# Assignment 04
# TTK4255 - Robotsyn

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# Contents

# 1 PART I - Transformation between a plane and its image

## 1.1 Task 1.1

given the following equation:

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = k\mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

where scaling factor $k \neq 0$ and $\mathbf{H}$ is:

$$\mathbf{H} = \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix}$$

such that:

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = k \begin{bmatrix} r_{11}X + r_{12}Y + t_x \\ r_{21}X + r_{22}Y + t_y \\ r_{31}X + r_{32}Y + t_z \end{bmatrix}$$

dividing by $\tilde{z}$ to get dehomogenized coordinates:

$$x = \frac{\tilde{x}}{\tilde{z}} = \frac{k(r_{11}X + r_{12}Y + t_x)}{k(r_{31}X + r_{32}Y + t_z)} = \frac{r_{11}X + r_{12}Y + t_x}{r_{31}X + r_{32}Y + t_z}$$

$$y = \frac{\tilde{y}}{\tilde{z}} = \frac{k(r_{21}X + r_{22}Y + t_y)}{k(r_{31}X + r_{32}Y + t_z)} = \frac{r_{21}X + r_{22}Y + t_y}{r_{31}X + r_{32}Y + t_z}$$

Hence the homography matrix $\mathbf{H}$ defines the same mapping for the 2D dehomogenized coordinates for any k different from zero.

## 1.2 Task 1.2

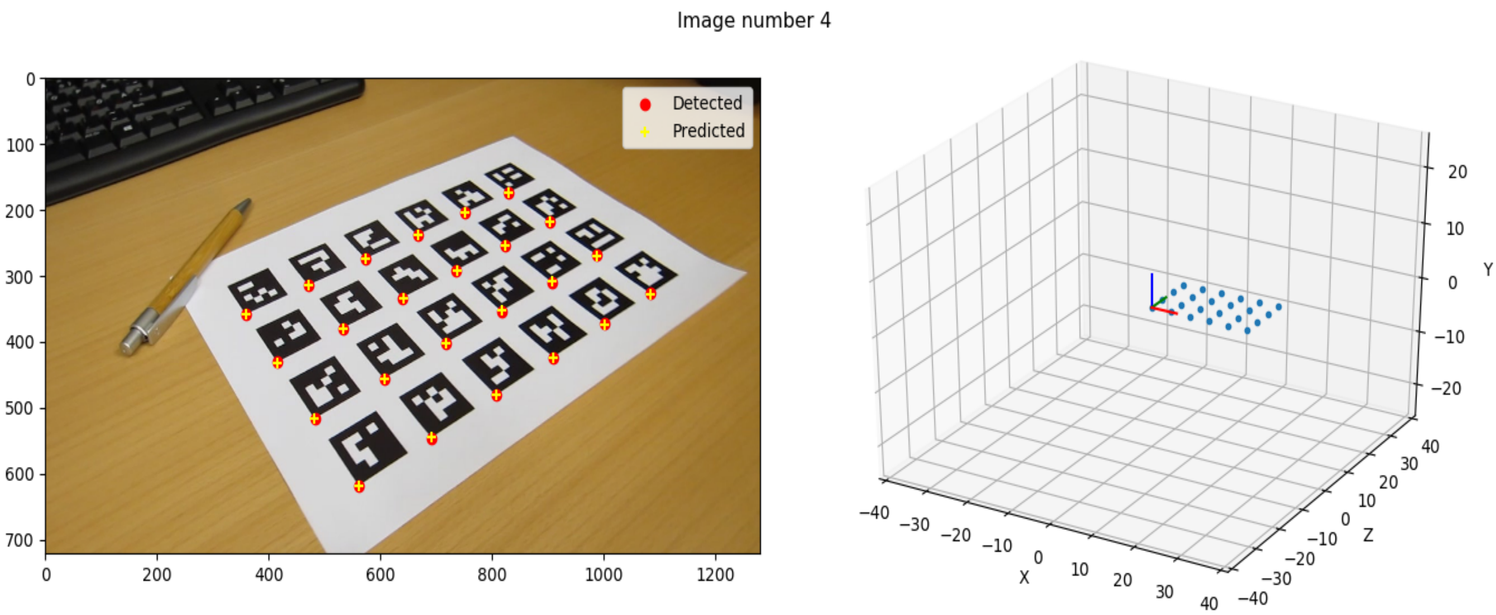
To understand why this particular homography is restricted, it is important to understand how transformations of euclidean, similarity, affine and projective work.

The euclidean transformation performs only rotation and/or translation, and does not affect distances between points in a picture. Hence there are 5 degrees of freedom, in our case 3 for translaton and 2 for rotation. So whilst an arbitrary homography can have up to 8 degrees of freedom because of transformations that allow for that sort of behavior, our case is restricted.

More theoretically, we know that the first two column vectors of $\mathbf{H}$ are orthogonal and hence the dot product between the two should be zero. The norm of each of these two first column vectors on their own is one, which also indicates that they are part of the SO(3) rotation group. Hence the first two columns are dependant on each other allowing for only two degrees of freedom. Whilst each element in the third vector represents translation and gives 3 degrees of freedom. Hence this specific homography is more constrained.

# 2 PART II - The direct linear transform

## 2.1 Task 2.1




There is no body frame as this is implemented in task 3.

## 2.2 Task 2.2

The mean reprojection error was found to be 0.428895pixels, between the detected and predicted coordinates of image 4.

The minimum reprojection error was found to be 0.123860pixels, between the detected and predicted coordinates of image 4.

The maximum reprojection error was found to be 0.936816pixels, between the detected and predicted coordinates of image 4.

# 3 PART III - Recover the pose

## 3.1 Task 3.1

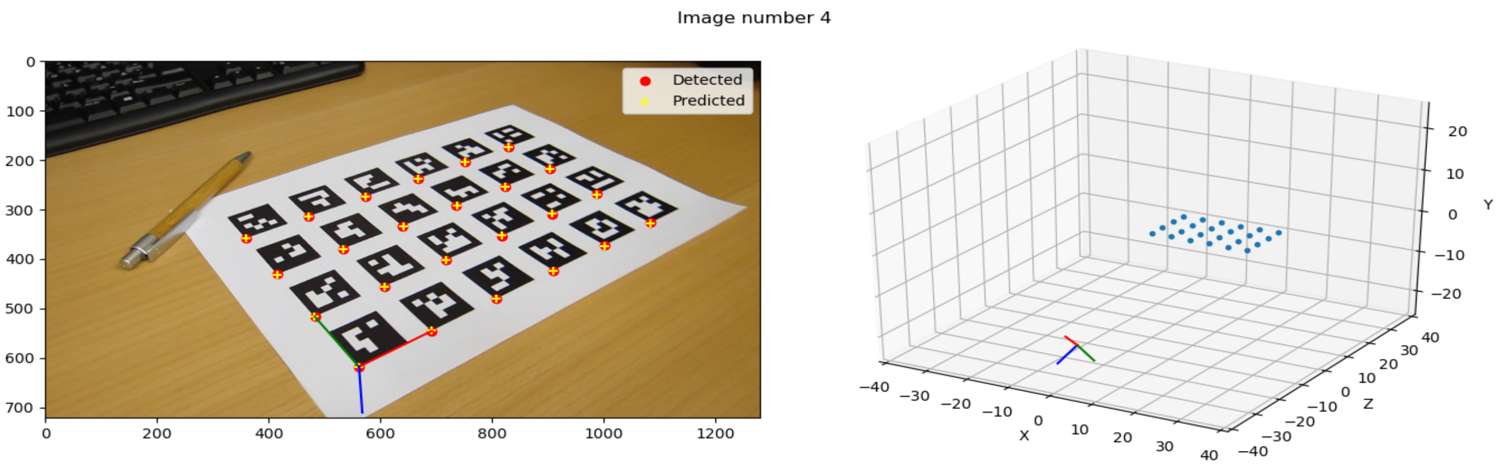



Figure 1: Transformation matrix T1, where k is positive.

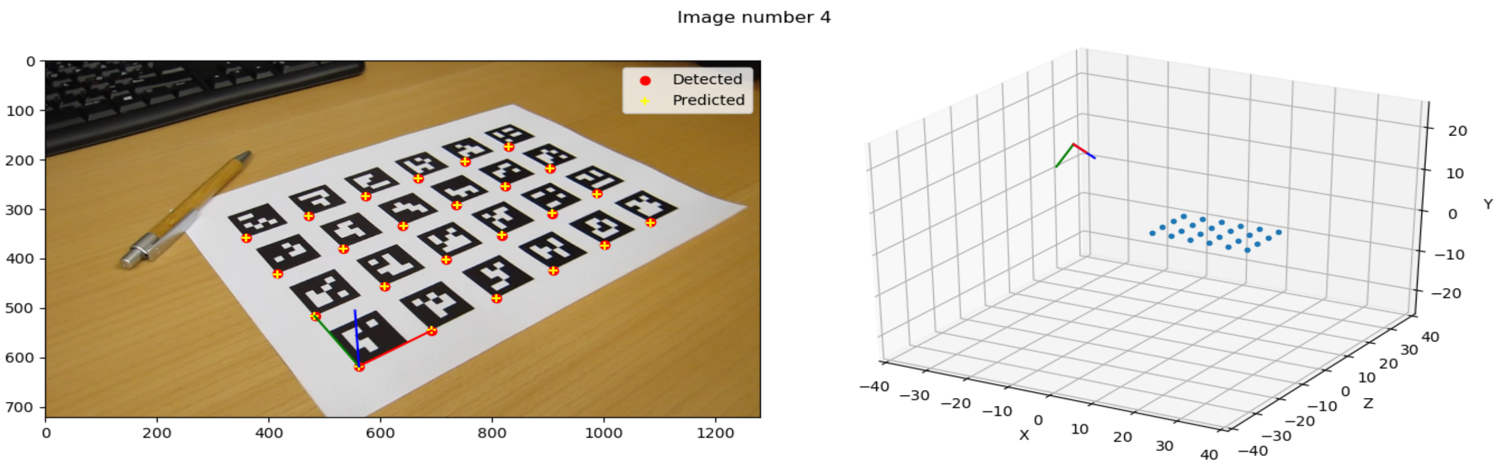



Figure 2: Transformation matrix T2, where k is negative.

## 3.2 Task 3.2

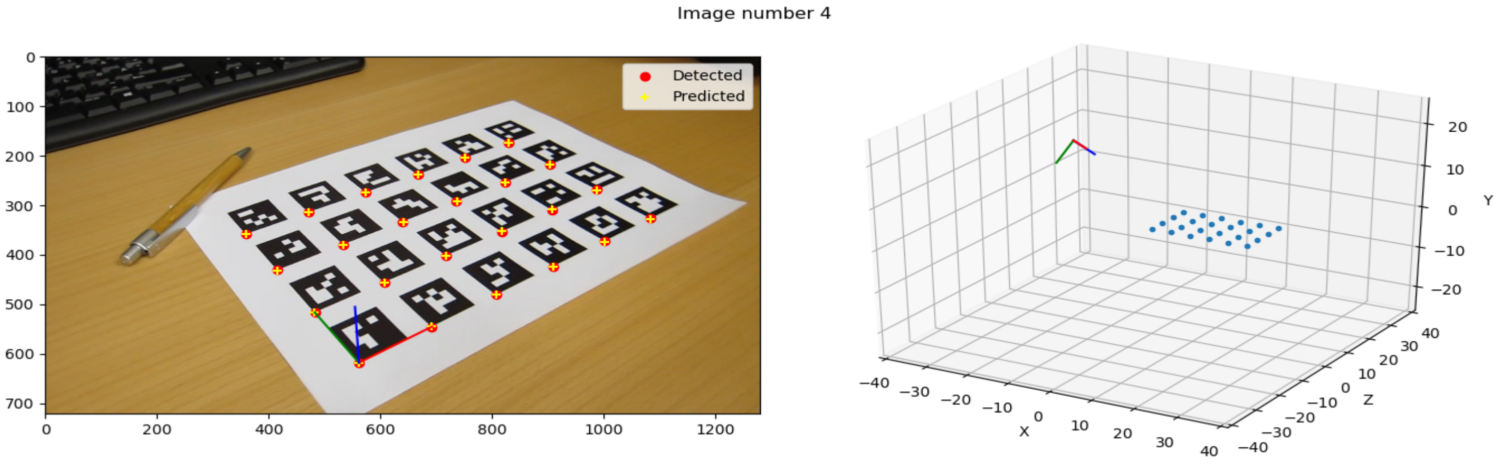



Figure 3: Picture 4

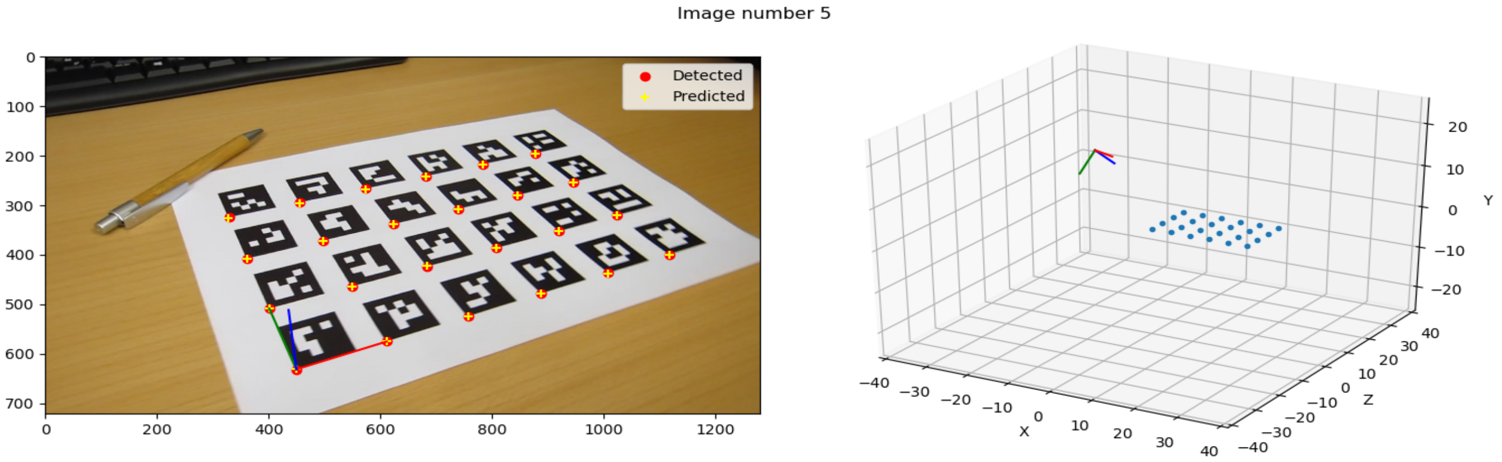



Figure 4: Picture 5

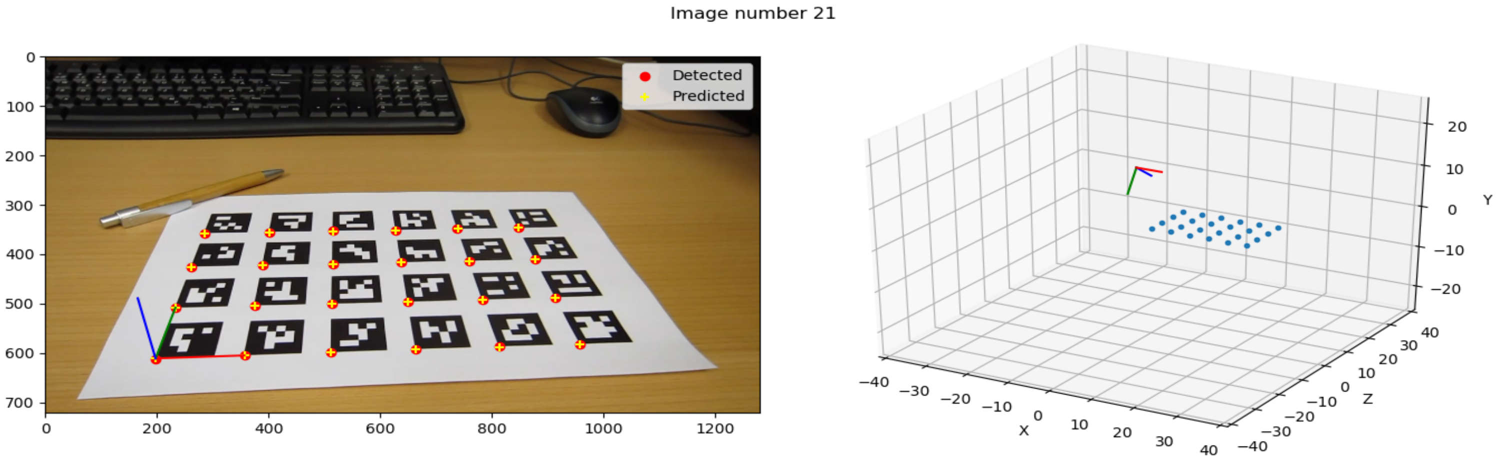



Figure 5: Picture 21

### 3.3 Task 3.3

Known properties of a rotation matrix comes from the fact that a 3x3 rotation matrix is made of 3 orthogonal vectors such that:

$$\mathbf{R}\mathbf{R}^T = \mathbf{I} \implies \mathbf{R}^{-1} = \mathbf{R}^T$$

The best way to quantify how well a matrix satisfies the 3x3 rotation matrix properties is to use the determinant which in theory should be the following:

$$det(\mathbf{R}\mathbf{R}^T) = det(\mathbf{R})^2 = det(\mathbf{I}) = 1 \implies det(\mathbf{R}) = \pm 1$$

Given that $\mathbf{Q} = \mathbf{U}\mathbf{S}\mathbf{V}^T$ is the rotation matrix without the correction and $\mathbf{R} = \mathbf{U}\mathbf{V}^T$ is the corrected rotation matrix, the following determinants were found:

$$\mathbf{Q} = 0.9959559111979558$$

$$\mathbf{R} = 0.9999999999999994 \approx 1$$

# 4 PART VI - Derive your own linear algorithm

## 4.1 Task 4.1

We have the following relationship between $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{x}}$, looking at the simple case for derivation purposes:

$$\tilde{\mathbf{x}} = \mathbf{K}^{-1}\tilde{\mathbf{u}} = \mathbf{R}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \mathbf{t}$$

Expanding the equation further:

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

This gives the following equations when dividing by the last element:

$$x = \frac{\tilde{x}}{\tilde{z}} = \frac{r_{11}X + r_{12}Y + r_{13}Z + t_x}{r_{31}X + r_{32}Y + r_{33}Z + t_z}$$

$$y = \frac{\tilde{y}}{\tilde{z}} = \frac{r_{21}X + r_{22}Y + r_{23}Z + t_y}{r_{31}X + r_{32}Y + r_{33}Z + t_z}$$

where if we multiply both sides we get:

$$r_{11}X + r_{12}Y + r_{13}Z - (r_{31}X + r_{32}Y + r_{33}Z + t_z)x = -t_x$$

$$r_{21}X + r_{22}Y + r_{23}Z - (r_{31}X + r_{32}Y + r_{33}Z + t_z)y = -t_y$$

from this we are able to get $\mathbf{Am} = \mathbf{b}$:

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 0 & 0 & 0 & -X_1x_1 & -Y_1x_1 & -Z_1x_1 \\ 0 & 0 & 0 & X_1 & Y_1 & Z_1 & -X_1y_1 & -Y_1y_1 & -Z_1y_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_n & Y_n & Z_n & 0 & 0 & 0 & -X_nx_n & -Y_nx_n & -Z_nx_n \\ 0 & 0 & 0 & X_n & Y_n & Z_n & -X_ny_n & -Y_ny_n & -Z_ny_n \end{bmatrix} \begin{bmatrix} r_{11} \\ r_{12} \\ r_{13} \\ r_{21} \\ r_{22} \\ r_{23} \\ r_{31} \\ r_{32} \\ r_{33} \end{bmatrix} = \begin{bmatrix} t_zx_1 - t_x \\ t_zy_1 - t_y \\ \vdots \\ t_zx_n - t_x \\ t_zy_n - t_y \end{bmatrix}$$

## 4.2 Task 4.2

A system of linear equations $\mathbf{Am} = \mathbf{b}$ is called homogeneous if $\mathbf{b} = \mathbf{0}$ and in-homogeneous if $\mathbf{b} \neq \mathbf{0}$. Since:

$$\mathbf{Am} = \mathbf{b}$$

where $\mathbf{b} \neq \mathbf{0}$, hence the system of linear equations is in-homogeneous.

### 4.3 Task 4.3

To recover the rotation matrix $\mathbf{R}$ from the equation above, we can find the pseudo inverse of $A$/least-square solution to the matrix equation:

$$\mathbf{A}\mathbf{m} = \mathbf{b}$$

$$\mathbf{A}^T\mathbf{A}\mathbf{m} = \mathbf{A}^T\mathbf{b}$$

$$\mathbf{m} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b} = \mathbf{A}^{+}\mathbf{b}$$

We do this as matrix $\mathbf{A}$ is not square and therefore not invertible. The elements in vector $\mathbf{m}$ can be now reshaped into the form of matrix $\mathbf{R}$.