PROBLEM 2 (47 points)

$$F(x_1, x_2, x_3, x_4) = \begin{cases} x_1^3 x_2^3 x_3^3 x_4^3 & 0 < x_i < 1;\ i = 1,2,3,4 \\ 0 & \text{otherwise} \end{cases}$$

$\boxed{A}$

$x_1^3 x_2^3 x_3^3 x_4^3 \, dx_1 dx_2 dx_3 dx_4$

$3x_1^3 x_2^3 x_3^3 x_4^2 \, dx_1 dx_2 dx_3$

$9x_1^3 x_2^3 x_3^2 x_4^2 \, dx_1 dx_2$

$27x_1^3 x_2^2 x_3^2 x_4^2 \, dx_1$

$81 x_1^2 x_2^2 x_3^2 x_4^2$

$$\boxed{f(x_1, x_2, x_3, x_4) \begin{cases} 81 x_1^2 x_2^2 x_3^2 x_4^2 & 0 < x_i < 1;\ i = 1,2,3,4 \\ 0 & \text{other} \end{cases}}$$

$\boxed{B}$

$$\int_0^1\int_0^1\int_0^1\int_0^1 81 x_1^2 x_2^2 x_3^2 x_4^2 \, dx_1 dx_2 dx_3 dx_4 = 1$$

$$81 \int_0^1\int_0^1\int_0^1 \frac{1}{3} x_2^2 x_3^2 x_4^2 \, dx_2 dx_3 dx_4 = 1$$

$$81 \int_0^1\int_0^1 \frac{1}{9} x_3^2 x_4^2 \, dx_3 dx_4 = 1$$

$$81 \int_0^1 \frac{1}{27} x_4^2 \, dx_4 = 1$$

$$81 \cdot \frac{1}{81} = 1$$

$$1 = 1$$

$\boxed{C}$

$$\int_0^1\int_0^1 81 x_1^2 x_2^2 x_3^2 x_4^2 \, dx_2 \, dx_4$$

$$\int_0^1 81 \cdot \frac{1}{3} x_1^2 x_3^2 x_4^2 \, dx_4$$

$$81 \cdot \frac{1}{9} x_1^2 x_3^2$$

$$f_X(x_1, x_3) \begin{cases} 9x_1^2 x_3^2 & 0 < x_i < 1;\ i = 1,3 \\ 0 & \text{otherwise} \end{cases}$$

D

$$E x_1 x_3 = \int_0^1\int_0^1 x_1 x_3 (9x_1^2 x_3^2)\, dx_1 dx_3$$

$$E x_1 x_3 = \int_0^1\int_0^1 9x_1^3 x_3^3\, dx_1 dx_3$$

$$E x_1 x_3 = 9\int_0^1 \tfrac{1}{4} x_3^3\, dx_3$$

$$E x_1 x_3 = 9\cdot\tfrac{1}{4}\int_0^1 x_3^3\, dx_3$$

$$E x_1 x_3 = \frac{9}{4}\cdot\frac{1}{4}$$

$$\boxed{E x_1 x_3 = \frac{9}{16} = 0.5625}$$

E

$$\frac{\text{joint}}{\text{marginal}} = \frac{A}{C}$$

$$\frac{81 x_1^2 x_2^2 x_3^2 x_4^2}{9 x_1^2 x_3^2} = 9 x_2^2 x_4^2$$

$$P\left(x_2 > \tfrac{3}{4},\ x_4 < \tfrac{1}{2} \,\middle|\, x_1 = \tfrac{1}{3},\ x_3 = \tfrac{2}{3}\right)$$

$$\int_0^{1/2}\int_{3/4}^{1} 9x_2^2 x_4^2\, dx_2 dx_4$$

$$9x_4^2 \int_0^{1/2} \left[\tfrac{1}{3}x^3\right]_{3/4}^{1} \longrightarrow \tfrac{1}{3} - \tfrac{1}{3}\cdot\left(\tfrac{3}{4}\right)^3 = .192$$

$$9(.192)\int_0^{1/2} x_4^2\, dx_4$$

$$9(.192)\left[\tfrac{1}{3}x^3\right]_0^{1/2} \rightarrow \tfrac{1}{3}\cdot\tfrac{1}{8} = .042$$

$$9(.192)(.042) = 0.072$$

$$\boxed{P\left(x_2 > \tfrac{3}{4},\ x_4 < \tfrac{1}{2} \,\middle|\, x_1 = \tfrac{1}{3},\ x_3 = \tfrac{2}{3}\right) = 0.072}$$

PROBLEM 1 (35 points)

A.
B.

	x = 6	x = 7	x = 8	x = 9	margin y
y = 3	144/960 .15	96/960 .1	144/960 .15	72/960 .075	.475
y = 6	168/960 .175	192/960 .2	96/960 .1	48/960 .05	.525
margin x	.325	.3	.25	.125	1

A. Use this table for problems 1A and 1B. For it to be valid, all joint probabilities should add up to 1.

$.15 + .1 + .15 + .075 + .175 + .2 + .1 + .05 = 1$

Therefore, it is a valid joint distribution.

B. Marginal of y would be the probabilities of each possible instance taking either value of Y. For Y=3, the marginal probability is .475. For y = 6, the marginal probability is .525.

In the context of the problem, this tells us that it is more likely for fans to like 6 movies than it is for them to like 3 movies.

C. $E[Y] = 3(.475) + 6(.525) = 1.425 + 3.15 = 4.575$

D.

$f_{x|y}(x=6|y=3) = .15/.475 = .316$

$f_{x|y}(x=7|y=3) = .1/.475 = .211$

$f_{x|y}(x=8|y=3) = .15/.475 = .316$

$f_{x|y}(x=9|y=3) = .075/.475 = .158$

1

$f_{x|y}(x=6|y=6) = .175/.525 = .33$

$f_{x|y}(x=7|y=6) = .2/.525 = .381$

$f_{x|y}(x=8|y=6) = .1/.525 = .19$

$f_{x|y}(x=9|y=6) = .05/.525 = .095$

1

The conditional distribution where y=3 tells us that it's most likely that a person who liked 3 of the movies has read a series with either 6 or 8 books.

The conditional distribution where y=6 tells us that from the people who liked 6 movies, most of them read either 6 (.33) or 7 (.381) books.

If the producer wants to maximize satisfaction and profits, I would recommend they select a series with 7 books since that is where the likelihood of all movies being liked is highest.