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Practical SAT Solving

Lecture 5

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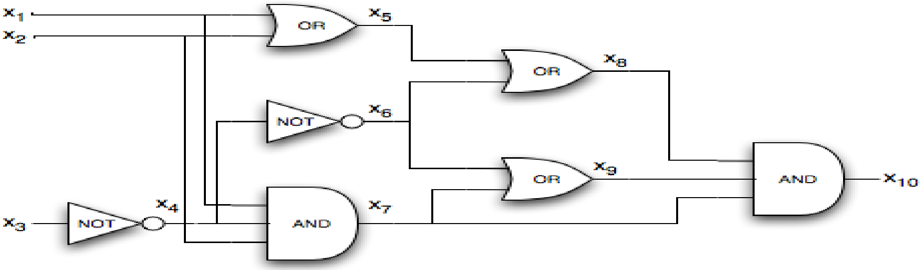
INSTITUTE FOR THEORETICAL COMPUTER SCIENCE



Lecture Outline

- Details on implementing DPLL
  - Decision Heuristics
  - Restarts
  - Implementing Unit Propagation

DPLL Algorithm: Outline

- DPLL: Davis-Putnam-Logemann-Loveland
- Basic idea: case splitting (Depth first search on the partial assignments) and simplification
- Simplification: unit propagation and pure literal deletion
- Unit propagation: 1-clauses (unit clauses) fix variable values: if $\{x\} \in S$, in order to satisfy S, variable x must be set to 1.
- Pure literal deletion: If variable x occurs only positively (or only negatively) in S, it may be fixed, i.e. set to 1 (or 0).

DPLL Algorithm

```
boolean DPLL(ClauseSet S)
{
  while ( S contains a unit clause {L} ) {
    delete from S clauses containing L;  // unit-subsumption
    delete ¬L from all clauses in S;     // unit-resolution
  }
  if ( ⊥ ∈ S ) return false;             // empty clause?
  while ( S contains a pure literal L )
    delete from S all clauses containing L;
  if ( S = ∅ ) return true;              // no clauses?
  choose a literal L occurring in S;     // case-splitting
  if ( DPLL(S ∪ {{L}} ) return true;     // first branch
  else if ( DPLL(S ∪ {{¬L}} ) return true;  // second branch
  else return false;
}
```

DPLL: Implementation Issues

- How can we implement unit propagation efficiently?
- How can we implement pure literal elimination efficiently?
- Which literal L to use for case splitting?
- How can we efficiently implement the case splitting step?

“Modern” DPLL Algorithm with “Trail”

```
boolean mDPLL(ClauseSet S, PartialAssignment α)
{
  while ( (S,α) contains a unit clause {L} ) {
    add {L = 1} to α
  }
  if ( a literal is assigned both 0 and 1 in α ) return false;
  if ( all literals assigned ) return true;
  choose a literal L not assigned in α occurring in S;
  if ( mDPLL(S, α ∪ {L = 1} ) return true;
  else if ( mDPLL(S, α ∪ {L = 0} ) return true;
  else return false;
}
```

(S, α): clause set S as “seen” under partial assignment α

Properties of a good decision heuristic

Properties of a good decision heuristic

- Fast to compute
- Yields efficient sub-problems
  - More short clauses?
  - Less variables?
  - Partitioned problem?

Bohm's Heuristic

- Best heuristic in 1992 for random SAT (in the SAT competition)
- Select the variable x with the maximal vector $(H_1(x), H_2(x), \dots)$

$$H_i(x) = \alpha \max(h_i(x), h_i(\overline{x})) + \beta \min(h_i(x), h_i(\overline{x}))$$

- where $h_i(x)$ is the number of unsatisfied clauses with i literals that contain the literal x.
- α and β are chosen heuristically ($\alpha = 1$ and $\beta = 2$).
- Goal: satisfy or reduce size of many preferably short clauses

MOMS Heuristic

- Maximum Occurrences in clauses of Minimum Size
- Popular in the mid 90s
- Choose the variable x with a maximum $S(x)$.

$$S(x) = (f^*(x) + f^*(\overline{x})) \times 2^k + (f^*(x) \times f^*(\overline{x}))$$

- where $f^*(x)$ is the number of occurrences of x in the smallest unsatisfied clauses, k is a parameter
- Goal: assign variables with high occurrence in short clauses

Jeroslow-Wang Heuristic

- Considers all the clauses, shorter clauses are more important
- Choose the literal x with a maximum $J(x)$.

$$J(x) = \sum_{x \in c, c \in F} 2^{-|c|}$$

- Two-sided variant: choose variable x with maximum $J(x) + J(\overline{x})$
- Goal: assign variables with high occurrence in short clauses
- Much better experimental results than Bohm and MOMS
- One-sided version works better

(R)DLCS and (R)DLIS Heuristics

- (Randomized) Dynamic Largest (Combined | Individual) Sum
- Dynamic = Takes the current partial assignment in account
- Let C_P (C_N) be the number of positive (negative) occurrences
- DLCS selects the variable with maximal $C_P + C_N$
- DLIS selects the variable with maximal $\max(C_P, C_N)$
- RDLCS and RDLIS does a random selection among the best
  - Decrease greediness by randomization
- Used in the famous SAT solver GRASP in 2000

LEFV Heuristic

- Last Encountered Free Variable
- During unit propagation save the last unassigned variable you see, if the variable is still unassigned at decision time use it otherwise choose a random
- Very fast computation: constant memory and time overhead
  - Requires 1 int variable (to store the last seen unassigned variable)
- Maintains search locality
- Works well for pigeon hole and similar formulas

Restarts

- What is a restart?

Restarts

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  - Clear the partial assignment
  - Unassign all the variables
  - Backtrack to level 0

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- Why would anybody want to do restarts in DPLL?

Restarts

- What is a restart?
  - Clear the partial assignment
  - Unassign all the variables
  - Backtrack to level 0
- Why would anybody want to do restarts in DPLL?
  - To recover from bad branching decisions
  - You solve more instances
  - Might decrease performance on easy instances

When to Restart?

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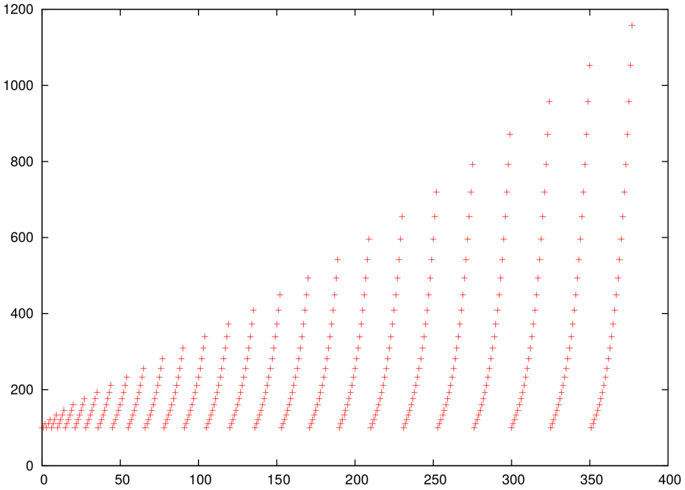
- After a given number of decisions
- The number of decision between restarts should grow
  - To guarantee completeness
- How much increase?
  - Linear increase – too slow
  - Exponential increase – ok with small exponent
  - MiniSat: k-th restart happens after 100×1.1^k

Inner/Outer Restart Scheduling



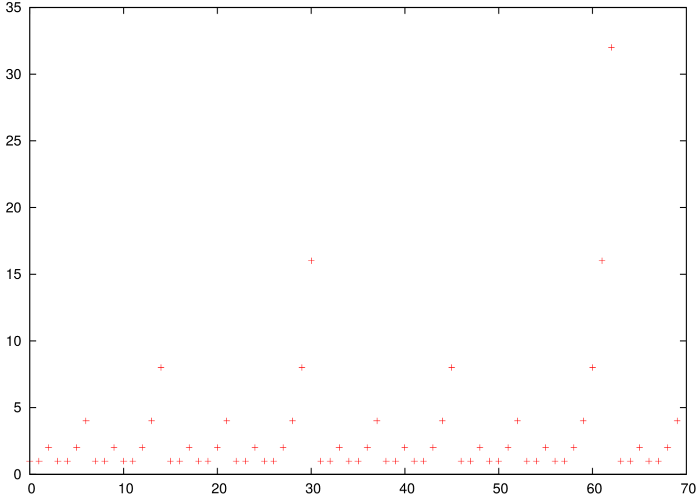
Inner/Outer Restart Scheduling

Inner/Outer Restart Algorithm

int inner = 100
int outer = 100

forever do
    . . . do DPLL for *inner* conflicts . . .
    restarts++
    if inner >= outer **then**
        outer *= 1.1
        inner = 100
    else
        inner *= 1.1

Luby Sequence Restart Scheduling



Luby Sequence Restart Scheduling

Luby Sequence Algorithm

```
unsigned luby (unsigned i)
    for (unsigned k = 1; k < 32; k++)
        if (i == (1 << k) - 1) then return 1 << (k - 1)
    for (k = 1;; k++)
        if ((1 << (k - 1)) <= i && i < (1 << k) - 1) then
            return luby(i - (1 << (k-1)) + 1);

limit = 512 * luby (++restarts);
... // run SAT core loop for limit conflicts
```

- Complicated, not trivial to compute

Reluctant Doubling

- A more efficient implementation of the Ruby sequence
- Use the v_n of the following pair

$$(u_1, v_1) = (1, 1) \tag{1}$$

$$(u_{n+1}, v_{n+1}) = u_n \& -u_n = v_n ? (u_n + 1, 1) : (u_n, 2v_n) \tag{2}$$

- Example: (1,1), (2,1), (2,2), (3,1), (4,1), (4,2), (4,4), (5,1), ...
- Invented by Donald Knuth

How to Implement Unit Propagation

The Task

Given a partial truth assignment ϕ and a set of clauses F identify all the unit clauses, extend the partial truth assignment, repeat until fix-point.

Simple Solution

- Check all the clauses
- Too slow
- Unit propagation cannot be efficiently parallelized (is P-complete)

How to Implement Unit Propagation

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Simple Solution

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In the context of DPLL the task is actually a bit different

- The partial truth assignment is created incrementally by adding (decision) and removing (backtracking) variable value pairs
- Using this information we will avoid looking at all the clauses

How to Implement Unit Propagation

The Real Task

We need a data structure for storing the clauses and a partial assignment ϕ that can efficiently support the following operations

- detect new unit clauses when ϕ is extended by $x_i = v$
- update itself by adding $x_i = v$ to ϕ
- update itself by removing $x_i = v$ from ϕ
- support restarts, i.e., un-assign all variables at once

Observation

- We only need to check clauses containing x_i

Occurrences List and Literals Counting

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The Data Structure

- For each clause remember the number unassigned literals in it
- For each literal remember all the clauses that contain it

Operations

- If $x_i = T$ is the new assignment look at all the clauses in the occurrence list of $\overline{x_i}$. We found a unit if the clause is not SAT and counter=2
- When $x_i = v$ is added or removed from ϕ update the counters

2 watched literals

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The Data Structure

- In each non-satisfied clause "watch" two non-false literals
- For each literal remember all the clauses where it is watched

Maintain the invariant: two watched non-false literals in non-sat clauses

- If a literal becomes false find another one to watch
- If that is not possible the clause is unit

Advantages

2 watched literals

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The Data Structure

- In each non-satisfied clause "watch" two non-false literals
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Advantages

- visit less clauses: when $x_i = T$ is added only visit clauses where $\overline{x_i}$ is watched
- no need to do anything at backtracking and restarts
  - watched literals cannot become false

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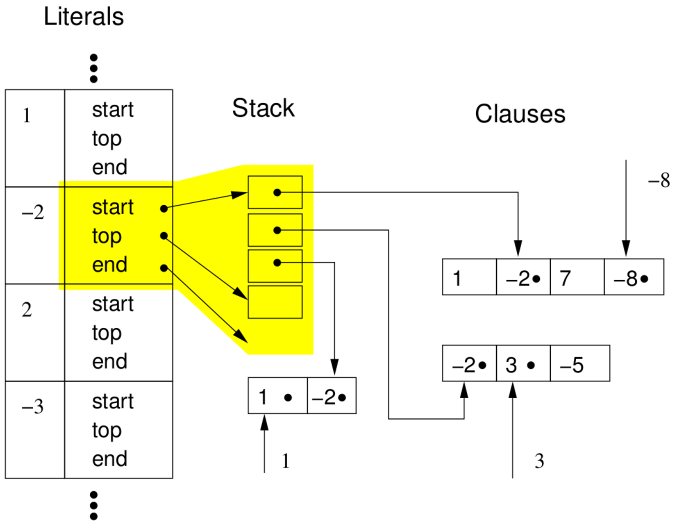
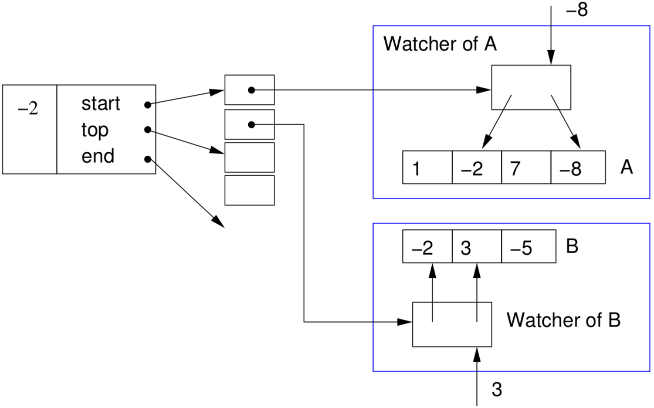


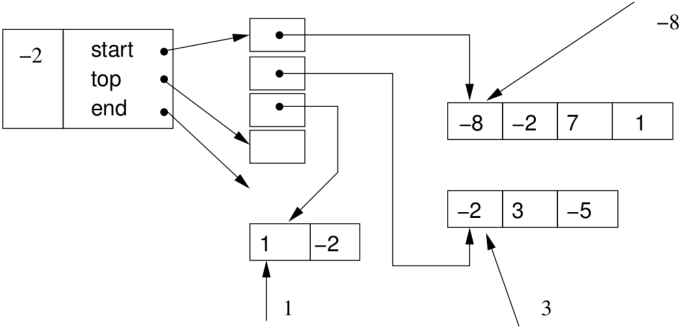
Literals
1
start
top
end
−2
start
top
end
2
start
top
end
−3
start
top
end
Stack
Clauses
−8
1
−2•
7
−8•
−2•
3 •
−5
1 •
−2•
1
3

Limmat



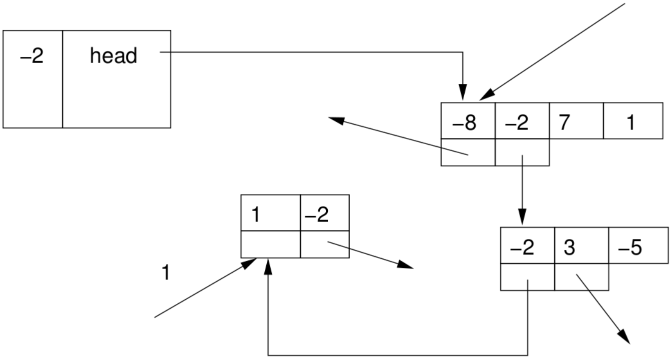
- Good for parallel SAT solvers with shared clause database

MiniSat



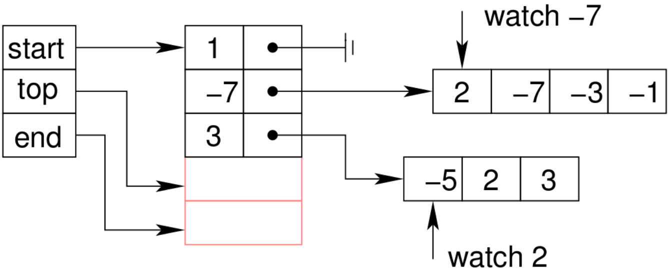
invariant: first two literals are watched

PicoSat



invariant: first two literals are watched

Lingeling



- often the other watched literal satisfies the clause
- for binary clauses no need to store the clause