

Show your steps clearly and note that this is a closed book test.

1. Let  $\bar{a}, \bar{b} \in \mathbb{R}^3$ . Then  $\bar{a} \times \bar{b} = \bar{b} \times \bar{a}$ . True or False? Support your answer in detail.

[2]

2. Let

$$A = \begin{bmatrix} 1 & 4 & 3 \end{bmatrix}.$$

Find, if possible,  $AA^T$  and  $A^T A$

[3]

3. Let  $A$  be a  $2 \times 2$  matrix. Prove or disprove the following statement: if  $A \neq 0$ , then  $A^2 \neq 0$ .

[2]

4. Find, if possible, conditions on  $a, b \in \mathbb{R}$  such that the following system of linear equations has only one solution, by using Gaussian elimination: [3]

$$-x + 3y + 2z = -8$$

$$x + z = 2$$

$$3x + 3y + az = b.$$