

PROBLEM 2 (47 points)

$$f(x_1, x_2, x_3, x_4) = \begin{cases} x_1^3 x_2^3 x_3^3 x_4^3 & 0 < x_i < 1; i = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

A  $x_1^3 x_2^3 x_3^3 x_4^3 dx_1 dx_2 dx_3 dx_4$

$$3x_1^3 x_2^3 x_3^3 x_4^2 dx_1 dx_2 dx_3$$

$$9x_1^3 x_2^3 x_3^2 x_4^2 dx_1 dx_2$$

$$27x_1^3 x_2^2 x_3^2 x_4^2 dx_1$$

$$81x_1^2 x_2^2 x_3^2 x_4^2$$

$$f(x_1, x_2, x_3, x_4) = \begin{cases} 81x_1^2 x_2^2 x_3^2 x_4^2 & 0 < x_i < 1; i = 1, 2, 3, 4 \\ 0 & \text{other} \end{cases}$$

B  $\int_0^1 \int_0^1 \int_0^1 \int_0^1 81x_1^2 x_2^2 x_3^2 x_4^2 dx_1 dx_2 dx_3 dx_4 = 1$

$$81 \int_0^1 \int_0^1 \int_0^1 \frac{1}{3} x_2^2 x_3^2 x_4^2 dx_2 dx_3 dx_4 = 1$$

$$81 \int_0^1 \int_0^1 \frac{1}{9} x_3^2 x_4^2 dx_3 dx_4 = 1$$

$$81 \int_0^1 \frac{1}{27} x_4^2 dx_4 = 1$$

$$81 \cdot \frac{1}{27} = 1$$

$$1 = 1$$

C  $\int_0^1 \int_0^1 81x_1^2 x_2^2 x_3^2 x_4^2 dx_2 dx_4$

$$\int_0^1 81 \cdot \frac{1}{3} x_1^2 x_3^2 x_4^2 dx_4$$

$$81 \cdot \frac{1}{9} x_1^2 x_3^2$$

$$f_{X_1, X_3}(x_1, x_3) = \begin{cases} 9x_1^2 x_3^2 & 0 < x_i < 1; i = 1, 3 \\ 0 & \text{otherwise} \end{cases}$$