

## Closed Form Solution for Linear Regression Ident. Act. With Regularization:

$$W = (D^T D + \lambda I)^{-1} D^T Y, \quad \text{where: } W_{(d+1) \times 1}, D_{N \times (d+1)}, Y_{N \times 1}$$

## 1D Example: Polynomial Curve Fitting (Identity Activation fn.) (Regression)

$$y(x, w) = w_0 + w_1 x + w_2 x^2 + \dots + w_N x^N = \sum_{j=0}^N w_j x^j \quad \left\{ \begin{array}{l} \text{Gradient Descent Method (Mean Square Loss fn.)} \\ \text{In Closed Form Solution} \rightarrow D_{N \times (N+1)} W_{(N+1) \times 1} = Y_{N \times 1} \rightarrow W = (D^T D)^{-1} D^T Y \end{array} \right.$$

$$\text{Precision: } \frac{TP}{TP + FP}, \quad \text{Accuracy: } \frac{TP + TN}{TP + TN + FP + FN}$$

## Batch Normalization

$$\begin{array}{ll} \text{Input} & \rightarrow x: (N \times D) \\ \text{Intermediates} & \rightarrow \mu, \sigma: D, \hat{x}: (N \times D) \\ \text{Output} & \rightarrow y: (N \times D) \end{array}$$

$$\mu_i = \frac{1}{N} \sum_{l=1}^N x_{li}$$

$$\sigma_i^2 = \frac{1}{N} \sum_{l=1}^N (x_{li} - \mu_i)^2$$

$$\hat{x}_{li} = \frac{x_{li} - \mu_i}{\sqrt{\sigma_i^2 + \epsilon}}$$

$$y_{li} = \gamma_i \hat{x}_{li} + \beta_i$$

## Backpropagation (Vectorization Rule):

Backward		Forward	
$\frac{\partial L}{\partial W_3} = \frac{\partial L}{\partial A_3} \cdot \frac{\partial A_3}{\partial Z_3} \cdot \left( \frac{\partial Z_3}{\partial W_3} \right)^T$	$\frac{\partial L}{\partial b_3} = \frac{\partial L}{\partial A_3} \cdot \frac{\partial A_3}{\partial Z_3} = \theta$	$Z_3 = W_3 \cdot X + b_3$	$A_3 = f_3(Z_3)$
$\frac{\partial L}{\partial W_2} = \left( \frac{\partial Z_3}{\partial A_2} \right)^T \cdot \theta = \frac{\partial A_2}{\partial Z_2} \cdot \left( \frac{\partial Z_2}{\partial W_2} \right)^T$	$\frac{\partial L}{\partial b_2} = \left( \frac{\partial Z_3}{\partial A_2} \right)^T \cdot \theta = \frac{\partial A_2}{\partial Z_2} = \delta$	$Z_2 = W_2 \cdot A_1 + b_2$	$A_2 = f_2(Z_2)$
$\frac{\partial L}{\partial W_1} = \left( \frac{\partial Z_2}{\partial A_1} \right)^T \cdot \delta = \frac{\partial A_1}{\partial Z_1} \cdot \left( \frac{\partial Z_1}{\partial W_1} \right)^T$	$\frac{\partial L}{\partial b_1} = \left( \frac{\partial Z_2}{\partial A_1} \right)^T \cdot \delta = \frac{\partial A_1}{\partial Z_1} = \lambda$	$Z_1 = W_1 \cdot X + b_1$	$A_1 = f_1(Z_1)$

$$\text{CNN: } W_{n+1} = \frac{W_n - F + 2P}{S} + 1; \quad H_{n+1} = \frac{H_n - F + 2P}{S} + 1, \quad D_{n+1} = K$$

$$\text{Centroid: } z = \frac{\sum_{i=1}^N x_i \cdot A_i}{\sum_{i=1}^N A_i}$$

## Basic requirements:

	Boundary	Monotonicity	Commutativity	Associativity:
<b>Fuzzy Intersection:</b> (T - norm)	$T(0, 0) = 0$ $T(a, 1) = T(1, a) = a$	$T(a, b) \leq T(c, d)$ if $a \leq c$ & $b \leq d$	$T(a, b) = T(b, a)$	$T(a, T(b, c)) = T(T(a, b), c)$
<b>Fuzzy Union:</b> (S - norm) or (S - Conorm)	$S(1, 1) = 1$ $S(a, 0) = S(0, a) = a$	$S(a, b) \leq S(c, d)$ if $a \leq c$ & $b \leq d$	$S(a, b) = S(b, a)$	$S(a, S(b, c)) = S(S(a, b), c)$

$$\text{Concentration: } CON(A) = A^2 \quad (\text{very})$$

$$\text{Dilation: } DIL(A) = A^{0.5} \quad (\text{more or less} \approx \text{Approximately})$$

$$\begin{aligned} LAE &= \int_a^b |e(t)| dt \\ ITAE &= \int_a^b t |e(t)| dt \\ ISE &= \int_a^b e(t)^2 dt \\ ITSE &= \int_a^b t e(t)^2 dt \end{aligned}$$

Criterion for Comparison (Objective Function == Error) ----->

$$\text{Genetic: } \text{Step} = \frac{\text{range}}{d_{\max}}, \quad d = \frac{r - r_{\min}}{\text{range}} d_{\max}$$

## Particle Swarm Optimization:

$$v_i^{t+1} = v_i^t + \underbrace{c_1 U_i^t (p_i^t - p_i^t)}_{\text{Diversification}} + \underbrace{c_2 U_i^t (g_i^t - p_i^t)}_{\text{Intensification}}$$

$$p_i^{t+1} = p_i^t + v_i^{t+1}$$