Assignment 1

EE 468: Selected Topics on Communications and Signal Processing
Neural Networks and Deep Learning
Due on: Sunday Feb 6th, 2022

Problem: Regression with Basis Functions

We have learned in class how to fit a linear model to some data points (i.e., the dataset), but we have restricted ourselves in the choice of model to the linear family of functions. In this assignment, you will see that such restriction is not extreme; although the relation between observed variables and desired responses is not linear in many real-world applications, linear models could still be used to handle some of those inherently non-linear relations. They do so using what is call *basis functions*.

Theoretical Part

Assume an observed variable $x \in \mathbb{R}$, a desired response $y \in \mathbb{R}$, and a dataset of pairs $\mathcal{D} = \{(x,y)_u\}_{u=1}^{U}$. Let $\phi_i(x) = x^i$ for $i \in \{1,\dots,N'\}$ be a transformation of x. This function ϕ_i is referred to as the *basis function*. Using $N' = 3$, we can express our model as follows

$$y(x) = w_3\phi_3(x) + w_2\phi_2(x) + w_1\phi_1(x) + w_0 \tag{1}$$
$$= w_3x^3 + w_2x^2 + w_1x + w_0 \tag{2}$$

Now, answer the following questions:

- **Q.1** Is the model in Equation 2 still linear?
- **Q.2** What is the design matrix in this case? What are the dimensions of the design matrix?
- **Q.3** Can you express the regression problem in vector notation (similar to Equation 1.6 in lecture notes 2)?
- **Q.4** Following similar steps to those followed in lecture notes 2, can you derive the least squares solution (normal equation) $\mathbf{w}^\star$ in this case? **Hint:** Start by expressing your loss function in vector notation (Equations 1.8 to 1.11 in lecture notes 2), and then, solve using the first derivative for $\mathbf{w}$ (Equations 1.13 to 1.16 in lecture notes 2)

Coding Part

The goal of this part is to get you to develop two machine learning algorithms and, then, get you to compare the two on the same dataset. The first algorithm is an implementation of what we have discussed in class while the other is an implementation to the theoretical part.

Visit the course GitHub account at: `https://github.com/ModernMLCourse`. Go to the repository (repo) "Assignment_1," and download it. You should get a script file named "main_temp.py," a " README.md" file, and a data file named "assig_1_dataset_1." The latter is the dataset, which includes training and validation. Most of the script is written for you, and only a few important lines are left to you to complete.

> **REMARK:** you are required to understand the whole script and not only those lines you complete; moving forward, I will assume you are familiar with everything you have seen in an assignment script