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|  | **Universidade de Aveiro**  **Ano 2024** |  | |
| MOHAMED  HADDADI | TÍTULO (MÁXIMO 130 CARACTERES) | | |
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|  | Projeto final apresentado à Universidade de Aveiro para cumprimento dos requisitos necessários à obtenção do grau de Licenciatura em física, realizado sob a orientação científica do Doutor Gil Fernandes, Professor (categoria do orientador) do Departamento de (designação do departamento) da Universidade de Aveiro | |

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| presidente | Prof. Doutor João Antunes da Silva  professor associado da Universidade de Aveiro |
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C 1

**Introduction**

Communication plays a crucial role in the current and future scenario, driving globalization, innovation and collaboration in various sectors, while also facilitating the dissemination of knowledge and education around the world. In the business world, effective communication is the backbone of successful operations, allowing coordination between globally distributed teams and enabling rapid adaptation to market changes. Furthermore, in the educational sphere, communication plays a vital role in delivering educational content to a wide audience, allowing students and teachers to interact efficiently, regardless of physical distance.

In this context, advanced technologies such as fiber optics play a crucial role, offering high bandwidth and low latency to support the growing demand for fast and reliable communication. However, optical fibers face significant challenges, such as chromatic dispersion, which can distort the transmitted signals, and attenuation, which limits the transmission distance. In addition, the non-linearity of optical fibers can introduce distortions in signals at high transmission powers.

To overcome the inherent limitations of optical fibers and advance the capacity and efficiency of communication networks, innovative solutions are being developed that address specific challenges.

Structured light

Structured light involves the deliberate manipulation of light across its various properties. This manipulation can include shaping light in terms of its timing and frequency to generate precise pulses, or more commonly, controlling its spatial characteristics like polarization, intensity, and phase.

The use of structured light is one such solution, where the spatial pattern of light is manipulated to optimize data transmission, and this can involve various techniques such as the following :

Bessel beams, which are being explored because of their ability to maintain their shape during propagation, which minimizes dispersion and preserves signal integrity, thus enabling more reliable communication over long distances. This unique property of Bessel beams makes them particularly valuable in applications that require high-quality transmission in challenging conditions.

In addition, Hermite-Gauss and Laguerre-Gauss beams are being developed to mitigate chromatic dispersion, a phenomenon that causes distortion in transmitted signals at different wavelengths. These beams have been designed with specific intensity distributions that minimize chromatic dispersion, thus improving the efficiency of optical fiber communication and allowing the use of a wider range of wavelengths.

As we know, an electromagnetic wave is capable of carry both energy and momentum, and momentum comprises linear momentum **P** and angular momentum **L**. Addictively, angular momentum has one more component linked to polarization, spin angular momentum (SAM), and another one, orbital angular momentum (OAM), associated with spatial distribution.

Orbital Angular Momentum Multiplexing (OAM) technology, allows multiple channels of information to be transmitted simultaneously on a single optical fiber, significantly increasing data transmission capacity. Light is encoded with different values of orbital angular momentum, that can take on an infinite number of values. Each value of orbital angular momentum can be used to encode a separate channel of information, thus allowing the simultaneous transmission of multiple channels of information on a single optical fiber.

And this results in a significant increase in available bandwidth, enabling faster and more efficient communication.

However, despite its many advantages, OAM technology still faces several technical challenges that need to be overcome. For example, generating and detecting light beams with orbital angular momentum requires complex and precise optical devices. In addition, light beams with orbital angular momentum can be affected by atmospheric turbulence, which can degrade transmission quality.

The aim of this work is to generate beams with OAM, analyze their purity and make any corrections to obtain a beam with OAM of greater purity.

Structure: ...

C 2

**Theoretical foundation:**

2.1 Paraxial Helmoltz equation:

Maxwell's equations are fundamental equations in classical electromagnetism. They describe how electric and magnetic fields interact and propagate in space. The four Maxwell's equations are:

Gauss's Law for Electricity:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1) |

Gauss's Law for Magnetism:

|  |  |  |
| --- | --- | --- |
|  |  | (2.2) |

Faraday's Law of Electromagnetic Induction:

|  |  |  |
| --- | --- | --- |
|  |  | (2.3) |

Ampère's Law with Maxwell's Addition:

|  |  |  |
| --- | --- | --- |
|  |  | (2.4) |

Where:

- is the electric field,

- is the magnetic field,

- is the vacuum permittivity (electric constant),

- is the vacuum permeability (magnetic constant),

And , is the speed of light in vacuum.

These equations govern the behavior of electromagnetic fields in vacuum.

Helmoltz equation:

If we apply the rotational operator to equation 2.1, we get

|  |  |  |
| --- | --- | --- |
|  |  | (2.5) |

And by replacing the second term of the above equation with equation 2.2 we get

|  |  |  |
| --- | --- | --- |
|  |  | (2.6) |

Substituting the identity

|  |  |  |
| --- | --- | --- |
|  |  | (2.7) |

In the equation 2.6, and since we know that is zero (from equation 2.3) we can get the equation of propagation of the electric field:

|  |  |  |
| --- | --- | --- |
|  |  | (2.8) |

The above equation in general can be known as the equation for a quantity that propagates like a wave.

To study monochromatic waves, we know that the wave phasor only depends on the position and provides a sufficient description of the disturbance.

The variable separation process can be used in equation 2.8 to obtain the time independent equation of propagation, also known as the Helmholtz equation.

(2.9)

|  |  |  |
| --- | --- | --- |
|  |  | (2.9) |

eqnm

Where is the wave number.

In that derivation we are assuming a medium with known sources or losses, and since we are studying beams which propagate in air and are monochromatic, these assumptions are still valid.

Paraxial Helmholtz equation:

A paraxial wave is essentially a type of wave where we can simplify the governing equation due to certain assumptions. We can Imagine a wave traveling straight ahead in the z-direction, with its intensity fluctuating slowly compared to its overall strength. In simpler terms, it's like a gentle wave that doesn't change too rapidly over short distances.

So, when we're dealing with such waves, we can approximate the equation that describes them. Picture a flat, single-colored wave with a certain height (amplitude) and a specific length (wavelength) λ. Now, if we consider that the change is smaller than U itself, in a distance , it may be asserted that:

(2.10)

|  |  |  |
| --- | --- | --- |
|  |  | (2.10) |

And we know that , so

(2.11)

|  |  |  |
| --- | --- | --- |
|  | . | (2.11) |

To complete that approximation, we need to assume that the derivative of U must be itself a slow varying function of z such that:

(2.12)

|  |  |  |
| --- | --- | --- |
|  | , | (2.12) |

So, consequently we have that:

|  |  |  |
| --- | --- | --- |
|  |  | (2.13) |

When we plug the expression for the paraxial wave envelope into equation 2.9 and disregard the smaller terms, we end up with what’s called the paraxial Helmholtz equation.

|  |  |  |
| --- | --- | --- |
|  |  | (2.14) |

Where is the transverse Laplacian operator.

In simpler terms, it’s like swapping in our simplified wave description a bigger equation and ignoring the less significant bits, what we are left with is a more focused equation that describes the behavior of paraxial eaves specifically.

changing the boundary conditions, we arrived at several solutions for this equation, we will focus on the most notable ones in this work.

Beams

Gaussian beams

The gaussian beam can be interpreted as a shifted version of the paraboloidal witch is given by:

(2.15)

|  |  |  |
| --- | --- | --- |
|  |  | (2.15) |

We can confirm that the above equation satisfies both equation 2.9 and 2.14 due to its small variation across distance comparable to its wavelength.

When we replace z in equation 2.15 by , where represents a simple shift in position, and represent the same wave but centered in .

It remains a solution of the Helmholtz equation even if ξ is complex. If that is the case, the solution acquires very different properties and in the particular case that ξ is a pure imaginary, we get the complex envelope of the Gaussian beam,

(2.16)

|  |  |  |
| --- | --- | --- |
|  |  | (2.16) |

Where is the Rayleigh range and is the cartesian distance.

(2.17)

|  |  |  |
| --- | --- | --- |
|  |  | (2.17) |

and as

|  |  |  |
| --- | --- | --- |
|  |  | (2.18) |

|  |  |  |
| --- | --- | --- |
|  |  | (2.19) |

Combining equations 2.18, 2.19 and equation 2.16, we can write equation 2.16 as

(2.20)

|  |  |  |
| --- | --- | --- |
|  |  | (2.20) |

And can be simplified to

|  |  |  |
| --- | --- | --- |
|  |  | (2.21) |

Hermite-Gaussian beams:

As we know, the Gaussian beam is not the only beam-like solution of the paraxial Helmholtz equation (2.14),

There are solutions that have non-Gaussian intensity distributions but share the same wave fronts as gaussian beams, this means they have the same curvature of wavefronts as gaussian beams. One example of non-gaussian beams with gaussian wavefronts are Hermite-gaussian beams. They are solutions to the paraxial Helmholtz

equation in the cartesian coordinate system. These beams have intensity distributions that exhibit a petaloid structure, meaning they have lobes or petals in their intensity profile.

Laguerre-Gaussian beams:

In the other hand, there is another famous solution to the paraxial helm eq, an aproximate solution in the cylindrical coordinate system. They have different intensity distribution compared to hermite-gaussian beams, with concentric ring instead of petals.

Bessel beams:

When searching for beam-like waves, we often seek waves with planar wavefronts but nonuniform intensity distributions in the transverse plane. This means that the phase of the wave remains constant along a plane (the wavefront), but the intensity varies across the cross-section of the beam.

One way to achieve this is by considering a wave with a complex amplitude 𝑈(𝑟), where 𝑟 represents the position vector in the transverse plane. In other words, the wave is described by a complex-valued function that varies across the transverse plane.

|  |  |  |
| --- | --- | --- |
|  |  | (2.22) |

In order fot the wave described by the complex amplitude 2.22 satisfy the Helmholtz equation 2.9, the quantity A(x,y) must satisfy the condition:

|  |  |  |
| --- | --- | --- |
|  |  | (2.23) |

Where and is the transverse Laplacian operator.

The quation 2.23 is known as the two-dimensional Helmholtz equation, may be solver by employing the method of separation of variables using polar coordinates:

|  |  |  |
| --- | --- | --- |
|  |  | (2.24) |

In simpler terms, the conditions described ensure that the wave has planar wavefronts, meaning that the wavefronts (the surfaces where the phase of the wave is constant) are all flat and parallel to the z-axis. This implies that the rays of the wave (the lines perpendicular to the wavefronts that indicate the direction of energy propagation) are also parallel to the z-axis.

Additionally, the intensity distribution of the wave is circularly symmetric, meaning that it looks the same when viewed from any direction around the z-axis and varies with .Consequently, the intensity distribution does not change as the wave propagates along the z-axis, meaning there is no spread of the optical power in the z-direction.

This particular type of wave, with planar wavefronts and circularly symmetric intensity distribution that remains constant along the z-axis, is known as a Bessel beam.

A black and red squares

Description automatically generated

|  |  |  |
| --- | --- | --- |
|  |  | (2.25) |

sss

**Sketch**:

𝑘:

𝑘𝑟 2 + 𝑘𝑧 2 = 𝑘2 = 4𝜋2 𝜆2 (5)

Because the Bessel beams have infinitely extended light field structure, they are only ideal theoretical models and cannot really exist. In practice, the Bessel-Gaussian beams are generally used as the approximation of the Bessel beams [44]. The Bessel-Gaussian beams have the non-diffraction characteristic similar to that of the Bessel beams in the finite propagation distance. When beyond the maximum propagation distance, the Bessel-Gaussian beams will no longer exist [45,46]. The complex amplitude of l order Bessel-Gaussian beams can be expressed as 𝐵𝐺𝑙(𝑟, 𝜑, 𝑧) = 𝐴𝑙𝐽𝑙 ( 𝑘𝑟𝑟 ) 𝑒𝑥𝑝( 𝑖𝑘𝑧𝑧 ) 𝑒𝑥𝑝(𝑖𝑙𝜑)𝑒𝑥𝑝( − 𝑟2 𝜔0 2 ) (6) In the formula, 𝜔0 is the limited aperture size and z represents the propagation direction of the beam. Formula (6) holds only at the initial light source plane (z = 0). When the off-axis Bessel-Gaussian beam propagates for a certain distance, according to the Fresnel diffraction integral formula, the expression of the light field at the observation surface (the plane after the propagation distance z) is [47–49] 𝑬(𝒙, 𝒚, 𝒛) = 1 𝒊𝝀𝒛 exp (𝒊𝒌𝒛) ∫ ∫ 𝑬 ( 𝒙0, 𝒚0, 0 ) exp [ 𝒊𝒌 2𝒙 ( 𝒙 − 𝒙0 )2 + 𝒊𝒌 2𝒙 ( 𝒚 − 𝒚0 )2 ] 𝒅𝒙0𝒅𝒚0

It can be seen that compared with the expression of complex amplitude of the Bessel beams, this expression only has more real term 𝑒𝑥𝑝(− 𝑟2 𝜔0 2 ) than the ideal Bessel beams, indicating that the BesselGaussian beams have the same phase structure as the ideal Bessel beams. When 𝑙 ≠ 0, each photon contained in the Bessel-Gaussian beams carries OAM. Fig. 4 shows the light field distribution of different orders of Bessel-Gaussian beams. The intensity of the Bessel-Gaussian beams can be obtained from the complex amplitude of them: 𝐼𝐵𝐺 = | |𝐵𝐺𝑙 | | 2 ∝ 𝐽𝑙 2( 𝑘𝑟𝑟 ) 𝑒𝑥𝑝( − 2𝑟2 𝜔0 2 ) (8) For a beam of Bessel-Gaussian beams, both 𝑘𝑟 and 𝜔0 are fixed values. Bessel-Gaussian beams only exist in limited propagation distance, which is different from the existence of Bessel beams in the whole space. The reasons for these differences can be explained by the interference field [50–52]. The Bessel beams are superimposed by many plane wavelets with equal amplitude and the direction of the wave vector is at 𝛽 angle to the optical axis. The wavefronts of these plane wavelets continue infinitely and have no aperture. For Bessel-Gaussian beams, although these plane wavelets are also superimposed by these plane wavelets, these plane wavelets have aperture limitations and do not have an infinitely extended wave surface. This makes the BesselGaussian light field range related to the aperture size and diffraction angle 𝛽 [53,54].

ref

[Exjobbsframsida (arxiv.org)](https://arxiv.org/ftp/arxiv/papers/0905/0905.0190.pdf) paraxial eq, gaussian beam, maybe L\_G to

[Fundamentals of Photonics (ysu.am)](http://lib.ysu.am/disciplines_bk/1e50d8144d6e0c3ffea3ae655684c626.pdf) Helmholtz

L-G, H-G and Bessel

[OAM beam generation in space and its applications: A review (sciencedirectassets.com)](https://pdf.sciencedirectassets.com/271471/1-s2.0-S0143816621X00127/1-s2.0-S0143816621003924/main.pdf?X-Amz-Security-Token=IQoJb3JpZ2luX2VjEIT%2F%2F%2F%2F%2F%2F%2F%2F%2F%2FwEaCXVzLWVhc3QtMSJHMEUCIHd%2F2kptGxGX9OueAxLOgRu3xWKa2DbXVfUljM3DlS%2FmAiEA%2B0WweXyDdAcrPgh0Gr%2BHxbWFYvT9JRpQh3tCBy1GIjcqvAUIvf%2F%2F%2F%2F%2F%2F%2F%2F%2F%2FARAFGgwwNTkwMDM1NDY4NjUiDMD4KMqq91GhVFb7UCqQBbcJ%2B1AZEF%2BY%2BD5cdA6oZYYxJC%2BtvwjSx8xhdT9fWEPiW2b7jHIM7zlCC2LwiXyzsFaFmzccRGABkfS3qZZfC2mWrG422qzf6cvslZhpV0QEz5U6eiGqzmYVqMIGIeLXw%2BxdPhv37Kqgh1BBswjkkcSrR9tBruOd4UIUgmlBwd5pdr4c%2FNp14Ih5%2BoKYB2%2BfcpwlPXMiwQBeHiKYUW2VHrLwqrd9OCVytzudbAoTtBgs9g2%2Fqz6VVKHMyWlYmG2nnzQg%2FA%2B2wzSyx%2BVzbxXwxZPohrtjRBw0QyRYKE%2F7EcaF4YOnPhtIiqzKRnaelphWynyRn%2Bb3CG3%2F%2F5BmlZx6sHfI5O6UOk0qYQxYyMlBk8B0YeDAFss81%2Bm7zc4gQ99UIb6Dw7D1BmjnWe8NKT3oF%2FEznhz5VxCyYmaKOgCKAgQN0Q46uc3ztEQenskH03AmhNcA3r4muKuw3sZBuiQ11vdmlTTsBG0Rg5OZdzhAz92nbrQp%2BiI06xqFlCxFyjTsw5JfRTzUsE8dmF1u645Zdk3MyNYtLaVp%2FzE5SnuuAzWz4HvxVEu8pCDfZxQeORDrVYSPuS3c0mnrXMzeWZNn862xXsGc%2F84PlptZaEZ2hac3oy%2BHYC%2FA%2BboKbFsx2YwLn2%2Bf174GNlGziqzankUs%2FlWMGH7q5RINiXc4We%2FNnsr1ZIOKwyQR0o2X9MASXV%2BaOycqmmTWKAA3hf6U1TUdFzJ04Np5yU%2FPX6nbAo1k20WfwDvA1Nbk3ddNkOilNnGqdDSyNcPin%2FKc1KcVFF66ur1iy3y9Aj3ErhuclSb00W5SIIwIpM4IHgd4sMyahHMY0VQRRbTQd2uCMEBG9VHnaQ70vzUbBJEgiUWDl%2Fx5RUV3MJfK%2BbAGOrEBKdaMS8Ja4ifeHoDyaipb36fcr5LFahaVLBFt5DtCXA%2BHHhBzME04KwdPrKOfM9aVguhgXWyE6C0lLN10Na1HwFWkV72NUeSIbWerkSBLP332AvJYwkYWQuyVr4%2B1cc202In%2BrWm5xyoN9XspU1bVc0mpmMuZ11UqTegLiIOqudonU6Q%2FLTa2ps7YFCBkb%2BqYOFMZqGD5eB6o7G8hN3dMStcWht4rNnmjQqemnFKY9Elv&X-Amz-Algorithm=AWS4-HMAC-SHA256&X-Amz-Date=20240416T120852Z&X-Amz-SignedHeaders=host&X-Amz-Expires=299&X-Amz-Credential=ASIAQ3PHCVTYT3TX7T7W%2F20240416%2Fus-east-1%2Fs3%2Faws4_request&X-Amz-Signature=b2486495ed6621fc57e8057fd35e1636015df0173ba6c6fa08bd8e226f2e23bd&hash=0451458168a4426d37e9478744562bc5e1e246aa4f9aac0a735fb17cf891cbe4&host=68042c943591013ac2b2430a89b270f6af2c76d8dfd086a07176afe7c76c2c61&pii=S0143816621003924&tid=spdf-186692b5-b4ef-4ab1-95d7-2f4de484f659&sid=b136733e7b9c5843697b64e050a615b4)