



CAT 1- Probability and Statistics 2

Bsc. mathematics and computer science (Jomo Kenyatta University of Agriculture and Technology)



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JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

CAT 1 STA 2200 PROBABILITY AND STATISTICS II Time 1.25 Hours

Instructions: Answer all questions

//this question paper has 2 pages//

Question One

- a) The number of days in the summer months that a construction crew cannot work because of the weather has the probability distribution

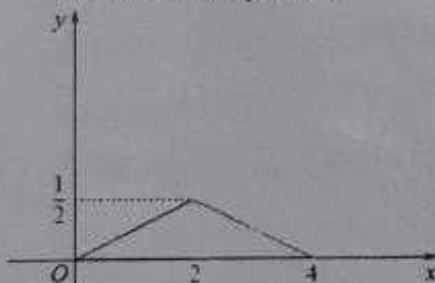
x	5	6	7	8	9	10	11	12
$P(X = x)$	p	0.08	$3p$	$2q$	0.20	$3p$	q	0.07

If $P(X > 9) = 0.32$, determine;

- the probability that at least eight days will be lost next summer. (4 marks)
 - the probability of losing fewer than 10 days next summer given that at least 8 days will be lost. (2 marks)
 - the mean and standard deviation of number of days that the construction crew cannot work because of the weather. Interpret the mean in the context of the problem. (4 marks)
- b) The lifetime in tens of hours of a certain delicate electrical component can be modelled by the random variable T with cumulative distribution function

$$F(t) = \begin{cases} 0 & T < 0 \\ \frac{1}{84}t^2 & 0 \leq T \leq 6 \\ \frac{1}{7}(t-3) & 6 \leq T \leq 10 \\ 1 & T > 10 \end{cases}$$

- Find the pdf of T , median lifetime and the probability that a component last at least 50 hours (3 marks)
 - A particular device requires 3 of these components and it will not operate if one or more of the components fail. The device has just been fitted with 3 new components and the lifetime of these three components are independent. Find the probability that the device breaks down within the next 50 hours (1 mark)
- c) The figure below shows a sketch of the probability density function $f(x)$ of the random variable X . the part of the sketch from $x = 0$ to $x = 4$ consist of an isosceles triangle with maximum at $(2, 0.5)$



This PDF can be written in the form

$$f(x) = \begin{cases} ax, & 0 \leq X \leq 2 \\ b - ax, & 2 \leq X \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

- Find the values of a and b hence Show that the standard deviation $\sigma \approx 0.8165$ to 4 significance figures. (4 marks)
- Find the lower quartile of X hence State giving reason whether $P(2 - \sigma < X < 2 + \sigma)$ is more or less than 0.5 (3 marks)

Colossians 3:23-24 "Whatever you do, work heartily, as for the Lord and not for men, knowing that from the Lord you will receive the inheritance as your reward. You are serving the Lord Christ."
 Galatians 6:9 "And let us not grow weary of doing good, for in due season we will reap, if we do not give up."

Question Two

- a) A loss for a company has moment-generating function $M(t) = \frac{0.16}{0.16 - t}$, $t < 0.16$. An insurance policy pays a benefit equal to 80% of the loss.
- What is the moment-generating function of the benefit? (3 marks)
 - What is the expected benefit payable to the company in case of a loss occur, (2 marks)
- b) Exclusive wines imports Argentinian wines into Australia. They have begun advertising on Facebook to direct traffic into their website where customers can order wine online. The number of click through sale from the ad is Poisson distributed with a mean of 12 click through sales per day. Find the probability of getting;
- strictly between 10 and 15 click through sale in the first day (3 marks)
 - More than one click through sale in the first hour (2 marks)
- c) The lifespan of each laptop produced by company C can be modelled by an exponential distribution with an average 7.5 years.
- What are the chances that a laptop will last for between 5 and 10 years? (3 marks)
 - If a laptop has lasted for eight years, what are the chances that this laptop will last for at least five more years? (2 marks)
 - Calculate the probability that the lifespan of a randomly selected laptop is at most 10 years given that it has already lasted for 5 years. (3 marks)
 - Seventy five percent of all laptops last at least how long? (2 marks)