Exam solutions

02506
Spring 2021

Question 1 Scale normalized Laplacian is $t\nabla^2 L = t(L_{xx} + L_{yy})$. For $t = 12$ we have $t\nabla^2 L = 12 * (5.12 + 7.16) = 147.36$.

The correct answer is (g).

Question 2 Smoothing effect is needed in both direction. Kernels are separable so no need to use 2D kernels. Instead of only smoothing in both direction, we need the second order derivatives in x direction, and smoothing in y. This leaves one correct answer.

The correct answer is (b).

Question 3 Pixel $(4, 2)$ is 2D local maximum for all t. To find maximum in 3D we need to scale-normalize, so we get the sequence

$$1 \cdot 28.9\,,\ 2 \cdot 19.9\,,\ 3 \cdot 13.7\,,\ 4 \cdot 9.8\,,\ 5 \cdot 7.0\,.$$

The larges number in this sequence is 41.1 at the scale $t = 3$. Radius is $\sqrt{2t} = \sqrt{6}$. Since it is a large positive value, the blob is dark.

The correct answer is (c).

Question 4 Cluster 2 contains pixels

$$1, 2, 3, 5, 7, 8, 9, 10, 12, 14, 15, 16, 17$$

Those 13 pixels have labels

$$1, 2, 1, 2, 2, 2, 1, 1, 1, 2, 2, 1, 1$$

So seven out of 13 cluster pixels have label 1, and probability is $\frac{7}{13} = 0.538$.

The correct answer is (f).

Question 5 Before smoothing, probabilities are $[0.04, 0.90, 0.06]$, so second probability is largest and the label is 2. After smoothing probability for label 2 is $\frac{1}{5}(0.01 + 0.02{+}0.15{+}0.01{+}0.9)$ and probability for label 3 is $\frac{1}{5}(0.98{+}0.98{+}0.85{+}0.98{+}0.06)$. So probability for label 3 is larger.

The correct answer is (c).

Question 6 The model for transformation which takes $\mathbf{q}_i$ and returns $\mathbf{p}_i$ is

$$\mathbf{p}_i = \frac{1}{s}\mathbf{R}^{-1}(\mathbf{q}_i - \mathbf{t})$$

and we also know that $\mathbf{R}^{-1} = \mathbf{R}^T$. For calculation of points see code.

The correct answer is (c).

Question 7 SIFT features are invariant to scale, rotation, and change in illumination but not mirroring. SIFT descriptor is used to find correspondence between images.

The correct answer is (a).

Question 8 The Gaussian and its derivatives up to third order consist of

$$[g, g_x, g_y, g_{xx}, g_{xy}, g_{yy}, g_{xxx}, g_{xxy}, g_{xyy}, g_{yyy}]$$

This is a 10-element vector for each scale, and with 3 scales we have 30-element feature vector for every image pixel. So size is $1024 \times 1024 \times 30$.

The correct answer is (g).

Question 9 $g = \sqrt{(209-147)^2 + (158-215)^2} = 84.22$

The correct answer is (e).

Question 10 We consider contribution of a single pixel to the likelihood and the prior

$$E_{\text{new}} = (30-52)^2 + 3 \cdot 125 + \text{const}$$

$$E_{\text{current}} = (20-52)^2 + 1 \cdot 125 + \text{const}$$

where const includes all other contributions. So

$$\Delta E = (30-52)^2 - (20-52)^2 + 3 \cdot 125 - 1 \cdot 125 = -290$$

The correct answer is (d).

Question 11 See code

The correct answer is (h).

Question 12 See code

The correct answer is (c).

Question 13 Internal forces depend only on curve.

The correct answer is (a).

Question 14 See code

The correct answer is (f).

Question 15

$$a = (1.2, 5.4) - 2(0.1, 2.9) + (1.4, 1.1) = (2.4, 0.7)$$

$$b = -(3.3, 7.1) + 4(1.2, 5.4) - 6(0.1, 2.9) + 4(1.4, 1.1) - (3.5, 0.2) = (3, 1.3)$$

$$p = (0.1, 2.9) + 0.05a + 0.1b = (0.52, 3.065)$$

The correct answer is (d).

Question 16 Δ_x needs to be larger than 0. Cost $c_{\text{on}} = 10 - I$ would seek larger values and would not pass 0 in the third column. Cost $c_{\text{on}} = \text{abs}(I - 2)$ would seek 3 instead 0 in the third column and 2 instead of 1 in the sixth column.

The correct answer is (d).

Question 17 We compute cost for all horizontal lines. First compute row-wise sum of cost for bright and dark as

$$b = [67, 35, 53, 101, 96]$$

$$d = [53, 85, 67, 19, 24]$$

The cost for lines from bottom upwards are

$$c(s = 1) = 67 + 85 + 67 + 19 + 24 = 262$$

$$c(s = 2) = 67 + 35 + 67 + 19 + 24 = 212$$

$$c(s = 3) = 67 + 35 + 53 + 19 + 24 = 198$$

$$c(s = 4) = 67 + 35 + 53 + 101 + 24 = 280$$

$$c(s = 5) = 67 + 35 + 53 + 101 + 96 = 352$$

and the smallest cost is 198.

The correct answer is (d).

Question 18 Computing element wise exponential gives

$$\exp \hat{\mathbf{y}} = [1.65\,,\ 3640.95\,,\ 992.27\,,\ 0.90\,,\ 1.34]$$

So we have

$$y_2 = \frac{3640.95}{1.65 + 3640.95 + 992.27 + 0.90 + 1.34} = 0.78517$$

and

$$L = -\ln 0.78517 = 0.24185$$

The correct answer is (b).

Question 19 We have

$$\begin{aligned} h_1 &= 0 \qquad (\text{since } 0.2 - 1.3 \cdot 2.5 = -3.05 < 0) \\ h_2 &= -0.3 + 1.8 \cdot 2.5 = 4.2 \\ h_3 &= -1.7 + 1.6 \cdot 2.5 = 2.3 \end{aligned}$$

and

$$\begin{aligned} \hat{y}_1 &= -1.4 + 1.5 \cdot 0 - 0.5 \cdot 4.2 + 0.9 \cdot 2.3 = -1.43 \\ \hat{y}_2 &= 0.2 + 1.2 \cdot 0 - 0.9 \cdot 4.2 + 1.7 \cdot 2.3 = 0.33 \end{aligned}$$

So

$$y_2 = \frac{\exp 0.33}{\exp 0.33 + \exp(-1.43)} = 0.8532$$

The correct answer is (f).

Question 20 The size after convolution is $10 \times 10 \times 8$, and after max pooling the size is $5 \times 5 \times 8 = 200$. So input for fully connected layer is $200 + 1$ neurons, where one is a bias, and the number of parameters in fully connected part is $201 \cdot 10 = 2010$. The number of parameter in the convolutional part is $5 \cdot 5 \cdot 8 + 8 = 208$, since we have one bias for every convolution channel. In total $208 + 2010 = 2218$ parameters.

The correct answer is (f).