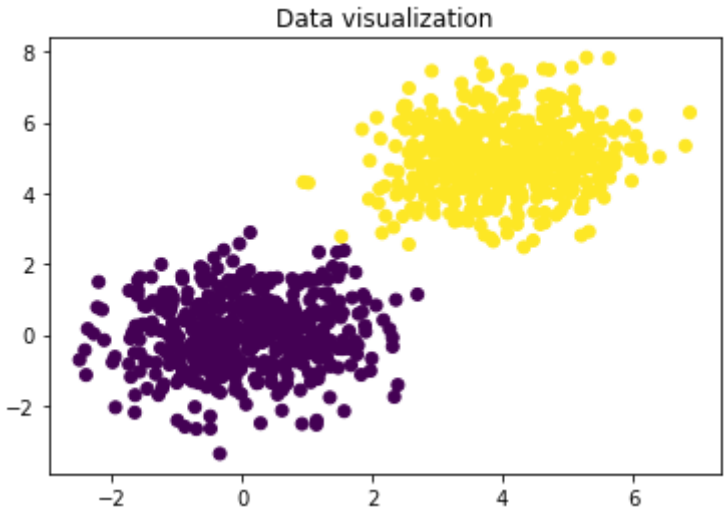
LAB 11 : Dimensionality Reduction

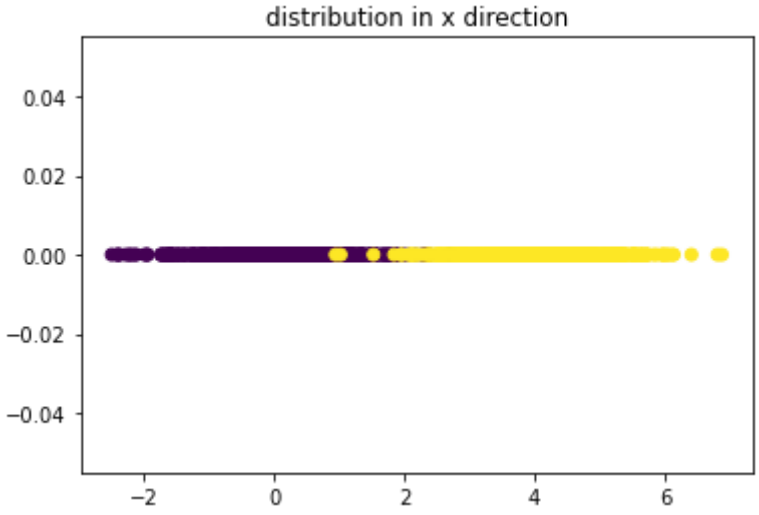
1. Principal Component Analysis (PCA)
2. Linear Discriminant Analysis (LDA)

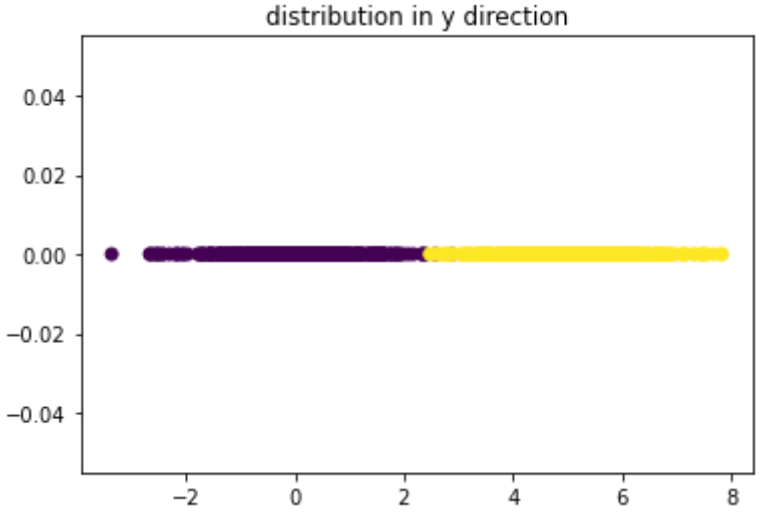
In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

PCA

In [ ]:
```
mean1 = np.array([0,0])
mean2 = np.array([4,5])
var = np.array([[1,0.1],[0.1,1]])
np.random.seed(0)
data1 = np.random.multivariate_normal(mean1,var,500)
data2 = np.random.multivariate_normal(mean2,var,500)
data = np.concatenate((data1,data2))
label = np.concatenate((np.zeros(data1.shape[0]),np.ones(data2.shape[0])))

plt.figure()
plt.scatter(data[:,0],data[:,1],c = label)
plt.title('Data visualization')
plt.figure()
plt.scatter(data[:,0],np.zeros(data.shape[0]),c = label)
plt.title('distribution in x direction')
plt.figure()
plt.scatter(data[:,1],np.zeros(data.shape[0]),c = label)
plt.title('distribution in y direction')
plt.show()
```

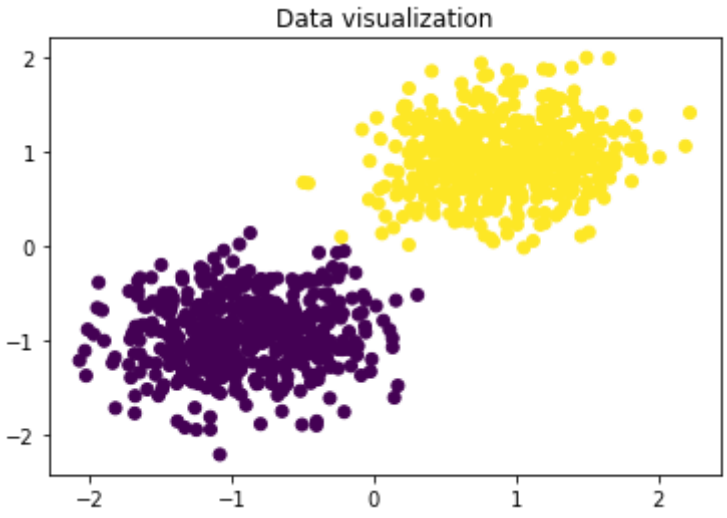






In [ ]:
```
# Data normalization

# Perform data normalization here using mean substraction and std division
data = (data - np.mean(data, axis=0)) / np.std(data, axis=0)

plt.figure()
plt.scatter(data[:,0],data[:,1],c = label)
plt.title('Data visualization')

print(f"New Mean: {np.mean(data, axis=0)}, New Std: {np.std(data, axis=0)}")
```

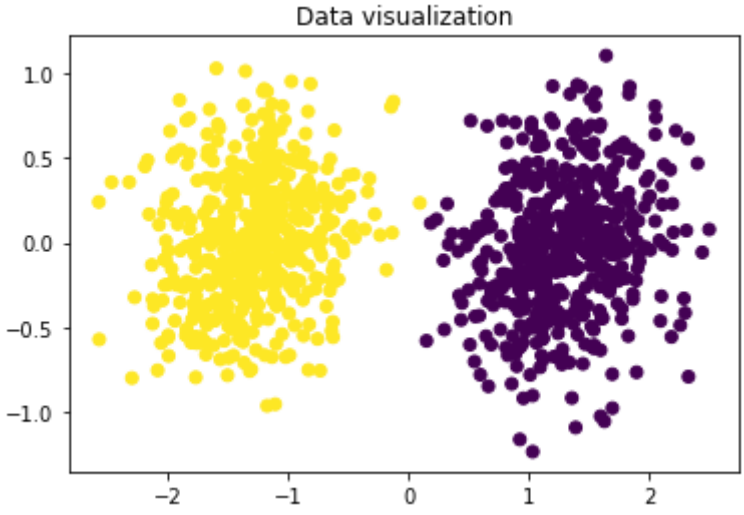
```
New Mean: [-4.25215418e-16  1.79334325e-15], New Std: [1. 1.]
```

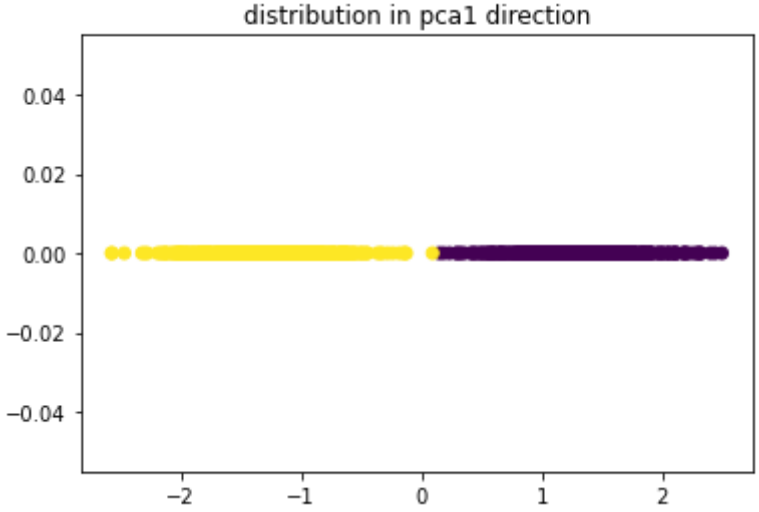


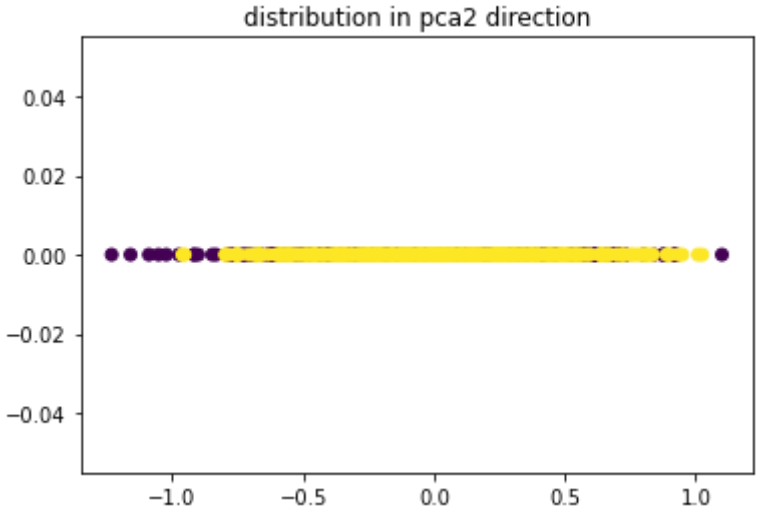
In [ ]:
```
# PCA
```

```
# coverance matrix
cov = data.T @ data


# using sigular value decomposition
u, s, v = np.linalg.svd(cov)

trans_data= data @ u

var_pca1 = np.var(trans_data[:,0])
var_pca2 = np.var(trans_data[:,1])

print('variance along pca1 direction = ', var_pca1)
print('variance along pca2 direction = ', var_pca2)


plt.figure()
plt.scatter(trans_data[:,0],trans_data[:,1], c=label)
plt.title('Data visualization')
plt.figure()
plt.scatter(trans_data[:,0],np.zeros(data.shape[0]), c=label)
plt.title('distribution in pca1 direction')
plt.figure()
plt.scatter(trans_data[:,1],np.zeros(data.shape[0]), c=label)
plt.title('distribution in pca2 direction')
plt.show()
```

```
variance along pca1 direction =  1.847766384345973
variance along pca2 direction =  0.152233615654027
```

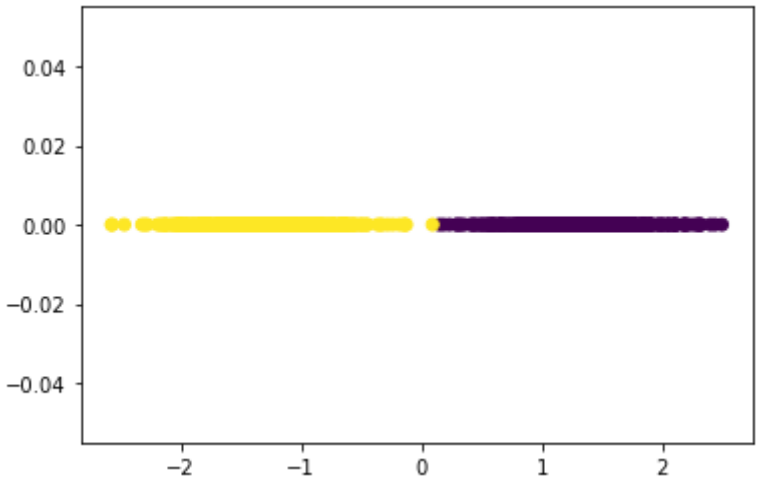






In [ ]:
```
class pca:
        # Constructor
        def __init__(self, name = 'reg', data = None, retain_dim = None):
                self.name = name  # Create an instance variable
                self.data = data
                self.retain_dim = retain_dim if retain_dim is not None else

        # compute pca transform value
        def pca_comp(self,data):
                data = self.pre_process(data)
                cov = data.T @ data
                u, _, _ = np.linalg.svd(cov) # singular value decomposition
                u_req = u[:, :self.retain_dim]
                trans_data = data @ u_req
                return trans_data, u_req

        # compute the required retain dimension
        def ret_dim(self,data):
                data = self.pre_process(data)
                cov = data.T @ data
                _, s, _ = np.linalg.svd(cov)
                ind = np.cumsum(s)/np.sum(s) >= 0.9
                ind = np.where(ind == True)[0][0]
                return ind+1

        # normalize the data
        def pre_process(self,data):
                data1 = (data-np.mean(data,axis = 0))
                data = data1/(np.std(data1,axis = 0)+10**(-30)) # avoid div
                return data
```

In [ ]:
```
# pca transformation
PCA = pca(data=data)
trans_data, trans_mat = PCA.pca_comp(data)
plt.scatter(trans_data, np.zeros(trans_data.shape), c=label)
plt.show()
```



In [ ]:
```
# classification using pca
# use k-nearest neighbour classifier after dimensionality reduction

from sklearn.neighbors import KNeighborsClassifier
k = 5
knn = KNeighborsClassifier(n_neighbors = k)
knn.fit(trans_data, label)

print('KNN Training accuracy  = ', knn.score(trans_data,label)*100)

# test data
np.random.seed(0)
data1 = np.random.multivariate_normal(mean1,var,50)
data2 = np.random.multivariate_normal(mean2,var,50)
data = np.concatenate((data1,data2))
tst_label = np.concatenate((np.zeros(data1.shape[0]),np.ones(data2.shape[0])

print('KNN Testing accuracy  = ', knn.score(PCA.pre_process(data) @ trans_ma
```

```
KNN Training accuracy  =  99.9
KNN Testing accuracy  =  100.0
```

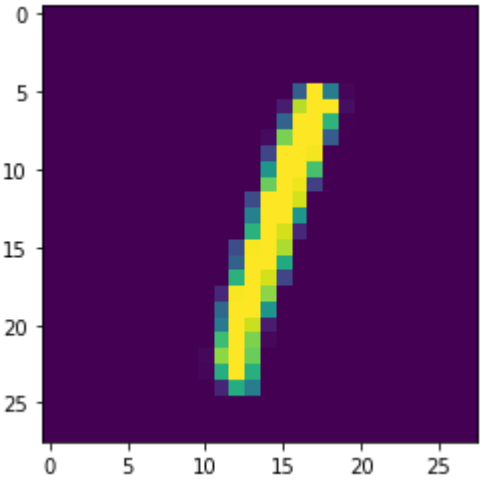
PCA on MNIST

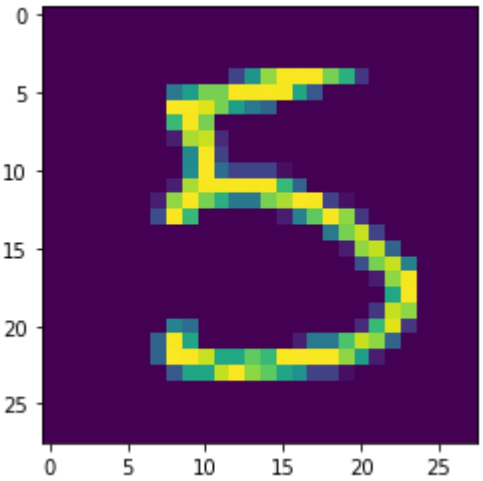
In [ ]:
```
# %pip install idx2numpy
```

In [ ]:
```
# MNIST data

file1 = 't10k-images-idx3-ubyte'
file2 = 't10k-labels-idx1-ubyte'

import idx2numpy

Images= idx2numpy.convert_from_file(file1)
labels= idx2numpy.convert_from_file(file2)

cl = [1,5]

# for class 1
id_1 = np.where(labels == cl[0])
id1 = id_1[0]
id1 = id1[:50]
Im_1 = Images[id1,:,:]
lab_1 = labels[id1]

# for class 5
```

```
id_5 = np.where(labels == cl[1])
id5 = id_5[0]
id5 = id5[:50]
Im_5 = Images[id5,:,:]
lab_5 = labels[id5]

plt.imshow(Im_1[1,:,:])
plt.figure()
plt.imshow(Im_5[1,:,:])

#print(Im_5.shape)

data = np.concatenate((Im_1,Im_5))
data = np.reshape(data,(data.shape[0],data.shape[1]*data.shape[2]))
print(data.shape)
G_lab = np.concatenate((lab_1,lab_5))
print(G_lab.shape)

data = data.astype('float32')

data /= 255
```

```
(100, 784)
(100,)
```



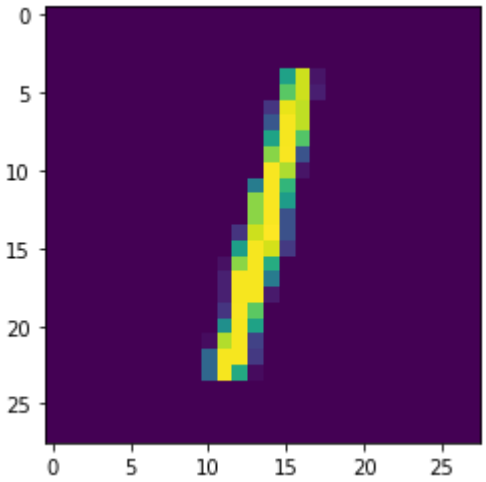


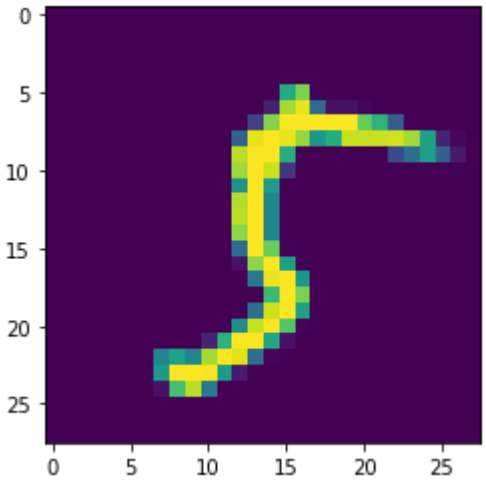
In [ ]:

```
print('Initial data dimension = ',data.shape[1])
PCA = pca(data = data)

trans_data, trans_mat = PCA.pca_comp(data)
print('Retained dimesion after PCA = ',trans_mat.shape[1])
k = 5
knn = KNeighborsClassifier(n_neighbors = k)
knn.fit(trans_data, G_lab)
```

```
print('KNN Training accuracy  = ',knn.score(trans_data,G_lab)*100)


## testing
## data preparation
id_1 = np.where(labels == cl[0])
id1 = id_1[0]
id1 = id1[100:150]
Im_1 = Images[id1,:,:]
lab_1 = labels[id1]

# for class 5
id_5 = np.where(labels == cl[1])
id5 = id_5[0]
id5 = id5[100:150]
Im_5 = Images[id5,:,:]
lab_5 = labels[id5]

plt.imshow(Im_1[1,:,:])
plt.figure()
plt.imshow(Im_5[1,:,:])

print(Im_5.shape)

data_tst = np.concatenate((Im_1,Im_5))
data_tst = np.reshape(data_tst,(data_tst.shape[0],data_tst.shape[1]*data_tst

tst_lab = np.concatenate((lab_1,lab_5))


# final testing
print('KNN Testing accuracy  = ', knn.score(PCA.pre_process(data_tst) @ trar
```

```
Initial data dimension =  784
Retained dimesion after PCA =  34
KNN Training accuracy  =  96.0
(50, 28, 28)
KNN Testing accuracy  =  98.0
```



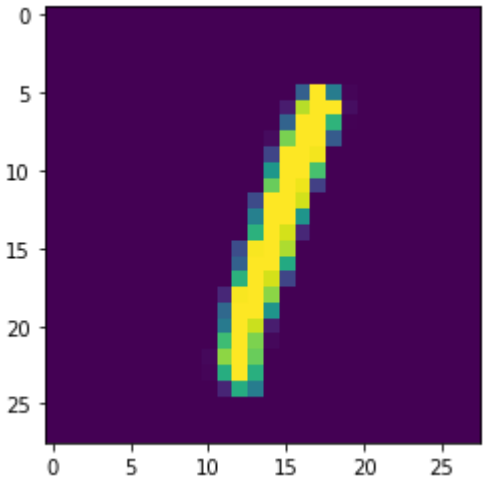


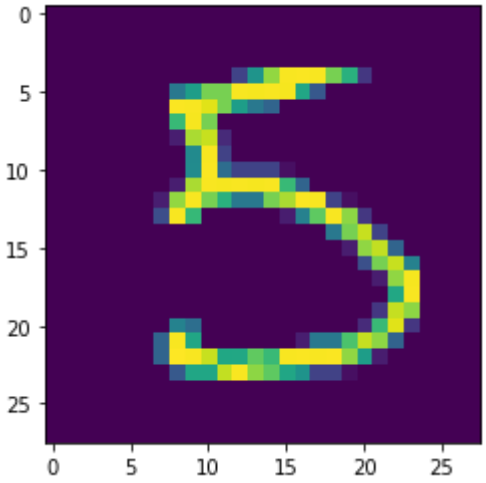
Perform PCA on MNIST and Classify taking the data with any 3 Classes

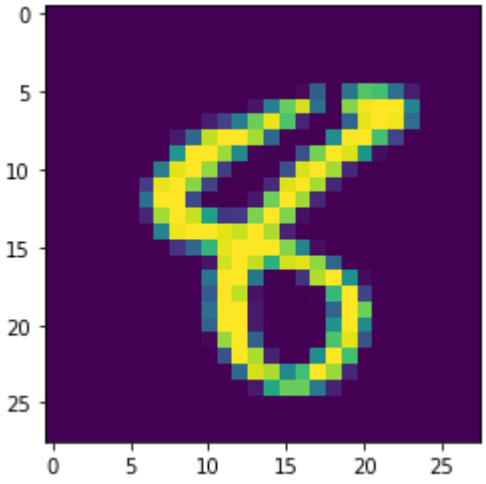
In [ ]:

```
# Prepare data for classification
classes = [1, 5, 8]

# for class 1
id_1 = np.where(labels == classes[0])[0][:50]
Im_1 = Images[id_1,:,:]
lab_1 = labels[id_1]

# for class 5
id_5 = np.where(labels == classes[1])[0][:50]
Im_5 = Images[id_5,:,:]
lab_5 = labels[id_5]

# for class 8
id_8 = np.where(labels == classes[2])[0][:50]
Im_8 = Images[id_8,:,:]
lab_8 = labels[id_8]

# concatenate the data
data = np.concatenate((Im_1,Im_5,Im_8))
data = np.reshape(data,(data.shape[0],data.shape[1]*data.shape[2]))
print(data.shape)
G_lab = np.concatenate((lab_1,lab_5,lab_8))
print(G_lab.shape)

# normalize the data
data = data.astype('float32')
data /= 255

# Show the data
plt.imshow(Im_1[1,:,:])
plt.figure()
plt.imshow(Im_5[1,:,:])
plt.figure()
plt.imshow(Im_8[1,:,:])
plt.show()
```

```
(150, 784)
(150,)
```







In [ ]:

```
# PCA then KNN
PCA = pca(data = data)

trans_data, trans_mat = PCA.pca_comp(data)
print(f"Retained dimesion after PCA = {trans_mat.shape[1]}")
k = 5
knn = KNeighborsClassifier(n_neighbors = k)
knn.fit(trans_data, G_lab)

print(f"KNN Training accuracy  = {knn.score(trans_data,G_lab)*100}")

# testing
# data preparation
# for class 1
id_1 = np.where(labels == classes[0])[0][100:150]
Im_1 = Images[id_1,:,:]
```

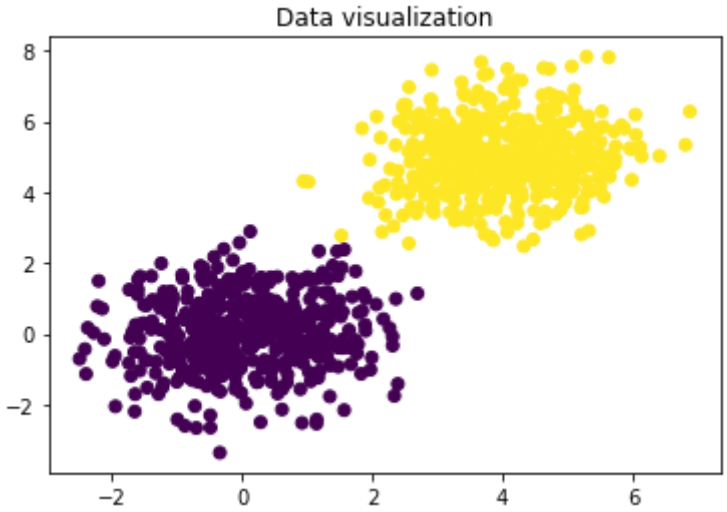
```
lab_1 = labels[id_1]

# for class 5
id_5 = np.where(labels == classes[1])[0][100:150]
Im_5 = Images[id_5,:,:]
lab_5 = labels[id_5]

# for class 8
id_8 = np.where(labels == classes[2])[0][100:150]
Im_8 = Images[id_8,:,:]
lab_8 = labels[id_8]

# concatenate the data
data_tst = np.concatenate((Im_1,Im_5,Im_8))
data_tst = np.reshape(data_tst,(data_tst.shape[0],data_tst.shape[1]*data_tst

tst_lab = np.concatenate((lab_1,lab_5,lab_8))

# final testing
print(f"KNN Testing accuracy  = {knn.score(PCA.pre_process(data_tst) @ trans
```

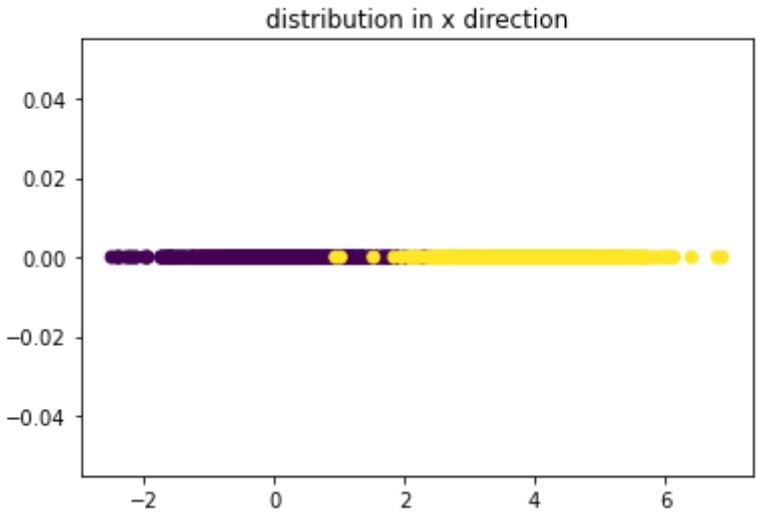
```
Retained dimesion after PCA = 50
KNN Training accuracy  = 91.33333333333333
KNN Testing accuracy  = 86.66666666666667
```

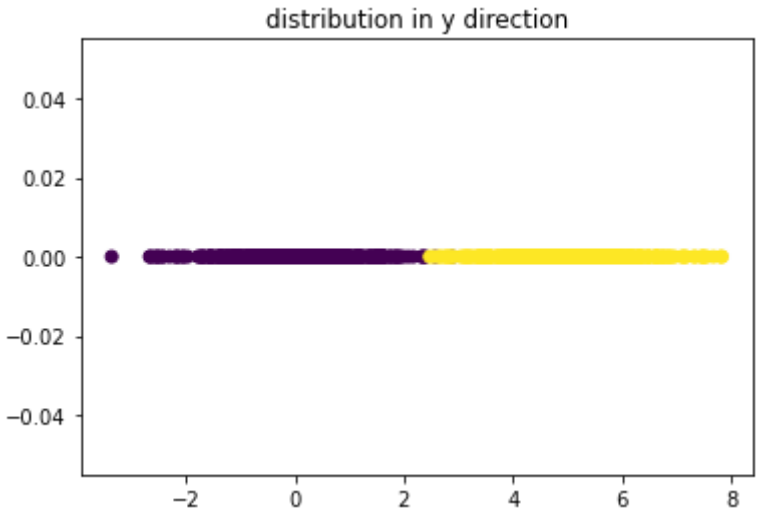
LDA

In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt

# data generation

mean1 = np.array([0,0])
mean2 = np.array([4,5])
var = np.array([[1,0.1],[0.1,1]])
np.random.seed(0)
data1 = np.random.multivariate_normal(mean1,var,500)
data2 = np.random.multivariate_normal(mean2,var,500)
data = np.concatenate((data1,data2))
label = np.concatenate((np.zeros(data1.shape[0]),np.ones(data2.shape[0])))

plt.figure()
plt.scatter(data[:,0],data[:,1],c = label)
plt.title('Data visualization')
plt.figure()
plt.scatter(data[:,0],np.zeros(data.shape[0]),c = label)
plt.title('distribution in x direction')
plt.figure()
plt.scatter(data[:,1],np.zeros(data.shape[0]),c = label)
plt.title('distribution in y direction')
```

Out[ ]:
```
Text(0.5, 1.0, 'distribution in y direction')
```





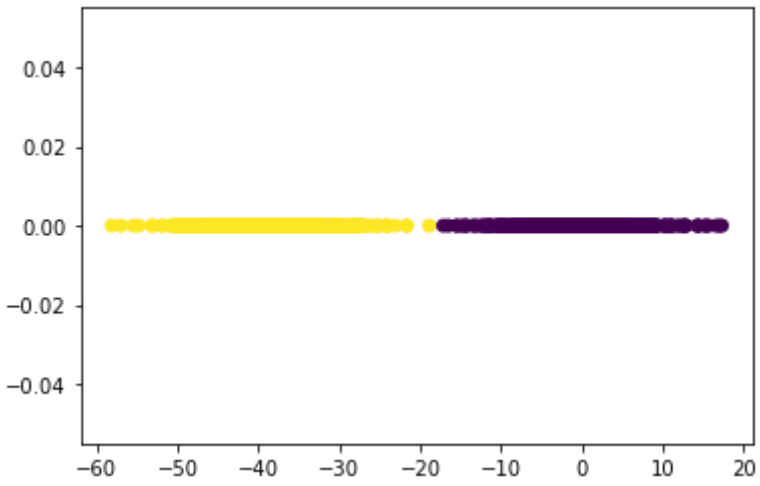


In [ ]:
```python
# perform 2-class and m-class LDA
def LDA(data,label):
        id = {}
        data_l = {}
        mean_l = {}
        cov_l = {}
        S_w = np.zeros((data.shape[1],data.shape[1]))
        cls = np.unique(label)

        for i in cls:
                id[i] = np.where(label == i)[0]
                data_l[i] = data[id[i],:]
                mean_l[i] = np.mean(data_l[i],axis = 0)
```

```
                cov_l[i] = ((data_l[i] - mean_l[i]).T @ (data_l[i] - mean_l
                S_w = S_w+cov_l[i]

        S_w = S_w/len(data_l)

        if len(data_l) == 2:
                S_b = ((mean_l[0] - mean_l[1]).T @ (mean_l[0] - mean_l[1]))
                w = np.linalg.pinv(S_w) @ (mean_l[0] - mean_l[1]).T

        else:
                S_t = np.cov(data,rowvar = False)
                S_b = S_t - S_w
                u, _, _= np.linalg.svd(np.linalg.pinv(S_w) @ S_b)
                w = u[:,:len(data_l)-1]

        return w
```

In [ ]:
```
# after LDA projection

w = LDA(data,label)
plt.figure()
plt.scatter(data @ w,np.zeros(data.shape[0]),c = label)
```

Out[ ]:
```
<matplotlib.collections.PathCollection at 0x7fbfd30a6260>
```



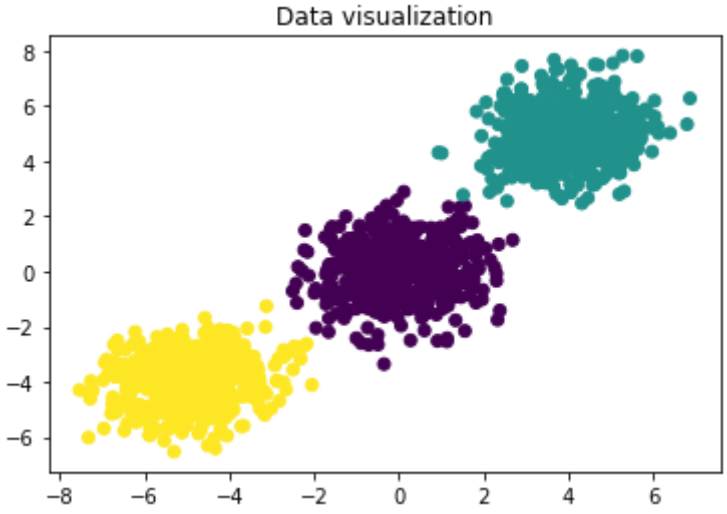
In [ ]:
```
# Classification using LDA
# Use k-nearest neighbour classifier (Scikit Learn) after dimensionality re

from sklearn.neighbors import KNeighborsClassifier

k = 5
knn = KNeighborsClassifier(n_neighbors = k)
knn.fit((data @ w.reshape(-1,1)),label)

print(f"KNN Training accuracy  = {knn.score(data @ w.reshape(-1,1),label)*10

# testing
# data preparation
mean1 = np.array([0,0])
mean2 = np.array([4,5])
var = np.array([[1,0.1],[0.1,1]])
np.random.seed(1)
data1 = np.random.multivariate_normal(mean1,var,100)
data2 = np.random.multivariate_normal(mean2,var,100)
data_tst = np.concatenate((data1,data2))
label_tst = np.concatenate((np.zeros(data1.shape[0]),np.ones(data2.shape[0]
print(f"KNN Testing accuracy  = {knn.score(data_tst @ w.reshape(-1,1),label_
```

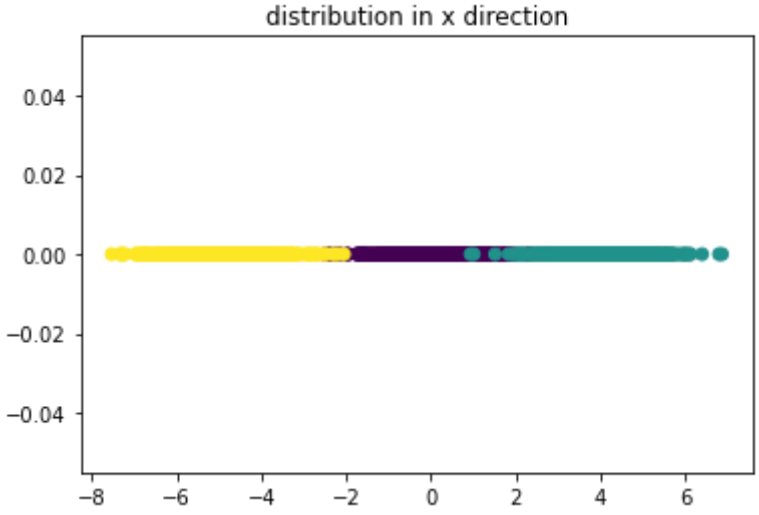
```
KNN Training accuracy  = 100.0
KNN Testing accuracy  = 100.0
```

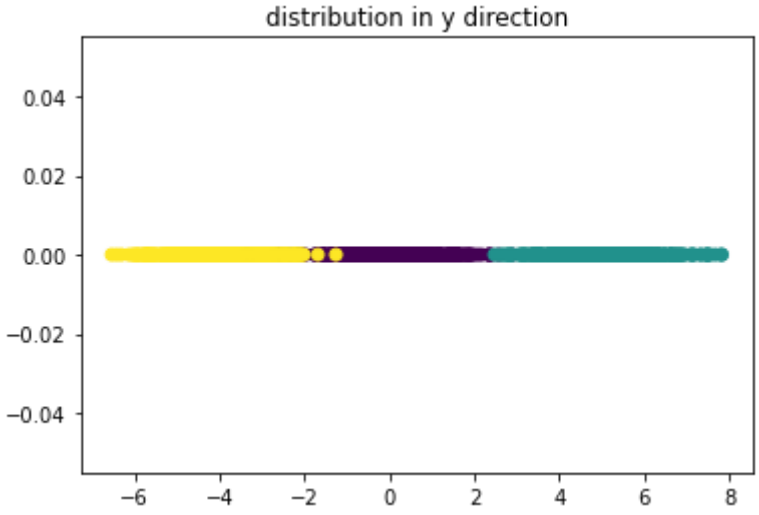
LDA Multiclass

In [ ]:
```
mean1 = np.array([0,0])
mean2 = np.array([4,5])
mean3 = np.array([-5,-4])
var = np.array([[1,0.1],[0.1,1]])
np.random.seed(0)
data1 = np.random.multivariate_normal(mean1,var,500)
data2 = np.random.multivariate_normal(mean2,var,500)
data3 = np.random.multivariate_normal(mean3,var,500)
data = np.concatenate((data1,data2,data3))
label = np.concatenate((np.zeros(data1.shape[0]),np.ones(data2.shape[0]),np.

plt.figure()
plt.scatter(data[:,0],data[:,1],c = label)
plt.title('Data visualization')
plt.figure()
plt.scatter(data[:,0],np.zeros(data.shape[0]),c = label)
plt.title('distribution in x direction')
plt.figure()
plt.scatter(data[:,1],np.zeros(data.shape[0]),c = label)
plt.title('distribution in y direction')
```

Out[ ]:
```
Text(0.5, 1.0, 'distribution in y direction')
```

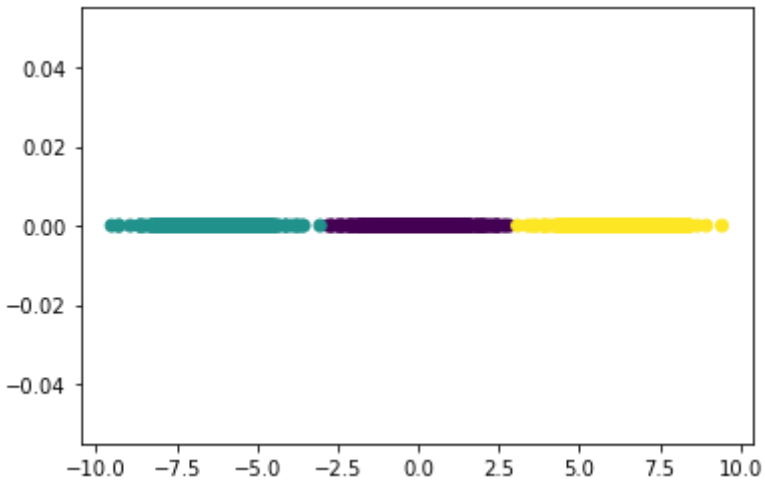






In [ ]:
```
# after projection
w = LDA(data,label)
print(w.shape)
plt.figure()
plt.scatter(data @ w[:,0],np.zeros(data.shape[0]),c = label) # by performing
```

Out[ ]:
```
(2, 2)
<matplotlib.collections.PathCollection at 0x7fbfd3ca73d0>
```



In [ ]:
```
# Testing (using KNN)
# Use k-nearest neighbour classifier (Scikit Learn) after dimensionality red

from sklearn.neighbors import KNeighborsClassifier

k = 5
knn = KNeighborsClassifier(n_neighbors = k)
knn.fit(data @ w,label)

print(f"KNN Training accuracy  = {knn.score(data @ w,label)*100}")

# testing
# data preparation
mean1 = np.array([0,0])
mean2 = np.array([4,5])
mean3 = np.array([-5,-4])
var = np.array([[1,0.1],[0.1,1]])
np.random.seed(1)
data1 = np.random.multivariate_normal(mean1,var,100)
data2 = np.random.multivariate_normal(mean2,var,100)
data3 = np.random.multivariate_normal(mean3,var,100)
```

```
data_tst = np.concatenate((data1,data2,data3))
label_tst = np.concatenate((np.zeros(data1.shape[0]),np.ones(data2.shape[0])
print(f"KNN Testing accuracy  = {knn.score(data_tst @ w,label_tst)*100}")
```

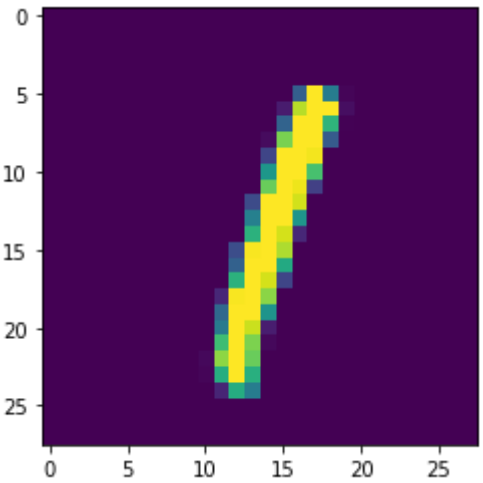
```
KNN Training accuracy  = 99.93333333333332
KNN Testing accuracy  = 100.0
```

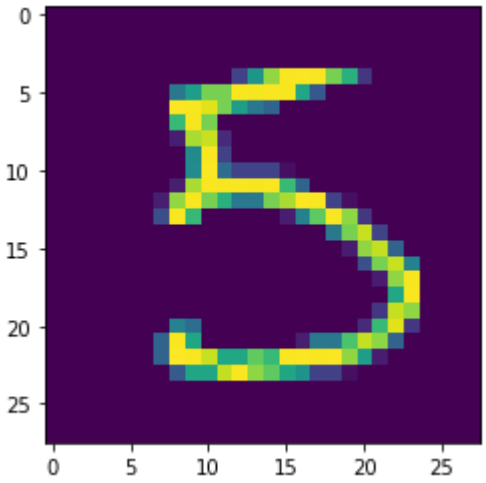
Perform LDA on MNIST and Classify using the data of any 3 classes

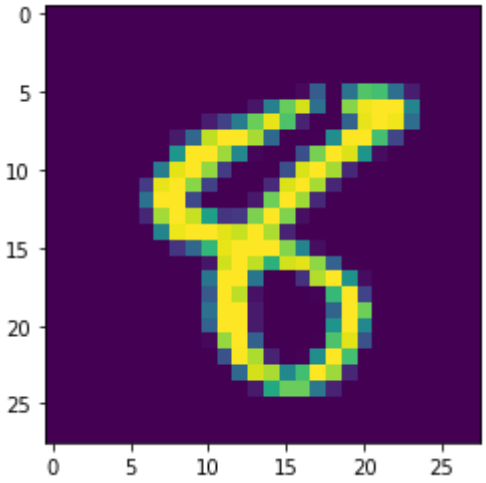
In [ ]:

```
# Prepare data for classification
classes = [1, 5, 8]

# for class 1
id_1 = np.where(labels == classes[0])[0][:50]
Im_1 = Images[id_1,:,:]
lab_1 = labels[id_1]

# for class 5
id_5 = np.where(labels == classes[1])[0][:50]
Im_5 = Images[id_5,:,:]
lab_5 = labels[id_5]

# for class 8
id_8 = np.where(labels == classes[2])[0][:50]
Im_8 = Images[id_8,:,:]
lab_8 = labels[id_8]

# concatenate the data
data = np.concatenate((Im_1,Im_5,Im_8))
data = np.reshape(data,(data.shape[0],data.shape[1]*data.shape[2]))
print(data.shape)
G_lab = np.concatenate((lab_1,lab_5,lab_8))
print(G_lab.shape)

# normalize the data
data = data.astype('float32')
data /= 255

# Show the data
plt.imshow(Im_1[1,:,:])
plt.figure()
plt.imshow(Im_5[1,:,:])
plt.figure()
plt.imshow(Im_8[1,:,:])
plt.show()
```

```
(150, 784)
(150,)
```







In [ ]:

```
# LDA
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis

lda = LinearDiscriminantAnalysis(n_components=2)
lda.fit(data,G_lab)

# KNN
k = 5
knn = KNeighborsClassifier(n_neighbors = k)
knn.fit(lda.transform(data), G_lab)

print(f"KNN Training accuracy  = {knn.score(lda.transform(data),G_lab)*100}"

# testing
# data preparation
# for class 1
id_1 = np.where(labels == classes[0])[0][100:150]
Im_1 = Images[id_1,:,:]
lab_1 = labels[id_1]

# for class 5
id_5 = np.where(labels == classes[1])[0][100:150]
Im_5 = Images[id_5,:,:]
lab_5 = labels[id_5]

# for class 8
id_8 = np.where(labels == classes[2])[0][100:150]
Im_8 = Images[id_8,:,:]
lab_8 = labels[id_8]

# concatenate the data
data_tst = np.concatenate((Im_1,Im_5,Im_8))
```

```
data_tst = np.reshape(data_tst,(data_tst.shape[0],data_tst.shape[1]*data_tst

tst_lab = np.concatenate((lab_1,lab_5,lab_8))

print(f"KNN Testing accuracy  = {knn.score(lda.transform(data_tst),tst_lab)*
```

```
KNN Training accuracy  = 100.0
KNN Testing accuracy  = 72.0
```