Mathematical Logic: Assignment 2

Oct 16, 2022

Attention: To get full credits, you *must provide explanations to your answers*! You will get at most $1/3$ of the points if you only provide the final results without any explanation.

1. (5pt) For each set Σ of formulas below, describe all truth assignments that satisfy it:

   - (2pt) $\Sigma = \{A_1, A_2 \to A_1, A_3 \to (A_2 \to A_1)\}$;
   - (3pt) $\Sigma = \{\neg A_1, \neg(A_2 \to A_1), \neg(A_3 \to A_2 \to A_1), \ldots, \neg(A_i \to A_{i-1} \to \ldots \to A_1), \ldots\}$ (note that Σ is an infinite sequence).

2. (5pt) Determine whether or not the following wffs are tautologies:

   - (2pt) $((P \to Q) \to P) \to P$;
   - (3pt) $(A \leftrightarrow B) \to \neg((A \to B) \to \neg(B \to A))$.

3. (5pt) Assume a knowledge base containing the following information:

   KB = {"If dogs want attention and show you affection, then they lick you."}.

   If you ask KB "Is it true that 'if dogs want attention then they lick you, or else if dogs show you affection then they lick you'?", what answer do you get? (Hint: translate KB and the question into wff with A_1 denoting "Dogs want attention", A_2 denoting "Dogs show you affection" and A_3 denoting "Dogs lick you".)

4. (6pt) Prove the following statement:

   For any wff α, β and γ, if $\beta \models\!\dashv \gamma$ and we replace all the occurrences of β in α with γ to get a wff α', then $\alpha \models\!\dashv \alpha'$.

   (Hint: use structural induction on α.)

5. (4pt) For each of the following formulas, write a formula that is tautologically equivalent to it and whose only connectives are $\neg$ and $\vee$ (Hint: replace subformulas with tautologically equivalent wffs):

   - (2pt) $(\neg A \wedge (B \to C)) \to \neg(\neg B \vee C)$;
   - (2pt) $A \wedge \neg(D \to (\neg A \wedge E))$;

6. (5pt) Prove the following property (called the cut rule):

   If $\Sigma \models \alpha$ and $\Delta; \alpha \models \beta$, then $\Sigma \cup \Delta \models \beta$.