Assignment 03
TTK4255 - Robotsyn

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Februar, 2021

Contents

1 Task 1 - Choosing a sensor and a lens

1.1 Task 1.1

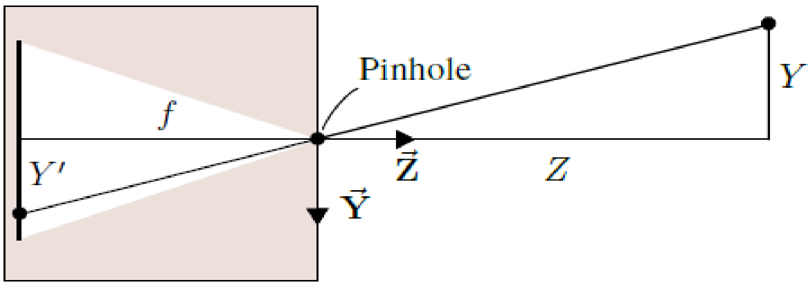


Figure 1: source: homework 3 figure for this course

Using the pinhole model and the fact that the camera is pointing down towards the ground which means that in this case $Z = 50m$ distance from the camera to the ground. The ground distance is $X = 1cm$ and the camera sensor pixel distance is 10 microns. Using similar triangles we get:

$$\frac{X'}{f} = \frac{X}{Z}$$

$$f = \frac{X'}{X}Z = \frac{10 \cdot 10^{-3}}{10}50000 = \underline{\underline{50mm}}$$

1.2 Task 1.2

The difference of the pixel coordinates in the x-axis denoted by $u_2 - u_1$ is used. Note that calculating $v_2 - v_1 = 0$ since there is no change in the Y coordinate from 1 timestep to another.

$$u_1 = c_x + s_x f \frac{X_1}{Z}$$

$$u_2 = c_x + s_x f \frac{X_2}{Z}$$

Everything here remains constant except for the X points as the displacement is happening in the X direction.

$$u_2 - u_1 = c_x - c_x + \frac{s_x f}{Z}(X_2 - X_1)$$

$$u_2 - u_1 = \frac{s_x f}{Z}(X_2 - X_1)$$

Defining $X_1 = 0$ and $X_2 = \frac{50}{5} = 10m$

$$\frac{s_x f}{Z}(X_2 - X_1) = \frac{\frac{1}{10\cdot 10^{-3}}50}{50000}(10000 - 0) = 1000$$

Finding the percentage of this gives me:

$$\frac{1000}{1024} \cdot 100 = \underline{\underline{97.65\%}}$$

This comes from the fact that:

$$\frac{1}{s_x} = 10 microns$$

2 Task 2 Implementing the pinhole camera model

2.1 Task 2.1

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \underbrace{\begin{bmatrix} s_x f & 0 & c_x \\ 0 & s_y f & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{2.1}$$

Expanding the matrix:

$$\tilde{u} = c_x Z + s_x f X$$

$$\tilde{v} = c_y Z + s_y f Y$$

$$\tilde{w} = Z$$

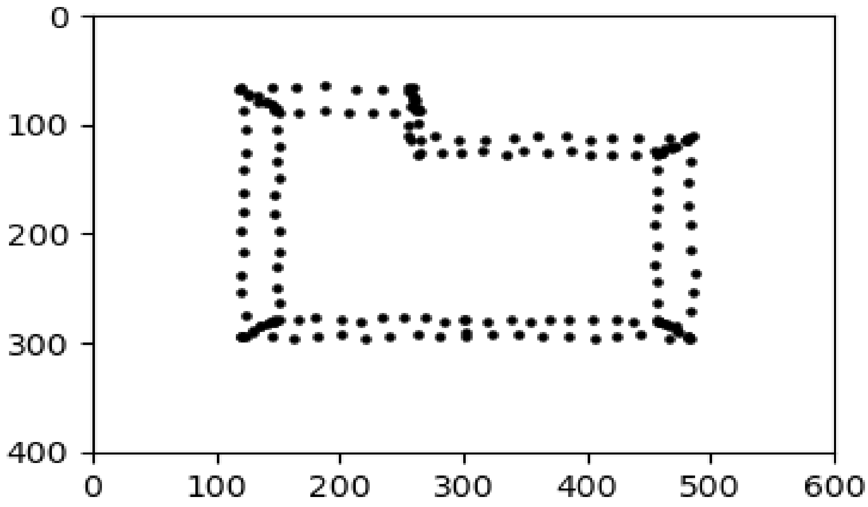
Dividing $\tilde{u}$ and $\tilde{v}$ with $\tilde{w} = Z$:

$$u = \underline{\underline{c_x + s_x f \frac{X}{Z}}}$$

$$v = \underline{\underline{c_y + s_y f \frac{Y}{Z}}}$$

Ending up with the dehomogenized coordinates u and v.

2.2 Task 2.2



3 Task 3 - Homogeneous coordinates and transformations

3.1 Task 3.1

from definition we know that the transformation that yields the coordinate vector in the camera frame is given by:

$$\mathbf{X}^c = \mathbf{R}\mathbf{X}^\mathbf{o} + \mathbf{t}$$

Specifically if one looks at the transformation in task 3.2 it would be as such:

$$\mathbf{X}^c = (\mathbf{R}_\mathbf{x}\mathbf{R}_\mathbf{y})\mathbf{X}^\mathbf{o} + \mathbf{t}_\mathbf{z}$$

where the vector $t_z = [0;0;6]^T$ and the rotation matrices are rotated 15 and 45 respectively. This can be written more compactly in a homogeneous transformation format:

$$\tilde{\mathbf{X}}^c = \mathbf{T}_o^c \tilde{\mathbf{X}}^\mathbf{o}$$

where:

$$\begin{bmatrix} \mathbf{X}^c \\ 1 \end{bmatrix} = \begin{bmatrix} (\mathbf{R}_\mathbf{x}\mathbf{R}_\mathbf{y}) & \mathbf{t}_\mathbf{z} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X}^\mathbf{o} \\ 1 \end{bmatrix}$$

And so we can implement either the homogeneous ord the regular format of the camera frame coordinates to the homogeneous projection, With normal format this gives:

$$\begin{bmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{w}\end{bmatrix} = \underbrace{\begin{bmatrix}s_x f & 0 & c_x\\ 0 & s_y f & c_y\\ 0 & 0 & 1\end{bmatrix}}_{\mathbf{K}} \begin{bmatrix}X^c\\ Y^c\\ Z^c\end{bmatrix} \tag{3.1}$$

or we can use the 4x1 vector implementation:

$$\begin{bmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{w}\\ 1\end{bmatrix} = \begin{bmatrix}s_x f & 0 & c_x & 0\\ 0 & s_y f & c_y & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}X^c\\ Y^c\\ Z^c\\ 1\end{bmatrix} \tag{3.2}$$

$$\implies u = c_x + s_x f \frac{X^c}{Z^c}$$

$$\implies v = c_y + s_y f \frac{Y^c}{Z^c}$$

One can see in the second implementation that $u = \tilde{u}/\tilde{w}$ and that $v = \tilde{v}/\tilde{w}$ are still both only dependant on the same dehomogenization factors as shown in task 2.1. This is because dividing by 1 is similar to just skipping this step/removing the last row altogether.

we can prove this for also values of $W^c \neq 1$ Proof:
Given u and v:

$$u = c_x + s_x f \frac{X^c}{Z^c}$$

$$v = c_y + s_y f \frac{Y^c}{Z^c}$$

and that:

$$\tilde{u} = \tilde{w}u = c_x Z^c + s_x f X^c$$

$$\tilde{v} = \tilde{w}v = c_y Z^c + s_y f Y^c$$

this implies that:

$$\begin{bmatrix}\tilde{u}\\ \tilde{v}\\ \tilde{w}\end{bmatrix} = \begin{bmatrix}s_x f & 0 & c_x\\ 0 & s_y f & c_y\\ 0 & 0 & 1\end{bmatrix} \begin{bmatrix}X^c\\ Y^c\\ Z^c\end{bmatrix}$$

Expanding further:

$$\implies \begin{bmatrix}s_x f & 0 & c_x\\ 0 & s_y f & c_y\\ 0 & 0 & 1\end{bmatrix} \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix} \begin{bmatrix}X^c\\ Y^c\\ Z^c\\ W^c\end{bmatrix}$$

$$\implies \begin{bmatrix} s_x f & 0 & c_x \\ 0 & s_y f & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R_o^c} & \mathbf{t} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X^o \\ Y^o \\ Z^o \\ W^o \end{bmatrix} \blacksquare$$

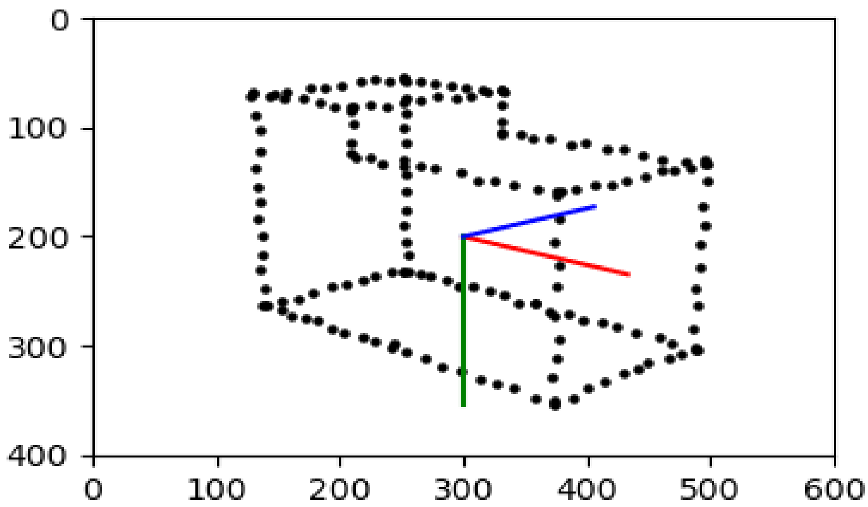
Hence we can see that W^c does not affect the camera coordinates in any way even if it is different than 1.

3.2 Task 3.2

The following transformation in this order yields the figure below:

$$\mathbf{T}_o^c = T_z(6) R_x(15°) R_y(45°)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(15°) & -sin(15°) & 0 \\ 0 & sin(15°) & cos(15°) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(45°) & 0 & sin(45°) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(45°) & 0 & cos(45°) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



4 Task 4 - Image formation model for the Quanser helicopter

4.1 Task 4.1

The 4 defined vectors in the platform coordinate frame is defined as such:

$$\tilde{\mathbf{X}}_{\mathbf{1}}^{\mathbf{P}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

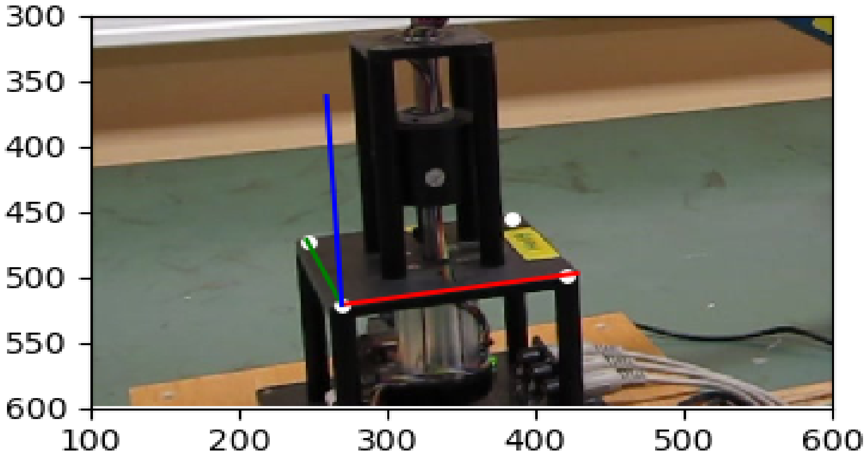
$$\tilde{\mathbf{X}}_{\mathbf{2}}^{\mathbf{P}} = \begin{bmatrix} 0.1145 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{X}}_{\mathbf{3}}^{\mathbf{P}} = \begin{bmatrix} 0 \\ 0.1145 \\ 0 \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{X}}_{\mathbf{4}}^{\mathbf{P}} = \begin{bmatrix} 0.1145 \\ 0.1145 \\ 0 \\ 1 \end{bmatrix}$$

where $\tilde{\mathbf{X}}_{\mathbf{1}}^{\mathbf{P}}$ is the screw at the origin of the platform frame, $\tilde{\mathbf{X}}_{\mathbf{2}}^{\mathbf{P}}$ is the screw on the x-axis and $\tilde{\mathbf{X}}_{\mathbf{3}}^{\mathbf{P}}$ is the screw on the y-axis and $\tilde{\mathbf{X}}_{\mathbf{4}}^{\mathbf{P}}$ is the farthest screw that is defined in the x-y plane of the platform frame. Note that the length of 11.45cm is defined in meters instead to match python requirements.

4.2 Task 4.2



Where the scale of the drawn frame is 0.12 and the scale of the white circles that is marking the screws is 60.

4.3 Task 4.3

First i translate 5.725 cm in the x direction then 5.725 cm in the y direction and then rotate the z axis by ψ.

$$\mathbf{T_B^P} = \begin{bmatrix} 1 & 0 & 0 & \frac{0.1145}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{0.1145}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 & 0 \\ sin(\psi) & cos(\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation for coordinates from the base frame to the camera frame is:

$$\mathbf{T_B^C} = \mathbf{T_P^C T_B^P}$$

4.4 Task 4.4

To find the transformation $\mathbf{T_H^B}$ we need to first translate 32.5 cm in the z-axis then rotate the y-axis by θ

$$\mathbf{T_H^B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.325 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & 0 & sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\theta) & 0 & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation for coordinates from the hinge frame to the camera frame is:

$$\mathbf{T_H^C} = \mathbf{T_P^C T_B^P T_H^B}$$

4.5 Task 4.5

The transformation from the arm to the hinge frame is found by translating -5 cm in the z.axis, such that:

$$\mathbf{T_A^H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.05 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation for coordinates from the arm frame to the camera frame:

$$\mathbf{T_A^C} = \mathbf{T_P^C T_B^P T_H^B T_A^H}$$

4.6 Task 4.6

The transformation from the rotor to the arm frame is achieved by translating 65 cm in x-axis then -3 cm in z-axis and then rotating the x-axis with an angle ϕ.

$$\mathbf{T_R^A} = \begin{bmatrix} 1 & 0 & 0 & 0.65 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.03 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\phi) & -sin(\phi) & 0 \\ 0 & sin(\phi) & cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation for coordinates from the rotor frame to the camera frame:

$$\mathbf{T_R^C} = \mathbf{T_P^C T_B^P T_H^B T_A^H T_R^A}$$

4.7 Task 4.7

Here the size of the orange markers is chosen to be 500, and the angles for the transformation as 11.6°, 28.9°, 0° respectively for ψ, θ, ϕ.

Helicopter Coordinate frames