Applied Mathematical Modelling 35 (2011) 1322–1330

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Multi-criteria decision making with fuzzy linguistic preference relations

Ying-Hsiu Chen [a], Tien-Chin Wang [b,*], Chao-Yen Wu [c]

[a] Department of Information Engineering, I-Shou University, Kaohsiung 840, Taiwan
[b] Department of International Business, National Kaohsiung University of Applied Sciences, Kaohsiung 807, Taiwan
[c] Department of Information Management, I-Shou University, Kaohsiung 840, Taiwan

ARTICLE INFO

Article history:
Received 31 July 2009
Received in revised form 19 August 2010
Accepted 1 September 2010
Available online 24 September 2010

Keywords:
Fuzzy LinPreRa
Fuzzy linguistic preference relations
AHP
Extent analysis method
Consistency
Decision analysis

ABSTRACT

Although the analytic hierarchy process (AHP) and the extent analysis method (EAM) of fuzzy AHP are extensively adopted in diverse fields, inconsistency increases as hierarchies of criteria or alternatives increase because AHP and EAM require rather complicated pairwise comparisons amongst elements (attributes or alternatives). Additionally, decision makers normally find that assigning linguistic variables to judgments is simpler and more intuitive than to fixed value judgments. Hence, Wang and Chen proposed fuzzy linguistic preference relations (Fuzzy LinPreRa) to address the above problem. This study adopts Fuzzy LinPreRa to re-examine three numerical examples. The re-examination is intended to compare our results with those obtained in earlier works and to demonstrate the advantages of Fuzzy LinPreRa. This study demonstrates that, in addition to reducing the number of pairwise comparisons, Fuzzy LinPreRa also increases decision making efficiency and accuracy.

1. Introduction

Multi-criteria decision making (MCDM) methods have been proposed in recent decades for assisting decision making with multiple objectives. MCDM involves determining the most suitable optimal alternative, in the sense that, when the problem involves multiple conflicting criteria, there are several Pareto-optimal solutions, but only one solution (the preferred one) should be selected. Pairwise comparison is often used in the decision making process. By using pairwise comparisons, judges are not required to explicitly define a measurement scale for each attribute [1]. Since pairwise comparison values are the judgments obtained from an appropriate semantic scale, in practice the decision-maker(s) usually give some or all pair-to-pair comparison values with an uncertainty degree rather than precise ratings. Hence, pairwise comparisons provide a flexible and realistic way to accommodate real-life data. One common MCDM method is analytic hierarchy process (AHP), which was proposed by Saaty [2]. The main advantage of AHP is its systematic organization of tangible and intangible factors, which provides a structured, yet relatively simple solution to decision-making problems [3]. The AHP has been applied in many different domains, including project management [4], enterprise resource planning (ERP) system selection [5], risk assessment [6] and knowledge management tools evaluation [7].

However, due to the complexity and uncertainty of real-world decision making problems and the inherent subjectivity of human judgment, exact judgments are often unrealistic or infeasible. Decision makers often find it more natural or easier to assign linguistic variables to judgments rather than to make fixed value judgments. It is more appropriate to present data using fuzzy numbers instead of crisp numbers. Therefore, alternative methods have been proposed to improve AHP. These

* Corresponding author. Address: Department of International Business, National Kaohsiung University of Applied Sciences, No. 415, Jiangong Rd., Sanmin Dist., Kaohsiung City 807, Taiwan. Tel.: +886 7 3814526x6126.
E-mail addresses: ivychen31@gmail.com (Y.-H. Chen), tcwang@cc.kuas.edu.tw, dr.deor@gmail.com (T.-C. Wang), cywu@isu.edu.tw (C.-Y. Wu).

0307-904X/$ - see front matter
doi:10.1016/j.apm.2010.09.009

methods are systematic approaches to solving the alternative selection and justification problem by applying fuzzy set theory and hierarchical structure analysis.

Early work in fuzzy AHP by van Laarhoven and Pedrycz [8] compared fuzzy ratios described by triangular membership functions. Buckley [9] investigated the use of fuzzy weights and fuzzy utility to extend AHP by the geometric mean method to derive the fuzzy weight. Chang [10] introduced a new approach using triangular fuzzy numbers for a pairwise comparison scale in fuzzy AHP. Extent analysis method (EAM) has also been used for synthetic extent values of pairwise comparisons. Cheng [11] proposed a new algorithm for evaluating naval tactical missile systems using fuzzy AHP based on grade value of membership function. Cheng et al. [12] proposed a new method for evaluating weapons systems by AHP based on linguistic variable weight. Zhu et al. [13] discussed EAM and applications of fuzzy AHP. Leung and Cao [14] proposed a fuzzy consistency definition with tolerance deviation.

Among the above approaches, EAM has been employed in many applications due to its computational simplicity. For example, Bozdag et al. [15] used the EAM to evaluate computer integrated manufacturing alternatives. Kahraman et al. [16,17] also used this approach for selecting facility locations and evaluating catering firms in Turkey. Bozbura and Beskese [18] also applied the Chang EAM to improve the quality of prioritization of organizational capital measurement indicators under uncertain conditions. In addition to those mentioned above, numerous other studies [19–26] have applied this method.

Although AHP and EAM are employed in diverse fields, inconsistency increases as hierarchies of criteria or alternatives increase. Decision makers often have difficulty ensuring a consistent pairwise comparison between voluminous decisions because consistency ratios (CRs) are produced after the evaluation process and global acceptance criteria are limited. To address this dilemma, Herrera-Viedma et al. [27] developed consistent fuzzy preference relations to avoid the inconsistent solutions in the decision-making processes. Moreover, using AHP requires $n(n-1)/2$ pairwise comparisons, whereas consistent fuzzy preference relations require only $\boldsymbol{n}-1$ comparisons. Wang and Chen [28–30], Chen [31] developed a method that adopts fuzzy linguistic assessment variables rather than crisp values to construct fuzzy linguistic preference relations (Fuzzy LinPreRa) matrices based on consistent fuzzy preference relations [27]. Their method assures consistency and only requires $\boldsymbol{n}-1$ judgments from a set of n elements. In this study, Fuzzy LinPreRa is applied to re-examine three numerical examples investigated by Kahraman et al. [17], Erensal et al. [21] and Bozbura and Beskese [18] in order to compare our results with those of earlier studies and demonstrate the advantage of Fuzzy LinPreRa.

The rest of this paper is organized as follows. Section 2 briefly reviews the EAM on fuzzy AHP, consistent fuzzy preference relations and Fuzzy LinPreRa. Section 3 presents three numerical studies to demonstrate applications of Fuzzy LinPreRa. Finally, discussion and concluding remarks are presented in Sections 4 and 5, respectively.

2. Methodology

This section presents three different MCDM methods. The first is EAM on fuzzy AHP proposed by Chang in 1996. The second is consistent fuzzy preference relations developed by Herrera-Viedma et al. [27]. Finally, a fuzzy linguistic preference relation is introduced [28–31].

2.1. The extent analysis method (EAM) on fuzzy AHP

Let $X = \{x_1,\ldots,x_n, n \geqslant 2\}$ be an object set, and $U = \{g_1,g_2,\ldots,g_m\}$ be a goal set. According to the Chang [10] extent analysis, each object is considered separately, and for each object, the analysis is carried out for each of the possible goals, g_i. Therefore, m extent analysis values for each object can be obtained as follows:

$$\widetilde{M}^1_{gi}, \widetilde{M}^2_{gi}, \ldots, \widetilde{M}^m_{gi}, \quad i = 1,2,\ldots,n, \tag{1}$$

where $\widetilde{M}^j_{gi}$ $(j = 1,2,\ldots,m)$ are all triangular fuzzy numbers (TFNs), and parameters l, m and u are the least possible value, the most possible value, and the largest possible value respectively. A TFN is represented as (l,m,u).

The EAM step can be summarized as follows:

Step 1: The value of fuzzy synthetic extent with respect to the ith object is defined as

$$S_i \approx \sum_{j=1}^{m} \widetilde{M}^j_{gi} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \widetilde{M}^j_{gi}\right]^{-1}, \tag{2}$$

where $\otimes$ denotes the extended multiplication of two fuzzy numbers.

Step 2: The degree of possibility of $\widetilde{M}_2 = (l_2,m_2,u_2) \geqslant \widetilde{M}_1 = (l_1,m_1,u_1)$ is defined as

$$V(\widetilde{M}_2 \geqslant \widetilde{M}_1) = \sup_{y \geqslant x}\left[\min(\mu_{\widetilde{M}_1}(x), \mu_{\widetilde{M}_2}(y))\right] \tag{3}$$

and can be equivalently expressed as follows:

$$V(\widetilde{M}_2 \geqslant \widetilde{M}_1) = hgt(\widetilde{M}_1 \cap \widetilde{M}_2) = \mu_{\widetilde{M}_2}(d) = \begin{cases} 1 & \text{if } m_2 \geqslant m_1, \\ 0 & \text{if } l_1 \geqslant u_2, \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise}, \end{cases} \tag{4}$$

where d is the ordinate of the highest intersection point D between $\mu_{\widetilde{M}_1}$ and $\mu_{\widetilde{M}_2}$ (see Fig. 1). To compare $\widetilde{M}_1$ and $\widetilde{M}_2$, the values of both $V(\widetilde{M}_1 \geqslant \widetilde{M}_2)$ and $V(\widetilde{M}_2 \geqslant \widetilde{M}_1)$ must be determined.

Step 3: The possibility of a convex fuzzy number $\widetilde{M}$ exceeding k convex fuzzy number $\widetilde{M}_i (i = 1, 2, \ldots, k)$ can be defined as

$$\begin{aligned} & V(\widetilde{M} \geqslant \widetilde{M}_1, \widetilde{M}_2, \ldots, \widetilde{M}_k) \\ & = V[(\widetilde{M} \geqslant \widetilde{M}_1), \text{ and } (\widetilde{M} \geqslant \widetilde{M}_2), \text{ and } \ldots, \text{and } (\widetilde{M} \geqslant \widetilde{M}_k)] \\ & = \min_i V(\widetilde{M} \geqslant \widetilde{M}_i), \quad i = 1, 2, \ldots, k. \end{aligned} \tag{5}$$

Assume that

$$d'(A_i) = \min_i V(S_i \geqslant S_k). \tag{6}$$

For $k = 1, 2, \ldots, n$, $n \neq i$. The weight vector is then given by

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T, \tag{7}$$

where $A_i (i = 1, 2, \ldots, n)$ are n elements.

Step 4: via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), \ldots, d(A_n))^T, \tag{8}$$

where W is a non-fuzzy number.

2.2. Consistent fuzzy preference relations

Consistent fuzzy preference relations, was proposed by Herrera-Viedma et al. [27], to avoid inconsistent solutions in the decision-making processes. This method enables a decision maker to determine values for a set of criteria and a set of alternatives. The value represents the degree of preference for the first alternative over the second alternative. Two major preference relations are (1) multiplicative preference relations and (2) fuzzy preference relations [32–34].

(1) Multiplicative preference relations: The preference of a decision maker for a set of alternatives X is denoted by a positive preference relation matrix $A \subset X \times X$, $A = (a_{ij})_{n \times n}$, $a_{ij} \in [1/9, 9]$ where a_{ij} is the ratio of degree of preference for alternative x_i over x_j. As $a_{ij} = 1$ indicates the indifference between x_i and x_j, $a_{ij} = 9$ indicates that the x_i is highly preferable to x_j. The A is assumed to be a multiplicative reciprocal given by $a_{ij} \cdot a_{ji} = 1$ $\forall i, j \in \{1, \ldots, n\}$.

(2) Fuzzy preference relations: The preference of a decision maker for a set of alternatives X is denoted by a positive preference relation matrix $P \subset X \times X$ with membership function μ_p: $X \times X \to [0, 1]$ where $\mu_p\,(x_i, x_j) = p_{ij}$ indicates the ratio of the preference intensity of alternative x_i to that of x_j. If $p_{ij} = \frac{1}{2}$ implies there is no difference between x_i and $x_j (x_i \sim x_j)$, $p_{ij} > \frac{1}{2}$ implies x_i is preferred to $x_j (x_i > x_j)$, $p_{ij} = 1$ indicates that x_i is absolutely preferred to x_j and $p_{ij} = 0$ indicates that x_j is absolutely preferred to x_i. The P is assumed to be an additive reciprocal given by $p_{ij} + p_{ji} = 1$ $\forall i, j \in \{1, \ldots, n\}$.

Fedrizzi [35] proved that, by means of function g, a "multiplicative" formulation of a problem can be transformed into an "additive" one. For a set of alternatives $X = \{x_1, \ldots, x_n\}$ associated with a reciprocal multiplicative preference relation $A = (a_{ij})$

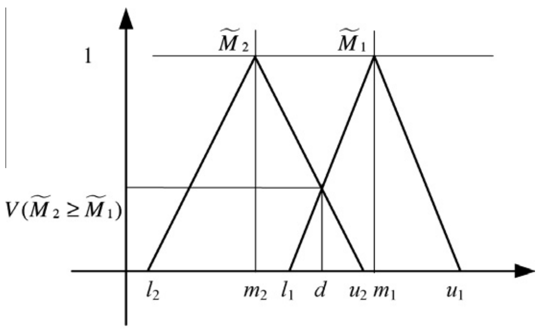

Fig. 1. The intersection of $\widetilde{M}_1$ and $\widetilde{M}_2$.

and $a_{ij} \in [1/9,9]$, transformation function g can be used as in Eq. (9) to find the corresponding reciprocal additive fuzzy preference relation $P = (p_{ij})$ and $p_{ij} \in [0,1]$

$$p_{ij} = g(a_{ij}) = 1/2 \cdot (1 + \log_9 a_{ij}), \tag{9}$$

$\log_9 a_{ij}$ considered because a_{ij} is between 1/9 and 9. If a_{ij} is between 1/7 and 7, then $\log_7 a_{ij}$ is used [34].

Herrera-Viedma et al. [27] proved that, for a reciprocal additive fuzzy preference relation $P = (p_{ij})$, the following statements are equivalent:

$$p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \quad \forall i < j < k, \tag{10}$$

$$p_{i(i+1)} + p_{(i+1)(i+2)} + \cdots + p_{(j-i)j} + p_{ji} = \frac{j-i+1}{2} \quad \forall i < j. \tag{11}$$

According to above Eq. (11), therefore, it can be deduced that

$$p_{ji} = \frac{j-i+1}{2} - p_{i(i+1)} - p_{(i+1)(i+2)} - \cdots - p_{(j-1)j} \tag{12}$$

and, based on the additive reciprocal,

$$p_{ij} + p_{ji} = 1 \ \forall i,\ j \in \{1, \ldots, n\}. \tag{13}$$

The steps of Fuzzy PreRa as follows:

Step 1. Compute the set of preference values B as

$$B = \{p_{ij}, i < j \wedge p_{ij} \notin \{p_{12}, p_{23}, \ldots, p_{n-1n}\}\} \tag{14}$$

Step 2. Find P

$$P = \{p_{12}, p_{23}, \ldots, p_{n-1n}\} \cup B \cup \{1 - p_{12}, 1 - p_{23}, \ldots, 1 - p_{n-1n}\} \cup \neg B. \tag{15}$$

Step 3. The consistent fuzzy preference relation P' is obtained as $P' = f(P)$ such that f: $[-a, 1+a] \to [0,1]$

$$f(x) = \frac{x+a}{1+2a} \tag{16}$$

The concept of this method is that, for n attributes $X = \{x_1, \ldots, x_n, n \geqslant 2\}$, we can obtain the pairwise preference relation data $\{p_{12}, p_{23}, \ldots, p_{n-1n}\}$ comparing $n-1$ and constructing a consistent reciprocal fuzzy preference relations P'. This method is similar to that for traditional AHP characteristics, which is preference relation satisfied transitivity property.

2.3. Fuzzy linguistic preference relations (Fuzzy LinPreRa)

Wang and Chen [28–30] developed a method using fuzzy linguistic assessments variables to construct fuzzy linguistic preference relations (Fuzzy LinPreRa) matrices based on consistent fuzzy preference relations. The fuzzy linguistic assessments variables are given by $\widetilde{P} = (\tilde{p}_{ij}) = \left(p_{ij}^L, p_{ij}^M, p_{ij}^R\right)$ where p_{ij}^L and p_{ij}^R denote the lower and upper bounds of the fuzzy number $\widetilde{P}$, respectively and p_{ij}^M denotes the median value, rather than the crisp values $P = (p_{ij})$. The Fuzzy LinPreRa is adopted to improve the consistency of fuzzy AHP. Some important propositions given below have been presented [28–31].

Proposition 2.3.1. *Consider a set of alternatives $X = \{x_1, \ldots, x_n\}$ associated with a fuzzy reciprocal multiplicative preference relation $\widetilde{A}_{ij} = (\tilde{a}_{ij})$ for $\tilde{a}_i \in [1/9,9]$, and the corresponding fuzzy reciprocal linguistic preference relation $\widetilde{P} = (\tilde{p}_{ij})$ with $\tilde{p}_{ij} \in [0,1]$. The following statements are equivalent:*

(1) $p_{ij}^L + p_{ji}^R = 1 \quad \forall i,j \in \{1, \ldots, n\}$.
(2) $p_{ij}^M + p_{ji}^M = 1 \quad \forall i,j \in \{1, \ldots, n\}$.
(3) $p_{ij}^R + p_{ji}^L = 1 \quad \forall i,j \in \{1, \ldots, n\}$.

Proposition 2.3.2. *For a consistent reciprocal fuzzy linguistic preference relation $\widetilde{P} = (\tilde{p}_{ij}) = (p_{ij}^L, p_{ij}^M, p_{ij}^R)$ the following statements are equivalent:*

(1) $p_{ij}^L + p_{jk}^L + p_{ki}^R = \frac{3}{2} \ \forall i < j < k$,
(2) $p_{ij}^M + p_{jk}^M + p_{ki}^M = \frac{3}{2} \ \forall i < j < k$,
(3) $p_{ij}^R + p_{jk}^R + p_{ki}^L = \frac{3}{2} \ \forall i < j < k$,

(4) $p^L_{i(i+1)} + p^L_{(i+1)(i+2)} + \cdots + p^L_{(j-1)j} + p^R_{ji} = \frac{j-i+1}{2}\ \forall i < j,$
(5) $p^M_{i(i+1)} + p^M_{(i+1)(i+2)} + \cdots + p^M_{(j-1)j} + p^M_{ji} = \frac{j-i+1}{2}\ \forall i < j,$
(6) $p^R_{i(i+1)} + p^R_{(i+1)(i+2)} + \cdots + p^R_{(j-1)j} + p^L_{ji} = \frac{j-i+1}{2}\ \forall i < j.$

Notably, a decision matrix with entries that are not in the interval [0,1] but are in the interval $[-c, 1+c]$, given $c > 0$, can transform the obtained fuzzy numbers using the following transformation function to preserves reciprocity and additive consistency f: $[-c, 1+c] \to [0,1]$.

$$f(x^L) = \frac{x^L + c}{1+2c}, \quad f(x^M) = \frac{x^M + c}{1+2c}, \quad f(x^R) = \frac{x^R + c}{1+2c}. \tag{17}$$

The procedure of Fuzzy LinPreRa is described as follows:

Step 1. For a decision making problem, let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives. The decision maker compares each pair of alternatives by fuzzy linguistic assessment variable and constructs an incomplete consistent fuzzy linguistic preference relation $\widetilde{P} = (\tilde{p}_{ij})_{n\times n}$ with only $n-1$ judgments $\{p_{12}, p_{23}, \ldots, p_{n-1n}\}$.

Step 2. Adopt the known elements in $\widetilde{P}$ and Propositions 2.3.1 and 2.3.2 to calculate all unknown elements in $\widetilde{P}$ and obtain the corresponding complete consistent fuzzy linguistic preference relations $\bar{\widetilde{P}} = (\bar{\tilde{p}}_{ij})_{n\times n}$.

Step 3. Utilize the linguistic averaging operator

$$\widetilde{A}_i = \frac{\sum_{j=1}^{n} \bar{\tilde{p}}_{ij}}{n} \quad \text{for all } i \tag{18}$$

to compute the averaged $\widetilde{A}_i$ of the ith criterion (alternative) over all other criteria (alternatives). The weight is calculated as

$$\widetilde{W}_i = \widetilde{A}_i \Big/ \sum_{i=1}^{n} \widetilde{A}_i. \tag{19}$$

Step 4. Group integration and defuzzification
The final fuzzy weight values of alternatives are represented in terms of fuzzy numbers. The optimum alternative is determined by constructing a crisp value from the fuzzy number. Hence, defuzzification needs to be performed to arrange the fuzzy numbers for ranking. The fuzzy mean and spread method [36] is adopted to defuzzify and rank the fuzzy numbers. This method ranks fuzzy numbers by means of the probabilities of fuzzy events, assuming that $\widetilde{U}_i$ denotes a TFN (l, m, u) with uniform distribution. Its mean $x(\widetilde{U}_i)$ is defined as

$$x(\widetilde{U}_i) = (l + m + u)/3. \tag{20}$$

Step 5. The optimum alternative is determined from the value of the fuzzy mean $x(\widetilde{U}_i)$. A higher ranking value indicated an alternative with a higher priority.

3. Numerical examples

This section employs three numerical examples to demonstrate the Fuzzy LinPreRa and compares the results with the EAM. The first example is a comparison of catering service companies. The second example is a determination of key capabilities in technology management. The third example is an evaluation of organizational capital measurement indicators.

Example 1. The numerical example investigated by Kahraman et al. [17] is as follows.

A large Turkish textile company wishes to contract with a catering firm. The goal is to select the best catering firm from three alternatives: Durusu (*D*), Mertol (*M*) and Afiyetle (*A*). The three criteria are hygiene (*H*), quality of meal (ingredients) (*QM*), and quality of service (*QS*). The eleven sub-criteria include hygiene of meal (*HM*), hygiene of service personnel (*HSP*), hygiene of service vehicles (*HSV*), variety of meals (*VM*), complementary meals in a day (*CoM*), calorie of meal (*CaM*), taste of meal (*TM*), service time (*ST*), communication on phone (*CP*), problem solving ability (*PS*) and behavior of service personnel (*BSP*). Their linguistic scale was defined as follows: equal (1,1,1), weak (2/3,1,3/2), fairly strong (3/2,2,5/2), very strong (5/2,3,7/2) and absolute (7/2,4,9/2). The fuzzy evaluation matrix relevant to the goal is given in Table 1.

Table 1
Fuzzy comparison matrix with respect to the goal (EAM).

Criteria	*H*	*QM*	*QS*	Weight
H	(1,1,1)	(3/2,2,5/2)	(2/3,1,3/2)	0.43
QM	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	0.37
QS	(2/3,1,3/2)	(2/5,1/2,2/3)	(1,1,1)	0.20

Table 2
Fuzzy linguistic assessment variables.

Linguistic term	Triangle fuzzy numbers	Triangular fuzzy reciprocal scale
Equal (1, 1, 1)	(0.3, 0.5, 0.7)	
Weak (2/3, 1, 3/2)	(0.5, 0.65, 0.8)	(0.2, 0.35, 0.5)
Fairly strong (3/2, 2, 5/2)	(0.6, 0.8, 1)	(0, 0.2, 0.4)
Very strong (5/2, 3, 7/2)	(0.7, 0.9, 1)	(0, 0.1, 0.3)
Absolute (7/2, 4, 9/2)	(0.8, 1, 1)	(0, 0, 0.2)

Table 3
Pairwise comparison of three criteria with respect to the goal.

	H	*QM*	*QS*
H	x	Fairly strong	
QM		x	Fairly strong
QS			x

Table 4
Decision matrix for fuzzy linguistic preference relation with three criteria.

	H	*QM*	*QS*
H	(0.5, 0.5, 0.5)	(0.6, 0.8, 1.0)	(0.7, 1.1, 1.5)
QM	(0.0, 0.2, 0.4)	(0.5, 0.5, 0.5)	(0.6, 0.8, 1.0)
QS	(−0.5, −0.1, 0.3)	(0.0, 0.2, 0.4)	(0.5, 0.5, 0.5)

This problem can be solved by using the Fuzzy LinPreRa [28–31]. The linguistic terms used in those conversion scales and their corresponding representations of fuzzy numbers are given in Table 2 [37].

Table 3 displays the pairwise comparison matrix for the goal. Three criteria were adopted, and only two comparison judgments were needed to construct the fuzzy preference relation decision matrix.

According to Propositions 2.3.1 and 2.3.2, the entire calculation is as follows:

$$p_{31}^{L} = 1.5 - p_{12}^{R} - p_{23}^{R} = 1.5 - 1.0 - 1.0 = -0.5,$$
$$p_{31}^{M} = 1.5 - p_{12}^{M} - p_{23}^{M} = 1.5 - 0.8 - 0.8 = -0.1,$$
$$p_{31}^{R} = 1.5 - p_{12}^{L} - p_{23}^{L} = 1.5 - 0.6 - 0.6 = 0.3.$$

Table 4 shows the complete consistent fuzzy linguistic preference relation matrix relevant to main criteria. The matrix has entries that are not included in the interval [0, 1]. Therefore, the transforming function Eq. (17) is applied.

The entire preference relation matrix can be transformed as in Table 5. According to Eqs. (18) and (19), the average $(\widetilde{A}_i)$ in Table 5 is calculated as H (0.55, 0.65, 0.75), QM (0.43, 0.50, 0.57) and QS (0.25, 0.35, 0.45). The weight $(\widetilde{W}_i)$ for each criterion is calculated as H (0.31, 0.43, 0.61), QM (0.25, 0.33, 0.46) and QS (0.14, 0.23, 0.36). Similarly, other decision matrices were also calculated with the same steps. The final fuzzy weight values of alternatives are represented in terms of fuzzy numbers. The optimum alternative is determined by using Eq. (20) to construct a crisp value from the fuzzy number.

In this example, Afiyetle is the selected catering firm. The results of this numerical example are the same as those in the original example. However, in [38], the weights determined by the EAM did not represent the relative importance of decision criteria when an irrational zero weight was assigned to some useful decision criteria or sub-criteria. Clearly, the true corresponding weight can be obtained by Fuzzy LinPreRa.

Example 2. The numerical example investigated by Erensal et al. [21] is as follows.

The management of technology is a vital determinant of long-run success or failure of organization. Erensal et al. [21] proposed a framework to explore the links between competitive advantages, competitive priorities and competencies of a firm in a context of technology management. Their linguistic scale was defined as follows: equal importance (1, 1, 1),

Table 5
Transforming results of the five criteria from Table 4.

	H	*QM*	*QS*
H	(0.5, 0.5, 0.5)	(0.55, 0.65, 0.75)	(0.60, 0.80, 1.00)
QM	(0.25, 0.35, 0.45)	(0.5, 0.5, 0.5)	(0.55, 0.65, 0.75)
QS	(0.00, 0.20, 0.40)	(0.25, 0.35, 0.45)	(0.5, 0.5, 0.5)

Table 6
Pairwise comparison of five criteria with respect to the goal.

	Growth	Profit	Return on investment
Growth	x	(0,0.1,0.3)	
Profit		x	(0,0.1,0.3)
ROI			x

Table 7
Evaluation of key capabilities with respect to competitive priority and competitive advantages (Fuzzy LinPreRa).

	Growth (0.221)		Profit (0.337)		ROI (0.463)	
Cost	(0.09,0.14,0.21)	0.148	(0.13,0.24,0.43)	0.264	(0.12,0.21,0.36)	0.228
Price	(0.12,0.16,0.22)	0.164	(0.15,0.24,0.39)	0.259	(0.07,0.13,0.24)	0.148
Quality	(0.15,0.20,0.26)	0.205	(0.13,0.21,0.34)	0.226	(0.13,0.21,0.32)	0.224
Flexibility	(0.18,0.23,0.31)	0.240	(0.08,0.15,0.27)	0.164	(0.18,0.27,0.41)	0.288
Time	(0.19,0.27,0.36)	0.275	(0.08,0.18,0.34)	0.200	(0.10,0.17,0.32)	0.200

moderate importance (1,3,5), strong importance (3,5,7), very strong importance (5,7,9) and demonstrated importance (7,9,11). This example was also re-examined by Fuzzy LinPreRa. The linguistic terms used in those conversion scales and their corresponding representations of fuzzy numbers are given in Table 2 [37]. Table 6 displays the pairwise comparison matrix for the goal.

Using Fuzzy LinPreRa, the weight $(\widetilde{W}_i)$ for each criterion in Table 6 are as follows: Growth is (0.15,0.20,0.31), Profit is (0.27,0.33,0.41) and ROI is (0.36,0.47,0.56). Similarly, all comparison matrices in their paper were also re-examined using the Fuzzy LinPreRa.

In [21], they indicated that in order to increase the sales growth rate; time, flexibility and quality appear to be more important than cost and price. For the profit of a company, price and cost play a much more important role than other criteria. The most important criteria for the ROI are flexibility, cost and quality.

Table 7 shows the result obtained by the evaluation of key capabilities with respect to competitive priority and competitive advantages and defuzzification using Eq. (20). The calculation result is as same as the original example. Meanwhile, the results of defuzzifying using the maximizing set and minimizing set method [39] yielded the same ranking, but different weights. Thus, the defuzzification method does not affect evaluation results.

Example 3. The numerical example investigated by Bozbura and Beskese [18] is as follows.

Organizational capital is a sub-dimension of the intellectual capital which is the sum of all assets that make the creative ability of the organization possible. A hierarchical model consisting of three main criteria, seven sub-criteria, and ten indicators is proposed. Three main criteria are deployment of the strategic values (*DS*), investments in the technology (*IT*) and flexibility of the organizational structure (*FS*). Seven sub-criteria contain useableness of values in processes (*UV*), fitness of values to daily working environment (*FV*), Reliability (*RE*), ease of use (*EU*), relevance (*RV*), supporting development (*SD*) and innovation (*IN*).

Ten indicators are defined as: implementation rate of new ideas (*IND*1), quick access to information (*IND*2), R&D investment rate per employee (*IND*3), access to all information without any limitation (*IND*4), increasing rate of revenue per employee (*IND*5), updating rate of the databases (*IND*6), MIS contains all information (*IND*7), decreasing rate of cost per revenue (*IND*8), knowledge sharing rate (*IND*9), index of transaction time of the processes (*IND*10).

Their linguistic scale was defined as follows: just equal (1,1,1), equally important (*EI*) (1/2,1,3/2), weakly more important (*WMI*) (1,3/2,2), strongly more important (*SMI*) (3/2,2,5/2), very strongly more important (*VSMI*) (2,5/2,3) and absolutely more important (*AMI*)(5/2,3,7/2). Fig. 2 illustrates the hierarchical structure explained above.

This example was also re-examined by Fuzzy LinPreRa. The Fuzzy LinPreRa requires $n - 1 = 10 - 1 = 9$ comparisons, and consistency is ensured. The ranking is *Ind*.1 > 10 > 2 > 9 > 4 > 3 > 6 > 7 > 5 > 8. The indicator *Implementation rate of new ideas* is the most important indicator for organizational capital measurement. This decision result is the same as that obtained using EAM. However, the overall ranking of priorities in this study differs from Bozbura and Beskese [18]. Further, the correlation between the results in that and the current study is determined by Spearman rank analysis. The Spearman correlation coefficient is 0.7455. We also calculate $t = 3.163 > 1.860$, when $\alpha = .05$, it means the correlation between the results have significant correlation.

4. Discussion

Three numerical examples, including catering service company comparison, key capabilities in technology management determination and organizational capital measurement indicators ranking were re-examined to demonstrate the application of the Fuzzy LinPreRa.

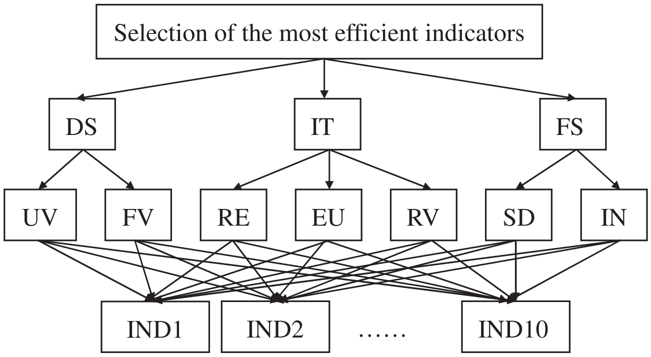

Fig. 2. The hierarchy of the problem.

From the perspective of pairwise comparison times in the various approaches, for ten indicators in Example 3, the EAM needs $\frac{n\times(n-1)}{2}=\frac{10\times9}{2}=45$ pairwise comparisons for each decision matrix, which may cause an inconsistency problem. However, the Fuzzy LinPreRa only requires $n-1 = 10-1 = 9$ comparisons, and consistency is ensured.

Additionally, Wang et al. [38] proposed that EAM is effective for showing to what degree the priority of one decision criterion or alternative is higher than those of all the others in a fuzzy comparison matrix. The method cannot derive priorities from a fuzzy comparison matrix. In three examples, the weights of some decision criteria were zero, which was irrational, and the criteria were excluded from the decision analysis. However, using Fuzzy LinPreRa can avoid such unreasonable conditions.

5. Conclusion

Perfect consistency is difficult to obtain in practice, particularly when measuring preferences on a set with many alternatives. The concept of EAM is applied to solve the fuzzy reciprocal matrix for determining the criteria importance and alternative performance. However, as in traditional AHP, inconsistency increases as hierarchies of criteria or alternatives increase. This study adopts Fuzzy LinPreRa, a new approach for handling fuzzy AHP. This method can avoid inconsistent conditions as the number of criteria (alternative) increase.

Compared to EAM, the Fuzzy LinPreRa introduced here provides greater flexibility for solving MCDM problems with preference information about alternatives and/or attributes. The EAM requires $n(n-1)/2$ pairwise comparisons, but the Fuzzy LinPreRa only requires $n-1$ comparisons and ensures their consistency. The numerical illustrations clearly demonstrate the accuracy and efficiency of this method. The consistency of the fuzzy preference relations provided by decision makers is improved, such that inconsistent solutions in decision making processes are avoided. In conclusion, in addition to reducing the number of pairwise comparison, Fuzzy LinPreRa also enhances decision making efficiency and accuracy.

Acknowledgement

The authors gratefully acknowledge the Editor and anonymous reviewers for their valuable comments and constructive suggestions. This research was supported in part by the National Science Council of the Republic of China under the grant NSC 97-2410-H-151-017 and NSC 98-2410-H-151-006.

References

[1] E.E. Spires, Using the analytic hierarchy process to analyze multiattribute decisions, Multivar. Behav. Res. 26 (1991) 345–361.
[2] T.L. Saaty, The Analytic Hierarchy Process: Planning, Priority Setting, Resources Allocation, McGraw-Hill, London, 1980.
[3] M.J. Skibniewski, L.-C. Chao, Evaluation of advanced construction technology with AHP method, J. Constr. Eng. Manage. 118 (1992) 577–593.
[4] K.M.A.-S. Al-Harbi, Application of the AHP in project management, Int. J. Project Manage. 19 (2001) 19–27.
[5] C.-C. Wei, C.-F. Chien, M.-J.J. Wang, An AHP-based approach to ERP system selection, Int. J. Prod. Econ. 96 (2005) 47–62.
[6] M.-C. Tsai, C.-H. Su, Political risk assessment of five East Asian ports—the viewpoints of global carriers, Mar. Policy 29 (2005) 291–298.
[7] E.W.T. Ngai, E.W.C. Chan, Evaluation of knowledge management tools using AHP, Expert Syst. Appl. 29 (2005) 889–899.
[8] P.J.M. Laarhoven, W. Pedrycz, A fuzzy extension of Saaty's priority theory, Fuzzy Sets Syst. 11 (1983) 229–241.
[9] J.J. Buckley, Fuzzy hierarchical analysis, Fuzzy Sets Syst. 17 (1985) 233–247.
[10] D.-Y. Chang, Applications of the extent analysis method on fuzzy AHP, Eur. J. Oper. Res. 95 (1996) 649–655.
[11] C.-H. Cheng, Evaluating naval tactical missile systems by fuzzy AHP based on the grade value of membership function, Eur. J. Oper. Res. 96 (1997) 343–350.
[12] C.-H. Cheng, K.-L. Yang, C.-L. Hwang, Evaluating attack helicopters by AHP based on linguistic variable weight, Eur. J. Oper. Res. 116 (1999) 423–435.
[13] K.-J. Zhu, Y. Jing, D.-Y. Chang, A discussion on extent analysis method and applications of fuzzy AHP, Eur. J. Oper. Res. 116 (1999) 450–456.
[14] L.C. Leung, D. Cao, On consistency and ranking of alternatives in fuzzy AHP, Eur. J. Oper. Res. 124 (2000) 102–113.
[15] C.E. Bozdag, C. Kahraman, D. Ruan, Fuzzy group decision making for selection among computer integrated manufacturing systems, Comput. Ind. 51 (2003) 13–29.
[16] C. Kahraman, D. Ruan, I. Dogan, Fuzzy group decision-making for facility location selection, Inform. Sci. 157 (2003) 135–153.

[17] C. Kahraman, U. Cebeci, D. Ruan, Multi-attribute comparison of catering service companies using fuzzy AHP: the case of Turkey, Int. J. Prod. Econ. 87 (2004) 171–184.
[18] F.T. Bozbura, A. Beskese, Prioritization of organizational capital measurement indicators using fuzzy AHP, Int. J. Approx. Reason. 44 (2007) 124–147.
[19] F.T. Bozbura, A. Beskese, C. Kahraman, Prioritization of human capital measurement indicators using fuzzy AHP, Expert Syst. Appl. 32 (2007) 1100–1112.
[20] F.T.S. Chan, N. Kumar, Global supplier development considering risk factors using fuzzy extended AHP-based approach, Omega 35 (2007) 417–431.
[21] Y.C. Erensal, T. Öncan, M.L. Demircan, Determining key capabilities in technology management using fuzzy analytic hierarchy process: a case study of Turkey, Inform. Sci. 176 (2006) 2755–2770.
[22] T. Ertay, G. Büyüközkan, C. Kahraman, D. Ruan, Quality function deployment implementation based on analytic network process with linguistic data: an application in automotive industry, J. Intell. Fuzzy Syst. 16 (2005) 221–232.
[23] C. Kahraman, T. Ertay, G. Büyüközkan, A fuzzy optimization model for QFD planning process using analytic network approach, Eur. J. Oper. Res. 171 (2006) 390–411.
[24] O. Kulak, C. Kahraman, Fuzzy multi-attribute selection among transportation companies using axiomatic design and analytic hierarchy process, Inform. Sci. 170 (2005) 191–210.
[25] Y.C. Tang, M.J. Beynon, Application and development of a fuzzy analytic hierarchy process within a capital investment study, J. Econ. Manage. 1 (2005) 207–230.
[26] E. Tolga, M.L. Demircan, C. Kahraman, Operating system selection using fuzzy replacement analysis and analytic hierarchy process, Int. J. Prod. Econ. 97 (2005) 89–117.
[27] E. Herrera-Viedma, F. Herrera, F. Chiclana, M. Luque, Some issues on consistency of fuzzy preference relations, Eur. J. Oper. Res. 154 (2004) 98–109.
[28] T.-C. Wang, Y.-H. Chen, A new method on decision-making using fuzzy linguistic assessment variables and fuzzy preference relations, in: The 9th World Multi-Conference on Systemics, Cybernetics and Informatics (WMSCI 2005), July 10–13, 2005, Orlando, Florida, USA, 2005.
[29] T.-C. Wang, Y.-H. Chen, Applying fuzzy linguistic preference relations to the improvement of consistency of fuzzy AHP, Inform. Sci. 178 (2008) 3755–3765.
[30] T.-C. Wang, Y.-H. Chen, Fuzzy linguistic preference relations under uncertain environment, Inform. Fusion 11 (2010) 201–207.
[31] Y.-H. Chen, On Consistency of Fuzzy Analytic Hierarchy Process, Ph.D. Thesis, I-Shou University, Kaohsiung, Taiwan, 2008.
[32] F. Chiclana, F. Herrera, E. Herrera-Viedma, L. Martinez, A note on the reciprocity in the aggregation of fuzzy preference relations using OWA operators, Fuzzy Sets Syst. 137 (2003) 77–83.
[33] T.-C. Wang, T.-H. Chang, Forecasting the probability of successful knowledge management by consistent fuzzy preference relations, Expert Syst. Appl. 32 (2007) 801–813.
[34] T.-C. Wang, Y.-H. Chen, Applying consistent fuzzy preference relations to partnership selection, Omega 35 (2007) 384–388.
[35] M. Fedrizzi, On a consensus measure in a group MCDM problem, in: J. Kacprzyk, M. Fedrizzi (Eds.), Multiperson Decision Making Models using Fuzzy Sets and Possibility Theory, Kluwer Academic, New York, 1990, pp. 231–241.
[36] E.S. Lee, R.-J. Li, Comparison of fuzzy numbers based on the probability measure of fuzzy events, Comput. Math. Appl. 15 (1988) 887–896.
[37] A.İ. Ölçer, A.Y. Odabaşi, A new fuzzy multiple attributive group decision making methodology and its application to propulsion/manoeuvring system selection problem, Eur. J. Oper. Res. 166 (2005) 93–114.
[38] Y.-M. Wang, Y. Lou, Z. Hua, On the extent analysis method for fuzzy AHP and its applications, Eur. J. Oper. Res. 186 (2008) 735–747.
[39] S.H. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets Syst. 17 (1985) 113–129.