Non-Thermal Continuum Radiation

Rybicki & Lightman Ch 5,6,7

Equation of Radiative Transfer

The equation for Radiative Transfer can be expressed as:

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

So far, we have look at blackbody radiation as the source function in this equation, where the Source Function is given as:

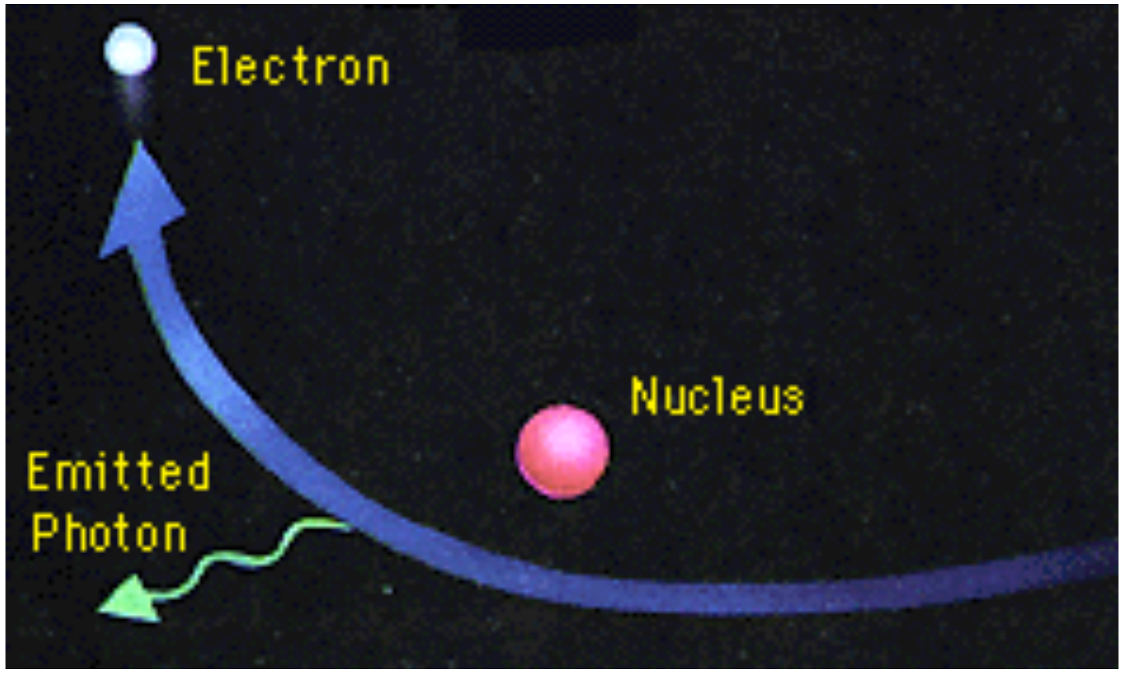
$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{hv/kT} - 1}$$

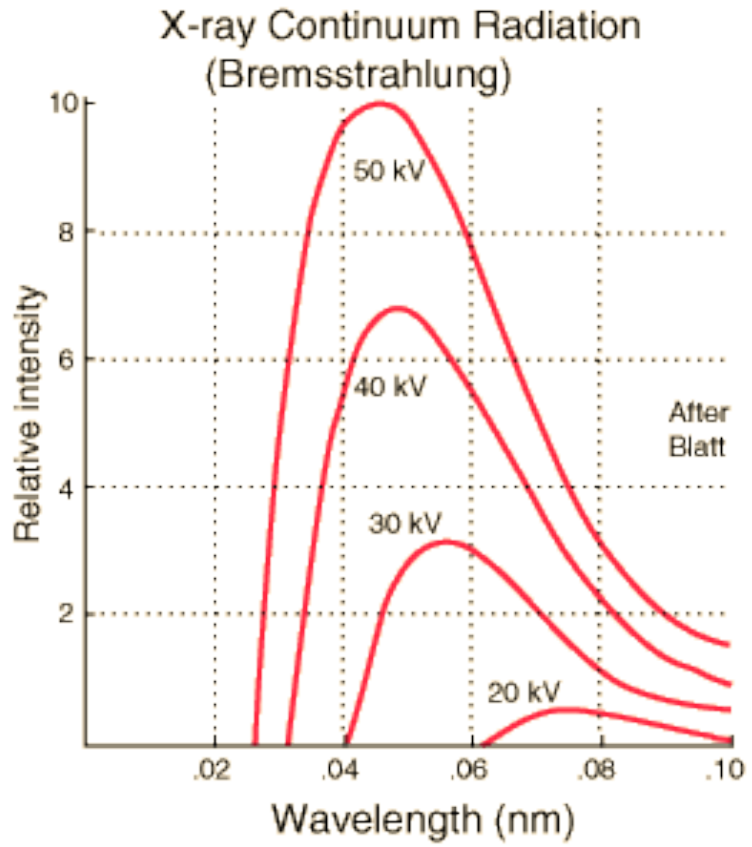
Non-Thermal Radiation for the source function

- Bremsstrahlung Radiation
- Synchrotron Radiation
- Compton Scattering

Bremsstrahlung

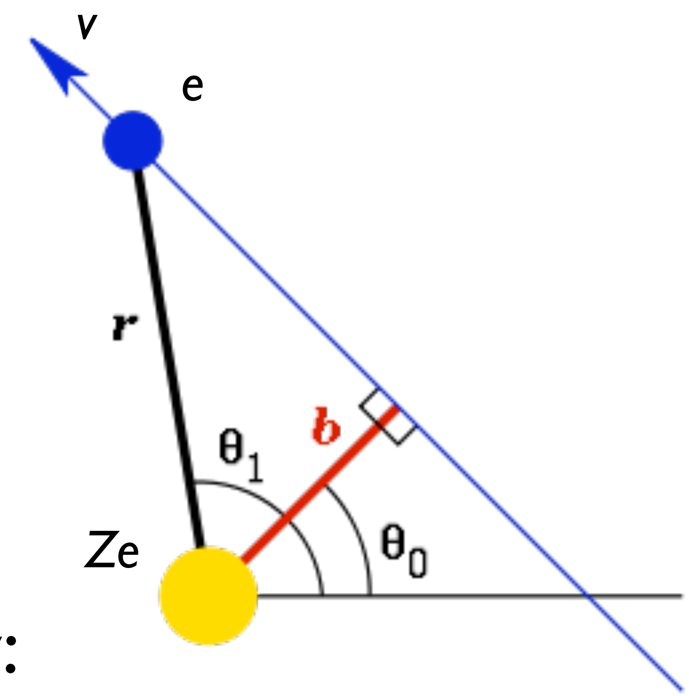
Bremsstrahlung emission is due to the change in acceleration of a charged particle due to another particle. It is is also sometimes referred to as 'free-free' emission and still retains its German name meaning 'breaking radiation.'





It can produce emission at any wavelength, but is very common in hot gas in x-ray sources.

Classical Treatment



The energy radiated by an accelerating charge is given by:

$$\frac{dE}{d\omega} = \frac{8\pi\omega^4}{3c^3}|\hat{d}(\omega)|^2$$

where the Fourier transform of the second derivative of the dipole moment, **d**=-e**R** is given by:

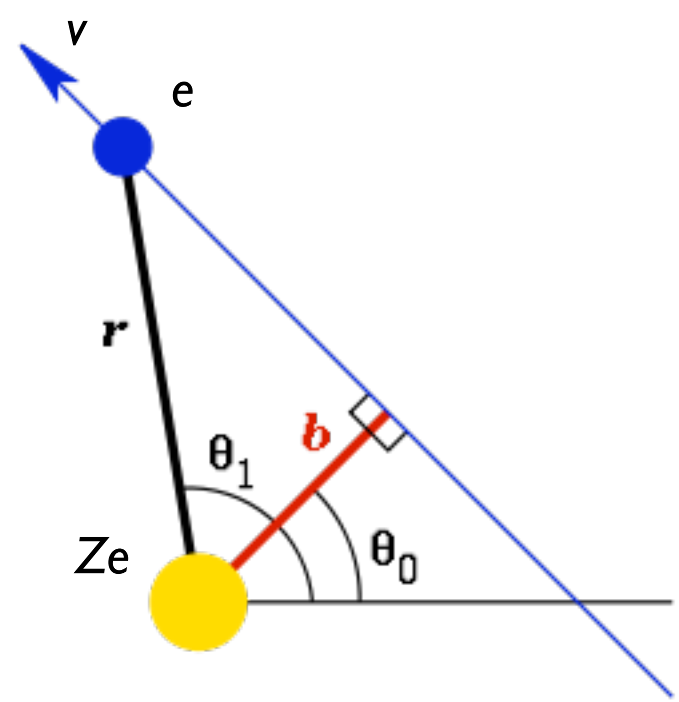
$$-\omega^2\hat{\mathbf{d}}(\omega) = -\frac{e}{2\pi}\int_{-\infty}^{\infty}\dot{\mathbf{v}}e^{i\omega t}dt$$

When $\omega b/v \ll 1$, then

$$\hat{\mathbf{d}}(\omega) \approx \frac{e}{2\pi\omega^2}\Delta\mathbf{v}$$

However $\omega b/v \gg 1$, then $\hat{\mathbf{d}}(\omega) = 0.$

Classical Treatment



So the energy radiated by an accelerating charge is given by:

$$\frac{dE}{d\omega} = \frac{2e^2}{3\pi c^3}|\mathbf{\Delta v}|^2$$

We can estimate the change in velocity if we assume that the particle is moving in a single direction:

$$\Delta v = \frac{Ze^2}{m}\int_{-\infty}^{\infty}\frac{bdt}{(b^2+(vt)^2)^{3/2}} = \frac{2Ze^2}{mbv}$$

So plugging this into the above equation gives:

$$\frac{dE(b)}{d\omega} = \frac{8Z^2e^6}{3\pi c^3 m^2 v^2 b^2}\ for\ b << v/\omega$$

Classical Treatment

Of course, what we want though, is the radiation given by a medium with a collection of ions and electrons. For this, the energy density radiated would be:

$$\frac{dE(b)}{d\omega dV dt} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{dW(b)}{d\omega} 2\pi b db$$

Classical Treatment

Of course, what we want though, is the radiation given by a medium with a collection of ions and electrons. For this, the energy density radiated would be:

$$\frac{dE(b)}{d\omega dV dt} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{dW(b)}{d\omega} 2\pi b db$$

Number of ions

Classical Treatment

Of course, what we want though, is the radiation given by a medium with a collection of ions and electrons. For this, the energy density radiated would be:

$$\frac{dE(b)}{d\omega dV dt} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{dW(b)}{d\omega} 2\pi b db$$

Number of ions

Flux of electrons

Classical Treatment

Of course, what we want though, is the radiation given by a medium with a collection of ions and electrons. For this, the energy density radiated would be:

$$\frac{dE(b)}{d\omega dV dt} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{dW(b)}{d\omega} 2\pi b db$$

Number of ions

Flux of electrons

Energy radiated from a single electrons

Classical Treatment

Of course, what we want though, is the radiation given by a medium with a collection of ions and electrons. For this, the energy density radiated would be:

$$\frac{dE(b)}{d\omega dV dt} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{dW(b)}{d\omega} 2\pi b db$$

Number of ions

Flux of electrons

Area

Energy radiated from a single electrons

Classical Treatment

Of course, what we want though, is the radiation given by a medium with a collection of ions and electrons. For this, the energy density radiated would be:

$$\frac{dE(b)}{d\omega dV dt} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{dW(b)}{d\omega} 2\pi b db$$

Classical Treatment

Of course, what we want though, is the radiation given by a medium with a collection of ions and electrons. For this, the energy density radiated would be:

$$\frac{dE(b)}{d\omega dV dt} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{dW(b)}{d\omega} 2\pi b db$$

Substituting for $dW(b)/d\omega$ and integrating gives:

$$\frac{dE(b)}{d\omega dV dt} = \frac{16e^6}{3c^3m^2v} n_e n_i Z^2 ln(\frac{b_{max}}{b_{min}})$$

where the limits on b are given by:

$$b_{max} = \frac{v}{\omega} \; and \; b_{min}^{classical} = \frac{4Ze^2}{\pi m v^2} \; or \; b_{min}^{QM} = \frac{h}{mv}$$

Classical Treatment

Of course, what we want though, is the radiation given by a medium with a collection of ions and electrons. For this, the energy density radiated would be:

$$\frac{dE(b)}{d\omega dV dt} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{dW(b)}{d\omega} 2\pi b db$$

Classical Treatment

Of course, what we want though, is the radiation given by a medium with a collection of ions and electrons. For this, the energy density radiated would be:

$$\frac{dE(b)}{d\omega dV dt} = n_i n_e v \int_{b_{min}}^{b_{max}} \frac{dW(b)}{d\omega} 2\pi b db$$

These can be stated in terms of the Gaunt factor and the radiated energy can be written as:

$$\frac{dE}{d\omega dV dt} = \frac{16\pi e^6}{3\sqrt{3}c^3 m^2 v} n_e n_i Z^2 g_{ff}$$

The Gaunt factor is defined as:

$$g_{ff}(v,\omega) = \frac{\sqrt{3}}{\pi} ln(\frac{b_{max}}{b_{min}})$$

Thermalized Bremsstrahlung

However, the previous expression is for a single velocity value. Most material in interstellar space is thermalized and exists at a range of velocities which are well described by a Maxwelliandistribution:

$$f(v) = 4\pi(\frac{m}{2\pi kT})^{3/2} v^2 \exp[-\frac{mv^2}{kT}]$$

Averaging the energy emitted over all velocities with the requirement that v_{min}^2=(2hv/m) leads to the radiative energy density:

$$\frac{dE}{dV\,dt\,d\nu} = \frac{32\pi e^6}{3mc^3}(\frac{2\pi}{3km})^{1/2} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \overline{g_{ff}}$$

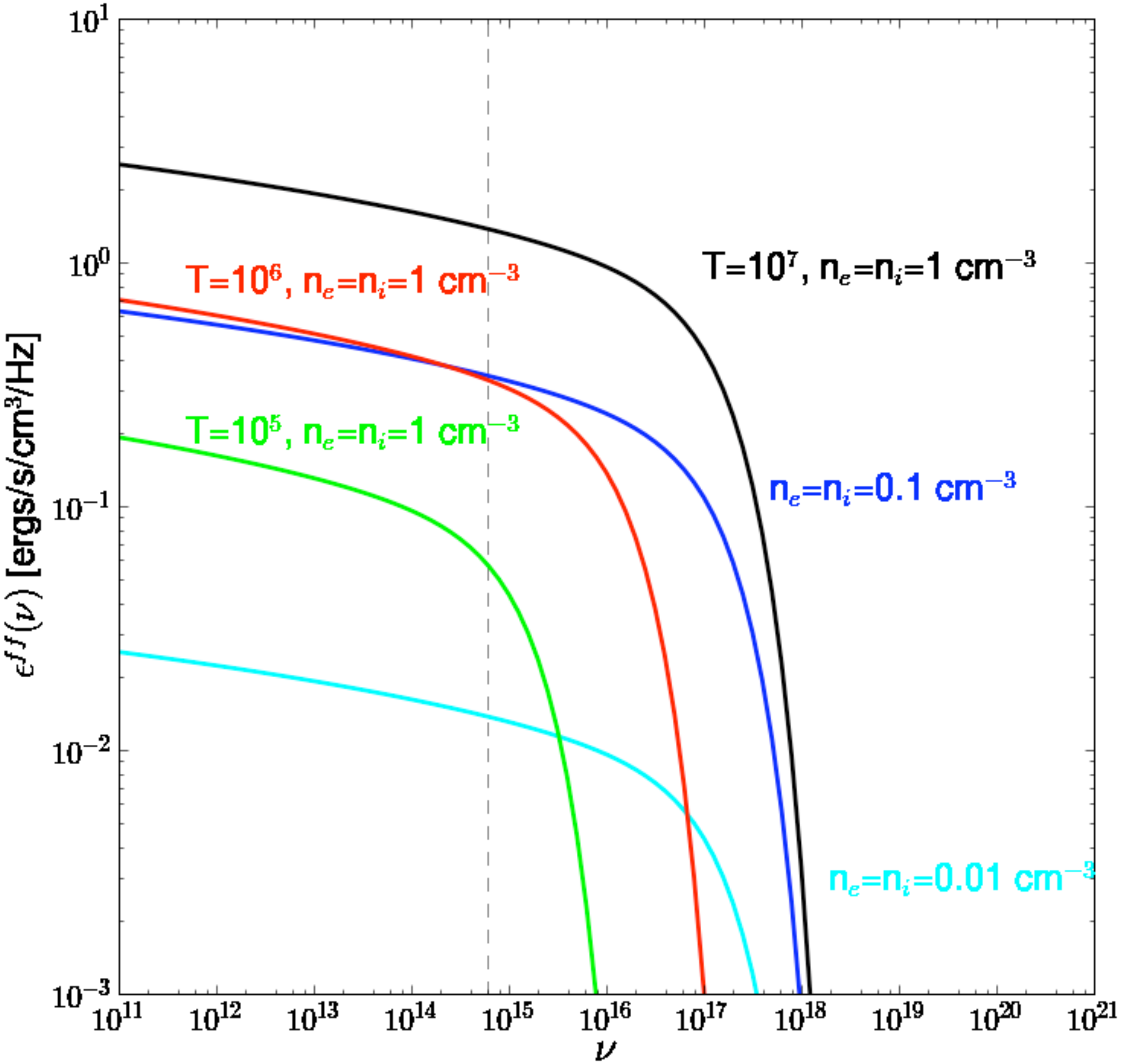
$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \overline{g_{ff}}$$

Averaging over all frequencies gives the emission (ergs s^{-1} cm^{-3}):

$$\epsilon^{ff} = 1.4 \times 10^{-27} Z^2 n_e n_i T^{1/2} \overline{g_B}$$

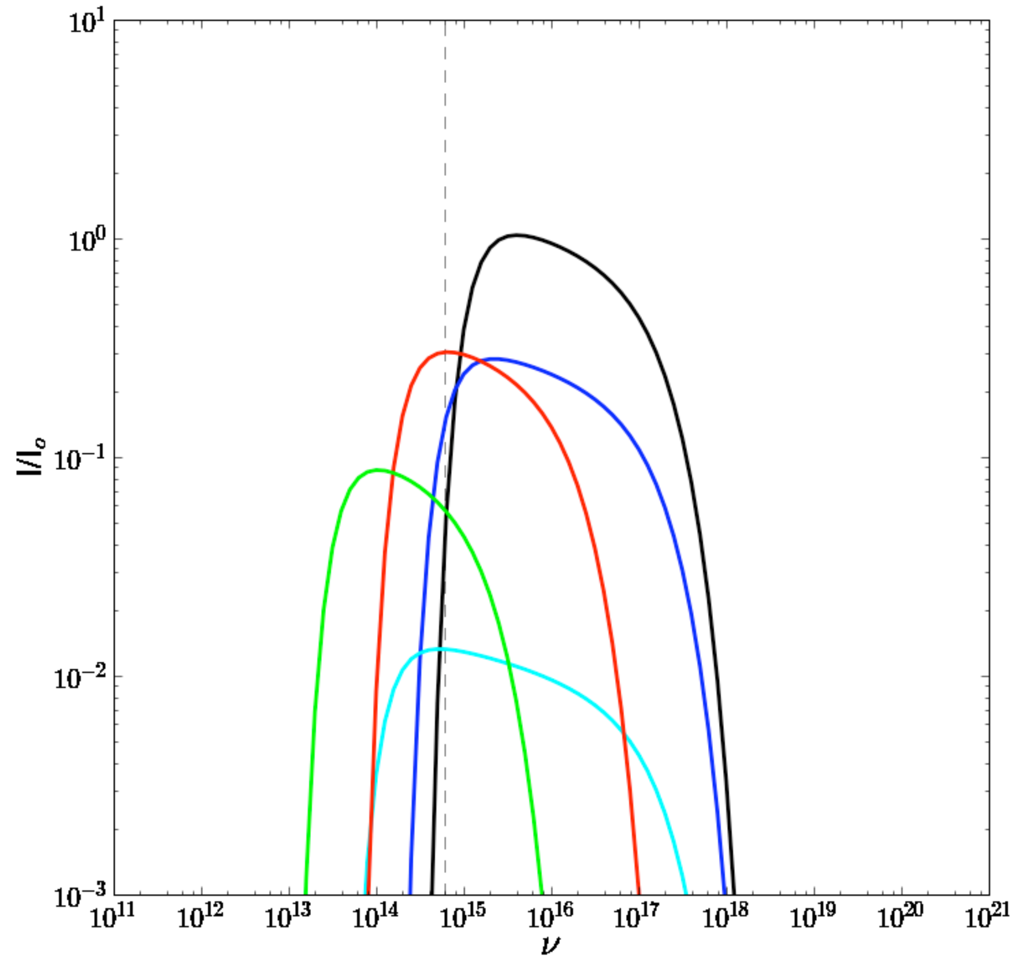
g_B typically has a value of 1.2.

Bremsstrahlung Emission



Bremsstrahlung in Absorption

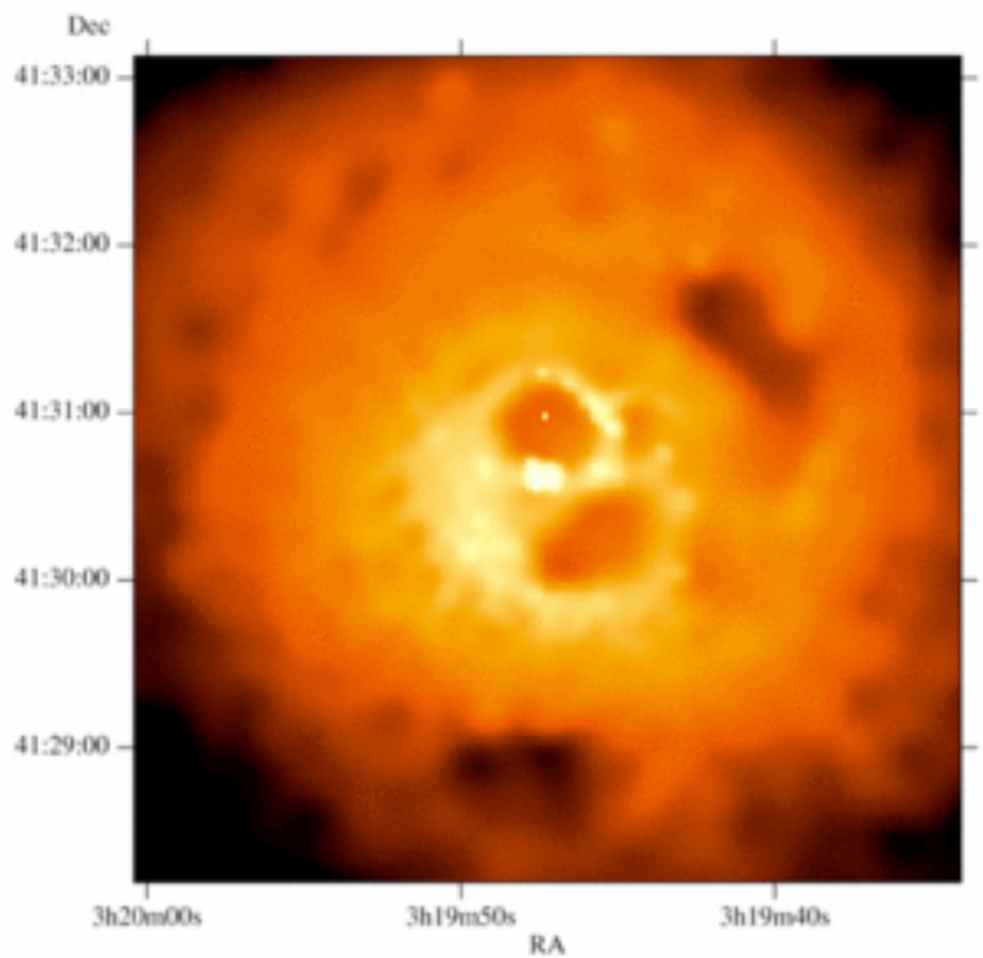
If radiation is passing through a hot plasma, the radiation can be absorbed through the same two particle interaction. At short wavelengths, the absorption falls off as ~ν^2. The full equation for the absorption as a function of wavelength is:



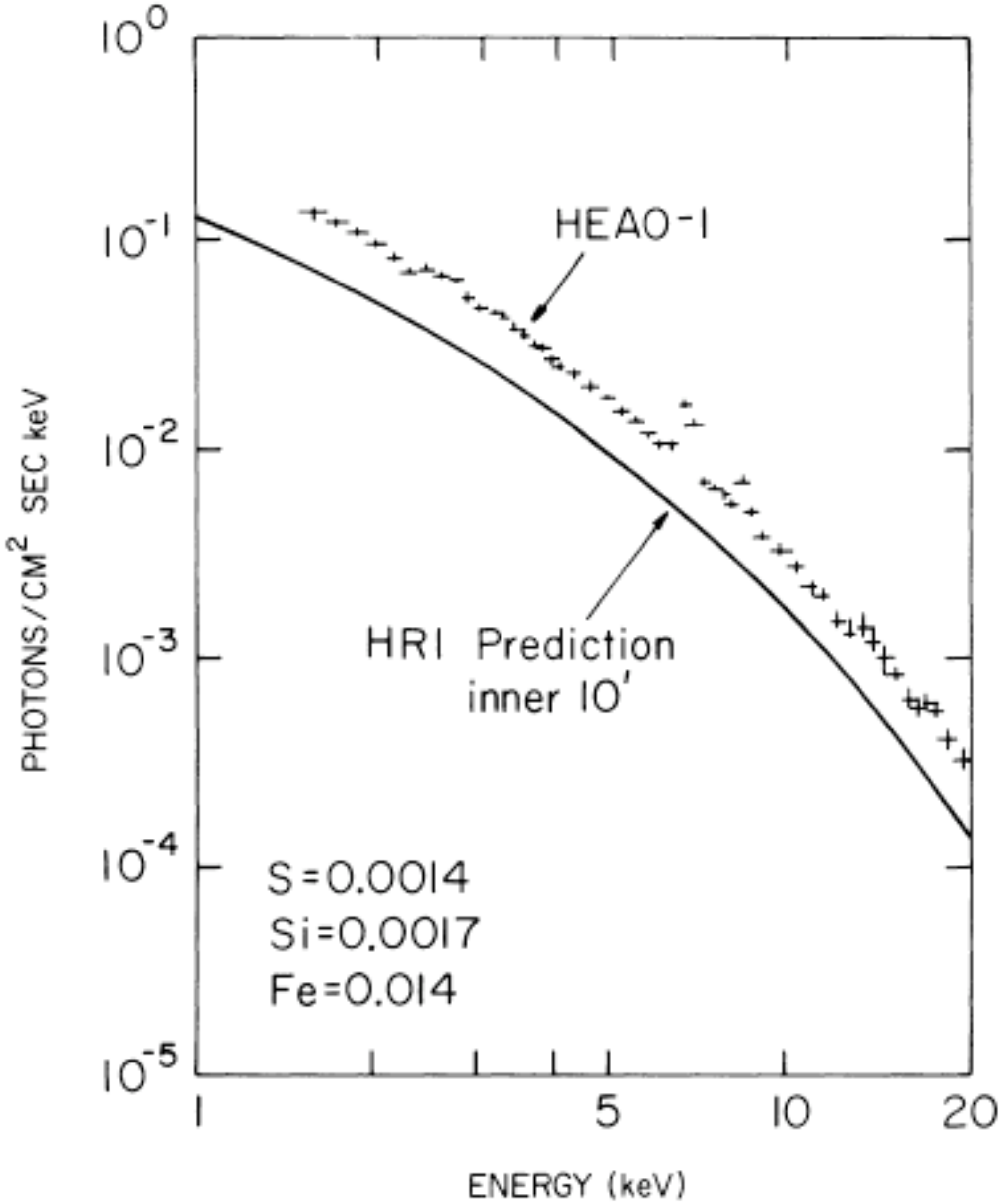
$$\alpha_v^{ff} = \kappa_v^{ff}\rho \quad = \frac{4e^6}{3mhc}\left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} Z^2 n_i n_e \nu^{-3}(1 - e^{-hv/kt})\overline{g_{ff}}$$

$$= 3.7 \times 10^8 \frac{1}{cm}\, T^{-1/2} Z^2 n_i n_e \nu^{-3}(1 - e^{-hv/kt})\overline{g_{ff}}$$

Optically Thin X-ray Emission in the Perseus Cluster

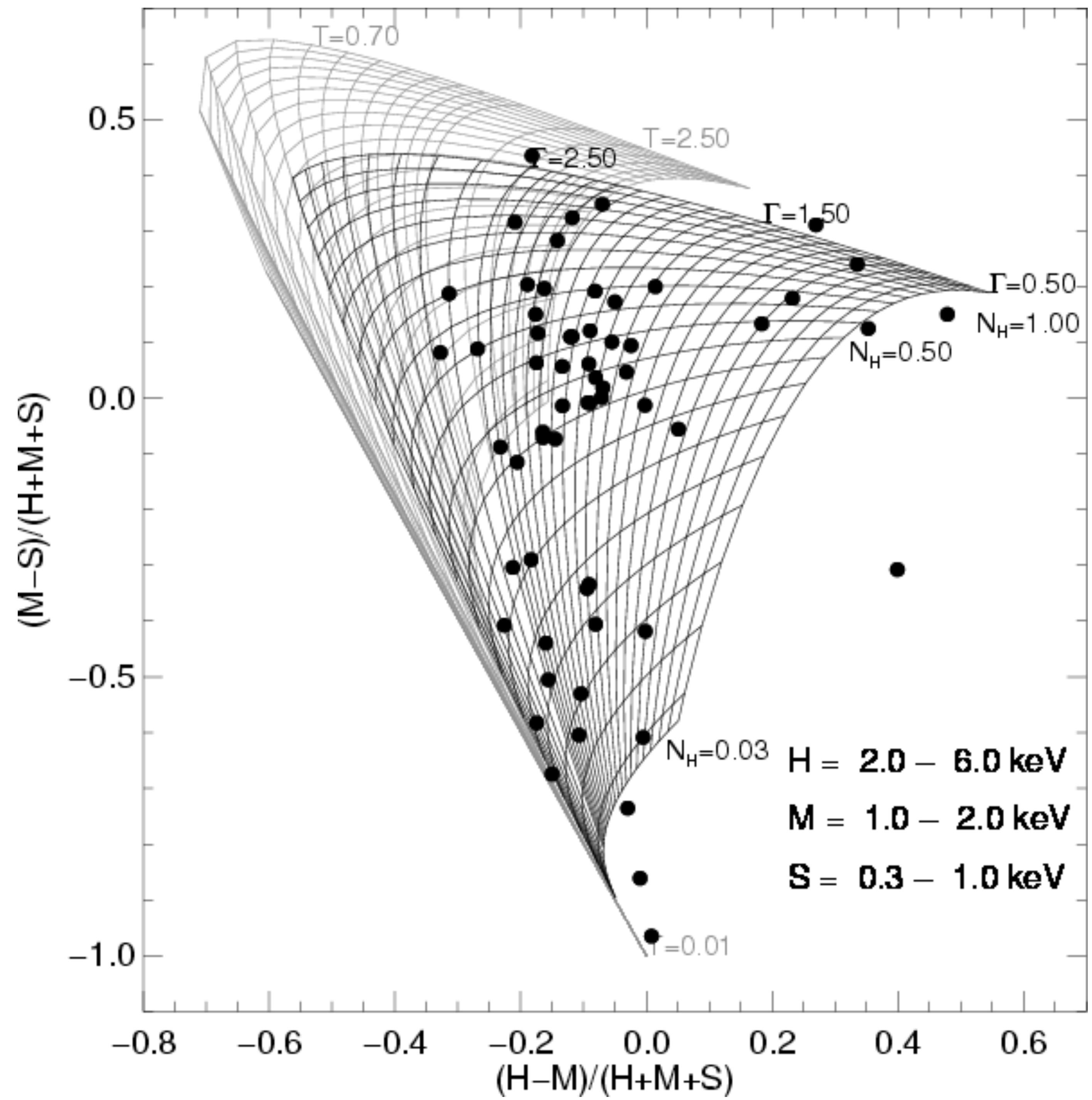


Chandra (SAO)



Fabian et al. 1981

X-Ray colors of Sources



Carpano et al. 2005

Cooling time in a Cluster

Example: What is the cooling time for the hot gas in a cluster with T=5e7 K, n=0.1 cm^{-3}, and R=100 kpc?

Cooling time in a Cluster

Example: What is the cooling time for the hot gas in a cluster with T=5e7 K, n=0.1 cm^{-3}, and R=100 kpc?

Rate of Cooling

$$\epsilon^{ff} = 1.4 \times 10^{-27} Z^2 n_e n_i T^{1/2} \overline{g_B}$$

dE/dt=1.4e-27*0.1^2*5e7^0.5*1.2*4/3*3.141*(100*3e21)^3

=1.3e46 ergs/s

Cooling time in a Cluster

Example: What is the cooling time for the hot gas in a cluster with T=5e7 K, n=0.1 cm^{-3}, and R=100 kpc?

Rate of Cooling

$$\epsilon^{ff} = 1.4 \times 10^{-27} Z^2 n_e n_i T^{1/2} \overline{g_B}$$

dE/dt=1.4e-27*0.1^2*5e7^0.5*1.2*4/3*3.141*(100*3e21)^3

=1.3e46 ergs/s

Total Energy

E= Ee * Ne = 3/2 kT * n *V

=1.5*1.4e-16*5e7 * 0.1 * 4/3*3.141*(100*3e21)^3

=9 e61 ergs

Cooling time in a Cluster

Example: What is the cooling time for the hot gas in a cluster with T=5e7 K, n=0.1 cm^{-3}, and R=100 kpc?

Rate of Cooling

$$\epsilon^{ff} = 1.4 \times 10^{-27} Z^2 n_e n_i T^{1/2} \overline{g_B}$$

dE/dt=1.4e-27*0.1^2*5e7^0.5*1.2*4/3*3.141*(100*3e21)^3
=1.3e46 ergs/s

Total Energy

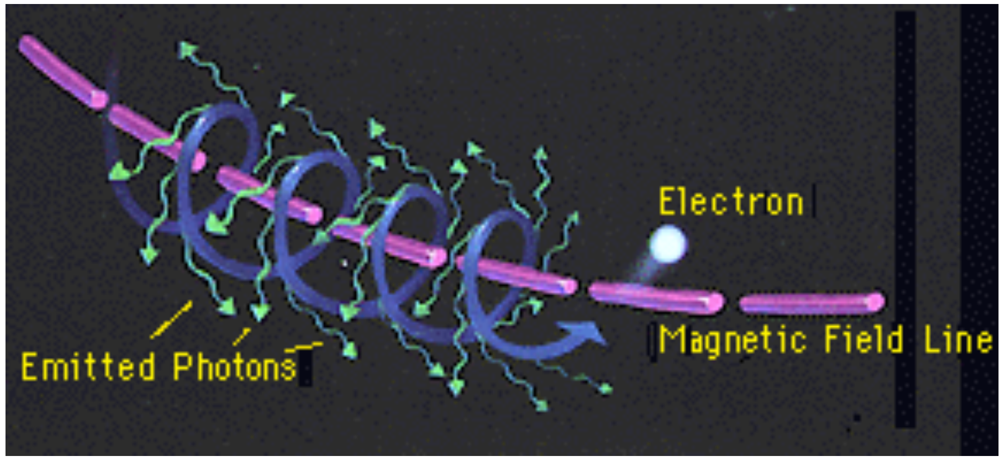
E= Ee * Ne = 3/2 kT * n *V
=1.5*1.4e-16*5e7 * 0.1 * 4/3*3.141*(100*3e21)^3
=9 e61 ergs

Cooling time

t= E/(dE/dt)
=9e61/1.3e46=220 Myr

Synchrotron Radiation

Synchrotron radiation is due to the movement of an electron charge in a magnetic field. As a particle gyrates around a magnetic field, it will emit radiation at a frequency proportional to the strength of the magnetic field and its velocity.



Synchrotron radiation is highly polarized and is seen at all wavelengths. At relativistic speeds, the radiation can also be beamed. It is very common in radio spectrum, but can be seen in x-rays. It is usually fit as a power law. For full details, see the review by Ginzburg & Syrovatskii (1969)

A single electron

The frequency of synchrotron radiation is:

$$\omega_B = \frac{qB}{\gamma mc}$$

The total power emitted by each electron is:

$$\frac{dE}{dt} = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B$$

Where the following definitions have been used:

$$U_B = B^2/8\pi$$

$$\beta = (1 - \frac{1}{\gamma^2})^{1/2}$$

$$\sigma_T = \frac{8\pi r_o^2}{3}$$

$$\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$$

Power-Law Distribution

The spectra can often be defined by a power law of form:

$$P \propto \nu^{-s}$$

For an input electron distribution given by:

$$N(E)dE = CE^{-p}dE$$

It can be shown that the power will be related to the frequency by:

$$P \propto \nu^{-(p-1)/2}$$

And so the spectral index s is related to the power law of the input electron index p, by:

$$s = \frac{p-1}{2}$$

Self Absorption

$$\kappa\rho \sim \nu^{-(p+4)/4}$$

In the case where it is optically thick:

$$S_\nu \sim \nu^{5/2}$$

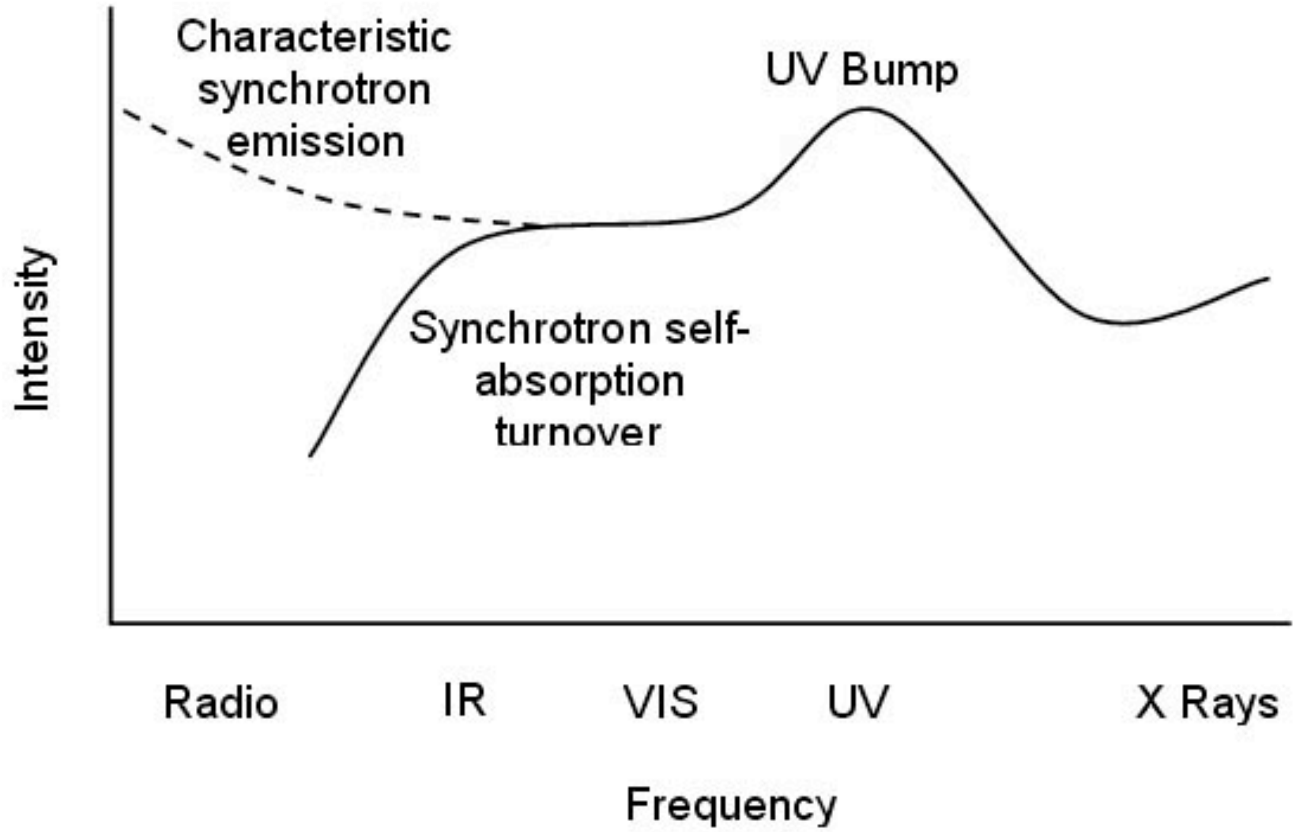
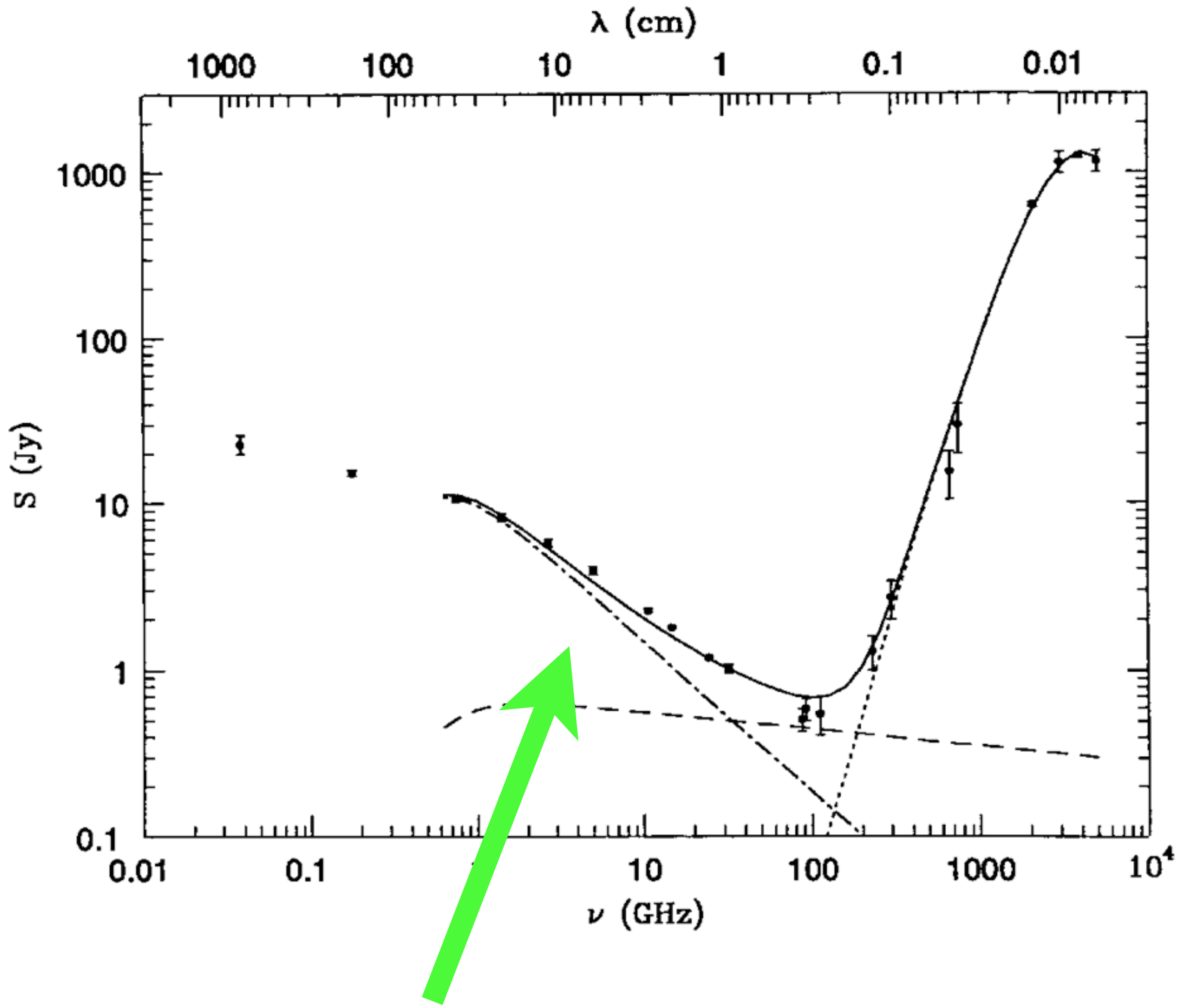


Figure 4: A sketch of the continuum observed for many AGN with intensity vs. frequency.

The Starburst galaxy M82



Synchrotron Emission

Review

Different values for the emission are:

Blackbody

$$B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{hv/kT} - 1}$$

Bremsstrahlung

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} \frac{ergs}{s\ cm^3\ Hz}\ Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT}\overline{g_{ff}}$$

Synchrotron

$$\frac{dE}{dtdVd\nu} = \frac{\sqrt{3}q^3}{mc^2(p+1)}\left(\frac{3q}{2\pi mc}\right)^{\frac{p-1}{2}} C_\gamma B_\perp^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \Gamma_1 \Gamma_2$$

Review

Different values for the emission are:

Blackbody

$$B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{hv/kT} - 1}$$

Bremsstrahlung

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} \frac{ergs}{s\ cm^3\ Hz}\ Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \overline{g_{ff}}$$

Synchrotron

$$\frac{dE}{dtdVd\nu} = \frac{\sqrt{3}q^3}{mc^2(p+1)}(\frac{3q}{2\pi mc})^{\frac{p-1}{2}} C_\gamma B_\perp^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \Gamma_1 \Gamma_2$$

Review

Different values for the emission are:

Blackbody

$$B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{h\upsilon/kT}-1}$$

Bremsstrahlung

$$\epsilon_\nu^{ff} = 6.8\times10^{-38}\frac{ergs}{s\ cm^3\ Hz}\ Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT}\overline{g_{ff}}$$

Synchrotron

$$\frac{dE}{dtdVd\nu} = \frac{\sqrt{3}q^3}{mc^2(p+1)}\left(\frac{3q}{2\pi mc}\right)^{\frac{p-1}{2}} C_\gamma B_\perp^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}}\Gamma_1\Gamma_2$$

Review

Different values for the emission are:

Blackbody

$$B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{hv/kT} - 1}$$

Bremsstrahlung

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} \frac{ergs}{s\ cm^3\ Hz}\ Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \overline{g_{ff}}$$

Synchrotron

$$\frac{dE}{dtdVd\nu} = \frac{\sqrt{3}q^3}{mc^2(p+1)}\left(\frac{3q}{2\pi mc}\right)^{\frac{p-1}{2}} C_\gamma B_\perp^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \Gamma_1 \Gamma_2$$

Review

Different values for the emission are:

Blackbody

$$B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{h\upsilon/kT} - 1}$$

Optically Thick

Bremsstrahlung

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} \frac{ergs}{s\ cm^3\ Hz}\ Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \overline{g_{ff}}$$

Synchrotron

$$\frac{dE}{dtdVd\nu} = \frac{\sqrt{3}q^3}{mc^2(p+1)}\left(\frac{3q}{2\pi mc}\right)^{\frac{p-1}{2}} C_\gamma B_\perp^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \Gamma_1 \Gamma_2$$

Review

Different values for the emission are:

Blackbody

Optically Thick

$$B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{hv/kT} - 1}$$

Bremsstrahlung

Optically Thin

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} \frac{ergs}{s\ cm^3\ Hz}\ Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT}\overline{g_{ff}}$$

Synchrotron

$$\frac{dE}{dtdVd\nu} = \frac{\sqrt{3}q^3}{mc^2(p+1)}\left(\frac{3q}{2\pi mc}\right)^{\frac{p-1}{2}} C_\gamma B_\perp^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \Gamma_1 \Gamma_2$$

Review

Different values for the emission are:

Blackbody

$$B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{h\upsilon/kT} - 1}$$

Optically Thick

Bremsstrahlung

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} \frac{ergs}{s\ cm^3\ Hz}\ Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \overline{g_{ff}}$$

Optically Thin

Synchrotron

$$\frac{dE}{dtdVd\nu} = \frac{\sqrt{3}q^3}{mc^2(p+1)}\left(\frac{3q}{2\pi mc}\right)^{\frac{p-1}{2}} C_\gamma B_\perp^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \Gamma_1 \Gamma_2$$

Optically Thin

Review

Different values for the emission are:

Blackbody

$$B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{h\upsilon/kT} - 1}$$

Optically Thick

Bremsstrahlung

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} \frac{ergs}{s\ cm^3\ Hz}\ Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \overline{g_{ff}}$$

Optically Thin

Synchrotron

$$\frac{dE}{dtdVd\nu} = \frac{\sqrt{3}q^3}{mc^2(p+1)}\left(\frac{3q}{2\pi mc}\right)^{\frac{p-1}{2}} C_\gamma B_\perp^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \Gamma_1 \Gamma_2$$

Optically Thin

Optically Thick

Review

Different values for the emission are:

Blackbody

Optically Thick

$$B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{h\upsilon/kT} - 1}$$

Bremsstrahlung

Optically Thin

$$\epsilon_\nu^{ff} = 6.8 \times 10^{-38} \frac{ergs}{s\ cm^3\ Hz}\ Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \overline{g_{ff}}$$

Synchrotron

Optically Thin

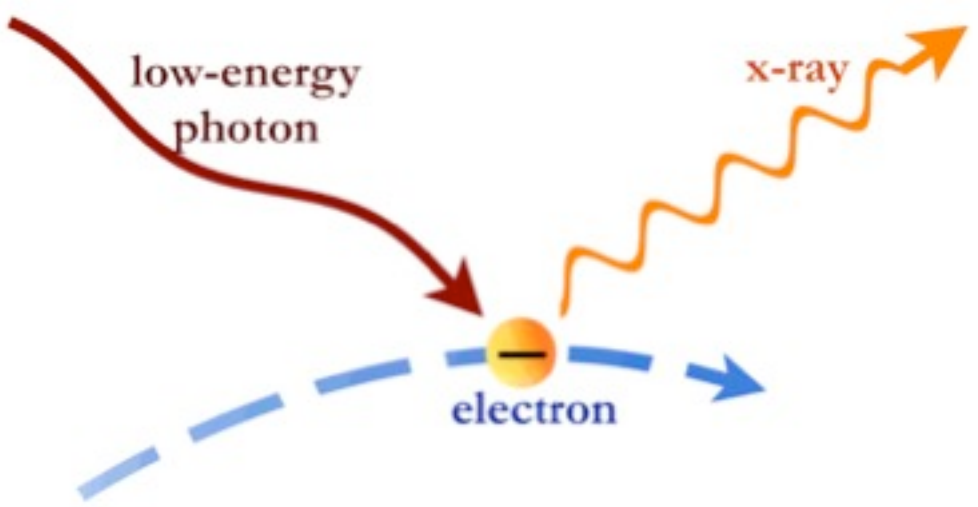
$$\frac{dE}{dtdVd\nu} = \frac{\sqrt{3}q^3}{mc^2(p+1)}\left(\frac{3q}{2\pi mc}\right)^{\frac{p-1}{2}} C_\gamma B_\perp^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \Gamma_1 \Gamma_2$$

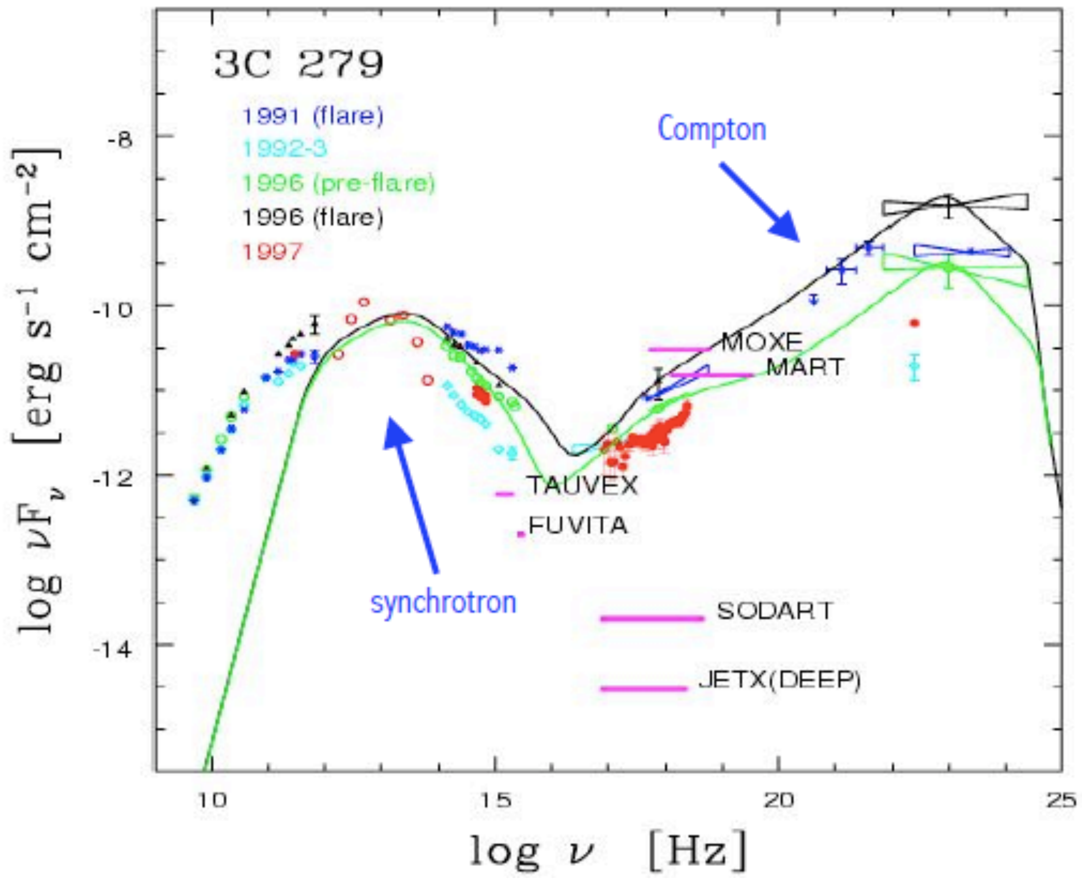
Optically Thick

$$S_\nu \sim \nu^{5/2}$$

Compton Scattering

Compton Scattering is due to the interaction between a low-energy photon and a relativistic electron. At non-relativistic speeds, it is known as Thomson scattering and has negligible effects on the energy.





Compton Scattering is due to the interaction between a low-energy photon and a relativistic electron. At non-relativistic speeds, it is known as Thomson scattering and has negligible effects on the energy. A very useful reference for Inverse Compton effects is the review by Blumenthal & Gould (1970).

Compton Scattering: Single photon

For a single photon interacting with an electron, the change in the wavelength of the photon due to Compton Scattering will be:

$$\lambda_o - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

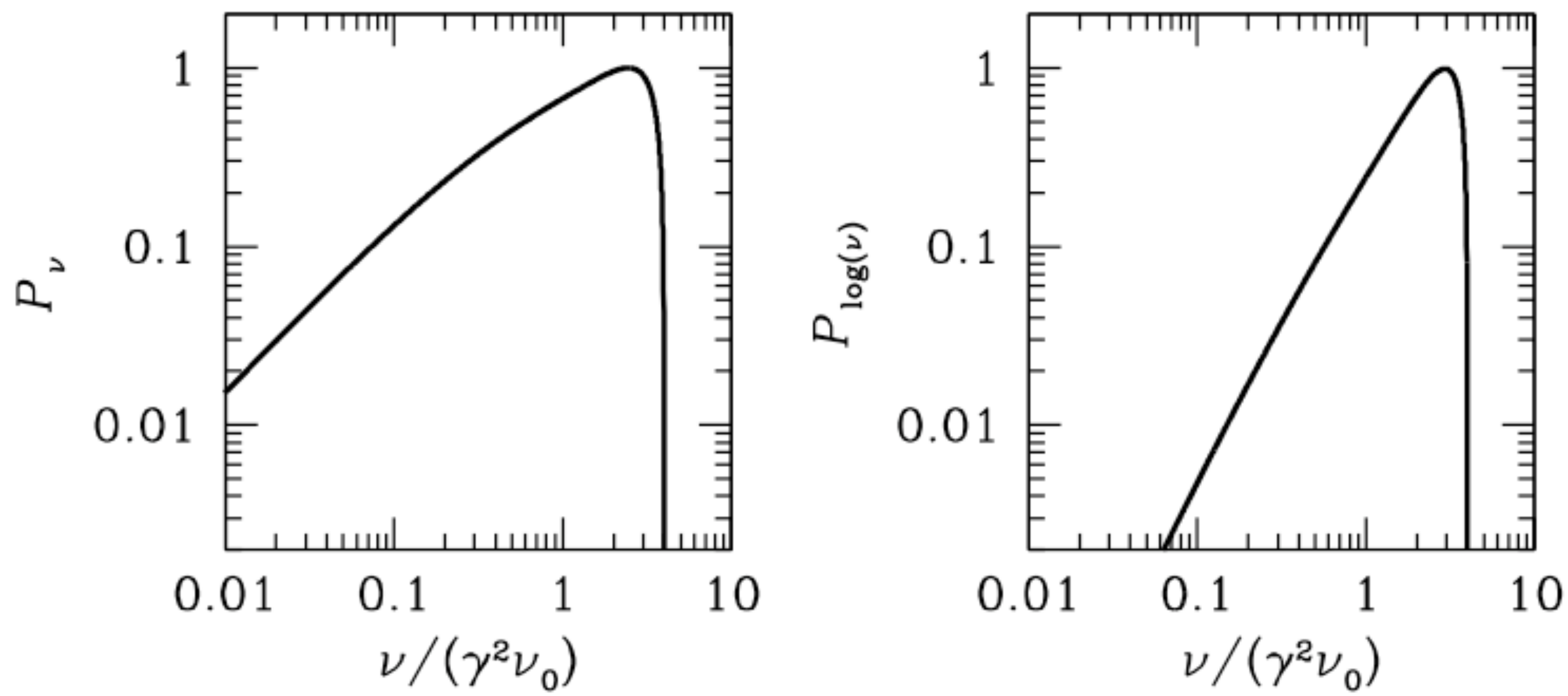
In the relativistic regime, the cross section for a single scattering is given by a correction to the Thomson scattering cross section where $x = h\nu/mc^2$:

$$\sigma = \frac{3}{8}\sigma_T x^{-1}(ln(2x) + \frac{1}{2})$$

And the total power radiated by this electron is:

$$\frac{dE}{dt} = \frac{4}{3}\sigma_T c \gamma^2 \beta^2 U_{photon}$$

Effect on a single frequency



From Blumenthal and Gould (1970), for a single frequency photon field, the change in the spectral will be given by:

$$I(\nu)d(\nu) = \frac{3\sigma_T c}{16\gamma^4}\frac{N(\nu_o)}{\nu_o^2}[2\nu ln(\frac{\nu}{4\gamma^2\nu_o}) + \nu + 4\gamma^2\nu_o - \frac{\nu^2}{2\gamma^2\nu_o}]d\nu$$

The maximum and average frequency of the scattered photons are:

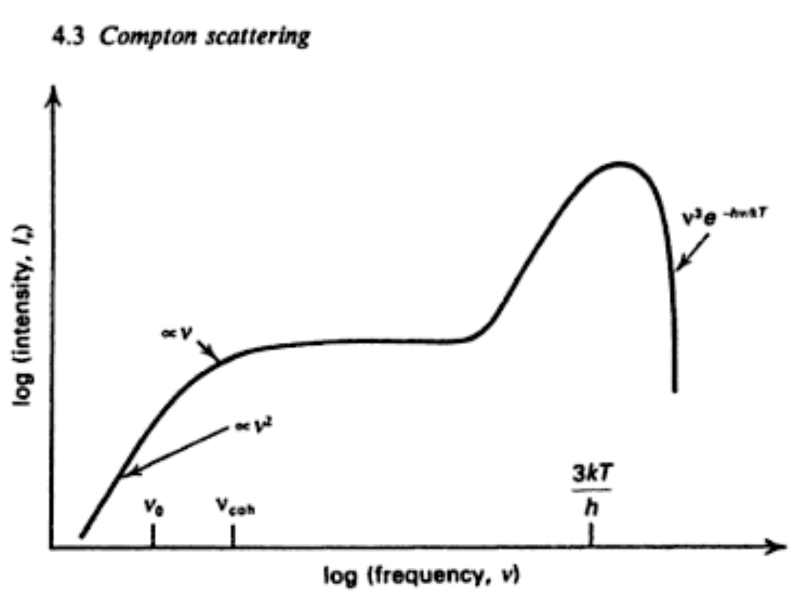
$$\nu_{max} \approx 4\gamma^2\nu_o \qquad <\nu>= \frac{4}{3}\gamma^2\nu$$

Comptonization

A solution has been presented for non-relativistic limit, which is the Kompaneets equation. Much of it depends on y, which is defined:
y=[average fractional energy changed per scattering] x
[mean number of scatters]

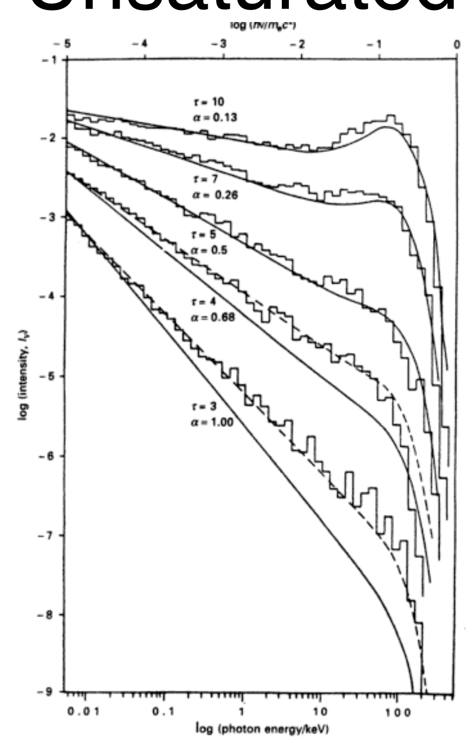
$$y \approx \int \frac{4kT_e}{mc^2} \sigma_T n_e dx$$

Saturated



Rybicki & Lightman

Unsaturated



Pozdnyakov et al. 1983

Effect on a Blackbody

For a blackbody, a solution to the Kompaneets equation can be found if cooling is ignored:

$$\frac{\Delta u_\nu}{u_\nu} = y \frac{x \exp x}{\exp x - 1} \left(x \frac{\exp x + 1}{\exp x - 1} - 4 \right)$$

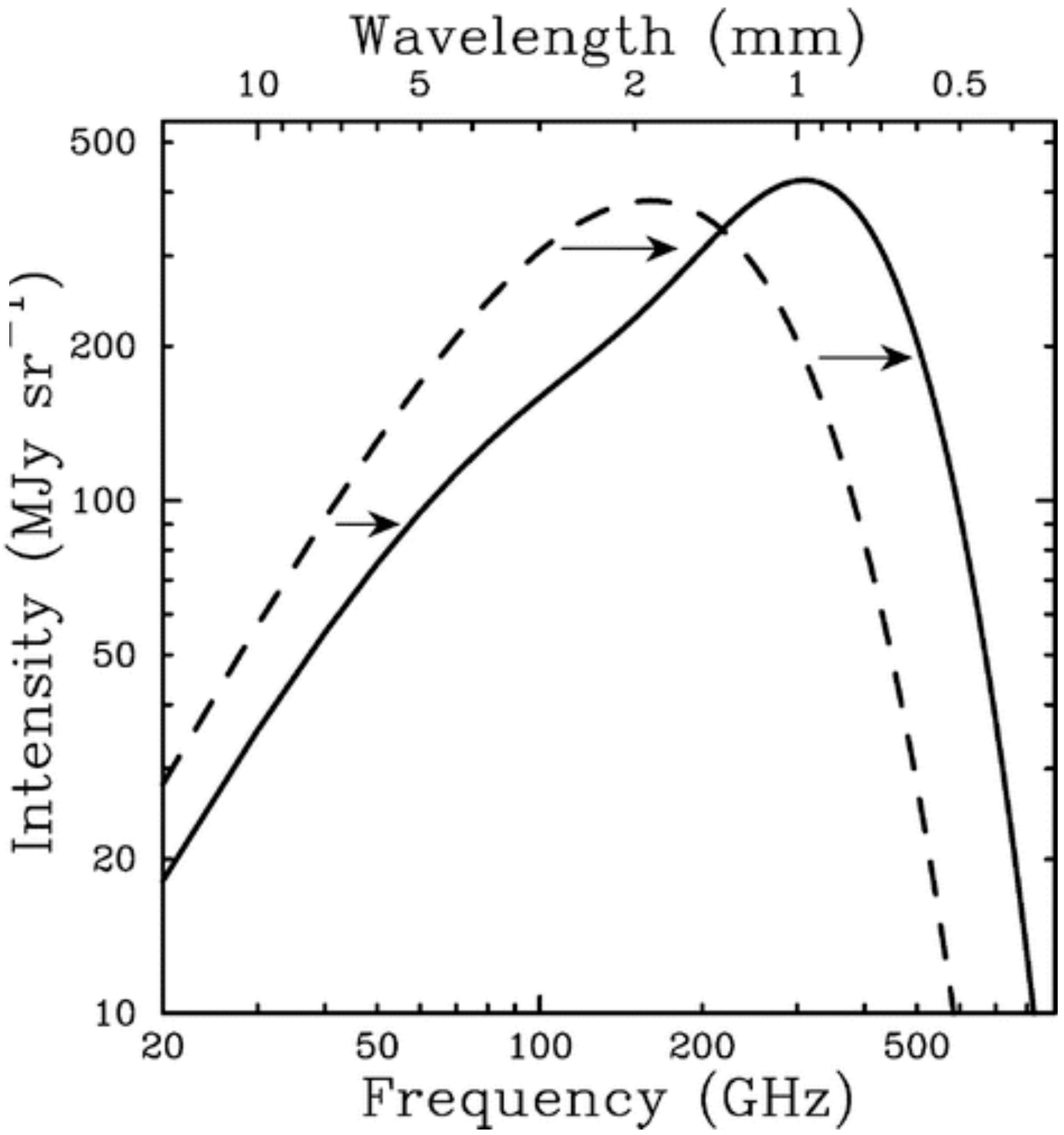
where

$$x = \frac{h\nu}{kT}$$

$$y \approx \int \frac{4kT_e}{mc^2} \sigma_T N_e dx$$

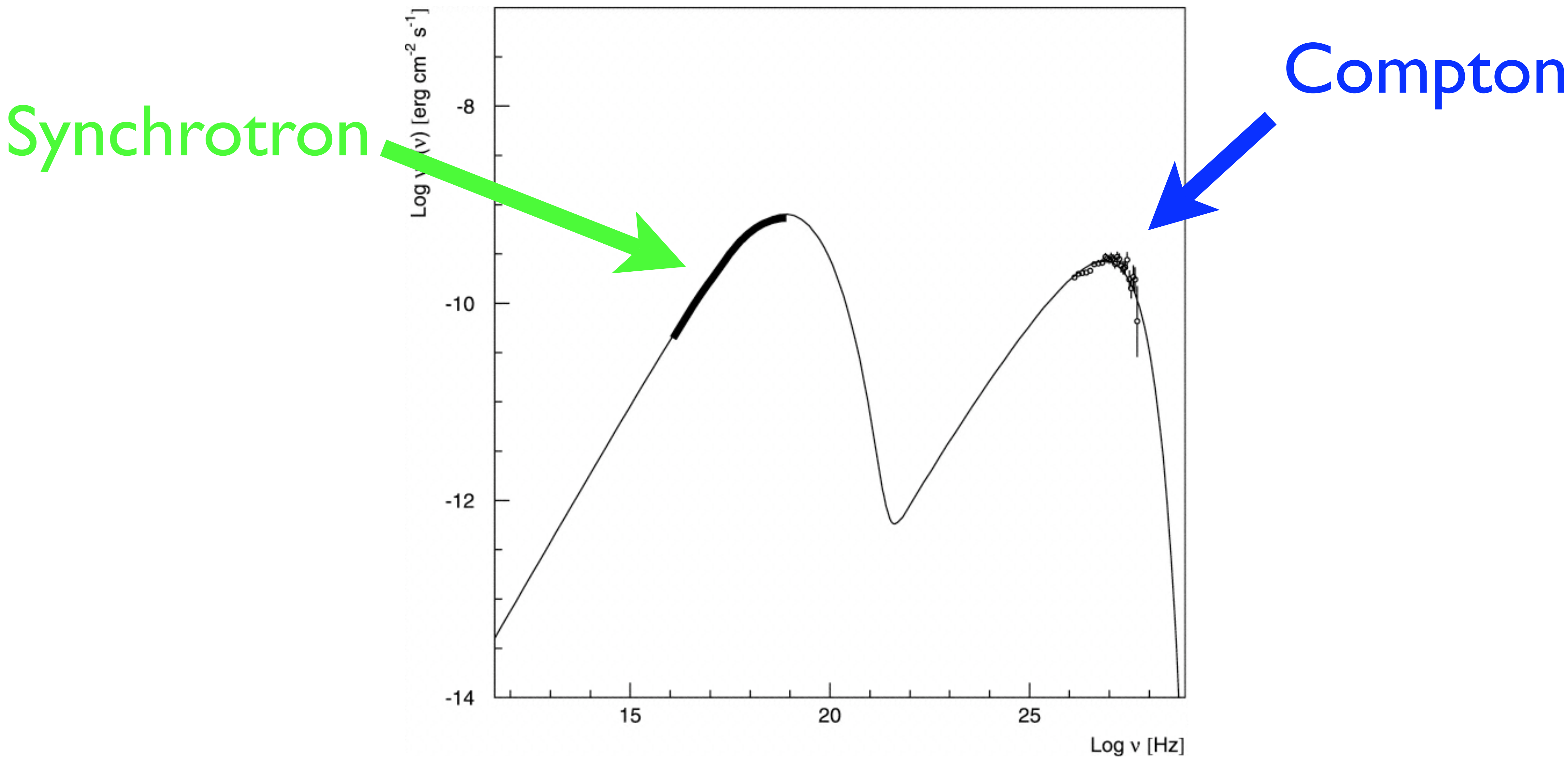
For low frequencies, it can be shown that

$$\frac{\Delta I_\nu}{I_\nu} = -2y$$



Carlstrom et al 2002

Comptonization of Synchrotron Radiation



Konopelko et al. 2003

Variability in micro-quasar

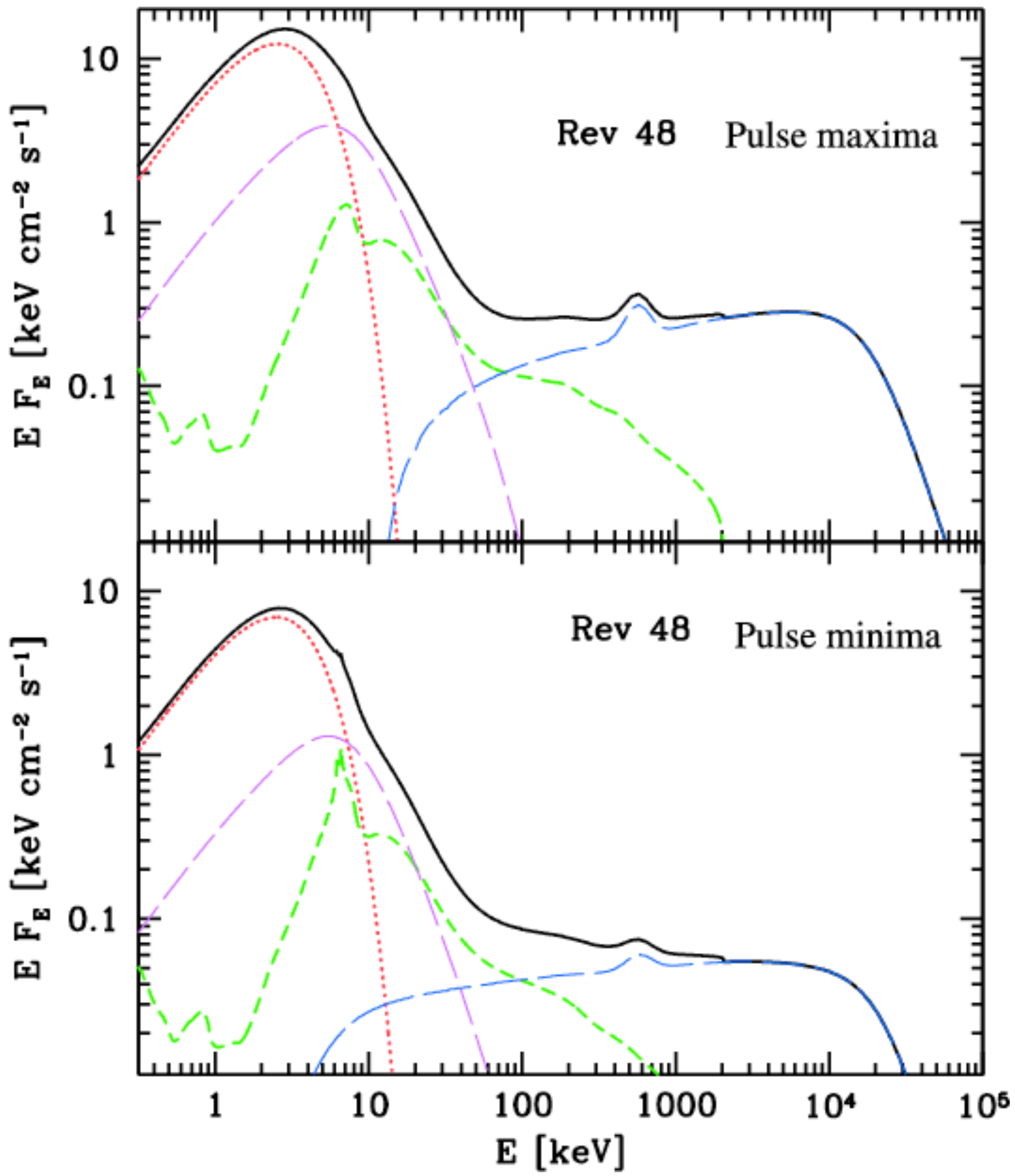
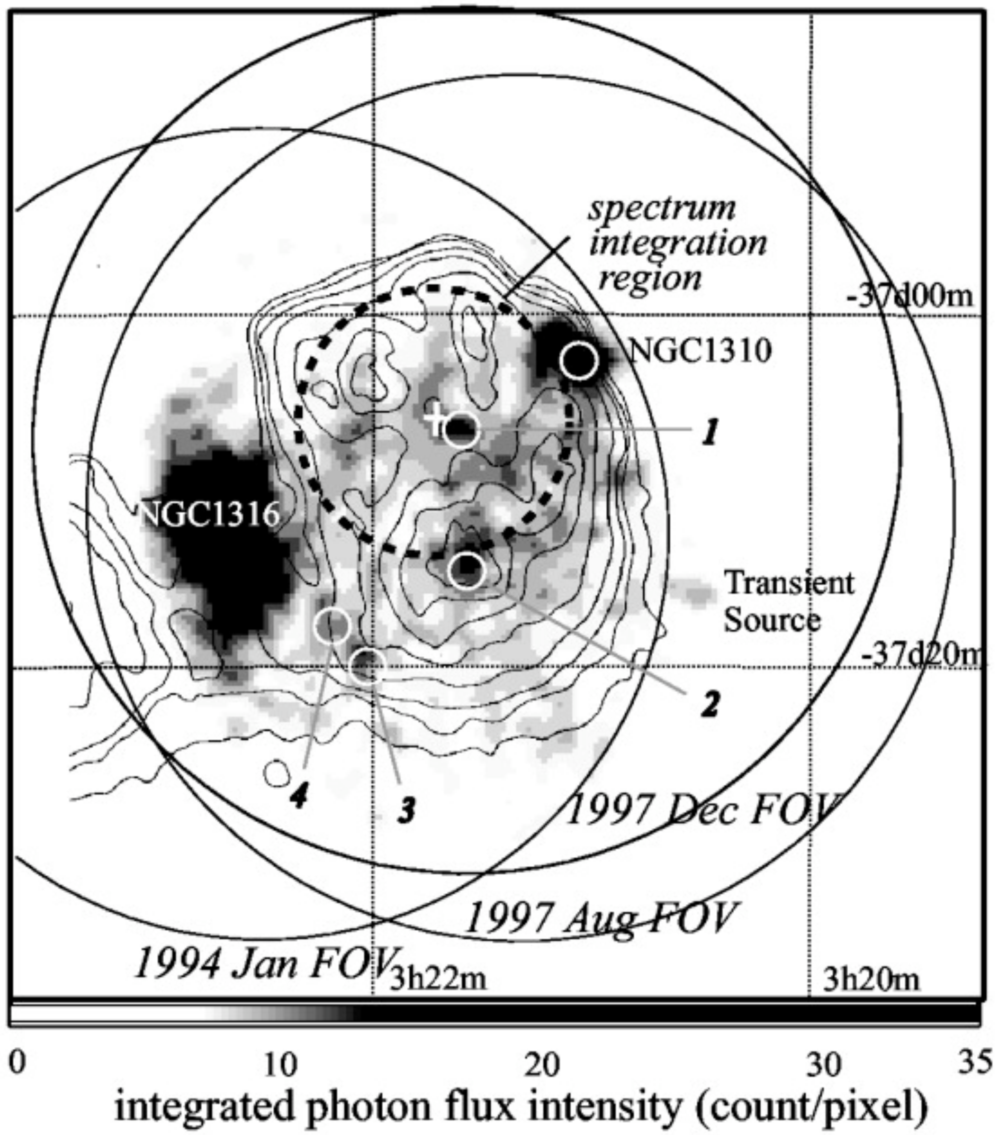
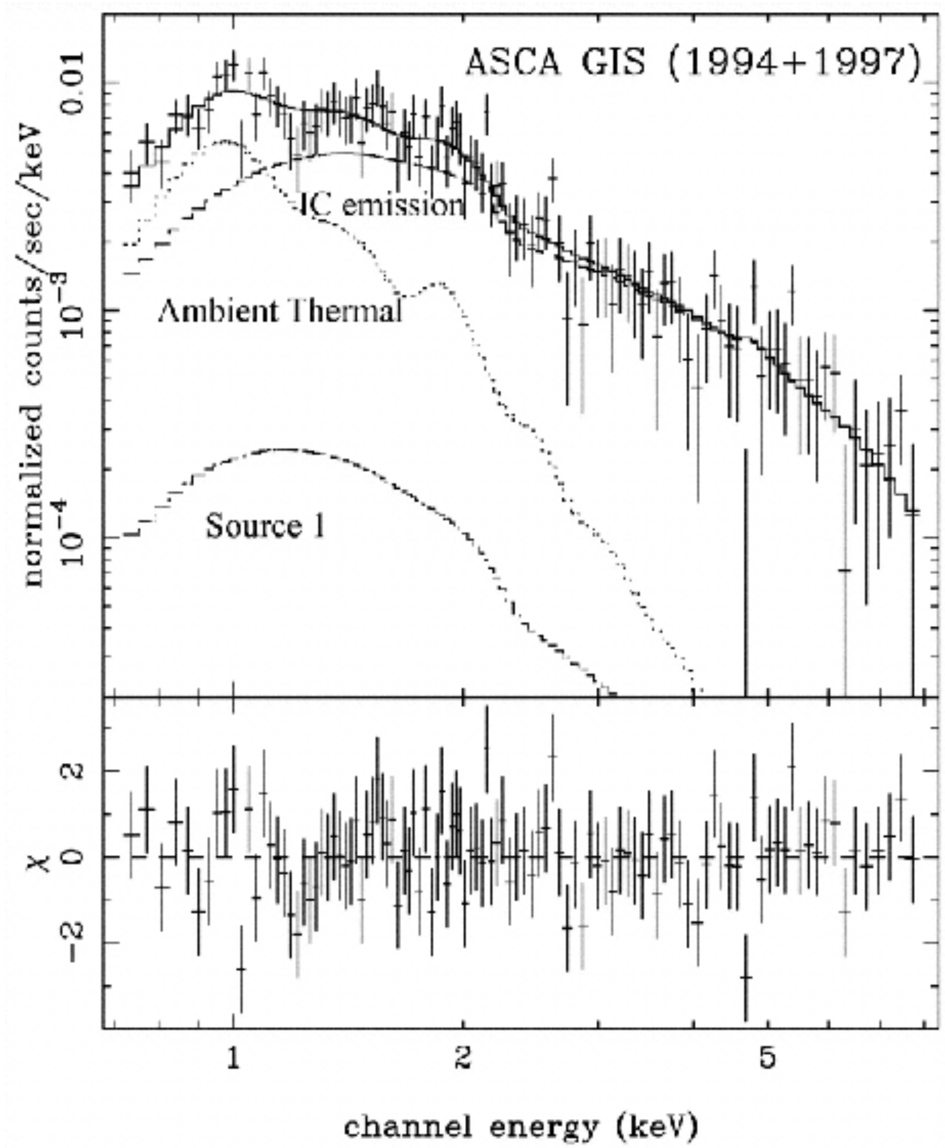


Figure 10: Spectral components of the fits to the pulse maxima (upper panel) and minima (lower panel). The dotted and long-dashed curves show the unscattered blackbody, and Compton scattering from thermal (red) and non-thermal electrons (cyan), including a component from e pair annihilation, important around 511 keV. The short-dashed (green) curve shows the component from Compton reflection, including the Fe Kα line.

Hannikainen et al 2005

In Lobes of Radio Galaxies



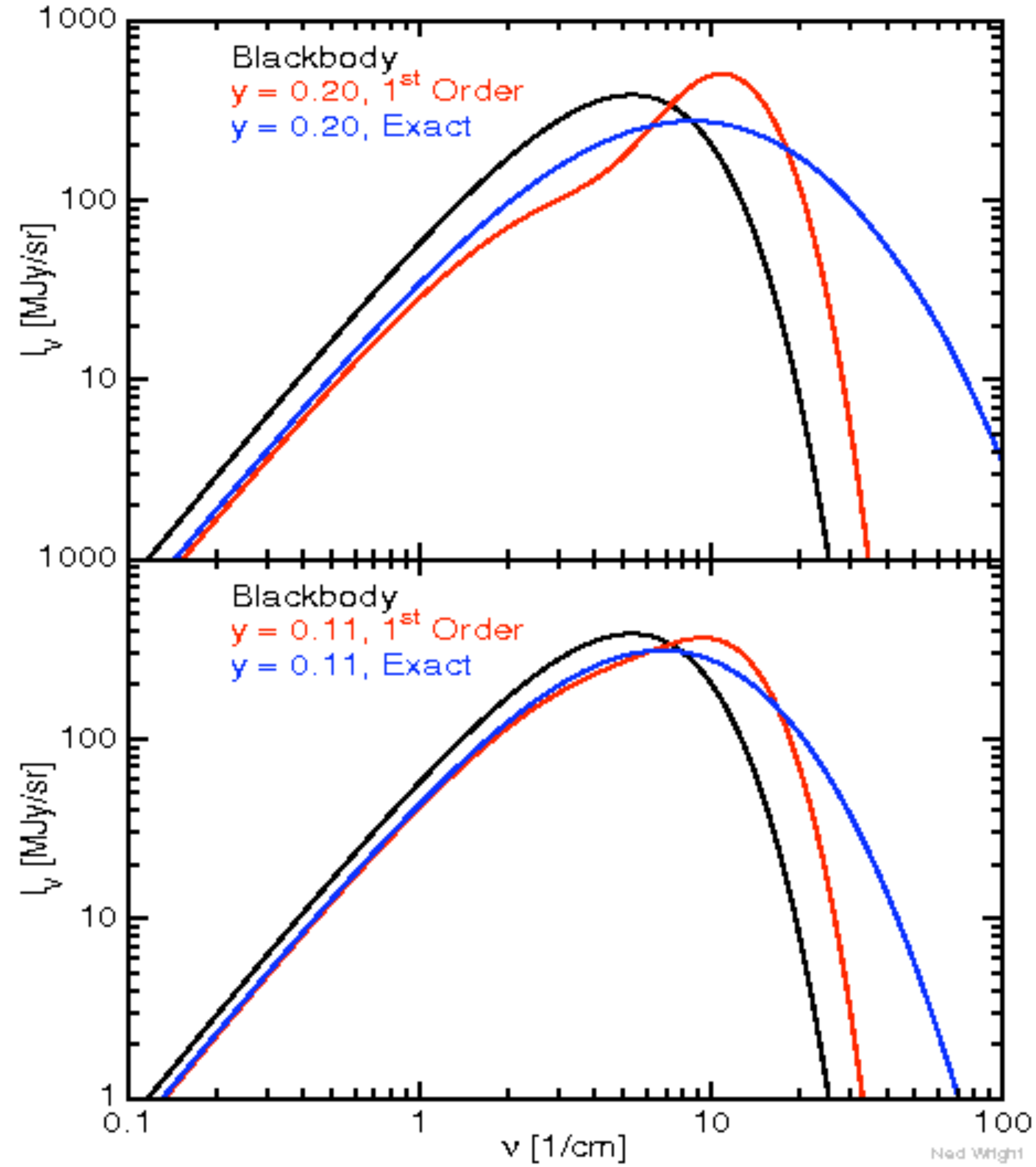


Tashiro et. al. 2000

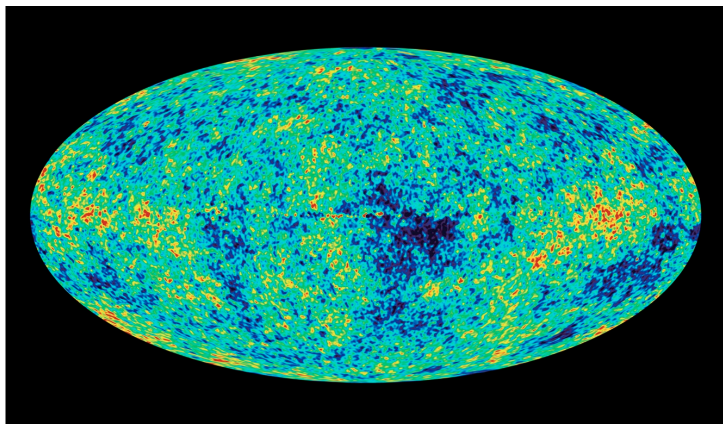
Summary

- From Bremsstrahlung, we can learn about an objects density and temperature
- From Synchrotron radiation, we can learn about the magnitude field and power spectra of underlying electron distribution
- From Inverse Compton Scattering, we can learn about the scattering particles.

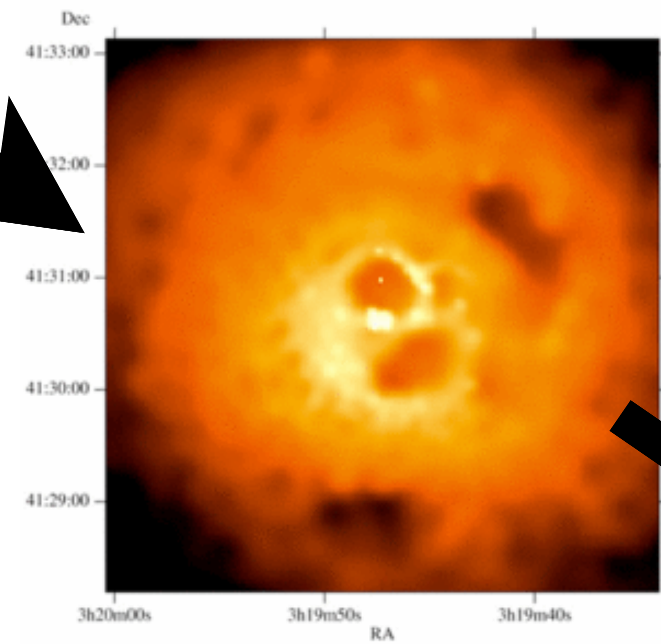
Example: Sunyeaz-Zeldovich effect

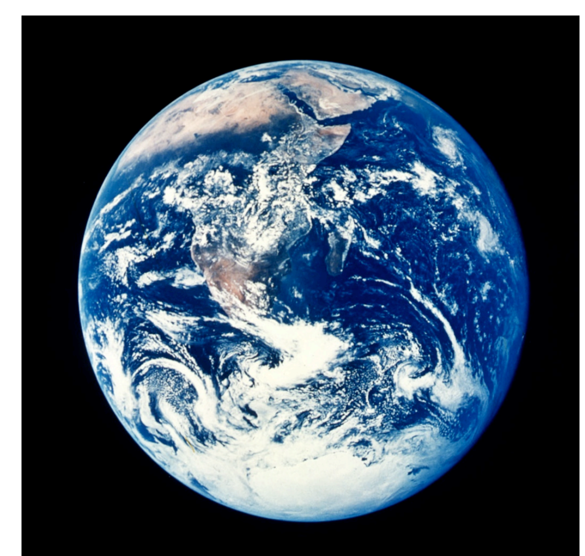


The Situation



Bremsstrahlung



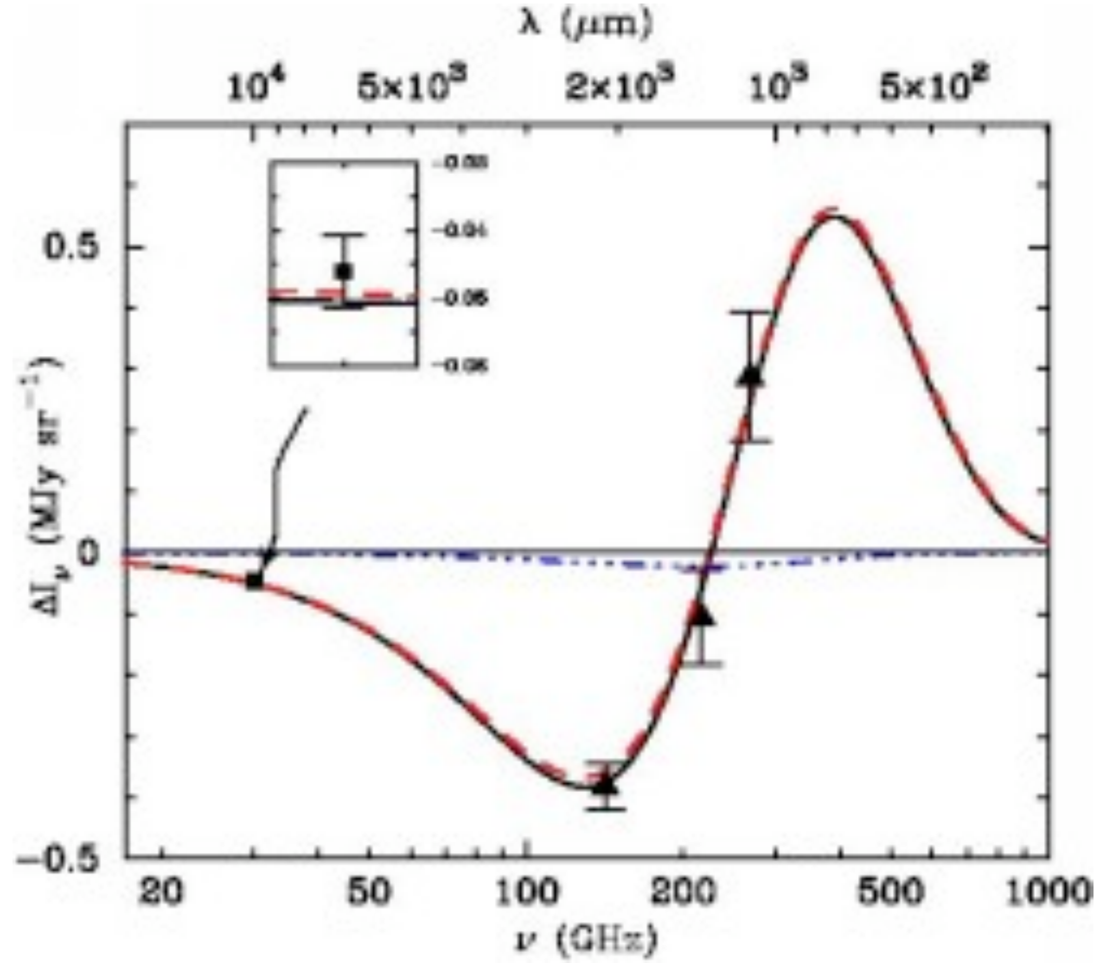


Compton

From the Compton Scattering

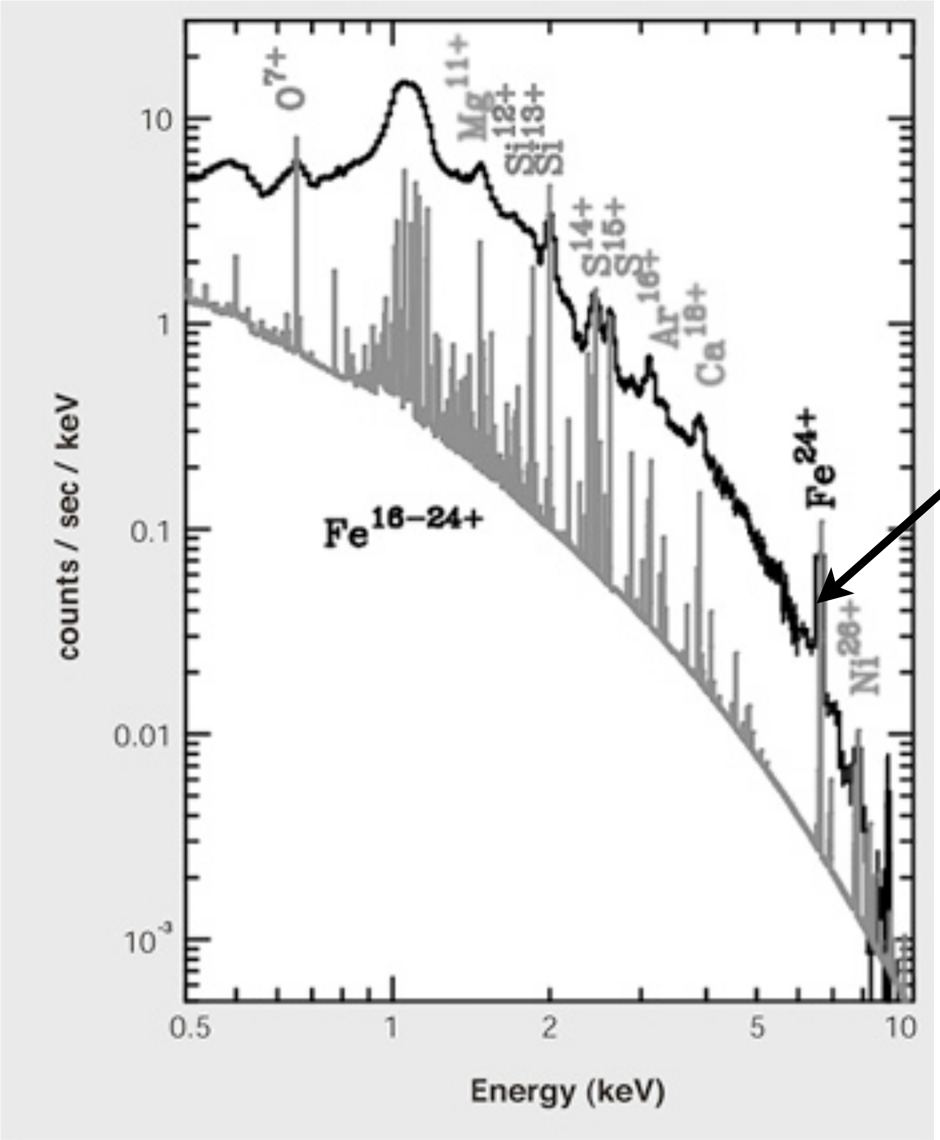
$$\frac{\Delta I_\nu}{I_\nu} = -2y$$

$$y \approx \int \frac{4kT_e}{mc^2}\sigma_T n_e dx$$



Carlstrom et al 2002

From the Bremsstrahlung



Get T from the Line emission

$$\epsilon^{ff} = 1.4 \times 10^{-27} \frac{ergs}{s\ cm^3}\ Z^2 n_e n_i T^{-1/2} \overline{g_B}$$

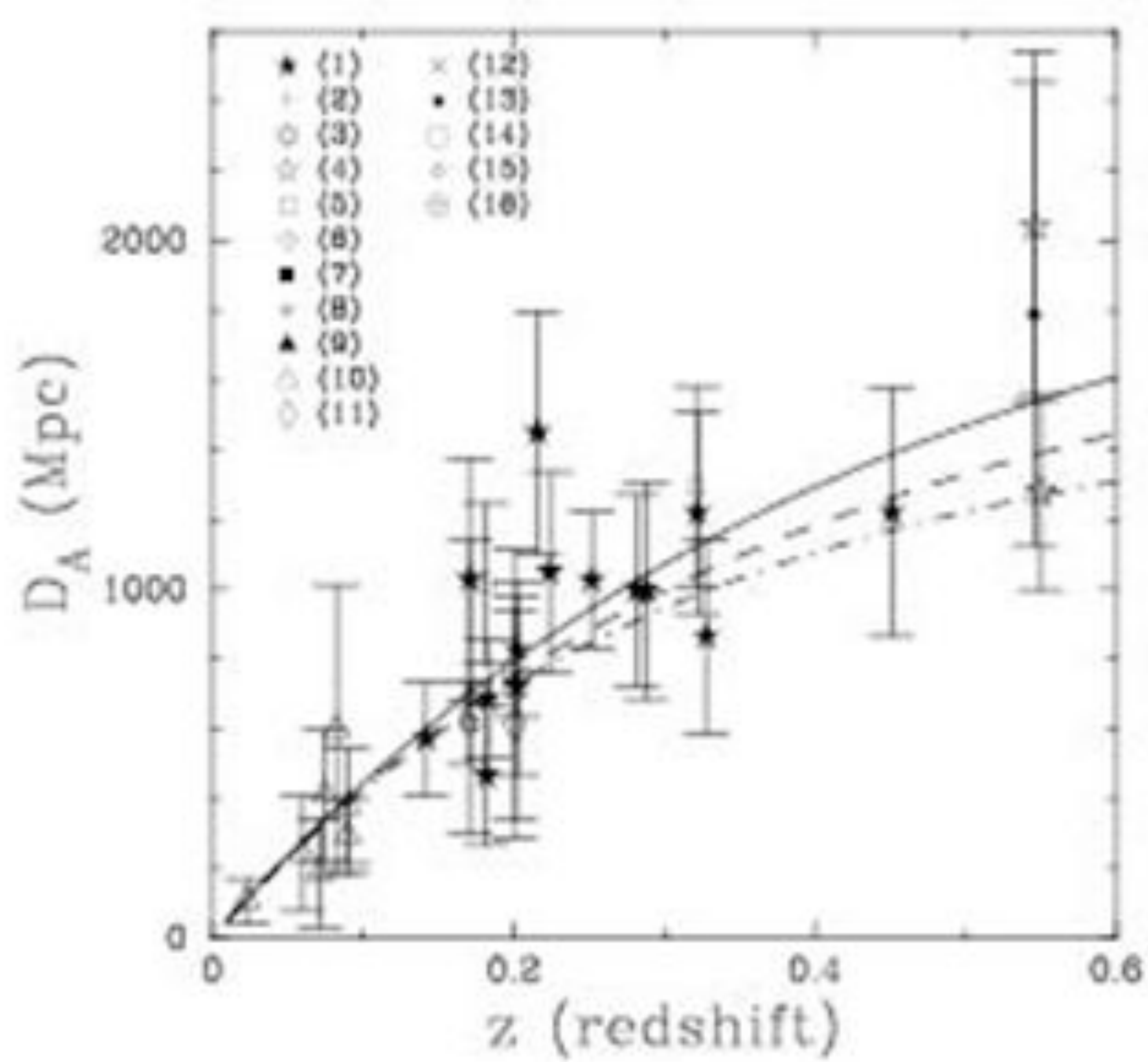
Gives us the Universe

$$\frac{\Delta I}{I} \propto n_e r$$

$$\epsilon \propto n_e^2$$

$$r \propto \frac{\Delta I}{I} \frac{1}{\sqrt{\epsilon}}$$

From imaging, we can get the size in arcseconds and hence derive the cosmology



Carlstrom, Holder, Reese 2002