Design & Analysis of Algorithms

Tutorial - 4

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Q1. $T(n) = 3T(n/2) + n^2$

$a = 3,\ b = 2,\ f(n) = n^2$

$c = \log_2 3 = 1.585$

$n^{1.585} < n^2 \longleftrightarrow n^c < f(n)$

$\therefore T(n) = \theta(f(n)) = \theta(n^2)$ [case 3]

Q2. $T(n) = 4T(\frac{n}{2}) + n^2$

$a = 4,\ b = 2,\ f(n) = n^2$

$c = \log_2 4 = 2$

$n^2 = n^2 \longleftrightarrow n^c = f(n)$ [case 2]

$\therefore T(n) = \theta(n^2 \log n)$

Q3. $T(n) = T(\frac{n}{2}) + 2^n$

$a = 1,\ b = 2,\ f(n) = 2^n$

$c = \log_2 1 = 0$

$n^c = 1 < 2^n \longleftrightarrow f(n) > n^c$ [case 3]

Q4. $T(n) = 2^n T(\frac{n}{2}) + n^n$

Master's Theorem can't be applied as $a = 2^n$ is not constant but depends on (n)

②

Q5 $T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16,\ b = 4,\ f(n) = n$

$c = \log_4 16 = 2$

$n^2 > n \longleftrightarrow n^2 > f(n)$ [case 1]

$\therefore T(n) = \Theta(n^2)$

Q6 $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$a = 2,\ b = 2,\ f(n) = n\log n$

$c = \log_2 2 = 1$

$n^1 = n \longleftrightarrow f(n) = n^c$ [case 2]

$T(n) = \Theta(n\log n)$

Q7. $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

$a = 2,\ b = 2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^1 > n/\log n \longleftrightarrow n^c > f(n)$ [case 1]

$T(n) = \Theta(n)$

Q8 $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.51}$

$c = \log_4 2 = 0.5$

$n^{0.5} < n^{0.51} \longleftrightarrow f(n) > n^c$ [case 3]

$T(n) = \Theta(n^{0.51})$

Q9. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

$a = 0.5,\ b = 2,\ f(n) = \frac{1}{n}$

$c = \log_2 0.5 = -1$

$n^{-1} = \frac{1}{n} \longleftrightarrow f(n) = n^c$ [Case 2]

$T(n) = \Theta\left(\frac{1}{n}\log n\right)$

Q10. $T(n) = 16T(\frac{n}{4}) + n!$

$a = 16, b = 4, c = n!$

$c = \log_4 16 = 2$

$n! > n^2 \longleftrightarrow f(n) > n^c$ [Case 3]

$\therefore T(n) = \theta(n!)$

Q11. $T(n) = 4T(\frac{n}{2}) + \log n$

$a = 4, b = 2, f(n) = \log n$

$c = \log_2 4 = 2$

$n^2 > \log n \longleftrightarrow n^c > f(n)$ [Case 1]

$\therefore T(n) = \theta(n^2)$

Q12. $T(n) = \sqrt{n}\, T(\frac{n}{2}) + \log n$

Masters Theorem is not applicable as $a = \sqrt{n}$ is not a constant but depends on n

Q13. $T(n) = 3T(\frac{n}{2}) + n$

$a = 3, b = 2, f(n) = n$

$c = \log_2 3 = 1.585 \longrightarrow n^c = n^{1.585}$

$n^{1.585} > n \longleftrightarrow n^c > f(n)$ [case 1]

$\therefore T(n) = \theta(n^{1.585})$

Q14. $T(n) = 3T(\frac{n}{3}) + sqrt(n)$

$a = 3, b = 3, f(n) = sqrt(n)$

$c = \log_3 3 = 1 \rightarrow n^1$

$\sqrt{n} < n \longleftrightarrow f(n) < n^c$ [Case 3]

$T(n) = \theta(n)$

④

Q 15. $T(n) = 4T\left(\frac{n}{2}\right) + cn$

$a = 4,\ b = 2,\ f(n) = cn$

$c = \log_2 4 = 2 \rightarrow n^2$

$cn < n^2 \longrightarrow Tn = \theta n^2$

On the assumption that (c) is a constant

Q 16 $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_4 3 = 0.792 \longrightarrow n^c = n^{0.792}$

$n\log n > n^{0.792} \longleftrightarrow n\log n > n^{0.792}$

[case 3]

$\therefore T(n) = \theta(n\log n)$

Q17 $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3,\ b = 3,\ f(n) = \frac{n}{2}$

$c = \log_3 3 = 1 \rightarrow n^1$

$n > \frac{n}{2} \longleftrightarrow n > f(n)$ [case 1]

$T(n) = \theta(n)$

Q 18 $T(n) = 6T\left(\frac{n}{3}\right) + n^2\log n$

$a = 6,\ b = 3,\ f(n) = n^2\log n$

$c = \log_3 6 = 1.631 \longrightarrow n^c = n^{1.631}$

$n^2\log n > n^{1.631} \longleftrightarrow f(n) = n^c$ [case 3]

$\therefore T(n) = \theta(n^2\log n)$

Q 19. $T(n) = 4T\left(\frac{n}{2}\right) + n/\log n$

$a = 4,\ b = 2,\ f(n) = n/\log n$

$c = \log_2 4 = 2 \rightarrow n^c = n^2$

$\frac{n}{\log n} < n^2 \longleftrightarrow f(n) < n^c$ [Case 1]

$\therefore T(n) = \Theta(n^2)$

Q 20. $T(n) = 64\,T(n/8) - n^2 \log n$

$a = 64,\ b = 8,\ f(n) = n^2 \log n$

$c = \log_8 64 = 2 \rightarrow n^c = n^2$

$n^2 \log n > n^2 \longleftrightarrow f(n) > n^2$ [Case 3]

$\therefore T(n) = \Theta(n^2 \log n)$

Q 21. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$c = \log_3 7 = 1.771 \rightarrow n^c = n^{1.771}$

$n^2 > n^{1.771} \longleftrightarrow f(n) > n^c$ [Case 3]

$\therefore T(n) = \Theta(n^2)$

Q 22. $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

$a = 1,\ b = 2,\ f(n) = n(2 - \cos n)$

$c = \log_2 1 = 0,\quad n^c = 1$

$n(2 - \cos n) > 1 \longleftrightarrow f(n) > n^c$ [Case 3]

$\therefore T(n) = \Theta(n(2 - \cos n))$

$n \leq f(n) \leq 3n \longrightarrow T(n) = \Theta(n)$