A

$$F(x,y) = \begin{cases} Cx^2y(x+y^2) & 0 \le y \le 3,\ 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$x^3y + x^2y^3$$

$$3x^2y + 2xy^3$$

$$\boxed{C\left[3x^2 + 6xy^2\right]}$$

B

$$\int_0^3 3x^2 \cdot dy + \int_0^3 6xy^2 \cdot dy$$

$$3x^2 \int_0^3 dy + 6x \int_0^3 y^2\, dy$$

$$3x^2[y]_0^3 + 6x\left[\tfrac{1}{3}y^3\right]_0^3$$

$$9x^2 + 54x$$

$$\int_0^2 9x^2\, dx + \int_0^2 54x\, dx$$

$$9\left[\tfrac{1}{3}x^3\right]_0^2 + 54\left[\tfrac{1}{2}x^2\right]_0^2$$

$$C(24 + 108) = 1$$

$$\boxed{C = \frac{1}{132}}$$

C

$$f_x(x) = \int_0^3 f(x,y) \cdot dy$$

$$\int_0^3 \frac{1}{132}\left(3x^2 + 6xy^2\right) dy$$

$$\frac{1}{132}\int_0^3 3x^2 + 6xy^2 \cdot dy$$

$$\frac{1}{132}\int_0^3 3x^2\, dy + \frac{1}{132}\int_0^3 6xy^2\, dy$$

$$\frac{3x^2}{132}\int_0^3 dy + \frac{6x}{132}\int_0^3 y^2\, dy$$

$$\frac{3x^2}{132}[y]_0^3 + \frac{6x}{132}\left[\tfrac{1}{3}y^3\right]_0^3$$

$$\frac{9x^2}{132} + \frac{54x}{132}$$

$$\frac{9x^2 + 54x}{132}$$

$$f_x(x) \begin{cases} \dfrac{9x^2+54x}{132} & ;\ 0 \le x \le 2 \\ 0 & \text{other} \end{cases}$$

Eduardo Fuenta

10.18.22