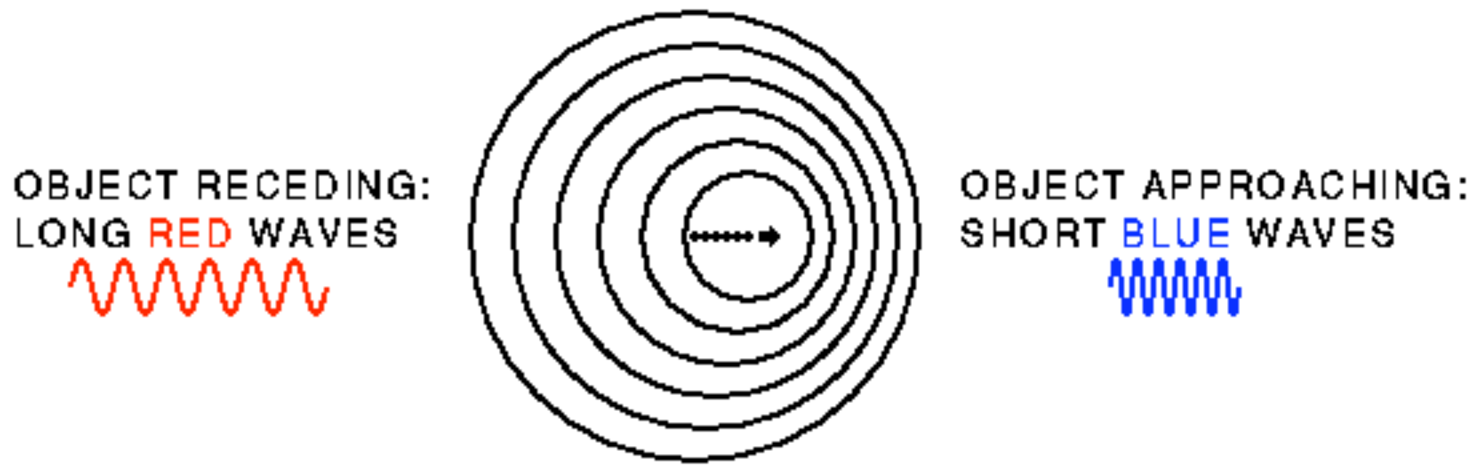
Spectroscopy

X. Velocity and Error Analysis

Shifts with velocity



As an object moves, the light from the source will be shifted either blueward or redward from that light source.

The redshift of the source will be the shift in the observed spectral line due to this effect.

$$1 + z = \frac{\lambda_{\rm obsv}}{\lambda_{\rm emit}}$$

Redshift

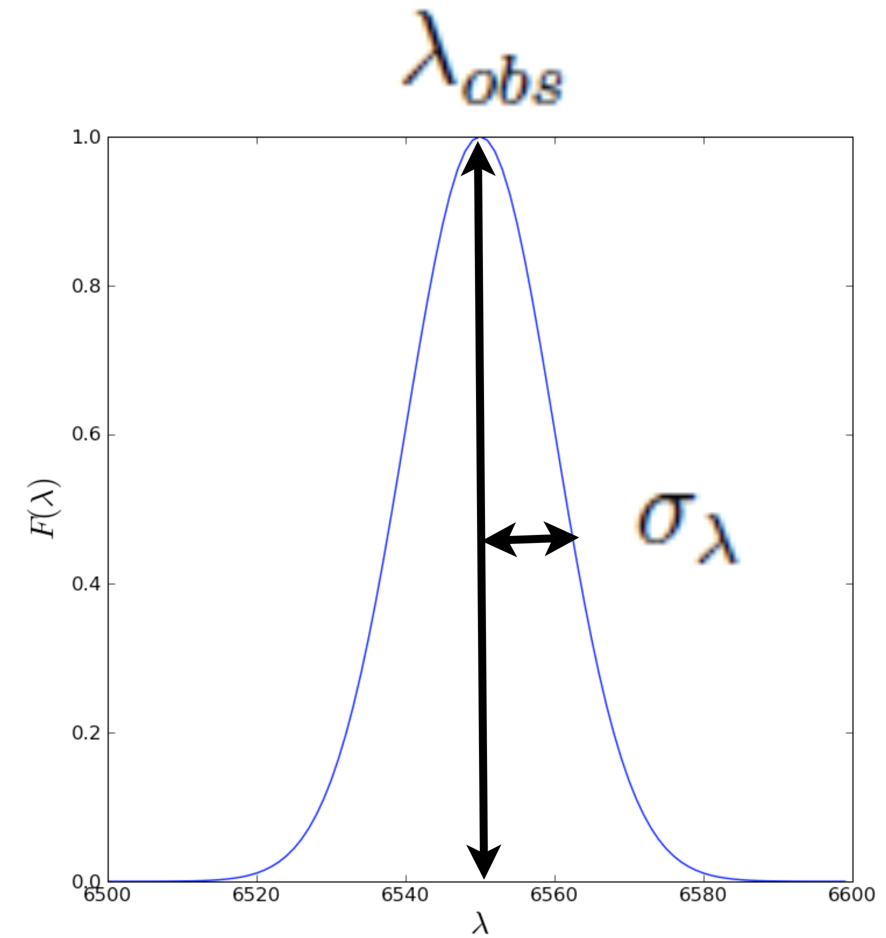
The relativistic doppler shift is given by

$$\frac{\lambda_{obs}}{\lambda_{em}} = \sqrt{\frac{1+\beta}{1-\beta}} = 1 + z \qquad \beta = \frac{v}{c}$$

where the velocity is positive for sources that are receding from us.

At low v, the redshift is given by:

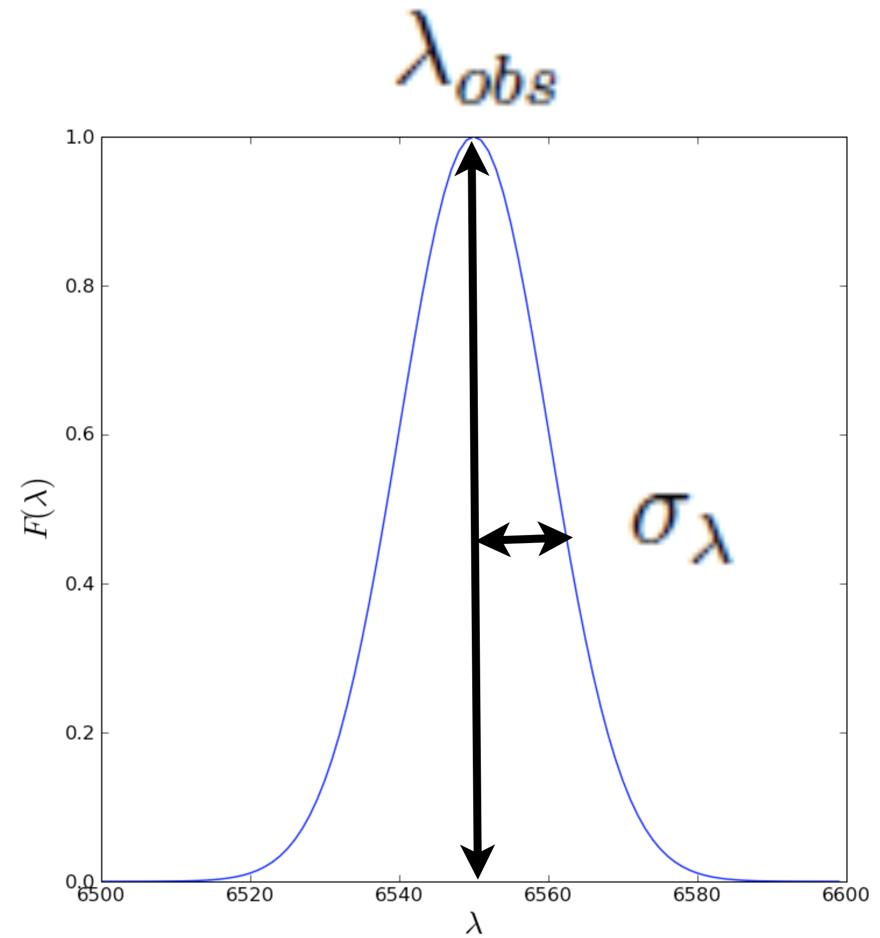
$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$



The redshift may also be due to the velocity, cosmological expansion, or gravitational redshift.

NB: Redshift should be measured relative to the vacuum speed of light (this is how SDSS does it), but what is detected is light at 1 atm

Velocity Width



If we assume a Gaussian distribution, then the velocity dispersion is:

$$\sigma_v = \frac{\sigma_\lambda}{(1+z)}\frac{c}{\lambda_{em}}$$

where the observed Full width at half maximum (FWHM) is given by:

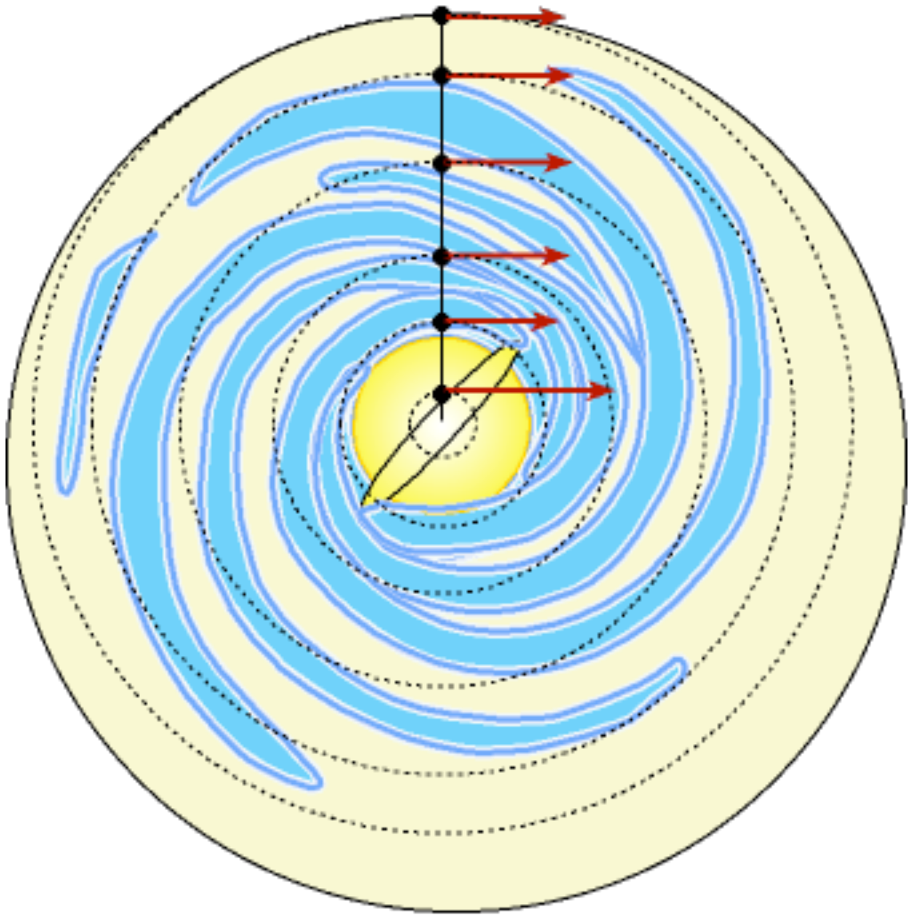
$$FWHM = 2.354\sigma_\lambda$$

For observations of a gas, the doppler broadening relates the line dispersion to the temperature

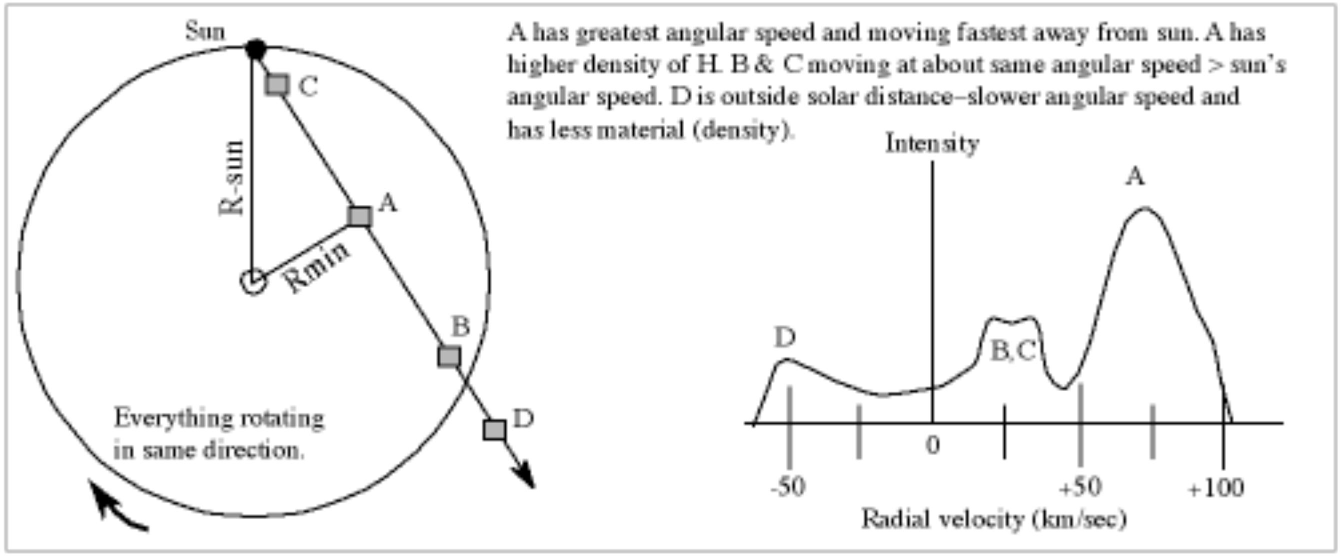
$$\sigma_\lambda = \sqrt{\frac{kT}{mc^2}}\lambda_0$$

For observations of systems like stellar systems, the velocity dispersion is related to the gravitation potential of the system.

Line of sight in the MW



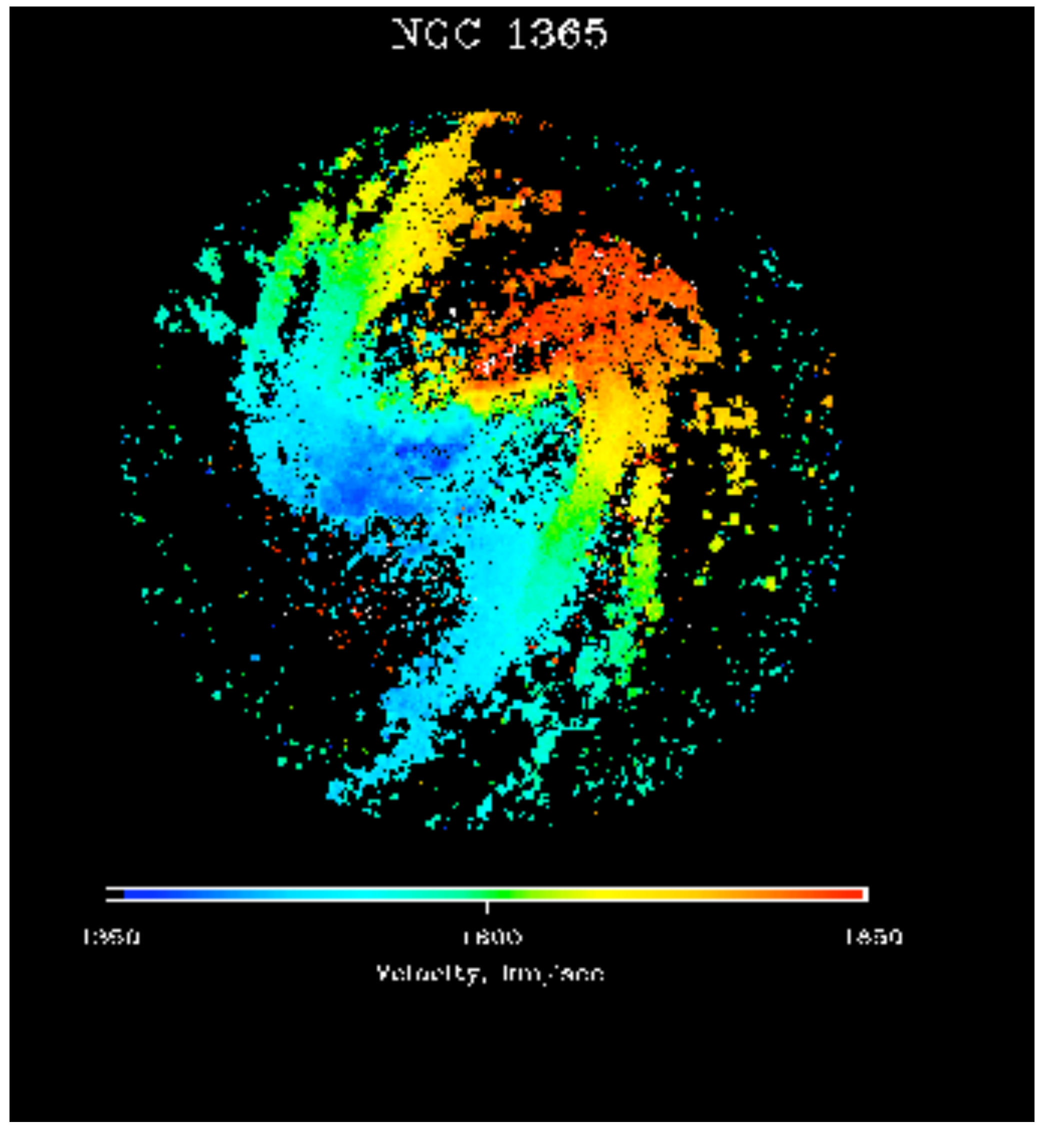
The mass inside an orbit can be found using the size of the orbit and the orbital speed. The arrows show the speeds for certain points on the **rotation curve** for this galaxy.



Four clouds all in the same direction. Use doppler shifts to distinguish one cloud from the other. Use the rotation curve to convert the doppler shifts of each cloud to distances from the center of the Galaxy. Do this for other directions to build up a map of the Galaxy strip by strip.

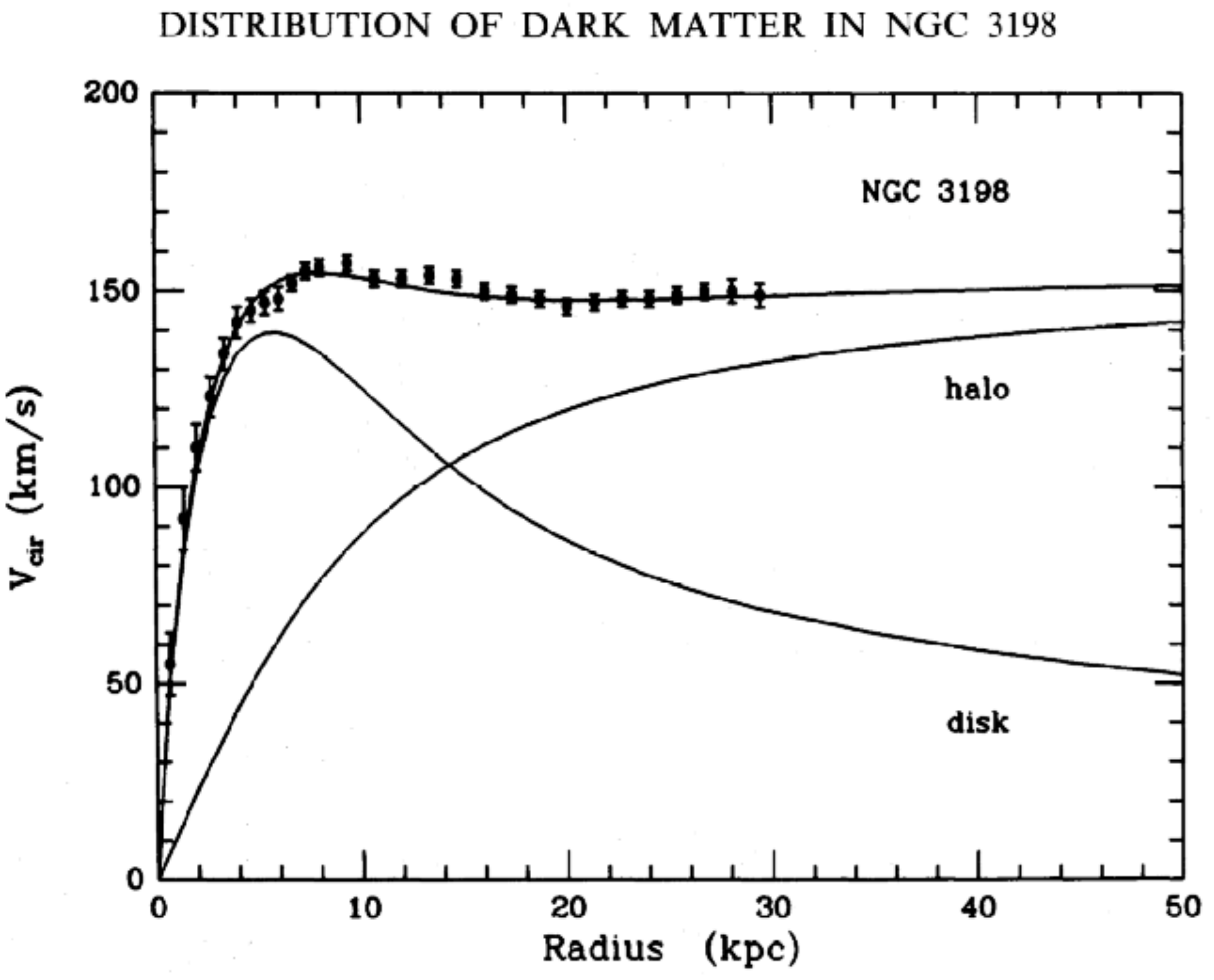
Nick Strobel

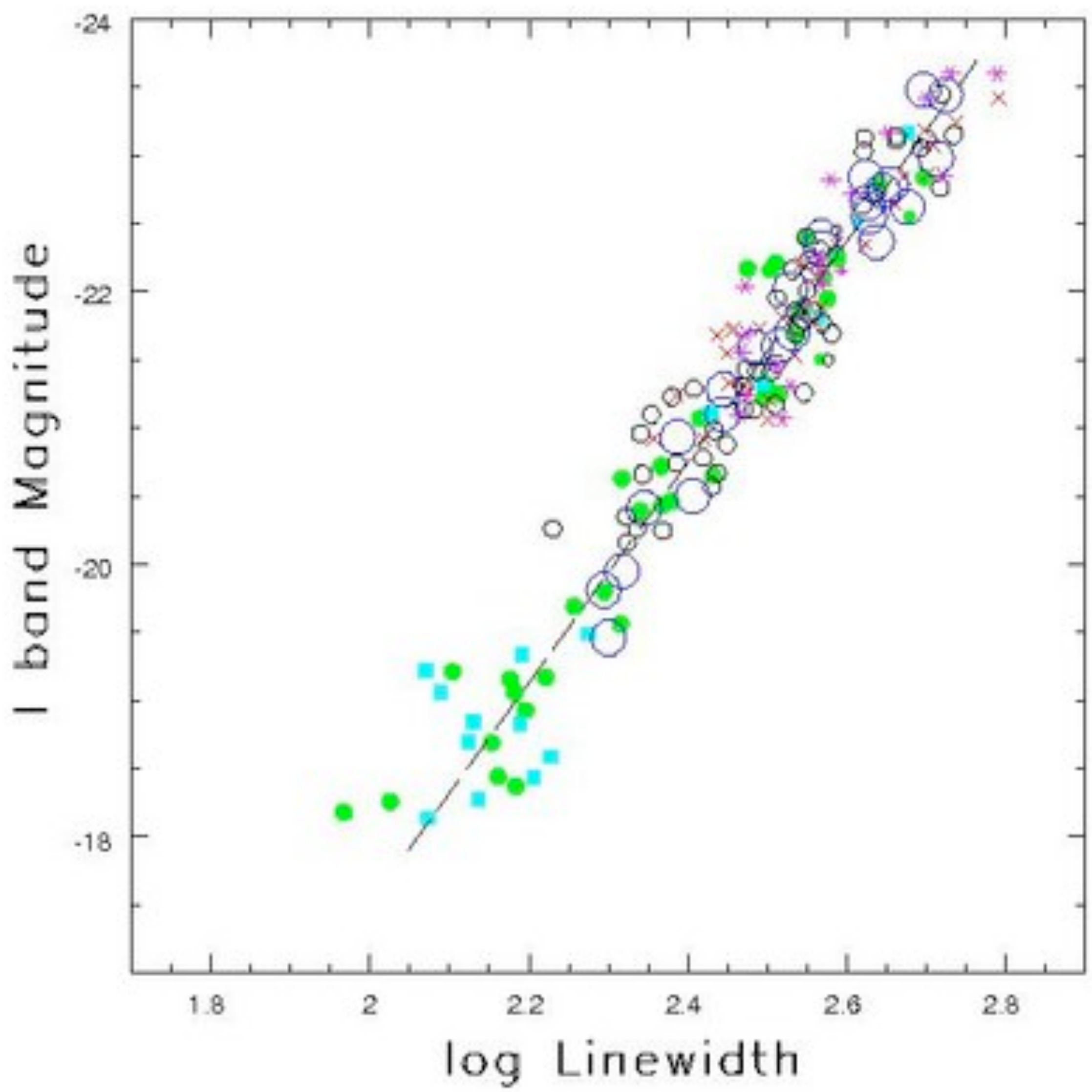
Velocity Curves



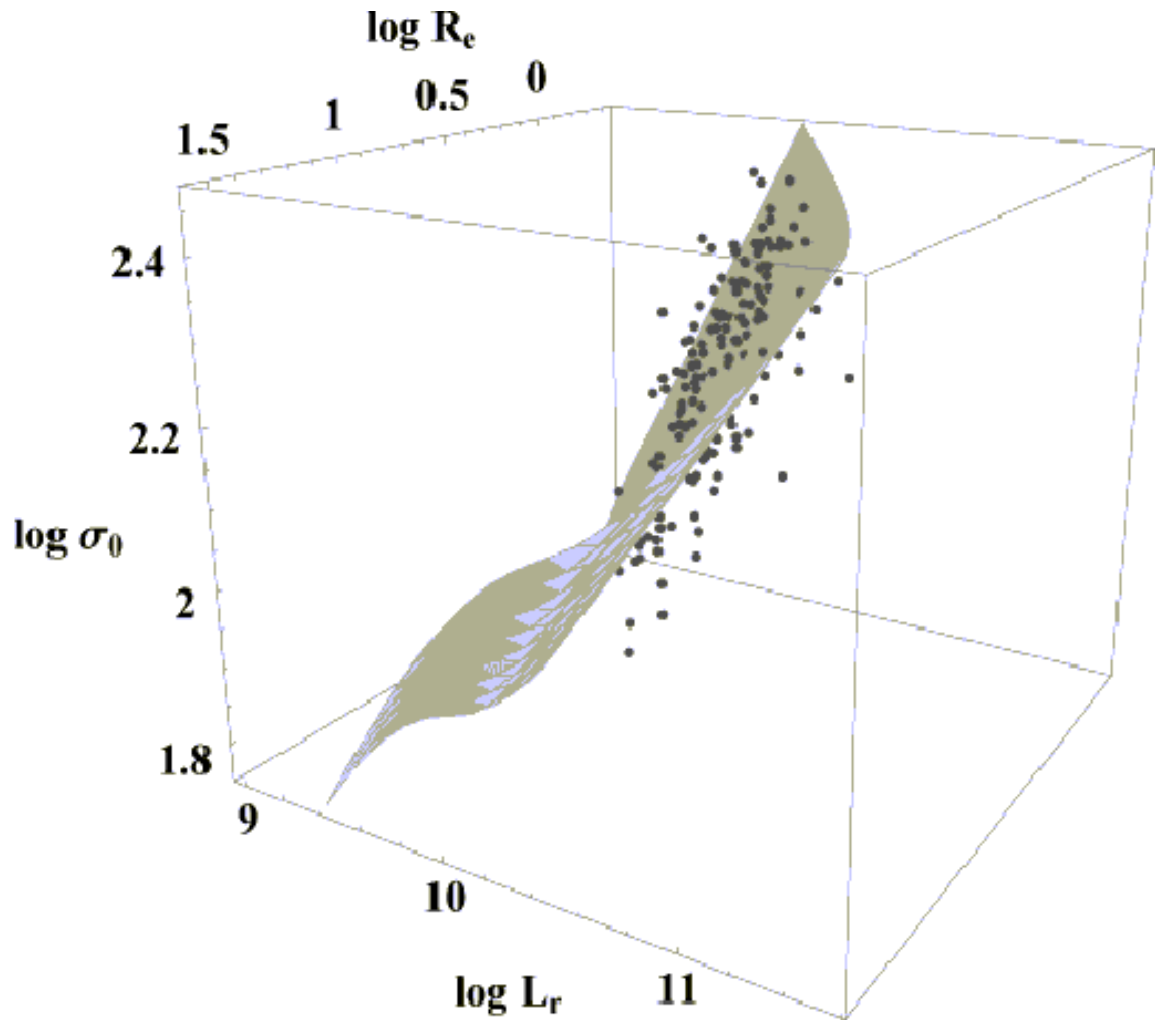
B. Weiner, T. Williams, and J. Sellwood.

Tully-Fisher





Fundamental Plane



Hubble Law

NEBULAE WHOSE DISTANCES HAVE BEEN ESTIMATED FROM STARS INVOLVED OR FROM MEAN LUMINOSITIES IN A CLUSTER

OBJECT	m_s	r	v	m_t	M_t
S. Mag.	..	0.032	+ 170	1.5	−16.0
L. Mag.	..	0.034	+ 290	0.5	17.2
N. G. C. 6822	..	0.214	− 130	9.0	12.7
598	..	0.263	− 70	7.0	15.1
221	..	0.275	− 185	8.8	13.4
224	..	0.275	− 220	5.0	17.2
5457	17.0	0.45	+ 200	9.9	13.3
4736	17.3	0.5	+ 290	8.4	15.1
5194	17.3	0.5	+ 270	7.4	16.1
4449	17.8	0.63	+ 200	9.5	14.5
4214	18.3	0.8	+ 300	11.3	13.2
3031	18.5	0.9	− 30	8.3	16.4
3627	18.5	0.9	+ 650	9.1	15.7
4826	18.5	0.9	+ 150	9.0	15.7
5236	18.5	0.9	+ 500	10.4	14.4
1068	18.7	1.0	+ 920	9.1	15.9
5055	19.0	1.1	+ 450	9.6	15.6
7331	19.0	1.1	+ 500	10.4	14.8
4258	19.5	1.4	+ 500	8.7	17.0
4151	20.0	1.7	+ 960	12.0	14.2
4382	..	2.0	+ 500	10.0	16.5
4472	..	2.0	+ 850	8.8	17.7
4486	..	2.0	+ 800	9.7	16.8
4649	..	2.0	+1090	9.5	17.0
Mean					−15.5

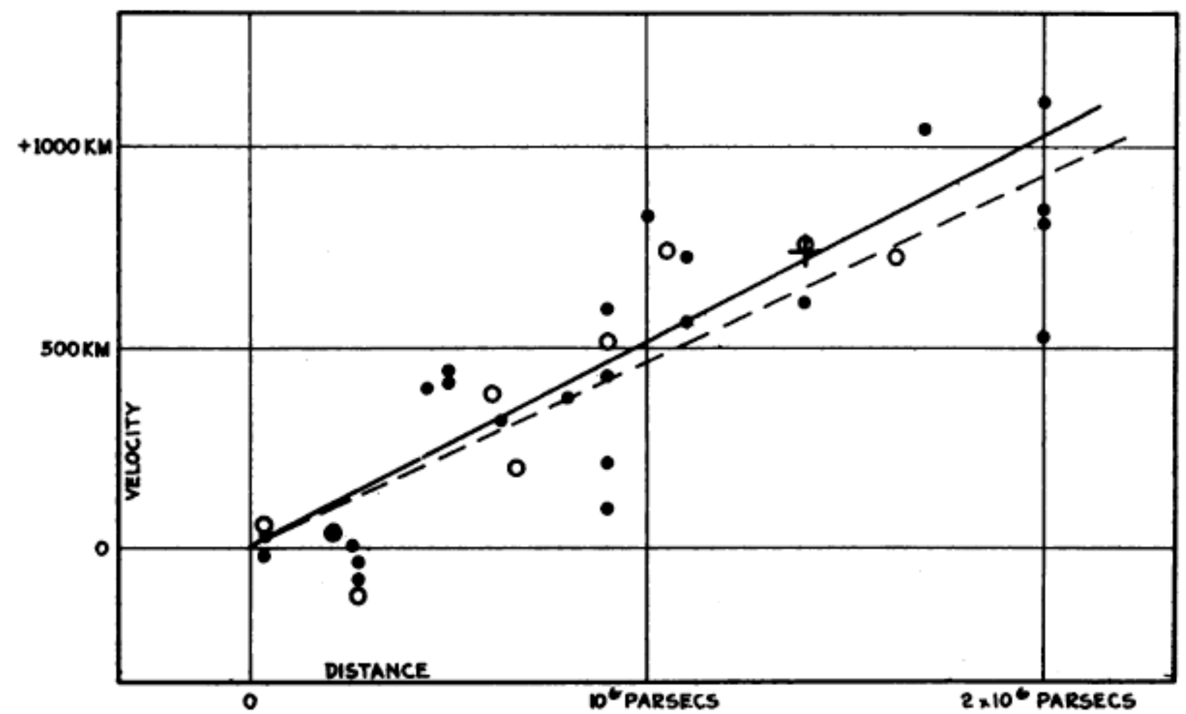


FIGURE 1

Hubble 1929

A relationship was found by Edwin Hubble between the distance to the source and its velocity. This relationship was the first evidence that our Universe was expanding!

$$v = H_o D \qquad H_o = 513 \pm 60 km/s/Mpc$$

But this value of H_o is nowhere close to the current value!

Types of Errors

Bevington & Robinson Ch 1

Accuracy		Precision
"How close the result of an experiment is to the true value" The correctness of the result	vs.	*"How well the result has been determined "* The reproducibility of a result

Systematic		Random
"Errors that will make our result different from the true value" Instrumental errors	vs.	*"Fluctuations...that yield results that differ from experiment to experiment"* Statistical errors

Random Errors

For a Gaussian distribution, the variance on the distribution is given by:

$$\sigma_x^2 = \lim_{N\to\infty} [\frac{1}{N} \sum (x_i - \overline{x}_i)^2]$$

Assuming x=f(u,v), then it can be shown:

$$\sigma_x^2 = \sigma_u^2 (\frac{dx}{du})^2 + \sigma_v^2 (\frac{dx}{dv})^2 + 2\sigma_{uv}^2 (\frac{dx}{du})(\frac{dx}{dv})$$

In most cases, the parent distribution is likely to be Gaussian or Poisson distribution. These have similar behaves for large N, but diverge for N<10. And very different error distributions can be produced when the parent distribution isn't Gaussian!

Example: Velocity

Assuming no covariance between the variables, find the random error for a velocity based on an error of σ_{obs} on λ_{obs}.

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$$v = c\frac{\lambda_{obs}}{\lambda_{em}}$$

$$\sigma_v = \frac{c}{\lambda_{em}}\sigma_\lambda$$

Sources of Error

$$v = H_o D$$

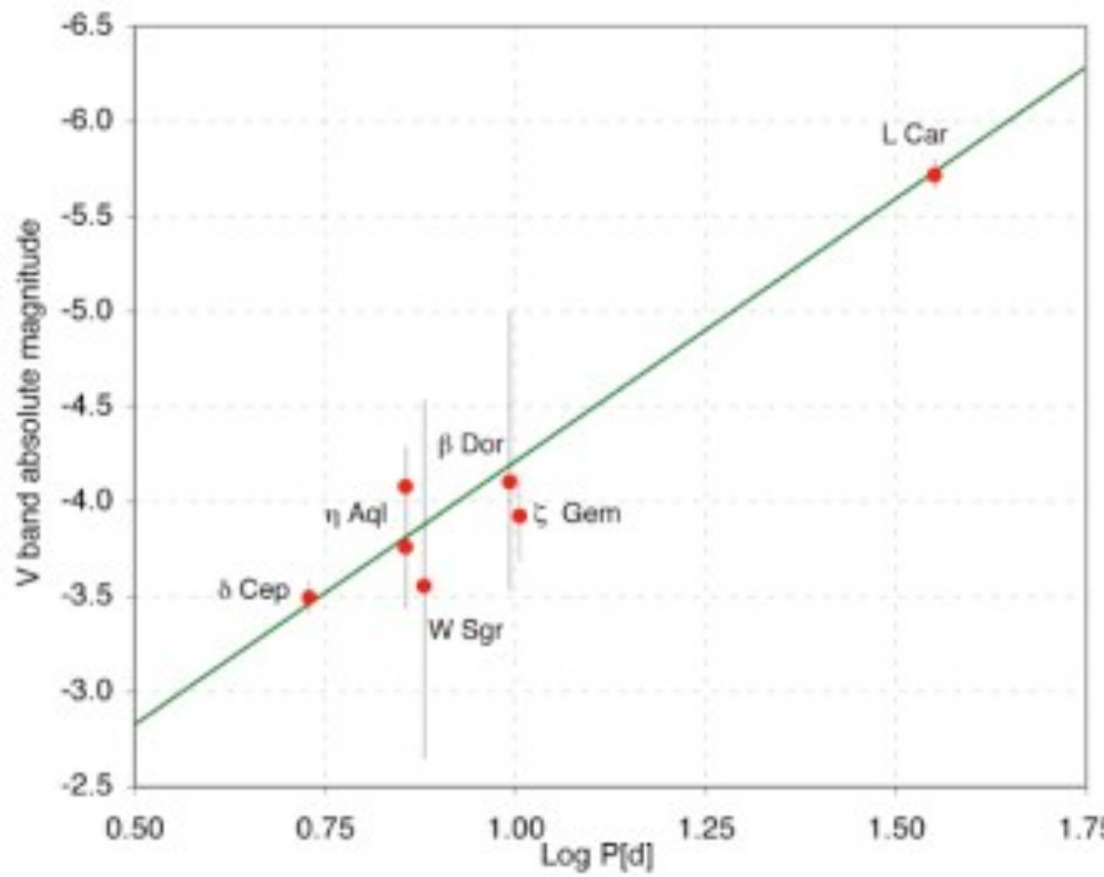
Calibration Errors?
Identification Errors?
Measurement errors?

Distances are based on the period-luminosity relationship for Cephied Variables.

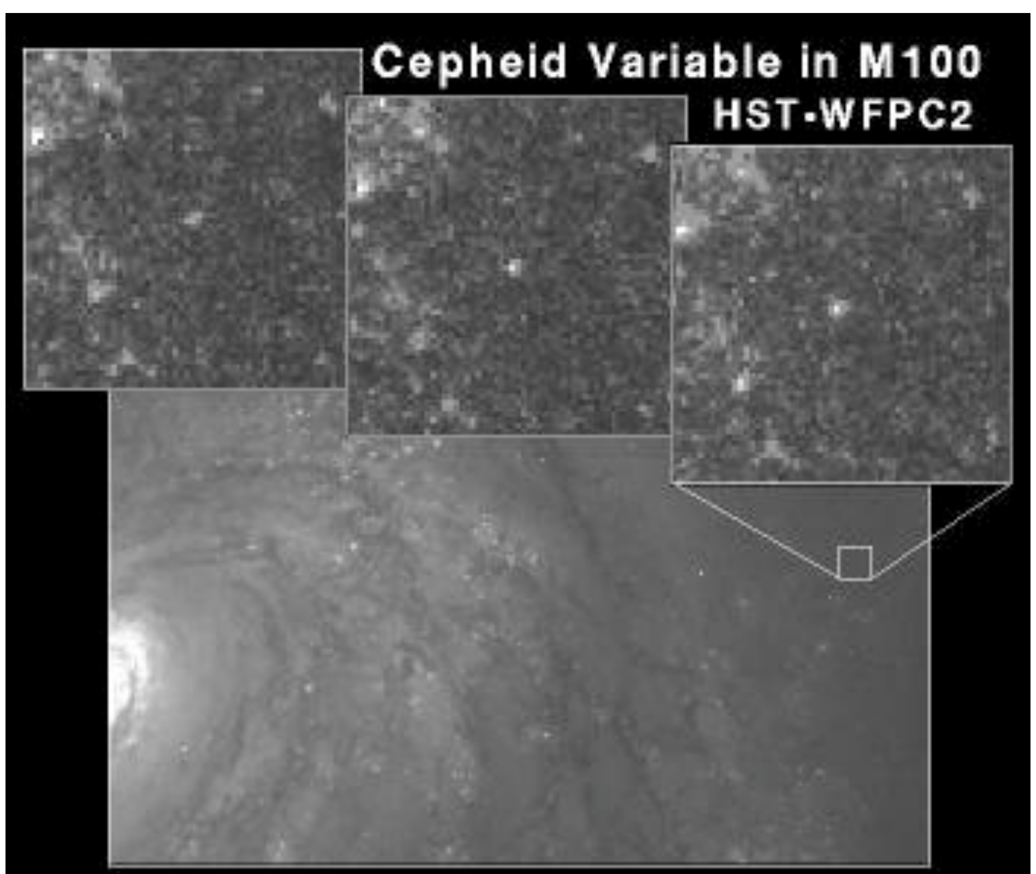
Calibration Error?

P-L Relationship

In 1908, Heinrietta Swan Leavitt showed that the maximum luminosity of a cephiod variable was correlated with its period. Early measurements of the P-L relationship were not very accurate, which led to the error in the distances to galaxies.



Gieren et al. 1998



But current versions have far smaller and improved errors:

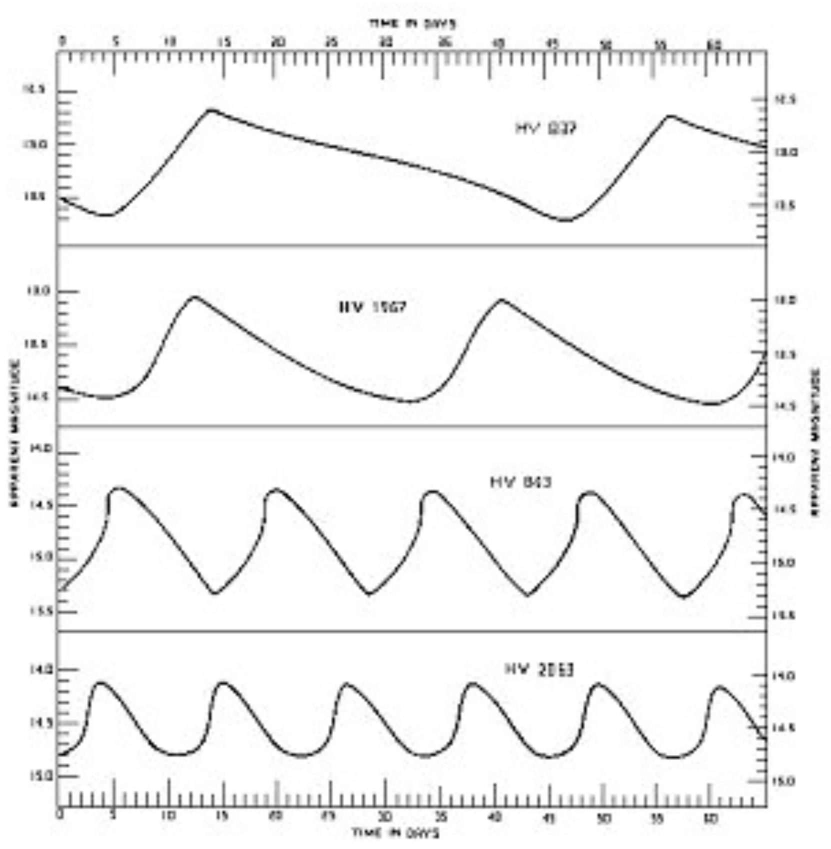
$$M_v = -2.81 log(P) - (1.43 \pm 0.1)$$

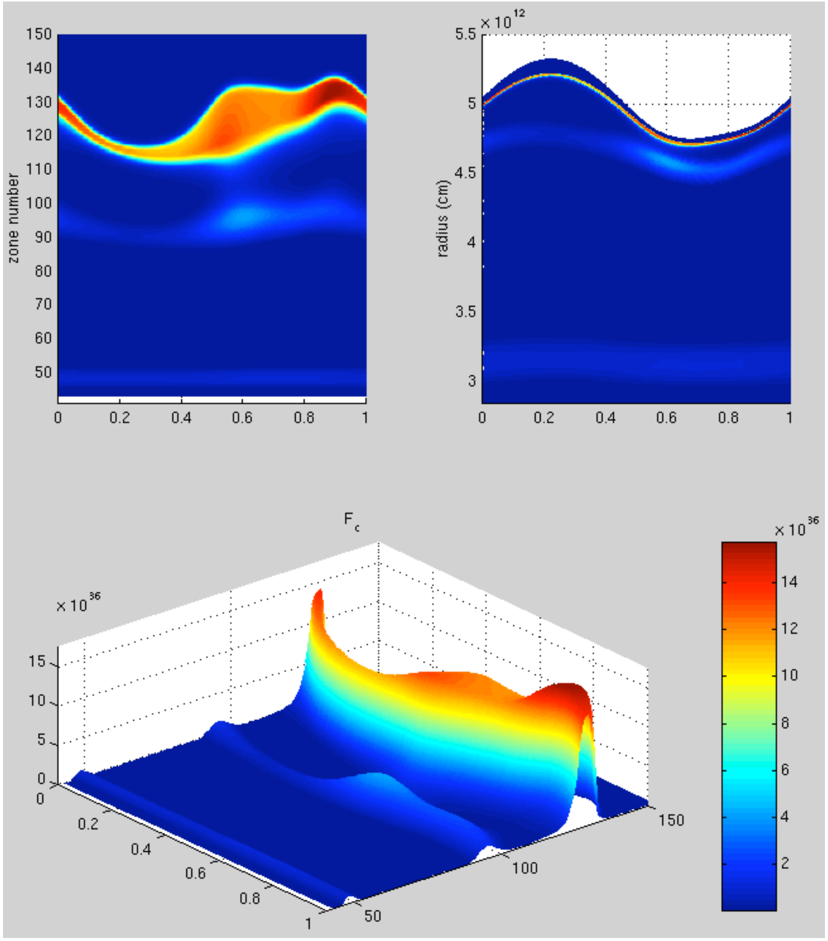
Feast & Catchpole 1997

Aside: Source of the variability

What is the mechanism driving the variability?

Many objects in the instability strip are around T~35-40k in temperature. This is the borderline temperature where He becomes fully ionized. Below these temperatures, He has one electron and the atmosphere is optically thick. So the stars pulsation depends on the **opacity of Helium**.





Fitting a Line

$$y(x) = y(x; a, b) = a + bx$$

The most common way to fit a straight line to data is to minimize the least squares function:

$$\chi^2(a,b) = \sum_{i=1}^{N} \left(\frac{y_i - a - bx_i}{\sigma_i} \right)^2$$

By finding the minimum to this equation in both a and b, we can determine an analytical solution for the two coefficients.

These solutions are usually implemented by various computer programs and a least squared program comes standard in most statistical packages (see leastsq in scipy.optimize for example).

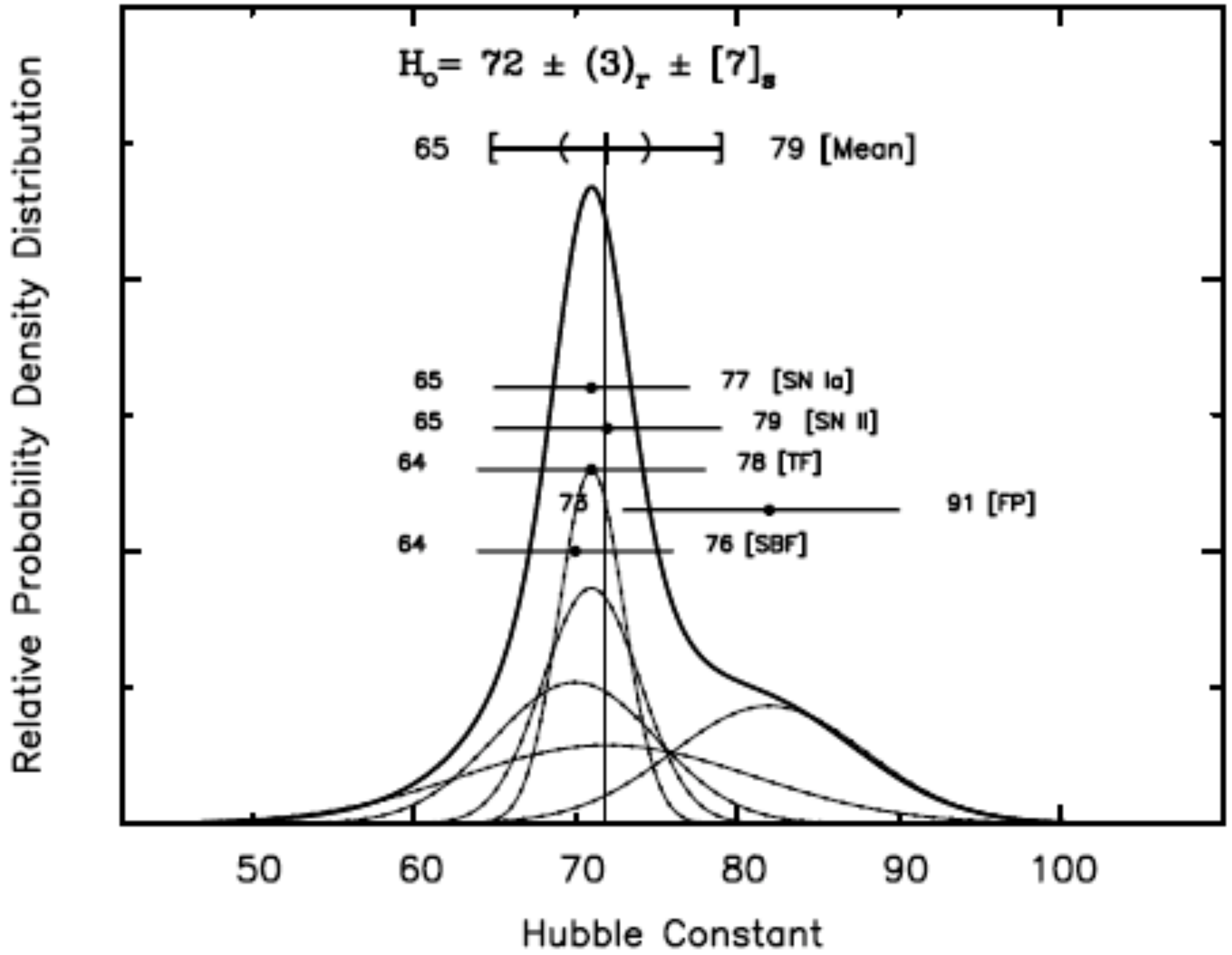
If you can simplify whatever needs to be fit to a line, it makes it much easier

Hubble Law

Using Hubble Space Telescope, Freedman et al. (2001) found:

$$H_o = 72 \pm 8km/s/Mpc$$

The error value here is based on multiple different methods and calibration procedures although still highly dependent on calibration of cephied variables and other earlier steps in the distance ladder (for example, the distance to the LMC). However, the error quoted here not only includes these different methods but the underlying errors in the calibration.

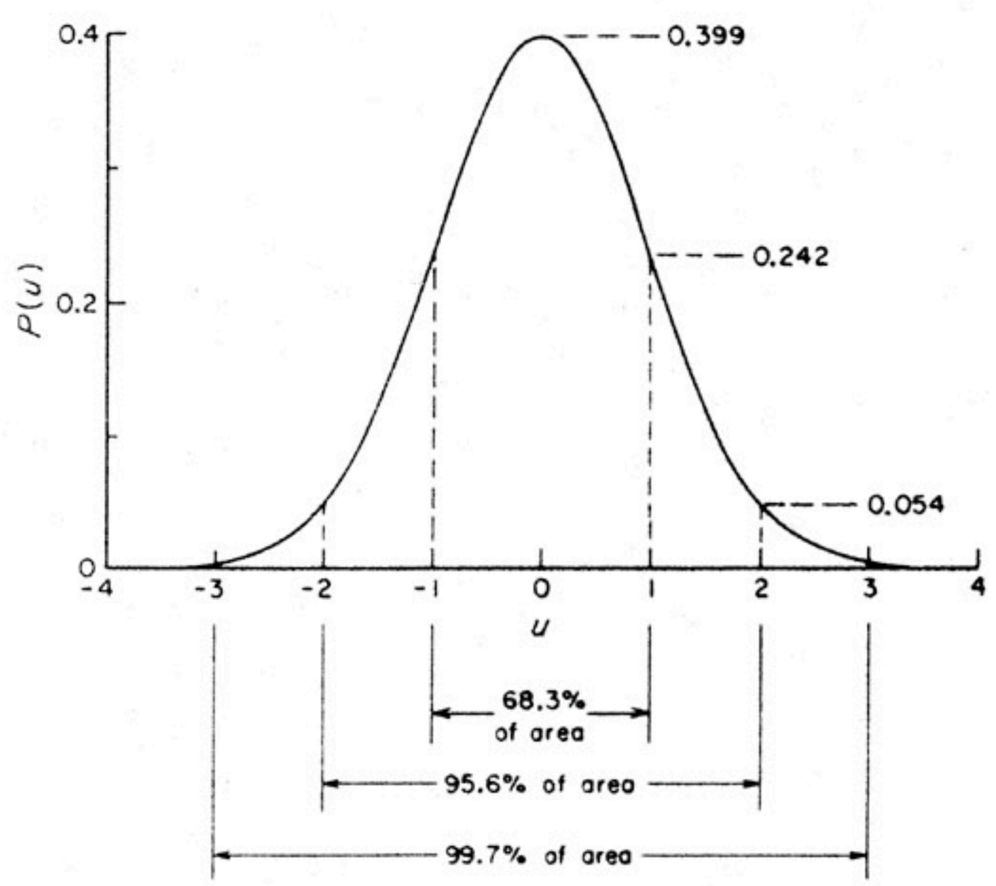


This work won the authors the 2009 Gruber prize in cosmology.

Confidence Limits

Freedman et al. (2001) quote an error of 8 km/s/Mpc on H_o. But what does that mean?

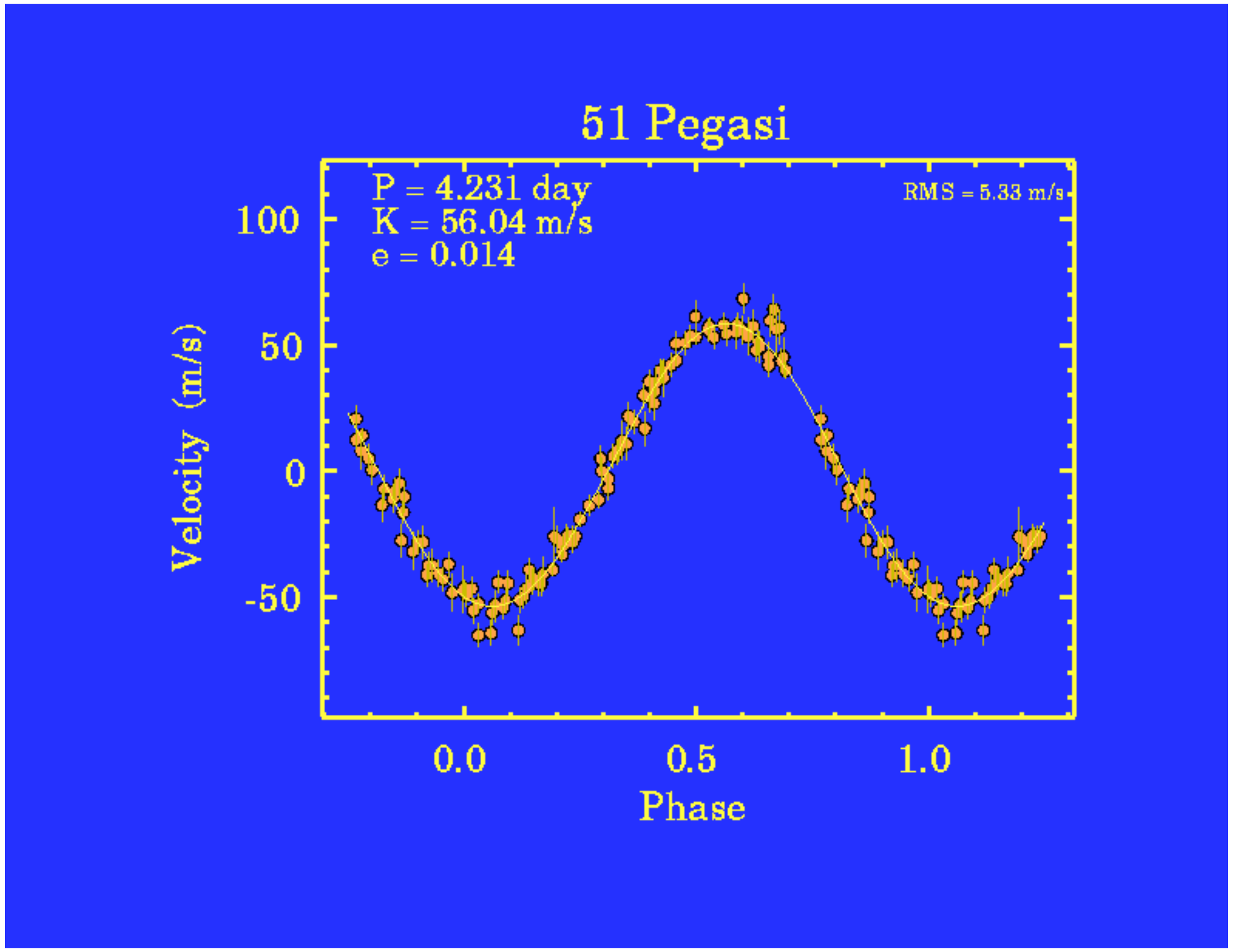
Errors are typically given by their 1-sigma confidence limits. For a Gaussian distribution, approximately 68.3% of the events fall within the 1-σ limit. Another way to put it, is that if the experiment was repeated, the value found would have a 68% of being within 1-sigma of the value found the first time.



σ	%
1	68.3
2	95.4
3	99.7
5	99.99999996

Finding Planets

Finding planets means fitting the complex orbitals to the observed small oscillations seen from the doppler shifts from spectra of star. These require very stable, very accurate spectrographs (<3 m/s) and long, repeated observations.



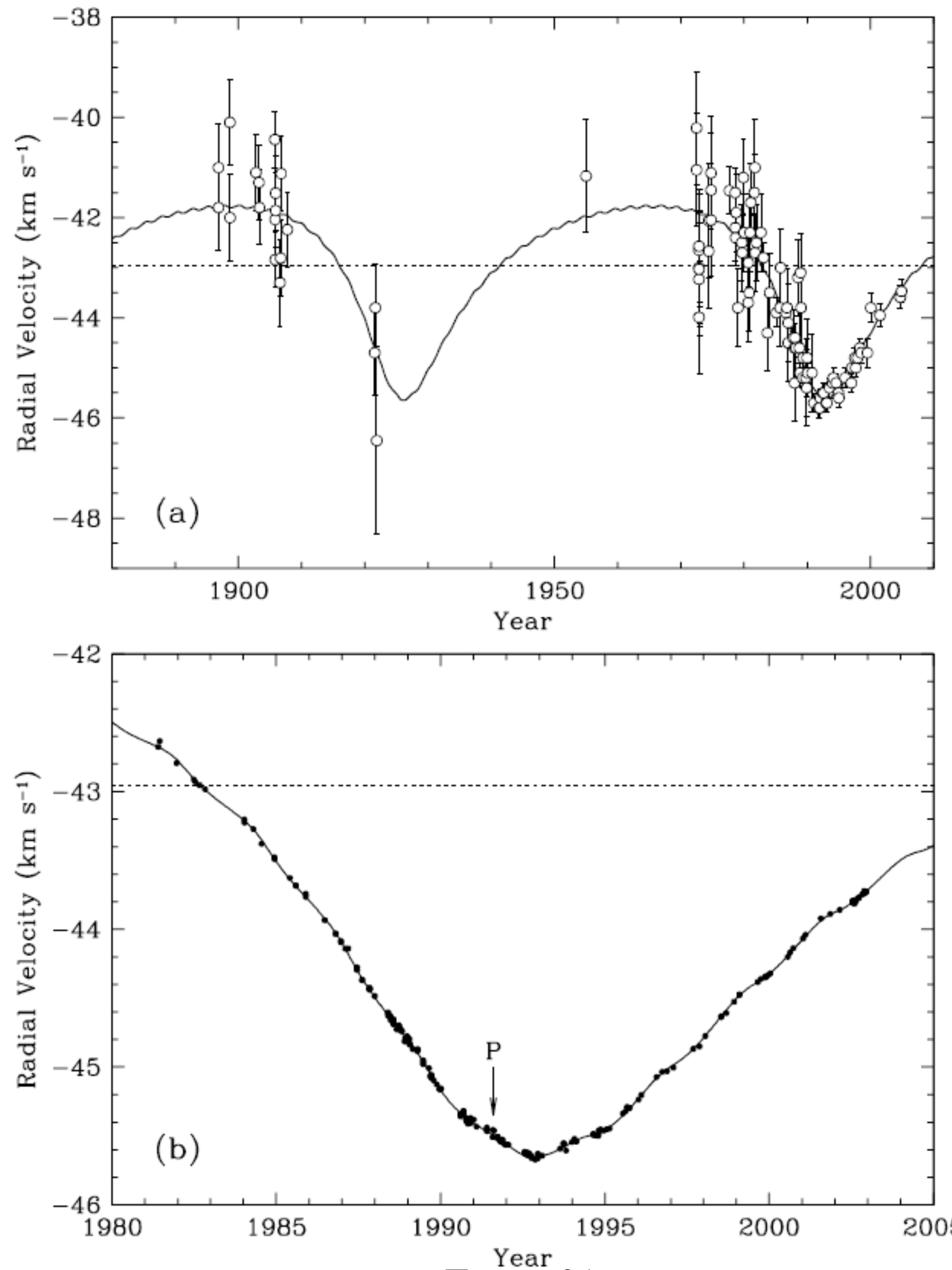
Marcy

Fitting Models

A more generally form of χ^2 can be used to fit any model to the data:

$$\chi^2 = \sum[\frac{y_i - f(x_i)}{\sigma_i}]^2$$

where f(x) is a model describing the observed data y. The minimum of this can be found in a number of different ways and there is a wide range of computer software to quickly find the minimum.



Torres 06