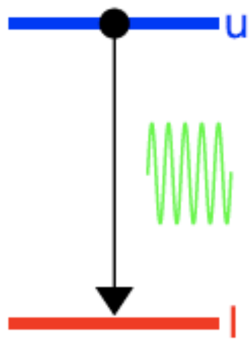
Line Emission

Bonus Addition

Einstein Coefficients

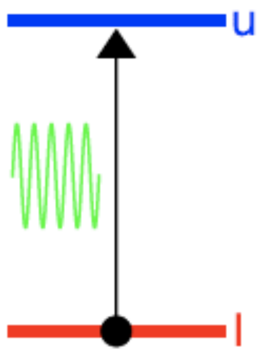
Spontaneous Emission



$$\frac{dn}{dt} = n_u A_{ul}$$

$$j_\nu \rho = h\nu n_u A_{ul} \Psi(\nu)$$

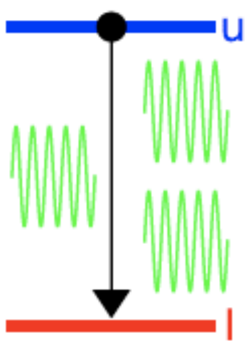
Absorption



$$\frac{dn}{dt} = n_l B_{lu} I_\nu$$

$$\kappa_\nu \rho I_\nu = h\nu n_l B_{lu} I_\nu \Phi(\nu)$$

Stimulated Emission



$$\frac{dn}{dt} = n_u B_{ul} I_\nu$$

$$\kappa_\nu \rho I_\nu = -h\nu n_u B_{ul} I_\nu \Phi(\nu)$$

Useful Relationships

Boltzman Equation

$$\frac{n_i}{n_j} = \frac{g_i}{g_j} e^{-(E_i - E_j)/kT}$$

Relationship between B coefficients

$$B_{ul} g_u = B_{lu} g_l$$

Relationship between A and B coefficients

$$\frac{A_{ul}}{B_{ul}} = \frac{2h\nu^3}{c^2}$$

The total absorption Coefficient

$$\kappa_\nu \rho = h\nu \Phi(\nu) n_l B_{lu} (1 - \frac{n_u B_{ul}}{n_l B_{lu}})$$

In equilibrium, the total absorption coefficient becomes

$$\kappa \rho = h\nu B_{lu} n_l \Phi(\nu) (1 - e^{-h\nu/kT})$$

If we take the following two limits:

$$h\nu >> kT -> \kappa\rho = h\nu n B_{lu} \Phi(\nu)$$

$$h\nu << kT -> \kappa\rho = 0$$

What we go back to the absorption coefficient and we aren't in equilibrium:

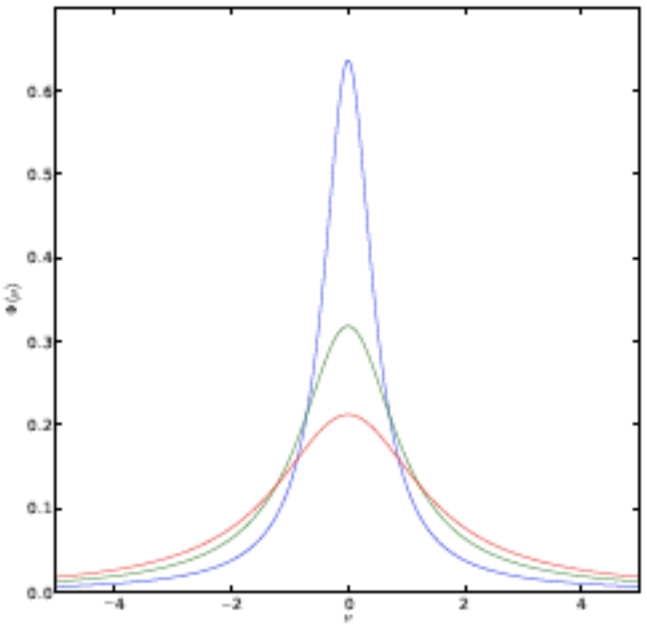
$$\kappa_\nu \rho = h\nu\Phi(\nu) n_l B_{lu}(1 - \frac{n_u B_{ul}}{n_l B_{lu}})$$

Take limit where nu>>nl*gu/gl?

Then kp is negative and we get emission. If the population can be maintained, we can get microwave amplification by stimulated emission radiation.

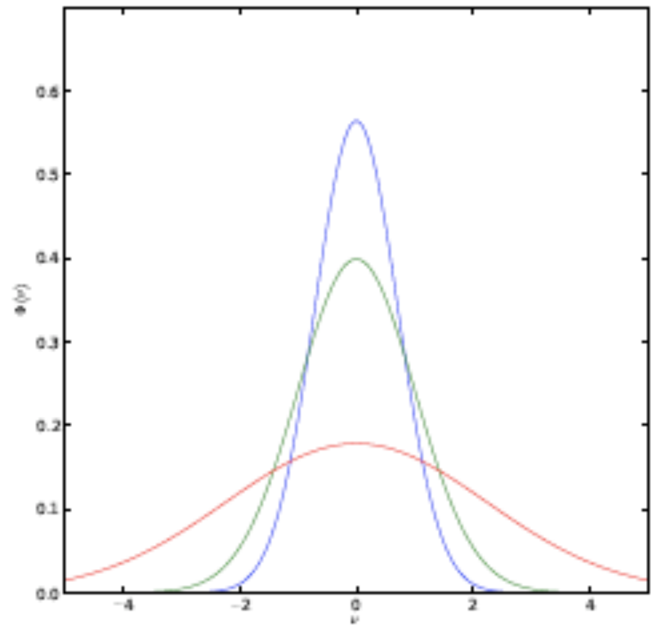
In the optical wavelengths, we get LASERs

Profiles



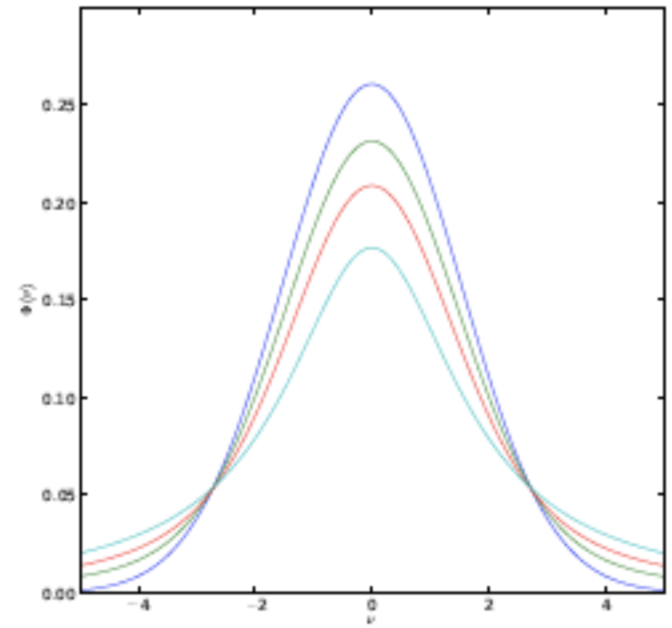
Natural
Lorentz

$$\Phi\nu = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\Gamma/4\pi}{(\nu-\nu_o)^2 + (\Gamma/4\pi)^2}$$



Thermal
Gaussian

$$\Phi\nu = \frac{\Delta\nu_D}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-(\frac{\nu-\nu_o}{\Delta\nu_D})^2} d\nu$$



Combined
Voigt

$$\Phi(\nu) = \frac{\alpha/\pi^{3/2}}{\Delta\nu_D}\int_{-\infty}^{\infty} \frac{e^{-y^2}dy}{(V-y)^2 + \alpha^2}$$

Questions for the next Lecture

Where does the ‘natural’ broadening come from?

$$\Phi\nu = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\Gamma/4\pi}{(\nu - \nu_o)^2 + (\Gamma/4\pi)^2}$$

And what sets or determines the value of the Einstein coefficients?

$$h\nu B_{ij} = \frac{e^2\pi}{4\pi\epsilon_o m_e} f_{ij}$$