A relation in number, which makes it divisible by three

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Abstract. A number of sequence based on a relationship of: $\frac{1}{2}n$, 2n, $\frac{1}{5}n$ or 5n, which were put together in odd iterations, is always divisible by three.

e.g.

$$17 \underbrace{,\ 34}_{1} \underbrace{,\ 68}_{2} \underbrace{,\ 136}_{3}$$

This is a three iteration sequence with relation of "2n"
We can see: 173,468,136 ÷ 3 = 57,822,712

- If the first number in sequence be a three multiplication itself, the number of iterations doesn't need to be necessarily odd; it would work for even numbers too.

- There are two examples, one for odd numbers and the other for even numbers, given at the end of this paper, you can check that to better understand the theory and conclusion and mainly how it works. (Page 13)

To understand this better and prove it step by step, we will start by finding a function that would help us represent a whole number as a result of adding each number to the previous number. (For example, sequence 3, 15, 75 would be defined as one number 31575)

Relation of 5n and 2n:

The general formula is shown below:

(1) $$\sum_{i=0}^{iterations} x5^i 10^{\sum_{j=i+1}^{iterations}[(\log x5^i)+1]}$$ | For 5n Relation

(2) $$\sum_{i=0}^{iterations} x2^i 10^{\sum_{j=i+1}^{iterations}[(\log x2^i)+1]}$$ | For 2n relation

$iterations$: Number of iterations you want the sequence be based on, and it should be odd.

x : The first number of sequence (May also refer as input number)

i : The current iteration number

j : The auxiliary variable

$10^{\sum_{j=i+1}^{iterations}[(\log x2or5^i)+1]}$ In this part, we multiply the current number by 10 to the power of the sum of the further iteration' s digits count, from the next iteration till the end of iterations. (Square brackets are used as a floor). Inside the brackets, there is a log with a base of 10, and adding 1 to the log' s result will show us the current iteration' s digits count.

Then we multiply it by 5 or 2 (depending on the relation) to the power of the current iteration, and next multiply it by x, so it will tell us the exact number that we will get in sequence by multiplying our input number, after that, we sum all the numbers with zeroes in front, so finally this formula provides a simple derivation to find the whole number at once.

Relation of 5n and 2n (proof):

The method used to prove the theory is based on "the rule of three": a number is divisible by three if and only if the sum of digits is divisible by 3. By considering the rule, we will first prove the theory based on 5n and 2n relation. Then, we will tell the proof based on divisions.

- We know that the number of iterations should be odd.
- We consider the input number as x.

- 5n relation:

i	Sum	Sequence
I	$6x$	x $5x$
II	$4x$	x $5x$ $25x$
III	$3x$	x $5x$ $25x$ $125x$
IV	$7x$	x $5x$ $25x$ $125x$ $625x$
V	$9x$	x $5x$ $25x$ $125x$ $625x$ $3125x$
VI	$1x$	x $5x$ $25x$ $125x$ $625x$ $3125x$ $15625x$
VII	$6x$	x $5x$ $25x$ $125x$ $625x$ $3125x$ $15625x$ $78125x$
VIII	$4x$	x $5x$ $25x$ $125x$ $625x$ $3125x$ $15625x$ $78125x$ $390625x$
IX	$3x$	x $5x$ $25x$ $125x$ $625x$ $3125x$ $15625x$ $78125x$ $390625x$ $1953125x$

TABLE 1

Table 1 shows that the sum of all digits with odd iteration (due to the concept of iteration on the first page) will always repeat the same pattern: 6-3-9.

- 2n relation:

i	Sum	Sequence
I	$3x$	$x\ 2x$
II	$7x$	$x\ 2x\ 4x$
III	$6x$	$x\ 2x\ 4x\ 8x$
IV	$4x$	$x\ 2x\ 4x\ 8x\ 16x$
V	$9x$	$x\ 2x\ 4x\ 8x\ 16x\ 32x$
VI	$1x$	$x\ 2x\ 4x\ 8x\ 16x\ 32x\ 64x$
VII	$3x$	$x\ 2x\ 4x\ 8x\ 16x\ 32x\ 64x\ 128x$
VIII	$7x$	$x\ 2x\ 4x\ 8x\ 16x\ 32x\ 64x\ 128x\ 256x$
IX	$6x$	$x\ 2x\ 4x\ 8x\ 16x\ 32x\ 64x\ 128x\ 256x\ 512x$

TABLE 2

Table 2 shows that the sum of all digits with odd iteration will always repeat the same pattern of 3-6-9.

We all know "the rule of three"; so by considering that rule, our number of sequences, in odd iterations, would have the sum that is divisible by three, so it will also be itself divisible by three.

- in both tables, shown iterations are from one to nine, but the conclusion that I said is based on a larger scale of iterations and sequences, but because of the large space it would acquire, I showed it on shorter sequences.

Relation of $\frac{1}{5}n$ and $\frac{1}{2}n$:

For this relation, I could not find a fixed formula like the last two formulas because we may get numbers with decimal points or with choosing a large value for iterations, we should divide too much, and we may get into smaller amounts than one (e.g., we will get 0.0875 based on iteration=5 and x=24 with $\frac{1}{2}n$ relation), and even some values need to be updated after each iteration.

Let us start with understanding these formulas below:

$$(3) \quad \delta = \sum_{m=1}^{\log_{\varepsilon} x} \mathrm{I}\{x \bmod \varepsilon^m = 0\}$$

δ: Counts how many times the number is divisible by 2 or 5.

ε: For $\frac{1}{2}n$ relation we replace "ε" with "2", and for $\frac{1}{5}n$ we replace "ε" with "5".

$\frac{1}{2}n$ Relation formula:

$$(4) \quad \mathrm{I}\{x \bmod 2^i = 0\} \times \frac{x}{2^i} + \mathrm{I}\{x \bmod 2^i \neq 1\} \times x5^i10^{-\delta}$$

$$(5) \quad \beta = \sum_{i=0}^{itteration} \mathrm{I}\{x \bmod 2^i = 0\} \times \frac{x}{2^i} + \mathrm{I}\{x \bmod 2^i \neq 1\} \times x5^i10^{-\delta}$$

$\frac{1}{2}n$ Relation formula:

$$(6) \quad \mathrm{I}\{x \bmod 5^i = 0\} \times \frac{x}{5^i} + \mathrm{I}\{x \bmod 5^i \neq 1\} \times x2^i10^{-\delta}$$

$$(7) \quad \beta = \sum_{i=0}^{itteration} \mathrm{I}\{x \bmod 5^i = 0\} \times \frac{x}{5^i} + \mathrm{I}\{x \bmod 5^i \neq 1\} \times x2^i10^{-\delta}$$

x: The first number of the sequence. (May also refer as input number)

i: The current iteration number.

I{ }: Indicators, returns 0 or 1 depending to the statement inside.

β: We will use it to counts digits till the end of iteration except for the current number.

In (4) and (6), we are calculating the iteration number, and you can see after the second indicator (when the current number isn't divisible by 5 or 2), for 2 division instead of $\frac{x}{2^i}$ we multiplied by $x5^i$ and precisely the opposite for 5 division, it has a simple reason why I have done that, with dividing the number by 2 or 5 when the current number isn't divisible by it we will get a number with a decimal value, the product of 2 and 5 is 10, just for reminding! So, if you have a non-divisible number consider as A, divide that number by two and name it A2; if you multiply that number by five and call it A5, the ratio of these numbers to each other is ten times, $\frac{A5}{A2} = 10$ And the opposite is precisely accurate for five divisions, and Indeed with this method, we are dropping the decimal point out.

$\frac{19}{2} = 9.5 \quad \rightarrow \quad 19 \times 5 = 95 \quad | \quad \frac{19}{5} = 3.8 \quad \rightarrow \quad 19 \times 2 = 38$

E.g. Input(x) = 18, iterations=3, division by 2, generation=2 (i=2)

- δ According to the formula will be: 1
- The number without using the method: $\frac{18}{4}$ = 4.5
- The number with using the method: $18 \times 25 \times 10^{-1}$ = 45

E.g. Input(x) = 18, iterations=3, division by 5, generation=2 (i=2)

- δ According to the formula will be: 0
- The number without using the method: $\frac{18}{25}$ = 0.72
- The number with using the method: $18 \times 4 \times 10^{0}$ = 72

$$(8) \quad output = \sum_{i=0}^{itteration} Theta1 \times 10^{\beta}$$

(8): This is our general formula for $\frac{1}{5}n$ and $\frac{1}{2}n$ relation, but we should update the values each iteration according to formulas below:

- It' s not a simultaneously update, so the order is essential.
- At the beginning, theta1 = input number (x)

$$output := output \times 10^{\,\mathrm{I}\left\{\frac{x}{2^{i+1}} < 0\right\} \times \left[-\left(\log \frac{x}{2^{i+1}}\right) + 1\right]}$$

$$\text{theta1} := \mathrm{I}\left\{x \bmod 2^{i} = 0\right\} \times \frac{theta1}{2} + \mathrm{I}\left\{x \bmod 2^{i} \neq 1\right\} \times theta1 \times 5$$

$$\beta := \beta - [(\log\ theta1\ ) + 1]$$

Relation of $\frac{1}{5}n$ and $\frac{1}{2}n$ (proof):

- $\frac{1}{5}n$ relation:

i	Sum	Sequence
I	$\frac{6}{5}x$	$x\ \frac{1}{5}x$
II	$\frac{31}{25}x$	$x\ \frac{1}{5}x\ \frac{1}{25}x$
III	$\frac{156}{125}x$	$x\ \frac{1}{5}x\ \frac{1}{25}x\ \frac{1}{125}x$
IV	$\frac{781}{625}x$	$x\ \frac{1}{5}x\ \frac{1}{25}x\ \frac{1}{125}x\ \frac{1}{625}x$
V	$\frac{3906}{3125}x$	$x\ \frac{1}{5}x\ \frac{1}{25}x\ \frac{1}{125}x\ \frac{1}{625}x\ \frac{1}{3125}x$
VI	$\frac{19531}{15625}x$	$x\ \frac{1}{5}x\ \frac{1}{25}x\ \frac{1}{125}x\ \frac{1}{625}x\ \frac{1}{3125}x\ \frac{1}{15625}x$
VII	$\frac{97656}{78125}x$	$x\ \frac{1}{5}x\ \frac{1}{25}x\ \frac{1}{125}x\ \frac{1}{625}x\ \frac{1}{3125}x\ \frac{1}{15625}x\ \frac{1}{78125}x$
VIII	$\frac{488281}{390625}x$	$x\ \frac{1}{5}x\ \frac{1}{25}x\ \frac{1}{125}x\ \frac{1}{625}x\ \frac{1}{3125}x\ \frac{1}{15625}x\ \frac{1}{78125}x\ \frac{1}{390625}x$
IX	$\frac{2441406}{1953125}x$	$x\ \frac{1}{5}x\ \frac{1}{25}x\ \frac{1}{125}x\ \frac{1}{625}x\ \frac{1}{3125}x\ \frac{1}{15625}x\ \frac{1}{78125}x\ \frac{1}{390625}x\ \frac{1}{1953125}x$

TABLE 3

According to table 3 sum of digits in numerator repeat, the same pattern of 6-4-3-7-9-1 every six iterations. And in odd iteration, the 6-3-9 pattern repeats. So by the "rule of three" it is always divisible by three.

- $\frac{1}{2}n$ Relation:

i	Sum	Sequence
I	$\frac{3}{2}x$	$x\ \frac{1}{2}x$
II	$\frac{7}{4}x$	$x\ \frac{1}{2}x\ \frac{1}{4}x$
III	$\frac{15}{8}x$	$x\ \frac{1}{2}x\ \frac{1}{4}x\ \frac{1}{8}x$
IV	$\frac{31}{16}x$	$x\ \frac{1}{2}x\ \frac{1}{4}x\ \frac{1}{8}x\ \frac{1}{16}x$
V	$\frac{63}{32}x$	$x\ \frac{1}{2}x\ \frac{1}{4}x\ \frac{1}{8}x\ \frac{1}{16}x\ \frac{1}{32}x$
VI	$\frac{127}{64}x$	$x\ \frac{1}{2}x\ \frac{1}{4}x\ \frac{1}{8}x\ \frac{1}{16}x\ \frac{1}{32}x\ \frac{1}{64}x$
VII	$\frac{255}{128}x$	$x\ \frac{1}{2}x\ \frac{1}{4}x\ \frac{1}{8}x\ \frac{1}{16}x\ \frac{1}{32}x\ \frac{1}{64}x\ \frac{1}{128}x$
VIII	$\frac{511}{256}x$	$x\ \frac{1}{2}x\ \frac{1}{4}x\ \frac{1}{8}x\ \frac{1}{16}x\ \frac{1}{32}x\ \frac{1}{64}x\ \frac{1}{128}x\ \frac{1}{256}x$
IX	$\frac{1023}{512}x$	$x\ \frac{1}{2}x\ \frac{1}{4}x\ \frac{1}{8}x\ \frac{1}{16}x\ \frac{1}{32}x\ \frac{1}{64}x\ \frac{1}{128}x\ \frac{1}{256}x\ \frac{1}{512}x$

TABLE 4

According to table 4, we can see the numerator of the sum of each iteration is $2^{i+1}-1$ and we now that $2^{i+1}=(-1)^{i+1}\, mod\, 3$ if iteration be an odd number, iteration+1 would be even and $(-1)^{i+1}$ will be +1, and the equation will become: $2^{i}-1=0\, mod\, 3$, and this number is always divisible by three. So here in odd iteration (i+1 was even, so iteration will be odd), we have a $2^{i+1}-1$ pattern cycling, and it is always divisible by 3.

In both proofs, if in dividing when we get to a decimal value, we should remove the decimal point (we shown with the formulas: if with dividing by two you get to a decimal value you should multiply it by five rather than dividing by two), so the denominator does not have a value to us in proving.

- CODE SECTION: (Python)

Theta0: is our input number (x) – the first number in the sequence.

Theta1: is our auxiliary variable, and at first, Theta1 = Theta0.

Iterations: is the number of iterations that we want. (Should be odd)

Choice: should be either 5 or 2, which one you want to divide with.

Delta: is δ, as in the formula shown.

Beta: is β, as in the formula shown.

- Relation of 5n or 2n according to the formulas:

```
def output_2(theta0, iteration):
    output = beta = 0

    for i in range (0 , iteration+1):

        for j in range (i+1, iteration+1):
            beta += math.floor(math.log10(theta0*pow(2,j))+1)

        output += theta0*pow(2,i)*pow(10,beta)

        beta = 0

    return output


def output_5(theta0, iteration):
    output = beta = 0

    for i in range (0 , iteration+1):

        for j in range (i+1, iteration+1):
            beta += math.floor(math.log10(theta0*pow(5,j))+1)

        output += theta0*pow(5,i)*pow(10,beta)

        beta = 0

    return output
```

- Relation of $\frac{1}{5}n$ and $\frac{1}{2}n$ according to the formulas:

```python
def output(theta0, iteration, choice):
    # values for a and b are set to default for 2 division
    a,b = 2,5

    # exchanging the values of a and b together if the choice was 5
    if choice==5:
        a,b = b,a # a,b = 5,2


    theta1 = theta0
    output = delta = beta = 0

    # finding how many dividable by a it has
    for m in range (0, math.floor(math.log(theta0,a))):
        if (theta0/(pow(a,m)))%a == 0:
            delta += 1

    # finding sum of the digits
    for j in range (1, iteration+1):
        beta += math.floor(math.log10( (theta0*pow(b,j)/pow(10,delta))
        \\ if theta0%pow(a,j) !=0 else (theta0*pow(b,j)/pow(10,j)) )+1)

    # calculating the output
    for i in range (0 , iteration+1):

        # adding the zeroes 0.007b adding 3 zeroes
        if theta0/pow(a,i) < 1:
            output = output*pow(10,int(-math.log10(theta0/pow(a,i))+1))

        # adding the values to output
        output += int(theta1*pow(10,beta))

        # updating the theta1 to the next number
        theta1 = int(theta1/a if theta1%a == 0 else  theta1*b )

        # updating alpha with removing digit counts of current number
        beta -= math.floor((math.log10(theta1)+1))

    return output
```

At first, I was trying to find the whole number according to the sequence, turn it into a code, then finding some relations and a way to prove it. So I know those parts were not necessary for proving the theory, but the coding part and inventing some formulas to find the whole number with being mathematically calculative were so challenging for me and the most enjoyable part that I wanted to show.

Finally, I want to thank my great teacher Mr.Makki and his friendly help and guides for finishing this project. He was always there for me, answering all my questions. Thank you so much.

* Examples:

- 5n Relation:

Input number (x): 7915

Iterations: 7

Result: 791539575197875989375494687524734375123671875618359375
= 3 * 2.638465250659586631251648958E+53

Division remainder = 0.000000

- 2n Relation:

Input number (x): 192

Iterations: 15

Result: 1923847681536307261441228824576491529830419660839321678643215
7286431457286291456
= 3 * 6.412825605121024204804096082E+78

Division remainder = 0.000000

- $\frac{1}{5}n$ Relation:

Input number (x): 13

Iterations: 7

Result: 1326052010400208000416000083200001664
= 3 * 4.420173368000693334720000277E+35

Division remainder = 0.000000

- $\frac{1}{2}n$ Relation:

Input number (x): 14

Iterations: 9

Result: 14735175087504375021875010937500546875002734375
= 3 * 4.911725029168125007291670313E+45

Division remainder = 0.000000

Let's have fun with a huge number:

$\frac{1}{5}n$ Relation

Input number (x): 531

Iterations: 179

Result:
5312655132756637533187516593758296875414843752074218751037109375518
5546875259277343751296386718756481933593753240966796875162048339843
7581024169921875405120849609375202560424804687510128021240234375506
4010620117187525320053100585937512660026550292968756330013275146484
3753165006637573242187515825033187866210937579125165939331054687539
5625829696655273437519781291484832763671875989064574241638183593754
9453228712081909179687524726614356040954589843751236330717802047729
4921875618165358901023864746093753090826794505119323730468751545413
3972525596618652343757727066986262798309326171875386353349313139915
4663085937519317667465656995773315429687596588337328284978866577148
4375482941686641424894332885742187524147084332071244716644287109375
1207354216603562235832214355468756036771083017811179161071777343753
0183855415089055895805358886718751509192770754452794790267944335937
5754596385377226397395133972167968753772981926886131986975669860839
8437518864909634430659934878349304199218759432454817215329967439174
6520996093754716227408607664983719587326049804687523581137043038324
9185979366302490234375117905685215191624592989683151245117187558952
8426075958122964948415756225585937529476421303797906148247420787811
2792968751473821065189895307412371039390563964843757369105325949476
5370618551969528198242187536845526629747382685309275984764099121093
7518422763314873691342654637992382049560546875921138165743684567132
7318996191024780273437546056908287184228356636594980955123901367187
5230284541435921141783182974904775619506835937511514227071796057089
1591487452387809753417968755757113535898028544579574372619390487670
8984375287855676794901427228978718630969524383544921875143927838397
4507136144893593154847621917724609375719639191987253568072446796577
4238109588623046875359819595993626784036223398288711905479431152343
7517990979799681339201811169914435595273971557617187589954898998406
6960090558495721779763698577880859375449774494992033480045279247860
8898818492889404296875224887247496016740022639623930444940924644470
2148437511244362374800837001131981196522247046232223510742187556221
8118740041850056599059826112352311611175537109375281109059370020925
0282995299130561761558055877685546875140554529685010462514149764956
5280880779027938842773437570277264842505231257074882478264044038951
3969421386718753513863242125261562853744123913202201947569847106933
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5769535671801548915027524344623088836669921875219616452632828847678

3590077445751376217231154441833496093751098082263164144238391795038
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4405430778861045837402343752745205657910360595979487596807189220271
5389430522918701171875137260282895518029798974379840359461013576947
1526145935058593756863014144775901489948718992017973050678847357630
7296752929687534315070723879507449743594960089865253394236788153648
3764648437517157535361939753724871797480044932626697118394076824188
2324218758578767680969876862435898740022466313348559197038412094116
2109375428938384048493843121794937001123315667427959851920604705810
5468752144691920242469215608974685005616578337139799259603023529052
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3671875536172980060617303902243671251404144584284949814900755882263
1835937526808649003030865195112183562570207229214247490745037794113
1591796875134043245015154325975560917812851036146071237453725188970
5657958984375670216225075771629877804589064255180730356187268625944
8528289794921875335108112537885814938902294532127590365178093634312
9724264144897460937516755405626894290746945114726606379518258904681
7156486213207244873046875837770281344714537347255736330318975912945
2340857824310660362243652343754188851406723572686736278681651594879
5647261704289121553301811218261718752094425703361786343368139340825
7974397823630852144560776650905609130859375104721285168089317168406
9670412898719891181542607228038832545280456542968755236064258404465
8584203483520644935994559077130361401941627264022827148437526180321
2920223292921017417603224679972795385651807009708136320114135742187
5130901606460111646460508708801612339986397692825903504854068160057
0678710937565450803230055823230254354400806169993198846412951752427
0340800285339355468753272540161502791161512717720040308499659942320
6475876213517040014266967773437516362700807513955807563588600201542
4982997116032379381067585200071334838867187581813504037569779037817
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1810718781491015359329938005927829181018751114606857299804687512783
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8649902343756391680002935138987329526796953727538398324845014819572
9525468777865171432495117187531958400014675694936647633984768637691
9916242250740978647627343889325857162475585937515979200007337847468
3238169923843188459958121125370489323813671944662928581237792968757
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4642906188964843753994800001834461867080954248096079711498953028134
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