LAB 7 : Regression Part 2

In this Lab we will look into the shortcomings of Linear Regression and see how those problems can be solved using Logistic Regression. We will also explore Polynomian Regression

1. Polynomial Regression
2. Linear Regression on a specific pattern of data to observe shortcomings
3. Logistic Regression to solve those problems

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt
```

Polynomial Regression

1. Generate data using relation $y = 0.25x^3 + 1.25x^2 - 3x - 3$
2. Corrupt y by adding random noise (uniformly sampled)
3. Fit the generated curve using different polynomial order. (Using matrix inversion and gradient descent)

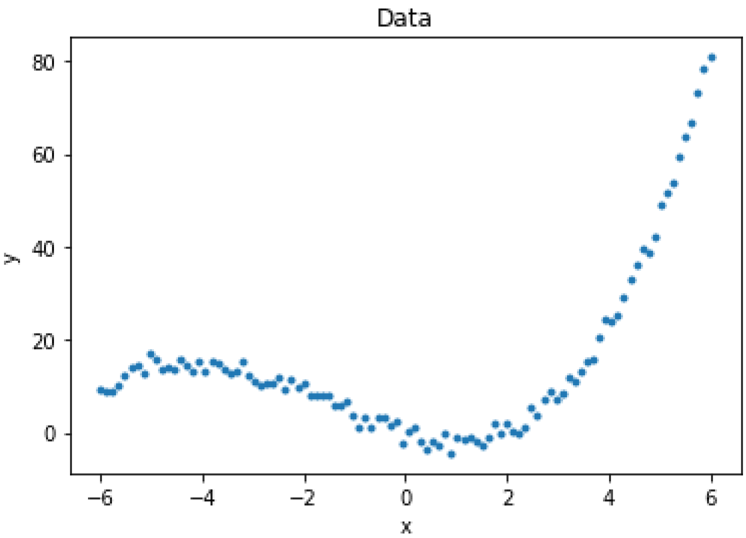
In [ ]:

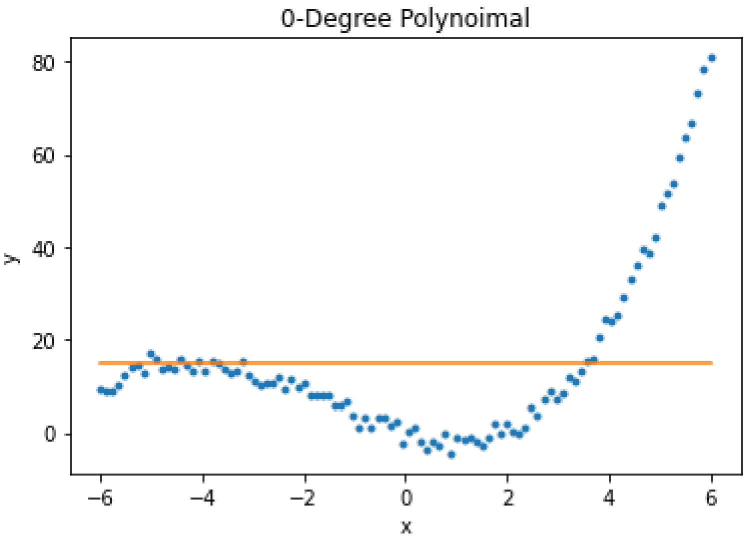
```
# Use the Regression class defined in the previous lab

class Regression:
        # Constructor
        def __init__(self, name='reg'):
                self.name = name  # Create an instance variable

        def grad_update(self,w_old,lr,y,x):
                w = w_old + 2 * lr / y.shape[0] * (x @ (y - x.T @ w_old))
                return w

        def error(self,w,y,x):
                return np.mean((y - x.T @ w)**2)

        def mat_inv(self,y,x_aug):
                return np.linalg.inv(x_aug @ x_aug.T) @ x_aug @ y

        # By Gradien descent

        def grad_descent(self,x,y,lr):
                err = []
                w_pred = np.random.uniform(-1, 1, (x.shape[0]))

                for i in range(int(1e+20)):
                        w_pred = self.grad_update(w_pred,lr,y,x)
                        err.append(self.error(w_pred,y,x))
                        if i > 1:
                                dev = np.abs(err[-2] - err[-1])
                        else:
                                dev = 1
                        if dev<=0.000001:
                                break
```

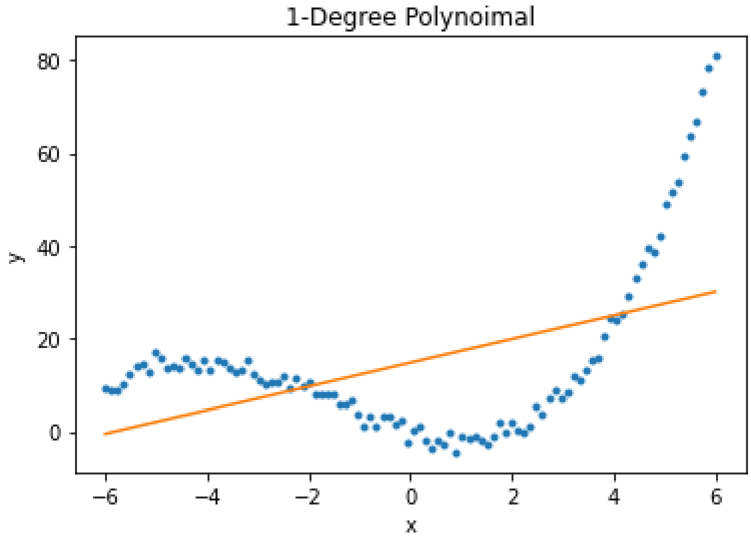
```
        return w_pred, err
```

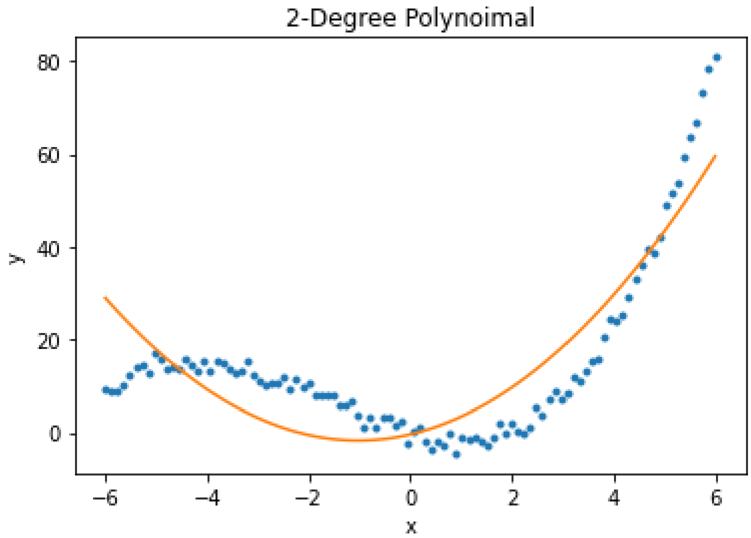
In [ ]:
```
# Data generation

x=np.linspace(-6,6,100)
x=x[np.newaxis,:]

# Define Weights as per the given equation
w = np.array([-3, -3, 1.25, 0.25]).T                    # w = [w0, w1, ...,

## Function to transform the data into polynomial

def data_transform(X,degree):
        # X = [x1, x2, ..., xn] 1xn
        X_new = np.array([np.squeeze(X)**i for i in range(degree+1)])

        # X_new = [[x0^0, x1^0, ..., xn^0], [x0^1, x1^1, ..., xn^1], ..., [

        return X_new


X = data_transform(x,3)

y = X.T @ w             # nx1

y = y+5*np.random.uniform(0,1,y.shape)

plt.plot(x.T,y,'.')
plt.title('Data')
plt.xlabel('x')
plt.ylabel('y')
plt.show()

reg=Regression()

# By computation

# Code for degree 0 polynomial fitting

degree = 0
X_1 = data_transform(x,degree)
w_mat=reg.mat_inv(y,X_1)
y_pred=X_1.T @ w_mat
plt.figure()
plt.plot(x.T,y,'.')
plt.plot(x.T,y_pred)
plt.title('0-Degree Polynoimal')
plt.xlabel('x')
plt.ylabel('y')
plt.show()

def polyComputationFit(x,y,degree):
        X_1 = data_transform(x,degree)
        w_mat=reg.mat_inv(y,X_1)
        y_pred=X_1.T @ w_mat
        plt.figure()
        plt.plot(x.T,y,'.')
        plt.plot(x.T,y_pred)
        plt.title(str(degree)+'-Degree Polynoimal')
        plt.xlabel('x')
        plt.ylabel('y')
        plt.show()
```

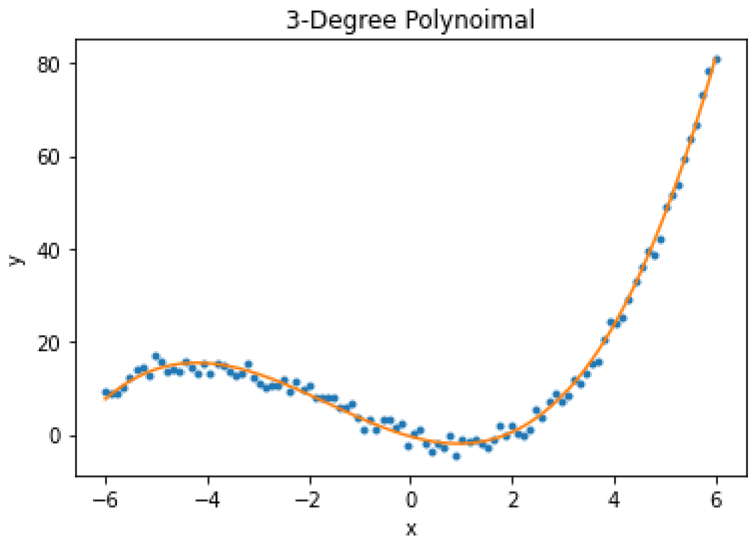
```
# Write the code for degree 1 polynomial fitting
polyComputationFit(x,y,1)

# Write the code for degree 2 polynomial fitting
polyComputationFit(x,y,2)

# Write the code for degree 3 polynomial fitting
polyComputationFit(x,y,3)

# Write the code for degree 4 polynomial fitting
polyComputationFit(x,y,4)
```

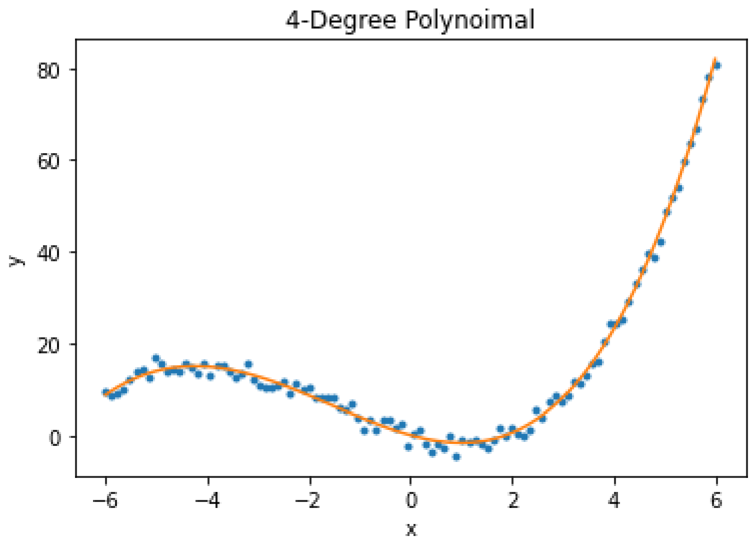





1-Degree Polynoimal
80
60
40
20
0
y
−6
−4
−2
0
2
4
6
x

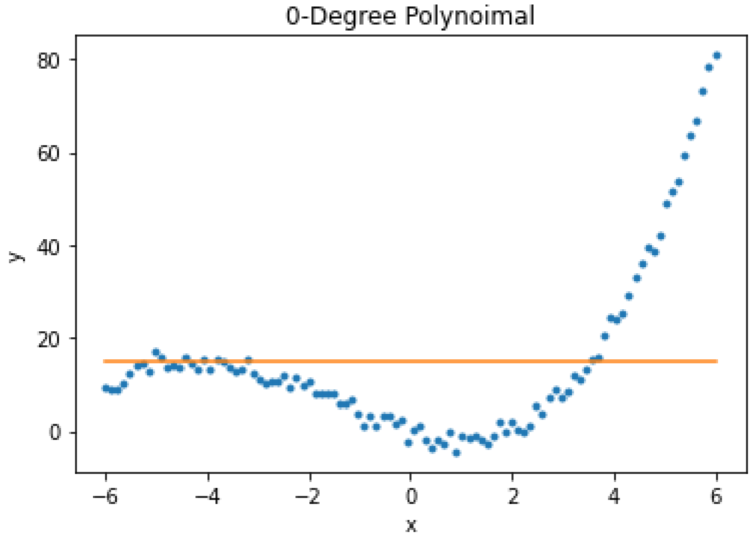

2-Degree Polynoimal
80
60
40
20
0
y
−6
−4
−2
0
2
4
6
x

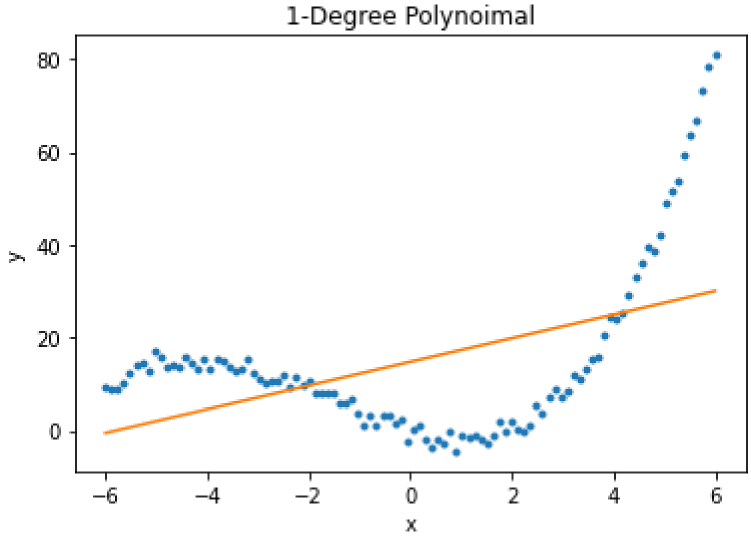

3-Degree Polynoimal
80
60
40
20
0
y
−6
−4
−2
0
2
4
6
x

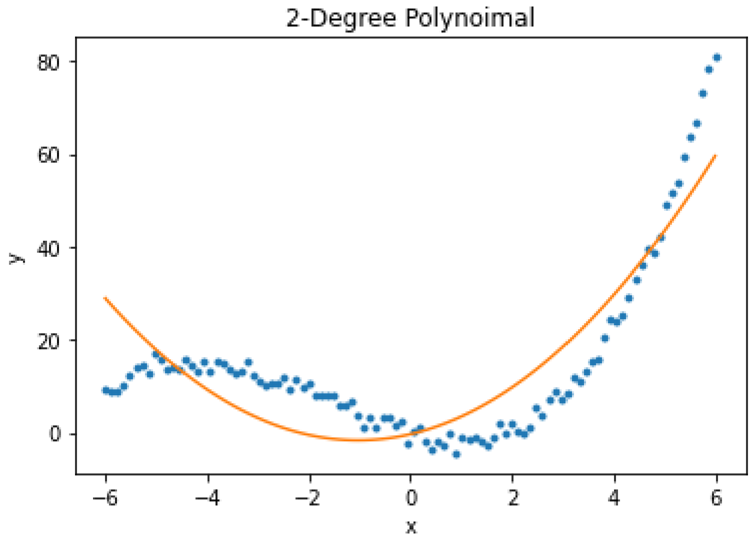


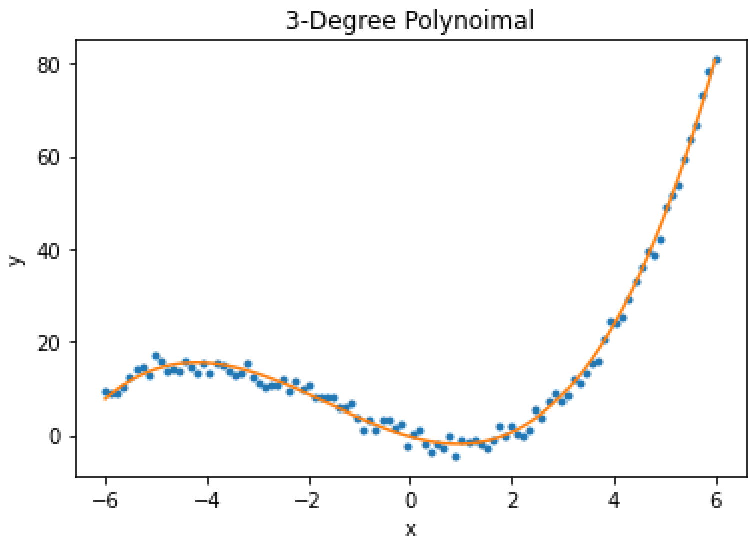
In [ ]:

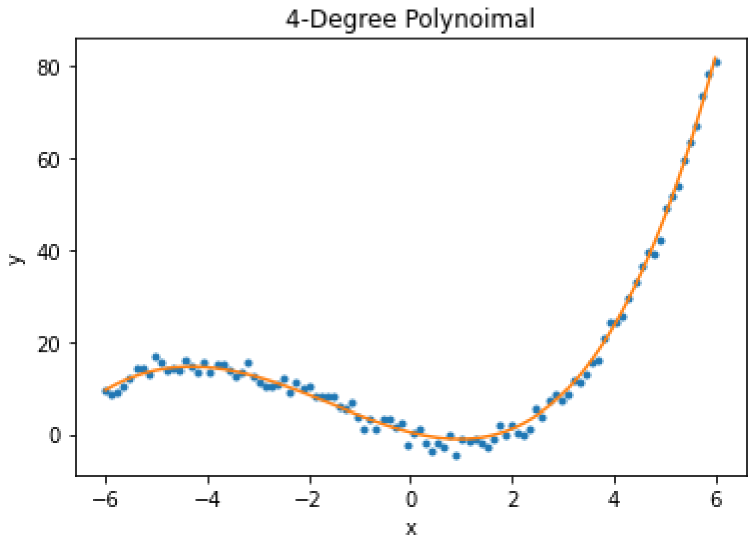
```
# By Gradient Descent

def polyGradientFit(x, y, degree, lr):
        X_1 = data_transform(x, degree)
        w_pred, err = reg.grad_descent(X_1, y, lr)
        y_pred=X_1.T @ w_pred
        plt.figure()
        plt.plot(x.T,y,'.')
        plt.plot(x.T,y_pred)
        plt.title(str(degree)+'-Degree Polynoimal')
        plt.xlabel('x')
        plt.ylabel('y')
        plt.show()

# Write the code for degree 0 polynomial fitting
polyGradientFit(x,y,0,1e-3)

# Write the code for degree 1 polynomial fitting
polyGradientFit(x,y,1,1e-3)

# Write the code for degree 2 polynomial fitting
polyGradientFit(x,y,2,1e-3)

# Write the code for degree 3 polynomial fitting
polyGradientFit(x,y,3,1e-4)

# Write the code for degree 4 polynomial fitting
polyGradientFit(x,y,4,1e-6)
```


0-Degree Polynoimal
y
x


1-Degree Polynoimal
y
x


2-Degree Polynoimal
y
x

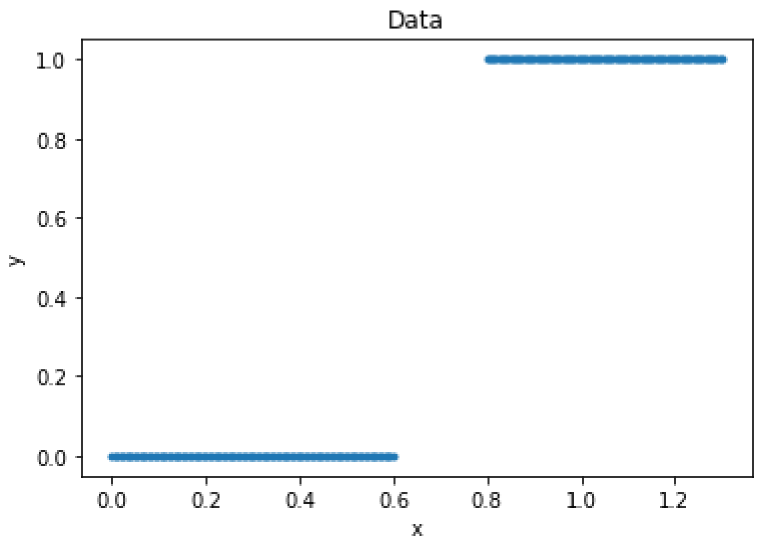




Linear Regression

Generate the data as shown in the figure below

In [ ]:
```
x1 = np.linspace(0, 0.6, 100)
x2 = np.linspace(0.8, 1.3, 100)
x = np.concatenate((x1, x2))
y = np.concatenate((np.zeros(100), np.ones(100)))

plt.plot(x, y, '.')
plt.title('Data')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```



Use the Regression class defined in the previous lab to fit the curve

In [ ]:
```
class Regression:
        # Constructor
        def __init__(self, name='reg'):
                self.name = name  # Create an instance variable

        def grad_update(self,w_old,lr,y,x):
                w = w_old + 2 * lr / y.shape[0] * (x @ (y - x.T @ w_old))
                return w

        def error(self,w,y,x):
                return np.mean((y - x.T @ w)**2)

        def mat_inv(self,y,x_aug):
                return np.linalg.inv(x_aug @ x_aug.T) @ x_aug @ y

        # By Gradien descent

        def grad_descent(self,x,y,lr):
                err = []
                w_pred = np.random.uniform(-1, 1, (x.shape[0]))

                for i in range(int(1e+10)):
                        w_pred = self.grad_update(w_pred,lr,y,x)
                        err.append(self.error(w_pred,y,x))
                        if i > 1:
                                dev = np.abs(err[-2] - err[-1])
                        else:
                                dev = 1
                        if dev<=0.000001:
                                break

                return w_pred, err
```

Augment the Data and generate optimal weights

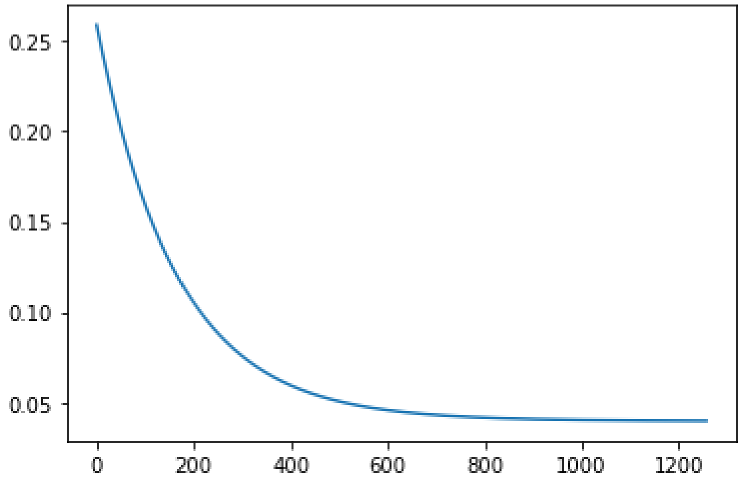
In [ ]:
```
# Augment the data
_x = x[np.newaxis,:]
print(f"Shape of x: {_x.shape}")

x_aug = np.vstack((np.ones((1,_x.shape[1])), _x))
print(f"Shape of x_aug: {x_aug.shape}")
```

```
Shape of x: (1, 200)
Shape of x_aug: (2, 200)
```

In [ ]:

```
# Generate optimal weights using gradient descent
reg = Regression()
w_pred, err = reg.grad_descent(x_aug, y, 1e-3)
print(f"Optimal weights: {w_pred}")

# Plot the error
plt.plot(err)
plt.show()
```

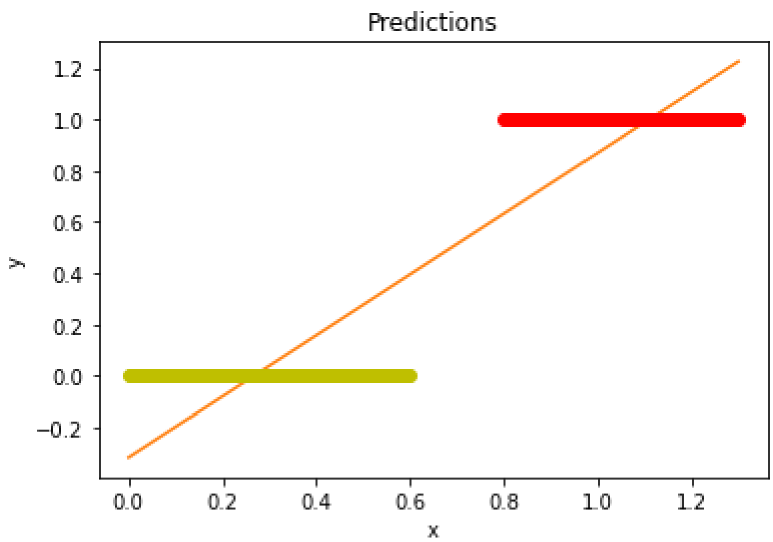
```
Optimal weights: [-0.31727127  1.1874253 ]
```



Using the optimal weights, fit the curve

In [ ]:

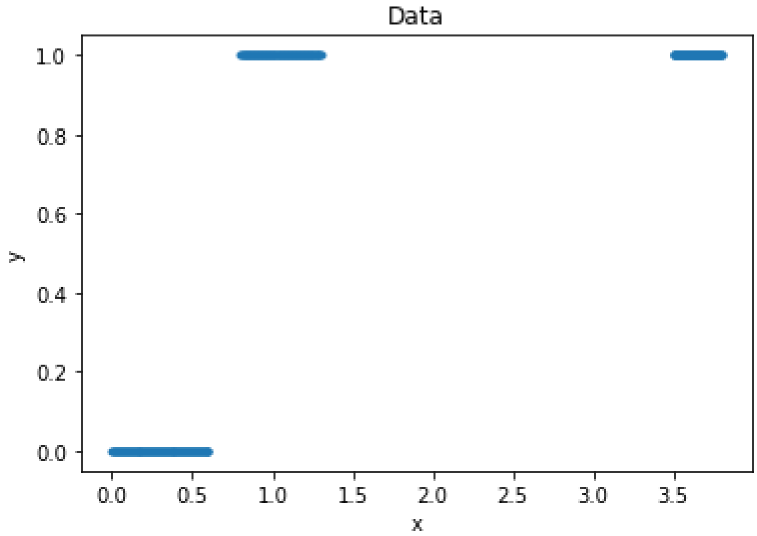
```
# Plot the data, predictions and the optimal line
plt.plot(x, y, '.')
y_pred = w_pred[0] + w_pred[1] * x
plt.plot(x,  y_pred, '-')
pred_zero = np.where(y_pred < 0.5)
pred_one = np.where(y_pred >= 0.5)
plt.plot(x[pred_zero], np.zeros(x[pred_zero].shape[0]), 'oy')
plt.plot(x[pred_one], np.ones(x[pred_one].shape[0]), 'or')
plt.title('Predictions')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```



Drawback of Linear regression based Classificaton

Generate the Data as shown in the figure and follow the same steps as above to fit a curve using regression class

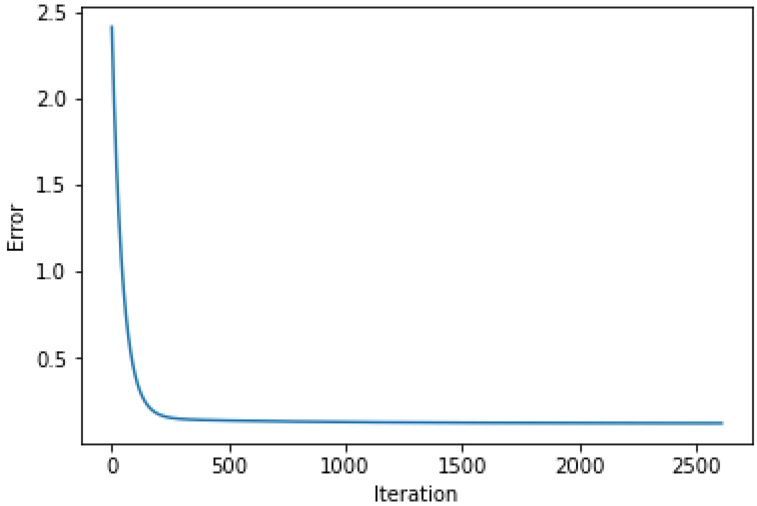
In [ ]:
```
x1 = np.linspace(0, 0.6, 100)
x2 = np.linspace(0.8, 1.3, 100)
x3 = np.linspace(3.5, 3.8, 100)
x = np.concatenate((x1, x2, x3))
y = np.concatenate((np.zeros(100), np.ones(100), np.ones(100)))

plt.plot(x, y, '.')
plt.title('Data')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

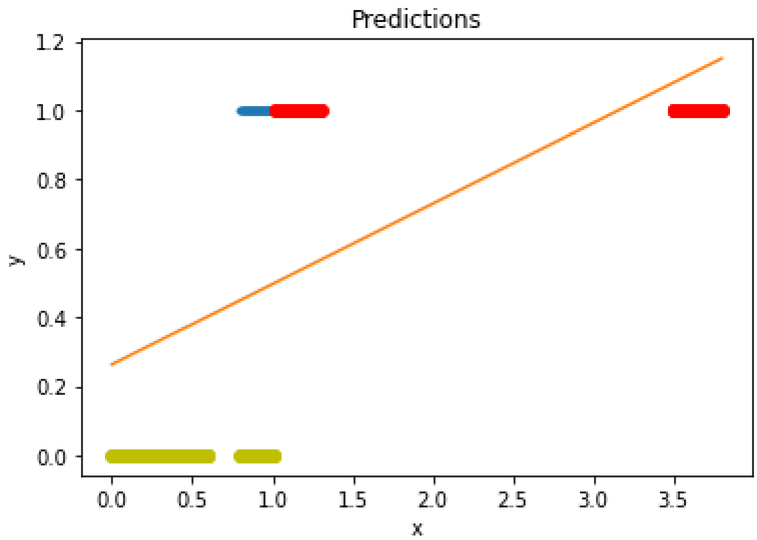


In [ ]:
```
# Augment the data
_x = x[np.newaxis,:]
print(f"Shape of x: {_x.shape}")

x_aug = np.vstack((np.ones((1,_x.shape[1])), _x))
print(f"Shape of x_aug: {x_aug.shape}")

# Generate optimal weights using gradient descent
reg = Regression()
w_pred, err = reg.grad_descent(x_aug, y, 1e-3)
print(f"Optimal weights: {w_pred}")

# Plot the error
plt.plot(err)
plt.xlabel('Iteration')
plt.ylabel('Error')
plt.show()

# Plot the data and the optimal line
plt.plot(x, y, '.')
y_pred = w_pred[0] + w_pred[1] * x
plt.plot(x,  y_pred, '-')
pred_zero = np.where(y_pred < 0.5)
pred_one = np.where(y_pred >= 0.5)
plt.plot(x[pred_zero], np.zeros(x[pred_zero].shape[0]), 'oy')
```

```
plt.plot(x[pred_one], np.ones(x[pred_one].shape[0]), 'or')
plt.title('Predictions')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

```
Shape of x: (1, 300)
Shape of x_aug: (2, 300)
Optimal weights: [0.2627255  0.23348291]
```





Logistic regression

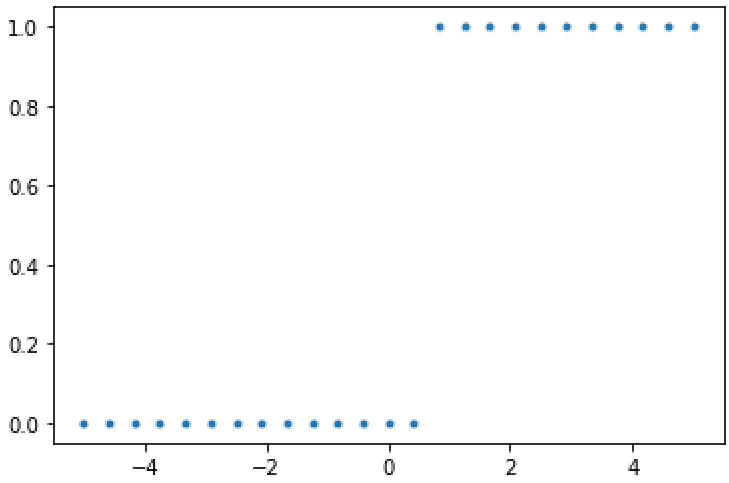
Error Surface (Comparison between Logistic Loss and Mean Squared Error)

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt


x=np.linspace(-5,5,25)
y=np.zeros(x.shape)
y[np.where(x>0.7314)]=1

plt.plot(x,y,'.')
```

Out[ ]:

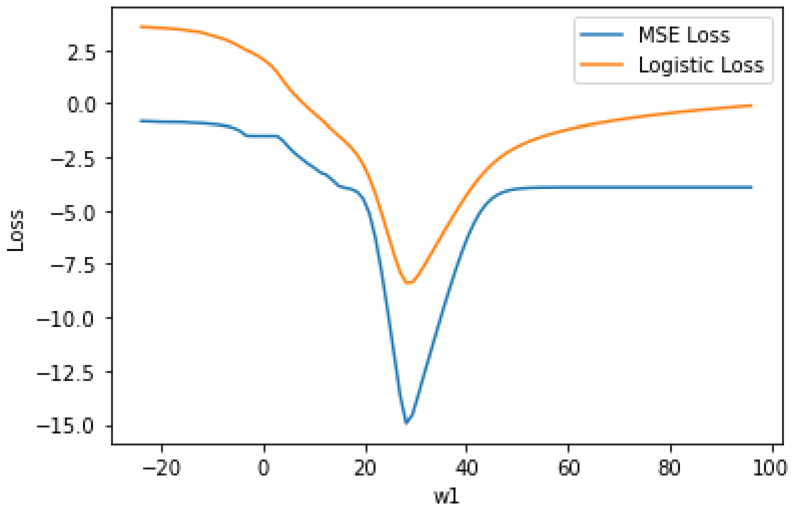
```
[<matplotlib.lines.Line2D at 0x7fcd24ba9420>]
```



1. MSE=$\frac{1}{2N}\sum_{i=1}^{N}(y_i^p - y_i)^2$, where $y^p = \frac{1}{1+e^{-w^Tx}}$
2. Logistic loss=$-\frac{1}{N}\sum_{i=1}^{N} y_i log(y_i^p) + (1-y_i)log(1-y_i^p)$

In [ ]:
```
# search space (only w1 is searched, where as w0 is fixed)
w1_in=10/(x[1]-x[0])
w0=-w1_in*0.7314
w1=np.linspace(-w1_in,4*w1_in,100)


cost_fn_mse=[]
cost_fn_logis=[]
for i in range(w1.shape[0]):

        # Compute Mean square error and logistic loss using cost function
        y_pred = (1 + np.exp(-w0 - w1[i] * x))**(-1)
        cost_fn_mse.append(np.mean((y - y_pred)**2)/2)
        cost_fn_logis.append(-np.mean(y * np.log(y_pred+1e-20) + (1 - y) * r
```

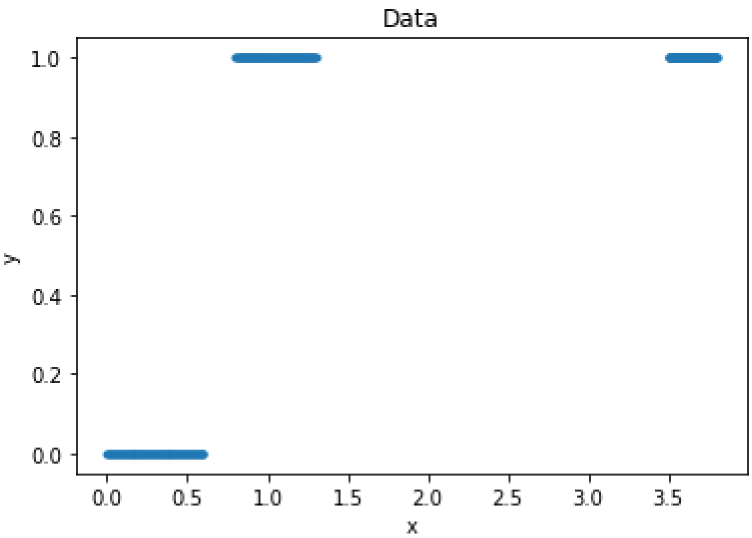
In [ ]:
```
# Ploting of error surface
plt.figure()
plt.plot(w1,np.log(cost_fn_mse),label='MSE Loss')
plt.plot(w1,np.log(cost_fn_logis),label = 'Logistic Loss')
plt.legend()
plt.xlabel('w1')
plt.ylabel('Loss')
plt.show()
```



Solving the Outlier Issue

Generate the Data as shown in the figure

```
In [ ]: x1 = np.linspace(0, 0.6, 100)
        x2 = np.linspace(0.8, 1.3, 100)
        x3 = np.linspace(3.5, 3.8, 100)
        x = np.concatenate((x1, x2, x3))
        y = np.concatenate((np.zeros(100), np.ones(100), np.ones(100)))

        plt.plot(x, y, '.')
        plt.title('Data')
        plt.xlabel('x')
        plt.ylabel('y')
        plt.show()
```



Define a Logistic Regression class

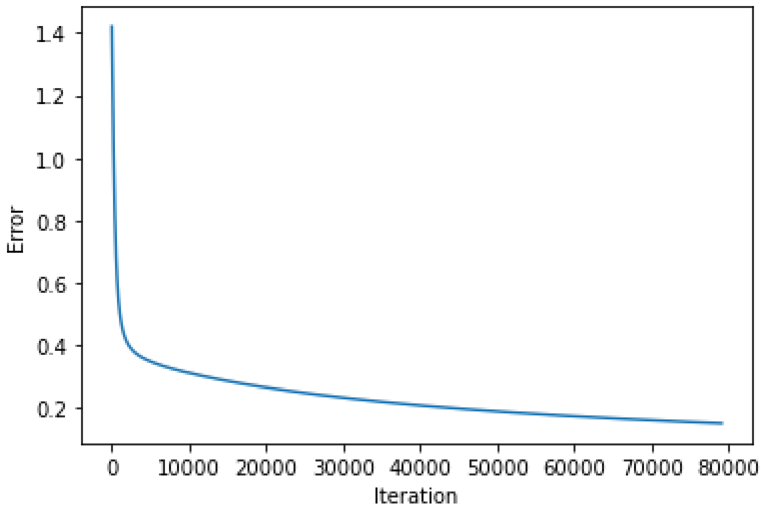
```
In [ ]: class logis_regression:
                # Constructor
                def __init__(self, name='reg'):
                        self.name = name  # Create an instance variable

                def logis(self,x,w_old):
                        op = 1/(1+np.exp(-x.T@w_old))
                        return op

                def grad_update(self,w_old,lr,y,x):
                        w = w_old + lr / y.shape[0] * (x @ (y - self.logis(x,w_old)
                        return w

                def error(self,w,y,x):
                        y_pred = self.logis(x,w)
                        return -np.mean(y * np.log(y_pred+1e-40) + (1 - y) * np.log

                def grad_descent(self,x,y,lr):
                        err = []
                        w_pred = np.random.uniform(-1, 1, (x.shape[0]))

                        for i in range(int(1e+10)):
                                w_pred = self.grad_update(w_pred,lr,y,x)
                                err.append(self.error(w_pred,y,x))
                                if i > 1:
                                        dev = np.abs(err[-2] - err[-1])
```

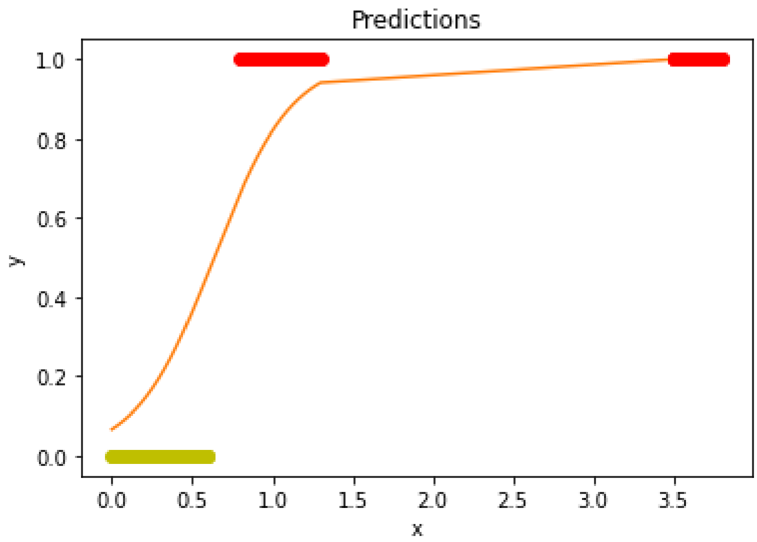
```
                        else:
                                dev = 1
                        if dev<=0.000001:
                                break

                return w_pred, err
```

Augment the data and fit the curve by obtaining optimal weights (Using Gradient Descent)

In [ ]:
```
# Augment the data
_x = x[np.newaxis,:]
print(f"Shape of x: {_x.shape}")

x_aug = np.vstack((np.ones((1,_x.shape[1])), _x))
print(f"Shape of x_aug: {x_aug.shape}")

print(f"Shape of y: {y.shape}")

# Generate optimal weights using gradient descent
reg = logis_regression()
w_pred, err = reg.grad_descent(x_aug, y, 1e-3)
print(f"Optimal weights: {w_pred}")

# Plot the error
plt.plot(err)
plt.xlabel('Iteration')
plt.ylabel('Error')
plt.show()

# Plot the data and the optimal line
plt.plot(x, y, '.')
y_pred = reg.logis(x_aug,w_pred)
plt.plot(x,  y_pred)
pred_zero = np.where(y_pred < 0.5)
pred_one = np.where(y_pred >= 0.5)
plt.plot(x[pred_zero], np.zeros(x[pred_zero].shape[0]), 'oy')
plt.plot(x[pred_one], np.ones(x[pred_one].shape[0]), 'or')
plt.title('Predictions')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

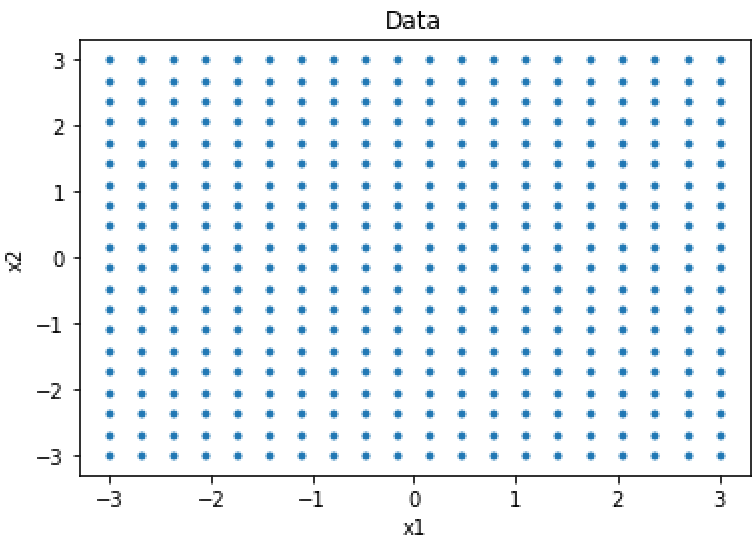
```
Shape of x: (1, 300)
Shape of x_aug: (2, 300)
Shape of y: (300,)
Optimal weights: [-2.63829824  4.15240555]
```





Classification of circularly separated data using logistic regression

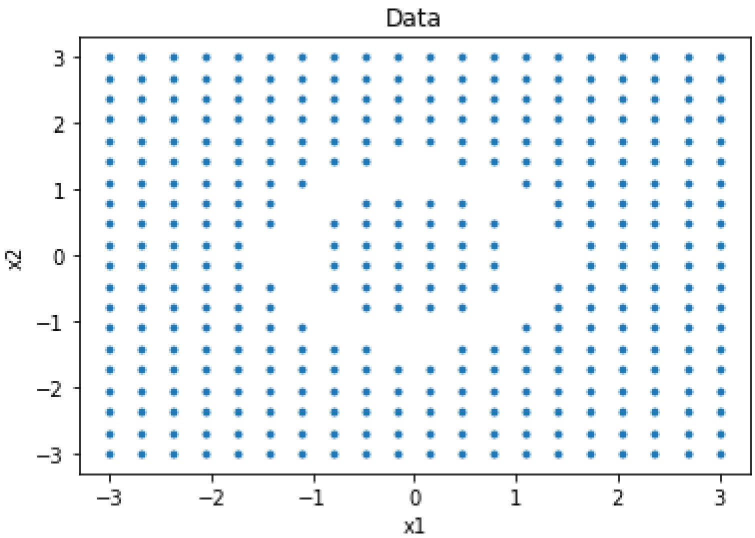
In [ ]:
```
x1=np.linspace(-3,3,20)
x2=np.linspace(-3,3,20)

x11,x22=np.meshgrid(x1,x2)
plt.plot(np.reshape(x11,(-1,1)),np.reshape(x22,(-1,1)),'.')
plt.title('Data')
plt.xlabel('x1')
plt.ylabel('x2')
plt.show()
```



Using the above data generate circular data

In [ ]:
```
x = np.hstack((x11.reshape(-1,1),x22.reshape(-1,1)))
in_points = np.where((x[:,0]**2+x[:,1]**2)<=1)
out_points = np.where((x[:,0]**2+x[:,1]**2)>=2.1)
in_points = x[in_points]
out_points = x[out_points]
points = np.concatenate((in_points,out_points))

plt.plot(points[:,0],points[:,1],'.')
plt.title('Data')
```

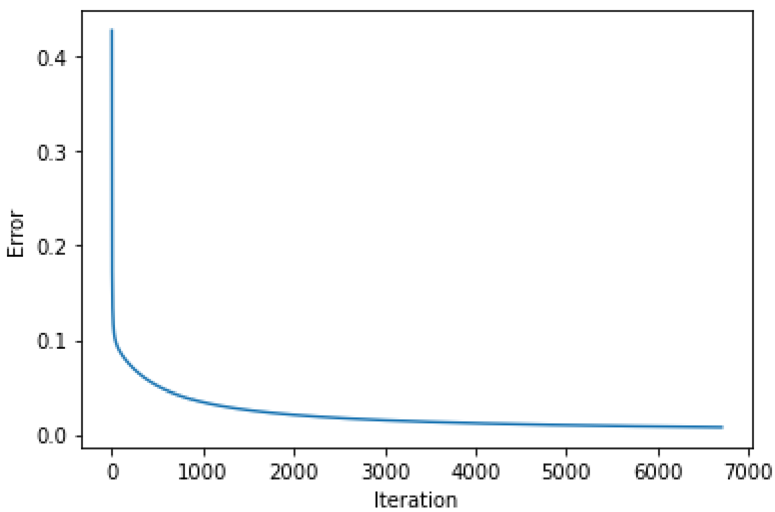
```
plt.xlabel('x1')
plt.ylabel('x2')
plt.show()
```



As in case of circularly separated data, the boundary is nonlinear, so squared feature is taken.

```
In [ ]: # perform logistic regression
reg = logis_regression()

x = np.hstack((np.ones((points.shape[0],1)),points**2)).T
print(f"Shape of x: {x.shape}")
y = np.concatenate((np.ones(in_points.shape[0]),np.zeros(out_points.shape[0]
print(f"Shape of y: {y.shape}")

w_pred, err = reg.grad_descent(x, y, 1e-1)
print(f"Optimal weights: {w_pred}")

# Plot the error
plt.plot(err)
plt.xlabel('Iteration')
plt.ylabel('Error')
plt.show()
```

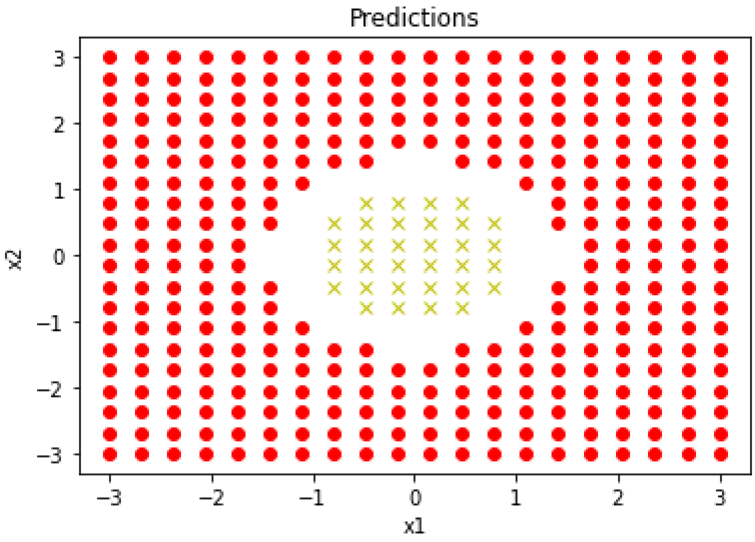
```
Shape of x: (3, 364)
Shape of y: (364,)
Optimal weights: [ 4.77626213 -3.28661612 -3.28681526]
```



Plot classification using 0.5 as threshold

In [ ]:

```
# Plot the classification using 0.5 threshold
y_pred = reg.logis(x,w_pred)
pred_zero = np.where(y_pred < 0.5)
pred_one = np.where(y_pred >= 0.5)
plt.plot(points[pred_zero,0], points[pred_zero,1], 'or')
plt.plot(points[pred_one,0], points[pred_one,1], 'xy')
plt.title('Predictions')
plt.xlabel('x1')
plt.ylabel('x2')
plt.show()
```



Multiclass logistic regression

1. Generate 1D data with 3 classes

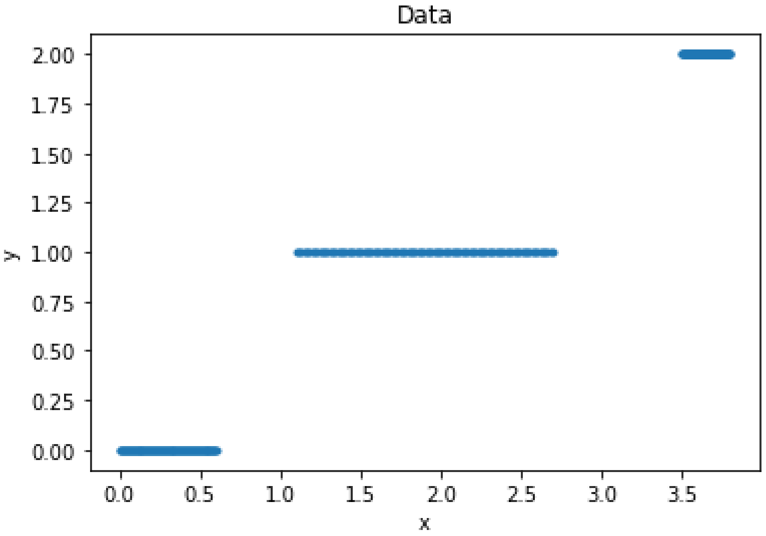
One vs rest classification

1. Lets take a polynomial of order 2 (by seeing the data distribution)

In [ ]:

```
## Write your code here

import numpy as np
import matplotlib.pyplot as plt

x1=np.linspace(0,0.6,100)
x2=np.linspace(1.1,2.7,100)
x3=np.linspace(3.5,3.8,100)

x=np.concatenate((x1,x2,x3))

y1=np.zeros(x1.shape)
y2=np.ones(x2.shape)
y3=np.tile([2],x3.shape)

y=np.concatenate((y1,y2,y3))

plt.figure()
plt.plot(x,y,'.')
plt.title('Data')
plt.xlabel('x')
```

```
plt.ylabel('y')
plt.show()
```



In [ ]:
```
# def data_transform(X,degree):
#   X_new=[]
#   for i in range(degree +1):
#     # write code here to generate a polynomial

def data_transform(X,degree):
  X_new=[]
  for i in range(degree +1):
    X_new.append(X**i)
  X_new = np.concatenate(X_new)
  return X_new

x_aug=data_transform(x[np.newaxis,:],2)
```

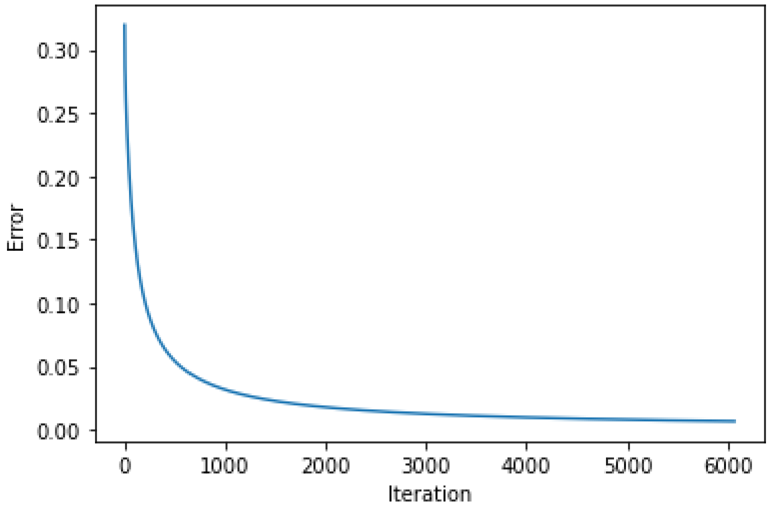
In [ ]:
```
# plot for classification
def plot_op(x,y_pred):

  ind0=np.where(y_pred<0.5)
  ind1=np.where(y_pred>=0.5)
  x0=x[ind0]
  x1=x[ind1]
  plt.plot(x0,np.zeros((x0).shape),'o',color='y')
  plt.plot(x1,np.ones((x1).shape),'x',color='r')
```

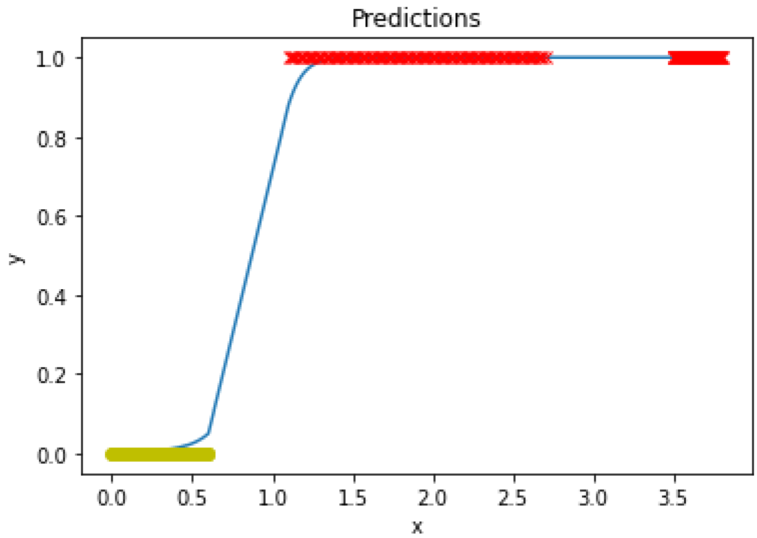
Using the above function for plotting, plot the curve using different configurations

In [ ]:
```
# take class 0 as '0' and other to '1'
zeros = np.where(y==0)
ones = np.where(y!=0)

ys = np.zeros(y.shape)
ys[ones] = 1

# perform logistic regression
reg = logis_regression()

w_pred, err = reg.grad_descent(x_aug, ys, 1e-1)
print(f"Optimal weights: {w_pred}")

# Plot the error
plt.plot(err)
```

```
plt.xlabel('Iteration')
plt.ylabel('Error')
plt.show()

# Plot the classification using 0.5 threshold
y_pred1 = reg.logis(x_aug,w_pred)
plt.plot(x,y_pred1)
plot_op(x,y_pred1)
plt.title('Predictions')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

```
Optimal weights: [-5.84700167  2.12841014  4.54526317]
```



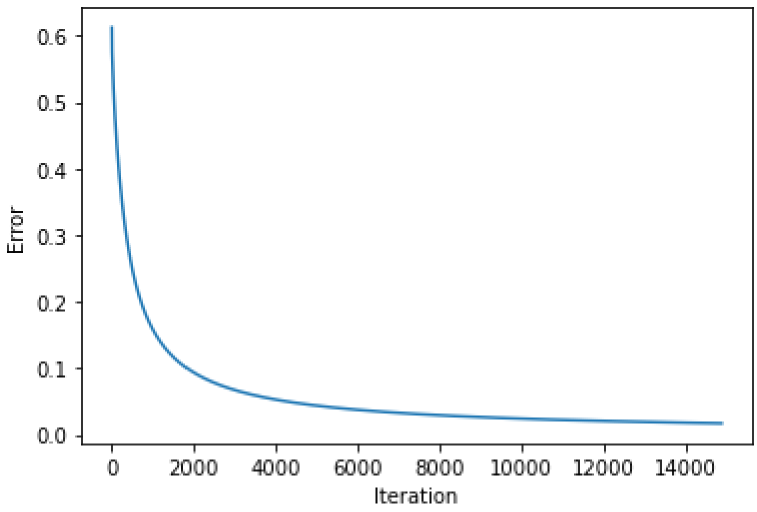


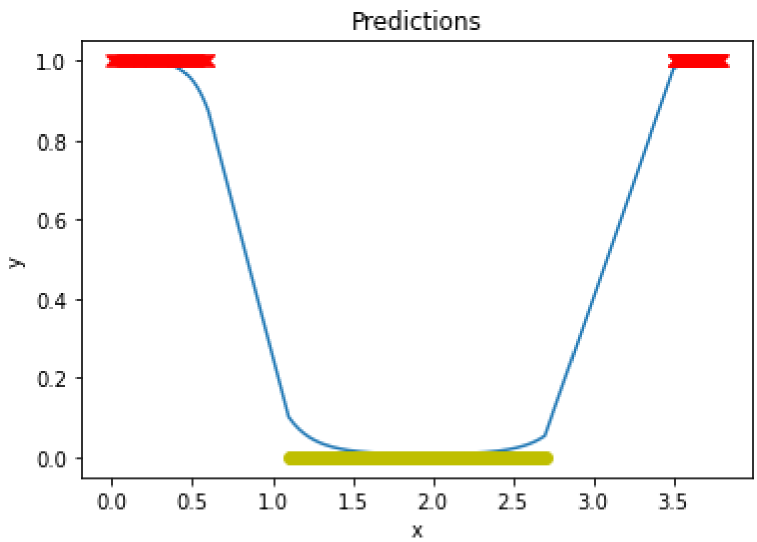
In [ ]:

```
# take class 1 as '0' and other to '1'
zeros = np.where(y==1)
ones = np.where(y!=1)

ys = np.zeros(y.shape)
ys[ones] = 1

# perform logistic regression
reg = logis_regression()

w_pred, err = reg.grad_descent(x_aug, ys, 1e-1)
print(f"Optimal weights: {w_pred}")

# Plot the error
plt.plot(err)
```

```
plt.xlabel('Iteration')
plt.ylabel('Error')
plt.show()

# Plot the classification using 0.5 threshold
y_pred2 = reg.logis(x_aug,w_pred)
plt.plot(x,y_pred2)
plot_op(x,y_pred2)
plt.title('Predictions')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

```
Optimal weights: [  9.25096248 -14.44342077   3.687699  ]
```



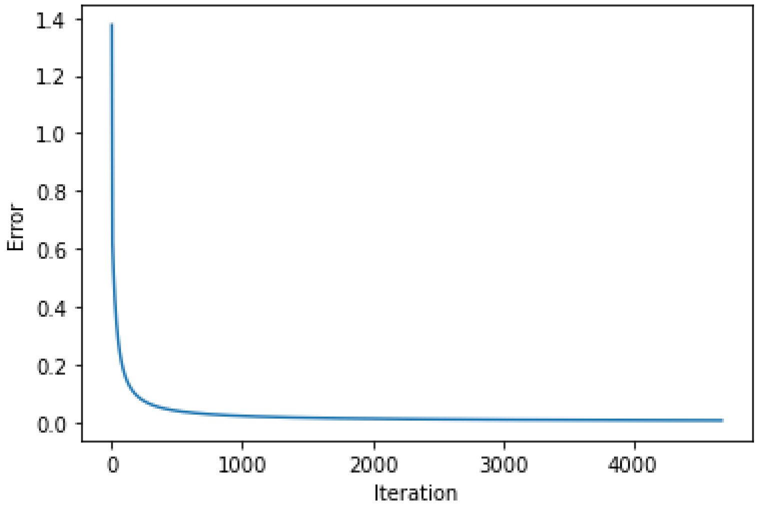


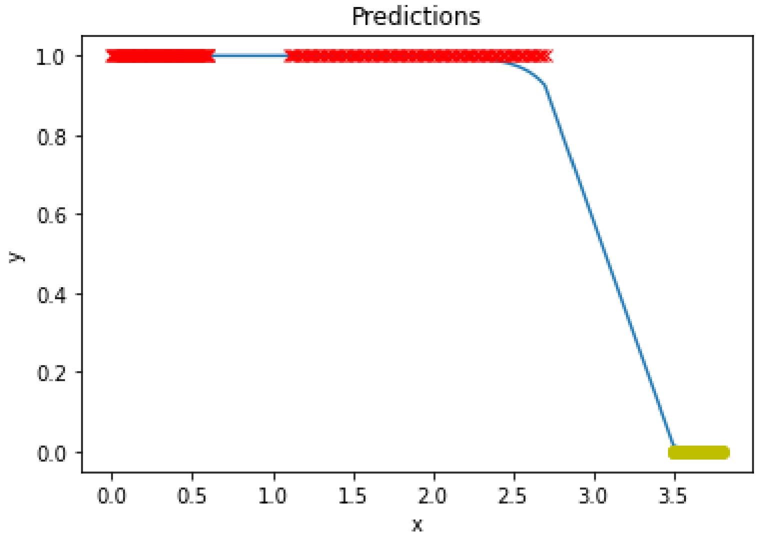
In [ ]:

```
# Take class 2 as '0' and other to '1'
zeros = np.where(y==2)
ones = np.where(y!=2)

ys = np.zeros(y.shape)
ys[ones] = 1

# perform logistic regression
reg = logis_regression()

w_pred, err = reg.grad_descent(x_aug, ys, 1e-1)
print(f"Optimal weights: {w_pred}")

# Plot the error
plt.plot(err)
```

```
plt.xlabel('Iteration')
plt.ylabel('Error')
plt.show()

# Plot the classification using 0.5 threshold
y_pred3 = reg.logis(x_aug,w_pred)
plt.plot(x,y_pred3)
plot_op(x,y_pred3)
plt.title('Predictions')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```

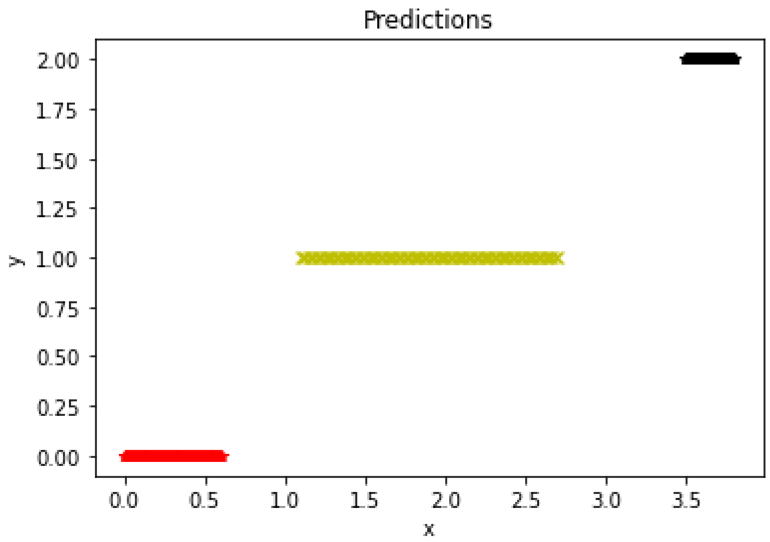
```
Optimal weights: [ 5.00985644  4.83691527 -2.13516568]
```





In [ ]:

```
# final classification
x1 = np.where(y_pred1<0.5)
x2 = np.where(y_pred2<0.5)
x3 = np.where(y_pred3<0.5)

plt.plot(x[x1],np.zeros((x[x1]).shape),'*r')
plt.plot(x[x2],np.ones((x[x2]).shape),'xy')
plt.plot(x[x3],np.tile([2],x[x3].shape),'*k')
plt.title('Predictions')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```


Predictions
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
x