CS500
Introduction to Computer Science

Lecture 3

Binary Number Representation

- Unsigned binary number
- Signed binary number
  - signed magnitude
  - signed complement
    - 1’s complement
    - 2’s complement

Unsigned number to decimal: Example

Find the Decimal Value of the Binary unsigned number $(1011)_2$

1011

$1^{8}0^{4}1^{2}1^{1} = 8+0+2+1=11$

Then the result is: 11

Unsigned binary number

Represention of unsigned binary numbers in 3 bits, i.e., n=3

BIT PATTERN	SIGNED MAGNITUDE DECIMAL VALUE
0 0 0	0
0 0 1	1
0 1 0	2
0 1 1	3
1 0 0	4
1 0 1	5
1 1 0	6
1 1 1	7

Range from 0 to 7

[0, $(2^n - 1)$]

Example: Show the upper and lower bound of decimal values that are allowed for binary bit patterns of lengths: 4, 5, 6, 7and 8 in the cases of unsigned.

PATTERN LENGTH	UNSIGNED RANGE $[0, 2^n - 1]$
4	[0 , 15]
5	[0 , 31]
6	[0 , 63]
7	[0 , 127]
8	[0 , 255]

From Decimal to unsigned number conversion

How to convert decimal number to a unsigned with n-bits?

first, find the range of the unsigned number for n-bits. $[0, 2^n - 1]$

second, check whether the decimal number is in the range of the signed number.

third, if it is out of range, no solution exist

if it is in the range,

convert the number to binary

Example:

Find in **8-bits** the Binary unsigned number for the decimal number **15** and **200**

Solution

$$range=[\ 0,\ 2^{n}-1\ ]$$

for 8-bit, range =[0, 127]

128 64 32 16 8 4 2 1

15 --> in the range

15 = 00001111

200 --> NOT in the range

Signed Binary Integer Numbers

The signed binary numbers are represented with a **bit placed in the leftmost position of the binary number to represent the sign.**

sign bit	value

Example

$$\underbrace{0}_{\text{sign}}\quad\underbrace{1\quad 0\quad 1}_{\text{value}}$$

The way of representing the signed binary numbers depends on the adopted notational system.

The most common notations are:

1. **Signed magnitude notation**

0 +ve 1 -ve	1	0	1

2. **Signed-2's complement notation**

0 +ve 1 -ve	1	0	1

3. **Excess notation**

1 +ve 0 -ve	1	0	1

Signed magnitude Binary Number

Example 1: the string of bits **01001** can be considered as:

- 9 (unsigned), or
- +9 (signed-magnitude) since the leftmost bit is 0.

Example 2: The string of bits **11001** can be considered as:

- 25 (unsigned), or
- –9 (signed-magnitude) since the leftmost bit is 1.

Signed magnitude to Decimal

Two steps are required:

1- check the sign of the number

In the leftmost bit,

0 means +ve number

1 means -ve number

2- convert the rest of the binary number to decimal via:

(a) +ve numbers--> convert the number to decimal

(b) -ve numbers--> - (convert the number to decimal)

Signed magnitude to decimal: Example

Find the Decimal Value of the Binary signed magnitude number $(1011)_2$

1011

sign(-ve)   magnitude $0^{4}1^{2}1^{1} = 0+2+1=3$

Then the result is: -3

Signed magnitude Binary Number...

Signed magnitude for representing binary numbers in 3 bits, i.e., n=3

the **leftmost bit** is reseved for the **sign**
0 means +ve
1 means -ve

BIT PATTERN	SIGNED-MAGNITUDE DECIMAL VALUE
0 0 0	+ 0
0 0 1	+ 1
0 1 0	+ 2
0 1 1	+ 3
1 0 0	– 0
1 0 1	– 1
1 1 0	– 2
1 1 1	– 3

Range from -3 to +3

[$-(2^{n-1})-1$, $+(2^{n-1})-1$]

Not Convenient

two representations for zero

000 : +0

100 : -0

Signed magnitude Binary Number...

- While the unsigned decimal range of the 3-bit binary pattern is [0 , 7], the corresponding signed-magnitude range is [– 3 , + 3].
- For a n-bit binary pattern, the allowed unsigned decimal range is $[\,0\,,\,(2^{n}-1)]$
- For a n-bit binary pattern, the allowed signed-magnitude decimal range is

  $$[-\,(2^{n-1}-1)\,,\,+\,(2^{n-1}-1)].$$

- Despite of the simplicity of the signed-magnitude system , it is not convenient to implement arithmetic operations in a computer since there are two different representations of 0 in the table, which are 000 (+0) and 100 (-0).

Example: Show the upper and lower bound of decimal values that are allowed for binary bit patterns of lengths: 4, 5, 6, 7and 8 in the cases of unsigned, signed magnitude, signed 1's complement.

PATTERN LENGTH	UNSIGNED RANGE $[0, 2^n - 1]$	SIGNED-MAGNITUDE $[-(2^{n-1} - 1), +(2^{n-1} - 1)]$
4	[0 , 15]	[– 7 , + 7]
5	[0 , 31]	[– 15 , + 15]
6	[0 , 63]	[– 31 , + 31]
7	[0 , 127]	[– 63 , + 63]
8	[0 , 255]	[– 127 , + 127]

From Decimal to signed magnitude number conversion

How to convert decimal number to a signed magnitude with n-bits?

first, find the range of the signed magnitude for n-bits. $[-(2^{n-1}-1), +(2^{n-1}-1)]$

second, check whether the decimal number is in the range of the 1's complement or not.

third, if it is out of range, no solution exist

if it is in the range,

+ve numbers-->convert the number to binary

assign the lefmost bit to 0

-ve numbers --> convert the number to binary

assign the leftmost bit to 1

Example:

Find in **8-bits** the Binary signed magnitude number for the decimal number **+15** and **-5**

Solution

$$range=[-(2^{n-1}-1), +(2^{n-1}-1)]$$

for 8-bit, range =[-127, +127]

128	64	32	16	8	4	2	1

+15 --> in the range

+15 = 00001111

-5 --> in the range

-5 = 10000101

Signed Complement Binary Numbers

Signed complement binary number

In this system, a negative number is indicated by its complement.

- Since positive numbers always start with 0;(i.e. +), in the leftmost position, the complement will always start with 1 (i.e. –).
- The signed-complement system can use either the 1's complement or the 2's complement notations.
- Changing from + to – and vise versa of the binary number in the 1's complement system is obtained by taking the 1's complement of the binary number.
- Changing from + to – and vise versa of the binary number in the 2's complement system is obtained by taking the 2's complement of the binary number.

Signed 1’s Complement

Signed 1's complement to Decimal

Two steps are required:

1- check the sign of the number

In the leftmost bit,

0 means +ve number

1 means -ve number

2- convert the binary number to decimal via:

(a) +ve numbers-->convert the number to decimal

(b) -ve numbers--> find its 1's complement

- (convert it to decimal)

Signed 1's complement to decimal: Example

Find the Decimal Value of the Binary signed **1's** complement Number **$(11101011)_2$** :

- 11101011 is the negative number
- 1's complement for 11101011 is 00010100
- Decimal value for (00010100)= $0^{128}0^{64}0^{32}1^{16}0^{8}1^{4}0^{2}0^{1}$ = 20
- Then the decimal value for 11101011 is $-(20)_{10}$ (-ve number)

Signed 1's complement to Decimal: Example 2

Find the Decimal Value of the Binary signed 1's complement Number $(0101.11)_2$

Solution

0101.11 is the positive number

Decimal value for (0101.11)= $0^{8}1^{4}0^{2}1^{1}.1^{0.5}1^{0.25}$ =5.75

Then the decimal value for 0101.11 is $(5.75)_{10}$ (+ve number)

Signed 1's complement binary number

Signed 1's complement for representing binary numbers in 3 bits, i.e., n=3

the **leftmost bit** is reseved for the **sign**
0 means +ve
1 means -ve

BIT PATTERN	SIGNED 1'S COMPLEMENT DECIMAL VALUE
0 0 0	+ 0
0 0 1	+ 1
0 1 0	+ 2
0 1 1	+ 3
1 0 0	– 3
1 0 1	– 2
1 1 0	– 1
1 1 1	– 0

Range from -3 to +3

$[\ -(2^{n-1}-1),\ \ +(2^{n-1}-1)\ ]$

Not Convenient

two representations for zero

000 : +0

111 : -0

Example: Show the upper and lower bound of decimal values that are allowed for binary bit patterns of lengths: 4, 5, 6, 7and 8 in the cases of unsigned, signed magnitude, signed 1's complement.

PATTERN LENGTH	UNSIGNED RANGE $[0, 2^n -1]$	SIGNED-MAGNITUDE $[-(2^{n-1}-1), +(2^{n-1}-1)]$	SIGNED-1'S COMPLELEMNT $[-(2^{n-1}-1), +(2^{n-1}-1)]$
4	[0 , 15]	[– 7 , + 7]	[– 7 , + 7]
5	[0 , 31]	[– 15 , + 15]	[– 15 , + 15]
6	[0 , 63]	[– 31 , + 31]	[– 31 , + 31]
7	[0 , 127]	[– 63 , + 63]	[– 63 , + 63]
8	[0 , 255]	[– 127 , + 127]	[– 127 , + 127]

From Decimal to signed 1's complement number conversion

How to convert decimal number to a signed 1's complement with n-bits?

first, find the range of the 1's complement for n-bits. $[-(2^{n-1}-1), +(2^{n-1}-1)]$

second, check whether the decimal number is in the range of the 1's complement or not.

third, if it is out of range, no solution exist

if it is in the range,

+ve numbers-->convert the number to binary

-ve numbers --> convert the number to binary

convert the binary number to its 1's complement

Example:

Find in **8-bits** the Binary signed 1's complement number for the decimal number **+15** and **-5**

Solution

$$range=[-(2^{n-1}-1), +(2^{n-1}-1)]$$

for 8-bit, range =[-127, +127]

128 64 32 16 8 4 2 1

+15 --> in the range

+15 = 00001111

-5 --> in the range

+5 = 00000101

-5 =(1's complement "00000101")=11111010

Signed 2’s complement

Signed 2's complement to Decimal

Two steps are required:

1- check the sign of the number

In the leftmost bit,

0 means +ve number

1 means -ve number

2- convert the binary number to decimal via:

(a) +ve numbers-->convert the number to decimal

(b) -ve numbers--> find its 2's complement

-(convert it to decimal)

Signed 2's complement to decimal: Example

Find the Decimal Value of the Binary signed **2's** complement Number **$(11101011)_2$** :

- 11101011 is the negative number
- 2's complement for 11101011 is 00010101
- Decimal value for (00010101)= $0^{128}0^{64}0^{32}1^{16}0^{8}1^{4}0^{2}1^{1}$ = 21
- Then the decimal value for 11101011 is -21 (-ve number)

Signed 2's complement to Decimal: Example 2

Find the Decimal Value of the Binary signed complement Number $(1101.11)_2$

Solution

1101.11 is the negative number

2's complement for 1101.11 is 0010.01

Decimal value for (0010.01)= $0^{8}0^{4}1^{2}0^{1}.0^{0.5}1^{0.25}$ = 2.25

Then the decimal value for 1101.11 is $-(2.25)_{10}$ (-ve number)

Signed 2's complement to Decimal: Example 3

Find the Decimal Value of the Binary signed complement Number $(010111)_2$

Solution

010111 is the positive number

Decimal value for (010111)= $0^{32}1^{16}0^{8}1^{4}1^{2}1^{1}$ =23

Then the decimal value for 010111 is $(23)_{10}$ (+ve number)

Signed 2's complement binary number

Signed 2's complement for representing binary numbers in 3 bits, i.e., n=3

the **leftmost bit** is reseved for the **sign**
0 means +ve
1 means -ve

BIT PATTERN	SIGNED 2'S COMPLEMENT DECIMAL VALUE
000	+ 0
001	+ 1
010	+ 2
011	+ 3
100	– 4
101	– 3
110	– 2
111	– 1

Range from -4 to +3

$[\ -(2^{n-1}),\ \ +(2^{n-1}-1)\ ]$

Example: Show the upper and lower bound of decimal values that are allowed for binary bit patterns of lengths: 4, 5, 6, 7and 8 in the cases of unsigned, signed 2's complement, exccess notations

PATTERN LENGTH	UNSIGNED RANGE $[0, 2^n -1]$	SIGNED-2'S COMPLEMENT $[-(2^{n-1}) , + (2^{n-1} -1)]$
4	[0 , 15]	[– 8 , + 7]
5	[0 , 31]	[– 16 , + 15]
6	[0 , 63]	[– 32 , + 31]
7	[0 , 127]	[– 64 , + 63]
8	[0 , 255]	[– 128 , + 127]

Signed 2's complement to Decimal: Example 1

Find the Decimal Value of the Binary signed 2's complement Number $(11101011)_2$

Solution

11101011 is the negative number

2's complement of 11101011 is 00010101

$0^{128}0^{64}0^{32}1^{16}0^{8}1^{4}0^{2}1^{1} = 16+4+1=21$

Then the decimal value for 11101011 is $-(21)_{10}$ (-ve number)

Signed 2's complement to Decimal: Example 1

Find the Decimal Value of the Binary signed 2's complement Number $(11101011)_2$

Another Solution

11101011 is the negative number

$1^{128}1^{64}1^{32}0^{16}1^{8}0^{4}1^{2}1^{1}$ =-128+64+32+8+2+1=-21

Then the decimal value for 11101011 is $-(21)_{10}$ (-ve number)

Signed 2's complement to Decimal: Example 2

Find the Decimal Value of the Binary signed 2's complement Number $(110111)_2$

Solution

110111 is the negative number

2's complement of 110111 is 001001

$0^{64}0^{32}1^{16}0^{4}0^{2}1^{1} = 1+16=17$

Then the decimal value for 110111 is $-(17)_{10}$ (-ve number)

Signed 2's complement to Decimal: Example 3

Find the Decimal Value of the Binary signed 2's complement Number $(010111)_2$

Solution

010111 is the positive number

$0^{32}1^{16}0^{8}1^{4}1^{2}1^{1} = 16+4+2+1 = 23$

Then the decimal value for 010111 is $(23)_{10}$ (+ve number)

Solved Excercises

Example:

obtain the decimal value of the binary number 11111001 in case of:

a) unsigned binary notation

$$1^{128}1^{64}1^{32}1^{16}1^{8}0^{4}0^{2}1^{1} = 128+64+32+16+8+1 = (249)_{10}$$

a) signed magnitude notation

$$1\,1^{64}1^{32}1^{16}1^{8}0^{4}0^{2}1^{1} = -\,(64+32+16+8+1) = -(121)_{10}$$

Solved Excercises...

Example:

obtain the decimal value of the binary number 11111001 in case of:

a) signed-1's complement notation 11111001 (-ve number)

1's complement for (11111001) is 00000110

$0^{128}0^{64}0^{32}0^{16}0^{8}1^{4}1^{2}0^{1} = 4+2 = -(6)_{10}$

a) signed-2's complement notation 11111001 (-ve number)

2's complement for (11111001) is 00000111

$0^{128}0^{64}0^{32}0^{16}0^{8}1^{4}1^{2}1^{1} = 4+2+1 = -(7)_{10}$

From Decimal to signed 2’s complement number conversion

How to convert decimal number to a signed 2’s complement with n-bits?

first, find the range of the 2’s complement for n-bits. $[-(2^{n-1}), +(2^{n-1}-1)]$

second, check whether the decimal number is in the range of the 2’s complement or not.

third, if it is out of range, no solution exist

if it is in the range,

+ve numbers-->convert the number to binary

-ve numbers --> convert the number to binary

convert the binary number to its 2’s complement

Example:

Find in **8-bits** the Binary signed 2's complement number for the decimal number **+15** and **-5**

Solution

$$range=[-(2^{n-1}), +(2^{n-1} -1)]$$

for 8-bit, range =[-128, +127]

128 64 32 16 8 4 2 1

+15 --> in the range

+15 = 00001111

-5 --> in the range

+5 = 00000101

-5 =(2's complement "00000101")=11111011

Thank You