Multilevel Models For Categorical Data

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Motivating Example

High School and Beyond Data

```
. use hsb.dta, clear

. describe

Contains data from hsb.dta
  obs:         7,185
 vars:             7                          27 Oct 2020 21:35
 size:       143,700
-------------------------------------------------------------------------------------------------
              storage   display    value
variable name   type    format     label      variable label
-------------------------------------------------------------------------------------------------
female          byte    %8.0g                 female
ses             float   %9.0g                 socioeconomic status
mathach         float   %9.0g                 math achievement
size            int     %8.0g                 school size
sector          byte    %8.0g                 Catholic vs. Public
schoolid        float   %9.0g                 School ID
mathgroup       float   %9.0g                 math group (Hi / Lo)
-------------------------------------------------------------------------------------------------
Sorted by:
```

A Multilevel Model

```
. melogit mathgroup female ses size sector || schoolid:

Fitting fixed-effects model:

Iteration 0:   log likelihood = -4565.8765
Iteration 1:   log likelihood = -4562.4746
Iteration 2:   log likelihood = -4562.4721
Iteration 3:   log likelihood = -4562.4721

Refining starting values:

Grid node 0:   log likelihood = -4513.3688

Fitting full model:

Iteration 0:   log likelihood = -4513.3688  (not concave)
Iteration 1:   log likelihood = -4489.5697
Iteration 2:   log likelihood = -4484.6285
Iteration 3:   log likelihood =  -4481.049
Iteration 4:   log likelihood = -4480.8848
Iteration 5:   log likelihood = -4480.8842
Iteration 6:   log likelihood = -4480.8842

Mixed-effects logistic regression               Number of obs     =      7,185
Group variable:        schoolid                 Number of groups  =        160

                                                Obs per group:
                                                              min =         14
```

```
                                                               avg =       44.9
                                                               max =         67
Integration method: mvaghermite                 Integration pts.  =          7
                                                Wald chi2(4)      =     393.35
Log likelihood = -4480.8842                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
   mathgroup |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -.3204768   .0579682    -5.53   0.000    -.4340924   -.2068611
         ses |   .6806318    .039101    17.41   0.000     .6039952    .7572684
        size |   .0001675   .0000892     1.88   0.061    -7.43e-06    .0003424
      sector |   .6718503   .1118137     6.01   0.000     .4526995    .8910011
       _cons |  -.3410853   .1410036    -2.42   0.016    -.6174473   -.0647234
-------------+----------------------------------------------------------------
schoolid     |
   var(_cons)|    .277578   .0485216                       .197057    .3910012
------------------------------------------------------------------------------
LR test vs. logistic model: chibar2(01) = 163.18      Prob >= chibar2 = 0.0000
```

Ask For Odds Ratios

```
. melogit, or
Mixed-effects logistic regression               Number of obs     =      7,185
Group variable:        schoolid                 Number of groups  =        160
                                                Obs per group:
                                                               min =         14
                                                               avg =       44.9
                                                               max =         67
Integration method: mvaghermite                 Integration pts.  =          7
                                                Wald chi2(4)      =     393.35
Log likelihood = -4480.8842                     Prob > chi2       =     0.0000
------------------------------------------------------------------------------
   mathgroup | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |   .7258029   .0420735    -5.53   0.000     .6478524    .8131326
         ses |   1.975125   .0772294    17.41   0.000     1.829413    2.132443
        size |   1.000167   .0000893     1.88   0.061     .9999926    1.000342
      sector |   1.957857   .2189152     6.01   0.000     1.572552    2.437569
       _cons |   .7109982   .1002533    -2.42   0.016     .5393194    .9373267
-------------+----------------------------------------------------------------
schoolid     |
   var(_cons)|    .277578   .0485216                       .197057    .3910012
------------------------------------------------------------------------------
Note: Estimates are transformed only in the first equation.
Note: _cons estimates baseline odds (conditional on zero random effects).
LR test vs. logistic model: chibar2(01) = 163.18      Prob >= chibar2 = 0.0000
```

Intra Class Correlation Coefficient (ICC)

```
. estat icc
Residual intraclass correlation
------------------------------------------------------------------------------
                       Level |        ICC   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
                    schoolid |   .0778086   .0125429      .0565131    .1062252
------------------------------------------------------------------------------
```

Visualizing The Idea Of A Random Intercept

```
. clear all
```

```
. twoway (function y = logistic(x), range(-5 5)) /// first school; random intercept 0
> (function y = logistic(x + 1), range(-5 5)) /// second school; random intercept 1
> (function y = logistic(x - 1), range(-5 5)), /// third school; random intercept -1
> title("Three Hypothetical Schools") ///
> sub("With Different Random Intercepts") ///
> legend(order(1 "random intercept 0" 2 "random intercept +1" 3 "random intercept -1")) ///
> scheme(michigan)

. graph export myMLM.png, width(1000) replace
(file myMLM.png written in PNG format)
```

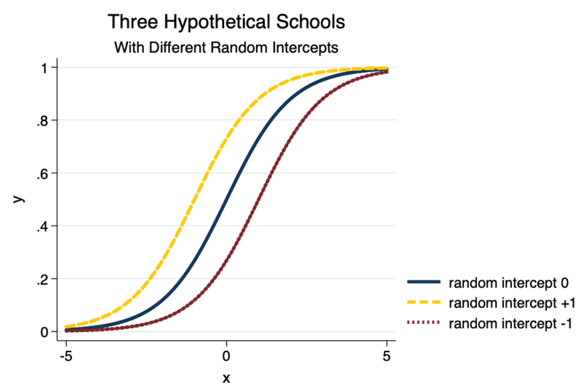

Figure 1: Simulated MLM of School Data

Multiple Uses For Multilevel Modeling

Multilevel modeling is useful in a number of situations with clustering.

Model	Clustering or Nesting
Nested or clustered cross-sectional data	People inside social units such as families, classrooms, schools or neighborhoods, … inside states, countries, etc.
Longitudinal data	Measurement occasions inside people (multiple time points; different people have very different time points)
Meta-Analysis	People inside multiple studies concerning a particular outcome
Meta-Analysis of Multiple Outcomes	People inside multiple studies concerning different outcomes
Dyadic analysis (e.g. couples; parent and child in family)	People inside dyads
Combinations of these approaches	

> Mathematics is the art of giving the same name to different things. —Henri Poincaré

Developing Some Notation

Our notation for logistic regression model is:

$$\ln\left(\frac{p(outcome)}{1-p(outcome)}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ...$$

which after *exponentiating* both sides, and some rearrangement, can be written:

$$p(outcome) = \frac{e^{\beta_0+\beta_1 x_1+\beta_2 x_2+...}}{1+e^{\beta_0+\beta_1 x_1+\beta_2 x_2+...}} =$$

$$F(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + ...)$$

where $F(z) = \frac{e^z}{1+e^z}$, which is the logistic distribution.

So in adapting this notation for the multilevel context, we are ultimately going to write the notation for the multilevel logistic regression model as:

$p(outcome|\text{unique intercept for each unit}) = F(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + u_{0j})$

Stata Commands

> Multilevel models have complicated likelihoods. As we move toward the middle to end of this table, models may have difficulty converging.

Single Level Command	Multilevel Command
`regress y x`	`mixed y x || id:`
`logit y x`	`melogit y x || id:`
`ologit y x`	`meologit y x || id:`
`mlogit y x`	`gsem...`
`poisson y x`	`mepoisson y x || id:`
`nbreg y x`	`menbreg y x || id:`