UNIVERSITY OF PRETORIA FACULTY OF NATURAL AND AGRICULTURAL SCIENCES DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

WTW 124 Mathematics

$\frac{\text{SEMESTER TEST 1}}{26 \text{ August } 2025}$

TIME: 120 mins MARKS: 38

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SURNAME:	
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STUDENT NUMBER:	
SIGNATURE:	
READ THE FOLLOV	VING INSTRUCTIONS
1. This paper consists of this cover page and 5 more pages containing questions 1-4. Check whether your paper is complete.	
	ronic equipment is forbidden: No candidate is allowed to use any i-pad, or, smart watch, etc. while writing this paper.
3. Do all scribbling on	the facing page. It will not be marked.
4. If you need more than the available space for an answer, use the facing page and please indicate it clearly.	
5. No pencil work or any work in red ink will be marked.	
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Question 1.

a) Find two unit vectors perpendicular to both vectors $\bar{u} = \langle 1, 2, -1 \rangle$ and $\bar{v} = \langle 3, 1, 2 \rangle$. [3]

b) Let \bar{u} and \bar{v} be two non-zero vectors such that $\bar{u} \cdot \bar{v} = ||\bar{u} \times \bar{v}||$. Find the magnitude of the angle between the rays determined by \bar{u} and \bar{v}

c) Let A, B and C be three matrices such that ABC exists, where A has size 3×3 and C has size 5×5 . Describe the sizes B and ABC.

Question 2.

a) Let \bar{u} and \bar{v} be vectors in \mathbb{R}^3 . Prove that if $\bar{u} + \bar{v} = \bar{0}$, then $\bar{u} \times \bar{v} = \bar{0}$.

[3]

[3]

b) Let A be a 2×2 matrix. Prove or disprove the following statement: If A(A-I)=0, then A=0 or A=I. [3]

c) Let A and B be $n \times n$ matrices. Prove that if AB = BA, then A^T commutes with B^T .

Question 3.

a) Use Gaussian elimination to find (if possible) conditions on real numbers a such that the following system of linear equations has no solution: [4]

$$x + ay - z = 1$$
$$-x + (a - 2)y + z = -1$$
$$2x + 2y + (a - 2)z = 1$$

b) Consider the following matrices:

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 3 \\ -1 & -2 & 5 \end{bmatrix}.$$

[3]

Do A and B commute? Show all steps.

Question 4.

Consider the following two lines in \mathbb{R}^3 :

$$L_1 = \{\langle 1, 2, -3 \rangle + t \langle 1, 2, -3 \rangle : t \in \mathbb{R} \} \text{ and } L_2 = \{\langle -3, 1, 0 \rangle + t \langle -2, 4, -6 \rangle : t \in \mathbb{R} \}.$$

a) If P is a plane with equation 2x - y - z = 3, then $L_1 \subseteq P$. Is this statement true or false? Explain with full details.

[3]

[4]

b) Give Cartesian equations for two parallel lines, each containing one of the lines above.

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c) Let L be a line passing through the point $\bar{p} = \langle 2, 0, 0 \rangle$ such that L is perpendicular to the line L_2 at their intersection. Find a vector equation of the line L. [4]

d) Find the equation of the line through the point $\bar{p} = \langle 2, -1, 4 \rangle$ and perpendicular to the plane 3x - 2y - z = 0. [3]