Logistic Regression

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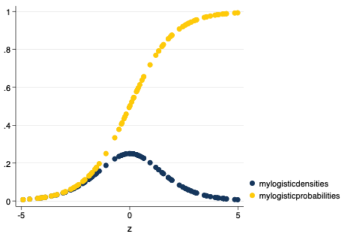
Key Concepts and Commands

- Fitting a Curve to 2 Possible Values



- Linear models, probit and logit
- `y x1 x2 ...` $\leftarrow \rightarrow F(y) = \beta_0 + \beta x_1 + \beta x_2 ...$
- `regress y x1 x2` OLS; Linear Model
- `logit y x1 x2` Logistic Regression
- `probit y x1 x2` Probit Regression
- `glm ...`

Limited Dependent Variables

- Categorical Dependent Variable
- Binary Dependent Variable
- Limited Dependent Variable

General Social Survey

```
. clear all

. set maxvar 10000

. use "/Users/agrogan/Box Sync/DATA WAREHOUSE/General Social Survey/GSS7218_R1.DTA", clea
> r
```

```
. * keep if year == 2018 // keep only most recent year

. codebook polviews // what does this variable look like?
-------------------------------------------------------------------------------------------
polviews                                         think of self as liberal or conservative
-------------------------------------------------------------------------------------------

                  type:  numeric (byte)
                 label:  POLVIEWS

                 range:  [1,7]                        units:  1
         unique values:  7                        missing .:  0/64,814
       unique mv codes:  3                       missing .*:  9,486/64,814

            tabulation:  Freq.   Numeric  Label
                         1,682         1  extremely liberal
                         6,514         2  liberal
                         7,010         3  slightly liberal
                        21,370         4  moderate
                         8,690         5  slghtly conservative
                         8,230         6  conservative
                         1,832         7  extrmly conservative
                         2,326        .d  DK
                         6,777        .i  IAP
                           383        .n  NA
```

Data Management

```
. recode polviews (1/3 = 1)(4/7 = 0), generate(liberal) // dichotomize
(53646 differences between polviews and liberal)

. generate coninc_10K = coninc / 10000 // income in $10K chunks
(6,520 missing values generated)

. label variable coninc_10K "Income 10K Chunks"

. egen income_group = cut(coninc), group(3) // divide income into three groups
(6520 missing values generated)
```

Reference group for income group is 0

```
. drop if class == 5
(1 observation deleted)

. recode hispanic (1 = 1)(else = 0), generate(latinx) // Latinx
(41258 differences between hispanic and latinx)

. keep year polviews liberal ///
> race latinx class ///
> coninc coninc_10K income_group // keep only some variables

. save GSSsmall.dta, replace
file GSSsmall.dta saved
```

Visualize

```
. twoway qfit liberal coninc, lwidth(thick) scheme(burd) ///
> title("Liberal Attitudes by Income")

. graph export liberal-income.png, width(500) replace
(file liberal-income.png written in PNG format)
```

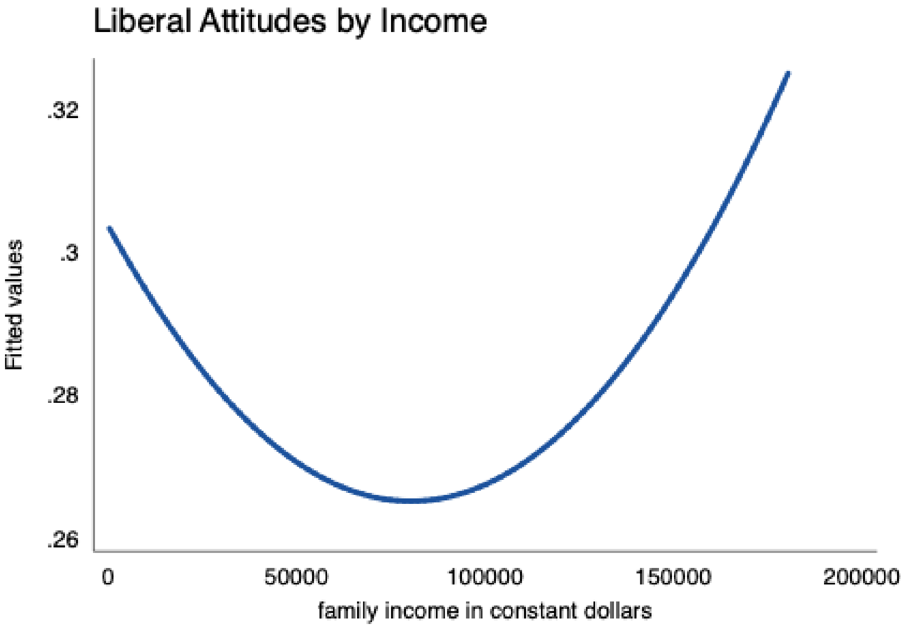


Figure 1: Liberal Attitudes and Income

Linear Probability Model

```
. regress liberal i.race i.income_group
      Source |       SS           df       MS      Number of obs   =    50,191
-------------+----------------------------------   F(4, 50186)     =     64.96
       Model |  52.1435055         4  13.0358764   Prob > F        =    0.0000
    Residual |  10071.8678    50,186  .200690786   R-squared       =    0.0052
-------------+----------------------------------   Adj R-squared   =    0.0051
       Total |  10124.0113    50,190  .201713713   Root MSE        =    .44799

------------------------------------------------------------------------------
     liberal |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        race |
      black  |   .0857774   .0059616    14.39   0.000     .0740926    .0974621
      other  |    .064563    .008817     7.32   0.000     .0472816    .0818444
             |
income_group |
          1  |  -.0082847   .0049636    -1.67   0.095    -.0180134     .001444
          2  |  -.0067437   .0049739    -1.36   0.175    -.0164925    .0030051
             |
       _cons |   .2701971   .0037985    71.13   0.000     .2627521    .2776422
------------------------------------------------------------------------------
```

Normal and Cumulative Normal Distribution

```
. clear all

. set obs 100 // 100 observations
number of observations (_N) was 0, now 100

. generate z = runiform(-5, 5) // randomly distributed z scores

. generate mynormaldensities = normalden(z) // normal densities
. generate myprobabilities = normal(z) // cumulative normal probabilities

. twoway scatter mynormaldensities myprobabilities z, scheme(michigan)

. graph export normal.png, width(500) replace
(file normal.png written in PNG format)
```

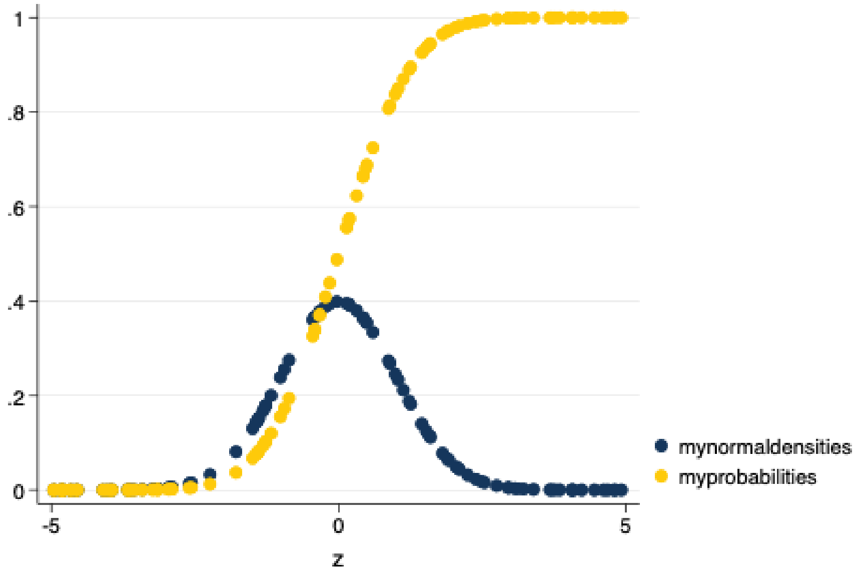


Figure 2: Standard and Cumulative Normal Curves

The Probit Model

```
. use GSSsmall.dta, clear

. probit liberal i.race i.latinx i.class i.income_group
Iteration 0:   log likelihood = -28929.993
Iteration 1:   log likelihood = -28779.708
Iteration 2:   log likelihood = -28779.659
Iteration 3:   log likelihood = -28779.659
Probit regression                               Number of obs     =     48,767
                                                LR chi2(8)        =     300.67
                                                Prob > chi2       =     0.0000
Log likelihood = -28779.659                     Pseudo R2         =     0.0052

--------------------------------------------------------------------------------
       liberal |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------------+----------------------------------------------------------------
          race |
        black  |   .2556235   .0176569    14.48   0.000     .2210165    .2902305
        other  |   .1917797   .0263808     7.27   0.000     .1400744    .2434851
               |
      1.latinx |  -.0105591   .0128091    -0.82   0.410    -.0356644    .0145462
               |
         class |
working class  |  -.0533243   .0268567    -1.99   0.047    -.1059624   -.0006861
 middle class  |   .0364691   .0275156     1.33   0.185    -.0174605    .0903987
  upper class  |   .1287644   .0426698     3.02   0.003     .0451331    .2123957
               |
  income_group |
             1 |  -.0277126   .0153164    -1.81   0.070    -.0577322     .002307
```

```
              2 |  -.0430226   .0159505    -2.70   0.007     -.074285   -.0117602
                |
          _cons |   -.597907   .0258768   -23.11   0.000    -.6486245   -.5471894
-------------------------------------------------------------------------------
```

The Logistic Distribution

```
. clear all

. set obs 100 // 100 observations
number of observations (_N) was 0, now 100

. generate z = runiform(-5, 5) // randomly distributed z scores

. generate mylogisticdensities = logisticden(z) // logistic densities

. generate mylogisticprobabilities = logistic(z) // cumulative logistic probabilities

. twoway scatter mylogisticdensities mylogisticprobabilities z, scheme(michigan)

. graph export logistic.png, width(500) replace
(file logistic.png written in PNG format)
```

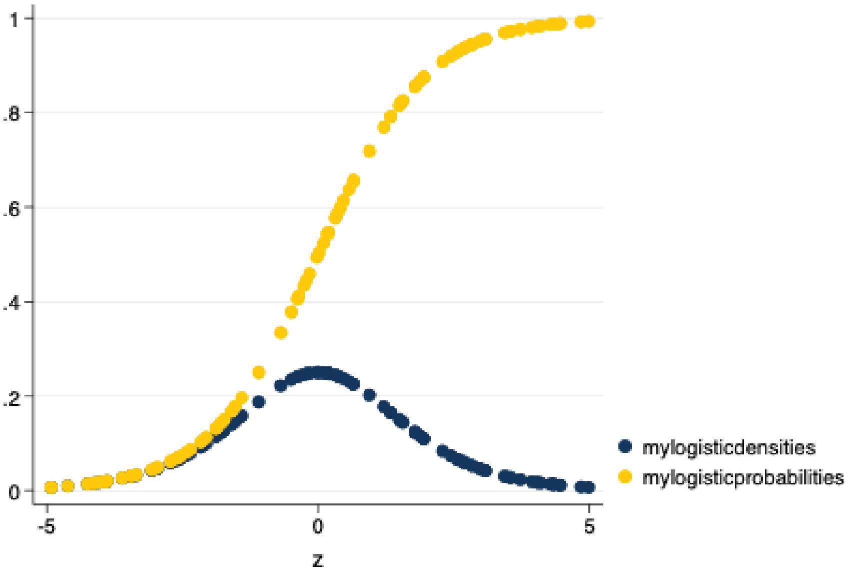


Figure 3: Standard and Cumulative Logistic Curves

The Logit (Logistic) Model

```
. use GSSsmall.dta, clear

. logit liberal i.race i.latinx i.class i.income_group

Iteration 0:   log likelihood = -28929.993
Iteration 1:   log likelihood = -28780.507
Iteration 2:   log likelihood = -28779.998
Iteration 3:   log likelihood = -28779.998
```

```
Logistic regression                             Number of obs     =     48,767
                                                LR chi2(8)        =     299.99
                                                Prob > chi2       =     0.0000
Log likelihood = -28779.998                     Pseudo R2         =     0.0052

------------------------------------------------------------------------------
     liberal |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        race |
      black  |   .4224471   .0289399    14.60   0.000     .3657258    .4791683
      other  |   .3178327   .0433273     7.34   0.000     .2329127    .4027526
             |
    1.latinx |   -.018475   .0214155    -0.86   0.388    -.0604486    .0234985
             |
       class |
working class|  -.0889014   .0446312    -1.99   0.046     -.176377   -.0014258
 middle class|   .0599663   .0456742     1.31   0.189    -.0295536    .1494862
  upper class|   .2126988   .0704279     3.02   0.003     .0746626    .3507349
             |
  income_group|
           1 |  -.0454226   .0255762    -1.78   0.076     -.095551    .0047057
           2 |  -.0697336   .0266137    -2.62   0.009    -.1218954   -.0175718
             |
       _cons |  -.9703756   .0430156   -22.56   0.000    -1.054685   -.8860666
------------------------------------------------------------------------------
```

Comparison of LPM, Probit and Logistic Coefficients

NB: Negative vs. positive β. Statistically significant vs. not statistically significant.

```
. quietly probit liberal i.race i.latinx i.class i.income_group

. est store myprobit

. quietly logit liberal i.race i.latinx i.class i.income_group

. est store mylogit

. est table myprobit mylogit, star

----------------------------------------------
    Variable |    myprobit         mylogit
-------------+--------------------------------
        race |
      black  |   .25562351***    .42244708***
      other  |   .19177974***    .31783265***
             |
      latinx |
          1  |   -.0105591      -.01847504
             |
       class |
 working c.. |  -.05332425*     -.08890139*
 middle cl.. |   .03646909       .05996631
 upper class |   .12876439**     .21269875**
             |
 income_group|
          1  |  -.02771262      -.04542261
          2  |  -.04302264**    -.06973358**
             |
       _cons |  -.59790698***    -.9703756***
----------------------------------------------
      legend: * p<0.05; ** p<0.01; *** p<0.001
```

Logistic Model (2)

Derivation of logistic model from linear probability model. Using instructor notes

$$\ln\left(\frac{P(y)}{1-P(y)}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ...$$

Interpretation of Odds Ratios (Robert Mare)

$$0 < OR < 1$$

indicates that an increase in x is associated with a decrease in y.

$$1 < OR < \infty$$

indicates that an increase in x is associated with an increase in y.

Logistic Model With Odds Ratios

```
. logit liberal i.race i.latinx i.class i.income_group, or

Iteration 0:   log likelihood = -28929.993
Iteration 1:   log likelihood = -28780.507
Iteration 2:   log likelihood = -28779.998
Iteration 3:   log likelihood = -28779.998

Logistic regression                             Number of obs     =     48,767
                                                LR chi2(8)        =     299.99
                                                Prob > chi2       =     0.0000
Log likelihood = -28779.998                     Pseudo R2         =     0.0052

------------------------------------------------------------------------------
     liberal | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        race |
      black  |    1.52569   .0441534    14.60   0.000      1.44156    1.614731
      other  |   1.374146    .059538     7.34   0.000     1.262271    1.495937
             |
    1.latinx |   .9816946   .0210234    -0.86   0.388     .9413422    1.023777
             |
       class |
working class|   .9149358   .0408347    -1.99   0.046     .8383019    .9985752
 middle class|   1.061801    .048497     1.31   0.189     .9708789    1.161237
  upper class|   1.237012   .0871201     3.02   0.003     1.077521    1.420111
             |
  income_group |
          1  |   .9555936   .0244404    -1.78   0.076      .908872    1.004717
          2  |   .9326423    .024821    -2.62   0.009      .885241    .9825817
             |
       _cons |   .3789407   .0163004   -22.56   0.000     .3483023    .4122742
------------------------------------------------------------------------------
Note: _cons estimates baseline odds.
```

A Poem About Logistic Regression

Complete Determination

See handout

Rare Events

- Statistical power
- Complete determination

Predicted Probabilities

Discussion

The General Linear Model

Interaction Terms

See interactive demo, or example script.

https://agrogan1.github.io/newstuff/categorical/logistic-interactions-2/logistic-interactions-2.html