Chapter 1

Radiative Transfer

Radiatrive transfer is a key component to understanding processeses in the Universe. It is critical for building a clear picture of structures in the Universe as it describes how light travels. It can be used to peel back the layers of a structure as different wavelengths will probe to different depths. From explaining the color of the sunset to the atmosphere of distant stars to the structure of material between galaxies, radiative transfer is the physics behind understanding how light travels.

Much of this material is based on Chapter 1 from Rybicki & Lightman and Chapter 5 and 7 from Gray.

1.1 Defintitions

1.1.1 Specific Intensity

We start with a slab of material (solid, plasma, gas, anything) that is radiating (see Figure 1). The specific intensity (or just intensity or brightness) of the radiation from the surface will be given by:

$$I_\nu = lim \frac{\Delta E_\nu}{cos\theta \Delta A \Delta\omega \Delta t \Delta\nu} = \frac{dE_\nu}{cos\theta dA d\omega dt d\nu} \tag{1.1}$$

In this equation, I_ν is given by the energy emitted at a specific wavelength, E_ν over an area (dA), solid angle ($d\omega$), time (dt), and frequency ($d\nu$). The factor of cos θ is due to the foreshortening of the area due to the angle of the observer. The units of I_ν in cgs are usually given as:

$$[I_\nu] = \frac{erg}{cm^2 rad^2 s Hz} \tag{1.2}$$

Alternatively, specific intensity can also be definite in terms of wavelength where:

$$I_\lambda d\lambda = I_\nu d\nu \tag{1.3}$$

and, the untis are:

$$[I_\lambda] = \frac{erg}{cm^2 rad^2 s \mathring{A}} \tag{1.4}$$

Often times, the mean intensity is used, which is defined as the directionally averaged specific intensity:

$$J_\nu = \frac{1}{4\pi} \oint I_\nu d\omega = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} I_\nu sin\theta d\theta d\phi \tag{1.5}$$

1.1.2 Flux

However nice it would be to have the specific intensity, what in reality we are able to measure is the flux. Flux is definide as the net energy flow across an infinitesimal small area, time, and spectral range:

$$F_\nu = lim \frac{\sum \Delta E_\nu}{\Delta A \Delta t \Delta \nu} = \frac{\int dE_\nu}{dAdtd\nu} \tag{1.6}$$

The flux is related to the specific intensity by integrating over all solid angle such that:

$$F_\nu = \oint I_\nu cos\theta d\omega \tag{1.7}$$

The difference between flux and intensity is the solid angle. From the perspective of an observer, the solid angle is given by $\omega \sim A_s/R^2$ where A_s is the area resoloved on the emitting surface and R is the distance to that surface. What happens when $R >> A_s$?

If a source of intensity is isotropic, then the $F_\nu = 0$. For that reason, the observed flux, which only considers the flux is a single direction, is often used.

1.1.3 Luminosity

Finally for unresolved sources, what is often measured is the luminosity. The luminosity is related to the specific intensity by integrating over the area, frequency, and solid angle.

$$L = \int I_\nu cos\theta dAd\omega d\nu \tag{1.8}$$

For an isotropically radiating source, the flux from the source can be related to the luminosity by the distance from the source:

$$F = \frac{L}{4\pi R^2} \tag{1.9}$$

1.2 Radiative Transfer

We are trying to determine what some source from some point A will look like by the time it arrives at our observer. A photon emitted from some point in a star will have to pass through its atmosphere, any surrounding gas cloud, it may have to pass through some intervening dust clouds or other material in the interstellar media (ISM), and, once it reaches Earth, it will have to pass through the Earth atmosphere, the system of the telescope, and onto the detector (a CCD, your eye). Along the way, the photon may be scattered, absorbed, stimulated, or reflected.

1.3 Emissivity

Emission is defined as the spontaneous increase in intensity which is occurring inside or due to the material though which our line of sight passes:

$$dI_\nu = j_\nu \rho ds \tag{1.10}$$

where j_ν is the emission coefficient [erg s^{-1} rad^{-2} Hz^{-1} g^{-1}] and ρ is the density [g cm^{-3}].

The emission can have a number of different sources include continuum and line emission along with scatterring.

Warning: Different work have different definitions for the coefficients with some including density and some averaged over solid angle. Always be mindful of the units being used

1.4 Absorption

Absorption represents the decrease in intensity as it travels through a material along our line of sight:

$$dI_\nu = -\kappa_\nu \rho I_\nu ds \tag{1.11}$$

Here, κ_ν is the absorption coefficient [cm^2 g^{-1}]. A number of different process can contribute to the absorption including continuum processes (bound-free, free-free, metals, dust), line processes, and scatterring. It is important to note, that κ_ν is cumulative and that many different processes may contribute to it.

In astrophysics, what we can measure is the optical depth. The optical depth is defined as:

$$\tau = \int_0^L \kappa_\nu \rho ds \tag{1.12}$$

It is how much material over some distance that the radiation sees. Two key terms:

- **Optical Thick:** $\tau >> 1$ and it is difficult to 'see' through the medium.
- **Optical Thin:** $\tau << 1$ and radiation passes through the medium with very little absorption.

1.4.1 Source Function

The source function is defined as:

$$S_\nu = j_\nu / \kappa_\nu q \tag{1.13}$$

This is to simplify the equation of radiative transfer and is far closer to what is physically observed.

1.4.2 Equation of Radiative Transfer

Combining emission and absorption gives the equation for radiative transfer:

$$dI_\nu = -\kappa_\nu \rho I_\nu ds + j_\nu \rho ds \tag{1.14}$$

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We can substitue in τ and S_ν to give:

$$\frac{dI_\nu}{d\tau} = -I_\nu + S_\nu \tag{1.15}$$

By starting at a solution of the form $I_\nu(\tau) = f(\tau)e^{-b\tau}$, it can be shown that the equation of radiative transfer has the following solution:

$$I_\nu = I_o e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t) e^{-(\tau_\nu - t)} dt \tag{1.16}$$

Solving the equation of radiative transfer is often done numerical (e.g DUSTY, CLOUDY, etc), but it can be very informative to solve the equation in certain limits like extinction-only (no emission), monochromatic or grey (assume no dependence on ν), and plane parrellel.

1.4.3 Example: Extinction

What is the fraction of sunlight that reaches the surface on a clear day?

First, we assume that we are measuring it in the optical at a single wavelength and we are only worried about emission so that j_ν=0. We also assume the following characteristics for the Earth's atmosphere: $\kappa = 0.0001$ cm^2/g, $\rho = 0.001$ g/cm^3, and h=8.5 km.

For the case of $j_\nu = 0$, the equation for radiative transfer reduces down to $dI/d\tau = I$ with the solution $I = I_0 e^{-\tau}$. Since we want the fraction of sunlight that reaches the Earth's surface, we have:

$$\frac{I}{I_o} = e^{-\tau} = exp(-\int_0^H \kappa\rho dx) = exp(-\kappa\rho H) \tag{1.17}$$

$$= exp(-0.001\frac{cm^2}{g}\ 0.001\frac{g}{cm^3}\ 8.5 \times 10^5 cm) = 0.92 \tag{1.18}$$