Elementär Talteori: 2015-03-11

Hjälpmedel: Papper, skrivdon och miniräknare.

1. (a) Solve the diophantine equation $35x - 55y + 77z = 23$.

   (b) Determine all continued fractions $K(a_0, ..., a_n) = \frac{89}{19}$.

2. Solve the congruence $251x \equiv 125 \mod (521)$.

3. Determine the zeros of the following polynomials

   (a) $X^2 - X$ in $\mathbb{Z}_{91}$,

   (b) $X^{11} - 2$ in $\mathbb{Z}_{125}$,

   (c) $X^2 + 25$ in $\mathbb{Z}_{125}$.

4. Determine whether the following residue classes are squares!

   (a) $\overline{328} \in \mathbb{Z}_{823}$

   (b) $\overline{823} \in \mathbb{Z}_{328}$.

5. (a) Find a primitive root in $\mathbb{Z}_{14641}^*$!

   (b) Does $\mathbb{Z}_{1001}^*$ admit a primitive root?

6. (a) Find all $(x, y) \in \mathbb{N}, x < y$, s.th. $x^2 + y^2 = 845$.

   (b) Which numbers $c \in \mathbb{N}_{>0}$ do occur in a pythagorean triple $(a, b, c) \in (\mathbb{N}_{>0})^3$?

7. (a) Find the continued fraction $K(a_0, ...) = \sqrt{15}$. Compute $K(a_0, a_1, a_2)^2$.

   (b) Find three solutions $(x, y) \in \mathbb{N}^2$ of $x^2 - 15y^2 = 1$.

   (c) Are there solutions $(x, y) \in \mathbb{N}^2$ of $x^2 - 15y^2 = -1$?

8. Let $\tau : \mathbb{N}_{>0} \longrightarrow \mathbb{C}$ be the arithmetic function with $\tau(n) :=$ the number of positive divisors of n. Find $\psi : \mathbb{N}_{>0} \longrightarrow \mathbb{C}$ with

$$\tau * \psi = \delta.$$

Here $\delta(n) = \delta_{n1}$.

<center>**L Y C K A   T I L L !**</center>