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COMP 3270
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Algorithm-1

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	n + 1
3	1	$\sum_{i=1}^{n}(i) + 1$
4	1	$\sum_{i=1}^{n} i$
5	1	$\sum_{i=1}^{n}\sum_{j=1}^{i}(j) + 1$
6	6	$\sum_{i=1}^{n}\sum_{j=1}^{i} j$
7	5	$\sum_{i=1}^{n} i$
8	2	1

Multiply col.1 with col.2, add across rows and simplify
$T_1(n)$ = 1 + n+1 + $\sum_{i=1}^{n}(i) + 1$ + $\sum_{i=1}^{n} i$ + $\sum_{i=1}^{n}\sum_{j=1}^{i}(j) + 1$ + 6 $\sum_{i=1}^{n}\sum_{j=1}^{i} j$ + 5 $\sum_{i=1}^{n}\sum_{j=1}^{i} j$ + 2$\sum_{i=1}^{n} i$
= 1+n+1+ (n(n+1)/2) +1 + (n(n+ 1)/2) + ((n(n+ 1) (n+2))/6) +1 + (6(n)(n+ 1) (n+2)/6) + 5((n(n+1))/2) + 2

= (7/6) n^3 + 7n^2 + (41/6)n + 6

T1(n): O(n3)

Algorithm-2

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	n+1
3	1	n
4	1	$\sum_{i=1}^{n}(i) + 1$

5	6	$\sum_{i=1}^{n} i$
6	5	$\sum_{i=1}^{n} i$
7	2	1

Multiply col.1 with col.2, add across rows and simplify

$T_2(n)$ = 1+n+1+n+ ((n(n+1))/2) +1+ (6n(n+1)/2) + (5n(n+1)/2) + 2

6n^2 + 8n + 5

T2(n): O(n2)

Algorithm-3

Step	Cost of each execution	Total # of times executed in any single recursive call
1	4	1
2	11	1
Steps executed when the input is a base case: 1. if L > U then return 0 /* zero- element vector */ 2. if L=U then return max(0,X[L]) /* one-element vector		
First recurrence relation: T(n=1 or n=0) = when n= 1 is 4 and when n = 1 is 11		
3	5	1
4	2	1
5	1	n+1
6	6	n
7	7	n
8	2	1
9	1	n+1
10	6	n
11	7	n
12	4	1
13	4	logn(cost excluding the recursive call)
14	5	logn(cost excluding the recursive call)
15	17	1
Steps executed when input is NOT a base case: 13		
Second recurrence relation: T(n>1) = 2T(n/2) + 56 + 14n		
Simplified second recurrence relation (ignore the constant term): T(n>1) = 2T(n/2) + 14n		

Solve the two recurrence relations using any method (recommended method is the Recursion Tree). Show your work below:

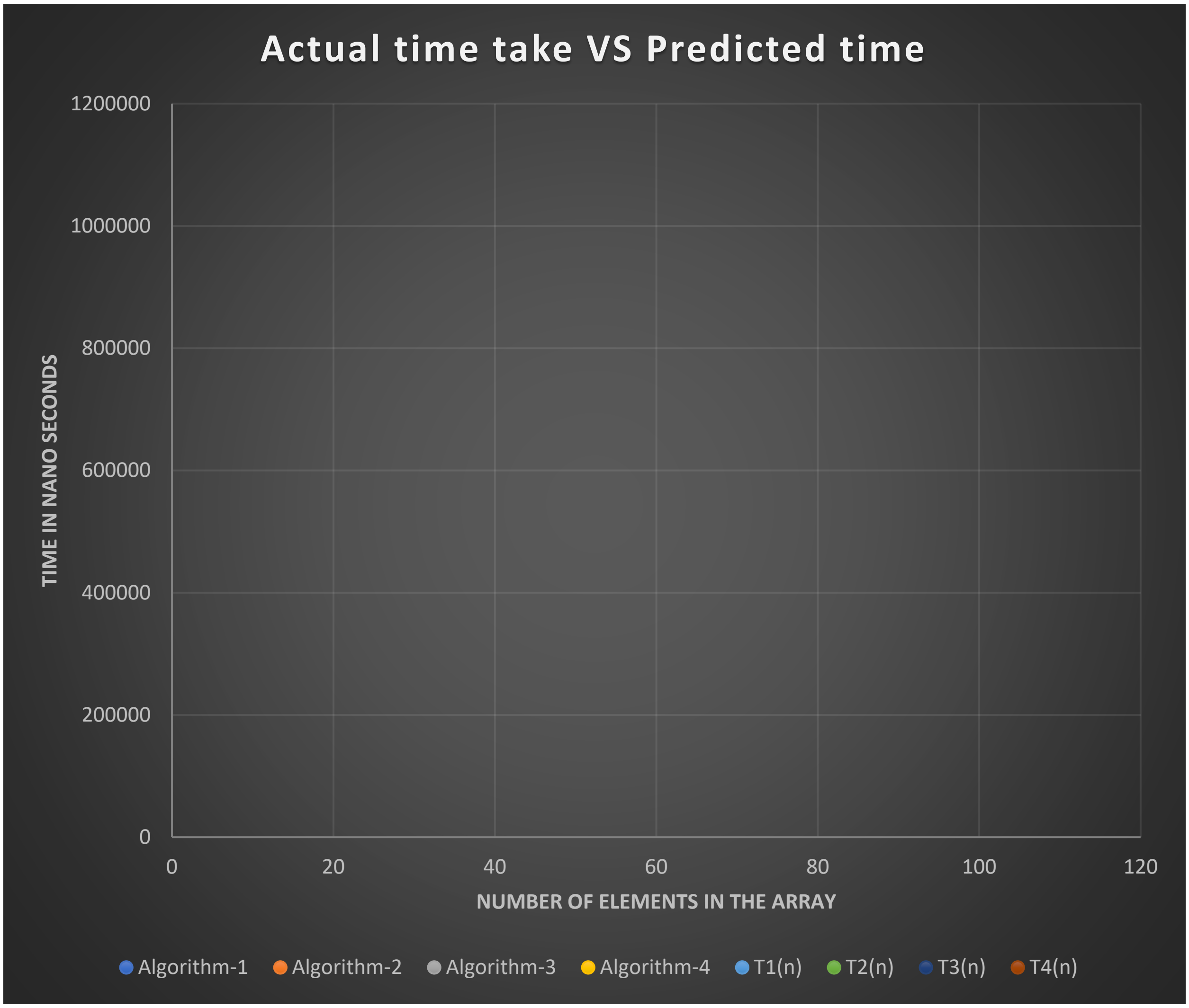
```
                14n
              /     \
          14n/2       14n/2
         /   \          /    \
    14n/4  14n/4   14n/4  14n/4
    /    \   /   \    /   \    /  \

                log2n
```

$T_3(n)$ = 14n log2n + 14n
T3(n) = O(n log(n)

Algorithm-4

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	1
3	1	n+1
4	7	n
5	7	n
6	2	1

Multiply col.1 with col.2, add across rows and simplify
$T_4(n)$ = 1 + 1 + n+1 + 7n + 7n + 2
= 15n + 5
T4(n): O(n)



This graph shows us that the Actual time taken follows and match the predicted time curve In algorithm 1, Algorithm 2, Algorithm 3, but not in Algorithm 4, since the actual time in algorithm 4 did not follow the predicted time curve for small values.