MAP 6264 Homework 2 (Reliable Intuition?) 20 Points

Consider a renewal process. Let X be the interrenewal times; and let I and R be the length of an interval interrupted at random and its remainder, respectively. The following BASIC simulation calculates the average values of X, I, and R (based on 10,000 replications of I and R, where T is a random interruption point).

```
100  FOR j=1 TO 10000
110  S=0
120  T = -1000*LOG(1-RND)
130  X=
140  c=c+1
150  SX=SX+X
160  S=S+X
170  IF S<T THEN 130
180  R=S-T: I=X
190  SR=SR+R: SI=SI+I
200  NEXT j
210  PRINT SX/c,SI/10000,SR/10000
```

a. Run the simulation for the case when X is exponentially distributed (that is, the renewal process is a Poisson process) with E(X) = 1. Fill in the following table.

E(X)		*E(I)*		*E(R)*	
theory	simulation	theory	simulation	theory	simulation

Comment on the assertion: "It is intuitively obvious that E(I) = E(X) and E(R) = E(X) / 2."

b. Let X = Y + u, where u is a constant (to be treated as a parameter), and Y is a random variable with probability distribution: P(Y=1) = 0.9, P(Y=11) = 0.1. Run the simulation and fill in the values indicated in the table. Calculate the theoretical values of E(I) and E(R) according to the formulas (to be derived later):
E(I) = E(X) + V(X) / E(X) and E(R) = E(I) / 2.

Draw the theoretical graph of E(R) versus u. On the same graph, plot the points produced by the simulation.

Comment on the assertion: "It is intuitively obvious that as the average length of the random interval X increases (that is, as u increases), the average lengths of the interrupted interval I and its remainder R will also increase."

	E(X)		*E(I)*		*E(R)*	
u	theory	simulation	theory	simulation	theory	simulation
0.0						
0.5						
1.0						
1.5						
2.0						
2.5						
3.0						
3.5						
4.0						
4.5						
5.0						