Assignment 3

杨乐天

Problem 1

The induction hypothesis is stated as follows: for any Σ, α satisfying $\Sigma \vdash \alpha$ within height $h-1$, we know that $\Sigma \vDash \alpha$.

Given the induction hypothesis, we work on the case when height is h.

(a)

Assume we prove $\Sigma \vdash \beta$ through $\perp$-introduction, let $\beta = \alpha \to \perp$, then we know that $\Sigma; \alpha \vdash \perp$ within height $h-1$. By induction hypothesis, for any truth assignment v, if $\forall \gamma \in \Sigma; \alpha$, $\bar{v}(\gamma) = T$, then $\bar{v}(\perp) = T$. However, $\bar{v}'(\perp) = F$ for any assingment v'. Therefore, $\Sigma; \alpha$ is unsatisfiable, which means any assignment v satisfying all elements in Σ has $\bar{v}(\alpha) = F$, and thus $\bar{v}(\alpha \to \perp) = \bar{v}(\alpha) \to \bar{v}(\perp) = F \to F = T$.

(b)

Assume we prove $\Sigma \vdash \alpha$ through $\perp$-elimination, then $\Sigma \vdash \perp$ within height $h-1$. Similarly, we know from $\Sigma \vDash \perp$ that Σ is unsatisfiable, which naturally leads to $\Sigma \vDash \alpha$.

Problem 2

(a)

With complexity seen in directly proving this wff, we prove it through the completeness of sentential logic. For any truth assignment v, $\bar{v}(A \to B) = F$ iff. $v(A) = T$ and $v(B) = F$. Therefore, if $v(A) = T$ and $v(B) = F$, $\bar{v}(A \wedge \neg B) = v(A) \wedge \neg v(B) = T$; otherwise, $\bar{v}(A \to B) = T$. Consequently, $\bar{v}((A \to B) \vee (A \wedge \neg b)) = T$. Then by completeness of sentential logic, this wff is provable.

(b)

Let $v(A) = T$ and $v(B) = F$, then $\bar{v}((A \to B) \vee (A \wedge B)) = \bar{v}(A \to B) \vee \bar{v}(A \wedge B) = F \vee F = F$. Similarly, by the completeness of sentential logic, this wff is not provable.

Problem 3

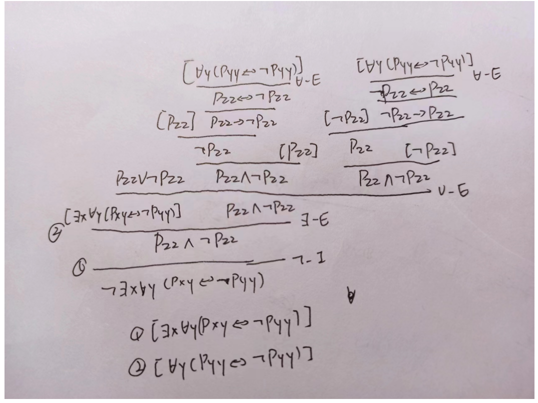
In $\forall y(Pxy \to \forall x Pxy)$, the first x occurs free.

In $\forall x(Qy \to \exists y Pxz)$, z and the first y occurs free.

In $(\neg \exists y R(fyz)) \wedge (\forall x \forall y R(fyz))$, the only free-occurring variable is z.

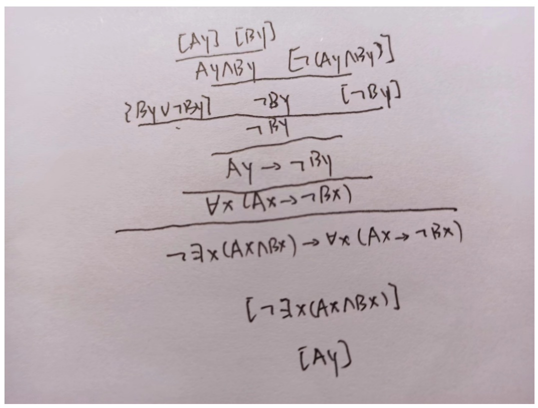
Problem 4

Let Pxy denotes x shaves y. Then the problem is to derive $\neg \exists x \forall y (Pxy \leftrightarrow \neg Pyy)$.

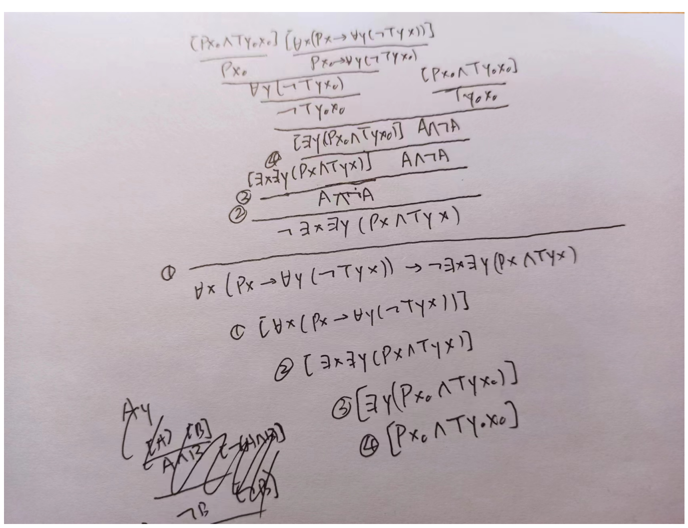


Problem 5

(a)

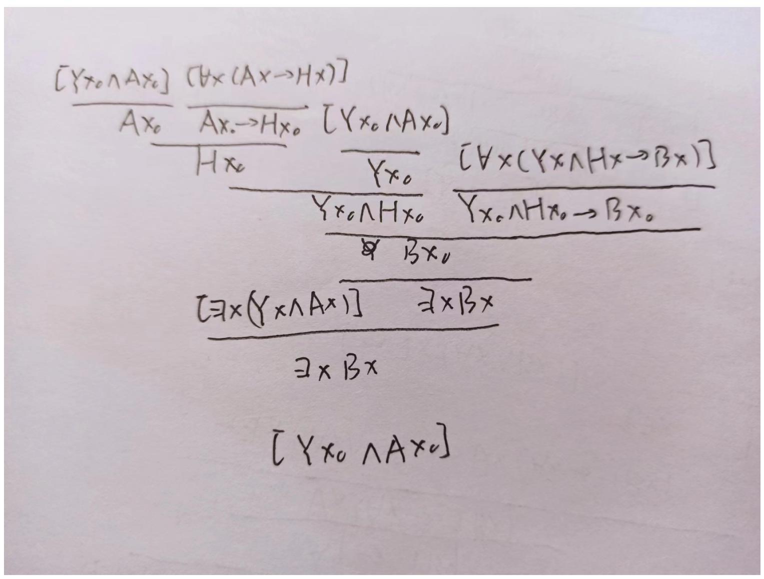


(b)



Problem 6

The four statements are (1) $\forall x(Yx \wedge Hx \to Bx)$, (2) $\forall x(Ax \to Hx)$, (3) $\exists x(Yx \wedge Ax)$, (4) $\exists x Bx$.


$[Yx_0 \wedge Ax_0]$ $[\forall x(Ax \to Hx)]$
Ax_0 $Ax_0 \to Hx_0$ $[Yx_0 \wedge Ax_0]$
Hx_0 Yx_0 $[\forall x(Yx \wedge Hx \to Bx)]$
$Yx_0 \wedge Hx_0$ $Yx_0 \wedge Hx_0 \to Bx_0$
Bx_0
$[\exists x(Yx \wedge Ax)]$ $\exists x Bx$
$\exists x Bx$
$[Yx_0 \wedge Ax_0]$