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K.N. Toosi University of Technology
Faculty of Electrical Engineering

System Identification

Assignment 1

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Deadline: 5th December 2021

Part I
Analytical

Problem 1: Weighting sequence and step response

Assume that the weighting sequence $\{h(k)\}_{k=0}^{\infty}$ of a system is given. Let $y(t)$ be the step response of the system. Show that $y(t)$ can be obtained by integrating the weighting sequence, in the following sense:

$$y(t) - y(t-1) = h(t)$$
$$y(-1) = 0$$

Assume that correlation analysis is applied in the case where $u(t)$ is zero-mean white noise. Evaluate the variances of the estimates $\hat{h}(k), k = 1, 2, \ldots$ using the results of Appendix A3.2

Problem 2: Accuracy of correlation analysis

Consider the noise-free first-order system

$$y(t) + ay(t-1) = bu(t-1) \quad |a| < 1$$

Assume that correlation analysis is applied in the case where $u(t)$ is zero-mean white noise. Evaluate the variances of the estimates $\hat{h}(k), k = 1, 2, \ldots$ using the results of Appendix A3.2 of the book "System Identification" by Torsten Söderström.

Problem 3

Consider the Laplace transform of the real signal $u(t)$ given by $U(s)$, If we apply this signal to a linear time-invariant system with a transfer function given by $H(s)$

$$U(s) = \frac{c_0 s^m + c_1 s^{m-1} + \cdots + c_m}{s^n + d_1 s^{n-1} + \cdots + s_n}$$
$$H(s) = \frac{b_0 s^q + b_1 s^{q-1} + \cdots + b_q}{s^r + a_1 s^{r-1} + \cdots + a_r}$$

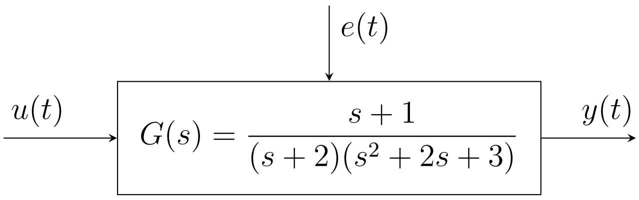
a. Given that $U(s)$ can be computed from $u(t)$ and $y(t)$ is observable in the time domain, and that $Y(s) = H(s)U(s)$, can we determine the transfer function $H(s)$? What conditions are needed in practice? Explain.

b. In your opinion, how does this method compare to system identification using frequency response diagrams? What are the main differences? Which method can be faster and which is more reliable?

Part II
Simulation (Nonparametric Identification)

Problem 1: Transient Analysis

Consider the following system



The noise $e(t)$ is white with a variance $\lambda_e^2 = 0.01$.

a. Identify the system by applying an impulse input and state whether an impulse input is suitable for identifying this system, with justification.

b. Determine the transfer function $G(s)$ in the given form for the second-order system $G(s) = \dfrac{K\omega_n^2}{s^2 + 2\zeta\omega_n + {\omega_n}^2} e^{\tau s}$, by applying a step input with an appropriate gain. Additionally, discuss potential applications of this approximation.

c. Compare the simulated step response of the estimated model obtained from the previous parts with the actual step response. Generate plots to visualize the error for both methods.

d. To analyze the effect of increased noise variance on identification, add white noise with variance $\lambda_e^2 = 0.1$ to the output and then repeat the previous steps. Compare the new simulated step response of the estimated model with the actual step response, and generate plots to visualize the error for both methods. Discuss the impact of the increased noise variance on the accuracy of the identification process.

Problem 2: Frequency Analysis

Consider the following system

$$y(t) = a_1 y(t-1) + a_2 y(t-2) = b_1 u(t-1) + b_2 u(t-2) + c_1 e(t)$$

Identify the given system with the provided coefficients ($a_1 = -1.4, a_2 = 0.6, b_1 = 1, b_2 = 0.3$) using frequency analysis with a sinusoidal input of suitable amplitude. Calculate $\hat{H}(e^{-j\omega})$ and compare it with the actual frequency response of the system.

a. Perform basic frequency analysis when $c1 = 0$.

b. Perform basic frequency analysis when $c1 \neq 0$ and $\lambda_e^2 = 0$.

c. Perform basic frequency analysis when $c1 \neq 0$ and $\lambda_e^2 = 0.1$.

d. Compare the previous sections together. Investigate the impact of white and colored noise on the system identification and, to improve the results in the presence of noise, follow the steps below:

e. Perform improved frequency analysis when $c1 \neq 0$ and $\lambda_e^2 = 0$.

f. Perform improved frequency analysis when $c1 \neq 0$ and $\lambda_e^2 = 0.1$.

g. Compare the results with the previous steps.

Problem 3: Correlation analysis

Consider the system described by the equation

$$y(t) = a_1 y(t-1) + a_2 y(t-2) + b_1 u(t-1) + b_2 u(t-2) + c_1 e(t)$$

where the assumption is made that $\lambda_e^2 = 0.1$. Generate a dataset of $n = 1000$ input and output data points. Next, apply correlation analysis to estimate the impulse response denoted as $\hat{h}(k)$ under the following conditions:

a. Utilize white noise as the input with an appropriate variance.

b. Apply a step input with a suitable amplitude and a rectangular window of length $M = 5\%$.

c. Employ a sinusoidal input with a suitable amplitude and a rectangular window of length $M = 5\%$.

d. After obtaining the impulse response for each part, compute the step response of the system using the estimated $\hat{h}(k)$ values. Subsequently, compare the obtained step response with the actual step response. Discuss the impact of using different input signals and windowing techniques on the accuracy of the impulse response estimation and the resulting system step response.

Problem 4: Spectral Analysis

Consider the system described by the equation with the assumption of $\lambda_e^2 = 0.1$:

$$y(t) = a_1 y(t-1) + a_2 y(t-2) + b_1 u(t-1) + b_2 u(t-2) + c_1 e(t)$$

a. For the input $u(t) = \sin(0.5t)$, plot $\hat{U}(\omega)$ over the frequency range $\omega \in [-\pi, \pi]$ and estimate $\hat{H}(j\omega)$ using spectral analysis. Plot the magnitude and phase response.

b. Repeat the previous step using ”Hamming” and ”Bartlett” windows of suitable lengths.

c. Compare the system identification results from the two previous steps with each other and the original system. Provide a thorough analysis of the comparisons.

d. Now, repeat part (a) with a white noise input with suitable variance. Based on the results, discuss which input is better for spectral analysis.

e. Finally, state which of the simulated identification methods used in the previous parts is most suitable for identifying the system, both with and without noise. Justify your choice with reasoning.

In the simulation section, personal reasoning and analysis play a vital role in solving the exercises. Ensure that you clearly explain the thought process and rationale behind your chosen approach in each exercise.

Please keep the following points in mind when submitting your assignment:

- Double-check and adhere to the specified deadline for assignment submission.
- Organize all the required files (report [in pdf and (tex or docx)] and MATLAB code [m-file]) in a single folder with the following format: `HW#_Name_StudentID.zip`.
- Only one file is allowed for submission, so ensure all necessary materials are included in the ZIP file.
- If you encounter any technical issues or face difficulties during the submission process, promptly contact the teaching assistant, Moein Sarabandi, via email for assistance.

Best regards,
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