Handout 8 (Writing)

One the next page is a (slightly doctored) excerpt from a *draft* of the review article "Sticky-sphere clusters," by Miranda Holmes-Cerfon, *Annual Reviews of Condensed Matter Physics*, 2017. There are many issues of writing style that can be improved. Do the following:

1. Read the 2nd and 3rd paragraphs of the Introduction, and pay attention to
   - Which words or phrases could be removed? (Concision)
   - Which pronouns (this, that, these, it, etc) are ambiguous?
2. Read Section 2.1 and think about:
   - Can any mathematical explanations be dropped?
   - Should any mathematical notation be added?
   - Is there any metadiscourse that should be removed?

Sticky-sphere clusters

1 Introduction

What can a small cluster of particles say about the materials we encounter in everyday life? While we cannot bang it with a hammer, wrap it around our shoulders, or throw it a ball, the information contained in the ground states of small systems is nevertheless critical to explaining many physical and biological properties of larger ones. Condensed-matter phenomena such as nucleation, the glass transition, gelation, epitaxial growth, aging, and the structure of liquids all have explanations rooted in the geometrically possible ways to arrange a small collection of particles. These possibilities also act as constraints on biological systems like proteins, viruses, chromatin, and microtubules, that fold, self-assemble, metabolize, or self-replicate. Small clusters have been used to design synthetic systems that perform these functions, bringing insight into the geometrical origins of biological complexity. Such synthetic systems are also of independent interest as we seek to design materials with new properties that may assemble or heal themselves.

There is still much that is unknown about clusters of any kind of particle, but a fair amount of progress has been made recently in describing small clusters of particles with very short-range attractive interactions. This is a good approximation for a wide range of nano- and micro-scale particles, like colloids, where longer-range interactions such as electrostatic forces may be screened by ions in the fluid medium. Common forces that cause them to stick to each other include depletion, and a popular synthetic method of coating them with strands of complementary DNA, which acts like velcro when they get close enough. The ranges of these are typically several times smaller than the particles' diameters. Colloids are convenient systems with which to study the scientific questions above because they are big – big enough that they can be treated theoretically as classical bodies, and big enough that they can be studied experimentally much more easily than atoms or molecules. There is also a real and exciting possibility of using them to design new substances and materials, since they can be synthesized to have a huge plethora of shapes, sizes and interaction structures so the parameter space of building blocks is very large.

This review describes the recent progress in understanding small clusters of particles interacting with a very short-range attractive potential, with colloids as the major motivation. It focuses mainly on theoretical and computational approaches, and the experimental measurements that validate and build upon these, doing little to discuss how these may be applied to glean insight into scientific questions. Nor does it broach the significant literature on simulating systems that are close to sticky. One reason for this is that the approach to be described is relatively new and under development. Another is that the ideas and tools are expected to apply to more general systems than clusters. The notion of points bound with constraints that are possibly harmonic has been used to study phenomena like jamming, structural glasses, silicates, self-folding polyhedra and origami, among many others. Nonlinearities in these models of the same nature as those in clusters have been particularly important in explaining dynamics near critical points and have been exploited in materials design. By focusing on the theoretical apparatus to describe "sticky" systems, it is hoped that connections to other fields may be easier to make.

[$\cdots$]

2 Rigid clusters

[· · ·]

2.1 Setup

To define the question precisely, let a cluster be represented as a vector $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) \in \mathbb{R}^{3n}$ where $\mathbf{x}_i = (x_{3i-2}, x_{3i-1}, x_{3i})$ is the center of the ith sphere. Each contact between a pair of spheres (i, j) with centers $\mathbf{x}_i$, $\mathbf{x}_j$ corresponds to an algebraic equation

$$|\mathbf{x}_i - \mathbf{x}_j|^2 = d_{ij}^2, \tag{1}$$

where d_{ij} is the sum of the radii. Hereafter we will consider identical spheres with unit diameters so $d_{ij} = 1$. A particular cluster can by identified by the set of pairs in contact, which gives a system of equations to solve for the coordinates of the cluster. This system can be represented by an adjacency matrix A by setting $A_{ij} = 1$ if spheres i, j are in contact, and $A_{ij} = 0$ otherwise.

It is sometimes convenient to remove the six rigid-body degrees of freedom of a cluster (three translational, three rotational), which can be done mathematically by fixing one sphere to the origin, one to the x-axis, and one to the xy-plane, as

$$x_s = 0, \qquad s \in \{1, 2, 3, 5, 6, 9\}. \tag{2}$$

(Alternatively, one can consider equivalence classes of rigid body motions, and interpret all of the statements to follow in the quotient space obtained by modding out by $SE(3)$.) A cluster is defined to be *rigid* if and only if it an isolated solution to the system (1), (2).

2.2 Testing for rigidity

[· · ·]