Assignment 02
TTK4255 - Robotsyn

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1 Task 1 - Theory

1.1 Task 1.1

The standard form might be problematic as the values of the slope parameter a in $y = ax + b$ might become unbounded as it becomes more vertical. Hesse Normal form solves this kind of issue of "infinite slopes" by having the straight line be represented by ρ and θ

1.2 Task 1.2

using the equation $\rho = x \cos\theta + y \sin\theta$

for a) $\theta = 0°$:

$$\rho = x \implies \rho \in [0, L]$$

for b) $\theta = 180°$:

$$\rho = -x \implies \rho \in [-L, 0]$$

for c) $\theta = 45°$:

$$\rho = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \implies \rho \in [0, \sqrt{2}L]$$

for d) $\theta = -45°$:

$$\rho = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y \implies \rho \in [-\frac{\sqrt{2}}{2}L, \frac{\sqrt{2}}{2}L]$$

1.3 Task 1.3

Given a radially symmetric w, Why is Harris-Stephens measure is invariant to intensity shifts? Answer:

$$\frac{\partial(I)}{\partial x} = \frac{\partial(I+c)}{\partial x} = I_x$$

$$\frac{\partial(I)}{\partial y} = \frac{\partial(I+c)}{\partial y} = I_y$$

$$A = \begin{bmatrix} w * I_x^2 & w * I_x I_y \\ w * I_x I_y & w * I_y^2 \end{bmatrix}$$

as seen here, the intensity shift invariance is independent of the weight w.
Given a radially symmetric w (example 2D gaussian function), why is Harris-Stephens measure invariant to image rotation? Answer:

A can be represented with eigenvalue matrix in the middle and eigenvector matrices as such, because of its symmetric property:

$$A = V^{-1} \begin{bmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{bmatrix} V$$

where the two eigenvectors determine the direction of largest and smallest changes of appearance in the image. This means as the image gets rotated, so will the eigenvectors to stay consistent with that definition and therefore not be affected by the rotation. On the other hand eigenvalues are the corresponding magnitude of the eigenvectors, and will therefore naturally remain constant under the rotation as long as shape/size/scale of the picture stays the same. This elaboration can be visualized as an ellipse with axes' length determined by the eigenvalues and axes' orientation by eigenvectors in lecture 2 slide 26. This ellipse can be represented mathematically as such:

$$\begin{bmatrix} u & v \end{bmatrix} A \begin{bmatrix} u \\ v \end{bmatrix} = const$$

Furthermore we know that the corner detection is only dependent on eigenvalues. Mathematically the Harris and Stephens corner measure can be represented as:

$$\lambda_0 \lambda_1 - \alpha(\lambda_0 + \lambda_1)^2 = \det(A) - \alpha trace(A)^2$$

And since the eigenvector magnitudes (eigenvalues) remain constant under rotational transformation of the image, this means that the corner classifactions will also remain the same. Hence all these properties mentioned in this answer allow for rotational invariance under symmetric weight.

Given that w is not radially symmetric (Box blur kernel), is the measure still invariant to these transformations? Answer:
The calculations to show intensity shift invariance stays the same whether the weight is symmetric or not and so i will reference my first answer for this.

How ever for rotation invariance, we need to first define what box blur kernel is. This is a rectangle shape defined in the continious domain, where it is constant within this rectanle and zero everywhere else. If this filter is applied to an image, it means that the response will be of a finite rotational value as the box blur does not cover the entire value space. And because of this limitation it will not be rotationally invariant, and the harris-corners detected will be different after a rotation vs before a rotation.

1.4 Task 1.4

Making the threshold a precentage of the highest occuring corner strength may help against effects coming from contrast changes. This way the threshold will always be relative/scaled and will therefore not run the risk of neglecting some of the corners that would otherwise be devoured by a flat threshold in the case of contrast adjustment. Therefore in theory the corners should stay consistent as the threshold would sink relatively to the max value of the corner strength response. This way if this value becomes lower then so does the threshold and vice versa. In short, corners will not be affected with a relative threshold.

2 Task 2 - Hough Transform

2.1 Task 2.1

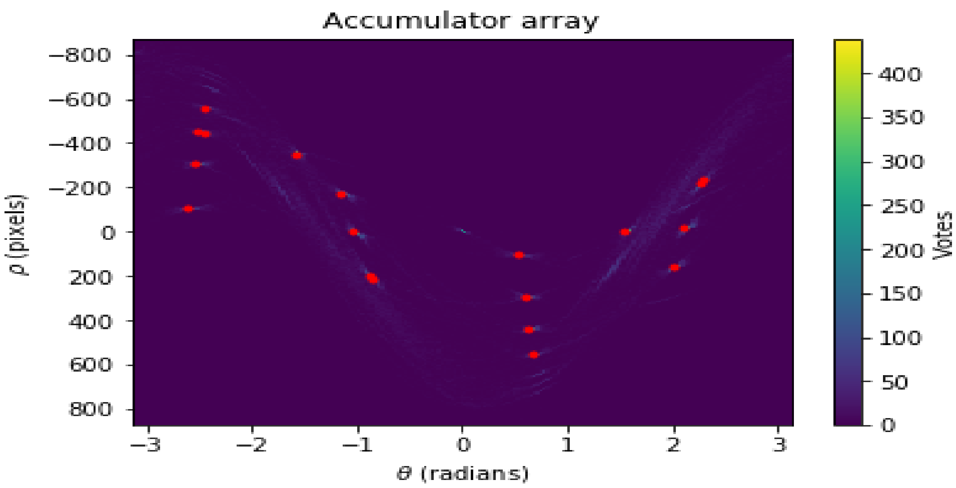
the range of ρ is defined to be:

$$\rho \in [-\sqrt{W^2 + H^2}, \sqrt{W^2 + H^2}]$$

where WxH is the width and height of the image, in this case 800x350. θ on the other hand is within the range:

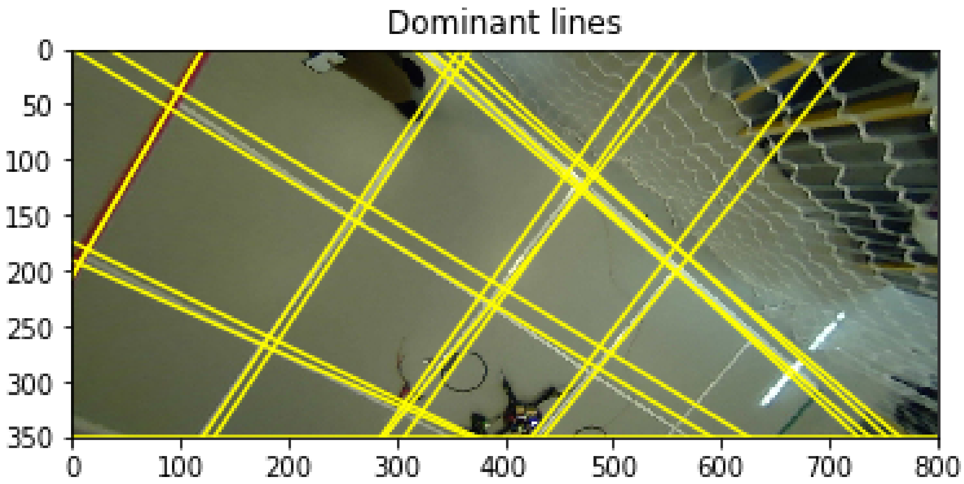
$$\theta \in [-\pi, \pi]$$

2.2 Task 2.2



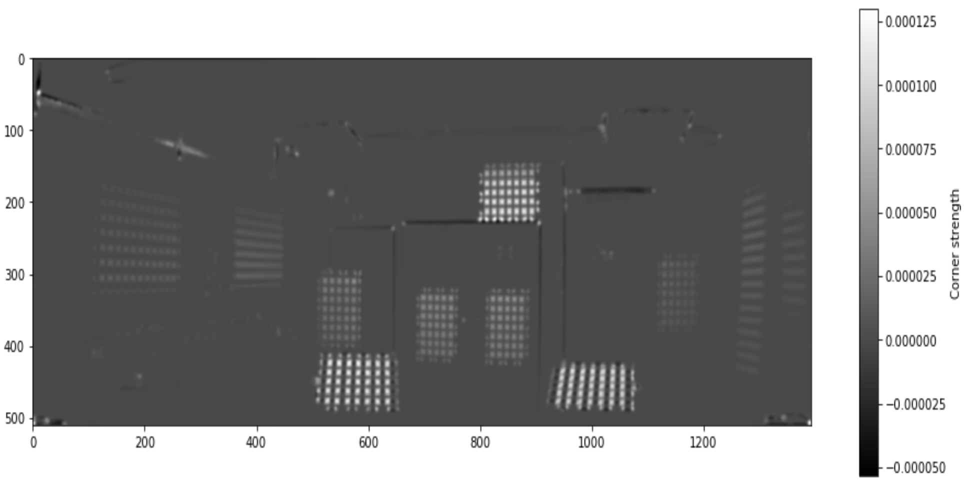
2.3 Task 2.3

The threshold used to extract local maxima is 0.2 as per request of the task.



3 Task 3 - Harris Detector

3.1 Task 3.1



3.2 Task 3.2

For each pixel we have eigenvalues represented as such:

$$Response = \lambda_0\lambda_1 - \alpha(\lambda_0 + \lambda_1)^2 = \det(A) - \alpha trace(A)^2$$

and so a negative response at those spots is trying to indicate that $\lambda_0 >> \lambda_1$ or $\lambda_1 >> \lambda_0$. Which in turn makes the response negative. The practical interpretation of this is that the response has most likely discovered an edge pixel.

3.3 Task 3.3

There are a lot different checkerboards rotated, with different brightness, contrast and scale because of the lighting and perspective in the image. This gives an interesting result in terms of response for the different checkerboards. Obviously since we are using a 2D-gaussian for the weight, we know that our harris-detector will have certain properties. For instance the method is invariant for rotational transformations and intensity shifts (brightness changes), but not invariant to scaling or contrast changes.

Naturally because of the lighting in the room, some of the checkerboards are either higher or lower contrast. For example, the left side of the room is a lot darker and therefore has lower contrast than the right side. As of yet the relative thresholding has not been applied, and will also be of little help to areas that has 0 corner strength. Another potential culprit could be the scaling of different checkerboards because of the perspective in the room, causing some of the checkerboards to appear small. How ever one should do tests to confirm these hypotheses.

3.4 Task 3.4

with a threshold value of 0.001 we get: