

Question 1.

a) Find two unit vectors perpendicular to both vectors $\bar{u} = \langle 1, 2, -1 \rangle$ and $\bar{v} = \langle 3, 1, 2 \rangle$. [3]

b) Let \bar{u} and \bar{v} be two non-zero vectors such that $\bar{u} \cdot \bar{v} = \|\bar{u} \times \bar{v}\|$. Find the magnitude of the angle between the rays determined by \bar{u} and \bar{v} [3]

c) Let A , B and C be three matrices such that ABC exists, where A has size 3×3 and C has size 5×5 . Describe the sizes B and ABC . [2]

Question 2.

a) Let \bar{u} and \bar{v} be vectors in \mathbb{R}^3 . Prove that if $\bar{u} + \bar{v} = \bar{0}$, then $\bar{u} \times \bar{v} = \bar{0}$. [3]

b) Let A be a 2×2 matrix. Prove or disprove the following statement: If $A(A - I) = 0$, then $A = 0$ or $A = I$. [3]

c) Let A and B be $n \times n$ matrices. Prove that if $AB = BA$, then A^T commutes with B^T . [3]

Question 3.

- a) Use Gaussian elimination to find (if possible) conditions on real numbers a such that the following system of linear equations has no solution: [4]

$$\begin{aligned}x + ay - z &= 1 \\ -x + (a - 2)y + z &= -1 \\ 2x + 2y + (a - 2)z &= 1\end{aligned}$$

- b) Consider the following matrices:

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 3 \\ -1 & -2 & 5 \end{bmatrix}.$$

Do A and B commute? Show all steps.

[3]

Question 4.

Consider the following two lines in \mathbb{R}^3 :

$$L_1 = \{\langle 1, 2, -3 \rangle + t\langle 1, 2, -3 \rangle : t \in \mathbb{R}\} \text{ and } L_2 = \{\langle -3, 1, 0 \rangle + t\langle -2, 4, -6 \rangle : t \in \mathbb{R}\}.$$

- a) If P is a plane with equation $2x - y - z = 3$, then $L_1 \subseteq P$.

Is this statement true or false? Explain with full details.

[3]

- b) Give Cartesian equations for two parallel lines, each containing one of the lines above.

[4]

- c) Let L be a line passing through the point $\bar{p} = \langle 2, 0, 0 \rangle$ such that L is perpendicular to the line L_2 at their intersection. Find a vector equation of the line L . [4]

- d) Find the equation of the line through the point $\bar{p} = \langle 2, -1, 4 \rangle$ and perpendicular to the plane $3x - 2y - z = 0$. [3]