Basic Probability

$$Var(X) = E(X - EX)^2$$

$$= EX^2 - (EX)^2$$

$$= EX(X - 1) + EX - (EX)^2$$

$$M_X(t) = E(e^{tX})$$

$$Cov(X,Y) = E(X - E(X))(Y - E(Y))$$

$$= E(XY) - E(X)E(Y)$$

$$Var(aX + bY) = a^2Var(X) + b^2Var(Y)$$

$$+ 2abCov(X,Y)$$

$$Correlation: \rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

$$f_{X_{(k)}}(x) = k\binom{n}{k}f(x)[F(x)]^{k-1}[1 - F(x)]^{n-k}$$
Independence: $F_{X,Y}(x,y) = F_X(x)F_Y(y), \forall x, y$

Cov(X,Y) = 0, if X, Y independent

Discrete Distributions

$$p_X(x) = \frac{1}{N}, \text{ if } x = 1, 2, ..., N, \quad F_X(x) = \frac{x}{N}$$

$$E(X) = \frac{N+1}{2}, \quad Var(X) = \frac{N^2 - 1}{12}$$

$$M_X(t) = \frac{e^t - e^{(N+1)t}}{N(1 - e^t)}$$

$$p_X(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}, \text{ if } x = 0, 1, ..., n$$

$$E(X) = \frac{nM}{N}, \quad Var(X) = \frac{nM}{N} \left(1 - \frac{M}{N}\right) \left(\frac{N-n}{N-1}\right)$$

$$p_X(x) = p^x(1-p)^{1-x}, \text{ if } x = 0, 1$$

$$E(X) = p, \quad Var(X) = p(1-P)$$

$$M_X(t) = (1-p) + pe^t$$

$$p_X(x) = \binom{n}{x} p^x(1-p)^{n-x}, \text{ if } x = 0, 1, ..., n$$

$$E(X) = np, \quad Var(X) = np(1-P)$$

$$M_X(t) = (1-p+pe^t)^n$$

$$p_X(x) = p(1-p)^{x-1}, \text{ if } x = 1, 2, ...$$

$$E(X) = \frac{1}{p}, \quad Var(X) = \frac{1-p}{p^2}$$

$$M_X(t) = \frac{pe^t}{1-(1-p)e^t}$$

Discrete Distributions cont.

$$p_X(x) = {x-1 \choose r-1} p^r (1-p)^{x-r}, \text{ if } x = r, r+1, \dots$$

$$E(X) = \frac{r}{p}, \quad Var(X) = \frac{r(1-p)}{p^2}$$

$$p_X(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \text{ if } x = 0, 1, 2, \dots$$

$$F_X(x) = e^{-\lambda} \sum_{i=0}^x \frac{\lambda^i}{i!}$$

$$E(X) = \lambda, \quad Var(X) = \lambda$$

$$M_X(t) = e^{\lambda(e^t - 1)}$$

Continuous Distributions

$$f_X(x) = \frac{1}{b-a}, \text{ if } a < x < b$$

$$E(X) = \frac{a+b}{2}, \quad Var(X) = \frac{(b-a)^2}{12}$$

$$M_X(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}, 1 \text{ if } t = 0$$

$$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-\frac{x}{\beta}}, \text{ if } x > 0$$

$$E(X) = \alpha\beta, \quad Var(X) = \alpha\beta^2$$

$$M_X(t) = (1-\beta t)^{-\alpha}$$

$$f_X(x) = \frac{1}{\beta^{\alpha}}e^{-\frac{x}{\beta}}, \text{ if } x > 0$$

$$X \sim \text{Gamma}(1, \beta)$$

$$E(X) = \beta, \quad Var(X) = \beta^2$$

$$M_X(t) = (1-\beta t)^{-1}$$

$$\chi^2 : X \sim \text{Gamma}(\frac{p}{2}, 2)$$

$$f_X(x) = \frac{1}{B(\alpha, \beta)}x^{\alpha-1}(1-x)^{\beta-1}, \text{ if } x \in (0, 1)$$

$$E(X) = \frac{\alpha}{\alpha+\beta}, \quad Var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$M_X(t) = \sum_{n=0}^{\infty} \frac{B(\alpha+n, \beta)}{B(\alpha, \beta)} \frac{t^n}{n!}$$

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \text{ if } x \in \mathbb{R}$$

$$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

- General Equations

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha - 1} e^{-t} dt, \quad \alpha > 0$$

$$\Gamma(\alpha + 1) = \alpha \Gamma(\alpha) \qquad \Gamma(\alpha) = (\alpha - 1)!$$

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)} = \int_0^1 x^{\alpha - 1} (1 - x)^{\beta - 1} dx$$

$$\sum_{x=1}^N x = \frac{N(N+1)}{2} \qquad \sum_{n=0}^\infty ar^n = \frac{a}{1 - r}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n}{r} \binom{n-1}{r-1}$$

$$\lim_{n \to \infty} \left(1 - \frac{c}{n}\right)^{-n} = 0$$

New Material

$$X \sim N_n(\mu, \Sigma) \& Y = AX + b$$

$$\implies Y \sim N_m(A\mu + b, A\Sigma A^T)$$

$$M_{X_n}(t) = M(t) \text{ for } |b| \leq h_1 \implies X_n \to DX$$

$$\lim_{n \to \infty} P[|X_n - X| \geq \epsilon] = 0 \implies X_n \to PX$$

$$\text{CLT: } \frac{\sqrt{n}(\bar{x}_n - \mu)}{\sigma}$$

$$\Delta \text{ Theorem: } \sqrt{n}(g(\bar{X}) - g(\mu)) \sim N(0, g'(\mu)^2) \sigma^2$$

$$\text{MLE: } L(\theta) = \prod_{i=1}^n f(x_i; \theta)$$

$$\frac{\delta \log(L(\theta))}{\delta \theta} = 0 \text{ gives } \hat{\theta}$$

$$\text{MME: } EX, \text{ solve for } \theta$$