HW 1

Exercises: 1.2.1, 1.3.1, 1.3.2, (1.3.3), (1.4.1), (1.5.1), 1.6.1, 1.6.2.

Exercises in parentheses are not required. (Please do not submit solutions to these exercises.)

1.2.1. Find a signature appropriate for the description of vector spaces over a give field $\mathcal{K}$.

1.3.1. Given $X \subseteq M$, let $\operatorname{Aut}_{\{X\}} \mathcal{M}$ be the set $\{h \in \operatorname{Aut} \mathcal{M} \colon h[X] = X\}$. Show that $\operatorname{Aut}_X \mathcal{M}$ is a normal subgroup of $\operatorname{Aut}_{\{X\}} \mathcal{M}$. What happens if, instead of $h[X] = X$, we require only $h[X] \subseteq X$?

1.3.2. Find a structure with a bijective endomorphism that is not an automorphism.

(1.3.3) Find an infinite structure $\mathcal{M}$ with a trivial automorphism group, i.e., $\operatorname{Aut} \mathcal{M} = \{\mathrm{id}_M\}$.

(1.4.1) Describe the difference between substructures of $\mathbb{Z}$ according to whether $\mathbb{Z}$ is considered in the signature $(0; +)$ or in the signature $(0; +, -)$.

(1.5.1) Given a signature σ, find a signature $\sigma_1 \supseteq \sigma$ such that all σ-sturctures $\mathcal{M}$ and $\mathcal{N}$ with $\mathcal{N} \leq \mathcal{M}$ have expansions $\mathcal{M}'$ and $\mathcal{N}'$ to σ_1 such that $\mathcal{N}' \subseteq \mathcal{M}'$ and $\operatorname{Aut} \mathcal{M}' = \operatorname{Aut}_{\{N\}} \mathcal{M}$

1.6.1. Show that $\mathcal{M} = \prod_{i \in I} \mathcal{M}_i$ is uncountable as soon as no M_i is empty and infinitely many of the M_i have at least two elements.

1.6.2. Find an embedding $e \colon \mathcal{M} \to \mathcal{M}^I$ such that $p_i e = \mathrm{id}_M$ for all $i \in I$.
