# Exam - 2024 April (SMA 2102) Prof. Okelo

Calculus 2 (Jomo Kenyatta University of Agriculture and Technology)

W1-2-60-1-6

**JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS 2023/2024**

**FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (MCS, PS, IC, AC, CS, BIT, COIN, GEOPHY, REEN).**

**SMA 2102: CALCULUS II**

**DATE: APRIL 2024** **TIME: 2 HOURS**

**INSTRUCTIONS: Answer Question ONE and any other TWO questions.**

## Question One (Compulsory – 30 Marks)

(a) Given $\frac{1}{x+1}+\frac{1}{y+1}=1$, find the equation of the tangent line at point (1,1). [3 Marks]

(b) Identify the asymptotes of the function $f(x)=\frac{x^2-3x+5}{x-4}$. [3 Marks]

(c) Evaluate the following integrals

i) $\int \frac{\sqrt[3]{x}-5\sqrt{x}+6x-1}{\sqrt{x}}dx$ [3 Marks]

ii) $\int \frac{e^{-4x}}{5-e^{-4x}}dx$ [3 Marks]

iii) $\int \frac{dx}{\sqrt{x}\left(1+\sqrt{x}\right)^3}$ [3 Marks]

iv) $\int \frac{x^3}{x^2+x-6}dx$ [5 Marks]

(d) Use trapezoidal rule with $h=0.5$ to estimate $I=\int_8^{10}\frac{5x}{2x+\ln(0.5x)}dx$ correct to 4 decimal places. [4 Marks]

(e) When a cake is removed from an oven, the temperature of the cake is 210°F. The cake is left to cool at a room temperature; which is 70°F. After 30 minutes, the temperature of the cake is 140°F. When will it be at a temperature of 100°F? [4 Marks]

(f) Given $z_1=1-3i$ and $z_2=1+4i$, find $z_1\bar{z}_2$. [2 Marks]

**Question Two (20 Marks)**

(a) Sketch the graph of $y(x) = \dfrac{2x^2 - 7x + 9}{x - 2}$ by stating the x-intercepts, y-intercept, vertical horizontal and slant asympotes. [8 Marks]

(b) Find the length of a smooth curve given by $12xy - 4y^4 = 3$ from a point A($\frac{7}{12}$,1) to point B($\frac{67}{24}$,2) in metres. [6 Marks]

(c) Evaluate the following integrals

i) $\displaystyle\int_2^3 x^3 \ln(4x)dx$ [3 Marks]

ii) $\displaystyle\int x\sqrt{1+3x}dx$ [3 Marks]

**Question Three (20 Marks)**

(a) Evaluate the following integrals

i) $\displaystyle\int \cos(5x)\sin(3x)dx$ [3 Marks]

ii) $\displaystyle\int \frac{x^2 + x}{x^3 - x^2 - 2x}dx$ [5 Marks]

iii) $\displaystyle\int x\tan^{-1}(x)dx$ [4 Marks]

(b) Using Simpson's 1/3 rule with $n = 8$, compute the integral $\displaystyle\int_0^{\pi/2} \sqrt{1 - 0.162\sin^2\theta}d\theta$ correct to 4 decimal places. [8 Marks]

**Question Four (20 Marks)**

(a) Evaluate the following integrals

i) $\displaystyle\int \frac{dx}{x^2 + 4x + 5}$ [4 Marks]

ii) $\displaystyle\int \sec^2(4x + 1)dx$ [3 Marks]

iii) $\displaystyle\int \frac{\sin\theta}{2 + \cos\theta}d\theta$ [4 Marks]

(b) Given $x^3 + 3x^2y - 6xy^2 + 2y^3 = 0$, find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at point (1,1). [4 Marks]

(c) If the marginal revenue function for output $x$ is given by MR $= \dfrac{3x^2 + 4x + 6}{x^3 + 2x^2 + 6x + 8}$, find

i) The total revenue function $R(x)$. [4 Marks]

ii) The demand equation $p(x)$. [1 Mark]