بخش (دو)

(سؤال ۲)

الف

A. $\frac{1}{2}(x[n] + x[n-1])$

$x[n] = \delta[n] \longrightarrow y[n] = \begin{cases} 0 & n \leq -1 \\ \frac{1}{2} & 0 < n \leq 1 \\ 0 & n > 1 \end{cases}$

$x[n] = u[n] \longrightarrow y[n] = \begin{cases} 0 & n \leq -1 \\ \frac{1}{2} & n = 0 \\ 1 & n > 0 \end{cases}$

B. $y[n] = x[n+1] - x[n]$

$x[n] = \delta[n] \longrightarrow y[n] = \begin{cases} 0 & n < -1 \\ 1 & n = -1 \\ -1 & n = 0 \\ 0 & n > 0 \end{cases}$

$x[n] = u[n] \longrightarrow y[n] = \begin{cases} 0 & n < -1 \\ 1 & n = -1 \\ 0 & n > -1 \end{cases}$

C. $y[n] = x[n+1] - 2x[n] + x[n-1]$

$x[n] = \delta[n] \longrightarrow y[n] = \begin{cases} 0 & n<-1 \\ 1 & n=-1 \\ -2 & n=0 \\ 1 & n=1 \\ 0 & n>1 \end{cases}$

$x[n] = u[n] \longrightarrow y[n] = \begin{cases} 0 & n<-1 \\ 1 & n=-1 \\ -1 & n=0 \\ 0 & n>0 \end{cases}$

D. $y[n] = \sum_{m=0}^{19} \cos\left(\frac{\pi m}{10}\right) x[n-m]$

$x[n] = \delta[n] \longrightarrow y[n] = \begin{cases} \cos\left(\frac{n\pi}{10}\right) & 0 \le n \le 19 \\ 0 & \text{otherwise} \end{cases}$

$x[n] = u[n] \longrightarrow y[n] = \begin{cases} 0 & n<0 \\ \sum_{i=0}^{n} \cos\left(\frac{i\pi}{10}\right) & 0 \le n < 20 \\ \sum_{i=n-19}^{n} \cos\left(\frac{i\pi}{10}\right) & 20 \le n \end{cases}$

بخش سوم
(سؤال سوم)

الف)

$$y[n] = x[n] * h[n] = \begin{cases} 0 & n < 0 \\ \sum_{i=0}^{n} 2^{-i} & 0 \le n \le 3 \\ \sum_{i=n-4}^{n} 2^{-i} & 4 \le n \end{cases}$$

بخش ۳
(سؤال چهارم
پیوسته زمان)

۱- ب)

$$y(t) = \int_{-\infty}^{+\infty} e^{-a\tau} u(\tau)\, u(t-\tau)\, d\tau$$

$$= \int_0^t e^{-a\tau} d\tau = -\frac{e^{-a\tau}}{a} \Big|_0^t = \frac{-e^{-at}}{a} + \frac{1}{a}$$

$a > 0$ x

$$= \frac{1 - e^{-at}}{a} \implies$$

۲- الف)

$$y(t) = x(t) * h(t)$$

$$y(t) = \begin{cases} 0 & ; t \leq -2 \\ \frac{(t+2)^2}{2} & ; -2 < t \leq -1 \\ \frac{1}{2} + t + 1 = t + \frac{3}{2} & ; -1 < t \leq 1 \\ \frac{5}{2} + \frac{1 + (2-t)}{2} = 4 - \frac{t}{2} & ; 1 < t \leq 2 \\ 3 & ; 2 < t \end{cases}$$