Logistic Regression

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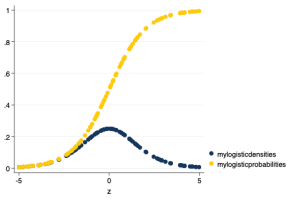
Key Concepts and Commands

- Fitting a Curve to 2 Possible Values



- Linear models, probit and logit
- `y x1 x2 ...` $\leftarrow \rightarrow F(y) = \beta_0 + \beta x_1 + \beta x_2 ...$
- `regress y x1 x2` OLS; Linear Model
- `logit y x1 x2` Logistic Regression
- `probit y x1 x2` Probit Regression
- `glm ...`

Limited Dependent Variables

- Categorical Dependent Variable
- Binary Dependent Variable
- Limited Dependent Variable

General Social Survey

```
. use "/Users/agrogan/Box Sync/DATA WAREHOUSE/General Social Survey Panel Data/GSS_panel
> 2010w123_R6 - stata.dta", clear
( )

. codebook happy_3 // what does this variable look like?

-------------------------------------------------------------------------------
happy_3                                                  happy_3: GENERAL HAPPINESS
-------------------------------------------------------------------------------

                  type:  numeric (byte)
```

```
          label:  HAPPY_3
          range:  [1,3]                        units:  1
  unique values:  3                         missing .:  0/2,044
unique mv codes:  3                        missing .*:  742/2,044

     tabulation:  Freq.   Numeric  Label
                    391         1  VERY HAPPY
                    758         2  PRETTY HAPPY
                    153         3  NOT TOO HAPPY
                      1        .d  DK
                    740        .i  IAP
                      1        .n  NA
```

Data Management

```
. recode happy_3 (1/2 = 1)(3=0), generate(happy_3_D)
(911 differences between happy_3 and happy_3_D)

. tabulate happy_3 happy_3_D // double check
```

happy_3: GENERAL HAPPINESS	RECODE of happy_3 (happy_3: GENERAL HAPPINESS) 0	1	Total
VERY HAPPY	0	391	391
PRETTY HAPPY	0	758	758
NOT TOO HAPPY	153	0	153
Total	153	1,149	1,302

```
. keep happy_3 happy_3_D coninc_3 // keep only some variables

. save GSSsmall.dta, replace
file GSSsmall.dta saved
```

Visualize

```
. twoway scatter happy_3_D coninc_3, scheme(burd) jitter(5)

. graph export happiness-income.png, width(500) replace
(file happiness-income.png written in PNG format)
```

Linear Probability Model

```
. regress happy_3_D coninc_3
```

Source	SS	df	MS		
Model	2.26477708	1	2.26477708	Number of obs	= 1,223
Residual	120.937185	1,221	.099047654	F(1, 1221)	= 22.87
				Prob > F	= 0.0000
Total	123.201962	1,222	.100819936	R-squared	= 0.0184
				Adj R-squared	= 0.0176
				Root MSE	= .31472

happy_3_D	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
coninc_3	9.69e-07	2.03e-07	4.78	0.000	5.72e-07	1.37e-06
_cons	.8368664	.0137133	61.03	0.000	.8099621	.8637706

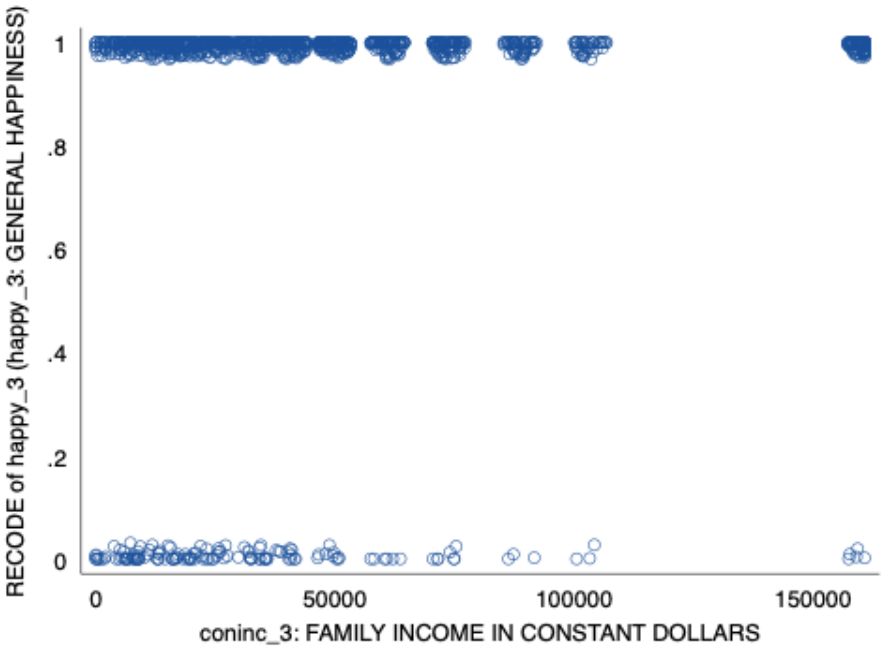


Figure 1: Happiness and Income

Normal and Cumulative Normal Distribution

```
. clear all

. set obs 100 // 100 observations
number of observations (_N) was 0, now 100

. generate z = runiform(-5, 5) // randomly distributed z scores

. generate mynormaldensities = normalden(z) // normal densities

. generate myprobabilities = normal(z) // cumulative normal probabilities

. twoway scatter mynormaldensities myprobabilities z, scheme(michigan)

. graph export normal.png, width(500) replace
(file normal.png written in PNG format)
```

The Probit Model

```
. use GSSsmall.dta, clear
( )

. probit happy_3_D coninc_3
Iteration 0:   log likelihood = -433.05123
Iteration 1:   log likelihood = -419.92819
Iteration 2:   log likelihood = -419.73499
Iteration 3:   log likelihood = -419.73484
Iteration 4:   log likelihood = -419.73484
Probit regression                                 Number of obs     =      1,223
                                                  LR chi2(1)        =      26.63
                                                  Prob > chi2       =     0.0000
```

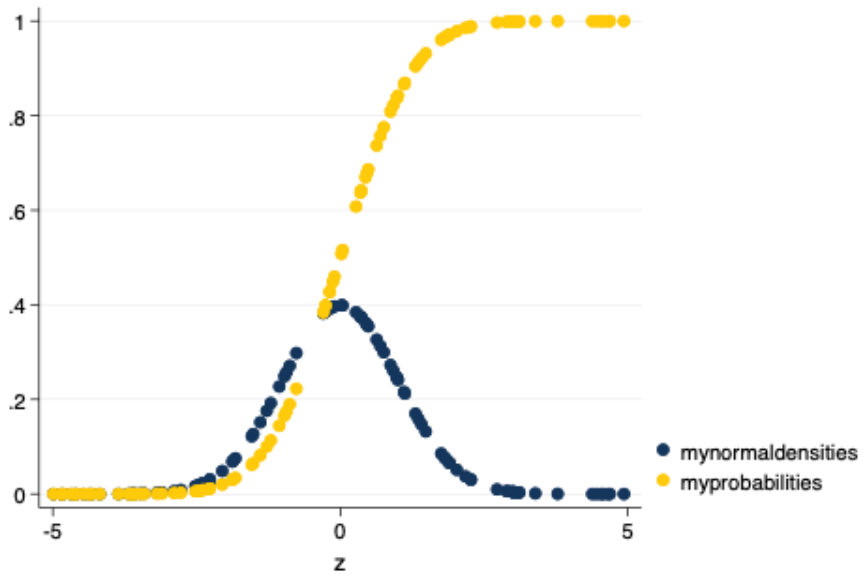


Figure 2: Standard and Cumulative Normal Curves

```
Log likelihood = -419.73484                     Pseudo R2         =     0.0308
```

happy_3_D	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
coninc_3	6.43e-06	1.35e-06	4.76	0.000	3.78e-06	9.08e-06
_cons	.9244086	.0721521	12.81	0.000	.7829931	1.065824

The Logistic Distribution

```
. clear all

. set obs 100 // 100 observations
number of observations (_N) was 0, now 100

. generate z = runiform(-5, 5) // randomly distributed z scores

. generate mylogisticdensities = logisticden(z) // logistic densities

. generate mylogisticprobabilities = logistic(z) // cumulative logistic probabilities

. twoway scatter mylogisticdensities mylogisticprobabilities z, scheme(michigan)

. graph export logistic.png, width(500) replace
(file logistic.png written in PNG format)
```

The Logit (Logistic) Model

```
. use GSSsmall.dta, clear
( )

. logit happy_3_D coninc_3
```

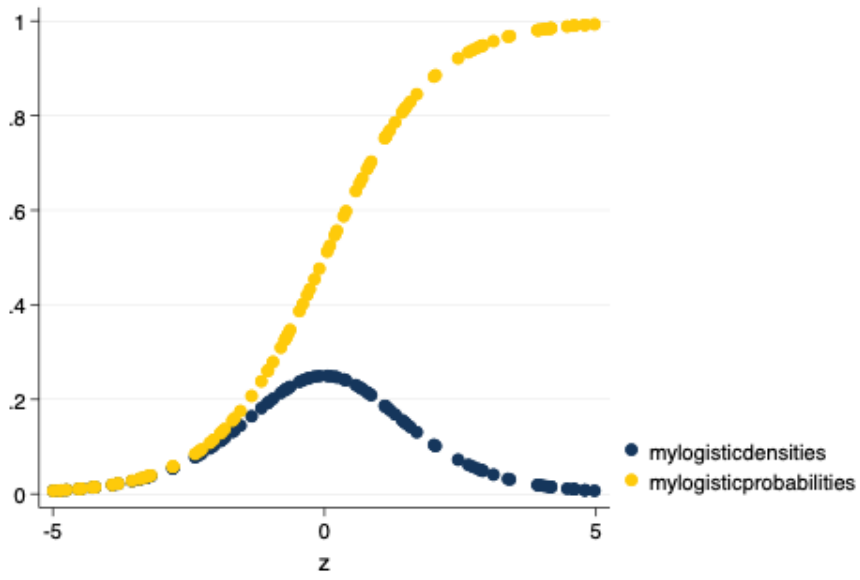


Figure 3: Standard and Cumulative Logistic Curves

```
Iteration 0:   log likelihood = -433.05123
Iteration 1:   log likelihood = -420.07608
Iteration 2:   log likelihood = -419.28644
Iteration 3:   log likelihood = -419.28513
Iteration 4:   log likelihood = -419.28513

Logistic regression                             Number of obs     =      1,223
                                                LR chi2(1)        =      27.53
                                                Prob > chi2       =     0.0000
Log likelihood = -419.28513                     Pseudo R2         =     0.0318

------------------------------------------------------------------------------
    happy_3_D |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------+---------------------------------------------------------------
    coninc_3 |   .0000134   2.93e-06     4.58   0.000     7.68e-06    .0000192
       _cons |   1.484066   .1381599    10.74   0.000     1.213277    1.754854
------------------------------------------------------------------------------
```

Comparison of LPM, Probit and Logistic Coefficients

NB: Negative vs. positive β.

```
. quietly probit happy_3_D coninc_3

. est store myprobit

. quietly logit happy_3_D coninc_3

. est store mylogit

. est table myprobit mylogit
```

Variable	myprobit	mylogit
coninc_3	6.430e-06	.00001343
_cons	.92440857	1.4840659

Logistic Model (2)

Derivation of logistic model from linear probability model. Using instructor notes

$$\ln\left(\frac{P(y)}{1-P(y)}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ...$$

Interpretation of Odds Ratios (Robert Mare)

$$0 < OR < 1$$

indicates that an increase in x is associated with a decrease in y.

$$1 < OR < \infty$$

indicates that an increase in x is associated with an increase in y.

A Poem About Logistic Regression

Complete Determination

See handout

Rare Events

- Statistical power
- Complete determination

Predicted Probabilities

Discussion

The General Linear Model

Interaction Terms

See interactive demo, or example script.

https://agrogan1.github.io/multilevel/logistic-interactions/logistic-interactions.html