## Girvan-Newman Benchmark

In this exercise we will generate the Girvan-Newman benchmark graph as described in Barabasi's book for multiple values of  $\mu$ , then we will use the Girvan-Newman algorithm to identify the communities. Then we calculate the information and compare the results from the ones mentioned in the book.

## Girvan-Newman benchmark graph generator

```
In [ ]: using Graphs, Plots, Colors, Statistics
In [ ]: function GN_generator(mu)
             #since communities are well connected, we choose a relatively large p
             p_{int} = 0.8
             p_ext = mu * p_int / (1 + mu)
             nc = 4
             Nc = 32
             graph = SimpleGraph(nc * Nc)
             #defining clusters for convinience
             cluster arr = []
             for i in 1:nc
                 cluster = Int[]
                 for j in ((i-1) * Nc + 1):((i-1) * Nc + Nc)
                     push!(cluster, j)
                 end
                 push!(cluster_arr, cluster)
             end
             #adding internal edges
             for i in 1:nc
                 for j in ((i-1) * Nc + 1):(i * Nc)
                     for k in j+1:((i-1) * Nc + Nc)
                         if rand()<= p int</pre>
                             add_edge!(graph, j, k)
                         end
                     end
                 end
             end
             #adding external edges
             for i in 1:nc-1
                 for j in ((i-1) * Nc + 1):(i * Nc)
                     for k in (i * Nc + 1):nc*Nc
                         if rand()<= p_ext</pre>
                             add edge!(graph, j, k)
                         end
                     end
                 end
             end
            return graph
        end
```

We hold our communities in a dictionory defined as below:

Now we create a function that generates the similarity matrix for our graph

```
In [ ]: function similarity matrix(g)
            nc = 4
            Nc = 32
            total nodes = nc * Nc
            adj matrix = adjacency matrix(g) \#gives\ values\ 0 and 1 so we don't ne
            degree arr = degree(g)
            sim matrix = zeros(total nodes, total nodes)
            for i in 1:total nodes-1
                for j in i+1:total nodes #we use the similarity amtrix defined in
                     sim matrix[i, j] = common neighbors(g, i, j) / (min(degree ar
                end
            end
            return sim matrix
        end
        #Function to calculate the common neighbors of two nodes
        function common neighbors(g, i, j)
            i neighbors = neighbors(g,i)
            j neighbors = neighbors(g,j)
            common_neighbor_count = length(findall(x->x in i_neighbors, j_neighbo
            if (j in i_neighbors) || (i in j_neighbors)
                 common neighbor count = common neighbor count + 1
            end
            if i == j
                 common_neighbor_count = 0
            return common neighbor count
        end
```

Now we create a function to recalculate the similarity matrix for the communities that have formed using our original similarity matrix.

```
for i in arr1
    for j in arr2
        sum = sum + mat[i, j]
    end
end
avg = sum / (length(arr1) * length(arr2))
return avg
end
```

Now we need a function that merges two communities which have the highest similarity and returns the new community table

```
In []:
    function community_merge_step(sim_mat, com_dict)
        community_arr = sort(collect(keys(com_dict)))
        max, indexes = findmax(sim_mat)
        min_index = min(indexes[1], indexes[2])
        max_index = maximum([indexes[1], indexes[2]])
        min_index_vals = com_dict[community_arr[min_index]]
        max_index_vals = com_dict[community_arr[max_index]]
        new_com_val = unique(vcat(min_index_vals, max_index_vals))
        com_dict[community_arr[min_index]] = new_com_val
        delete!(com_dict, community_arr[max_index])
    return com_dict
end
```

Now to find the accuracy we define the function below which shows the inaccuracy in percentages

```
In []: function accuracy(set_arr)
    set1 = floor.(Int, LinRange(1, 32, 32))
    set2 = floor.(Int, LinRange(33, 64, 32))
    set3 = floor.(Int, LinRange(65, 96, 32))
    set4 = floor.(Int, LinRange(97, 128, 32))
    diff = length(setdiff(set_arr[1], set1)) + length(setdiff(set_arr[2], return 100 - (diff/180) * 100)
end
```

Now we perform the steps until only four communities remain. We also calculate the accuracy according to the accuracy we defined. (We also average over 100 runs)

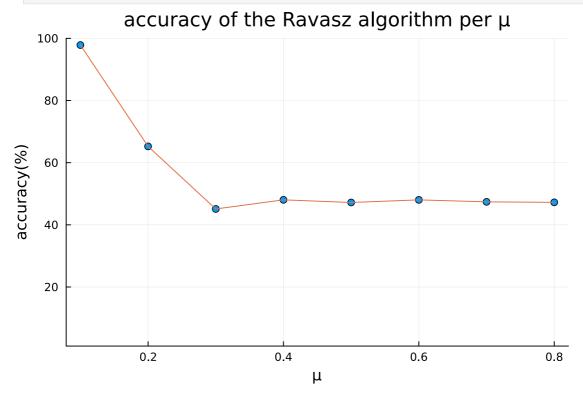
```
In []: mu \ arr = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8]
        accuracy_results = Float64[]
        for mu in mu_arr
            accuracy_arr = Float64[]
            for k in 1:100
                results = []
                GN benchmark = GN generator(mu)
                community_table = community_table_generator()
                sim_mat_org = similarity_matrix(GN_benchmark)
                sim_mat = sim_mat_org
                for i in 1:124
                    community table = community merge step(sim mat, community tab
                    sim mat = similarity matrix recalc(sim mat, community table,
                end
                val = collect(values(community table))
                for i in 1:4
                    val[i] = Set(val[i])
```

```
end
    push!(results, val)
    for i in 1:length(results)
        push!(accuracy_arr, accuracy(results[i]))
    end
    end
    push!(accuracy_results, mean(accuracy_arr))
end

accuracy_results
```

Now we plot the accuracy against  $\mu$ 

```
In [ ]: Plots.scatter(mu_arr, accuracy_results, ylims = (1,100), title = "accuracy_results)
```



As expected with the increase of  $\mu$ , the accuracy of our method drops but remains decently accurate for large  $\mu$  s. In the graph in the book, the accuracy remains higher for small  $\mu$  s, but that graph uses a different metric to gauge the accuracy of the algorithm. Overall we have been able to build the benchmark graph and identify it's communities using the Ravasz algorithm successfully from scratch.