Introduction to Computer Science

(CS500)

Lecture 1

References

- *" Introduction to Computer Sciences", Lecture Notes, by Prof. Hesham A. Hefny.*
- *Lecture Notes by Dr. Ahmed Hamza*

Course Outline

- Modern Computer Architecture
- Data Representation In Modern Computers
- Numbering Systems
- Computer Hardware
- Data Manipulation
- Computer Software

What is Computer Science

What is meant by Computer Science

- It is solving problems using computations
- In order to solve a problem computationally, we need:
  1. A representation captures all the relevant aspects of the problem
  2. An algorithm solves the problem by use of this representation
- A representation that leaves out details of what is being represented is a form of abstraction
- **Example**: The farmer, cabbage, goat and wolf problem

  Start State: **East: F,C,G,W**

  Wast:

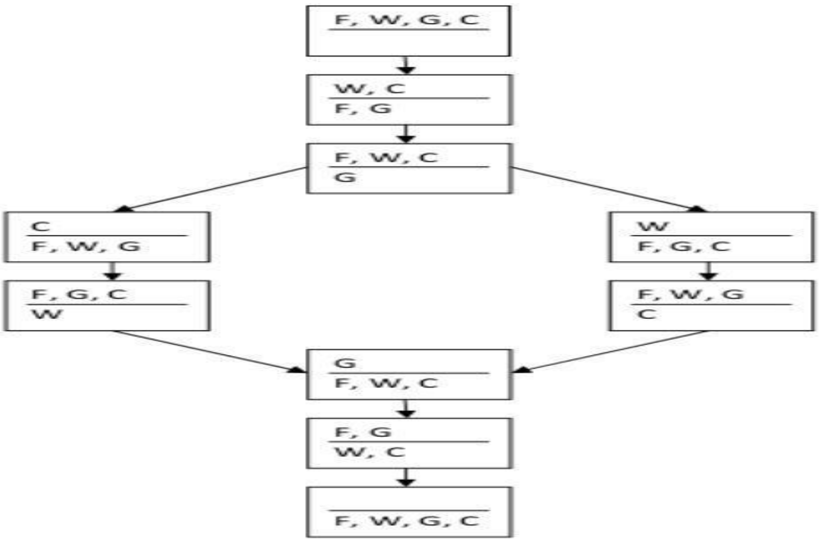
  Goal State: **East:**

  Wast: F,C,G,W

Algorithm

Algorithm **is a finite set of ordered, unambiguous and executable instructions that terminates with a desired output(s) for given input(s) in a finite amount of time**

Example: Farmer, Cabbage, Goat, Wolf problem



Algorithm

<u>Example:</u> find the average of two numbers

Solution Algorithm

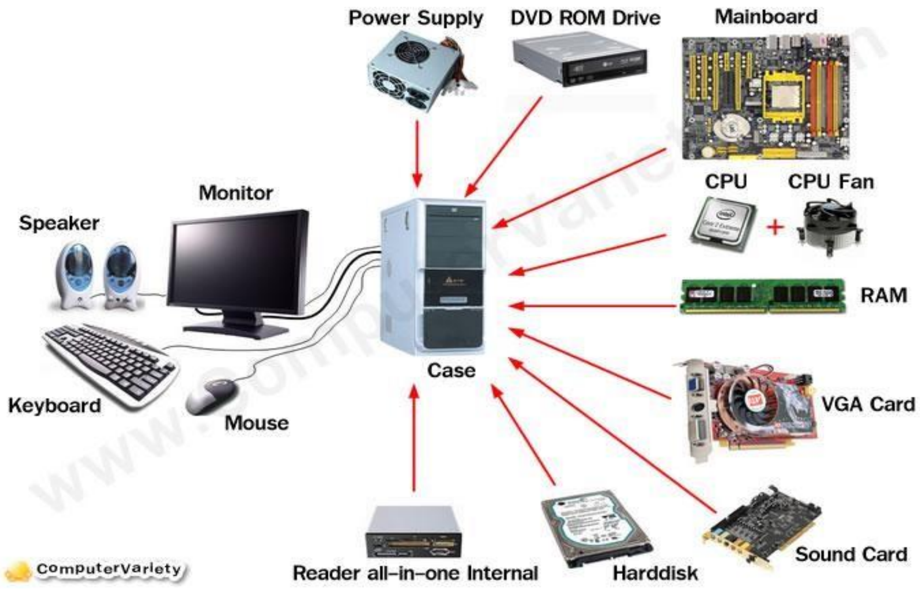
1. Read the two number (inputs)
2. Sum the two numbers
3. Divide the sum by 2
4. Store the division quotient
5. Display the stored quotient (output)

What is Computer

- A machine that can be instructed to carry out sequences of arithmetic or logical operations automatically via computer programming.
- It receives input, stores and manipulates data, and provides output in a useful format
- It includes hardware and software

Computer Hardware

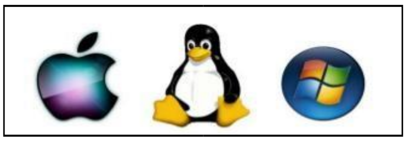
• The computer physical components



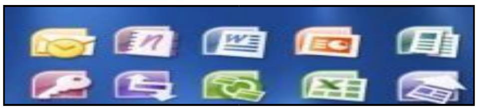
Computer Software

- Programs executed on the computer
- Computer software can be:
  - » system software

    Operating System



  - » applicatoin software
  - » It can be



    - » Desktop application

      Office package, calculator, games

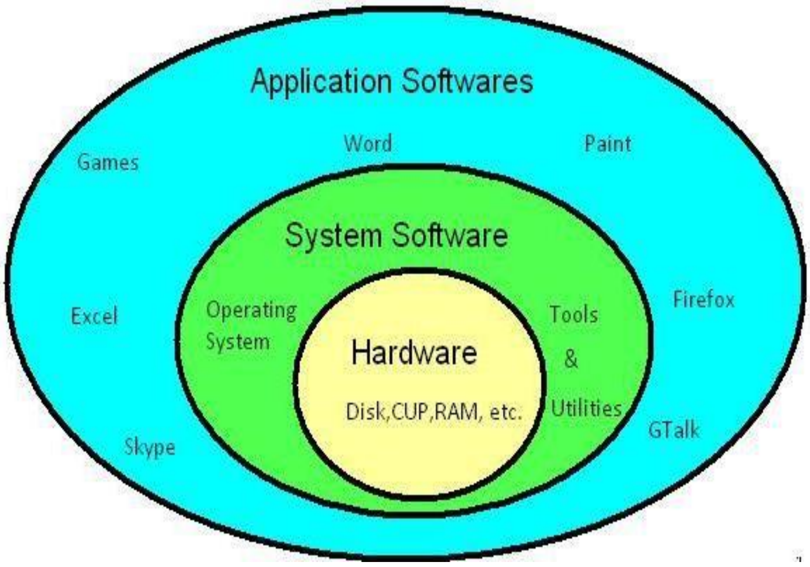
    - » Mobile application

      Android

    - » Web application

      Mail application, websites

Hardware Versus Software



Computer Categories

- Modern computers can be categorized respecting:
  - Usage
    - Special–purpose computers
    - Geneeral–purpose computers
  - Size and computations
    - Supercomputer
    - Microcomputer

Computer Categories: Size and computaional capabilities

Super computer

- the highest in speed, the most expensive
- used to process massive amounts of data.
- used in defense and military applications,
- wheather forecasting, complex scientific applications

Computer Categories: Size and computaional capabilities

Microcomputers

- CPU is manufactored as a single chip called microprocessor
- less powerful and cost
- can be categorized into:

  – **Desktop**

    –used for a variety of user applications

    (e.g., email, web browsing, document processing)

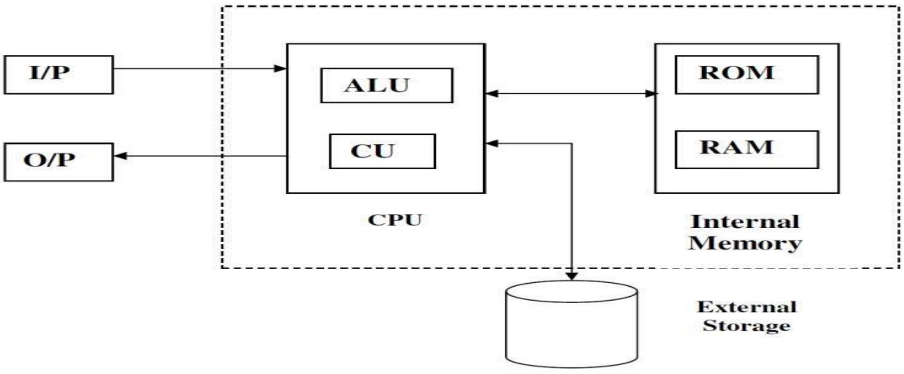
  – **Portable**

    categorized into:

    •Notebook (or laptop)

    •Tablet

Architecture of Modern Computer



Central Processing Unit (CPU)

Central Processing Unit (CPU):

the main unit of the computer responsible for executing programs stored in the internal memory. It consists of two parts:

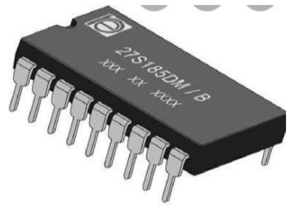
1. Arithmetic and Logic Unit (ALU) .
2. Control Unit (CU) .

1. **Arithmetic and Logic Unit (ALU)**: performs:
   - Arithmetic operations: Addition, Subtraction,Multiplication, Division, Modulus, Power,… etc.
   - Logical: AND, OR, XOR, SHIFT, ROTATE,…etc.
2. **Control Unit (CU):** responsible for controling all the CPUoperations, such as:
   - Decoding the instructions of programs stored in the internal memory.
   - Controlling the flow of information through the ALU, I/O, and internal memory.

Internal Memory

Composed of chips of integrated circuits which are capable of quickly storing and retrieving data. It can be:

1. Read Only Memory (ROM).
2. Random Access Memory (RAM).
3. Cache Memory



1) **Read Only Memory (ROM):**
   - It contains built-in programs (e.g., BIOS) that are needed by the computer to start operation (booting up) when powered on.
   - Theses programs are stored permanently during the manufacturing and can't be lost when power is turned off (***nonvolatile memory***)
   - The CPU can **only read from** the ROM.

Internal Memory

1. **Read Only Memory (ROM):**

- It contains built-in programs (e.g., BIOS) that are needed by the computer to start operation (booting up) when powered on.
- Theses programs are stored permanently during the manufacturing and can't be lost when power is turned off (nonvolatile memory)
- The CPU can only read from the ROM.

2. **Random Access Memory (RAM):**

- It's the main memory of the computer.
- It stores the programs and their data which are needed to be executed by the CPU.
- The CPU can read from and write to RAM .
- When the power is off, all the stored data in RAM are lost (volatile memory).
- When you buy a computer, you pay for RAM not ROM.

3. **Cache Memory:**

- Very high-speed memory acts as temporary area (volatile memory) between the CPU and RAM.
- store program instructions and data that are frequently re-referenced by the program during operation.

External Storage

External Storage (Auxiliary Memory):

- It's a secondary memory (nonvolatile storage).
- It holds the programs and data permanently even if the power is off.
- It has many forms such as Compact Disks (CDs), Digital Versatile Disks (DVDs), and Flash Memories.

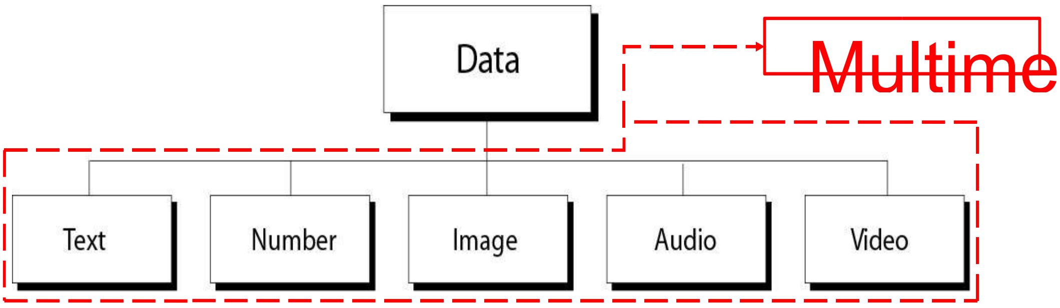
Input Devices:

- Devices that allow a user to enter data to a computer,
- Examples of these devices such as keyboard, mouse, light-pen, scanner, microphone and …etc.

Output Devices:

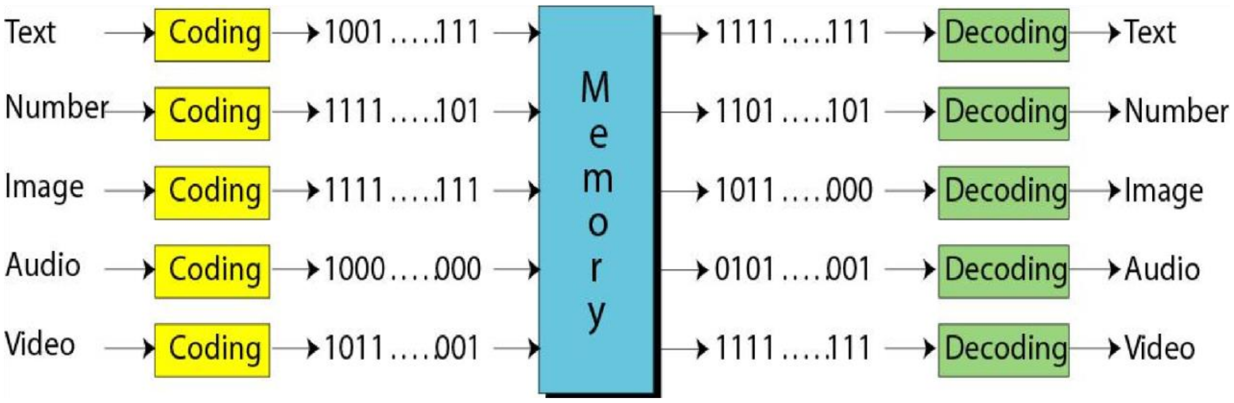
- Examples of these devices are screen, speaker, printer, and …etc.

Data Representation In Modern Computer



Data Coding & Decoding

- Human senses deal with a variety of data.
- Input devices of computer translates these data into electrical signals.
- Electrical signals are then translated into universal format (0s,1s), this is known as coding.
- After processing, output devices transform back data into their original form, this is known as decoding.



Text Representation: Extended ASCII

SYMBOL	DEC	HEX	BINARY
0	48	30	00110000
1	49	31	00110001
.	.	.	.
.	.	.	.
9	57	39	00111001
A	65	41	01000001
B	66	42	01000010
.	.	.	.
.	.	.	.
Z	90	5A	01011010
a	97	61	01100001
b	98	62	01100010
.	.	.	.
.	.	.	.
z	122	7A	01111010
!	33	21	00100001
"	34	22	00100010
#	35	23	00100011
$	36	24	00100100
%	37	25	00100101
&	38	26	00100110
*	42	2A	00101010
+	43	2B	00101011
-	45	2D	00101101
/	47	2F	00101111

Figure 1.13 The message "Hello." in ASCII

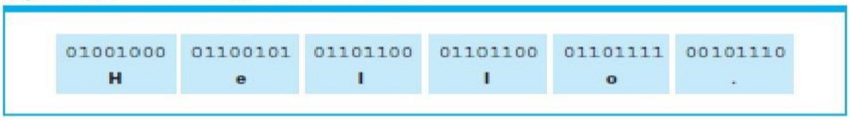
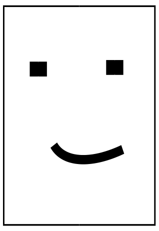
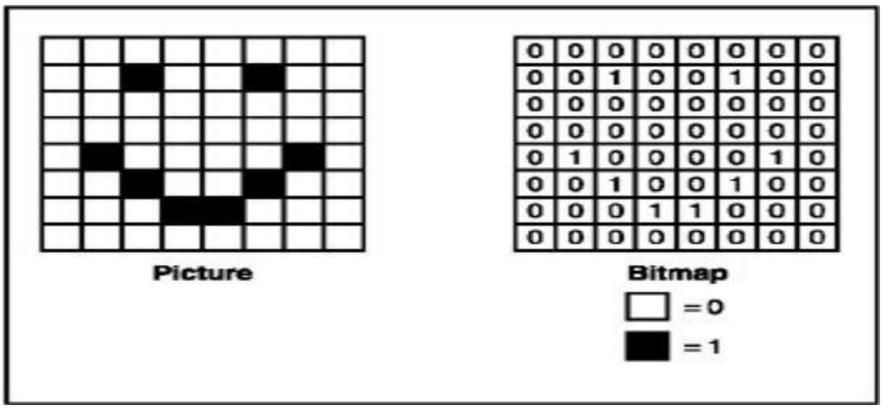


Image Representation

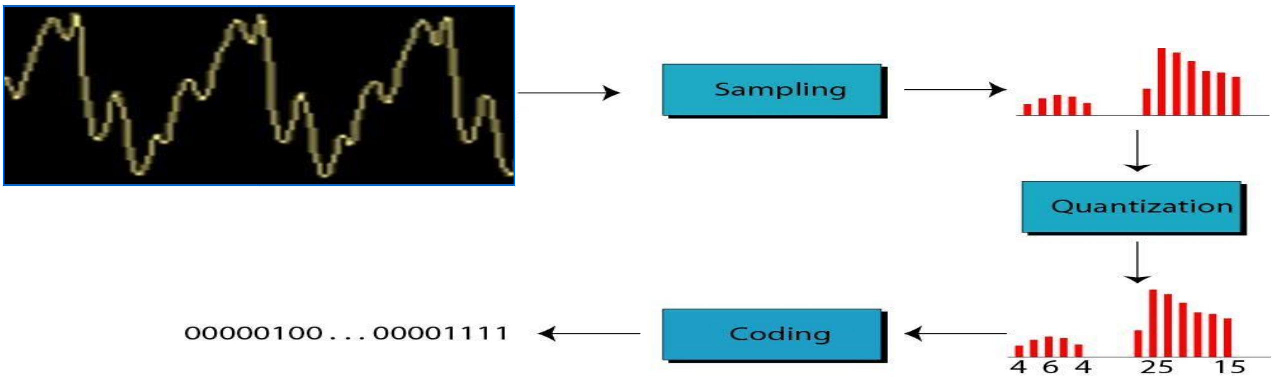
- Images can be **represented as a collection of dots (pixels)**. Such pixels are arranged in rows and columns over the image.
- **Image resolution depends on the number of pixels** in the image and **the higher the resolution the more information the image contains**
- In the case of a simple **black and white image**, each pixel can be represented by **a single bit** whose value depends on whether the corresponding pixel is black or white.





Audio Representation

- Audio is sound and the sound signal is analog signal.
- The representation of audio signal requires converting it from analog signal into digital signal (digitization) by the sampling and quantization processes.
- Videos can be **encoded as series of image frames with synchronized audio tracks also encoded using bits**.



Data representation in Computers

- All information entered to the modern digital computers, (regardless of being numbers, characters, symbols, colors,…), should be represented as a sequence of binary digits.
- A bit has one of two values (0 or 1)

Numbering system

- To define any numbering system, we have to determine:
- its base (radix)
- list of adopted symbols

Number System	**Base/ Radix**	**Symbols**	**Example**
Decimal	10	0,1,2,3,4,5,6,7,8,9	$(3)_{10}$, $(542)_{10}$, $(60.58)_{10}$
Binary	2	0,1	$(0)_2$, $(011)_2$, $(10101.010)_2$
Ternary	3	0,1,2	$(0)_3$, $(021)_3$, $(001.021)_3$
Octal	8	0,1,2,3,4,5,6,7	$(3)_8$, $(542)_8$, $(60.57)_8$
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9, A, B, C, D, E, F	$(AD)_{16}$, $(542)_{16}$, $(60.5A8)_{16}$

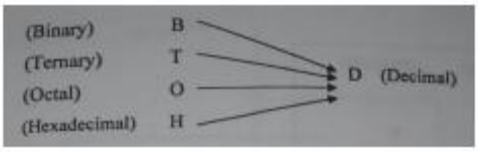
Numbering System Conversion

From any numbering system (R) to decimal numbering system (10)

$(d_n\, d_{n-1} \ldots\ldots\, d_0\, .d_{-1}\, d_{-2} \ldots\ldots\, d_{-m})_R =$

$d_n * R^n + d_{n-1} * R^{n-1} + \ldots\ldots + d_0 * R^0$

$d_{-1} * R^{-1} + d_{-2} * R^{-2} + \ldots\ldots + d_{-m} * R^{-m}$



Example

Example (1) (Decimal system)

The number 724.53 is calculated in the decimal system as follows:

$$(724.53)_{10} = 7*10^2 + 2*10^1 + 4*10^0 + 5*10^{-1} + 3*10^{-2}$$

$$= 700 + 20 + 4 + 0.5 + 0.03$$

Example: From binary to decimal

- The equivalent decimal value for the binary number:

$(11011.101)_2$ =

$$1 * 2^4 + 1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0 + 1 * 2^{-1} + 0 * 2^{-2} + 1 * 2^{-3}$$

$$= (27.625)$$

- Another solution:

1^{16} 1^{8} 0^{4} 1^{2} 1^{1} . $1^{0.5}$ $1^{0.25}$ $1^{0.125}$

= 1*16 +1*8 +0*4 +1*2 +1*1 +1*0.5 +0*0.25 +1.0.125

= $(27.625)_{10}$

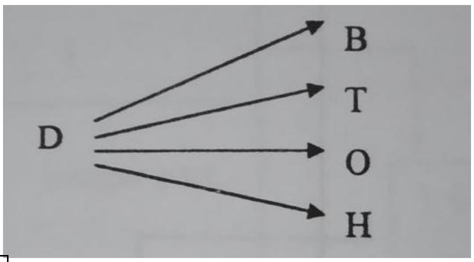
Exercise

Find the decimal equivalent for the following numbers:

- $(201.3)_3$
- $(127.64)_8$
- $(1CF.A3)_{16}$

Lecture 2

From Decimal Numbering system to others



$(502569)10$ is equivalent to $(???)_r$

r/

Integer part	Remainder
502569	

$(0.365)10$ is equivalent to $(???)_r$

r*

Fraction part	Integer
0.365	

Example: From Decimal to Binary (Integer part)

Convert $(41)_{10}$ to its equivalent binary value

i.e., $(41)_{10} = (??)_2$

2 ÷

Integer	Remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1

101001

The conversion from decimal integers to any base-*r* system is similar to the example, except that division is done by *r* instead of 2.

Example: From Decimal to Ternary

Convert $(41)_{10}$ to its equivalent Ternary value

i.e., $(41)_{10}$= $(??)_3$

	Integer part	Reminder part
3 ÷	41	
	13	2
	4	1
	1	1
	0	1

$(1112)_3$

Exercise

Convert $(41)10$ to its equivalent octal and Hexadecimal values

- **i.e., $(41)_{10}$= $(??)_8$**

- **$(41)_{10}$ =$(??)_{16}$**

From Decimal To Binary(Fraction Part)

Convert $(0.6875)_{10}$ to the binary system.

	Integer		Fraction	Coefficient
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} = 1$

$$(0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$$

To convert a decimal fraction to a number expressed in base r, a similar procedure is used. Multiplication is by r instead of 2, and the coefficients found from the integers may range in value from 0 to $r - 1$ instead of 0 and 1.

Example: From Decimal to Binary

Convert $(0.625)_{10}$ to its equivalent binary value

i.e., $(0.625)_{10}$= $(??)_2$

	Fraction part	Integer part
2 *	0.625	
	0.25	1
	0.5	0
	0	1

$(0.101)_2$

Therefore, the number $(0.1011)_2$ is the **exact** binary equivalent to the decimal number $(0.625)_{10}$

Example: From Decimal to Ternary

Convert $(0.012345679)_{10}$ to its equivalent Ternary value

i.e., $(0.012345679)_{10}$ = $(??)_3$

	Fraction part	Integer part
3 *	0.012345679	
	0.0370370375	0
	0.111111111	1
	0.3333333	0
	0.9999999	1

$(0.0101)_3$

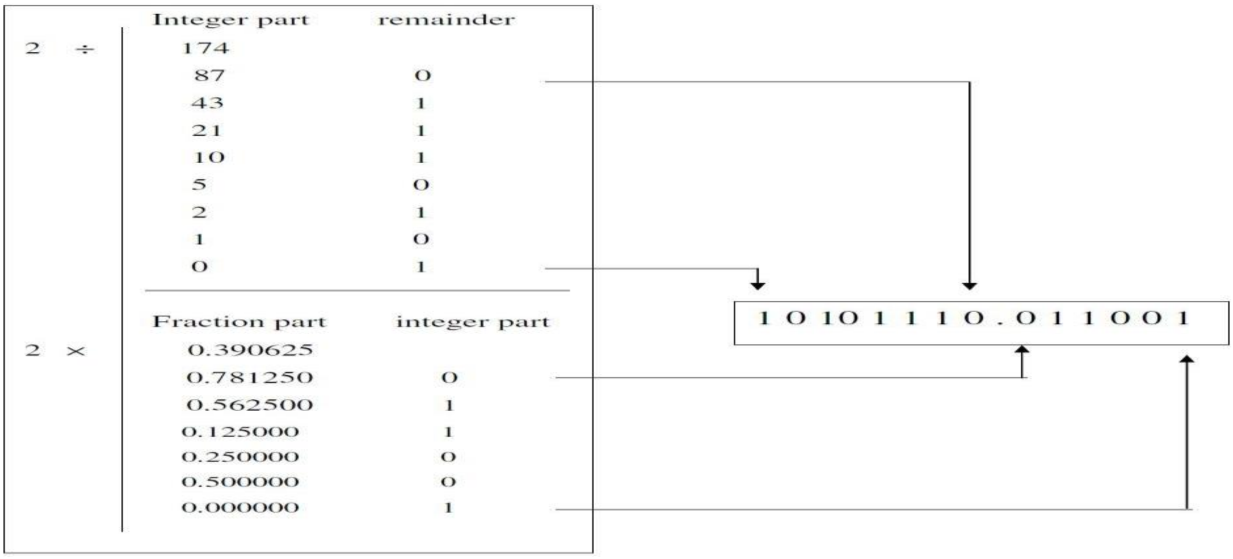
Therefore, the number $(0.0101)_2$ is the ternary equivalent to the decimal number $(0.012345679)_{10}$

<u>Exercise:</u> Convert the following:

- $(0.390625)_{10}$ = $(??)_8$
- $(0.25)_{10}$ = $(??)_{16}$

From Decimal to binary

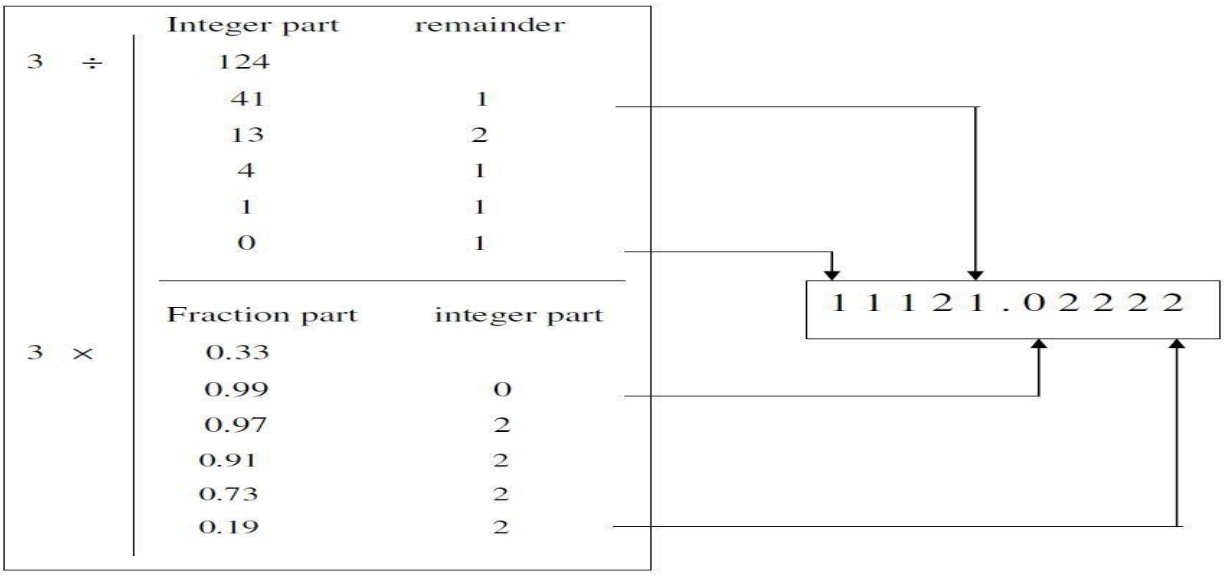
- Convert $(174.390625)_{10}$ to its binary value



Therefore, the number $(10101110.011001)_2$ is the **exact** binary equivalent to the decimal number $(174.390625)_{10}$

From decimal to ternary

Convert the decimal number $(124.33)_{10}$ to its equivalent ternary number



Therefore, the number $(11121.02222)_3$ is the approximate ternary equivalent to the decimal number $(124.33)_{10}$

From Decimal to Octal

Convert the decimal number $(167.390625)_{10}$ to its equivalent octal number.

	Integer part	remainder
8 ÷	167	
	20	7
	2	4
	0	2
	Fraction part	**integer part**
8 ×	0.390625	
	0.125	3
	0	1

2 4 7 . 3 1

Therefore, the number $(247.31)_8$ is the exact octal equivalent to the decimal number $(167.390625)_{10}$

From decimal10 to hexadecimal16

•Convert the decimal $(247.390625)_{10}$ to its equivalent hexadecimal number.

	Integer part	remainder
16 ÷	247	
	15	7
	0	15 = F
	Fraction part	**integer part**
16 ×	0.390625	
	0.250	6
	0.0	4

Therefore, the number $(F7.64)_2$ is the exact hexadecimal equivalent to the decimal number $(247.390625)_{10}$

Exercise

$(95.236)_{10} = (??)_8$

$(153.513)_{10} = (??)_8$

From Binary to Octal and Octal to Binar

There are two methods:

- First (Indirect Conversion):

$$B \longrightarrow D \longrightarrow O$$

$$O \longrightarrow D \longrightarrow B$$

- Second (Direct Conversion):

Each 3-binary digits are replaced by one octal digit , and vice versa , using the following table:

Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

Indirect Method: Example

$(1001101.1011)_2 = (??)_8$

Solution

- Step 1:

convert $(1001101.1011)_2$ to $(??)_{10}$

$$(1001101.1011)_2 = 1*2^6 + 0*2^5 + 0*2^4 + 1*2^3 + 1*2^2 + 0*2^1 + 1*2^0 + 1*2^{-1} + 0*2^{-2} + 1*2^{-3} + 1*2^{-4}$$

$$= 64 + 8 + 4 + 1 + 0.5 + 0.125 + 0.0625$$

$$= (77.6875)_{10}$$

Indirect Method: Example…

Step2:

Convert from $(77.6875)_{10} = (??)_8$

	Integer part	remainder
8 ÷	77	
	9	5
	1	1
	0	1
	Fraction part	**integer part**
8 ×	0.6875	
	0.5	5
	0	4

Thus, $(1001101.1011)_2 = (77.6875)_{10} = (115.54)_8$

From Binary to Octal and Octal to Binary

There are two methods:

- First (Indirect Conversion):

$$B \longrightarrow D \longrightarrow O$$

$$O \longrightarrow D \longrightarrow B$$

- Second (Direct Conversion):

Each 3−binary digits are replaced by one octal digit , and vice versa , using the following table:

Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

Direct Method: Example

Using the direct method, convert the binary number $(1001101.1011)_2$ to octal

1 0 0 1 1 0 1 . 1 0 1 1

0 0 1   0 0 1   1 0 1 . 1 0 1   1 0 0

1   1   5 . 5   4

Another Example

find the octal equivalent of $(10110001101011.111100000110)_2$

$$(\underbrace{10}_{2}\ \underbrace{110}_{6}\ \underbrace{001}_{1}\ \underbrace{101}_{5}\ \underbrace{011}_{3}\ .\ \underbrace{111}_{7}\ \underbrace{100}_{4}\ \underbrace{000}_{0}\ \underbrace{110}_{6})_2 = (26153.7460)_8$$

find the binary equivalent of $(374.26)_8$ and the octal equivalent of $(1110100.0100111)_2$.

Solution

- The given octal number = $(374.26)_8$
- The binary equivalent = $(011\ 111\ 100.010\ 110)_2 = (011111100.010110)_2$

Binary to Hexadecimal and Hexadecimal to Binary

<u>There are two Methods:</u>

- First (Indirect conversion):

$$B \longrightarrow D \longrightarrow H$$

$$H \longrightarrow D \longrightarrow B$$

- Second(Direct conversion):

Each 4-binary digits are replaced by one hexadecimal digit , and vice versa , using the following table:

Hexa	0	1	2	3	4	5	6	7	8	9
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

Hexa	A	B	C	D	E	F
Binary	1010	1011	1100	1101	1110	1111

Direct conversion Example

Convert the hexadecimal $(3BC.\ 2E)_{16}$ directly to its equivalent binary number.

binary

3	B	C	.	2	E
$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$	$\downarrow$
0011	1011	1100	.	0010	1110

Thus, $(3BC.2E)_{16} = (001110111100.00101110)_2$

Direct Conversion Example

- Convert the following:$(110101.101)_2 = (???)_{16}$

Solution

$(0011\ 0101.\ 1010)_2$

$(3 \quad 5 \quad . \quad A)_{16}$

- Convert the following:$(111010.11011)_2 = (???)16$

Solution

$$(111010.11011)_2 = (00111010.11011000)_2 = (3A.D8)_{16}$$

Direct Conversion Example

- Convert the following: $(10110001101011.1110010)_2 = (???)_{16}$

Solution

$$(\underbrace{10}_{2}\ \underbrace{1100}_{C}\ \underbrace{0110}_{6}\ \underbrace{1011}_{B}\ .\ \underbrace{1111}_{F}\ \underbrace{0010}_{2})_2 = (2C6B.F2)_{16}$$

- Convert the following: $(DEF.A1)_{16} = (???)_2$

Solution

$(D \quad E \quad F \; . \; A \quad 1)_{16}$

$(1101\ 1110\ 1111.\ 1010\ 0001)_2$

Hexa-Octal Conversion

There are two methods:

- First Method
  - 1- convert hexa to binary
  - 2- convert binary to Octal
- Second Method
  - 1- convert hexa to decimal
  - 2- convert decimal to octal

Hexa to octal conversion Example

- $(2F.C4)_{16} = (???)_8$

Solution

- The given hex number $= (2F.C4)_{16}$.
- The binary equivalent $= (0010\ 1111.1100\ 0100)_2 = (00101111.11000100)_2$
  $= (101111.110001)_2 = (101\ 111.110\ 001)_2 = (57.61)_8$.

Octal–Hexa Conversion

there are two methods:

- First Method
  1) convert octal to binary
  2) convert binary to hexa
- Second Method
  1) convert Octal to decimal
  2) convert decimal to hexa

Hexa to octal conversion Example

- $(762.013)_8 = (???)_{16}$

Solution

- The given octal number $= (762.013)_8$.
- The octal number $= (762.013)_8 = (111\ 110\ 010.000\ 001\ 011)_2$

$$= (111110010.000001011)_2$$

$$= (0001\ 1111\ 0010.0000\ 0101\ 1000)_2 = (1F2.058)_{16}.$$

Lecture 3

Addition & Subtraction In Binary System

Binary Addition: Rules for binary addition are:

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 0 with 1 to carry for the next column

Example 1: Find the sum of the binary numbers 1101 & 110

1 1 (carry)	
1 1 0 1	1 3
+ 0 1 1 0	+ 6
1 0 0 1 1	1 9

Addition in Binary System: Example

Example 2: Find the sum of the binary numbers 110101.101

+ 10110.111

1 1 1 1 11 1 1	
1 1 0 1 0 1 . 1 0 1	53 . 6 2 5
+ 0 1 01 1 0 . 1 1 1	+ 22 . 8 7 5
1 0 0 1 1 0 0. 1 0 0	7 6 . 5 0 0

Subtraction In Binary System

There are **three Methods** for **subtracting** binary numbers:

- Direct subtraction
- 1's complement
- 2's complement

1st: Direct Subtraction Method

Direct Subtraction Method

Binary Subtraction: Rules for binary addition are:

$0 - 0 = 0$

$1 - 0 = 1$

$1 - 1 = 0$

$0 - 1 = 1$, with 1 borrowed from the next column

Note that: (10) will be 1 after borrowing 1 from it as 10 – 1 = 1.

Example 3: use the direct binary subtraction to get the result of 1100101 – 0100111. Verify the result in decimal system.

$$
\begin{array}{ccccccccc}
 & \cancel{0} & 1 & 1 & & 1 & & \\
 & & & & & \cancel{0} & 10 & \\
 & \cancel{1} & \cancel{1} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} & 1 \\
- & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
\hline
 & 0 & 1 & 1 & 1 & 1 & 1 & 0
\end{array}
\qquad\qquad
\begin{array}{cccc}
 & \cancel{1} & \cancel{0} & \cancel{1} \\
- & & 3 & 9 \\
\hline
 & & 6 & 2
\end{array}
$$

2nd: 1's complement Subtraction Method

1's Complement

The **complement method** allows to **perform** the binary **subtraction** in the form of binary **addition.**

1's Complement: replace every 1 by 0 and every 0 by 1.

Example: find the 1's complement for: 100101

- Solution

011010

Subtraction using 1's Complement

Using the 1's complement:

1- **find** the **1's compelment** for the **second** number

2- **convert** the **subtraction** operation to **addition**

3- **add** the **first number** to the **1's complement** of the **second number.**

4- if the **result** has **no carry**, the result is **a negative number**

if the **result** has a **carry, add the carry** to the result and the result is a **positive number**

Subtraction using 1's Complement: Example

Calculate $11010.11001 - 1101.11110$ using 1's complement

$$
\begin{array}{r}
11010.11001 \\
- \; 01101.11110 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
11010\,.\,11001 \\
+ \; 10010\,.\,00001 \\
\hline
101100.11010 \\
+ \qquad\quad 1 \\
\hline
01100.11011
\end{array}
$$

1's complement of the second number

The result is the positive number: (1100.11011)

Subtraction using 1's Complement: Example 2

Calculate $11010.11001 - 11101.11110$ using 1's complement

$$
\begin{array}{r}
11010\,.\,11001 \\
- \; 11101\,.\,11110 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
11010\,.\,11001 \\
+ \; 00010\,.\,00001 \\
\hline
11100\,.\,11010 \\
\hline
- \; 00011\,.\,00101
\end{array}
$$

1's complement of the second number

no carry (−ve number)

1's complement of the result

The result is the negative number: $(-00011.00101) = (-11.00101)$

Subtraction using 1's Complement: Example 3

Calculate $1011.001 - 0110.100$, using 1's complement

Solution:

$$
\begin{array}{lr}
 & 1\,0\,1\,1\,.\,0\,0\,1 \\
 & +\;1\,0\,0\,1\,.\,0\,1\,1 \\
\hline
\text{Carry} & 1\,0\,1\,0\,0\,.\,1\,0\,0 \\
 & + \qquad\qquad 1 \\
\hline
 & 0\,1\,0\,0\,.\,1\,0\,1
\end{array}
$$

(1's complement of 0110.100)

Then the result is the positive number: 100.101

<u>**Exercise**</u>

Calculate the following using 1's complement

- $1011.001 - 110.101$
- $1001.1 - 11101.011$

3nd: 2's complement Subtraction Method

2's Complement Obtained via two Methods:

1) By **adding** 1 to the **1's complement.**

2) Start the binary number from right. Leave the binary digits unchanged until the first 1 appear, after it replace every 1 by 0 , and every 0 by 1.

<u>2's Complement: Example</u>

<u>Example:</u> find the 2's complement for: 1011010.110

<u>First solution</u>

```
  0 1 0 0 1 0 1 . 0 0 1   <---- 1's complement
+                     1
  ---------------------
  0 1 0 0 1 0 1 . 0 1 0
```

<u>Second solution</u>

```
1 0 1 1 0 1 0 . 1 1 0
          ⇩
0 1 0 0 1 0 1 . 0 1 0
```

2's Complement Subtraction Method <u>Rule:</u>

1- find the 2's compelment for the second number

2- convert the subtraction operation to addition

3- add the first number to the 2's complement of the second number.

4- if the result has no carry, the result is a negative number

if the result has a carry, omit the carry and the result is a positive number

2's Complement Subtraction Method: Example

culate 11101.101 – 10100.010, using 2's complement

```
   11111
   1 1 1 0 1 . 1 0 1
+  0 1 0 1 1 . 1 1 0
   -----------------
 1 0 1 0 0 1 . 0 1 1
```

the result is the positive number: 01001.011

2’s Complement Subtraction Method: Example 2

Calculate $\begin{array}{r} 11010.11001 \\ -\ \underline{01101.11110} \end{array}$ using 2’s complement

Solution

$$
\begin{array}{rl}
 & {\color{red}1}\ \ {\color{red}1} \\
 & 11010.11001 \\
+ & 10010.00010 \quad \textbf{2's of the second number} \\
\hline
\cancel{1} & 01100.11011
\end{array}
$$

the result is the positive number: 1100.11011

2’s Complement Subtraction Method: Example 3

Calculate $\begin{array}{r} 11010.11001 \\ -\ \underline{11101.11110} \end{array}$ using 2’s complement

– Solution:

$$
\begin{array}{rl}
 & \ \ \ {\color{red}1} \\
\hline
 & 11010.11001 \\
+ & 00010.00010 \quad \text{2's of the second number} \\
\hline
\text{no carry} & 11100.11011 \\
- & 00011.00101 \quad \text{2's of the result}
\end{array}
$$

the result is the negative number: (-11.00101)

Solved Excercise

Calculate the following using direct, 1’s, and 2’s complement subtraction method:

- 11101.101 – 1011.11

Direct subtraction

$$
\begin{array}{rr}
 & 11101.101 \\
- & 01011.110 \\
\hline
 & 10001.111
\end{array}
$$

1’s complement

$$
\begin{array}{rr}
 & 11101.101 \\
+ & 10100.001 \\
\hline
 & \boxed{1}\,10001.110 \\
+ & 1 \\
\hline
 & 10001.111
\end{array}
$$

2’s complement

$$
\begin{array}{rr}
 & 11101.101 \\
+ & 10100.010 \\
\hline
 & \cancel{\boxed{1}}\,10001.111
\end{array}
$$

Solved Excercise 2

Calculate the following using 1's, and 2's complement subtraction methods:

- 1101.101 – 11011.11

1's complement	2's complement
01101.101	01101.101
+ 00100.001	+ 00100.010
$\boxed{0}$ 10001.110	$\boxed{0}$ 10001.111
- 01110. 001	- 01110. 001

<u>Excercise</u>

Calculate the following:

- 1101.11101 + 10101.011
- 11011.101 − 1010.01 (using direct subtraction)
- 10101001.01 − 1001111.01 (using 1's complement)
- 01100.101 − 1101.100 (using 2's complement)

<u>9's complement</u> •

9's complement of a decimal number is the subtraction of it's each digits from 9

Example

a) 2 7 1 7 5 0

9	**9**	**9**	**9**	**9**	**9**
2	**7**	**1**	**7**	**5**	**0**
7	2	8	2	4	9

b) 8 5 2 8 7 4

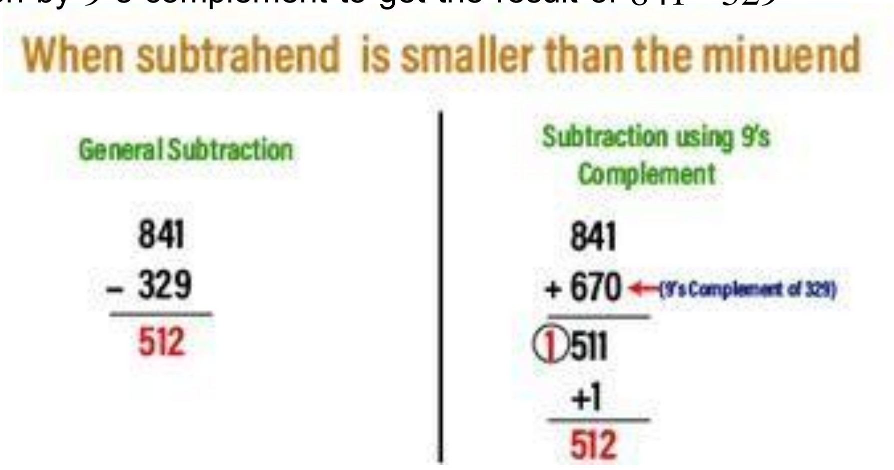
9	**9**	**9**	**9**	**9**	**9**
8	**5**	**2**	**8**	**7**	**4**
1	4	7	1	2	5

Subtraction using 9's complement

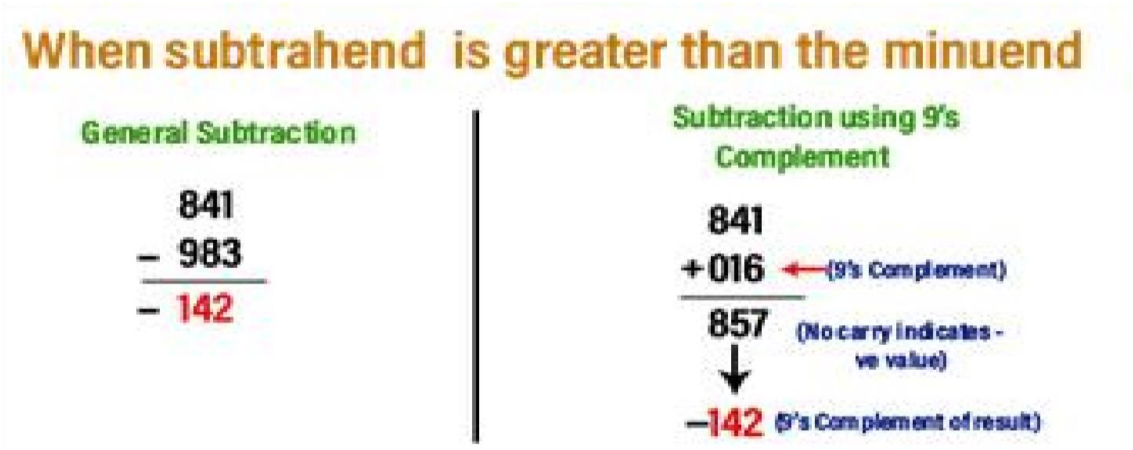
- Like 1's complement, 9's complement is used to subtract a number using addition.

Example

use subtraction by 9's complement to get the result of $841 - 329$



use subtraction by 9's complement to get the result of $841 - 98$



10's complement

- 10's complement of a decimal number can be found by adding 1 to the 9's complement of that decimal number.
- It is just like 2s compliment in binary number representation.
- 10's complement = 9's complement + 1

- **Example:** Find the 10's complement of the following numbers:

  2 7 7 5 0