

Problem 1 (35 points)

A.

B.

	x=6	x=7	x=8	x=9	marginY
y=3	144/960 .15	96/960 .1	144/960 .15	72/960 .075	.475
y=6	168/960 .175	192/960 .2	96/960 .1	48/960 .05	.525
marginX	.325	.3	.25	.125	1

A. Use this table for problems 1A and 1B. For it to be valid, all joint probabilities should add up to 1.

$$.15 + .1 + .15 + .075 + .175 + .2 + .1 + .05 = 1$$

Therefore, it is a valid joint distribution.

B. Marginal of Y would be the probabilities of each possible instance taking either value of Y. For $y=3$, the marginal probability is .475.

For $y=6$, the marginal probability is .525.

In the context of the problem, this tells us that it is more likely for fans to like 6 movies than it is for them to like 3 movies.

C

$$E[Y] = 3(.475) + 6(.525) = 1.425 + 3.15 = 4.575$$

D

$$f_{X|Y}(x=6|y=3) = .15 / .475 = .316$$

$$f_{X|Y}(x=7|y=3) = .1 / .475 = .211$$

$$f_{X|Y}(x=8|y=3) = .15 / .475 = .316$$

$$f_{X|Y}(x=9|y=3) = .075 / .475 = .158$$

1

$$f_{X|Y}(x=6|y=6) = .175 / .525 = .33$$

$$f_{X|Y}(x=7|y=6) = .2 / .525 = .381$$

$$f_{X|Y}(x=8|y=6) = .1 / .525 = .19$$

$$f_{X|Y}(x=9|y=6) = .05 / .525 = .095$$

1

The conditional distribution where $y=3$ tells us that it's most likely that a person who liked 3 of the movies has read a series with either 6 or 8 books.

The conditional distribution where $y=6$ tells us that from the people who liked 6 movies, most of them read either 6 (.33) or 7 (.381) books.

If the producer wants to maximize satisfaction and profits, I would recommend they select a series with 7 books since that is where the likelihood of all movies being liked is highest.