Take Home 5 (Due July 22nd, before class)

- You may get assistance from tutors.
- **Late work will not be accepted.**
- **All answers and work must be on seperate sheets of paper.**
- **Keep work organized.** Answer that are hard to find illegible work will be awarded no points.
- Answers without justification will be awarded no points.
- You may verify your answers with a calculator, but **all calculations must be done by hand.**
- Answers that look similar to another students work will be judged harshly.

1. Prove the following identity using the Principles of Mathematical Induction. The base case was shown in class. Assume the function $f(t)$ is exponentially bounded.

   $$\mathscr{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}(\mathscr{L}\{f\}(s))$$

2. Prove the following identity using the Principle of Mathematical Induction. Assume the function $f(t)$ is exponentially bounded.

   $$\mathscr{L}\{f^{(n)}\}(s) = s^n \mathscr{L}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - ... - f^{n-1}(0)$$

3. Find the Laplace transform of the following function.

   $$f(t) = 2^t$$

4. My Tesla model Y performance. Now that we have seen the application of the least-squares regression line in action, lets put it to the test. Here are more data (data is plural).

May 6th	May 8th	May 13th	May 15th	May 17th	May 20th
(0,51)	(0,32)	(0,12)	(0,36)	(0,30)	(0,43)
(12,70)	(7,50)	(6,33)	(7,51)	(6,46)	(7,54)
(20,81)	(15,66)	(13, 52)	(18,72)	(11,58)	(15, 67)
(28,90)	(20,75)	(18,62)	(31,90)	(17,70)	(20,75)
	(30,88)	(29,78)		(23,80)	(25,81)
	(32,90)	(35,85)		(32,90)	
		(40,90)			

From takehome #4 we now have a technique to model the differential equation based on an exponential model. But the problem is that the exponential model has no bound and as time increases to infinity the percent of battery full will also approach infinity. So instead of creating an exponential regression model as in takehome 4 let us create an logistic model using the following differential equation.

$$\frac{dB}{dt} = kB(100 - B)$$

Where B represents the percent of battery full after some time t, and t is amount of time charged measured in minutes. The 100 represent the carrying capacity or the upper bound on $B(t)$. To create a logistic equation requires two points, an initial point and some other point to describe the rate of increase of $B(t)$. Using the data given **describe how to pick an initial point and how to pick some other point**. **Use the two points to create the logistic equation**. There is no need to use any form of technology but please include documentation of your work. **Describe the strength and weakness of your logistic model**. **Determine the percent of battery full after 10 minutes of charging**.