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Section 2.2 Probability of independent trials

#Gacha #Minimum guarantee #Ceiling

independence of trials

When making a minimum guarantee for a 10-pull gacha (a system that guarantees that at least one card will be rated 2 stars or higher), consider the following two methods.

(1) Always the 10th consecutive ★Draw from 2 or more

(2) At least once in the first to ninth consecutive If you don't get 2 or more stars, the 10th time is ★Draw from 2 or more

In the case of (1), the results of the first 9th series do not affect the result of the 10th series, but in the case of (2), they do. In this way, trials that do not affect each other are called "independent," while trials that do affect each other are called "dependent." The probability of independent trials can be calculated by multiplying each probability.

Example 1

In a certain game's gacha, the drop rates for 3, 2, and 1 stars are 3.0%, 18.5%, and 78.5% for the first 9 pulls, and 3.0%, 97.0%, and 0% for the tenth pull. Find the probability of getting the minimum guaranteed value when pulling a 10-pull gacha once.

$$0.785^{9} \times 0.97 = 0.109799\cdots$$

Answer: 11.0%

↑Since each gacha attempt is independent, you can calculate it by multiplying the probability of drawing a specific rarity on the first, second, third, etc. attempts.

repeated attempts

Repeating independent trials under the same conditions, such as the first to ninth consecutive trials, is called a "repeated trial." The probability of drawing △ times out of ○ times is etc. can be calculated.

For example, let's consider the probability of drawing a 3 star twice out of 9 times. If we correspond the probability of drawing a 3 star to an O and the probability of drawing a 2 star or lower to an X, then:

The results of nine gacha attempts can be thought of as a "permutation containing the same item" of two circles and seven circles (9C2=36 combinations).

The probability of getting a 3 star on a specific draw (for example, the first or second draw) and not getting a 3 star on the third through ninth draws is the same as in Example 1: 0.032 x 0.97 = 0.000727...

The probability of it coming up in other draws is the same, so multiplying it by 36 permutations gives us 0.026178... or 2.6%.

Example 2

In the game in Example 1, spin the gacha 200 times and find the probability of getting a 3-star item 6 times.

$${}_{200}C_{6} \times 0.03^{6} \times 0.97^{194} = 0.163085\cdots$$

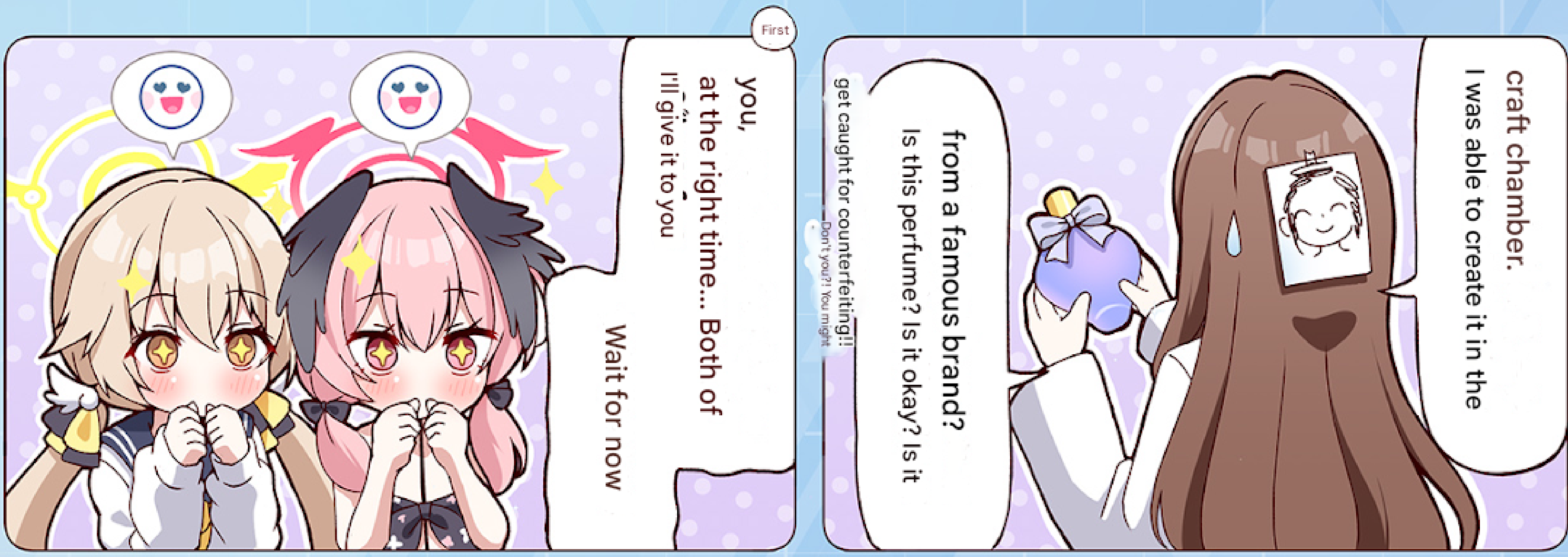
Answer: 16.3%

↑The appearance rate of ★3 is the same for the 1st to 9th consecutive tries and the 10th consecutive try, so we can assume that out of 200 repeated attempts, ★3 will appear 6 times and ★2 or below will appear 194 times.

practice questions

[1] If the drop rate of the featured character is 0.7%, find the probability (%) of not getting the featured character after spinning the gacha 200 times, rounding to two decimal places.

[2] In the games of Example 1 and Practice 1, spin the gacha 200 times and calculate the probability (%) of getting a slip-through (a 3-star item other than the featured item) 6 times. Round to two decimal places to find the answer.



Duplicate combination

When considering which locations to allocate your six Wanted Tickets or School Exchange Tickets to, you can think of it as "the number of times you can choose six times from three locations, allowing overlaps," since the items you get will be the same even if you change the order.

This is called an overlap combination, and is represented by the symbol "3H6."

The order doesn't matter, so let's think about it in a fixed order.

The order in which wanted tickets are used will now be fixed as Highway → Desert Railroad → School Building.

If we consider the six tickets as "●" and the separators as """, then the pattern of ticket use can be thought of as a permutation of six ● and two ", such as ●●●●●●. (When " comes at the beginning or end, it is a pattern where you have not gone to the highway or school building, and when there are two " in a row, it is a pattern where you have not gone to the desert railroad tracks.) In other words, there are 3H6=8C6=28 ways.

Using the same idea, nHr=n+r-1Cr.

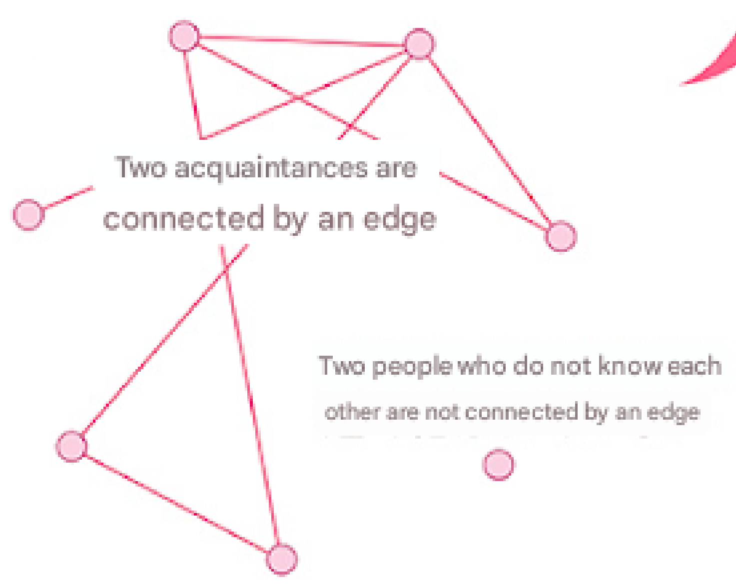
Example 2

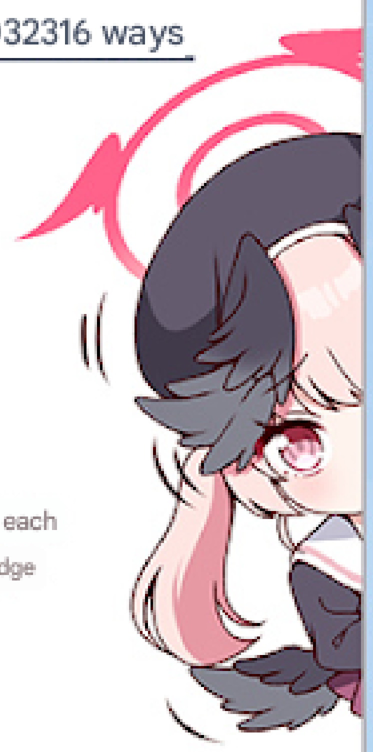
When all students with ★2 or below have joined the Petri, and you have recruited 10 times, and the minimum guarantee is reached, how many combinations of divine name characters can you obtain?

$$ {}_{11}H_9 \times 22 = {}_{19}C_9 \times 22 = 92378 \times 22 = 2032316 $$

Answer: 2032316 ways

graph





When there are seven students in a petri dish, there are 221 possible patterns of relationships between students, i.e., whether they know each other or not, since there are 7C2 possible pairs of students and two possible pairs for each pair.

If we represent the relationships between students and their acquaintances as points and the edges connecting them, we get the shape shown on the right. This is called a "graph."

Using graphs makes it easier to solve problems about the "relationships" between people and things.

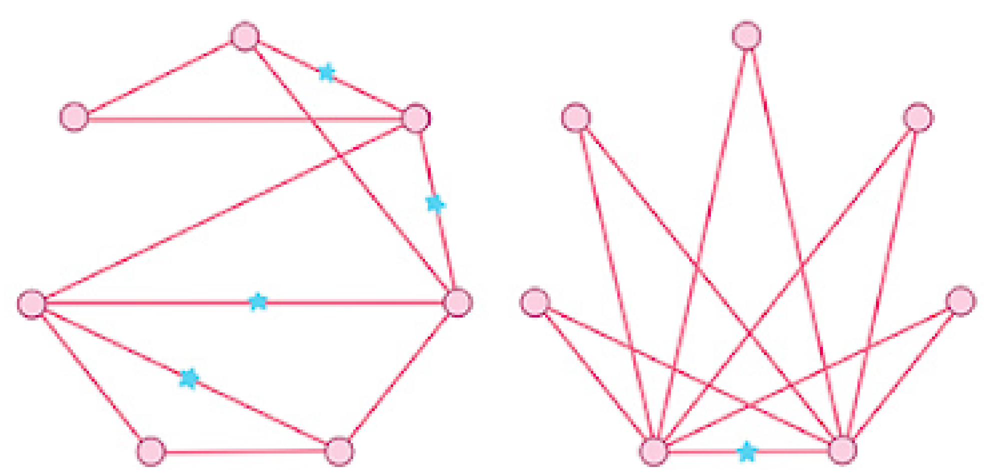
Example 3

If there are five pairs of three people in the seven people above where every two people know each other, what is the maximum number of pairs of two people who do not know each other?

In a triangle formed by three people, where any two people know each other, if two sides are shared, the other side is automatically shared as well, so the maximum number of sides that two pairs can share is one. In other words, the minimum number of sides that can be connected by five pairs is 15-4=11. Therefore, the maximum number of pairs that do not know each other is

So there are 7C2-11=10 pairs.

Answer 10 groups



★: Shared edge

practice questions

[3] How many ways can you give four Samuela "The Beyond"s to students with extra-high effect? (There are seven students with extra-high effect.)

[4] If there are 15 students in a petri dish, is it possible for all of them to know seven of them?

Column "Birthday Paradox"

Hanako
The problem is, there are 24 Trinity students in the Petri dish.
What is the probability that two people out of 24 have the same birthday?

Azusa
There are 365 days in a year, so that's about 7%.

Everyone is different, so 0%!

Koharu
All the teachers just remember!

Hifumi
I thought it was about that much too, but when I did the math, it was 53.8%.

The probability that everyone is different is

1st person, 2 people 3 people 4 people, 24th person

$$\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \cdots \times \frac{342}{365} = \frac{{}_{365}P_{24}}{}$$

Because it is 36524

With the first person With the first to second people With the first to third people

ちがう日 ちがう日 ちがう日

A different day from the 1st to 23rd person

$$1 - \frac{{}_{365}P_{24}}{365^{24}} = 0.5383442\cdots$$

53.8%

Azusa
That's a pretty counter-intuitive value

Hanako
That's right. This is the famous problem known as the "birthday paradox."

Koharu
So if we look at the whole petri dish, there are a lot more pairs with the same birthday?

Apart from the twins, the only ones are Hanako and Saya from Sankaikyo.

Hanako
Oh? That's pretty rare...

Hifumi
It's like one in four million.

There are 365×110C2 ways to choose one pair, and 364P108 ways to separate the other 108 people, so

$$\frac{365\times {}_{110}C_2 \times {}_{364}P_{108}}{365^{110}} = 2.5\times10^{-7}$$

*If we include students outside of Kivotos, Suzumi and Miku are the same, and if we include students outside of Shari, Saori and Momoka are the same.

Data variation

For example, if the test scores of four people are {40,45,55,60} and {10,30,70,90}, the mean and median will both be 50 points. However, it seems like there is something different that can't be measured by the mean and median, right?
This is the "variability" of the data, and is called "variance."

Variance literally indicates how dispersed the data is, and is calculated as the average of (values minus the mean). If the data are X1, X2, X3, ..., Xn, and the mean is μ, then the variance is $\{(x1-\mu)^2 + (x2-\mu)^2 + (x3-\mu)^2 + \ldots + (xn-\mu)^2\}/n$.

Using variance, you can calculate the "standard deviation."

The deviation value is (score - average) ÷ √ variance × 10 + 50, and if the difference from the average is the same, the smaller the variance, the greater the fluctuation in the deviation value. (The positive square root of the variance in the denominator is called the "standard deviation." We will see it again in 3.2, but standard deviation is a useful concept in many ways.

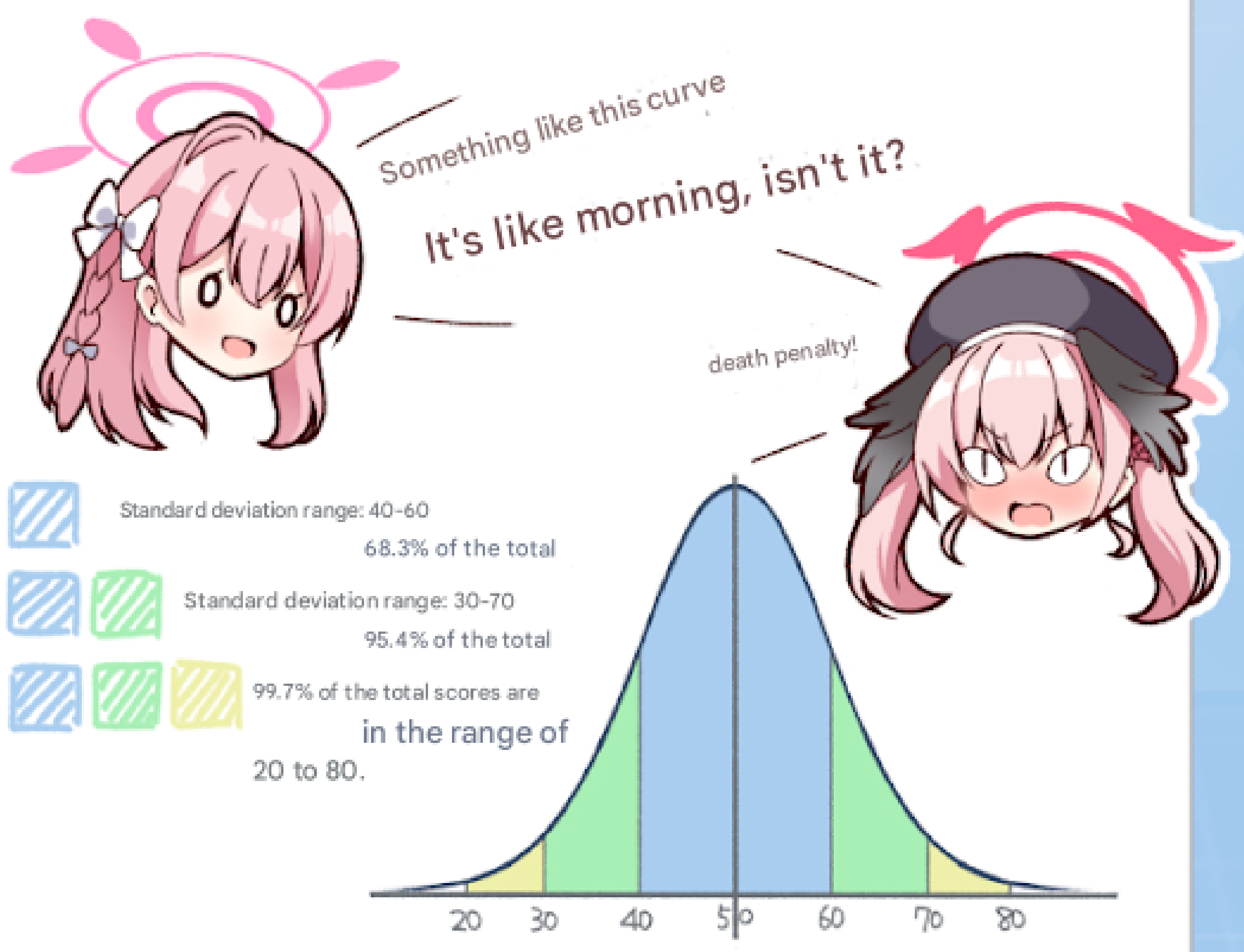
Example 2

Add one person who got 100 points to the two test results above, and find the average and variance of the five people. Also, what is the standard deviation of the person who got 100 points? Round to the nearest tenth.

Average = (50×4+100)/5 = 60 (both are the same)

First variance = (202+152+52+02+402)/5 = 450
Standard deviation 40/450×10+50 = 68.8561··· 68.8

Second variance = (502+302+102+302+402)/5 = 1200 Standard deviation 40/1200×10+50 61.5470 61.5



practice questions

[2] Find the variance and standard deviation for the height distribution in Exercise 1. Also, find the deviation values for a student who is 128 cm tall and a student who is 180 cm tall.