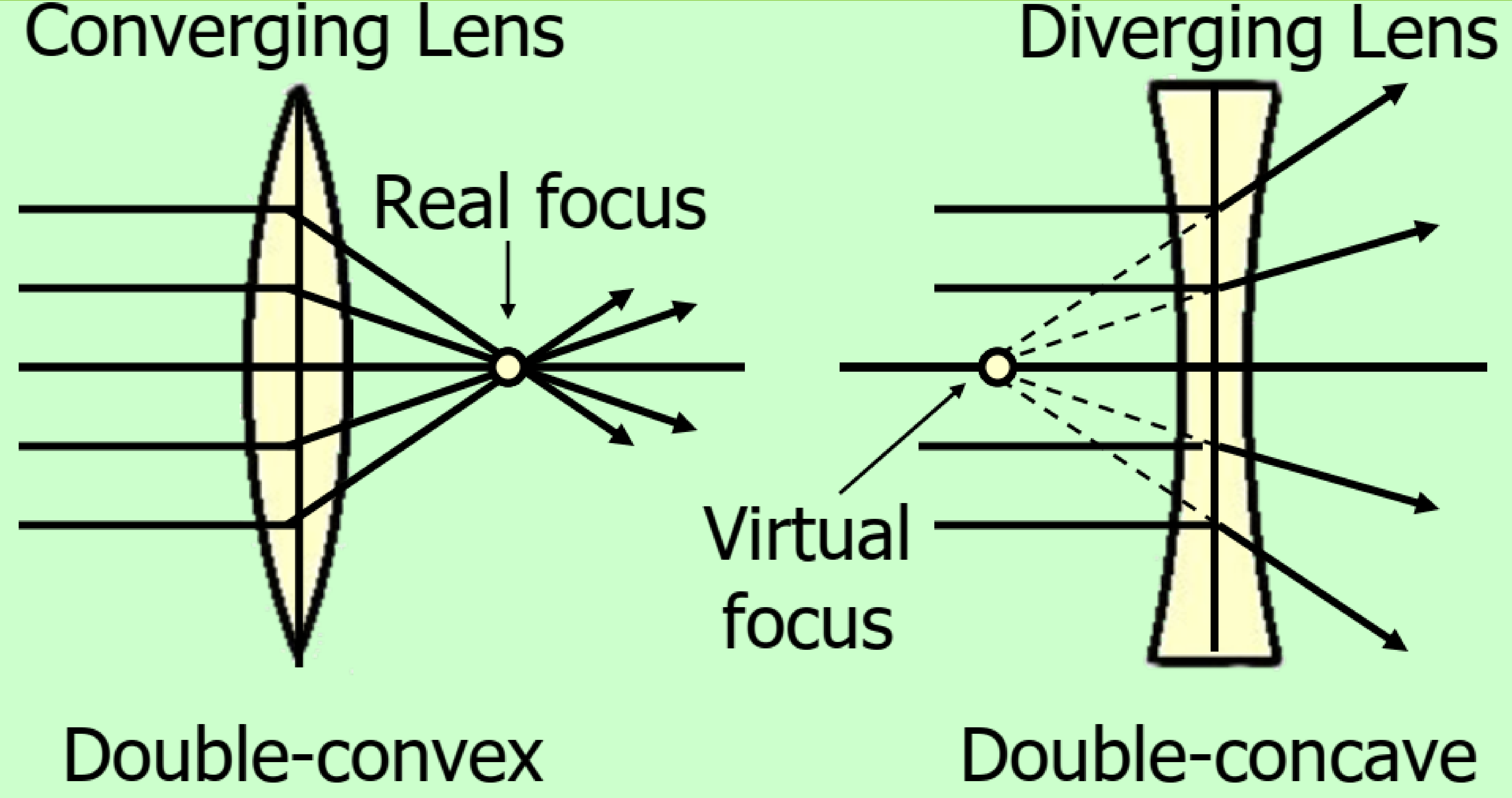


# Thin Lenses (Converging and Diverging Lenses)

**Types of Diverging Lenses:** In order for a lens to diverge light it must be thinner near the midpoint to allow more bending.

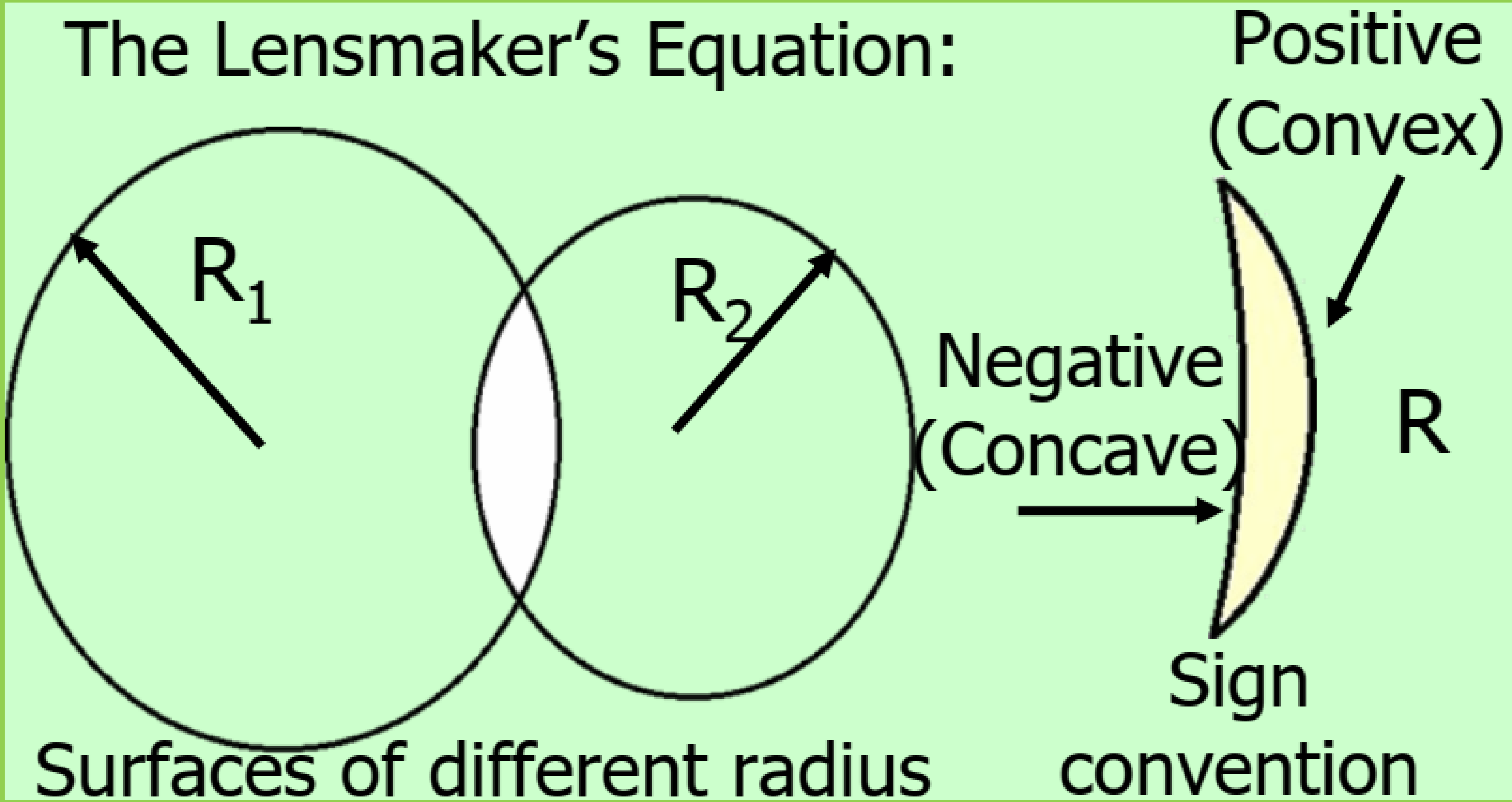


# Thin Lenses (Converging and Diverging Lenses)



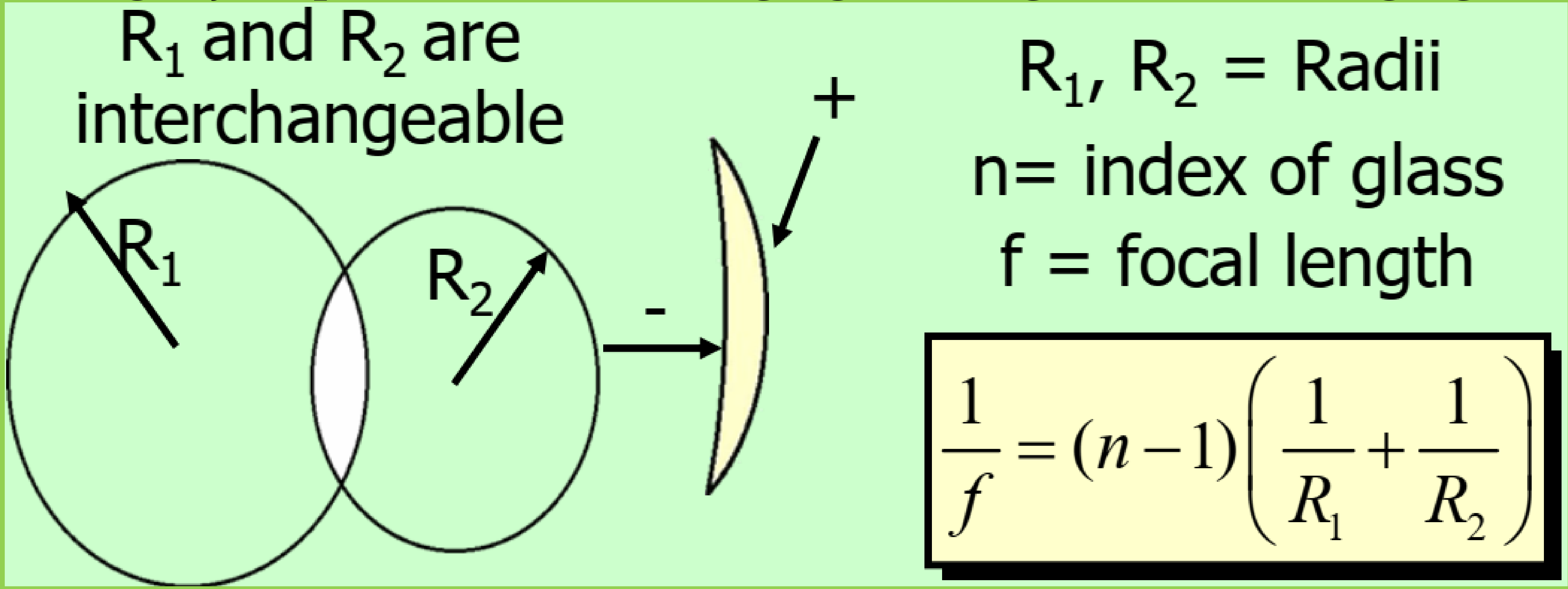
# Lensmaker's Equation

The focal length $f$ of a lens $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$



# Signs for Lensmaker's Equation

1. $R_1$ and $R_2$ are positive for convex outward surface and negative for concave surface.
2. Focal length $f$ is positive for converging and negative for diverging lenses

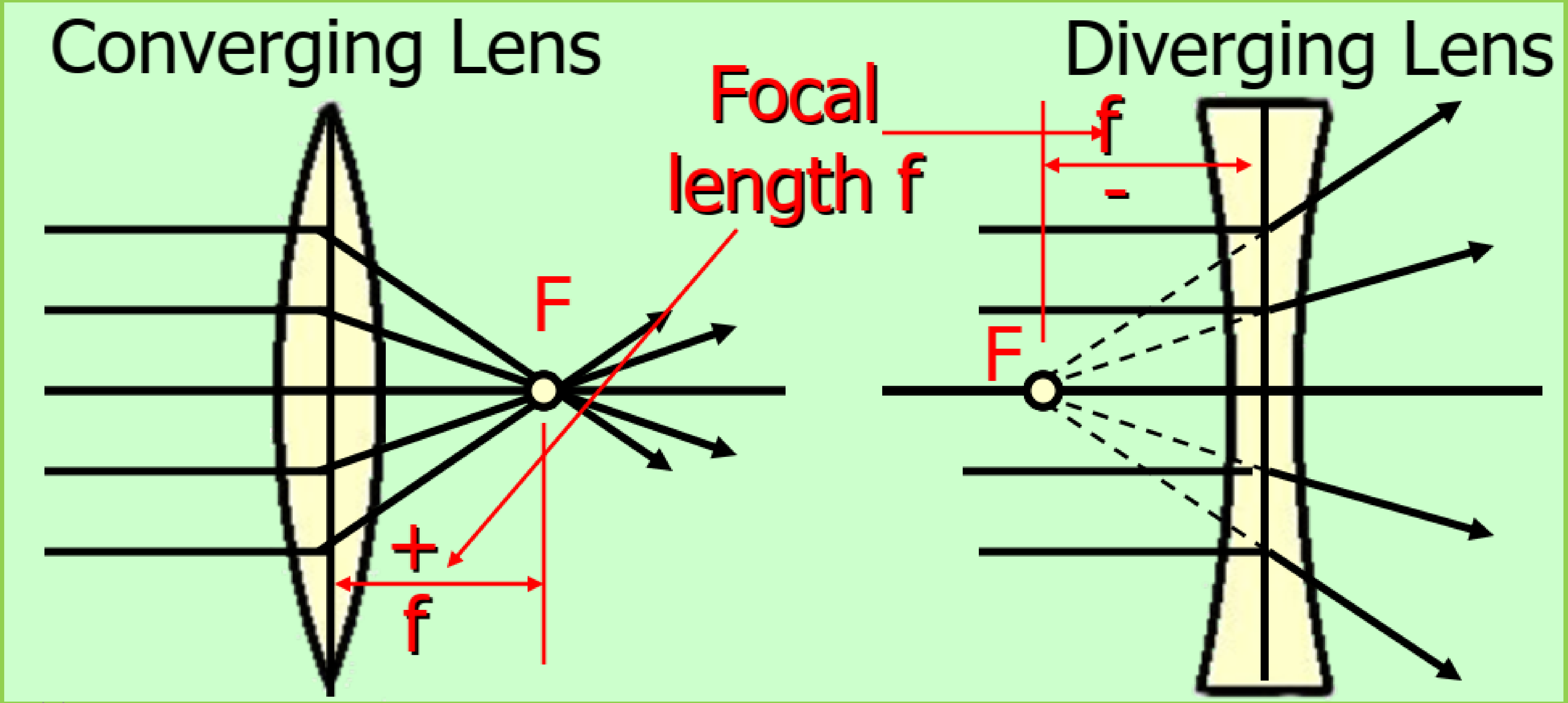


# Signs for Lensmaker's Equation *Example*

A glass meniscus lens (n = 1.5) has a concave surface of radius –40 cm and a convex surface whose radius is +20 cm. What is the focal length of the lens?

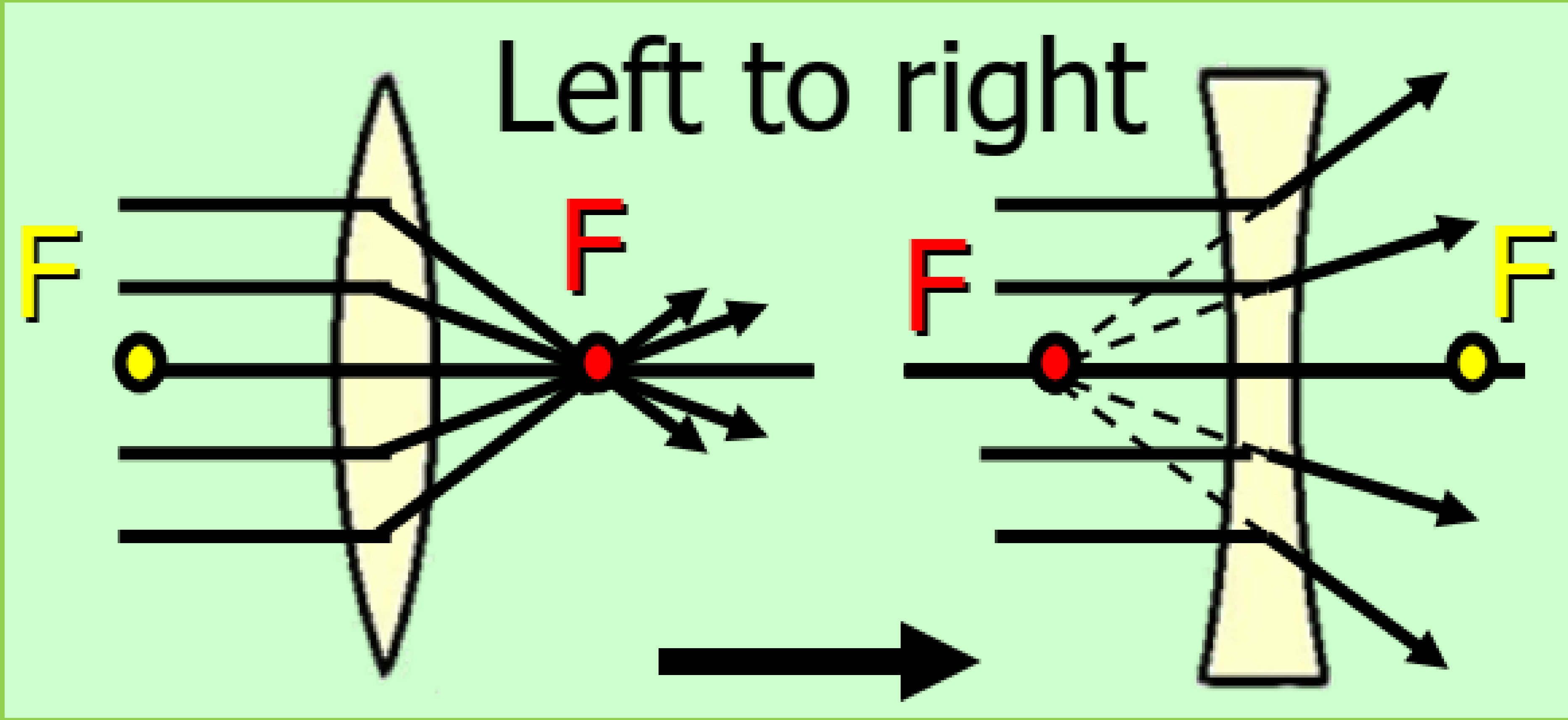
# The Focal Length of Lenses

The focal length $f$ is positive for a real focus (converging) and negative for a virtual focus.



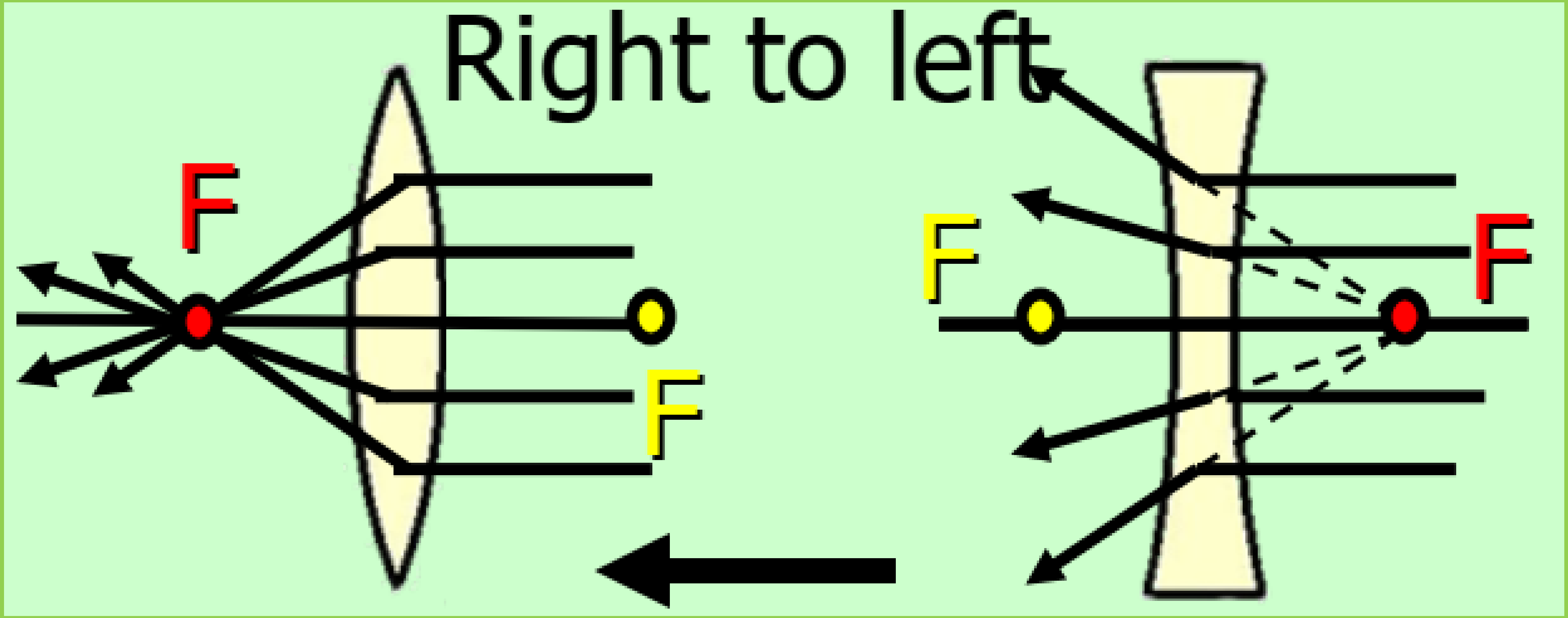
# The Principal Focus

Since light can pass through a lens in either direction, there are two focal points for each lens. The principal focal point *F* is shown here. Yellow *F* is the other one.



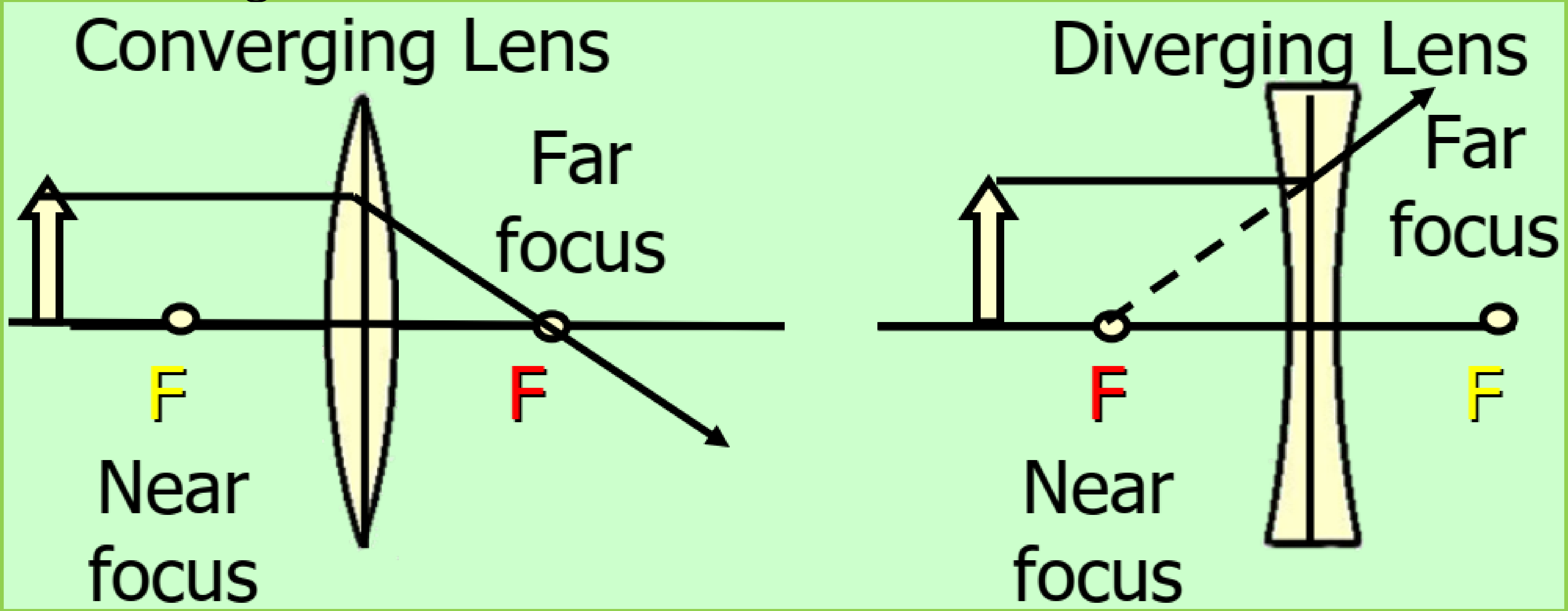
# The Principal Focus

Now suppose light moves from right to left instead.



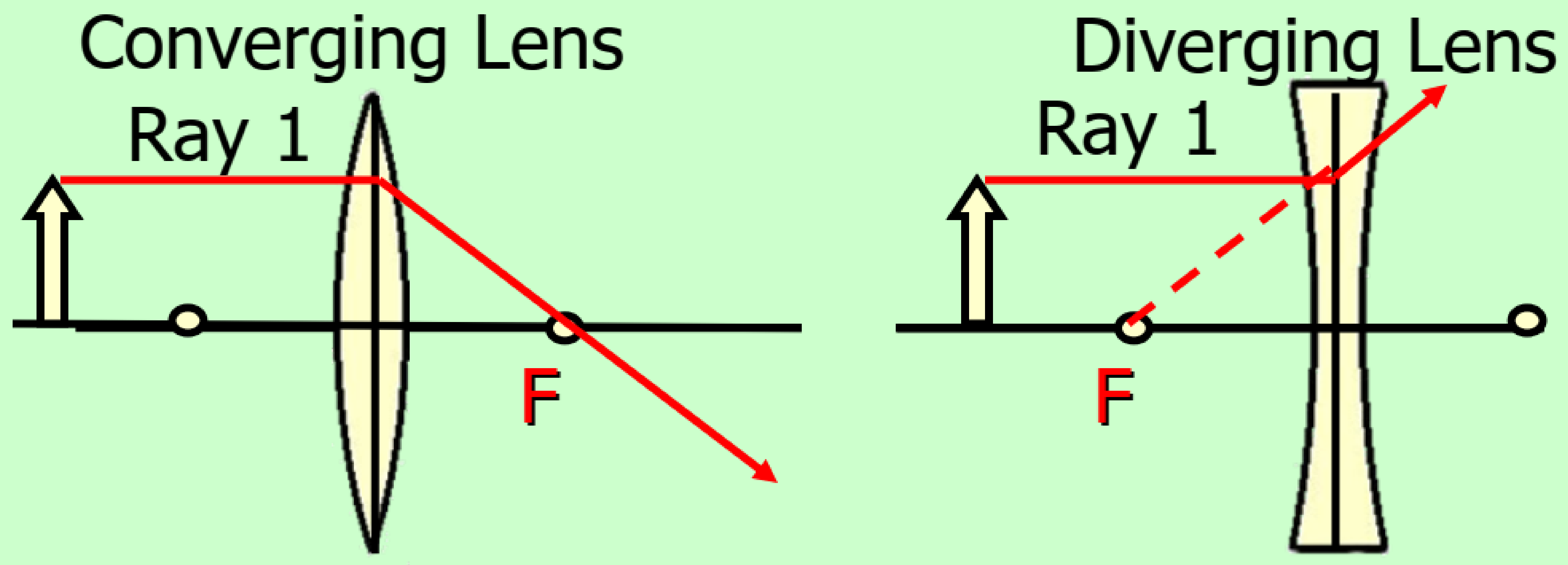
# Terms for Image Construction

The near focal point is the focus $F$ on the same side of the lens as the incident light. The far focal point is the focus $F$ on the opposite side to the incident light.



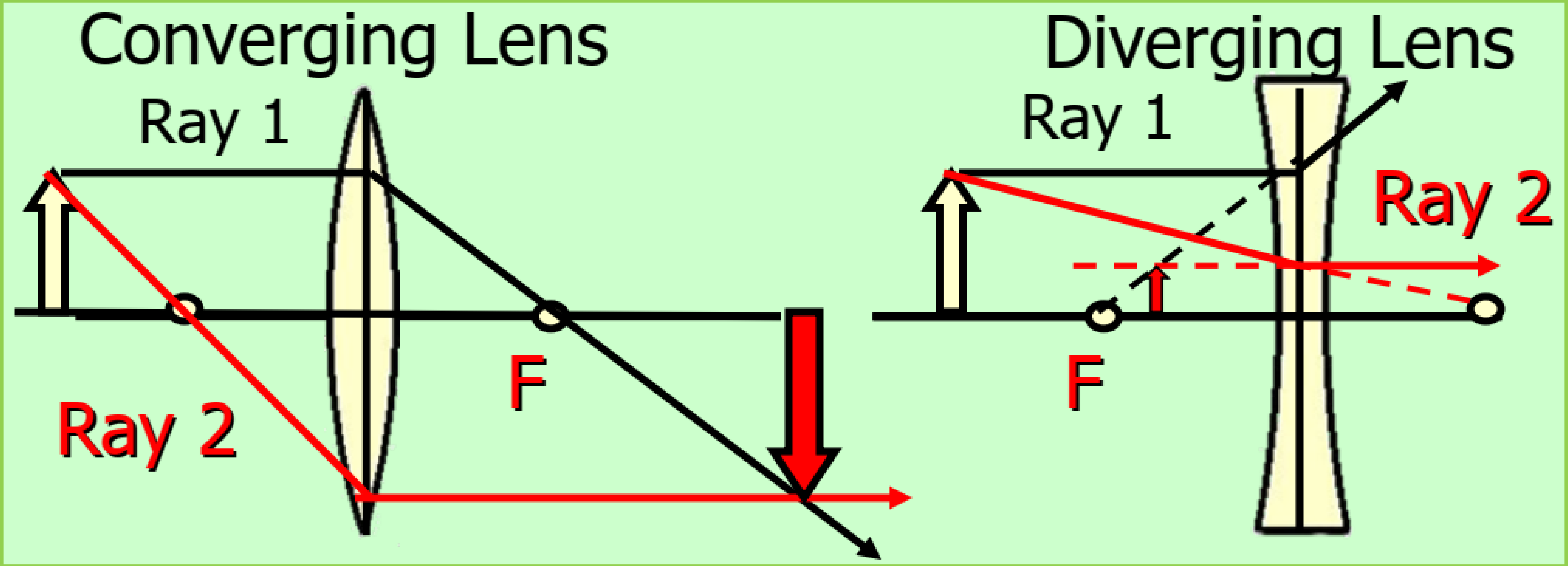
# Image Construction

Ray 1: A ray parallel to the lens axis passes through the far focus of a converging lens or appears to come from the near focus of a diverging lens.



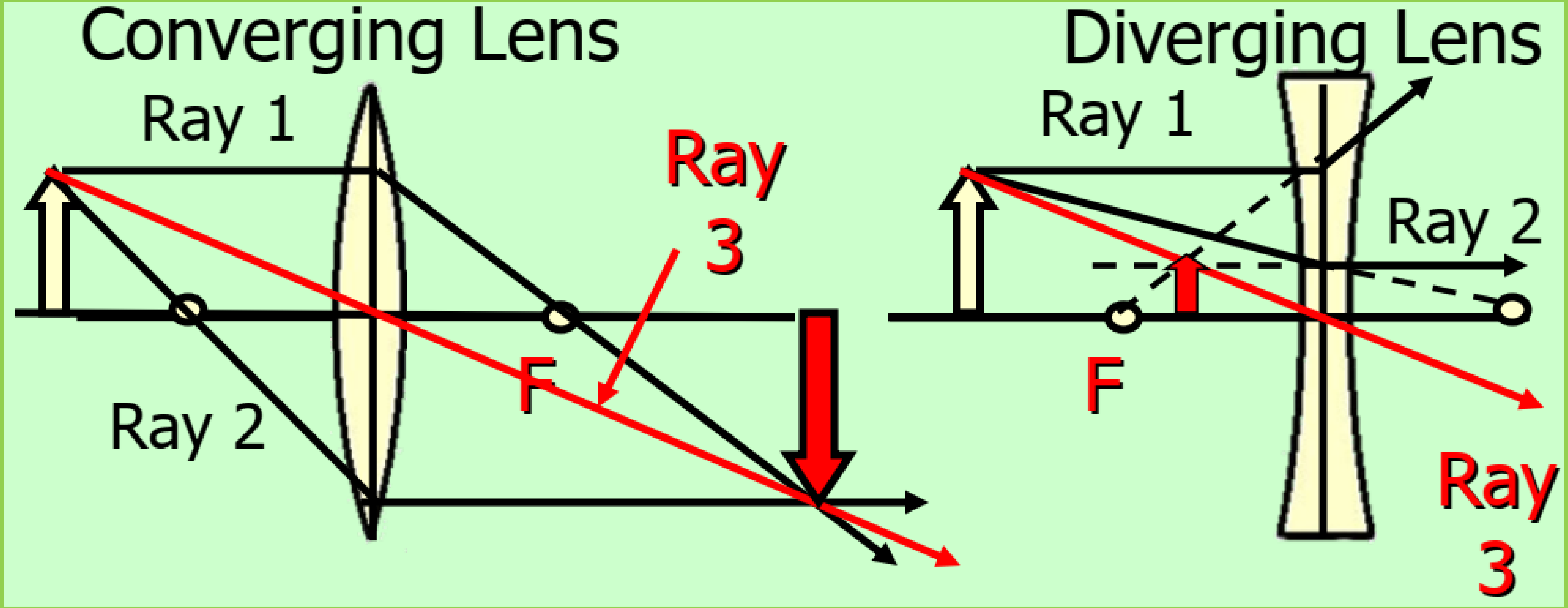
# Image Construction

Ray 2: A ray passing through the near focal point of a converging lens or proceeding toward the far focal point of a diverging lens is refracted parallel to the lens axis.



# Image Construction

Ray 3: A ray passing through the center of any lens continues in a straight line. The refraction at the first surface is balanced by the refraction at the second surface.

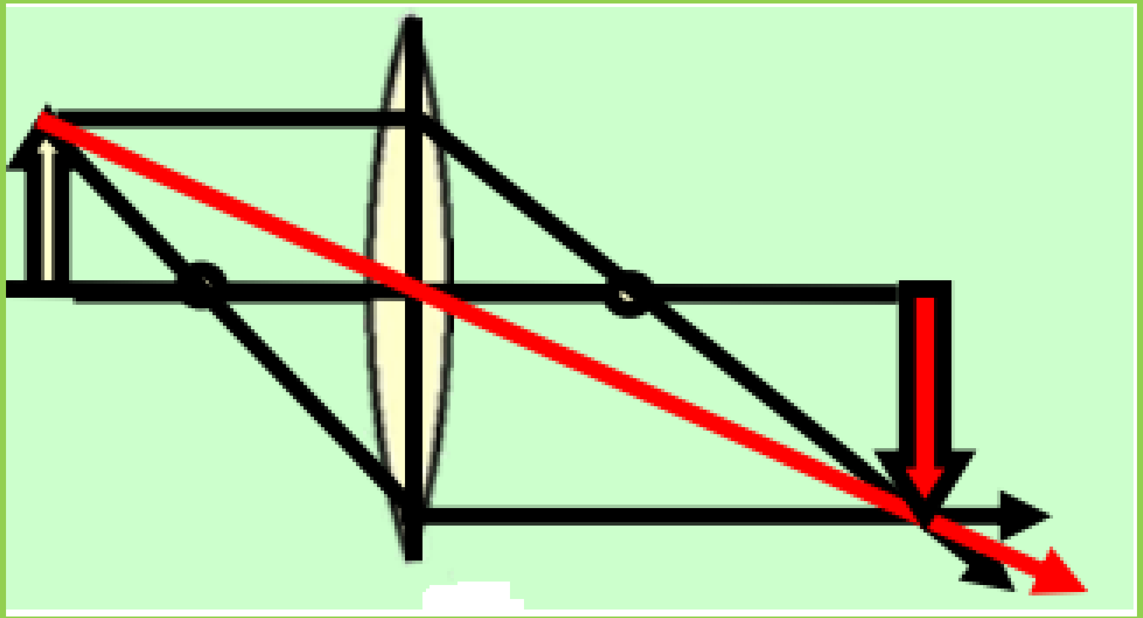


# Images' Tracing Points

Draw an arrow to represent the location of an object, then draw any two of the rays from the tip of the arrow. The image is where lines cross.

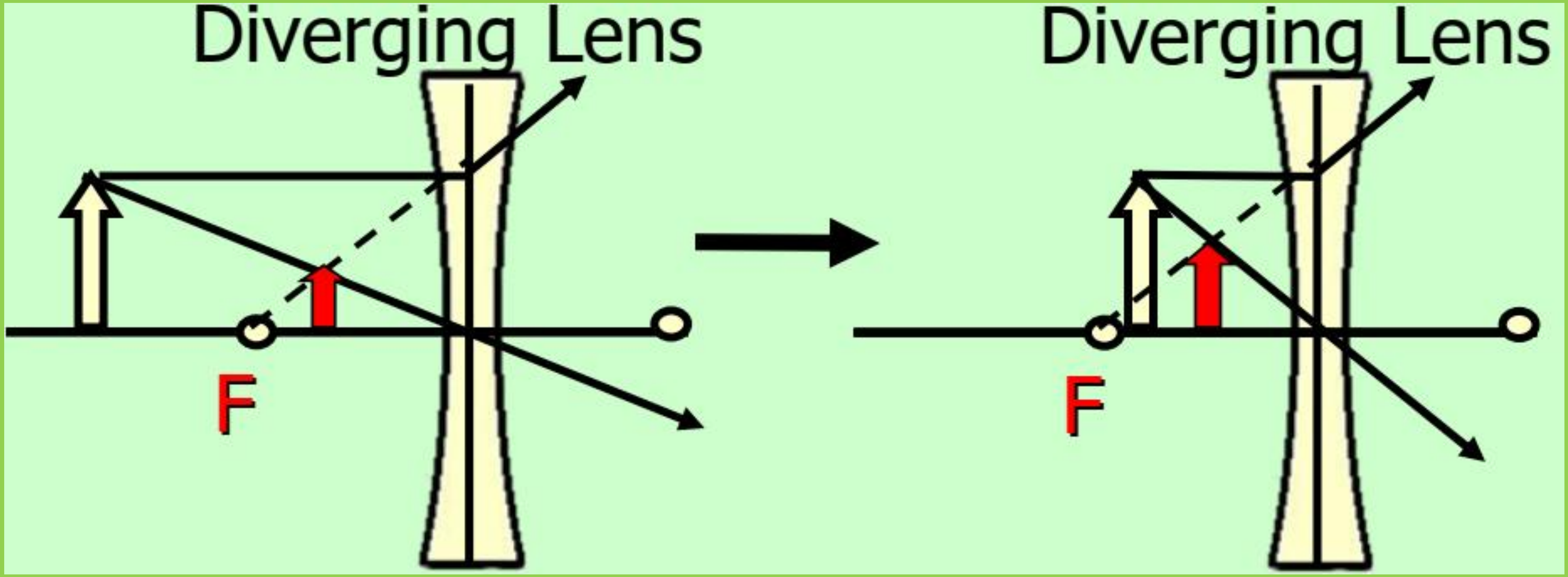
1. *Is the image erect or inverted? 2. Is the image real or virtual? 3. Is it enlarged, diminished, or same size?*

➢ Real images are always on the opposite side of the lens. Virtual images are on the same side.



# Diverging Lens Imaging

All images formed by diverging lenses are erect, virtual, and diminished. Images get larger as object approaches.



# Same Sign Convention as For Mirrors

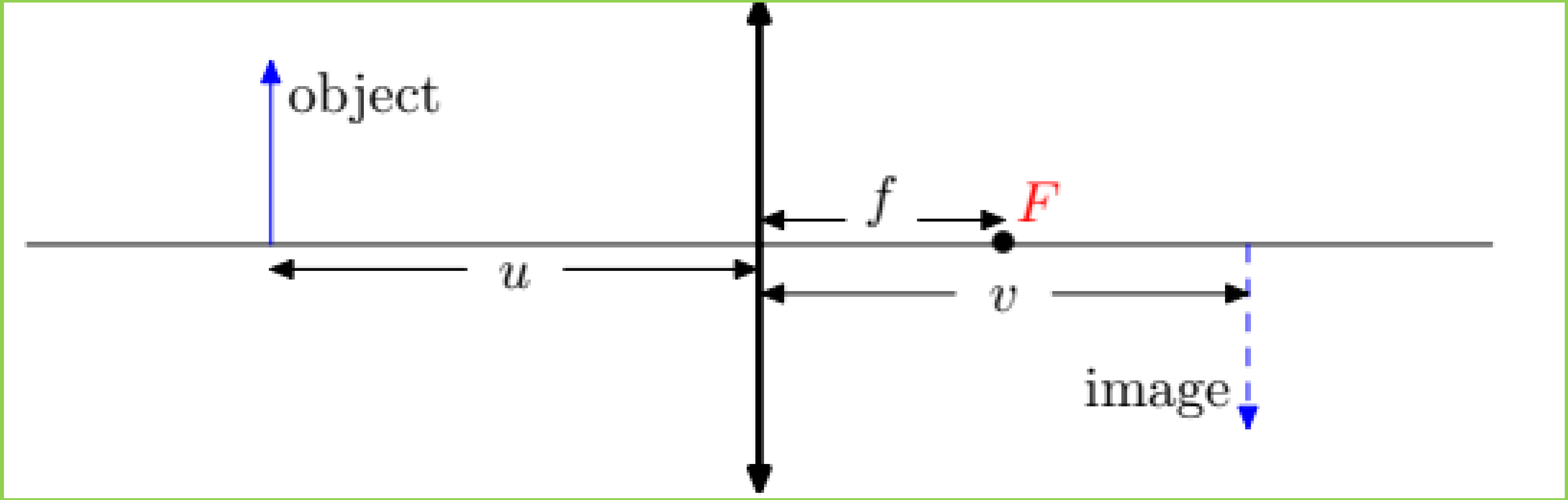
1. Object $p$ and image $q$ distances are positive for real and images negative for virtual images. $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

2. Image height $y'$ and magnification $M$ are positive for erect negative for inverted images. $M = \frac{y'}{y} = \frac{-q}{p}$

3. The focal length $f$ and the radius of curvature $R$ is positive for converging mirrors and negative for diverging mirrors. **OR**

An image formed by a convex lens is described by the **lens equation** $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, where $u$ is the distance of the object from the lens; $v$ is the distance of the image from the lens and $f$ is the *focal length*, i.e., the distance of the focus from the lens.

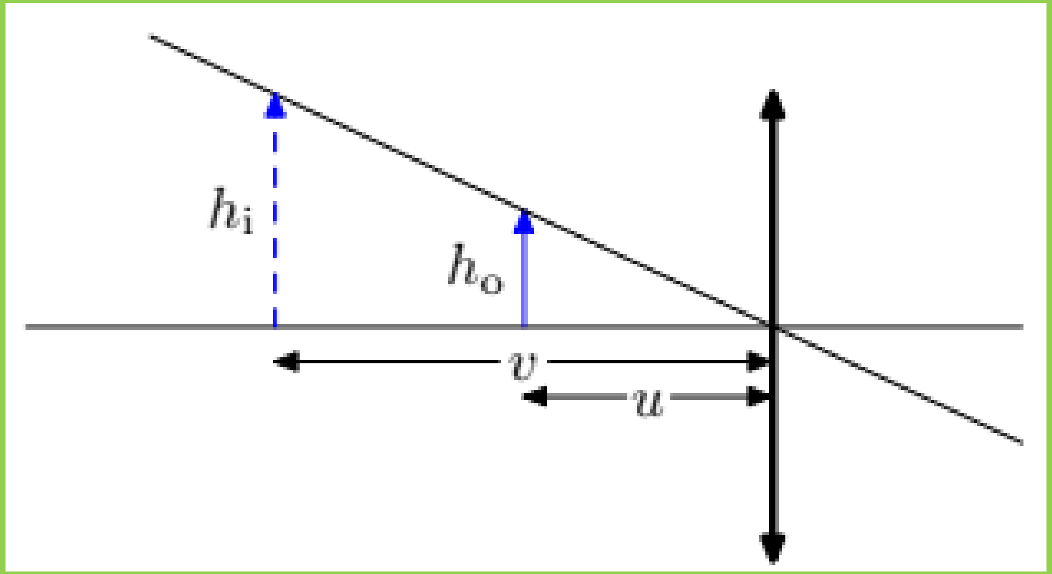
# Diagram for lens equation



# Magnification

The magnification $M$ of an image is the ratio of the height of the image to the height of the object: $M = \frac{Image\ height}{Object\ height} = \frac{Image\ distace}{Object\ distance}$. This number is a dimensionless ratio (a length over a length) and does not have any units. To calculate $M$ for virtual images, consider the diagram below:

From similar triangles: $\frac{h_i}{v} = \frac{h_o}{u} \Rightarrow \frac{h_i}{vh_o} = \frac{1}{u}, \therefore \frac{h_i}{h_o} = \frac{v}{u}$



# Thin Lenses

*Exercise: 3*

Use ray diagrams to find the image position and magnification for an object at each of the following distances from a converging lens with a focal length of 20 cm: (a) 50 cm; (b) 20 cm; (c) 15 cm;

*Exercise: 4*

A diverging lens of focal length 10.0 cm forms images of objects placed
(i) 30.0 cm,
(ii) 10.0 cm, and
(iii) 5.00 cm from the lens.
In each case, construct a ray diagram, find the image distance and describe the image.