Assignment 3

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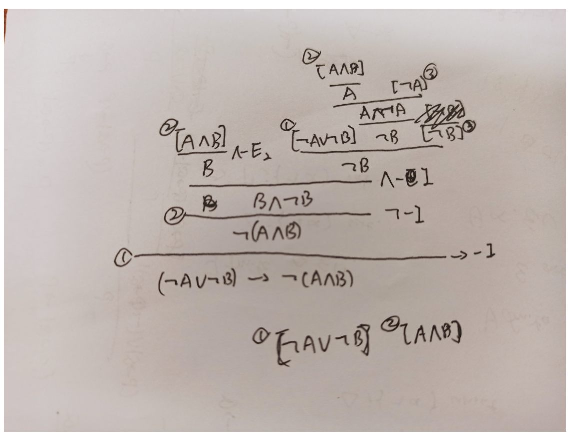
Problem 1

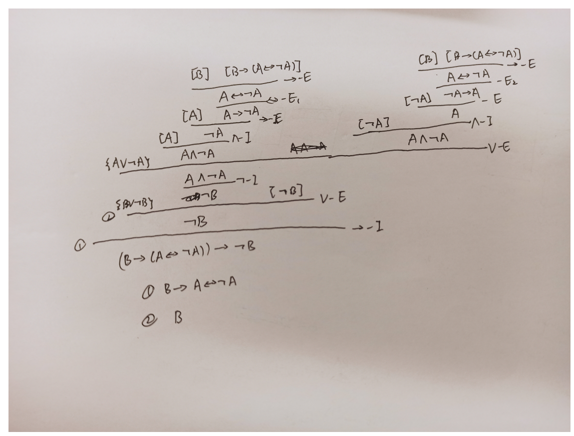
Suppose (a) holds but (b) doesn't. Then there exists a finitely satisfiable Γ that is not satisfiable.

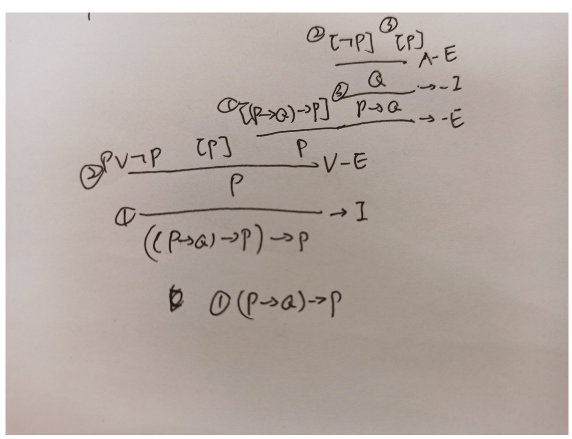
Notice that $\Sigma \vDash \alpha$ is equivalent to $\Sigma; \neg\alpha$ is unsatisfiable. Then the corollary is exactly saying that if $\Sigma; \beta$ is unsatisfiable, then there exists finite $\Delta \subseteq \Sigma$ such that $\Delta; \beta$ is unsatisfiable, where β is any wff.

Since Γ is unsatisfiable, then we let $\Gamma = \Gamma' \cup \{\beta\}$ such that $\Gamma' \cap \{\beta\} = \varnothing$. Then we know from the corollary that there exists finite $\Delta' \subseteq \Gamma'$ such that $\Delta'; \beta$ is unsatisfiable. However, $\Delta'; \beta$ is a finite subset of Γ, which contradicts with the assumption that Γ is finitely satisfiable.

Problem 2







Problem 3

3.a

We already have partial proof tree $\dfrac{\sigma_1\sigma_2...\sigma_n}{\alpha}$ where $\{\sigma_1, \sigma_2, ..., \sigma_n\} \subseteq \Sigma$ and $\dfrac{\alpha\delta_1...\delta_m}{\beta}$ where $\{\delta_1, ..., \delta_m\} \subseteq \Delta$.

Then we can construct new tree $\dfrac{\dfrac{\sigma_1\sigma_2...\sigma_n}{\alpha}\delta_1...\delta_m}{\beta}$ as a partial proof tree of $\Sigma \cup \Delta \vdash \beta$.

3.b

We have

$$\cfrac{\gamma_1 \qquad \gamma_2 \qquad \cdots \qquad \gamma_n \qquad \alpha}{\beta \leftrightarrow \neg\beta}$$

where $\{\gamma_1, \gamma_2, ..., \gamma_n\} \subseteq \Gamma$.

Then we build several partial proof trees.

$$\cfrac{\cfrac{\beta \qquad \cfrac{\cfrac{\gamma_1 \qquad \gamma_2 \qquad \cdots \qquad \gamma_n \qquad \alpha}{\beta \leftrightarrow \neg\beta}}{\beta \to \neg\beta} \leftrightarrow\text{-E}_1}{\neg\beta} \to\text{-E} \qquad \beta}{\beta \wedge \neg\beta} \wedge\text{-I}$$

Substituting β and $\neg\beta$ with each other, we can build a symmetric partial proof tree of $\Gamma \cup \{\alpha, \neg\beta\} \vdash \beta \wedge \neg\beta$.

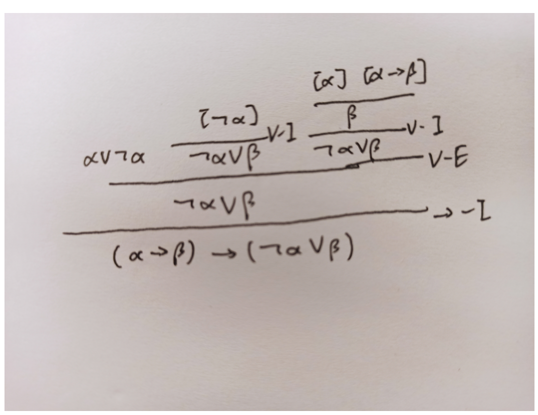
$$\cfrac{\alpha \vee \neg\alpha \qquad \cfrac{\cfrac{\beta \vee \neg\beta \qquad \cfrac{\Gamma; [\alpha], [\beta]}{\beta \wedge \neg\beta} \qquad \cfrac{\Gamma; [\alpha], [\neg\beta]}{\beta \wedge \neg\beta}}{\beta \wedge \neg\beta} \vee\text{-E}}{\neg\alpha} \wedge\text{-E} \qquad [\neg\alpha]}{\neg\alpha} \vee\text{-E}$$

Then we have shown that $\Sigma \vdash \neg\alpha$.

Problem 4

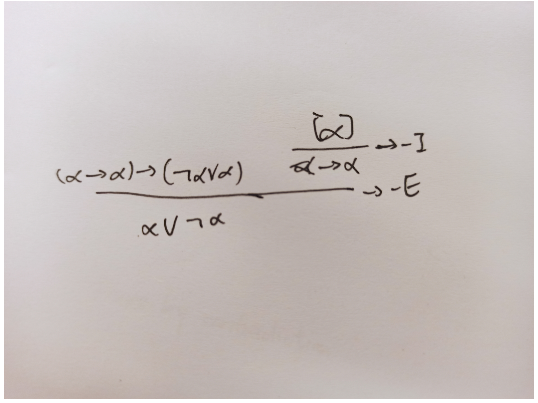
4.a

From the proof tree we know that the conclusion is dependent only on the assumption $\alpha \vee \neg\alpha$.



4.b

From the proof tree we know that the conclusion is dependent only on the assumption $(\alpha \to \beta) \to (\neg\alpha \vee \beta)$.



Problem 5

5.a

$$
\begin{aligned}
&(\neg A \vee \neg B) \rightarrow \neg(A \wedge B)\\
\Leftrightarrow &((A \rightarrow \perp) \vee (B \rightarrow \perp)) \rightarrow ((A \wedge B) \rightarrow \perp)
\end{aligned}
$$

5.b