Computational Physics: Problem Set 10

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1 Logistic Map

The logistic map is given by the equation below:

$$X(n+1) = 4rX(n)(1 - X(n))$$

We can easily write a program for this equation.

1.1 logisticMap() Function

This function simply takes the values of r and X_0 as input then it returns the results of the logistic map being applied on that value once.

1.2 Main Loop

We generate the values of r into an array. We take care to increase the number of data points after the frequency of bifurcations ramp up. Then we generate an array of random numbers between 0 and 1 as the initial state. Then we apply the logistic map 10^5 times and plot the results.

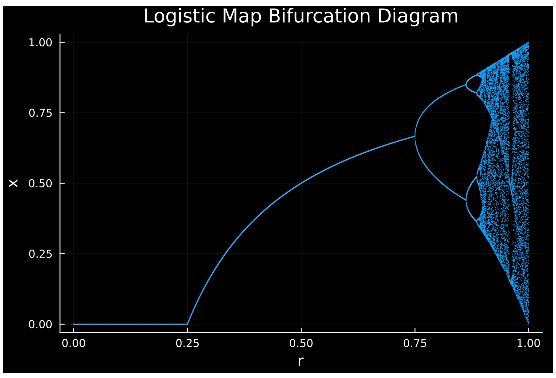

Figure 1: Diagram of the Logistic map

Then we will plot the same data again but this time we limit the x axis so the plot is zoomed in, then we will add some of the splitting points and the chaos point on top.

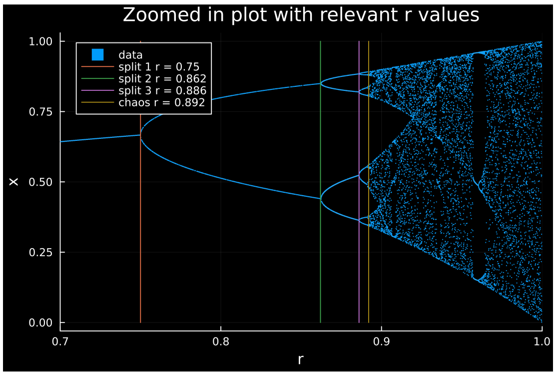

Figure 2: Diagram of the Logistic map with splitting points

2 Feigenbaum Constants

2.1 The First Constant

The first constant is defined as below:

$$\delta = \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}}$$

where a_n, a_{n-1} and a_{n-2} are the bifurcation points into 32, 16 and 8 respectively. We can find these values by generating the X points, 100 times and fitting all the data in a matrix. Then we can search the 100 data sets for the first point which for a certain r, The number of unique values of X is a certain amount. We do this using the findfirst command and then using the unique command to find the unique values of X. Then we will use the results and the above formula to calculate δ.

$$\delta = 4.463$$

2.2 The Second Constant

The second constant is defined as the width of a certain bifurcation divided by the width of the next bifurcation.

$$\alpha = \frac{\Delta X_1}{\Delta X_2}$$

We can find these value by finding the last point of a bifurcation, then finding the unique values of X at said point, then sorting the values and using two relevant values to calculate the width of each tine.

$$\alpha = 1.82$$