UNIVERSITY OF PRETORIA
FACULTY OF NATURAL AND AGRICULTURAL SCIENCES
DEPARTMENT OF MATHEMATICS AND APPLIED MATHEMATICS

WTW 124 Mathematics

SEMESTER TEST 1
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TIME: 120 mins
MARKS: 38

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SURNAME:

FIRST NAMES:

STUDENT NUMBER:

SIGNATURE: _____

READ THE FOLLOWING INSTRUCTIONS

1. This paper consists of this cover page and **5 more pages** containing questions 1-4. Check whether your paper is complete.
2. The use of all electronic equipment is forbidden: No candidate is allowed to use any i-pad, cell phone, calculator, smart watch, etc. while writing this paper.
3. Do all scribbling on the facing page. It will not be marked.
4. If you need more than the available space for an answer, use the facing page and please indicate it clearly.
5. No pencil work or any work in red ink will be marked.
6. If you use correcting fluid (Tipp-Ex or similar), you lose the right to question the marking or claim that work has not been marked.

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Question 1.

a) Find two unit vectors perpendicular to both vectors $\bar{u} = \langle 1, 2, -1 \rangle$ and $\bar{v} = \langle 3, 1, 2 \rangle$. [3]

b) Let $\bar{u}$ and $\bar{v}$ be two non-zero vectors such that $\bar{u} \cdot \bar{v} = \|\bar{u} \times \bar{v}\|$. Find the magnitude of the angle between the rays determined by $\bar{u}$ and $\bar{v}$ [3]

c) Let A, B and C be three matrices such that ABC exists, where A has size 3×3 and C has size 5×5. Describe the sizes B and ABC. [2]

Question 2.

a) Let $\bar{u}$ and $\bar{v}$ be vectors in $\mathbb{R}^3$. Prove that if $\bar{u} + \bar{v} = \bar{0}$, then $\bar{u} \times \bar{v} = \bar{0}$. [3]

b) Let A be a 2×2 matrix. Prove or disprove the following statement: If $A(A - I) = 0$, then $A = 0$ or $A = I$. [3]

c) Let A and B be $n \times n$ matrices. Prove that if $AB = BA$, then A^T commutes with B^T. [3]

Question 3.

a) Use Gaussian elimination to find (if possible) conditions on real numbers a such that the following system of linear equations has no solution: [4]

$$\begin{aligned} x + ay - z &= 1 \\ -x + (a-2)y + z &= -1 \\ 2x + 2y + (a-2)z &= 1 \end{aligned}$$

b) Consider the following matrices:

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 3 \\ -1 & -2 & 5 \end{bmatrix}.$$

Do A and B commute? Show all steps. [3]

Question 4.
Consider the following two lines in $\mathbb{R}^3$:

$$L_1 = \{\langle 1, 2, -3\rangle + t\langle 1, 2, -3\rangle : t \in \mathbb{R}\} \text{ and } L_2 = \{\langle -3, 1, 0\rangle + t\langle -2, 4, -6\rangle : t \in \mathbb{R}\}.$$

a) If P is a plane with equation $2x - y - z = 3$, then $L_1 \subseteq P$.
Is this statement true or false? Explain with full details. [3]

b) Give Cartesian equations for two parallel, each containing one of the lines above. [4]

c) Let L be a line passing through the point $\bar{p} = \langle 2, 0, 0 \rangle$ such that L is perpendicular to the line L_2 at their intersection. Find a vector equation of the line L. [4]

d) Find the equation of the line through the point $\bar{p} = \langle 2, -1, 4 \rangle$ and perpendicular to the plane $3x - 2y - z = 0$. [3]