

Mathematical Logic: Assignment 1

Sep 25, 2022

Attention: To get full credits, you *must provide explanations to your answers!* You will get at most 1/3 of the points if you only provide the final results without any explanation.

1. (6pt) Prove the following propositions:
 - (3pt) If the set A is countable and nonempty, then there is some listing with possible repetitions of A . (Hint: Think about the definition of *countable*).
 - (3pt) If there is some listing with possible repetitions of the set A , then A is countable and nonempty. (Hint: think about exploiting the lemma that a set A is countable iff there is an one-to-one mapping from A to \mathbb{N})
2. (3pt) Prove that if $f : \mathbb{N} \rightarrow A$ is surjective (i.e. f maps \mathbb{N} onto A) then A is countable. (Hint: you may use the above properties).
3. (3pt) Given a natural number n , the set S_n consists of expressions of length n in sentential logic. Prove that for any n , S_n is enumerable. (Hint: note that the set of all the symbols in sentential logic is enumerable).
4. (4pt) Translate the following sentences into well-formed formulas (please explain how you got the wffs):
 - (2pt) Cancer will not be cured unless its cause is determined and a new drug for cancer is found. (A_1 denotes “Cancer will be cured”; A_2 denotes “The cause of cancer is determined”; A_3 denotes “A new drug for cancer is found”)
 - (2pt) If Smith has installed central heating, then he has sold his car or he has not paid his mortgage. (A_1 denotes “Smith has installed central heating”; A_2 denotes “Smith has sold his car”; A_3 denotes “Smith has paid his mortgage”)
5. (4pt) Prove that if α is a wff not containing the negation symbol \neg , then more than a quarter of the symbols in α are sentence symbols. (Hint: Apply structural induction to show the length of α is $4k + 1$ and the number of sentence symbols in α is $k + 1$ for some k)