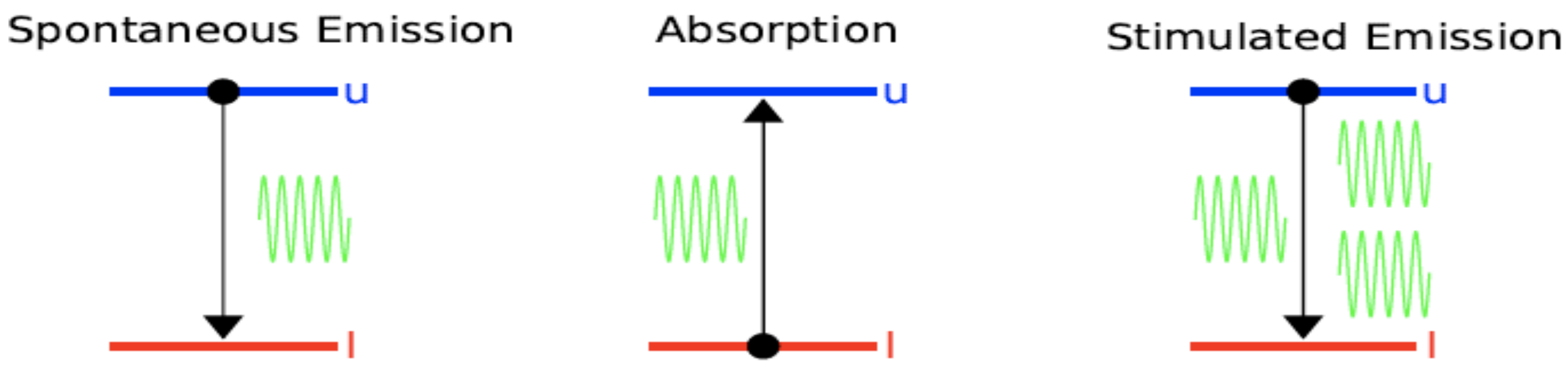
Spectroscopy

XII. Nebular Diagnostics

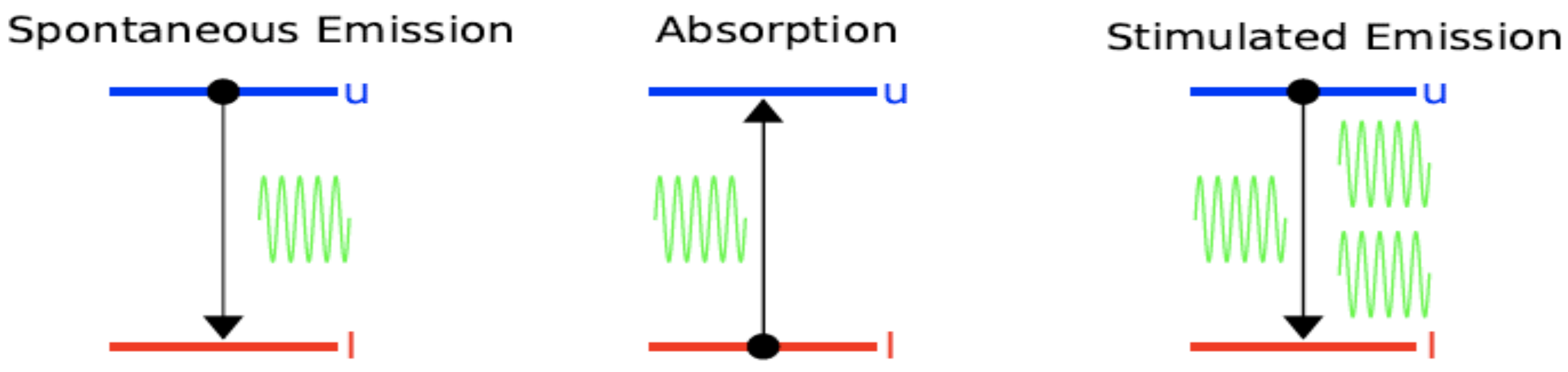
Review

From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.



Review

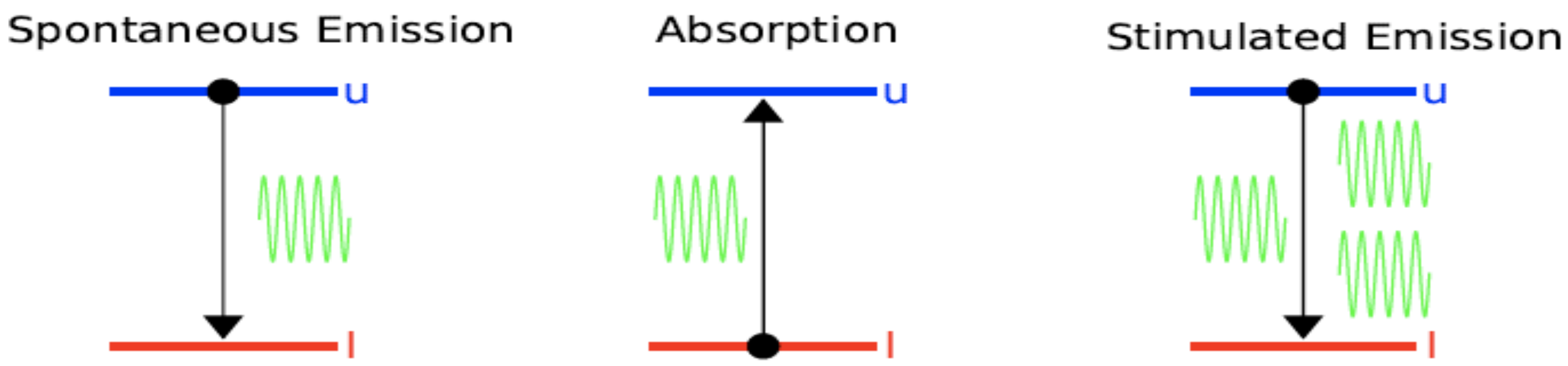
From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.



$$j_\nu \rho = h\nu n_u A_{ul} \Psi(\nu)$$

Review

From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.

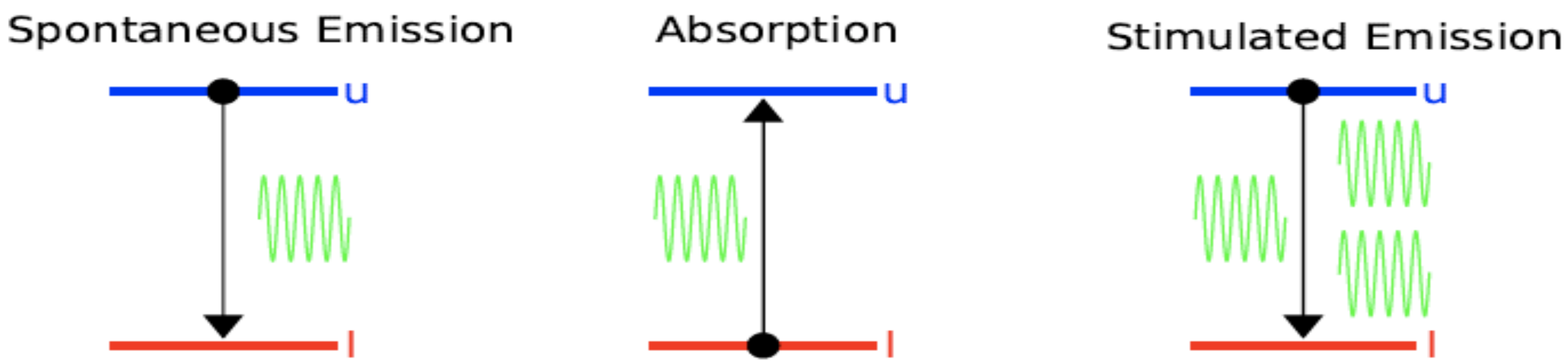


$$j_\nu \rho = h\nu n_u A_{ul} \Psi(\nu)$$

Energy

Review

From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.



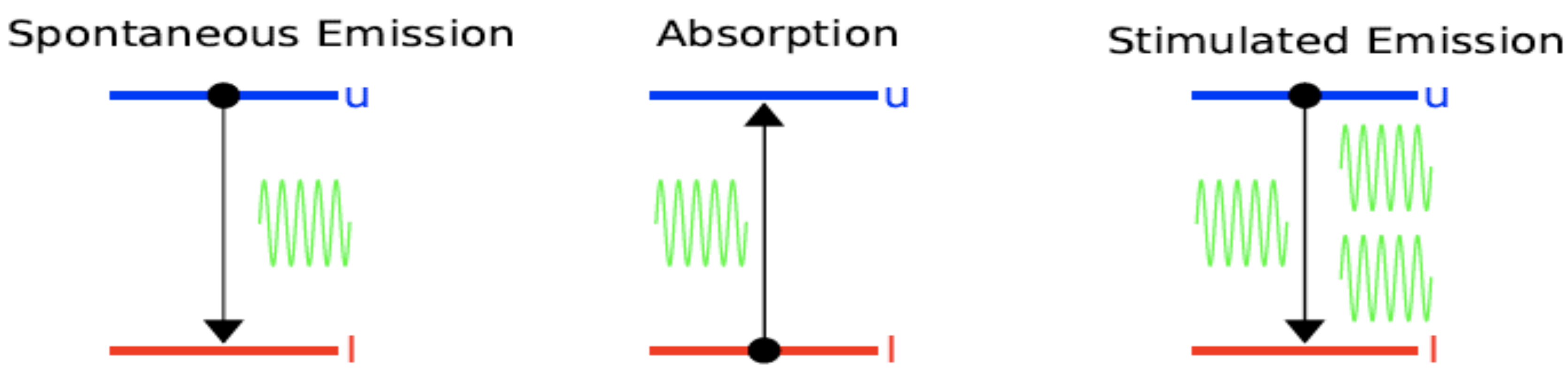
$$j_\nu \rho = h\nu n_v A_{ul} \Psi(\nu)$$

Energy

Density

Review

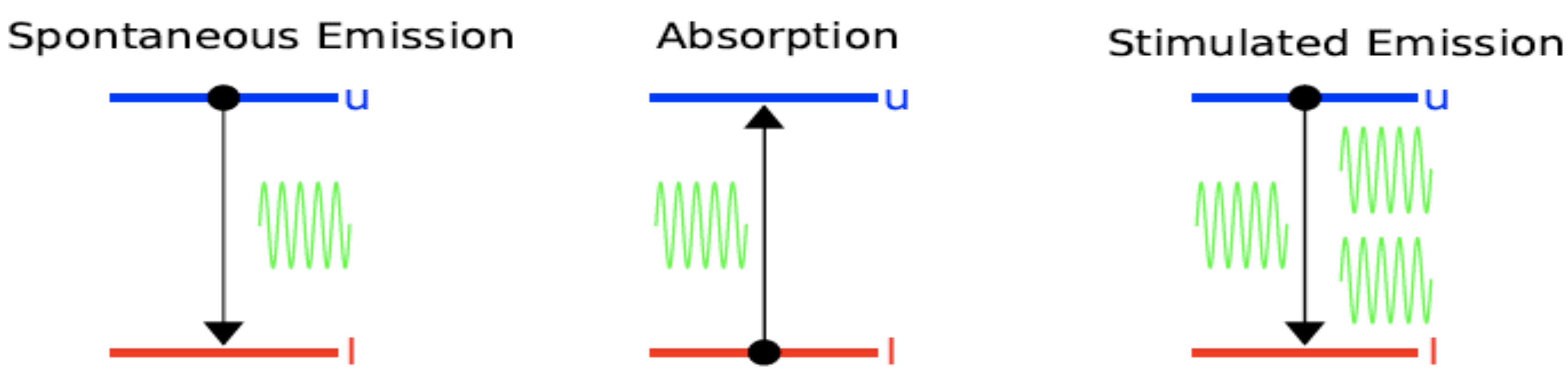
From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.



$$j_\nu \rho = \underbrace{h\nu}_{\text{Energy}} \underbrace{n_u}_{\text{Density}} \underbrace{A_{ul}}_{\text{Transition Rate}} \Psi(\nu)$$

Review

From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.



$$j_\nu \rho = h\nu \, n_u \, A_{ul} \, \Psi(\nu)$$

Energy

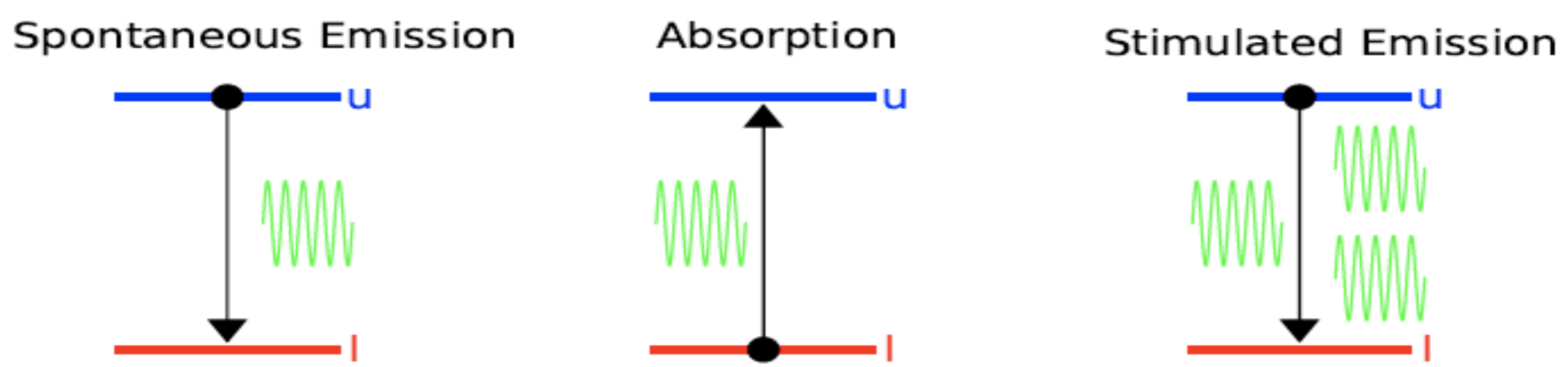
Density

Transition Rate

Line Profile

Review

From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.



$$j_\nu \rho = h\nu n_u A_{ul} \Psi(\nu) \qquad \kappa_\nu \rho I_\nu = h\nu n_l B_{lu} I_\nu \Phi(\nu)$$

Energy

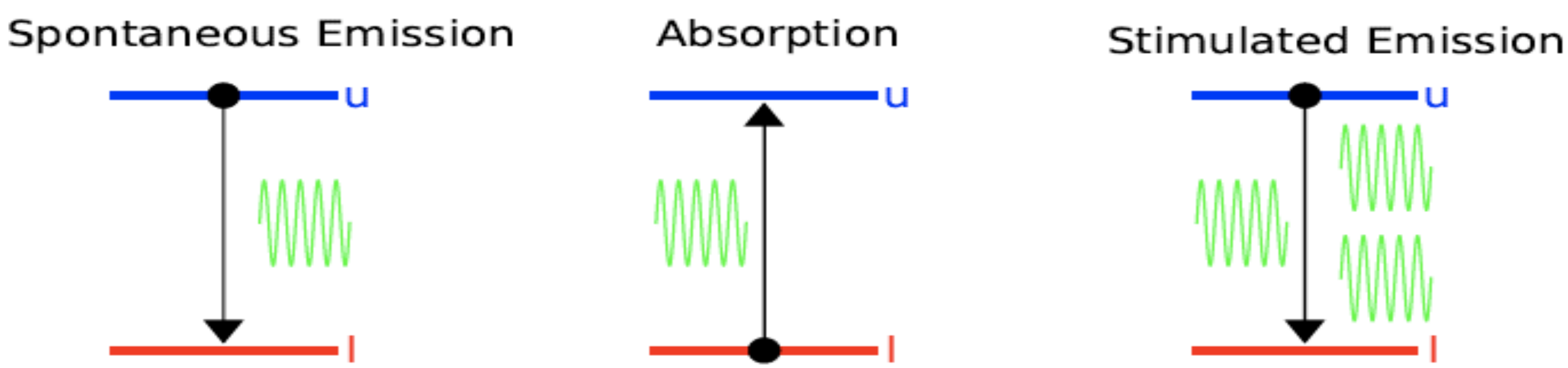
Density

Transition Rate

Line Profile

Review

From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.



$$j_\nu \rho = h\nu n_u A_{ul} \Psi(\nu) \qquad \kappa_\nu \rho I_\nu = h\nu n_l B_{lu} I_\nu \Phi(\nu)$$

Energy

Density

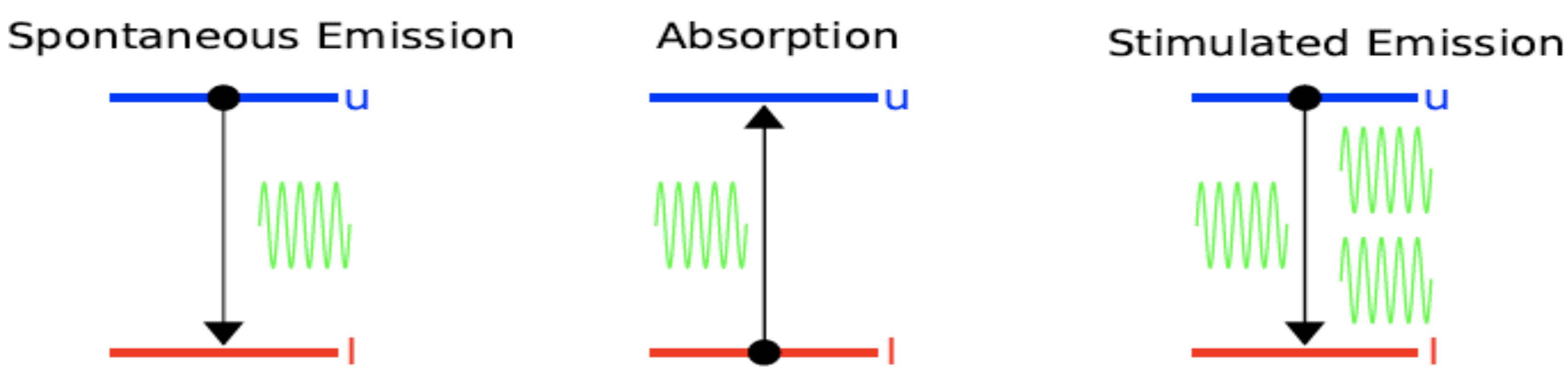
Transition Rate

Line Profile

Energy

Review

From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.



$$j_\nu \rho = h\nu\, n_u\, A_{ul}\, \Psi(\nu)$$

Energy

Density

Transition Rate

Line Profile

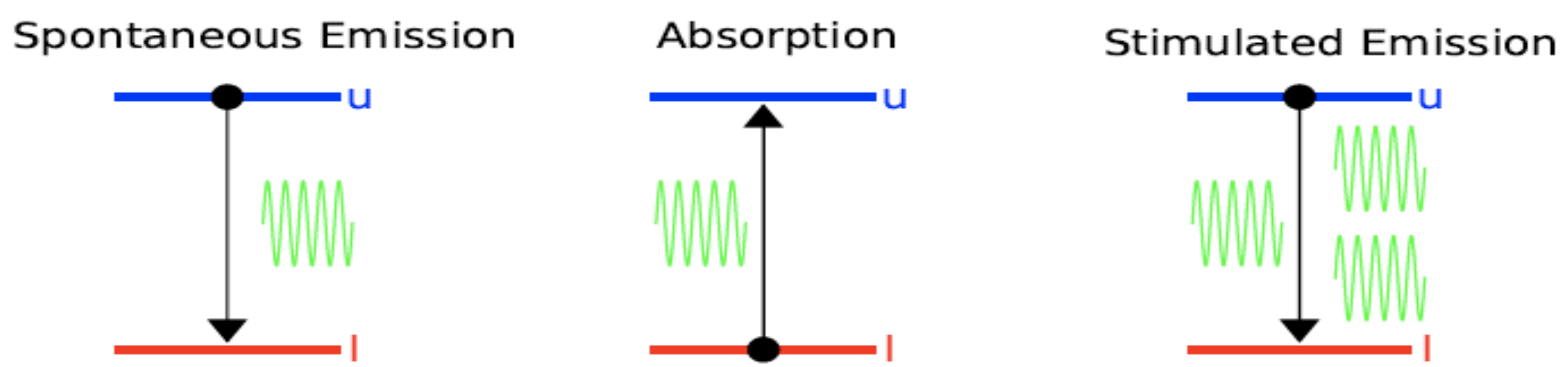
$$\kappa_\nu \rho I_\nu = h\nu\, n_l\, B_{lu} I_\nu \Phi(\nu)$$

Energy

Density

Review

From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.

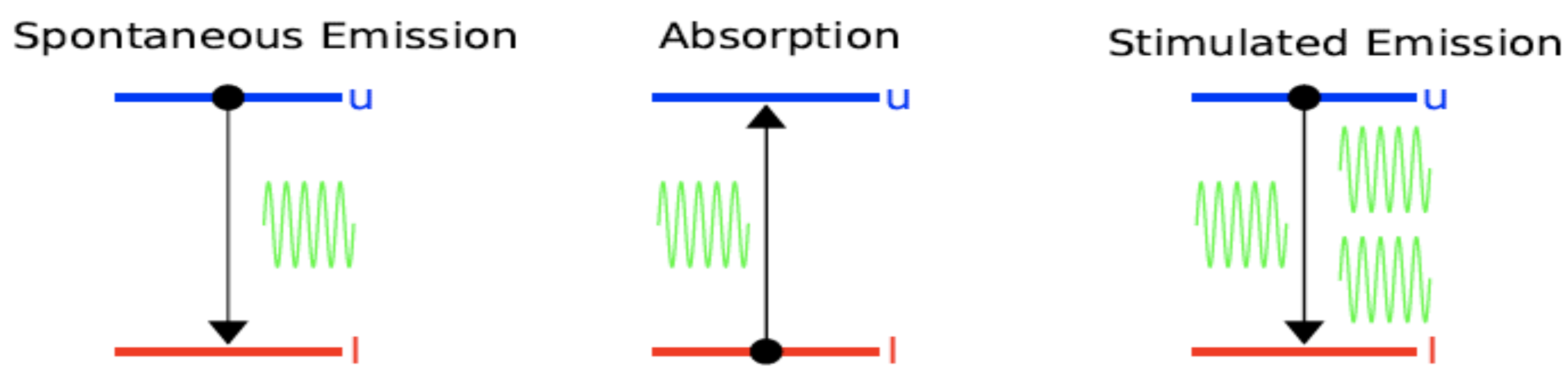


$$j_\nu \rho = \underbrace{h\nu}_{\text{Energy}}\underbrace{n_u}_{\text{Density}}\underbrace{A_{ul}}_{\text{Transition Rate}}\underbrace{\Psi(\nu)}_{\text{Line Profile}}$$

$$\kappa_\nu \rho I_\nu = \underbrace{h\nu}_{\text{Energy}}\underbrace{n_l}_{\text{Density}}\underbrace{B_{lu}}_{\text{Transition Rate}} I_\nu \Phi(\nu)$$

Review

From Lecture 6, we established that a system transitioning from one state to another will transmit a photon with energy equal to the difference between those two states. The three processes responsible for this transmission are spontaneous emission, absorption, and stimulated emission.



$$j_\nu \rho = \underbrace{h\nu}_{\text{Energy}}\,\underbrace{n_u}_{\text{Density}}\,\underbrace{A_{ul}}_{\text{Transition Rate}}\,\underbrace{\Psi(\nu)}_{\text{Line Profile}}$$

$$\kappa_\nu \rho I_\nu = \underbrace{h\nu}_{\text{Energy}}\,\underbrace{n_l}_{\text{Density}}\,\underbrace{B_{lu}}_{\text{Transition Rate}}\,I_\nu\,\underbrace{\Phi(\nu)}_{\text{Line Profile}}$$

What do we want to know?

What do we want to know?

Density

What do we want to know?

Density

Radiation Field

What do we want to know?

$\boxed{\text{Density}}$

$\left.\begin{array}{l}\boxed{\text{Density}}\\ \boxed{\text{Radiation Field}}\end{array}\right\}$

What do we want to know?

$\boxed{\text{Density}}$

$\boxed{\text{Radiation Field}}$

} These are the things that we hope to measure

What do we want to know?

Density

Radiation Field

These are the things that we hope to measure

Energy

What do we want to know?

Density

Radiation Field

}

These are the things that we hope to measure

Energy

Transition Rate

What do we want to know?

$\boxed{\text{Density}}$

$\boxed{\text{Radiation Field}}$

}

These are the things that we hope to measure

$\boxed{\text{Energy}}$

$\boxed{\text{Transition Rate}}$

}

What do we want to know?

Density

Radiation Field

$\Big\}$ These are the things that we hope to measure

Energy

Transition Rate

$\Big\}$ These are set by quantum mechanics

What do we want to know?

Density

Radiation Field

These are the things that we hope to measure

Energy

Transition Rate

These are set by quantum mechanics

Line Profile

What do we want to know?

Density

Radiation Field

These are the things that we hope to measure

Energy

Transition Rate

These are set by quantum mechanics

Line Profile

What do we want to know?

Density

Radiation Field

These are the things that we hope to measure

Energy

Transition Rate

These are set by quantum mechanics

Line Profile

And this is sometimes set by QM, and sometimes not

Energy Levels

For the case of a single electron orbiting around a bare nucleus the energy is given by:

$$E_n = -\frac{\mu Z^2 e^4}{8h^2\varepsilon_0^2}\frac{1}{n^2} = -R\frac{Z^2}{n^2}$$

However, in the presence of multiple electrons, the energy level is shifted by the different orbital states that are available. The energy becomes a function of both n and l.

$$E_n \approx -R_\infty \frac{Z_{eff}}{n^2 - \mu_{nl}}$$

The quantum defect, μ_{nl}, involves the effect of penetration to lower energy levels by electrons with l-orbits. This involves a rather complicated equation.

Profile Shapes

A number of different processes exist for the broadening of lines. The simplistic two level model for the atom is just that: simplistic. It does not include quantum mechanic effects like the uncertainty in the energy of a level or in proper relativistic treatment of the electron. Macroscopic effects like the temperature of the gas will also broaden lines as the Doppler shift of the emitting atoms relative to those that are absorbing with the same cloud. These effects are accounted for via the line profile function in the emissivity and absorption coefficients. The three main causes of line broadening are:

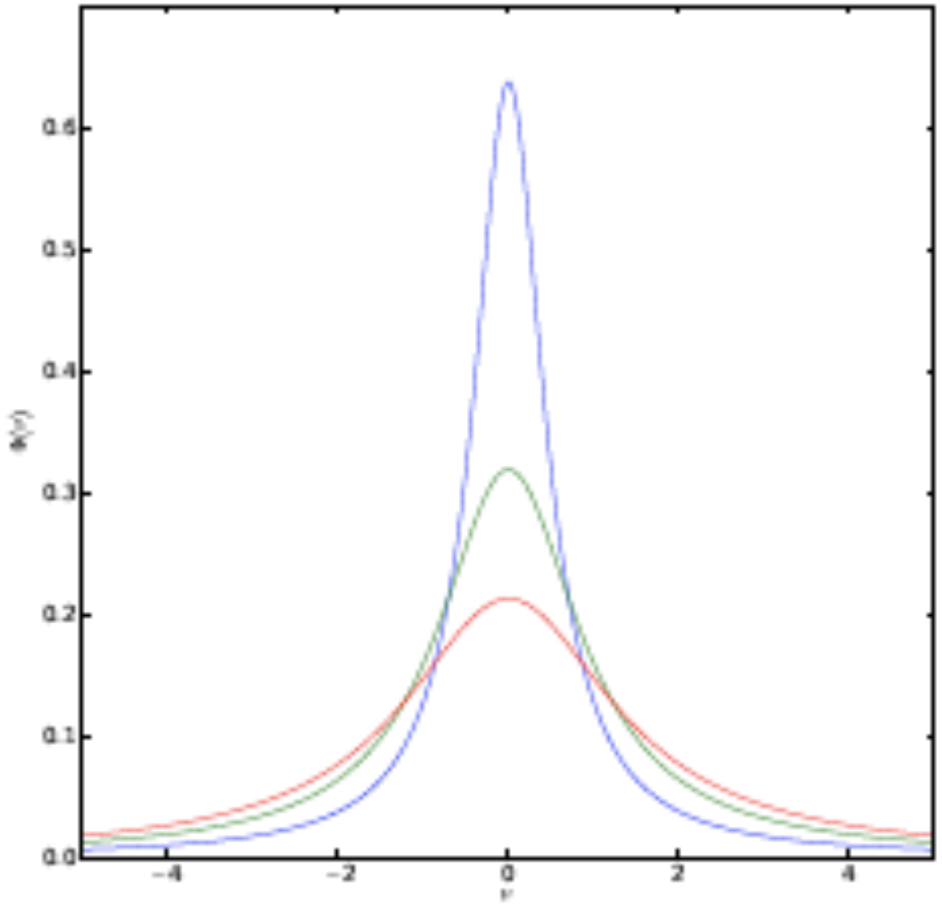
- Natural (Quantum Mechanical)
- Doppler (Thermal motions)
- Pressure (Collisional effects)

Natural Line Broadening

Natural line broadening of the line is due to the uncertainty in the energy of each level and the lifetime of that state. It is described by a Lorentzian Function, which has a narrow peak and broad wings.

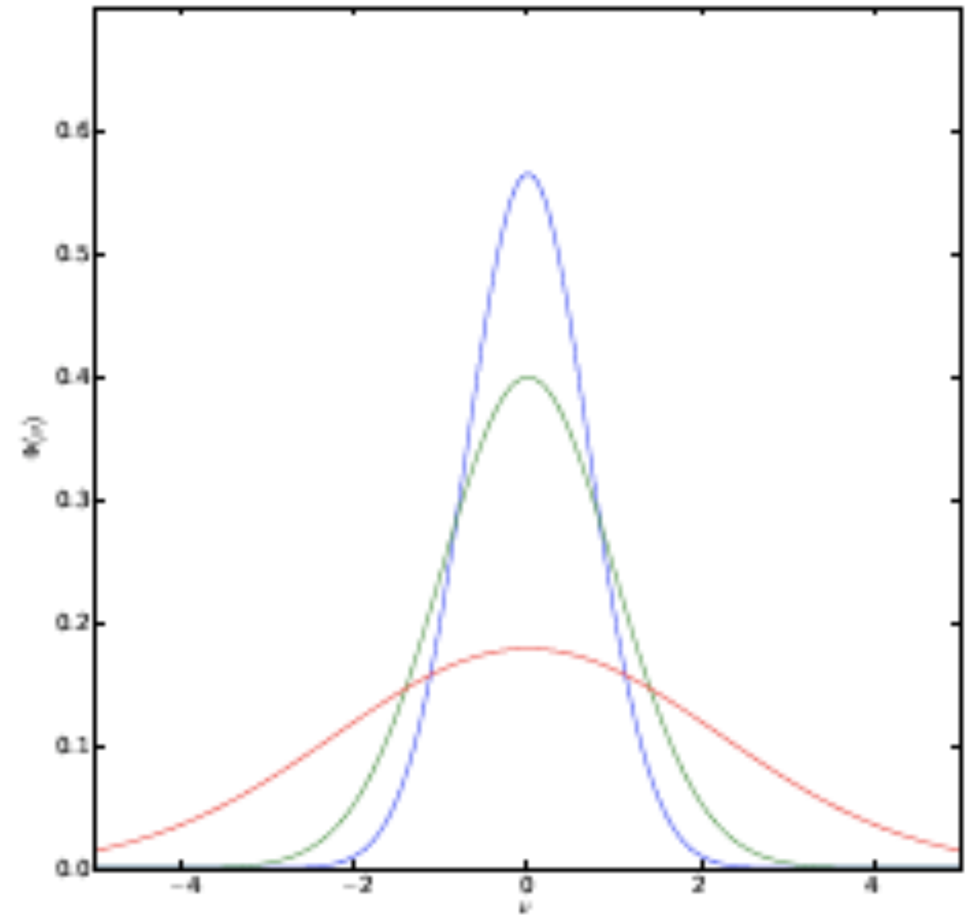
$$\Phi\nu = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\Gamma/4\pi}{(\nu-\nu_o)^2+(\Gamma/4\pi)^2}$$

In this equation, Γ is the natural broadening and is calculated by summing over all possible transitions between states.



Doppler Broadening

Doppler broadening is due to the thermal motion of the particles and the doppler shift in the line wavelength due to the different relative velocities of each particle.



$$\Phi\nu = \frac{\Delta\nu_D}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-(\frac{\nu-\nu_o}{\Delta\nu_D})^2} d\nu$$

where

$$\nu_D = \frac{b}{c}\nu_o$$

$$b = (\frac{2kT}{m})^{1/2}$$

Voigt Profile

The Voigt profile is given by the convolution of the Lorentzian and Gaussian profiles. In its extremes, it will behave as either a

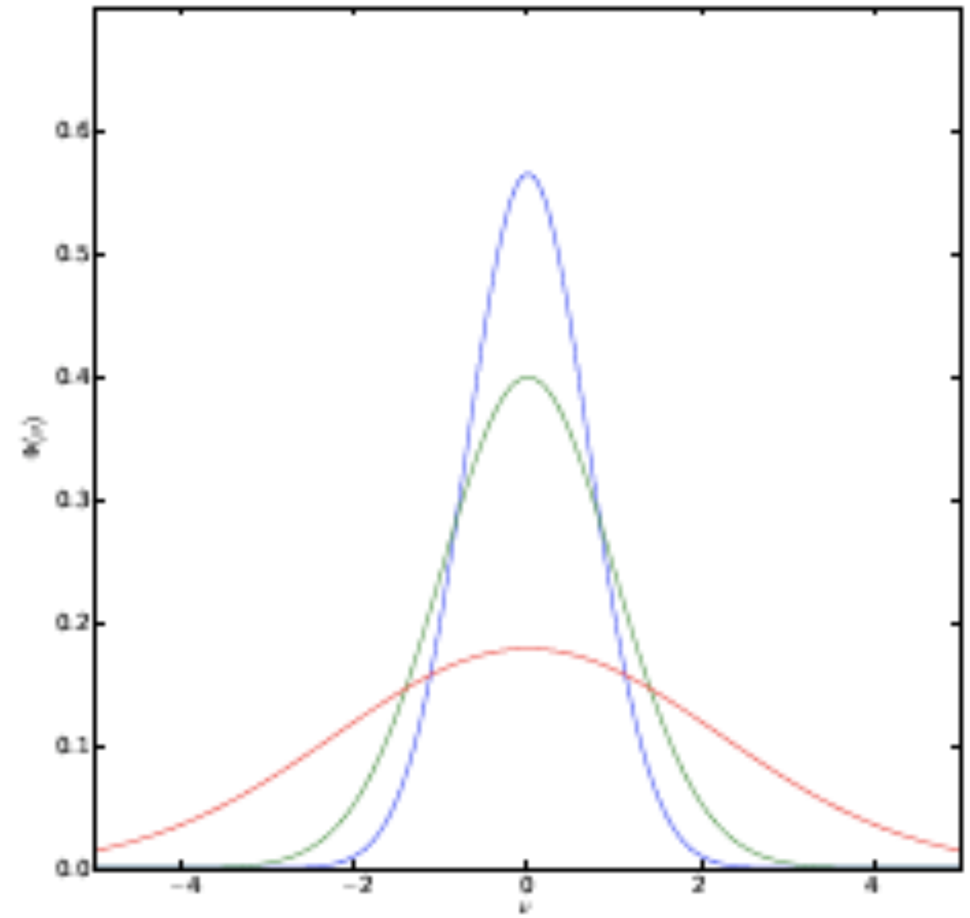
$$\Phi(\nu) = \frac{\alpha/\pi^{3/2}}{\Delta\nu_D} \int_{-\infty}^{\infty} \frac{e^{-y^2} dy}{(V-y)^2 + \alpha^2}$$

where

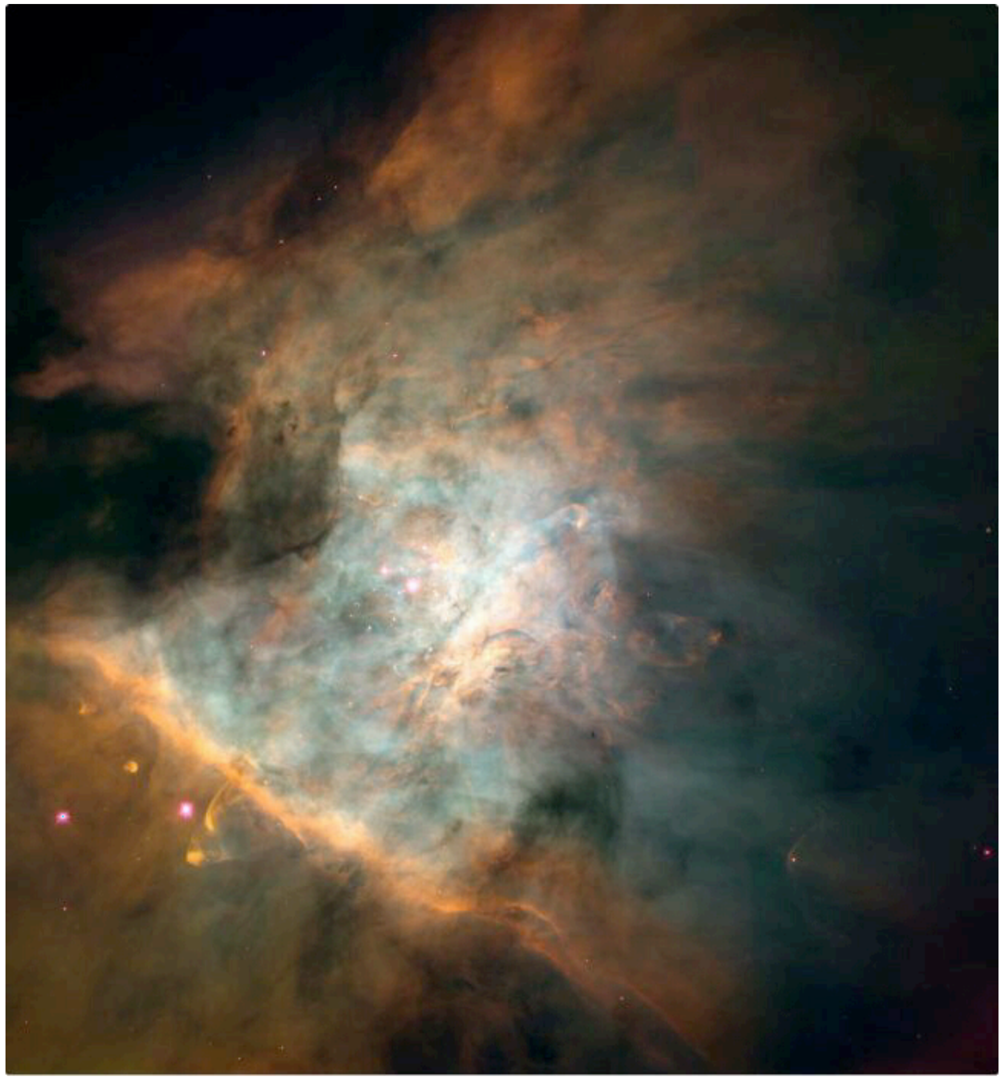
$$y = (\nu' - \nu_o')/\Delta\nu_D$$

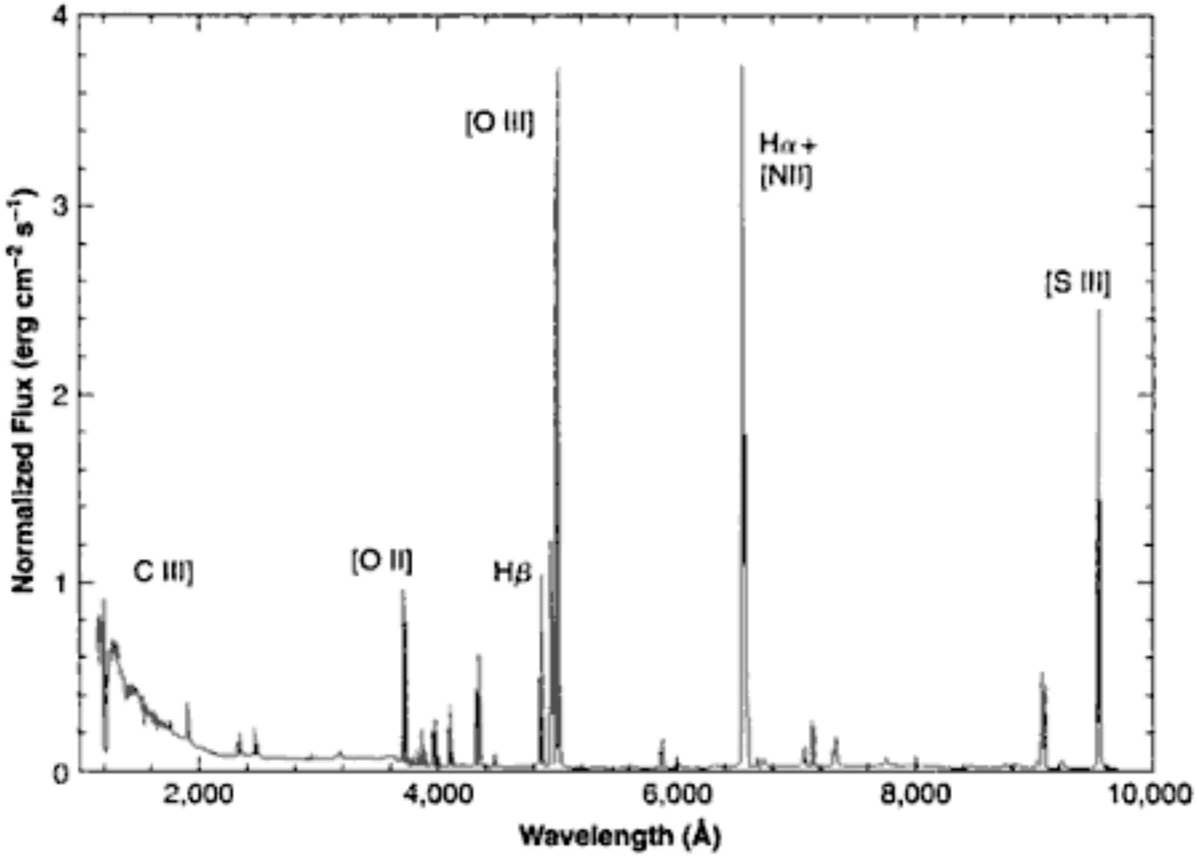
$$V = (\nu - \nu_o)/\Delta\nu_D$$

$$\alpha = \Gamma/4\pi/\Delta\nu_D$$



Emission Lines

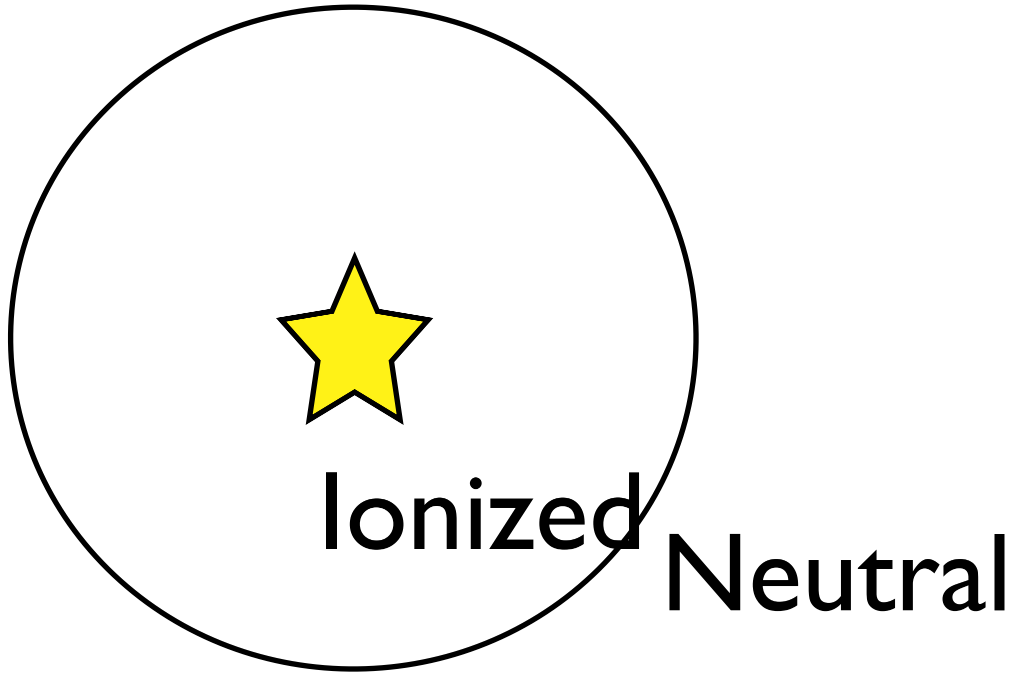




Stromgrën Sphere

A bright star produces a number of ionizing photons (Photons with E > 13.6 eV). If there is a cloud of neutral hydrogen around this star, the hydrogen will become ionized. If we assume all photons will eventually recombine, then we can assume that the number of ionizing photons will equal the number of recombination lines seen.

$$n(\mathrm{H}^0)\int_{\nu_0}^{\infty}\frac{4\pi J_\nu}{h\nu}a_\nu(\mathrm{H}^0)d\nu = n(\mathrm{H}^0)\int_{\nu_0}^{\infty}\phi_\nu a_\nu(\mathrm{H}^0)\ d\nu = n(\mathrm{H}^0)\,\Gamma(\mathrm{H}^0) \tag{2.1}$$
$$= n_e n_p\ \alpha(\mathrm{H}^0, T)[\mathrm{cm}^{-3}\ \mathrm{s}^{-1}]$$



Recombination Lines

Total Decay Rate:

$$A_{n\ell} = \sum_{n''=n_0}^{n-1} \sum_{\ell''=\ell\pm1} A_{n\ell n''\ell''}$$

Case A (no=1): All recombination photons escape from the cloud. A good assumption for very low mass, optical thin clouds. This is not very common though.

Case B (no=2): Assumes all Lyman lines higher than alpha are re-absorbed. Common assumption for most clouds.

Ionization Regions

$$\frac{n(\mathrm{H}^0)R^2}{r^2}\int_{\nu_0}^{\infty}\frac{\pi F_\nu(R)}{h\nu}a_\nu \exp(-\tau_\nu)\,d\nu = n_p n_e \alpha_B(\mathrm{H}^0, T)$$

$$4\pi R^2 \int_{\nu_0}^{\infty}\frac{\pi F_\nu}{h\nu}d\nu = \int_{\nu_0}^{\infty}\frac{L_\nu}{h\nu}d\nu$$

$$= Q(\mathrm{H}^0) = \frac{4\pi}{3} r_1^3 n_{\mathrm{H}}^2 \alpha_B.$$

Table 2.2

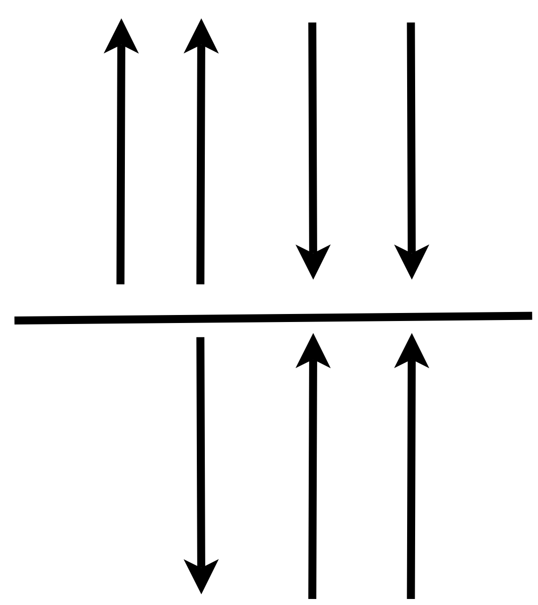
Calculated ionization distributions for model H II regions

	$T_* = 4 \times 10^{-4}$ K Blackbody Model		$T_* = 3.74 \times 10^{-4}$ K Model stellar atmosphere	
r (pc)	$\frac{n_p}{n_p + n(\mathrm{H}^0)}$	$\frac{n(\mathrm{H}^0)}{n_p + n(\mathrm{H}^0)}$	$\frac{n_p}{n_p + n(\mathrm{H}^0)}$	$\frac{n(\mathrm{H}^0)}{n_p + n(\mathrm{H}^0)}$
0.1	1.0	4.5×10^{-7}	1.0	4.5×10^{-7}
1.2	1.0	2.8×10^{-5}	1.0	2.9×10^{-5}
2.2	0.9999	1.0×10^{-4}	0.9999	1.0×10^{-4}
3.3	0.9997	2.5×10^{-4}	0.9997	2.5×10^{-4}
4.4	0.9995	4.4×10^{-4}	0.9994	4.5×10^{-4}
5.5	0.9992	8.0×10^{-4}	0.9992	8.1×10^{-4}
6.7	0.9985	1.5×10^{-3}	0.9985	1.5×10^{-3}
7.7	0.9973	2.7×10^{-3}	0.9973	2.7×10^{-3}
8.8	0.9921	7.9×10^{-3}	0.9924	7.6×10^{-3}
9.4	0.977	2.3×10^{-2}	0.979	2.1×10^{-2}
9.7	0.935	6.5×10^{-2}	0.940	6.0×10^{-2}
9.9	0.838	1.6×10^{-1}	0.842	1.6×10^{-1}
10.0	0.000	1.0	0.000	1.0

Detailed Balance

To fully consider all of the possibilities, we must consider all transitions into and out of a state. This includes those due to radiative and collisional processes.

$$\sum n_j n_e \gamma_{jk} + \sum n_j B_{jk} U_v + \sum n_j A_{jk} = \sum n_k n_e \gamma_{kj} + \sum n_k B_{kj} U_v + \sum n_k A_{kj}$$



Collision Excitation

Due to collisions with free electrons, bound electrons may become excited or de-excited. The rate at which they do, depends on :

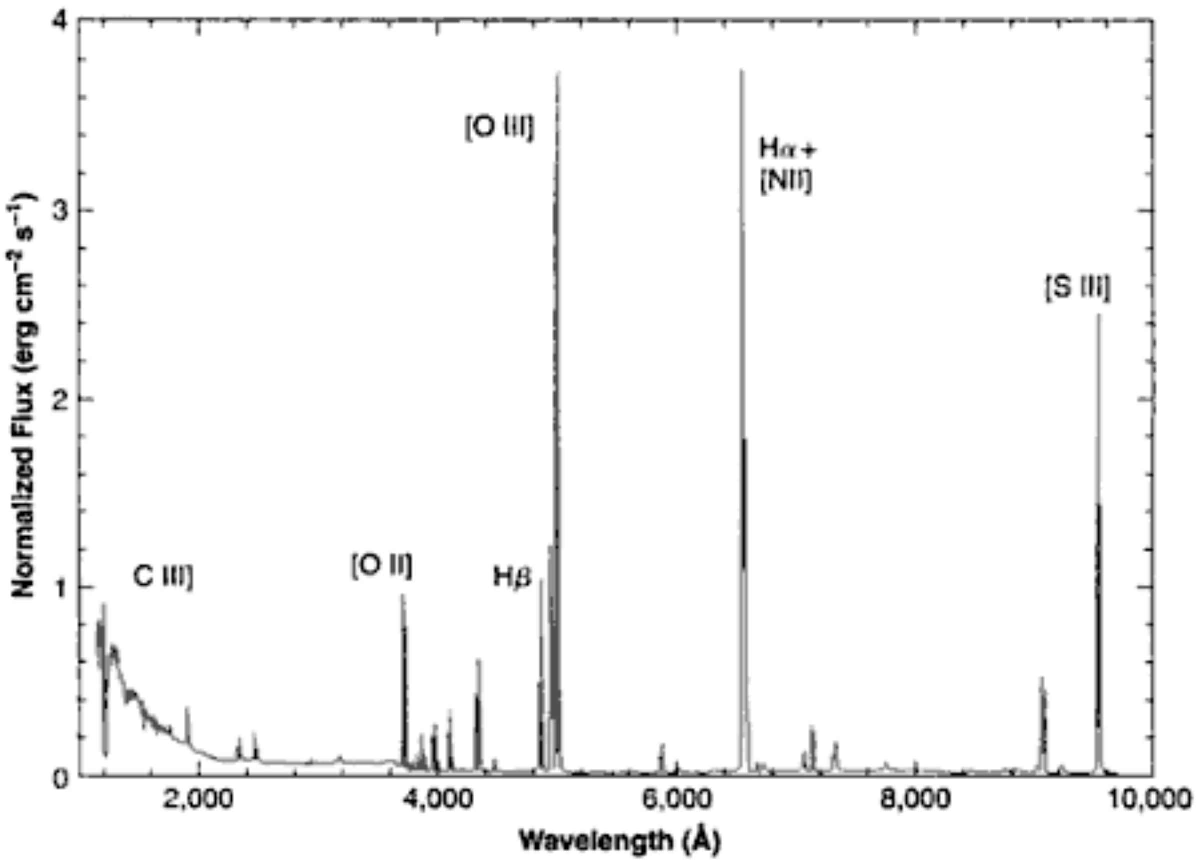
$$\gamma_{jk} =< v\sigma_{jk}(v) >= \frac{4\mu}{\pi^{1/2}} \int v^3 \sigma_{jk}(u) \exp^{-\mu v^2} dv$$

Which depends on the cross section, which is given by the collisional strength (which is usually tabulated for different collisions)

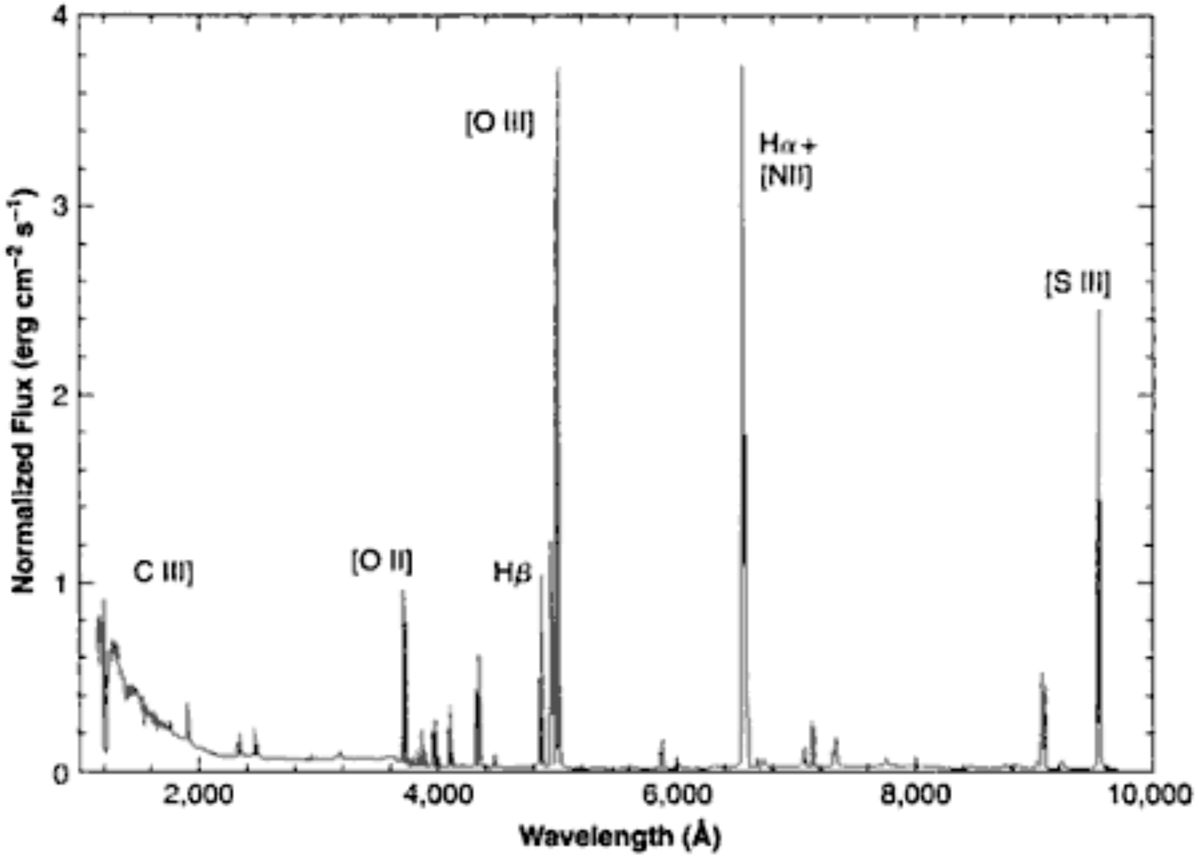
$$\sigma_{jk}(w_j) = \frac{\pi}{g_i} (\frac{h}{2\pi m_e w_j})^2 \Omega(j, k)$$

Temperature

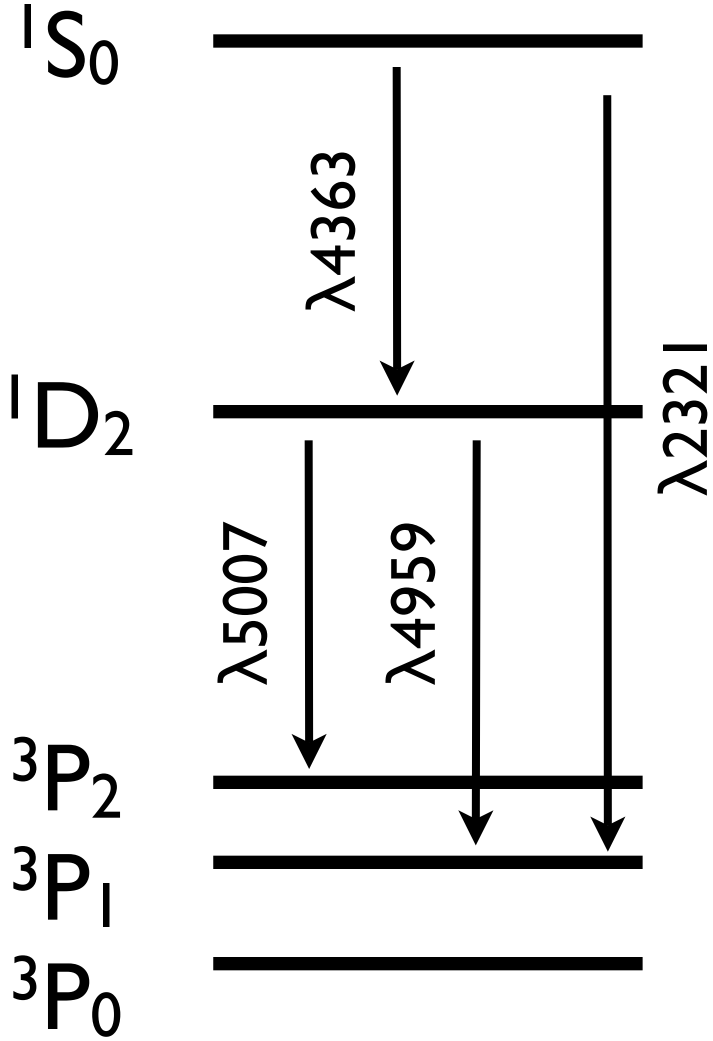
Temperature



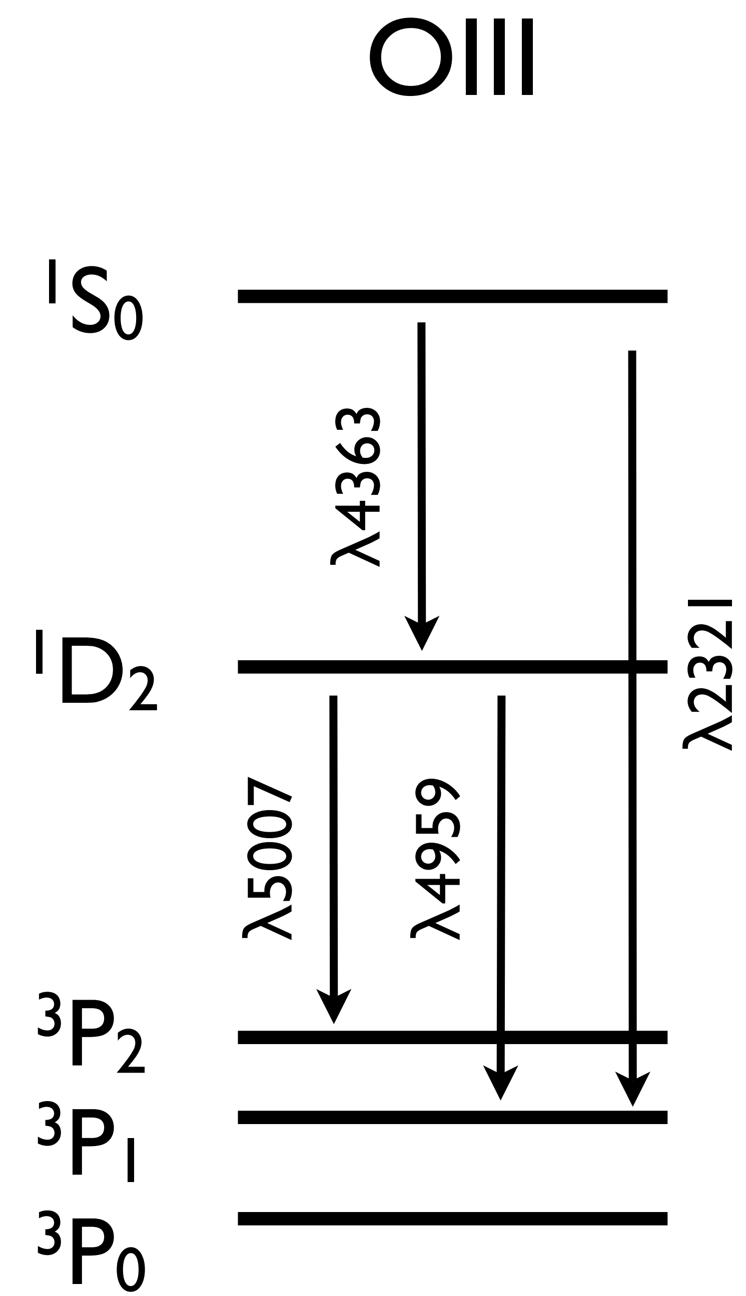
Temperature



OIII



Temperature



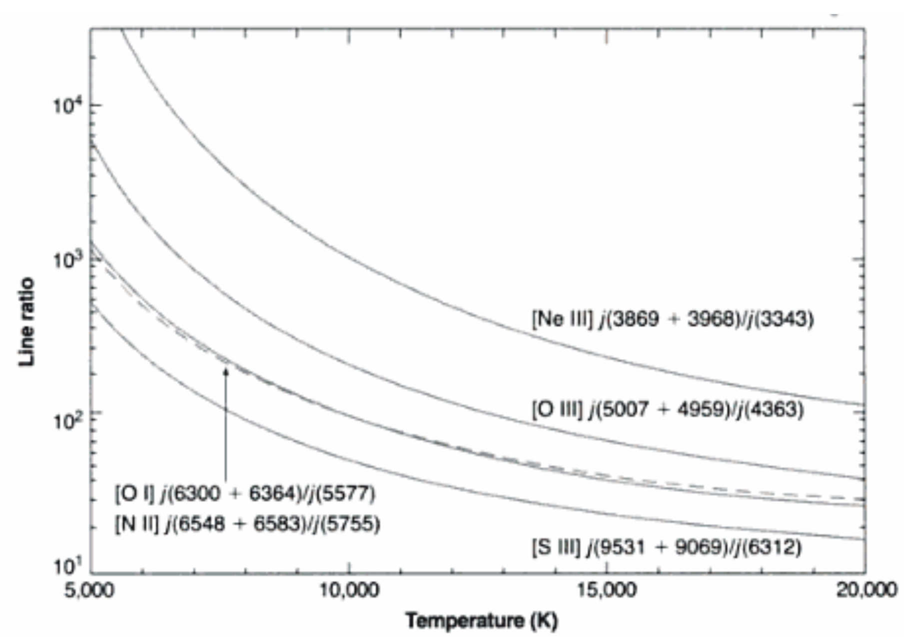
Temperature

By carefully considering all of the transitions into and out of the different levels, it can be shown that the ratio of the lines is given by:

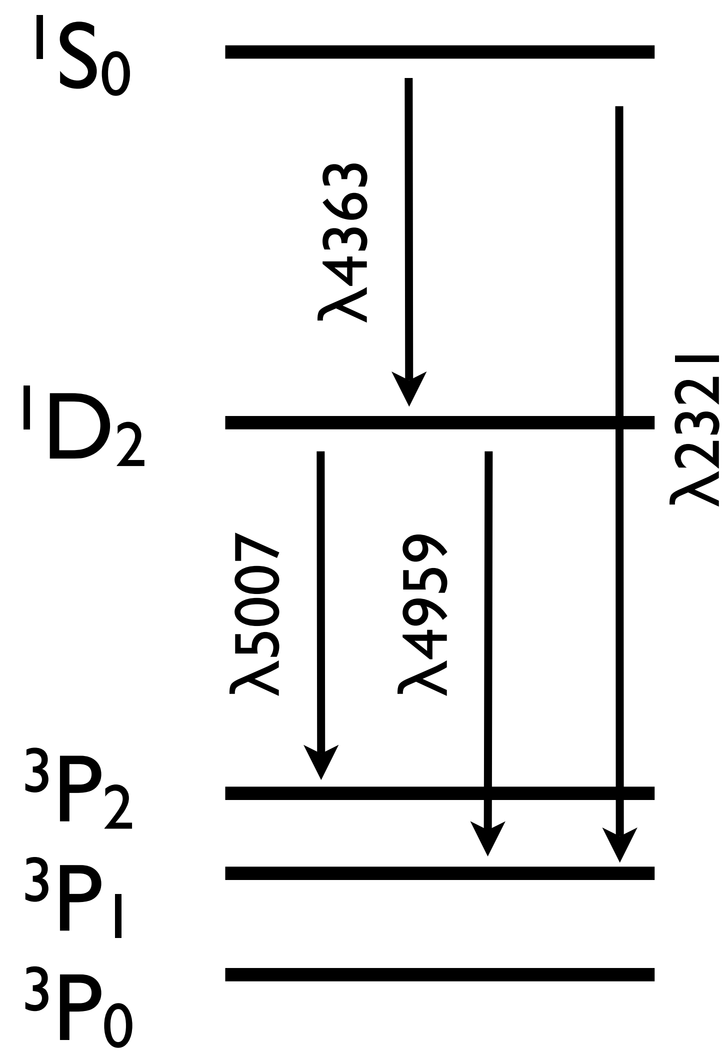
$$\frac{j_{4959}+j_{5007}}{j_{4363}}=\frac{\Omega(^3P,^1D)}{\Omega(^3P,^1S)}\frac{A_{^1S,^1D}+A_{^1S,^3P}}{A_{^1S,^1D}}\frac{\bar{\nu}(^3P,^1D)}{\nu(^1S,^1D)}e^{\Delta E/kT}$$

And this equation can be simplified so that it is mostly a fucntion of Temperature

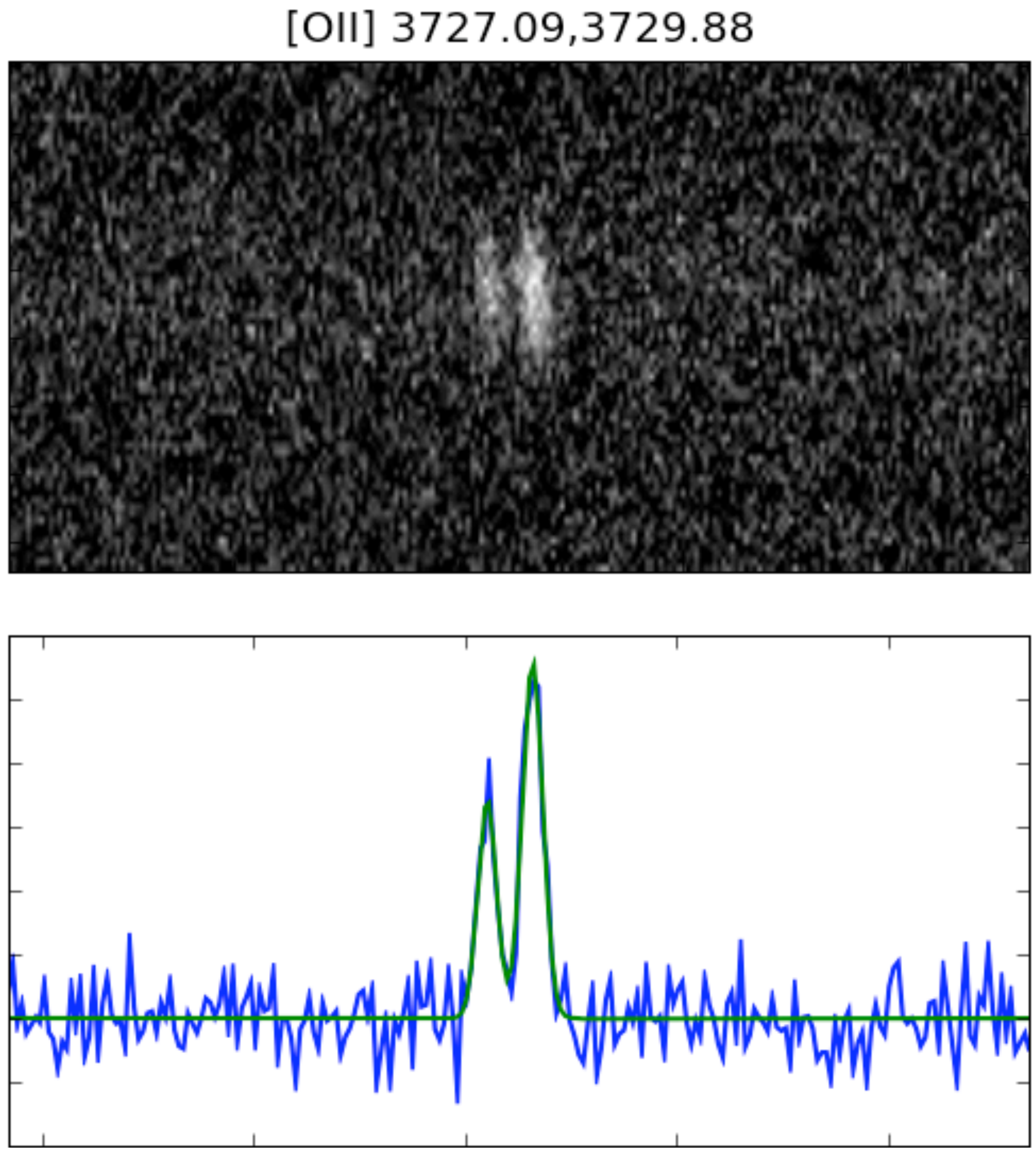
$$\frac{j_{4959}+j_{5007}}{j_{4363}}=\frac{6.91\times\exp\left[(2.5\times10^4)/T\right]}{1+2.5\times10^{-3}(n_e/T^{\frac{1}{2}})}$$



OIII



Electron Density



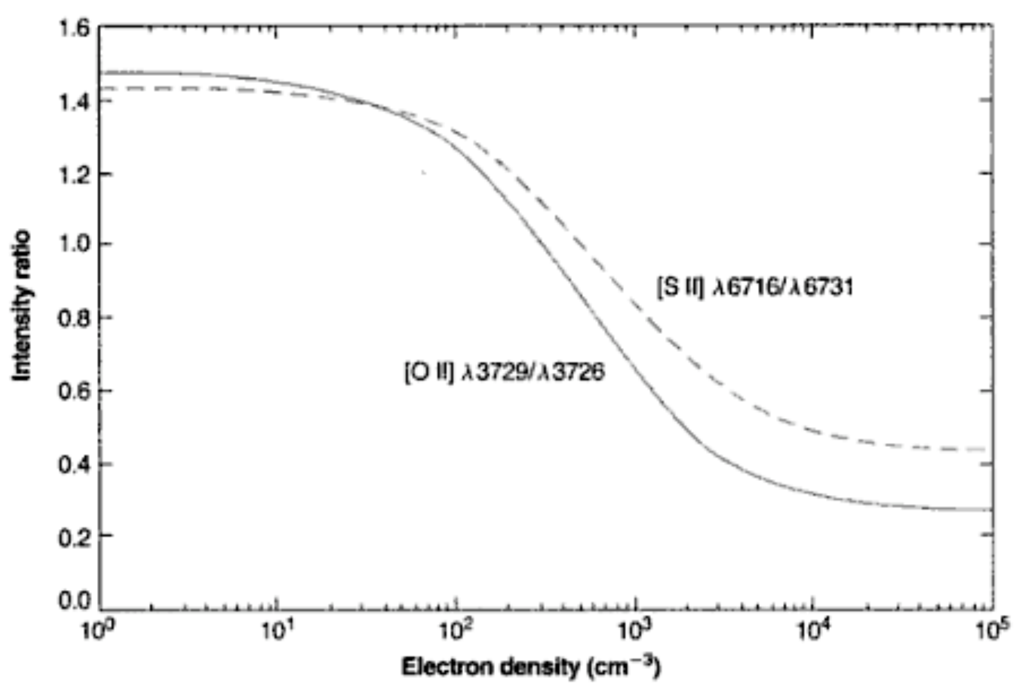
Electron Density

$$\frac{j_{3729}}{j_{3726}} \propto \frac{\Omega(^4S,^2D_{\frac{5}{2}})\ \nu(^4S,^2D_{\frac{5}{2}})}{\Omega(^4S,^2D_{\frac{3}{2}})\ \nu(^4S,^2D_{\frac{3}{2}})} e^{-\Delta E/kT}$$

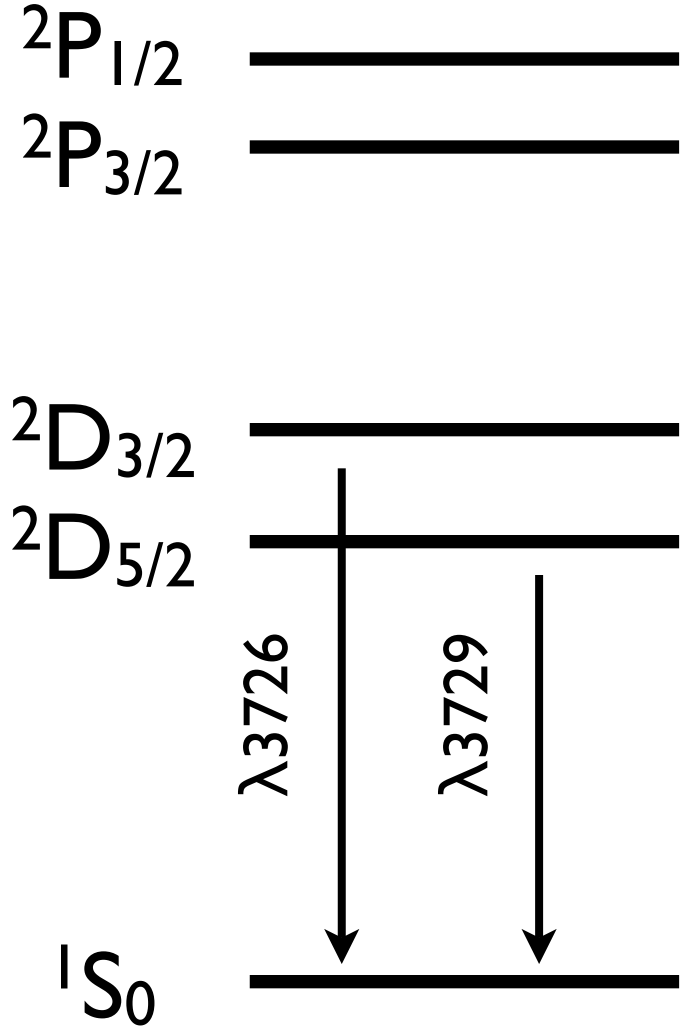
In the low density case, the lines intensities are given by: $$\frac{j_{3729}}{j_{3726}} \propto 1.5 \qquad (n_e \ll 10^3\,\mathrm{cm}^{-3})$$

In the high density case, the lines intensities are given by: $$\frac{j_{3729}}{j_{3726}} = \frac{g_{3729}}{g_{3726}}\frac{A_{3729}}{A_{3726}} = 0.3$$

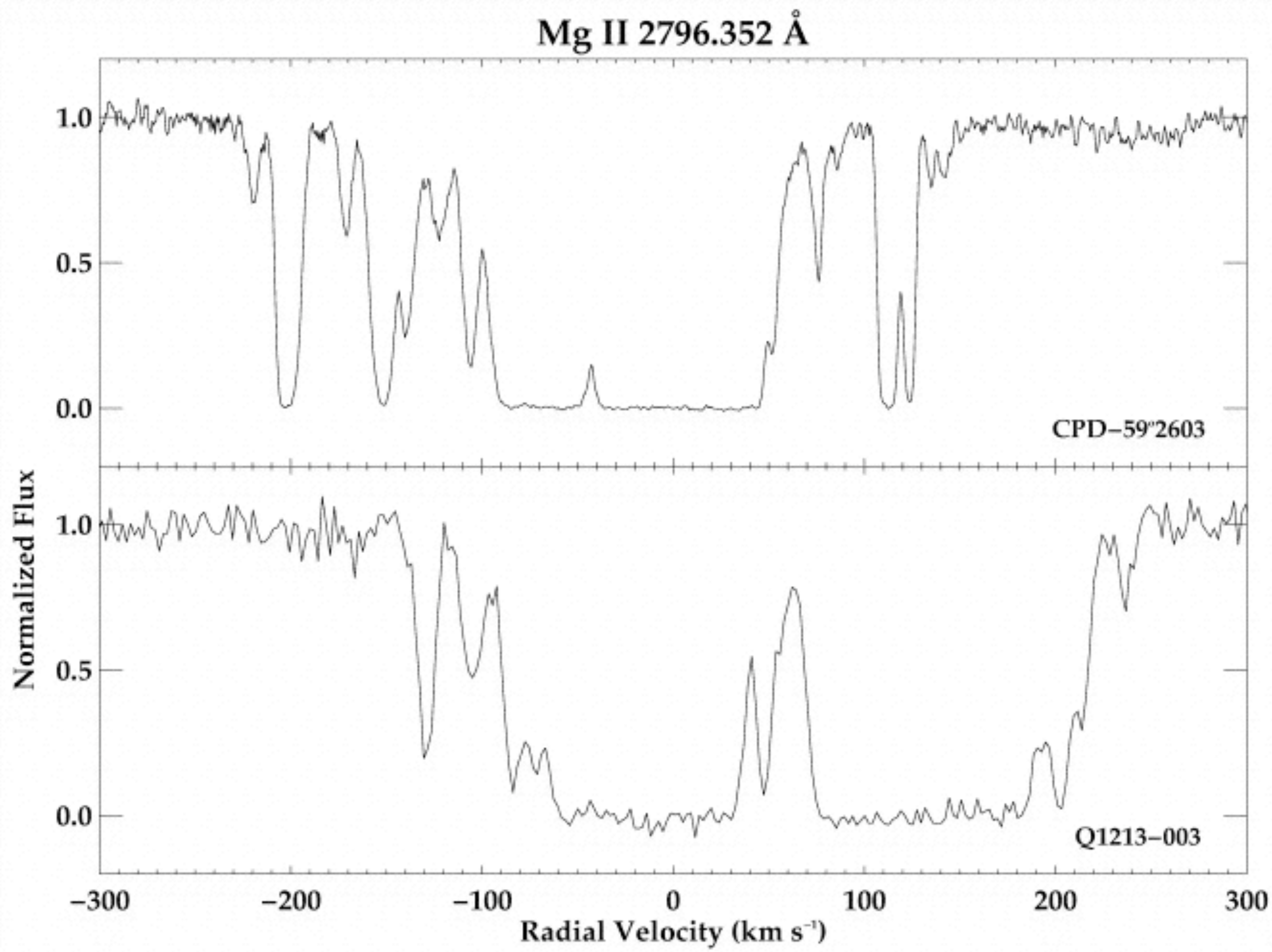
Detailed calculations lead to the relationship:



OII



Absorption Lines

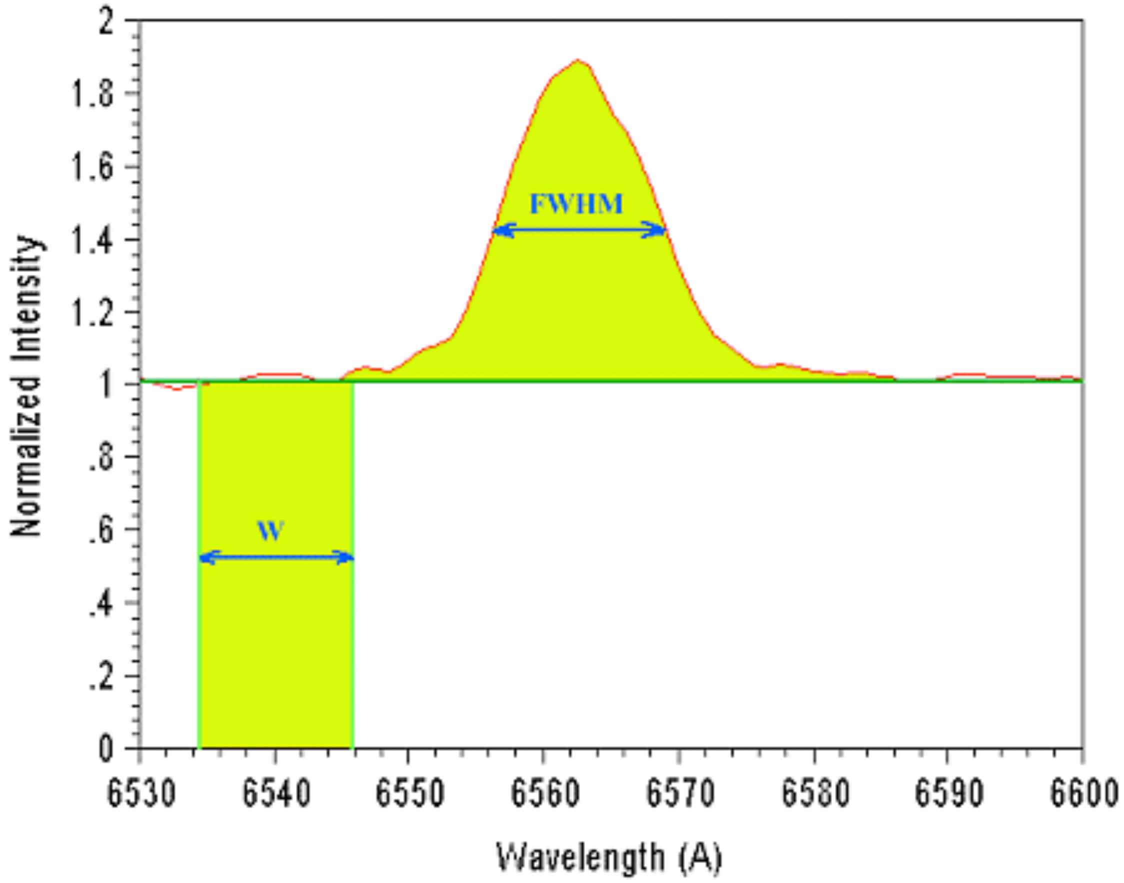


Danks et al. 2000

Equivalent Width

The equivalent width (EW) is described as the width a line if it was of unit length. Such that it is defined as:

$$W_\nu = \int \frac{I_c - I_\nu}{I_c} d\nu$$



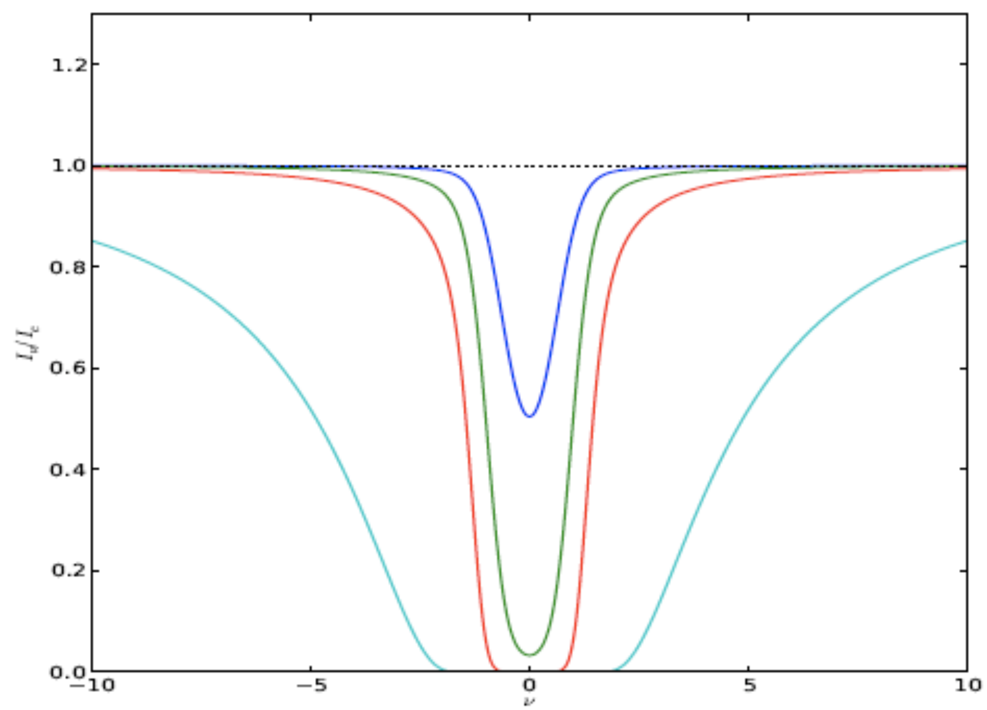
If absorption is the only effect, than the EW of an absorption feature can be given in terms of the optical depth:

$$W_\nu = \int (1 - e^{-\tau_\nu}) d\nu$$

In terms of wavelength, the EW is given by:

$$W_\nu / \nu = W_\lambda / \lambda$$

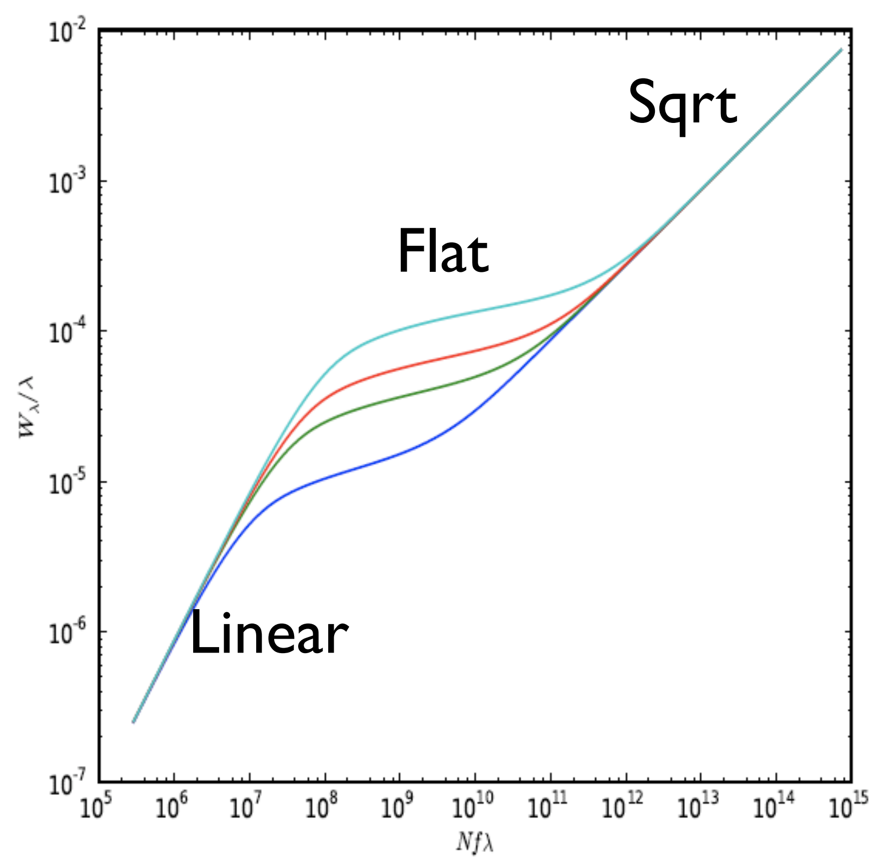
Optical Depth and EW



Reminder: The optical depth is given by the absorption coefficient, line profile, density, and path length

$$\tau_\nu = \int \kappa\rho ds = \int (\frac{\pi e^2}{m_e c}) f_{ij} \Phi(\nu) n ds$$

$$\tau_\nu = \frac{\pi e^2}{m_e c} f_{ij} \Phi(\nu) N$$



Curve of Growth: As the density of the absorber increases, so does the EW.

When the line is weak, it may be either thermally dominated. But as it becomes stronger, it becomes more dominated by the atomic line width.

Optically Thin

When $\tau_\nu << 1$, then $W_\nu = \int \tau_\nu d\nu$.

$$W_\nu = N \frac{\pi e^2}{m_e c} f_{ij} \int \Phi(\nu) d\nu = 0.02654\ cm^2/s\ N f_{ij}$$

Or in terms of the wavelength:

$$W_\lambda / \lambda = 8.85x10^{-13} cm\ N \lambda f_{ij}$$

This is known as the linear part of the curve because EW~N.

Thermally Dominated

When the line profile is dominated by thermal broadening, the optical depth is given by:

$$\tau_\nu = \tau_o e^{-((\nu-\nu_o)/\Delta\nu_D)^2}$$

$$\tau_o = \frac{N f_{ij}}{\sqrt{\pi}\Delta\nu_D}\frac{\pi e^2}{m_e c} = 0.01497 N f_{ij}\lambda/b$$

So the EW is then the integral of the function:

$$W_\nu = \int (1 - e^{-\tau_o e^{-((\nu-\nu_o)/\Delta\nu_D)^2}})d\nu$$

So the EW is then the integral of the function:

$$W_\lambda/\lambda = \frac{-2b}{c}F(\tau_o) \qquad\qquad F(\tau_o) \approx \sqrt{ln(N f_{ij}\lambda)}$$

This part of the curve of growth is described as the flat part because of the very weak dependance on density. However it can be used to determine the temperature of a region.

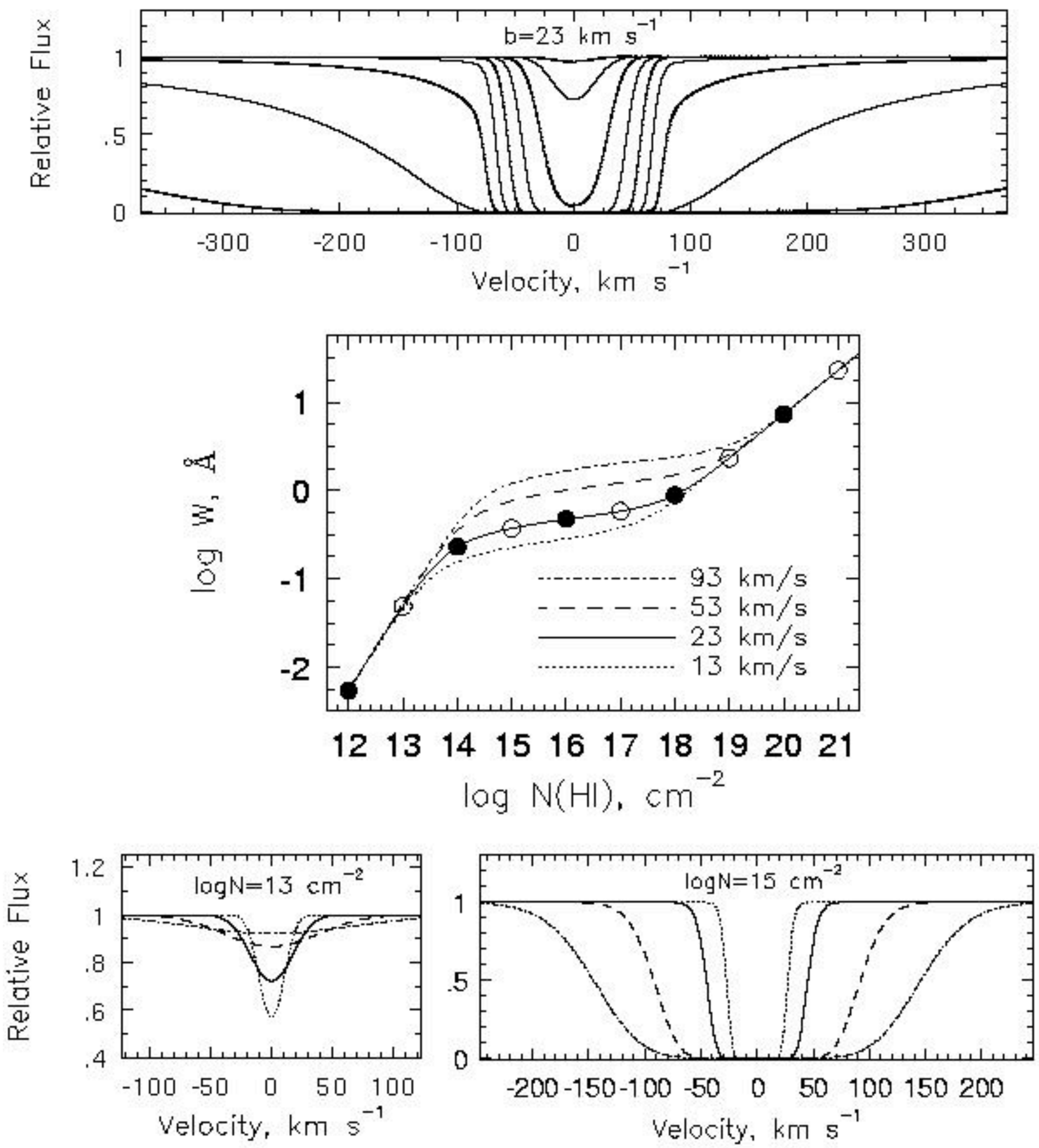
Very Optically Thick

When the line is very saturated, the wings are dominated by the Lorentz part of the profile. In this case, the EW is given by:

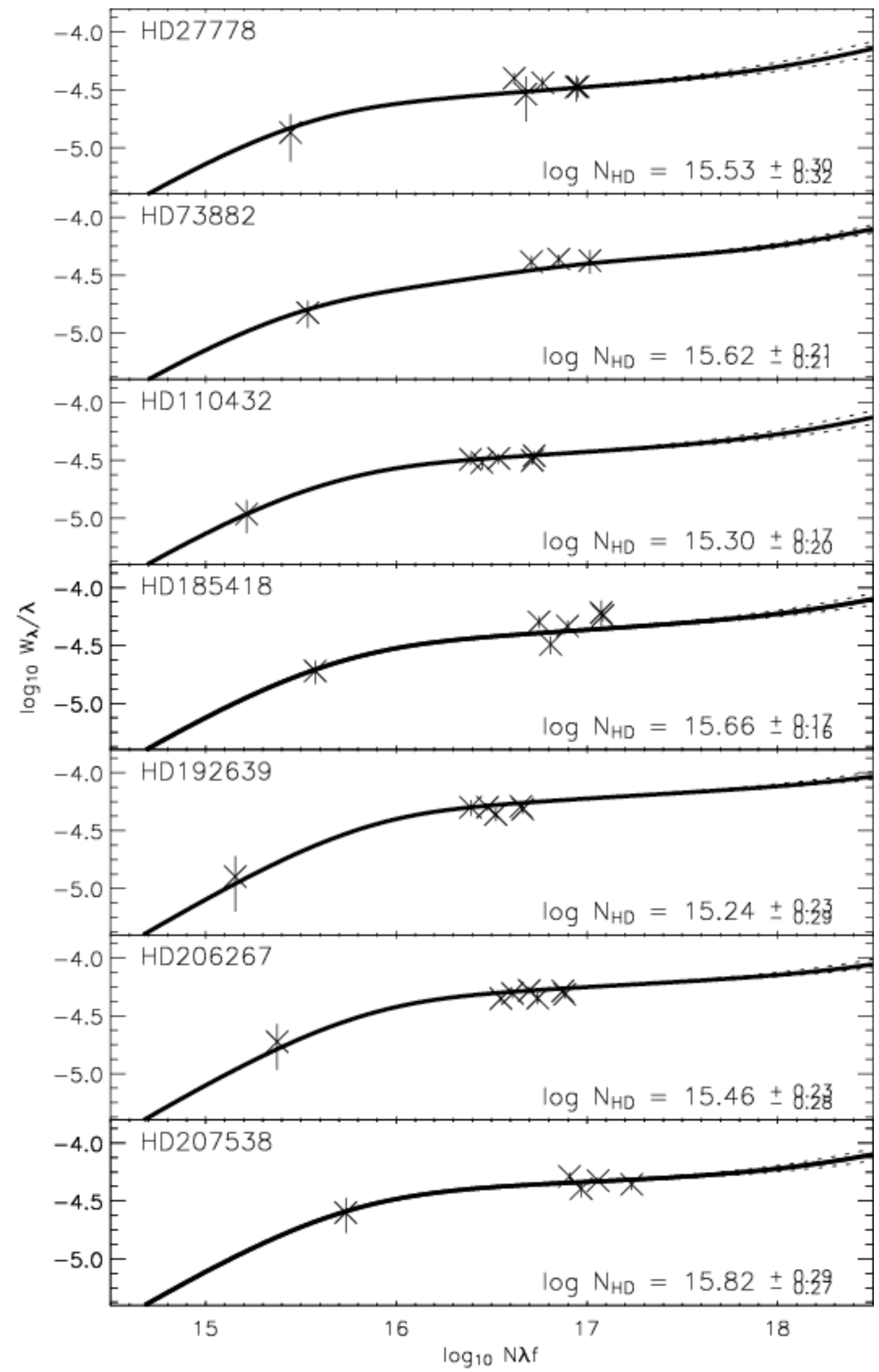
$$W_\lambda/\lambda = \frac{1}{c}((N f_{ij} \lambda)(\lambda)\frac{\Gamma}{\pi}(\frac{\pi e^2}{m_e c}))^{1/2}$$

This is referred as the Square-root part of the curve of growth.

Curve of Growth



Charlton and Churchwell 2000



Lacour et al 2005

The Einstein Coefficients

The one thing that we haven't had time to treat in this course is how to calculate the rate of transition. This requires a treatment of time-dependent perturbation theory for the quantum mechanical solution for the two level system. The probability for a electron to transition between state a and state b in the presence of an external force is:

$$P_{a->b} = \frac{|V_{ab}|^2}{\hbar^2} \frac{sin^2[(\omega_o - \omega)t/2]}{(\omega_o - \omega)^2]}$$

where the potential for an atom will be given by the electric dipole moment:

$$V_{ab} = -pE_o$$

If we sum over all possible transitions for incident photons of energy density ρ:

$$R_{a->b} = dP/dt = \frac{\pi}{3\epsilon\hbar^2}|p|^2\rho(\omega_o)$$

The Natural Broadening

The natural line broadening is related to treating the radiation as a dipole oscillator along with each level having a small width due to the uncertainty principle

$$\delta E \, \delta t = \hbar$$

The total width of the line in frequency will be given by:

$$\delta\nu = (\delta E_u - \delta E_l)/\hbar = \Gamma$$

where δE_u is the uncertainty in energy in the upper state and δE_l is the uncertainty in energy in the lower state.