

- 1 Following is the data on heights and weights of ten students in a class:

Heights (in cm)	140	142	140	160	150	155	160	157	140	170
Weights (in cm)	43	45	42	50	45	52	57	48	49	53

Calculate rank correlation coefficient between heights and weights of students.

- 2
- | Student | Chemistry Grade, $y$ | Test Score, $x_1$ | Classes Missed, $x_2$ |
|---------|----------------------|-------------------|-----------------------|
|---------|----------------------|-------------------|-----------------------|

1	85	65	1
2	74	50	7
3	76	55	5
4	90	65	2
5	85	55	6
6	87	70	3
7	94	65	2
8	98	70	5
9	81	55	4
10	91	70	3
11	76	50	1
12	74	55	4

- (a) Fit a multiple linear regression equation of the form  $\hat{y} = b_0 + b_1x_1 + b_2x_2$ .
- (b) Estimate the chemistry grade for a student who has an intelligence test score of 60 and missed 4 classes.

- 3 From the following data, obtain  $R_{1.23}$ ,  $R_{2.13}$  and  $R_{3.12}$

$X_1$	2	5	7	11
$X_2$	3	6	10	12
$X_3$	1	3	6	10

Q.1:-

H	W	H <sub>i</sub>	W <sub>i</sub>	d <sub>i</sub>	d <sub>i</sub> <sup>2</sup>
140	43	9	9	0	0
142	45	7	7.5	-0.5	0.25
140	42	9	10	-1	1
160	50	2.5	4	-1.5	2.25
150	45	6	7.5	-1.5	2.25
155	52	5	3	2	4
160	57	2.5	1	1.5	2.25
157	48	4	6	-2	4
140	49	9	5	4	16
170	53	1	2	-1	1
					$\Sigma d^2 = 33$

9n x → 140 repeats 3 times  
 $\therefore CF = \frac{m(m^2-1)}{12} = \frac{3(9-1)}{12}$

CF = 2

160 repeats 2 times

$CF = \frac{2(4-1)}{12} = 0.5$

9n y series 45 repeats 2 times

$\therefore CF = \frac{2(4-1)}{12}$

CF = 0.5

$P(x, y) = 1 - \frac{6(\Sigma d^2 + C.F)}{n(n^2-1)} \Rightarrow CF = \Sigma m$

$= 1 - \frac{6(33 + (2 + 0.5 + 0.5))}{10(10^2-1)}$

$1 - \frac{216}{990} = 1 - 0.218$

$= 0.78$

Heights and weights are positively correlated by 78.2%.



Q.2

y	$x_1$	$x_2$	$(x_2)^2$	$(x_1)^2$	$yx_1$	$yx_2$	$x_1x_2$
85	65	1	1	4225	5525	85	65
74	50	7	49	2500	3700	518	350
76	55	5	25	3025	4180	380	275
90	65	2	4	4225	5850	180	130
85	55	6	36	3025	4675	510	330
87	70	3	9	4900	6090	261	210
94	65	2	4	4225	6110	188	130
98	70	5	25	4900	6860	490	350
81	55	4	16	3025	4455	324	220
91	70	3	9	4900	6370	273	210
76	50	1	1	2500	3800	76	50
74	55	4	16	3025	4070	296	220
$\Sigma \Rightarrow$ 1011	725	43	195	44475	61685	3581	2540

$$d) \Sigma y = nb_0 + b_1 \Sigma x_1 + b_2 \Sigma x_2 \quad \text{--- (1)}$$

$$\Sigma yx_1 = b_0 \Sigma x_1 + b_1 \Sigma x_1^2 + b_2 \Sigma x_1x_2 \quad \text{--- (2)}$$

$$\Sigma yx_2 = b_0 \Sigma x_2 + b_1 \Sigma x_1x_2 + b_2 \Sigma x_2^2 \quad \text{--- (3)}$$

$$\Rightarrow 1011 = 12b_0 + 725b_1 + 43b_2 \quad \text{--- (a)}$$

$$\Rightarrow 61685 = 725b_0 + 44475b_1 + 2540b_2 \quad \text{--- (b)}$$

$$\Rightarrow 3581 = 43b_0 + 2540b_1 + 195b_2 \quad \text{--- (c)}$$

Solving (a), (b), (c) gives

$$b_0 = 27.54$$

$$b_1 = 0.921$$

$$b_2 = 0.284$$

$\therefore$  The regression equation is  $y = 27.54 + 0.921x_1 + 0.284x_2$



$$(b) x_1 = 60 \quad x_2 = 4$$

$$\begin{aligned} Y &= 27.54 + 0.921(60) + 0.284(4) \\ &= 27.54 + 55.26 + 1.136 \\ &= 83.936 \end{aligned}$$

∴ The chemistry grade of the student who missed 4 classes & has test score = 60 is 83.936.



Q.3

$x_1$	$x_2$	$x_3$	$(x_1)^2$	$(x_2)^2$	$(x_3)^2$	$x_1 x_2$	$x_2 x_3$	$x_1 x_3$
2	3	1	4	9	1	6	3	2
5	6	3	25	36	9	30	18	15
7	10	6	49	100	36	70	60	42
11	12	10	121	144	100	132	120	110
25	31	20	199	289	146	238	201	169

$$\begin{aligned}
 r_{12} &= \frac{n(\sum x_1 x_2) - \sum x_1 \sum x_2}{\sqrt{[n(\sum x_1^2) - (\sum x_1)^2][n(\sum x_2^2) - (\sum x_2)^2]}} \\
 &= \frac{4(238) - (25)(31)}{\sqrt{[4(199) - 625][4(289) - 961]}} \\
 &= \frac{952 - 775}{\sqrt{[171][195]}} = \frac{177}{\sqrt{33345}} = \frac{177}{182.61} = \boxed{0.9692}
 \end{aligned}$$

$$\begin{aligned}
 r_{13} &= \frac{n(\sum x_1 x_3) - \sum x_1 \sum x_3}{\sqrt{[n(\sum x_1^2) - (\sum x_1)^2][n(\sum x_3^2) - (\sum x_3)^2]}} \\
 &= \frac{4(169) - (25)(20)}{\sqrt{[4(199) - 625][4(146) - 400]}} \\
 &= \frac{676 - 500}{\sqrt{[171][184]}} \\
 &= \frac{176}{\sqrt{31464}} = \frac{176}{177.38} = \boxed{0.992}
 \end{aligned}$$



$$r_{23} = \frac{n(\sum x_2 x_3) - \sum x_2 \sum x_3}{\sqrt{[n(\sum x_2^2) - (\sum x_2)^2][n(\sum x_3^2) - (\sum x_3)^2]}}$$

$$= \frac{4(201) - (31)(20)}{\sqrt{[4(289) - 961][4(146) - 400]}}$$

$$= \frac{804 - 620}{\sqrt{[196][184]}}$$

$$= \frac{184}{\sqrt{35880}} = \frac{184}{189.42} = \boxed{0.971}$$

Now,

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{13}}{1 - r_{23}^2}}$$

$$= \sqrt{\frac{0.9393 + 0.984 - (2)(0.9692)(0.992)(0.971)}{1 - 0.942}}$$

$$= \boxed{0.98} \Rightarrow R_{1.23}$$

$$R_{2.13} = \sqrt{\frac{r_{21}^2 + r_{23}^2 - 2r_{12}r_{23}r_{13}}{1 - r_{13}^2}}$$

$$= \sqrt{\frac{0.9393 + 0.9428 - 1.867}{1 - 0.984}}$$

$$= \sqrt{\frac{0.0151}{0.016}}$$

$$= \sqrt{0.9437} = \boxed{0.9714} \Rightarrow R_{2.13}$$



$$R_{3.12} = \sqrt{\frac{r_{31}^2 + r_{32}^2 - 2r_{12}r_{23}r_{13}}{1 - r_{12}^2}}$$

$$= \sqrt{\frac{0.984 + 0.9428 - 1.867}{1 - 0.9393}}$$

$$= \sqrt{\frac{0.0598}{0.0607}}$$

$$= \sqrt{0.9851}$$

$$= \boxed{0.992} \Rightarrow R_{3.12}$$