Calculus 2 final exam

BSc. MATERIALS AND METALLURGICAL ENGINEERING (Jomo Kenyatta University of Agriculture and Technology)

W1-2-60-1-6

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2023/2024

SECOND YEAR SECOND SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN ENGINEERING (MINING, GEGIS, GIS, TIE, CIVIL, EEE, EEC AND ABE).

SMA 2173: CALCULUS II

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: Attempt question one and any other two questions.

QUESTION ONE (30 MARKS)

(a) Briefly explain the following terms

(i) Parametric differentiation **[1 mark]**

(ii) Implicit differentiation **[1 mark]**

Hence determine $\frac{dy}{dx}$ for the equations $x = te^t$ and $y = 4 + lnt$ **[3 marks]**

(b) Determine the equation of normal at $(3, -1)$ given that $-x^2y^2 + 2x^3 = 4x - y + 32$ **[5 marks]**

(c) State the assymptotes for the curve $y = \frac{2x^3}{x^2-1}$ **[3 marks]**

(d) Show that $\frac{d}{dx}\left(cosh^{-1}(e^x)\right) = \frac{e^x}{\sqrt{e^{2x}-1}}$ **[3 marks]**

(e) Evaluate the following integrals using an appropriate technique

(i) $\int sin^{-1}(x)dx$ **[3 marks]**

(ii) $\int cos(4x)cos(2x)dx$ **[3 marks]**

(iii) $\int e^{sinx}cosxdx$ **[2 marks]**

(iv) $\int \frac{x}{\sqrt{x+1}}dx$ **[3 marks]**

(f) Use Simpson's rule with $n = 4$ subintervals to approximate the integral $\int_{-2}^{2} 3^x dx$ to three decimal places. **[3 marks]**

QUESTION TWO (20 MARKS)

$2\pi\int y\left(1+\frac{dy}{dx}\right)^2$

(a) Determine the surface area of the solid obtained by rotating $y = \sqrt{16 - x^2}$, $-4 \leq x \leq 4$ about the x-axis. **[6 marks]**

(b) Evaluate the following integrals

(i) $\frac{dx}{\sqrt{x^2+2x+10}}$ **[5 marks]**

(ii) $\int tan^3 x sec^3 x dx$ **[5 marks]**

(c) Use trapezoidal rule with seven ordinates to estimate $\int \sqrt{2^x + 1} dx$ to four decimal place.

[4 marks]

QUESTION THREE (20 MARKS)

(a) Determine the area between the curves $x = y^2 - 4y$ and $y = x$

[4 marks]

(b) Sketch the graph of $f(x) = \frac{3x}{x^2-1}$ **[5 marks]**

(c) Determine the volume of the solid obtained by rotating the region $y = x^2$ between $x = 1$ and $x = 3$ about the x-axis. **[5 marks]**

(d) Evaluate $\int \frac{sin(x)dx}{1+cos(x)}$ using an appropriate method. **[6 marks]**

QUESTION FOUR (20 MARKS)

(a) Find the length of $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ between $x = 1$ and $x = 2$.

$L = \int \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

[9 marks]

(b) Show that

$\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)} dx = ln|x-4| + \frac{5}{x-4} - \frac{1}{2}ln|x^2+3| + \frac{2}{\sqrt{3}} tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$ **[9 marks]**

(c) Evaluate by separation of variables

$sin(x)sin(y)dx + cos(x)cos(y)dy = 0$ **[2 marks]**