Homework 4 Key

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Question 1: 16 points

Assume there are a total of 20 congressional seats up for election across the Unites States which has a multinomial distribution. We will assume that for every seat there are 3 candidates running, 1 from the Democratic party, 1 from the Republican party and 1 from an Independent party. We will also assume that for every seat there is a 45% chance the Democratic candidate will win, a 45% the Republican candidate will win and a 10% chance the Independent will win.

Let Democrat be denoted as D, Republican as R and Independent as I

Use a simulation with `rmultinom` to show that $P(D = 9, R = 9, I = 2) \approx 0.0529$. Confirm your results using `dmultinom`.

Question 2: 18 points

Suppose $X = (X_1, X_2, X_3)$ has a multinomial distribution with size $n = 10$ and probabilities $p_1 = .3, p_2 = .4, p_3 = .3$. Use a simulation with `sample` (not `rmultinom`) to show that $P(X_1 = 3, X_2 = 4, X_3 = 3) \approx 0.0784$. Confirm your results using **dmultinom**.

Question 3: 14 points

Let $X_1, \ldots, X_{12}$ be a random sample of size 12 from the $U(0, 1)$ distribution. Explain why $Z = X_1 + X_2 + \cdots + X_{12} - 6$ has an approximate standard normal distribution. You can either prove this theoretically by using CLT, or can use a simulation. *You will have to find or look up the variance of a single X_i.*

Question 4: 20 points

Problem 4.4 #14 a (8) and b(12) in Chihara/Hesterberg.

14. Let $X_1, X_2, \ldots, X_9 \overset{i.i.d.}{\sim} N(7, 3^2)$ and $Y_1, Y_2, \ldots, Y_{12} \overset{i.i.d.}{\sim} N(10, 5^2)$. Let $W = \bar{X} - \bar{Y}$.
    (a) Give the exact sampling distribution of W.
    (b) Simulate the sampling distribution of W in `R` and plot your results (adapt code from the previous exercise). Check that the simulated mean and the standard error are close to the theoretical mean and the standard error.
    (c) Use your simulation to find $P(W < -1.5)$. Calculate an exact answer and compare.

Hint:

Corollary A.2 *Let $X_1, X_2, \ldots, X_n$ be independent normal random variables with common mean μ and common variance σ^2. Let $\bar{X}$ denote the sample mean. Then $\bar{X}$ is normally distributed with mean μ and variance (σ^2/n).*

Theorem A.10 *Let X be a normal random variable with mean μ_1 and variance σ_1^2, and let Y be a normal random variable with mean μ_2, and variance σ_2^2. Assume that X and Y are independent. Then $X \pm Y$ is a normal random variable with mean $\mu_1 \pm \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$.*

Question 5 : 32 points, 8 for each

Problem 4.4 #18 in Chihara/Hesterberg.

18. Let $X_1, X_2, \ldots, X_{30} \overset{i.i.d.}{\sim} \text{Exp}(1/3)$ and let $\bar{X}$ denote the sample mean.
    (a) Simulate the sampling distribution of $\bar{X}$ in R.
    (b) Find the mean and standard error of the sampling distribution and compare to the theoretical results.
    (c) From your simulation, find $P(\bar{X} \leq 3.5)$.
    (d) Estimate $P(\bar{X} \leq 3.5)$ by assuming that the CLT approximation holds. Compare this result with the one in part (c).

Problem 6 (Bonus, 10 Points)

Let $X_1, \ldots, X_n$ be a random sample of size n from a $U(0, a)$ distribution, where $a > 0$. Find $E(X_1 + X_2 + \cdots + X_n)$. and find the approximate distribution of the sample mean, if n is large.