1 Quantities and units

1.1 (1.1)

quantity

property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference

NOTE 1 The generic concept ‘quantity’ can be divided into several levels of specific concepts, as shown in the following table. The left hand side of the table shows specific concepts under ‘quantity’. These are generic concepts for the individual quantities in the right hand column.

<table>
<tr><td rowspan="2">length, l</td><td>radius, r</td><td>radius of circle A, r_A or $r(A)$</td></tr>
<tr><td>wavelength, λ</td><td>wavelength of the sodium D radiation, λ_D or λ (D; Na)</td></tr>
<tr><td rowspan="2">energy, E</td><td>kinetic energy, T</td><td>kinetic energy of particle i in a given system, T_i</td></tr>
<tr><td>heat, Q</td><td>heat of vaporization of sample i of water, Q_i</td></tr>
<tr><td colspan="2">electric charge, Q</td><td>electric charge of the proton, e</td></tr>
<tr><td colspan="2">electric resistance, R</td><td>electric resistance of resistor i in a given circuit, R_i</td></tr>
<tr><td colspan="2">amount-of-substance concentration of entity B, c_B</td><td>amount-of-substance concentration of ethanol in wine sample i, $c_i(C_2H_5OH)$</td></tr>
<tr><td colspan="2">number concentration of entity B, C_B</td><td>number concentration of erythrocytes in blood sample i, C (Erys; B_i)</td></tr>
<tr><td colspan="2">Rockwell C hardness (150 kg load), HRC(150 kg)</td><td>Rockwell C hardness of steel sample i, HRC_i (150 kg)</td></tr>
</table>

NOTE 2 A reference can be a **measurement unit**, a **measurement procedure**, a **reference material**, or a combination of such.

NOTE 3 Symbols for quantities are given in the ISO 80000 and IEC 80000 series *Quantities and units*. The symbols for quantities are written in italics. A given symbol can indicate different quantities.

NOTE 4 The preferred IUPAC-IFCC format for designations of quantities in laboratory medicine is “System—Component; kind-of-quantity”.

EXAMPLE "Plasma (Blood)—Sodium ion; amount-of-substance concentration equal to 143 mmol/l in a given person at a given time".

NOTE 5 A quantity as defined here is a scalar. However, a vector or a tensor, the components of which are quantities, is also considered to be a quantity.

NOTE 6 The concept 'quantity' may be generically divided into, e.g. 'physical quantity', 'chemical quantity', and 'biological quantity', or **base quantity** and **derived quantity**.

1.2 (1.1, Note 2)
kind of quantity
kind
aspect common to mutually comparable **quantities**

NOTE 1 The division of the concept of 'quantity' according to 'kind of quantity' is to some extent arbitrary.

EXAMPLE 1 The quantities diameter, circumference, and wavelength are generally considered to be quantities of the same kind, namely of the kind of quantity called length.

EXAMPLE 2 The quantities heat, kinetic energy, and potential energy are generally considered to be quantities of the same kind, namely of the kind of quantity called energy.

NOTE 2 Quantities of the same kind within a given **system of quantities** have the same **quantity dimension**. However, quantities of the same dimension are not necessarily of the same kind.

EXAMPLE The quantities moment of force and energy are, by convention, not regarded as being of the same kind, although they have the same dimension. Similarly for heat capacity and entropy, as well as for number of entities, relative permeability, and mass fraction.

NOTE 3 In English, the terms for quantities in the left half of the table in 1.1, Note 1, are often used for the corresponding 'kinds of quantity'. In French, the term "nature" is only used in expressions such as “grandeurs de même nature” (in English, “quantities of the same kind”).

1.3 (1.2)

system of quantities

set of **quantities** together with a set of non-contradictory equations relating those quantities

NOTE **Ordinal quantities**, such as Rockwell C hardness, are usually not considered to be part of a system of quantities because they are related to other quantities through empirical relations only.

1.4 (1.3)

base quantity

quantity in a conventionally chosen subset of a given **system of quantities**, where no subset quantity can be expressed in terms of the others

NOTE 1 The subset mentioned in the definition is termed the “set of base quantities”.

EXAMPLE The set of base quantities in the **International System of Quantities (ISQ)** is given in 1.6.

NOTE 2 Base quantities are referred to as being mutually independent since a base quantity cannot be expressed as a product of powers of the other base quantities.

NOTE 3 ‘Number of entities’ can be regarded as a base quantity in any system of quantities.

1.5 (1.4)

derived quantity

quantity, in a **system of quantities**, defined in terms of the base quantities of that system

EXAMPLE In a system of quantities having the base quantities length and mass, mass density is a derived quantity defined as the quotient of mass and volume (length to the third power).

1.6
International System of Quantities
ISQ
system of quantities based on the seven **base quantities**: length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity

> NOTE 1 This system of quantities is published in the ISO 80000 and IEC 80000 series *Quantities and units*.

> NOTE 2 The **International System of Units (SI)** (see 1.16) is based on the ISQ.

1.7 (1.5)
quantity dimension
dimension of a quantity
dimension
expression of the dependence of a **quantity** on the **base quantities** of a **system of quantities** as a product of powers of factors corresponding to the base quantities, omitting any numerical factor

> EXAMPLE 1 In the **ISQ**, the quantity dimension of force is denoted by dim F = LMT^{-2}.

> EXAMPLE 2 In the same system of quantities, dim $\tilde{n}_{\mathrm{B}} = \mathrm{ML}^{-3}$ is the quantity dimension of mass concentration of component B, and ML^{-3} is also the quantity dimension of mass density, $\tilde{n}$, (volumic mass).

> EXAMPLE 3 The period T of a pendulum of length l at a place with the local acceleration of free fall g is

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad T = C(g)\sqrt{l}$$

where

$$C(g) = \frac{2\pi}{\sqrt{g}}$$

Hence dim $C(g)$ = $\mathsf{L}^{-1/2}\mathsf{T}$.

NOTE 1 A power of a factor is the factor raised to an exponent. Each factor is the dimension of a base quantity.

NOTE 2 The conventional symbolic representation of the dimension of a base quantity is a single upper case letter in roman (upright) sans-serif type. The conventional symbolic representation of the dimension of a **derived quantity** is the product of powers of the dimensions of the base quantities according to the definition of the derived quantity. The dimension of a quantity Q is denoted by dim Q.

NOTE 3 In deriving the dimension of a quantity, no account is taken of its scalar, vector, or tensor character.

NOTE 4 In a given system of quantities,

- quantities of the same **kind** have the same quantity dimension,
- quantities of different quantity dimensions are always of different kinds, and
- quantities having the same quantity dimension are not necessarily of the same kind.

NOTE 5 Symbols representing the dimensions of the base quantities in the ISQ are:

Base quantity	Symbol for dimension
length	L
mass	M
time	T
electric current	I
thermodynamic temperature	Θ
amount of substance	N
luminous intensity	J

Thus, the dimension of a quantity Q is denoted by dim Q = $\mathsf{L}^{\alpha}\,\mathsf{M}^{\beta}\,\mathsf{T}^{\gamma}\,\mathsf{I}^{\delta}\,\Theta^{\varepsilon}\,\mathsf{N}^{\zeta}\,\mathsf{J}^{\eta}$ where the exponents, named dimensional exponents, are positive, negative, or zero.

1.8 (1.6)

quantity of dimension one

dimensionless quantity

quantity for which all the exponents of the factors corresponding to the **base quantities** in its **quantity dimension** are zero

NOTE 1 The term “dimensionless quantity” is commonly used and is kept here for historical reasons. It stems from the fact that all exponents are zero in the symbolic representation of the dimension for such quantities. The term “quantity of dimension one” reflects the convention in which the symbolic representation of the dimension for such quantities is the symbol 1 (see ISO 31-0:1992, 2.2.6).

NOTE 2 The **measurement units** and **values** of quantities of dimension one are numbers, but such quantities convey more information than a number.

NOTE 3 Some quantities of dimension one are defined as the ratios of two quantities of the same **kind**.

EXAMPLES Plane angle, solid angle, refractive index, relative permeability, mass fraction, friction factor, Mach number.

NOTE 4 Numbers of entities are quantities of dimension one.

EXAMPLES Number of turns in a coil, number of molecules in a given sample, degeneracy of the energy levels of a quantum system.

1.9 (1.7)

measurement unit

unit of measurement

unit

real scalar **quantity**, defined and adopted by convention, with which any other quantity of the same **kind** can be compared to express the ratio of the two quantities as a number

NOTE 1 Measurement units are designated by conventionally assigned names and symbols.

NOTE 2 Measurement units of quantities of the same **quantity dimension** may be designated by the same name and symbol even when the quantities are not of the same kind. For example, joule per kelvin and J/K are respectively the name and symbol of both a measurement unit of heat capacity and a measurement unit of entropy, which are generally not considered to be quantities of the same kind. However, in some cases special measurement unit names are restricted to be used with quantities of a specific kind only. For example, the measurement unit ‘second to the power minus one’ (1/s) is called hertz (Hz) when used for frequencies and becquerel (Bq) when used for activities of radionuclides.

NOTE 3 Measurement units of **quantities of dimension one** are numbers. In some cases these measurement units are given special names, e.g. radian, steradian, and decibel, or are expressed by quotients such as millimole per mole equal to 10^{-3} and microgram per kilogram equal to 10^{-9}.

NOTE 4 For a given quantity, the short term “unit” is often combined with the quantity name, such as “mass unit” or “unit of mass”.

1.10 (1.13)
base unit
measurement unit that is adopted by convention for a **base quantity**

NOTE 1 In each **coherent system of units**, there is only one base unit for each base quantity.

EXAMPLE In the **SI**, the metre is the base unit of length. In the CGS systems, the centimetre is the base unit of length.

NOTE 2 A base unit may also serve for a **derived quantity** of the same **quantity dimension**.

EXAMPLE Rainfall, when defined as areic volume (volume per area), has the metre as a **coherent derived unit** in the SI.

NOTE 3 For number of entities, the number one, symbol 1, can be regarded as a base unit in any **system of units**.

1.11 (1.14)
derived unit

measurement unit for a **derived quantity**

EXAMPLES The metre per second, symbol m/s, and the centimetre per second, symbol cm/s, are derived units of speed in the SI. The kilometre per hour, symbol km/h, is a measurement unit of speed outside the SI but accepted for use with the SI. The knot, equal to one nautical mile per hour, is a measurement unit of speed outside the SI.

1.12 (1.10)

coherent derived unit

derived unit that, for a given **system of quantities** and for a chosen set of **base units**, is a product of powers of base units with no other proportionality factor than one

NOTE 1 A power of a base unit is the base unit raised to an exponent.

NOTE 2 Coherence can be determined only with respect to a particular system of quantities and a given set of base units.

EXAMPLES If the metre, the second, and the mole are base units, the metre per second is the coherent derived unit of velocity when velocity is defined by the **quantity equation** $\boldsymbol{v} = \mathrm{d}\boldsymbol{r}/\mathrm{d}t$, and the mole per cubic metre is the coherent derived unit of amount-of-substance concentration when amount-of-substance concentration is defined by the quantity equation $c = n/V$. The kilometre per hour and the knot, given as examples of derived units in 1.11, are not coherent derived units in such a system of quantities.

NOTE 3 A derived unit can be coherent with respect to one system of quantities but not to another.

EXAMPLE The centimetre per second is the coherent derived unit of speed in a CGS **system of units** but is not a coherent derived unit in the **SI**.

NOTE 4 The coherent derived unit for every derived **quantity of dimension one** in a given system of units is the number one, symbol 1. The name and symbol of the **measurement unit** one are generally not indicated.

1.13 (1.9)

system of units

set of **base units** and **derived units**, together with their **multiples** and **submultiples**, defined in accordance with given rules, for a given **system of quantities**

1.14 (1.11)

coherent system of units

system of units, based on a given **system of quantities**, in which the **measurement unit** for each **derived quantity** is a **coherent derived unit**

EXAMPLE Set of coherent **SI** units and relations between them.

NOTE 1 A system of units can be coherent only with respect to a system of quantities and the adopted **base units**.

NOTE 2 For a coherent system of units, **numerical value equations** have the same form, including numerical factors, as the corresponding **quantity equations**.

1.15 (1.15)

off-system measurement unit

off-system unit

measurement unit that does not belong to a given **system of units**

EXAMPLE 1 The electronvolt (about $1.602\,18 \times 10^{-19}$ J) is an off-system measurement unit of energy with respect to the **SI**.

EXAMPLE 2 Day, hour, minute are off-system measurement units of time with respect to the SI.

1.16 (1.12)

International System of Units

SI

system of units, based on the **International System of Quantities**, their names and symbols, including a series of prefixes and their names and symbols, together with rules for their use, adopted by the General Conference on Weights and Measures (CGPM)

NOTE 1 The SI is founded on the seven **base quantities** of the **ISQ** and the names and symbols of the corresponding **base units** that are contained in the following table.

Base quantity	Base unit	
Name	**Name**	**Symbol**
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

NOTE 2 The base units and the **coherent derived units** of the SI form a coherent set, designated the "set of coherent SI units".

NOTE 3 For a full description and explanation of the International System of Units, see the current edition of the SI brochure published by the Bureau International des Poids et Mesures (BIPM) and available on the BIPM website.

NOTE 4 In **quantity calculus**, the quantity 'number of entities' is often considered to be a base quantity, with the base unit one, symbol 1.

NOTE 5 The SI prefixes for **multiples of units** and **submultiples of units** are:

Factor	Prefix	
	Name	**Symbol**
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
	kilo	k

Factor	Prefix	
	Name	Symbol
10^{3}		
10^{2}	hecto	h
10^{1}	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

1.17 (1.16)

multiple of a unit

measurement unit obtained by multiplying a given measurement unit by an integer greater than one

EXAMPLE 1 The kilometre is a decimal multiple of the metre.

EXAMPLE 2 The hour is a non-decimal multiple of the second.

NOTE 1 **SI** prefixes for decimal multiples of SI **base units** and SI **derived units** are given in Note 5 of 1.16.

NOTE 2 SI prefixes refer strictly to powers of 10, and should not be used for powers of 2. For example, 1 kilobit should not be used to represent 1 024 bits (2^{10} bits), which is 1 kibibit.

Prefixes for binary multiples are:

Factor	Prefix	
	Name	Symbol
$(2^{10})^8$	yobi	Yi
$(2^{10})^7$	zebi	Zi
$(2^{10})^6$	exbi	Ei
$(2^{10})^5$	pebi	Pi
$(2^{10})^4$	tebi	Ti
$(2^{10})^3$	gibi	Gi
$(2^{10})^2$	mebi	Mi
$(2^{10})^1$	kibi	Ki

Source: IEC 80000-13.

1.18 (1.17)

submultiple of a unit

measurement unit obtained by dividing a given measurement unit by an integer greater than one

EXAMPLE 1 The millimetre is a decimal submultiple of the metre.

EXAMPLE 2 For a plane angle, the second is a non-decimal submultiple of the minute.

NOTE SI prefixes for decimal submultiples of SI **base units** and SI **derived units** are given in Note 5 of 1.16.

1.19 (1.18)

quantity value

value of a quantity

value

number and reference together expressing magnitude of a **quantity**

EXAMPLE 1 Length of a given rod:

5.34 m or 534 cm

EXAMPLE 2   Mass of a given body:

0.152 kg or 152 g

EXAMPLE 3   Curvature of a given arc:

112 m^{-1}

EXAMPLE 4   Celsius temperature of a given sample:

−5 °C

EXAMPLE 5   Electric impedance of a given circuit element at a given frequency, where j is the imaginary unit:

(7 + 3j) Ω

EXAMPLE 6   Refractive index of a given sample of glass:

1.32

EXAMPLE 7   Rockwell C hardness of a given sample (150 kg load):

43.5HRC(150 kg)

EXAMPLE 8   Mass fraction of cadmium in a given sample of copper:

3 μg ⁄ kg or 3 × 10^{-9}

EXAMPLE 9   Molality of Pb^{2+} in a given sample of water:

1.76 μmol ⁄ kg

EXAMPLE 10   Arbitrary amount-of-substance concentration of lutropin in a given sample of plasma (WHO international standard 80/552):

5.0 International Unit/l

NOTE 1 According to the type of reference, a quantity value is either

- a product of a number and a **measurement unit** (see Examples 1, 2, 3, 4, 5, 8 and 9); the measurement unit one is generally not indicated for **quantities of dimension one** (see Examples 6 and 8), or

- a number and a reference to a **measurement procedure** (see Example 7), or

- a number and a **reference material** (see Example 10).

NOTE 2 The number can be complex (see Example 5).

NOTE 3 A quantity value can be presented in more than one way (see Examples 1, 2 and 8).

NOTE 4 In the case of vector or tensor quantities, each component has a quantity value.

EXAMPLE Force acting on a given particle, e.g. in Cartesian components $(F_x; F_y; F_z) = (-31.5; 43.2; 17.0)$ N.

1.20 (1.21)
numerical quantity value
numerical value of a quantity
numerical value
number in the expression of a **quantity value**, other than any number serving as the reference

NOTE 1 For **quantities of dimension one**, the reference is a **measurement unit** which is a number and this is not considered as a part of the numerical quantity value.

EXAMPLE In an amount-of-substance fraction equal to 3 mmol/mol, the numerical quantity value is 3 and the unit is mmol/mol. The unit mmol/mol is numerically equal to 0.001, but this number 0.001 is not part of the numerical quantity value, which remains 3.

NOTE 2 For **quantities** that have a measurement unit (i.e. those other than **ordinal quantities**), the numerical value $\{Q\}$ of a quantity Q is frequently denoted $\{Q\} = Q / [Q]$, where $[Q]$ denotes the measurement unit.

EXAMPLE For a quantity value of 5.7 kg, the numerical quantity value is $\{m\}$ = (5.7 kg)/kg = 5.7. The same quantity value can be expressed as 5 700 g in which case the numerical quantity value $\{m\}$ = (5 700 g)/g = 5 700.

1.21
quantity calculus
set of mathematical rules and operations applied to **quantities** other than **ordinal quantities**

NOTE In quantity calculus, **quantity equations** are preferred to **numerical value equations** because quantity equations are independent of the choice of **measurement units**, whereas numerical value equations are not (see ISO 31-0:1992, 2.2.2).

1.22
quantity equation
mathematical relation between **quantities** in a given **system of quantities**, independent of **measurement units**

EXAMPLE 1 $Q_1 = \zeta Q_2 Q_3$ where Q_1, Q_2 and Q_3 denote different quantities, and where ζ is a numerical factor.

EXAMPLE 2 $T = (1/2)\, m v^2$ where T is the kinetic energy and v the speed of a specified particle of mass m.

EXAMPLE 3 $n = It/F$ where n is the amount of substance of a univalent component, I is the electric current and t the duration of the electrolysis, and where F is the Faraday constant.

1.23
unit equation
mathematical relation between **base units**, **coherent derived units** or other **measurement units**

EXAMPLE 1 For the **quantities** in Example 1 of item 1.22, $[Q_1] = [Q_2]\,[Q_3]$ where $[Q_1]$, $[Q_2]$ and $[Q_3]$ denote the measurement units of Q_1, Q_2 and Q_3, respectively, provided that these measurement units are in a **coherent system of units**.

EXAMPLE 2 J := kg $\mathrm{m}^2/\mathrm{s}^2$, where J, kg, m and s are the symbols for the joule, kilogram, metre and second, respectively. (The symbol := denotes "is by definition equal to" as given in the ISO 80000 and IEC 80000 series.)

EXAMPLE 3 1 km/h = (1/3.6) m/s.

1.24
conversion factor between units

ratio of two **measurement units** for **quantities** of the same **kind**

EXAMPLE km/m = 1 000 and thus 1 km = 1 000 m.

NOTE The measurement units may belong to different **systems of units**.

EXAMPLE 1 h/s = 3 600 and thus 1 h = 3 600 s.

EXAMPLE 2 (km/h)/(m/s) = (1/3.6) and thus 1 km/h = (1/3.6) m/s.

1.25
numerical value equation

numerical quantity value equation

mathematical relation between **numerical quantity values**, based on a given **quantity equation** and specified **measurement units**

EXAMPLE 1 For the **quantities** in Example 1 in item 1.22, $\{Q_1\} = \zeta\,\{Q_2\}\,\{Q_3\}$ where $\{Q_1\}$, $\{Q_2\}$ and $\{Q_3\}$ denote the numerical values of Q_1, Q_2 and Q_3, respectively, provided that they are expressed in either **base units** or **coherent derived units** or both.

EXAMPLE 2 In the quantity equation for kinetic energy of a particle, $T = (1/2)\,mv^2$, if m = 2 kg and v = 3 m/s, then $\{T\} = (1/2) \times 2 \times 3^2$ is a numerical value equation giving the numerical value 9 of T in joules.

1.26

ordinal quantity

quantity, defined by a conventional **measurement procedure**, for which a total ordering relation can be established, according to magnitude, with other quantities of the same **kind**, but for which no algebraic operations among those quantities exist

EXAMPLE 1 Rockwell C hardness.

EXAMPLE 2 Octane number for petroleum fuel.

EXAMPLE 3 Earthquake strength on the Richter scale.

EXAMPLE 4 Subjective level of abdominal pain on a scale from zero to five.

NOTE 1 Ordinal quantities can enter into empirical relations only and have neither **measurement units** nor **quantity dimensions**. Differences and ratios of ordinal quantities have no physical meaning.

NOTE 2 Ordinal quantities are arranged according to **ordinal quantity-value scales**.

1.27

quantity-value scale

measurement scale

ordered set of **quantity values** of **quantities** of a given **kind of quantity** used in ranking, according to magnitude, quantities of that kind

EXAMPLE 1 Celsius temperature scale.

EXAMPLE 2 Time scale.

EXAMPLE 3 Rockwell C hardness scale.

1.28 (1.22)

ordinal quantity-value scale

ordinal value scale

quantity-value scale for **ordinal quantities**

EXAMPLE 1 Rockwell C hardness scale.

EXAMPLE 2 Scale of octane numbers for petroleum fuel.

NOTE An ordinal quantity-value scale may be established by **measurements** according to a **measurement procedure**.

1.29
conventional reference scale
quantity-value scale defined by formal agreement

1.30
nominal property
property of a phenomenon, body, or substance, where the property has no magnitude

EXAMPLE 1 Sex of a human being.

EXAMPLE 2 Colour of a paint sample.

EXAMPLE 3 Colour of a spot test in chemistry.

EXAMPLE 4 ISO two-letter country code.

EXAMPLE 5 Sequence of amino acids in a polypeptide.

NOTE 1 A nominal property has a value, which can be expressed in words, by alphanumerical codes, or by other means.

NOTE 2 ‘Nominal property value’ is not to be confused with **nominal quantity value**.