Basic Probability

$$\begin{aligned} Var(X) &= E(X-EX)^2 \\ &= EX^2-(EX)^2 \\ &= EX(X-1)+EX-(EX)^2 \\ M_X(t) &= E(e^{tX}) \\ Cov(X,Y) &= E(X-E(X))(Y-E(Y)) \\ &= E(XY)-E(X)E(Y) \\ Var(aX+bY) &= a^2Var(X)+b^2Var(Y) \\ &\quad + 2abCov(X,Y) \end{aligned}$$

Correlation: $\rho_{X,Y} = \dfrac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$

$$f_{X_{(k)}}(x) = k\binom{n}{k} f(x)[F(x)]^{k-1}[1-F(x)]^{n-k}$$

Independence: $F_{X,Y}(x,y) = F_X(x)F_Y(y), \forall x,y$

$Cov(X,Y)=0,$ if X,Y independent

Discrete Distributions

$p_X(x) = \dfrac{1}{N}$, if $x=1,2,...,N$, $\quad F_X(x) = \dfrac{x}{N}$

$E(X) = \dfrac{N+1}{2}, \quad Var(X) = \dfrac{N^2-1}{12}$

$M_X(t) = \dfrac{e^t - e^{(N+1)t}}{N(1-e^t)}$

$p_X(x) = \dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$, if $x=0,1,...,n$

$E(X) = \dfrac{nM}{N}, \quad Var(X) = \dfrac{nM}{N}\left(1-\dfrac{M}{N}\right)\left(\dfrac{N-n}{N-1}\right)$

$p_X(x) = p^x(1-p)^{1-x}$, if $x=0,1$

$E(X)=p, \quad Var(X)=p(1-P)$

$M_X(t) = (1-p)+pe^t$

$p_X(x) = \dbinom{n}{x}p^x(1-p)^{n-x}$, if $x=0,1,\ldots,n$

$E(X)=np, \quad Var(X)=np(1-P)$

$M_X(t) = (1-p+pe^t)^n$

$p_X(x) = p(1-p)^{x-1}$, if $x=1,2,\ldots$

$E(X) = \dfrac{1}{p}, \quad Var(X) = \dfrac{1-p}{p^2}$

$M_X(t) = \dfrac{pe^t}{1-(1-p)e^t}$

Discrete Distributions cont.

$p_X(x) = \dbinom{x-1}{r-1}p^r(1-p)^{x-r}$, if $x=r,r+1,\ldots$

$E(X) = \dfrac{r}{p}, \quad Var(X) = \dfrac{r(1-p)}{p^2}$

$p_X(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$, if $x=0,1,2,\ldots$

$F_X(x) = e^{-\lambda}\displaystyle\sum_{i=0}^{x}\frac{\lambda^i}{i!}$

$E(X)=\lambda, \quad Var(X)=\lambda$

$M_X(t) = e^{\lambda(e^t-1)}$

Continuous Distributions

$f_X(x) = \dfrac{1}{b-a}$, if $a<x<b$

$E(X) = \dfrac{a+b}{2}, \quad Var(X) = \dfrac{(b-a)^2}{12}$

$M_X(t) = \dfrac{e^{tb}-e^{ta}}{t(b-a)}, 1$ if $t=0$

$f_X(x) = \dfrac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-\frac{x}{\beta}}$, if $x>0$

$E(X)=\alpha\beta, \quad Var(X)=\alpha\beta^2$

$M_X(t) = (1-\beta t)^{-\alpha}$

$f_X(x) = \dfrac{1}{\beta^\alpha}e^{-\frac{x}{\beta}}$, if $x>0$

$X \sim$Gamma$(1,\beta)$

$E(X)=\beta, \quad Var(X)=\beta^2$

$M_X(t) = (1-\beta t)^{-1}$

$\chi^2: X \sim$ Gamma$(\frac{p}{2}, 2)$

$f_X(x) = \dfrac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}$, if $x\in(0,1)$

$E(X) = \dfrac{\alpha}{\alpha+\beta}, \quad Var(X) = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

$M_X(t) = \displaystyle\sum_{n=0}^{\infty}\frac{B(\alpha+n,\beta)}{B(\alpha,\beta)}\frac{t^n}{n!}$

$f_X(x) = \dfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ if $x\in\mathbb{R}$

$M_X(t) = e^{\mu t+\frac{1}{2}\sigma^2t^2}$

General Equations

$\Gamma(\alpha) = \displaystyle\int_0^\infty t^{\alpha-1}e^{-t}dt, \quad \alpha>0$

$\Gamma(\alpha+1) = \alpha\Gamma(\alpha) \qquad \Gamma(\alpha) = (\alpha-1)!$

$B(\alpha,\beta) = \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \displaystyle\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx$

$\displaystyle\sum_{x=1}^{N}x = \frac{N(N+1)}{2} \qquad \sum_{n=0}^{\infty}ar^n = \frac{a}{1-r}$

$\dbinom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n}{r}\dbinom{n-1}{r-1}$

$\displaystyle\lim_{n\to\infty}\left(1-\frac{c}{n}\right)^{-n} = 0$

New Material

$X \sim N_n(\mu,\Sigma) \& Y = AX+b$

$\implies Y \sim N_m(A\mu+b, A\Sigma A^T)$

$M_{X_n}(t) = M(t)$ for $|b|\le h_1 \implies X_n \to DX$

$lim_{n\to\infty}P[|X_n - X| \ge \epsilon] = 0 \implies X_n \to PX$

CLT: $\dfrac{\sqrt{n}(\bar{x}_n - \mu)}{\sigma}$

Δ Theorem: $\sqrt{n}(g(\bar{X}) - g(\mu)) \sim N(0, g'(\mu)^2)\sigma^2$

MLE: $L(\theta) = \displaystyle\prod_{i=1}^{n} f(x_i;\theta)$

$\dfrac{\delta \log(L(\theta))}{\delta\theta} = 0$ gives $\hat{\theta}$

MME: $EX,$ solve for θ