Logistic Regression

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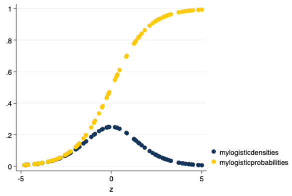
Key Concepts and Commands

- Fitting a Curve to 2 Possible Values



- Linear models, probit and logit
- `y x1 x2 ...` $\leftarrow\rightarrow F(y) = \beta_0 + \beta x_1 + \beta x_2 ...$
- `regress y x1 x2` OLS; Linear Model
- `logit y x1 x2` Logistic Regression
- `probit y x1 x2` Probit Regression
- `glm ...`

Limited Dependent Variables

- Categorical Dependent Variable
- Binary Dependent Variable
- Limited Dependent Variable

General Social Survey

```
. use "/Users/agrogan/Box Sync/DATA WAREHOUSE/General Social Survey Panel Data/GSS_panel
> 2010w123_R6 - stata.dta", clear
( )

. codebook happy_3 // what does this variable look like?
------------------------------------------------------------------------------------------
happy_3                                                           happy_3: GENERAL HAPPINESS
------------------------------------------------------------------------------------------

                  type:  numeric (byte)
```

```
                 label:  HAPPY_3

                 range:  [1,3]                        units:  1
         unique values:  3                        missing .:  0/2,044
       unique mv codes:  3                       missing .*:  742/2,044

            tabulation:  Freq.   Numeric  Label
                           391         1  VERY HAPPY
                           758         2  PRETTY HAPPY
                           153         3  NOT TOO HAPPY
                             1        .d  DK
                           740        .i  IAP
                             1        .n  NA
```

Data Management

```
. recode happy_3 (1/2 = 1)(3=0), generate(happy_3_D)
(911 differences between happy_3 and happy_3_D)

. tabulate happy_3 happy_3_D // double check

              |   RECODE of happy_3
     happy_3: |   (happy_3: GENERAL
      GENERAL |      HAPPINESS)
    HAPPINESS |         0          1 |     Total
--------------+----------------------+----------
   VERY HAPPY |         0        391 |       391 
 PRETTY HAPPY |         0        758 |       758 
NOT TOO HAPPY |       153          0 |       153 
--------------+----------------------+----------
        Total |       153      1,149 |     1,302 

. generate coninc_3_10K = coninc_3 / 10000
(820 missing values generated)

. label variable coninc_3_10K "Income 10K Chunks"

. keep happy_3 happy_3_D coninc_3 coninc_3_10K // keep only some variables

. save GSSsmall.dta, replace
file GSSsmall.dta saved
```

Visualize

```
. twoway scatter happy_3_D coninc_3, scheme(burd) jitter(5)

. graph export happiness-income.png, width(500) replace
(file happiness-income.png written in PNG format)
```

Linear Probability Model

```
. regress happy_3_D coninc_3_10K

      Source |       SS           df       MS      Number of obs   =     1,223
-------------+----------------------------------   F(1, 1221)      =     22.87
       Model |  2.26477699         1  2.26477699   Prob > F        =    0.0000
    Residual |  120.937185     1,221  .099047654   R-squared       =    0.0184
-------------+----------------------------------   Adj R-squared   =    0.0176
       Total |  123.201962     1,222  .100819936   Root MSE        =    .31472

------------------------------------------------------------------------------
   happy_3_D |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
             |
```

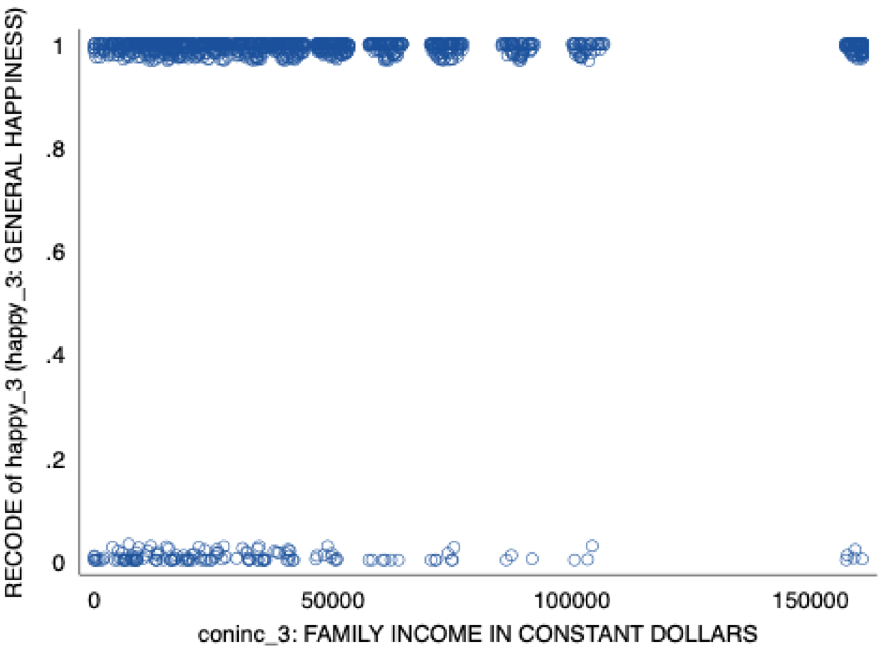


Figure 1: Happiness and Income

```
  coninc_3_10K |   .0096934    .0020272     4.78   0.000     .0057163     .0136705
         _cons |   .8368664    .0137133    61.03   0.000     .8099622     .8637706
------------------------------------------------------------------------------
```

Normal and Cumulative Normal Distribution

```
. clear all

. set obs 100 // 100 observations
number of observations (_N) was 0, now 100

. generate z = runiform(-5, 5) // randomly distributed z scores

. generate mynormaldensities = normalden(z) // normal densities

. generate myprobabilities = normal(z) // cumulative normal probabilities

. twoway scatter mynormaldensities myprobabilities z, scheme(michigan)

. graph export normal.png, width(500) replace
(file normal.png written in PNG format)
```

The Probit Model

```
. use GSSsmall.dta, clear
( )

. probit happy_3_D coninc_3_10K

Iteration 0:   log likelihood = -433.05123
Iteration 1:   log likelihood = -419.92819
```

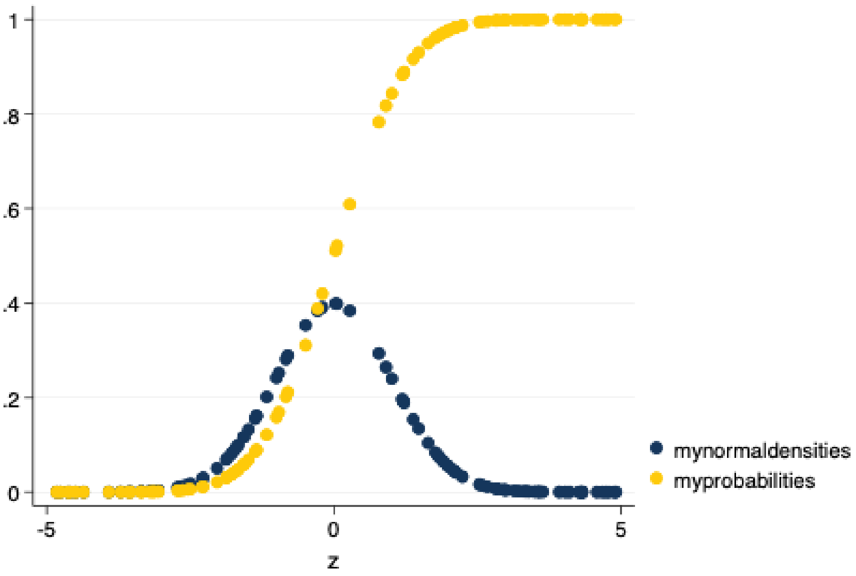


Figure 2: Standard and Cumulative Normal Curves

```
Iteration 2:   log likelihood = -419.73499
Iteration 3:   log likelihood = -419.73484
Iteration 4:   log likelihood = -419.73484
Probit regression                               Number of obs     =      1,223
                                                LR chi2(1)        =      26.63
                                                Prob > chi2       =     0.0000
Log likelihood = -419.73484                     Pseudo R2         =     0.0308

------------------------------------------------------------------------------
    happy_3_D |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  coninc_3_10K |   .0643046    .013517     4.76   0.000     .0378119    .0907974
        _cons |   .9244086   .0721521    12.81   0.000     .7829931    1.065824
------------------------------------------------------------------------------
```

The Logistic Distribution

```
. clear all

. set obs 100 // 100 observations
number of observations (_N) was 0, now 100

. generate z = runiform(-5, 5) // randomly distributed z scores

. generate mylogisticdensities = logisticden(z) // logistic densities

. generate mylogisticprobabilities = logistic(z) // cumulative logistic probabilities

. twoway scatter mylogisticdensities mylogisticprobabilities z, scheme(michigan)

. graph export logistic.png, width(500) replace
(file logistic.png written in PNG format)
```

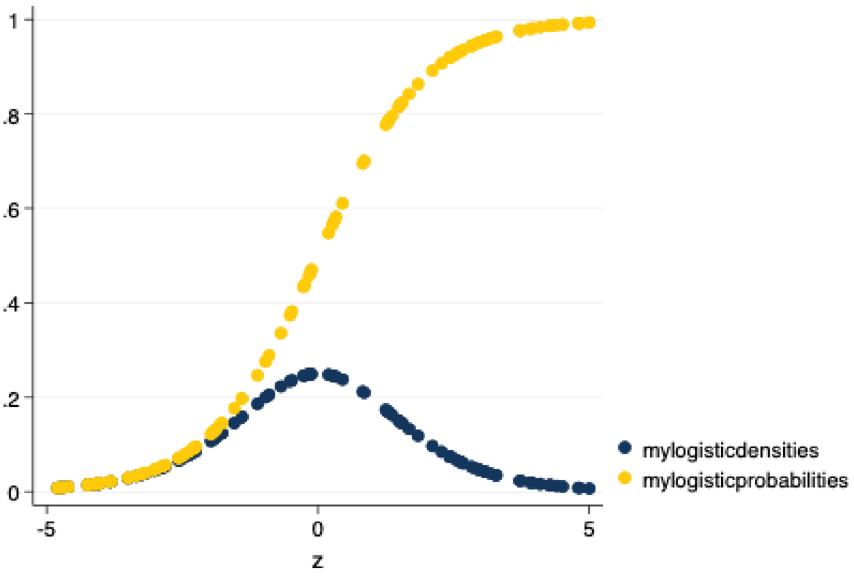


Figure 3: Standard and Cumulative Logistic Curves

The Logit (Logistic) Model

```
. use GSSsmall.dta, clear
( )

. logit happy_3_D coninc_3_10K

Iteration 0:   log likelihood = -433.05123  
Iteration 1:   log likelihood = -420.07608  
Iteration 2:   log likelihood = -419.28644  
Iteration 3:   log likelihood = -419.28513  
Iteration 4:   log likelihood = -419.28513  

Logistic regression                             Number of obs     =      1,223
                                                LR chi2(1)        =      27.53
                                                Prob > chi2       =     0.0000
Log likelihood = -419.28513                     Pseudo R2         =     0.0318

------------------------------------------------------------------------------
   happy_3_D |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
coninc_3_10K |   .1343328   .0293318     4.58   0.000     .0768437     .191822
       _cons |   1.484066   .1381599    10.74   0.000     1.213277    1.754854
------------------------------------------------------------------------------
```

Comparison of LPM, Probit and Logistic Coefficients

NB: Negative vs. positive β.

```
. quietly probit happy_3_D coninc_3_10K

. est store myprobit

. quietly logit happy_3_D coninc_3_10K

. est store mylogit
. est table myprobit mylogit

----------------------------------------
    Variable |  myprobit      mylogit   
-------------+--------------------------
 coninc_3_10K |  .06430462    .13433285  
       _cons |  .92440858    1.4840659  
----------------------------------------
```

Logistic Model (2)

Derivation of logistic model from linear probability model. Using instructor notes

$$\ln\left(\frac{P(y)}{1-P(y)}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ...$$

Interpretation of Odds Ratios (Robert Mare)

$$0 < OR < 1$$

indicates that an increase in x is associated with a decrease in y.

$$1 < OR < \infty$$

indicates that an increase in x is associated with an increase in y.

A Poem About Logistic Regression

Complete Determination

See handout

Rare Events

- Statistical power
- Complete determination

Predicted Probabilities

Discussion

The General Linear Model

Interaction Terms

See interactive demo, or example script.

https://agrogan1.github.io/multilevel/logistic-interactions/logistic-interactions.html