Leyes y desigualdades notables

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1. Frecuencia relativa

La frecuencia relativa es la cantidad de veces que un evento E ocurre a partir de n resultados mutuamente excluyentes (no ocurren resultados distintos en cada intento del experimento) de un experimento. La denotamos como:

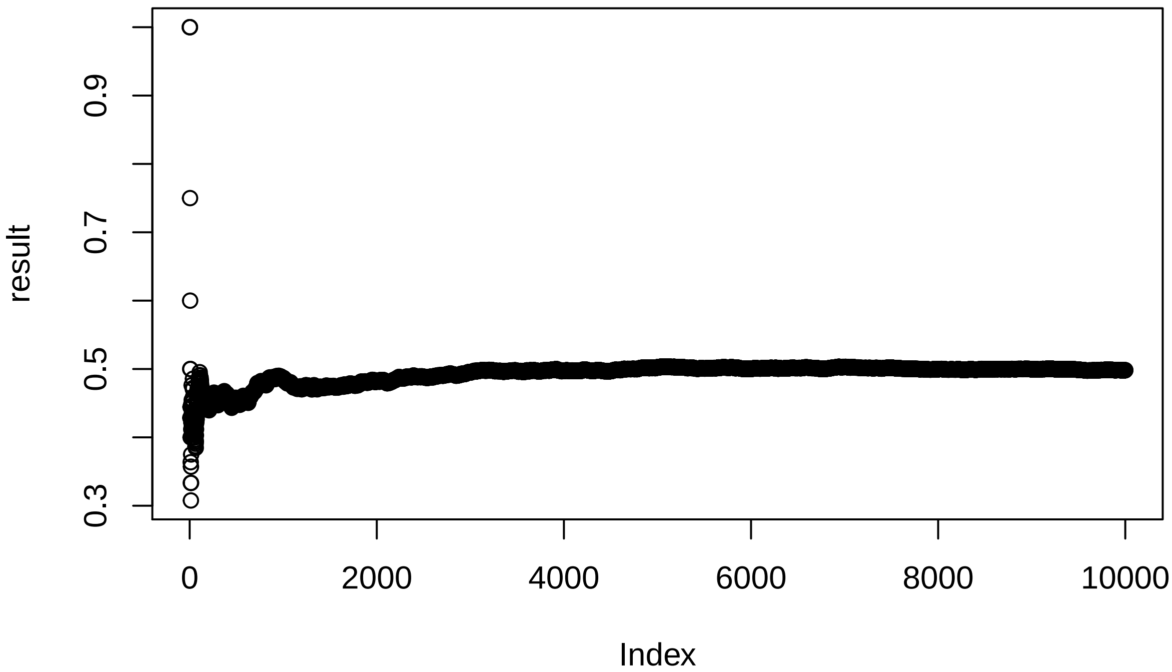
$$P(E) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_E(x_i)$$

```
relative_freq<-function(n,vars) {
  result<-c()
  heads<-0
  for (i in 1:n) {
    trial<-rbinom(vars,1,.5)
    if (trial==0) {
      heads<-heads+1
    }
    result<-append(result,heads/i)
  }
  return(result)
}

n<-10000
vars<-1
result<-relative_freq(n,vars)
result[n]
```

```
## [1] 0.4983
```

```
plot(result)
```




```
mean(result)
```

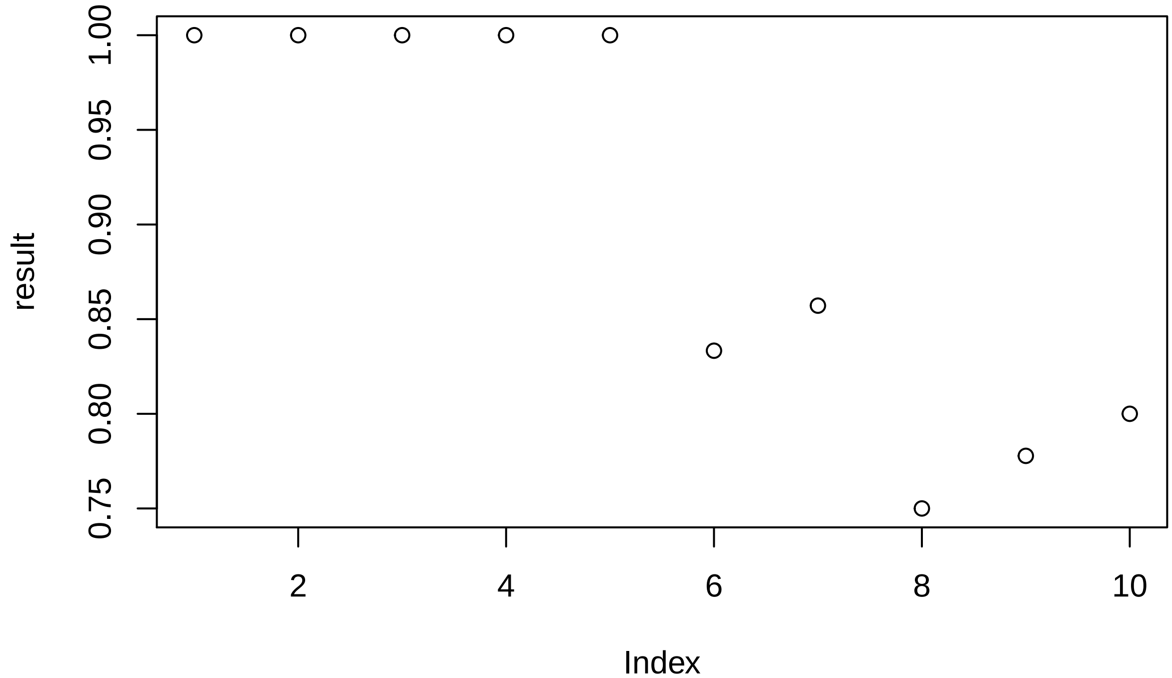
```
## [1] 0.4927526
```

```
sd(result)
```

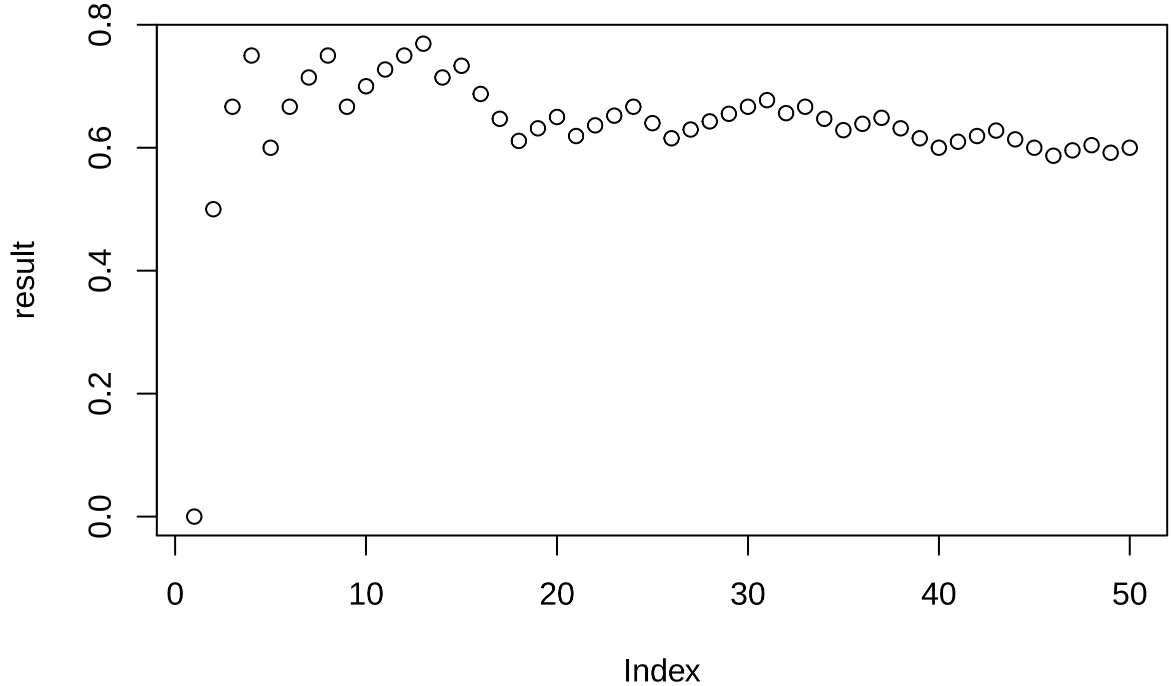
```
## [1] 0.01703201
```

```
vars<-1
for (n in c(10,50,100,500,1000,5000,10000)) {
  result<-relative_freq(n,vars)
  print(n)
  print(result[n])
  #write("n->",n," mean->", result[n])
  plot(result)
}
```

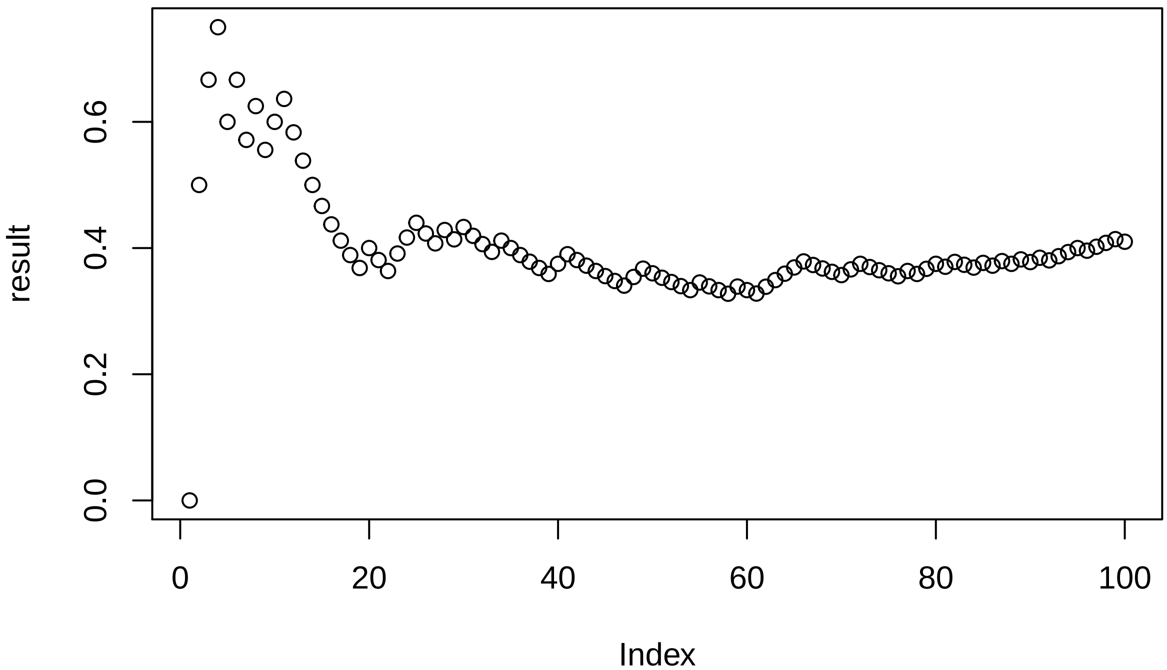
```
## [1] 10
## [1] 0.8
```



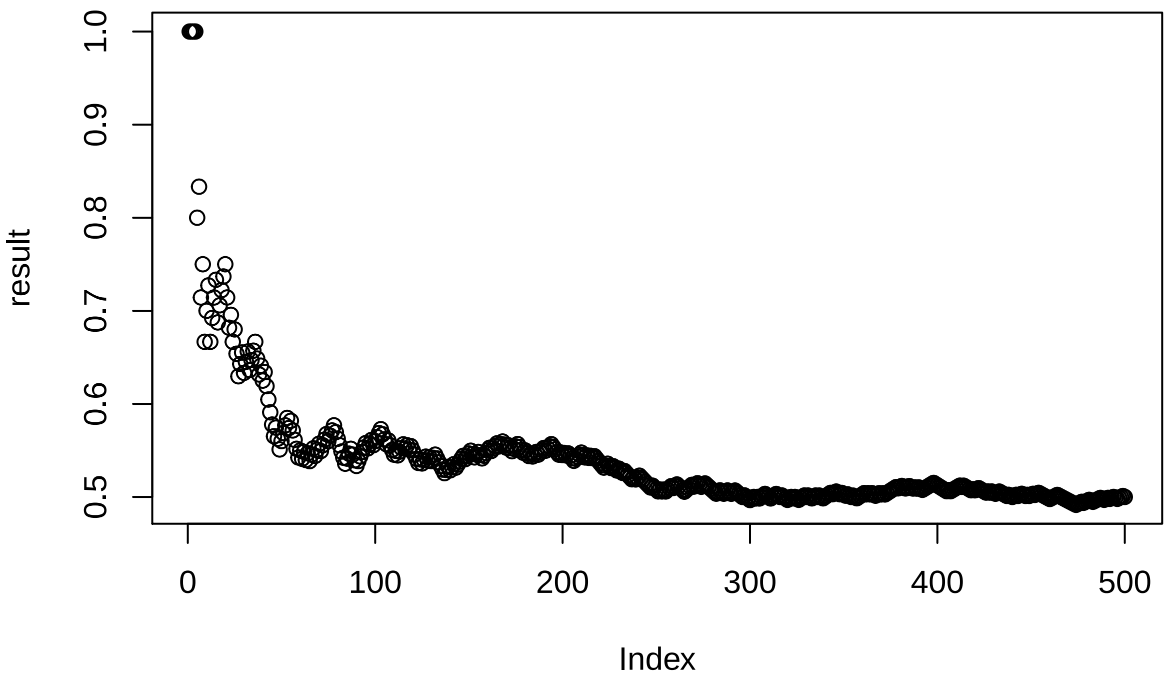

```
## [1] 50
## [1] 0.6
```



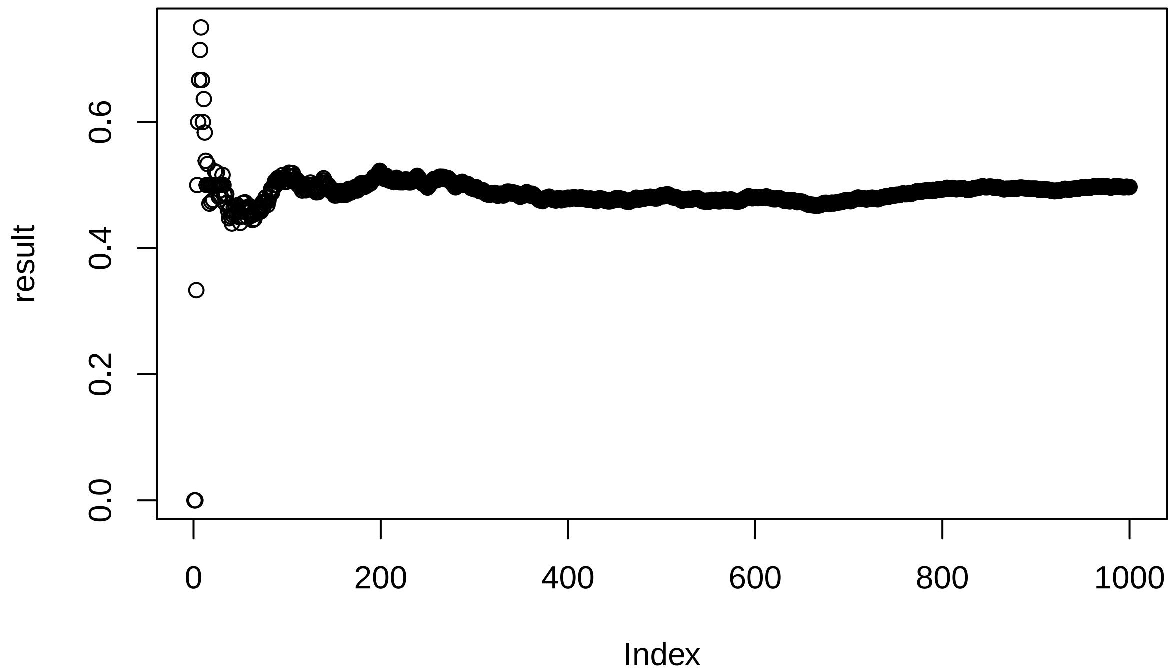

```
## [1] 100
## [1] 0.41
```



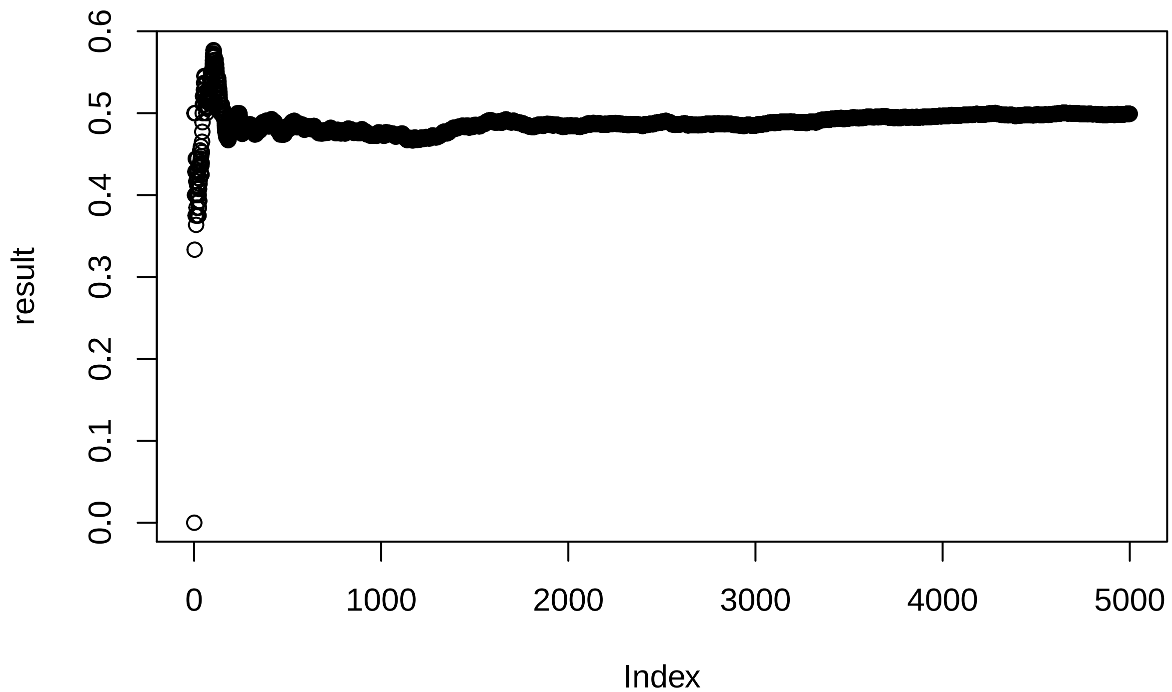

```
## [1] 500
## [1] 0.5
```



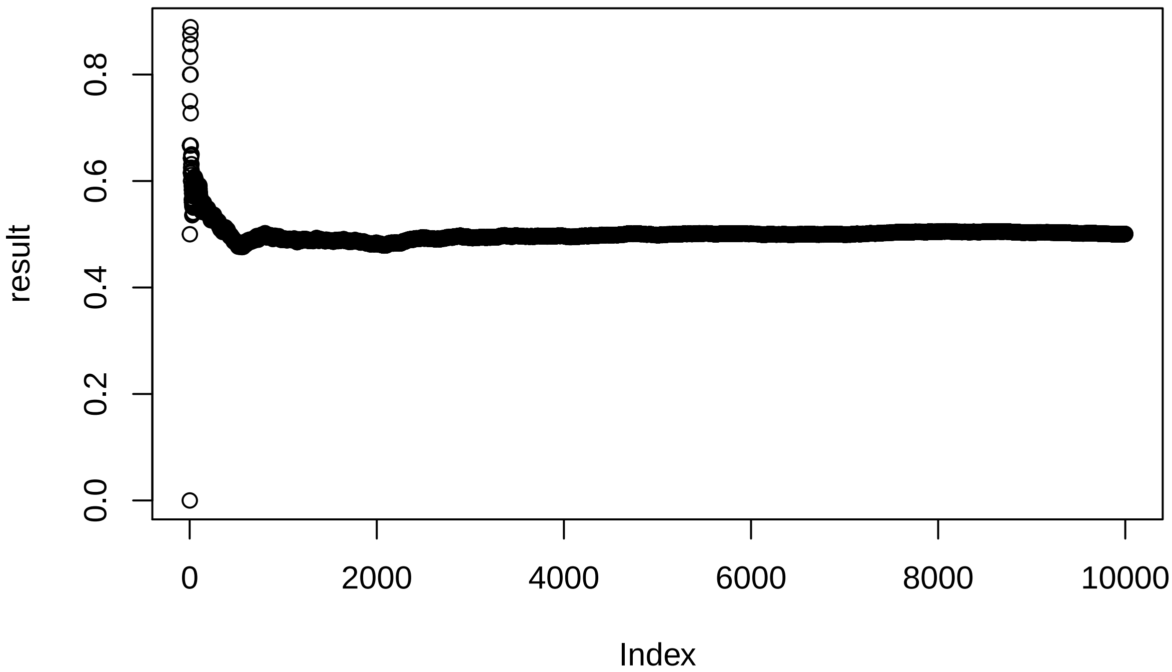

```
## [1] 1000
## [1] 0.497
```




```
## [1] 5000
## [1] 0.4992
```




```
## [1] 10000
## [1] 0.5005
```



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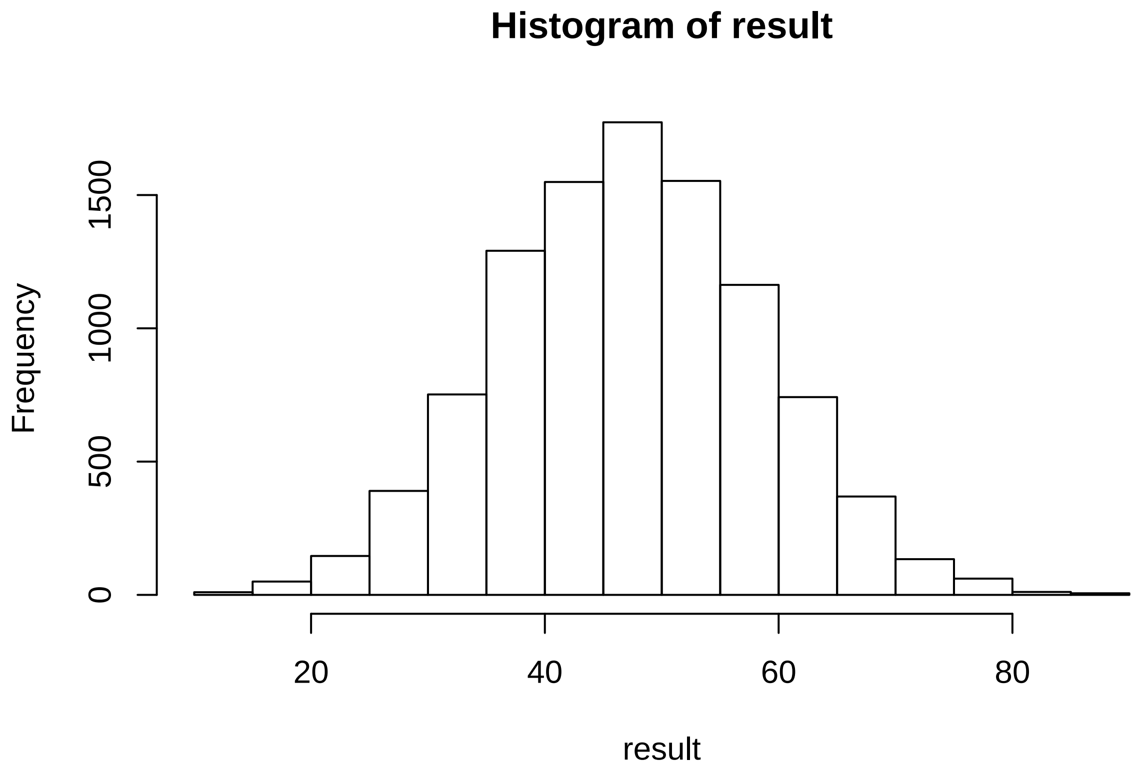
2. Suma de variables aleatorias Bernoulli

3. Media de variables aleatorias Bernoulli

```
meanof_bernoulli<-function(n,vars) {
  result<-c()
  for (i in 1:n) {
    result<-append(result,mean(rbinom(vars,1,.5))*sqrt(n))
  }
  return(result)
}

n<-10000
vars<-20
result<-meanof_bernoulli(n,vars)
hist(result)
```




```r
mean(result)
```

```
## [1] 49.847
```

```r
sd(result)
```

```
## [1] 11.28359
```

El promedio de n medias:

```r
mean(result)
```

```
## [1] 49.847
```

```r
sd(result)
```

```
## [1] 11.28359
```

---

4. Desigualdad de Markov

Sea X una variable aleatoria no negativa y $a > 0$,

$$P(X \geq a) \leq \frac{E(X)}{a}$$

o también

$$P(\mid X - \mu \mid \geq k\sigma) \leq \frac{1}{k^2}$$

5. Desigualdad de Tchebysheff

Sea X una variable aleatoria con media μ y varianza $\sigma^2 < \infty$. Entonces, para $k > 0$,

$$P(\mid X - \mu \mid < k\sigma) \geq 1 - \frac{1}{k^2}$$

o también

$$P(\mid X - \mu \mid \geq k\sigma) \leq \frac{1}{k^2}$$

Otra notación es la siguiente:

$$P(\mid X - E(X) \mid \geq k) \leq \frac{\sigma^2}{k^2}$$

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6. Ley de los grandes números

6.1. Ley débil

Sean $X_1, X_2, X_3, \ldots$ una sucesión infinita de variables aleatorias independientes y distribuidas idénticamente con $E(Y_i) = \mu$ y $Var(Y_i) = \sigma^2 < \infty$, entonces el promedio

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

converge en probabilidad a μ.

Es decir, sea $\epsilon > 0$ entonces

$$\lim_{n\to\infty} P(\mid \bar{X}_n - \mu \mid < \epsilon) = 1$$

Donde podemos decir que

$$P(\mid \bar{X}_n - \mu \mid < \epsilon) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{(-\epsilon+\mu, \epsilon+\mu)}(\bar{X}_n)$$

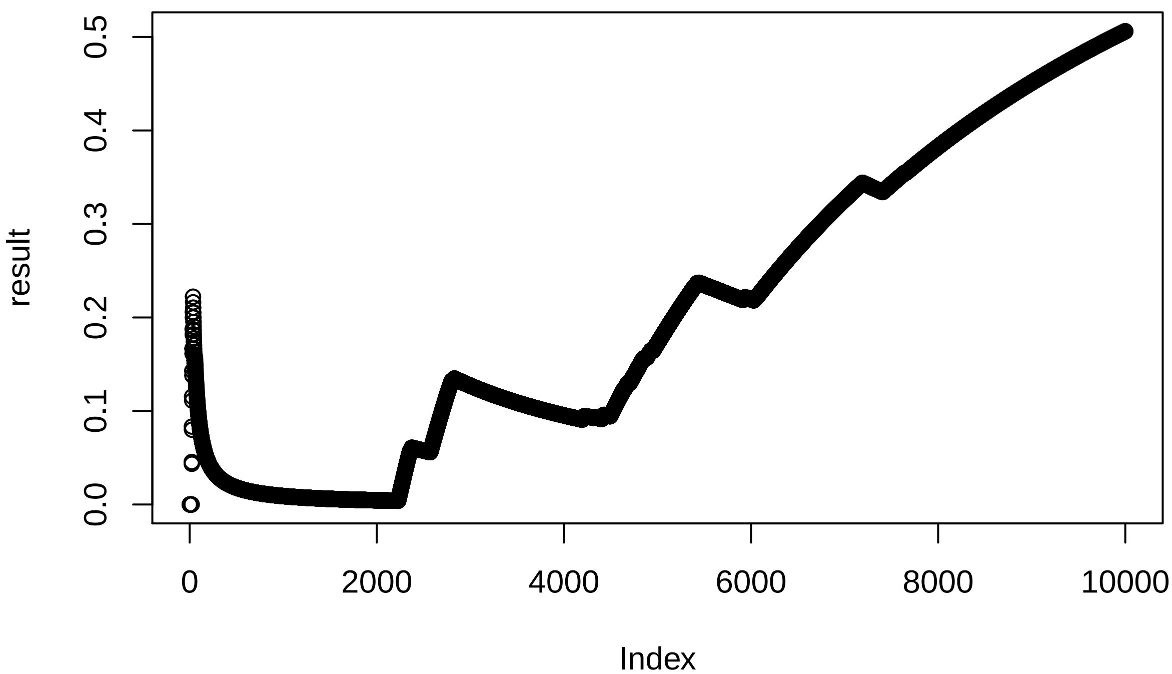
```
weak_law<-function(n,vars,epsilon) {
  result<-c()
  heads<-0
  f<-0
  for (i in 1:n) {
    trial<-rbinom(vars,1,.5)
    if (trial==0) {
      heads<-heads+1
    }
    if (abs((heads/i)-0.5)<epsilon) {
      f<-f+1
```

```
    }
    result<-append(result,f/i)
  }
  return(result)
}

n<-10000
vars<-1
epsilon<-0.01
result<-weak_law(n,vars,epsilon)
result[n]
```

```
## [1] 0.5061
```

```
plot(result)
```




```
mean(result)
```

```
## [1] 0.20673
```

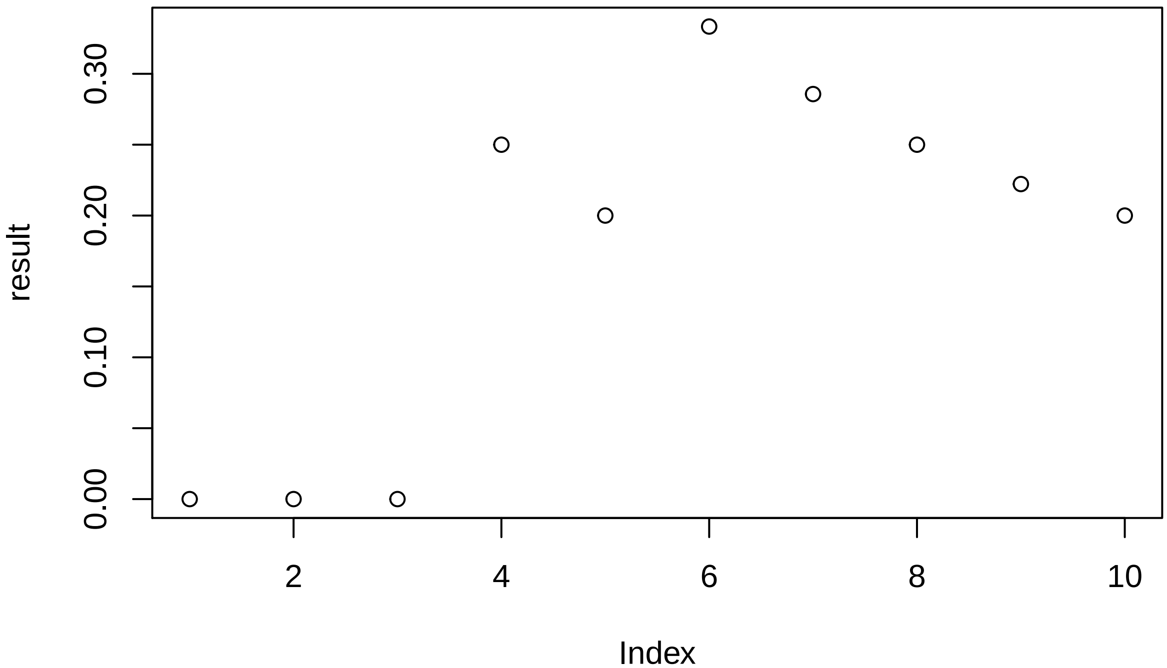
```
sd(result)
```

```
## [1] 0.1610966
```

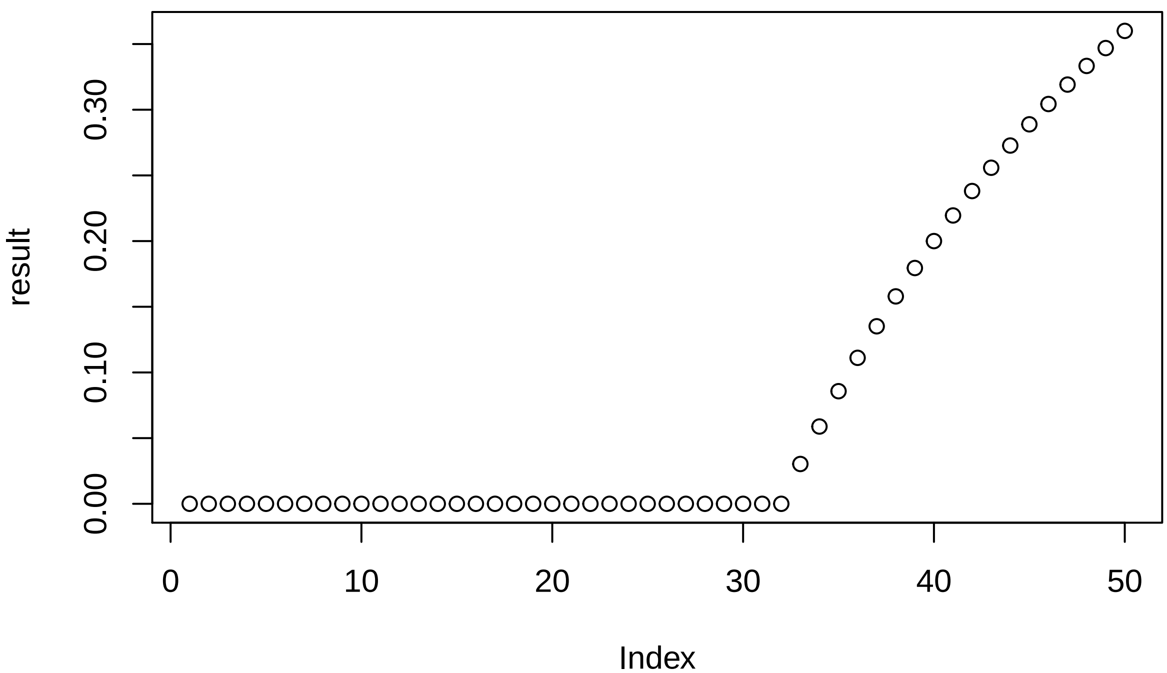
```
vars<-1
epsilon<-0.05
for (n in c(10,50,100,500,1000,5000,10000)) {
  result<-weak_law(n,vars,0.05)
  print(n)
  print(result[n])
  #write("n->",n," mean->", result[n])
  plot(result)
}
```

```
## [1] 10
```

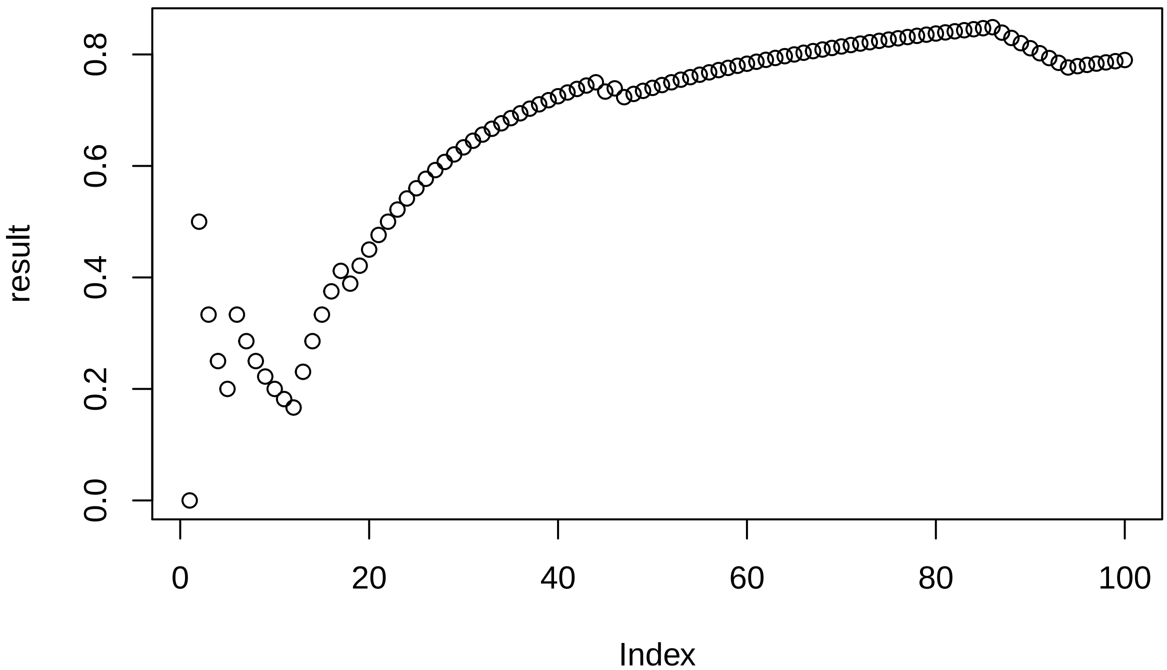
```
## [1] 0.2
```



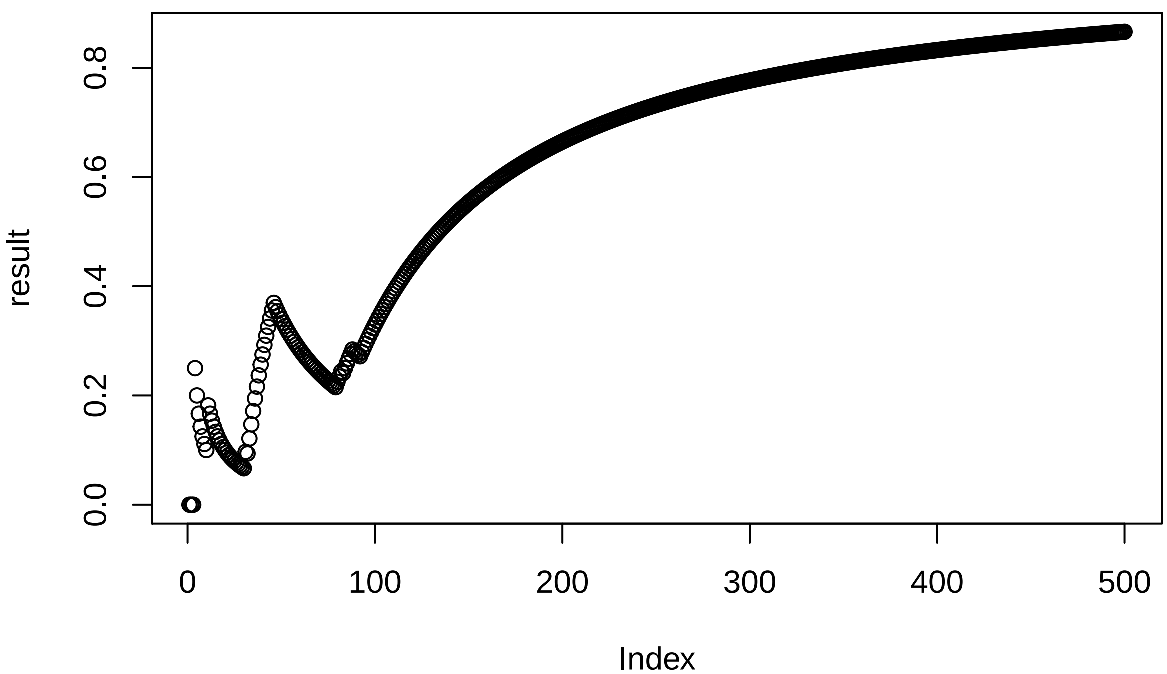

```
## [1] 50
## [1] 0.36
```



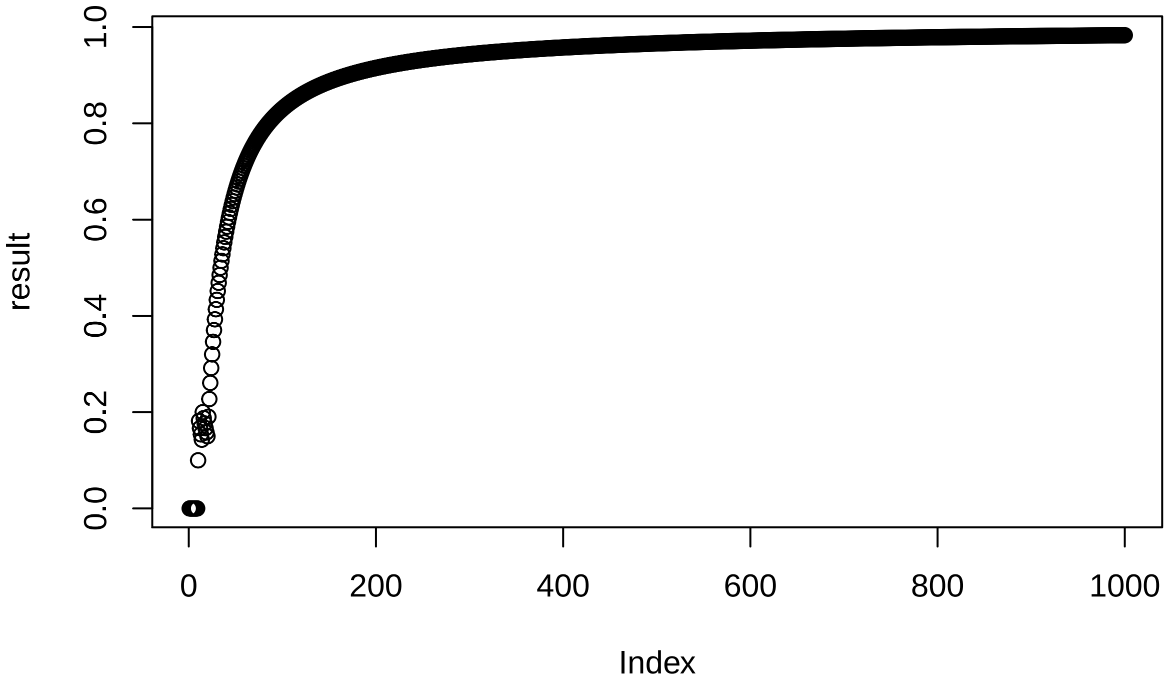

```
## [1] 100
## [1] 0.79
```



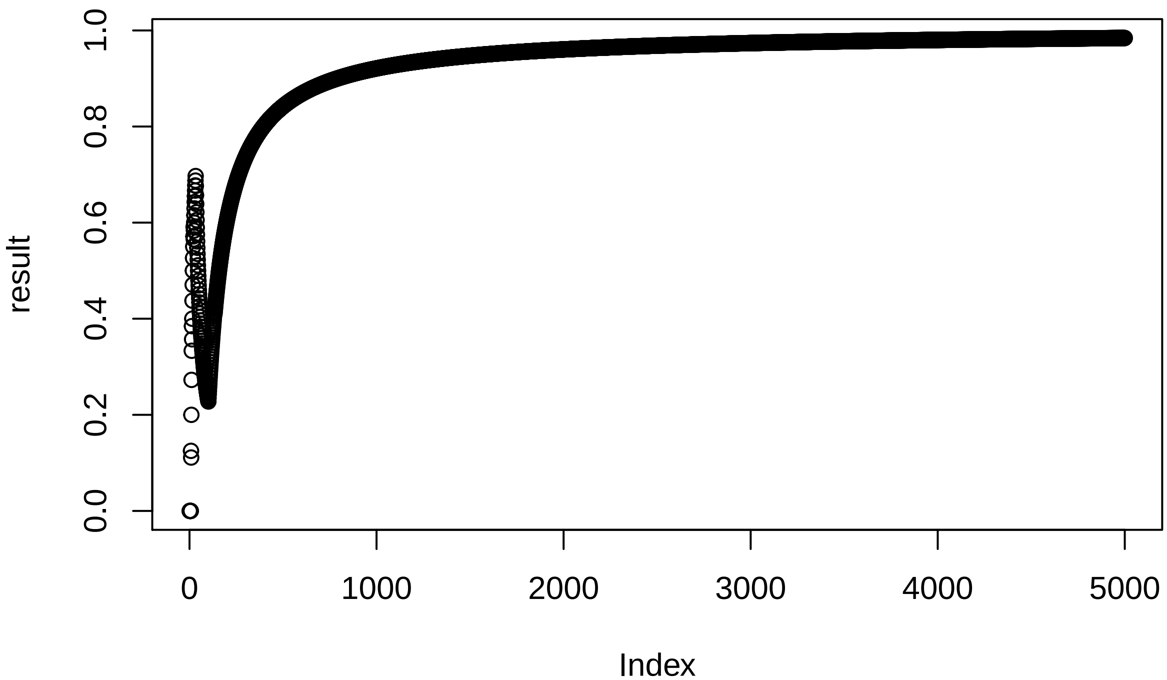

```
## [1] 500
## [1] 0.866
```



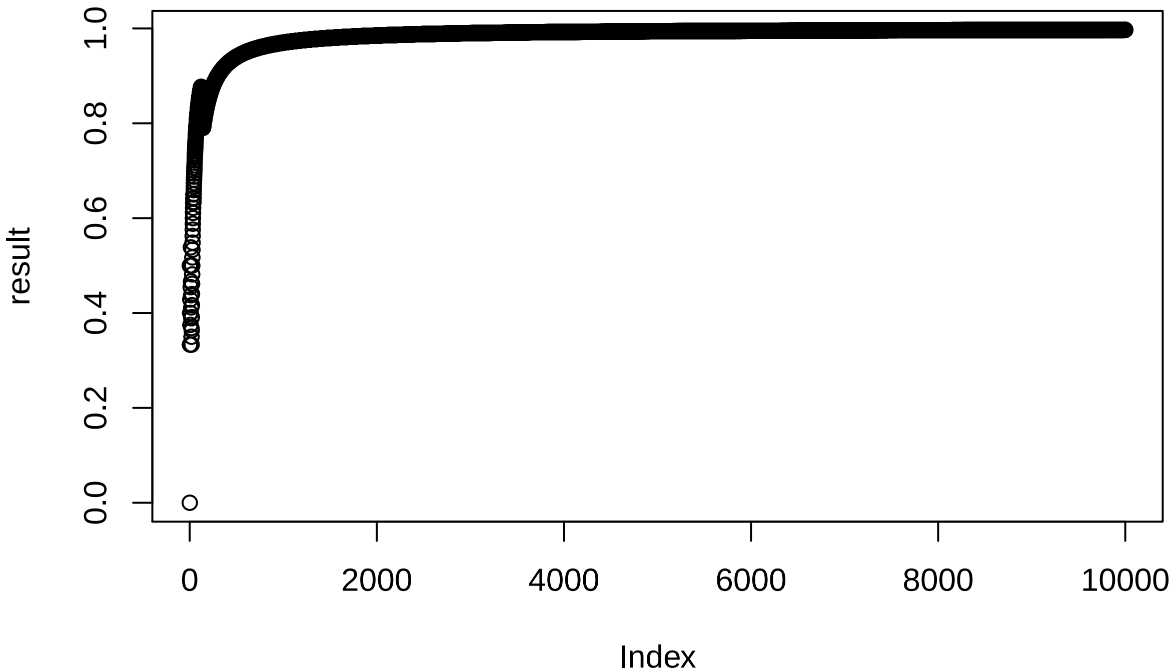

```
## [1] 1000
## [1] 0.983
```




```
## [1] 5000
## [1] 0.9842
```




```
## [1] 10000
## [1] 0.997
```



6.2. Ley fuerte

7. Teorema del límite central

Sean $Y_1, Y_2, , Y_n$ variables aleatorias independientes y distribuidas idénticamente con $E(Y_i) = \mu$ y $Var(Y_i) = \sigma^2 < \infty$. Definamos

$$U_n = \frac{\sum Y_i - n\mu}{\sigma\sqrt{n}} = \frac{\bar{Y} - \mu}{\sigma/\sqrt{n}}, \ \bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

Entonces la función de distribución de U_n converge hacia la función de distribución normal estándar cuando $n \to \infty$. Esto es,

$$\lim_{n\to\infty} P(U_n \leq u) = \int_{-\infty}^{u} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt, \text{ para toda u}$$