Math 6000: Homework 2

Due: Friday, Feb 2, 11am

Exercises: 2.2.1, 2.2.2, 2.2.3, 2.3.1, 2.3.2, 2.5.1, 2.6.1.

2.2.1. Show that any map h_0 from X to an L-structure $\mathcal{M}$ can be uniquely extended to a homomorphism h from $\mathrm{Term}_L(X)$ to $\mathcal{M}$.

2.2.2. Let h and h_0 be as above and let $t(\bar{x})$ be an L-term whose variables $\bar{x}$ are in X. Prove that $t^{\mathcal{M}}(h_0[\bar{x}]) = h(t(\bar{x}))$

2.2.3. (About unique legibility) Prove that no proper initial segment of a term (regarded as a string of symbols of the alphabet) can be a term. Derive that for every term there is a unique way of building it up from its constituents according to the above recursion.

2.3.1. Verify that there are only finitely many nested atomic sentences in L, provided the signature of L is finite.

2.3.2. (About unique legibility) Prove that no proper initial segment of a formula (regarded as a string of symbols of the alphabet) can be a formula. Derive that for every formula there is a unique way of building it up from its constituents according to the above recursion.

2.5.1. Find a recursive definition of “free variable” that recurses on the syntactic complexity of the formula under consideration.

2.6.1. What does the notation $\varphi(y, x)$ mean for a given $\varphi(x, y) \in L_2$?
