DEMAND ANALYSIS

The demand for a particular commodity means the quantity of a commodity which one acquires or is willing to acquire at a given price. Clearly, the demand for a commodity depends on a number of factors like price of the commodity, price of other commodities, income of the consumer, etc. an analysis of pattern of dependence of demand functions on such factors is very important for the government to control the prices of essential commodities and to maintain overall economic stability in the market. As every government these days has to formulate some economic policy in order to have a balanced economic structure in the country, the subject matter of demand analysis has gained considerable importance. The demand analysis deals with the following two important aspects of economic statistics:

(i) To study the relationship between the demand and market price on the basis of market data.

(ii) To study the relationship between demand and income of the consumers on the basis of family budget data

Demand and supply

In the theory of economics, mere for a commodity does not mean demand, unless one can pay and is willing to pay the necessary amount for it. By the term demand we mean the quantity of a commodity which one acquires or is able to acquires at a given price. Demand for any commodity depends on a number of factors such as the price of the commodity, the price of the other commodities, income of the consumers, time, place, etc.

"The functional relationship between consumption of a commodity and the factors responsible for the changes in consumption, is defined as the demand function of that commodity." From this function, one can find different quantities od a commodity demanded at different prices.

Similarly, the term supply means the amount of commodity available in the market at a given price. Obviously, if the market price is so low that the producer cannot realise the cost of production then he will not be interested in producing the required amount. Consequently, supply is also a function of the price at which commodity is sold in the market.

Thus, economic study of the market data is made to determine the relation between:

(i) The price of a given commodity and its absorption capacity for the market, i.e., demand, and

(ii) The price of the commodity and its output, i.e., supply.

Laws of Demand and Supply.

From the traditional concepts of economics, the laws of demand and supply may be stated as follows:

"Demand for a commodity, in general, varies is in the direction opposite to that of prise whereas supply in general varies in the same direction as price."

In other words, a rise in the price of a commodity results in decrease of its demand and increase in its supply and vice versa if the price falls down.

Demand (d) is a certain function of price (p) i.e., $d = f\ (p)$, the only assumption regarding $f\ (p)$ is that as a rule, it is a diminishing function of p, i.e., $f'(p) < 0$. Supply is also a function of p i.e., $s = \emptyset(p)$, say, where $\emptyset(p)$ is an increasing function of p I.e., $\emptyset'^{(p)} > 0$. The demand and supply functions can be represented graphically as shown in given figure.

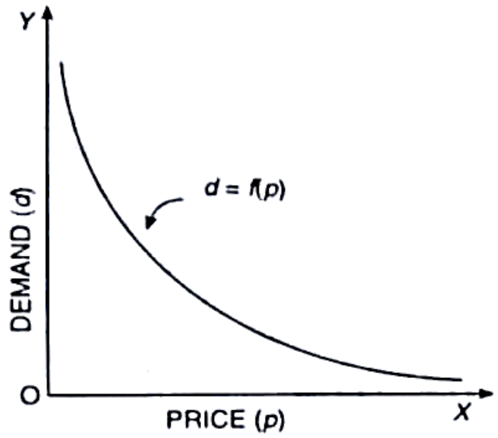

Fig. 4·1 : *Demand Curve*

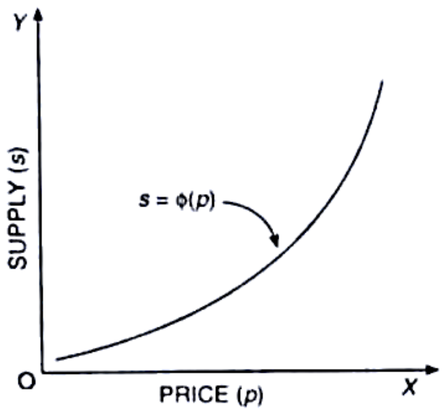

Fig. 4·2 : *Supply Curve*

These laws further state that the market price settles at a level at which supply and supply and demand are equal and is determined by the point of intersection of the two curves $d = f(p)$ and $s = \emptyset(p)$ as shown in graph OM and no other point has any chance of lasting in the market.

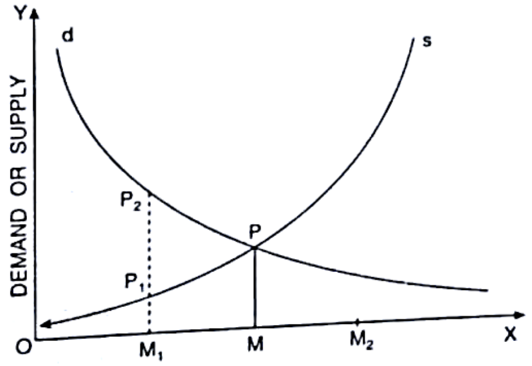

Fig. 4·3 : *Demand Curve and Supply Curve*

For example, suppose the price at any time 't' (say) is less than OM and equal to OM_1 (say). Then corresponding values of d and s from the figure are P_2M_1 and P_1M_1 respectively, i.e., demand is greater than supply and naturally the producer will be encouraged to increase the price since under those circumstances, there will be persons willing to pay a higher price in order to get the commodity they need. This will consequently result in reduction in the demand and ultimately balance between supply and demand will be achieved at point 'P' where demand and supply curves intersect.

On the other hand, if price is greater than OM and equal to OM_2 (say) then $s > d$ and there will be producers willing to sell the goods at a lower price in order to clear their stocks. Consequently, the prices of the goods are bound to come down which, in turn, will increase the demand and ultimately there will be balance at the point P.

Equilibrium Price. The price OM, determined by the point of intersection of the curves $d = f(p)$ and $s = \emptyset(p)$ is termed as equilibrium price. Thus the equilibrium price is the solution of the equation:

$$d = s$$

$$f(p) = \emptyset(p)$$

Que 1. The demand curve and the supply curve of a commodity are given by $d = 19 - 3p - p^2$ and $s = 5p - 1$. Find the equilibrium price and quantity exchanged.

Answer: $p =$ 2 and $d = s = 9$

Que2. The demand functions of two commodities A and B are: $D_A = 10 - p_A - 2p_B$ and $D_B = 6 - p_A - p_B$ respectively and the corresponding supply function are $S_A = -3 + p_A + p_B$, and $S_B = -2 + p_B$, where p_A *and* p_B are the prices of A and B respectively. Find.

(i) The equilibrium price and (ii) equilibrium quantities exchanged

Answer: $p_A = 2 \; and \; p_B = 3$, $D_A = S_A = 2$, $D_B = S_B = 1$.

PRICE ELASTICITY OF DEMAND

One of the most important characteristics of demand function is what is known as its 'elasticity'. According to the law of demand, the changes in price and demand are in opposite direction and it is a common experience that price changes affect the demand for different commodities in different degrees. In other words, the demand for some commodities is more sensitive to price changes then is the demand for others.

For example, demand for necessities decreases very little when their price rise whereas only a slight increase in the prices of luxuries will reduce their demand considerably. The quality of demand by virtue of which it extends or contracts with a fall or rise in price is known as 'price elasticity of demand', a term introduced by Marshall for evaluating the influence of variation in prices of the commodity on its demand. It shows the sensitiveness (संवेदनशीलता) or responsiveness (अनुक्रियाशीलता) of demand to the change changes in price.

Definition. Price elasticity of demand is defined as the value of the ratio of the relative (for proportionate) change in the demand to the relative (or proportionate) change in the price.

Mathematically, let x be the quantity demanded for a commodity 'A' such that the demand function of A is $x = f(p)$ where $f\ (.)$ is a continuous function. Let the increment in demand x corresponding to an increment δp in p, be δx. Then, by definition, of price elasticity of demand (η_v) is given by:

$$\frac{Proportionate\ change\ in\ demand}{Proportionate\ change\ in\ price} = \frac{(\delta x)/x}{(\delta p)/p} = \frac{p}{x}.\frac{\delta x}{\delta p}$$

This is the average elasticity of demand over the price range $(p, p + \delta p)$. The elasticity of demand (η_p) at a particular price level p is given by:

$$\eta_p = \lim_{\delta p \to 0} \frac{p}{x}.\frac{\delta x}{\delta p} = \frac{p}{x} \lim_{\delta p \to 0} \frac{\delta x}{\delta p} = -\frac{p}{x}.\frac{dx}{dp} = -\frac{p}{f(p)}.\frac{df}{dp}$$

$$-\frac{d \log f}{\mathrm{d} \log p}$$

negative sign being taken since demand and price move in opposite direction.

Interpretation. If the price rises (falls) by 1%, the demand will falls (rises) by η_p%.

If $\eta_p > 1$, demand is said to be elastic.

If $\eta_p < 1$, demand is said to be inelastic.

If $\eta_p = 1$, demand has unit elasticity.

Que 3. If the demand function is $p = 4 - 5x^2$, for what value of x, the elasticity of demand will be unity? Where x is the quantity demanded and p is price.

Solution: Here, $p = 4 - 5x^2$

Differentiating with respect to x.

$$\frac{dp}{dx} = -10\,x$$

$$\frac{dx}{dp} = \frac{-1}{10\,x} \quad \ldots\ldots\ldots.(1)$$

Elasticity

$$\eta_p = -\frac{p}{x}.\frac{dx}{dp}$$

$$\eta_p = -\frac{(4-5x^2)}{x}.\left(\frac{-1}{10\,x}\right)$$

$$\eta_p = \frac{(4-5x^2)}{10\,x^2}$$

elasticity of demand will be unity if $\eta_p = 1$.

$$1 = \frac{(4-5x^2)}{10\,x^2}$$

$$10\,x^2 = 4 - 5x^2$$

$$10\,x^2 + 5x^2 = 4$$

$$15x^2 = 4$$

$$x = \frac{2}{\sqrt{15}}$$

Price Elasticity of Supply. If $s = \emptyset(p)$ is the supply curve for a commodity 'A' then the price elasticity of supply of A at price p is given by:

$$\varepsilon_p = \frac{Relative\ Change\ in\ supply\ of\ A}{Realtive\ change\ in\ price\ of\ A} = \frac{\delta s/s}{\delta p/p} = \frac{p}{s} \cdot \frac{\delta s}{\delta p}$$

Where s is the quantity supplied of the commodity 'A' at price p and δs is the increment in supply s corresponding to an increase δp in price p. This is the average price elasticity of supply over the price range $(p, p + \delta p)$. The price elasticity of supply (ε_p) at a particular price level p is given by:

$$\varepsilon_p = \lim_{\delta p \to 0} \frac{p}{s} \cdot \frac{\delta s}{\delta p} = \frac{p}{s} \lim_{\delta p \to 0} \frac{\delta s}{\delta p} = \frac{p}{s} \cdot \frac{ds}{dp} = \frac{d \log s}{d \log p}$$

The positive sign being taken since price and supply change in the same direction. Taking $s = \emptyset(p)$, we get

$$\varepsilon_p = \frac{p}{s} \cdot \frac{d}{dp} \emptyset(p)$$

$$\varepsilon_p = p \ \frac{\emptyset'\ (p)}{\emptyset\ \ (p)}$$

If the price of a commodity falls (rises) by 1%, the supply will decrease (increase) by ε_p%.

TYPES OF DATA REQUIRED FOR ESTIMATING ELASTICITIES

The empirical demand analysis is based on the data obtained from two main sources of statistical observation:

(a) Family Budget Data. Family budget data are collected through sample surveys covering households which are representatives of different classes of people with respect to income, family size, social class etc., and the expenditure on different items of consumption like food, clothing, power and fuel etc., during a period of 12 months or over is collected.

In order to study the influence of income level on the expenditure habits of the people, we carry out an experiment consisting of the following main steps:

1. The first step is to select a group of households which are as homogeneous as possible with respect to regional environments, social and economic characteristics, family size and other factors that affect the demand, without making any reference to their family income.

2. The next steps consists in regulating the family income by allotting the households to different income levels at random, randomisation being resorted to neutralise the effect of factors other than income.
3. Finally, a detailed account of the expenditure of each household during a period of 12 months on various budget items is compiled.

In demand analysis, by family budget data we mean the data collected through an experiment with the above three steps. Here, expenditure (rather than income) is interpreted in terms of demand function.

(b) Time Series Data. The statistical market data are usually time series data relating to the prices of the commodities and their quantities bought (or sold) at that price at different points of time.

The treatment of such data is quite analogous to that of family budget data except that demand is now primarily regarded as a function of price and not of income. A variation in the price of a commodity over time means a shift in either or both of the demand and supply curves if both the curves $d = f(p),\ s = \emptyset(p)$ remain fixed the market data remains more or less static and does not provide enough number of points their determination. If both the demand and supply curves shift their positions then the market statistics would give us a picture of variation of demand (and supply) curves consequent upon the variations in the equilibrium price $'p_1'$ and in this case it is unlikely to trace either the supply or the demand function closely. However, if one of the two curves remains fixed and the other changes its position, then the family budget data provide a number of points on the fixed curve and hence the curve is determined.

PARETO'S LAW OF INCOME DISTRIBUTION

Vilfredo pareto (1848 to 1923) was the first man to make an extensive study from statistical point of view, the problem of the distribution of income among the citizen of a state. Pareto propounded his famous law of distribution of income in the $19^{th} - 20^{th}$ century, primarily basing his arguments and reasoning on the empirical study of the income data of various countries of the world at different times. Pareto's law of distribution of incomes can be cast, somewhat precisely, in the following statement:

"In all places and at all times, the distribution of income in a stable economy is given approximately by the empirical formula:

$$y = A(x - a)^{-v} \qquad \dots \dots (1)$$

where y is the number of people having income x or greater, a is a lowest income at which the curve begins and A and υ are certain parameters."

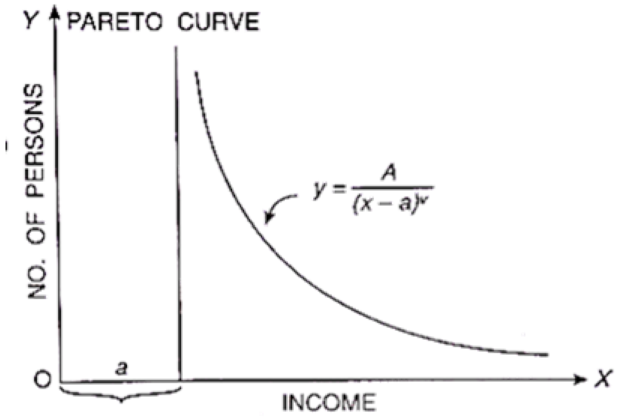

Fig. 4·7 : Pareto Curve

The curve represented by (1) is an extremely asymmetrical one like a hyperbola, the lines $x = a$ and $y = 0$ being the asymptotes to the curve since $x \to a$, $y \to \infty$ and as $x \to \infty$, $y \to 0$,

Shifting the origin to $x = a$, the pareto curve becomes:

$$y = Ax^{-\upsilon} \qquad \ldots\ldots(2)$$

Taking logarithm of both sides,

$$\log y = \log A - \upsilon \log x$$

Hence, if Parato curve is represented graphically on a double logarithmic scale the graph will be straight line with its slope equal to $-\upsilon$.

Formulation of the problem. The problem of distribution of income may be formulated precisely as follows:

(a) Determination of the form of the frequency distribution $\emptyset(x)$, (say), for the total distribution of income from the poorest to the richest member of society.
(b) Is $\emptyset(x)$ governed by the type of society from which income is derived or is its form inevitable(अवश्यंभावी)?
(c) Can any a priori (अनुमानतः) reason be assigned for the form of $\emptyset(x)$?

In order to formulate our ideas mathematically, let us suppose that there are N individuals to be distributed with respect to their income (X) which the frequency function (say), $\emptyset(x)$, $A < x < B$, where A and B are respectively the lowest and highest measure of incomes in the population. Then by definition, the distribution function is given by:

$$F(x) = \int_{x}^{A} \emptyset(x)\, dx \qquad \ldots\ldots (3)$$

which represents the total number of persons with income less than is equal to X. Obviously

$$F(B) = \int_{A}^{B} \emptyset(x)\, dx = \quad N$$

$$y(x) = N - F(x) = \int_{x}^{\infty} \emptyset(x)\, dx \qquad \ldots\ldots (4)$$

gives the number of persons with income x or more. It is the same function which Pareto assumed to be of form (1) $y = A\, x^{-v}$

where v is 1.5 and x is sufficiently large. From equation (2), (3) and (4)

$$\emptyset(x) = \frac{d}{dx}\, F(x) = -\frac{dy}{dx} = A\, v\, x^{-v-1}$$

$$\Rightarrow \quad \log \emptyset(x) \; = \log(Av) - (v+1) \log x$$

Also we have $\log y = \log A - \; v \log x$

Hence if the functions $y(x)$ and $\emptyset(x)$ are plotted on a double logarithmic scale, the graph will be straight lines, the ratio of their slopes being $\frac{v}{1+v}$.

Curves of Concentration. A very useful formulation of the problem of 'distribution of income' can be made even if the form of distribution, viz., $\emptyset(x)$ is not known. This is achieved through 'Curve of Concentration'.

Let the total frequency of the population be N and let its total income be I.

$$p_x = \frac{Number\ of\ persons\ with\ income\ \leq x}{N} \qquad \ldots .(1)$$

$$q_x = \frac{Total\ income\ of\ the\ persons\ with\ income\ \leq x}{I} \qquad \ldots \ldots .(2)$$

If there exists a functional relationship between the variables p and q, the equations:

$$p = \; p_x \qquad and \qquad q = q_x \qquad \ldots ..(3)$$

form a parametric system of determining it.

If the income distribution law is sufficiently well known then, eliminating x in (3) we get the equation

$$q = f(p) \qquad \text{....(4)}$$

The straight line $q = p$ is known as the 'line of equal distribution'. Since both p and q range from 0 to 1, the line of equal distribution is the diagonal of a square of unit side.

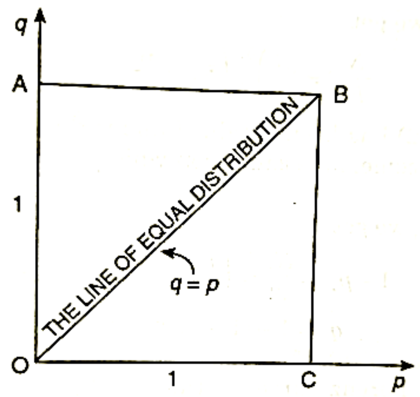

Fig. 4.8 : Line of Equal Distribution

another formulation, especially useful for income data may be given

Let us write: $N_x =$ Number of individuals with income x or more and

$I_x = T$otal amount of income possessed by N_x.

Then p_x and q_x as defined in equation (1) and (2) are given by:

$$p_x = \frac{N - N_x}{N} \qquad and \qquad q_x = \frac{I - I_x}{I}$$

These curves were suggested for the representation of distribution of income.

Lorenz Curve: the function

$$q = 1 - (1 - p)^{\frac{1}{\delta}}$$

Where $\delta = \frac{v}{v-1}$. which we assume as $q = f(p)$.

The graph of this function (known as Lorenz curve) is the design of income concentration and is compared with the line of equal distribution.

As a measure of the difference between distributions, Gini proposed a concentration ratio (ρ) defined as follows:

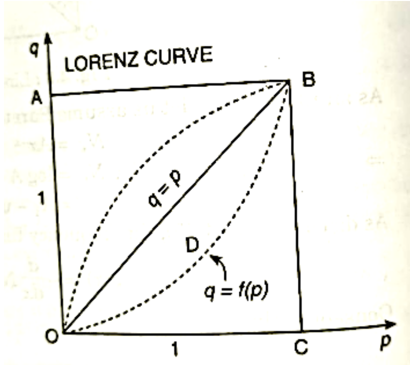

Fig. 4.9 : *Lorenz Curve*

$$\rho = \frac{Area\ BDOB}{Area\ of\ \Delta BOC}$$

$$\rho = \frac{Area\ of\ \Delta BOC - Area\ BDOCB}{Area\ of\ \Delta BOC}$$

$$\rho = 1 - \frac{Area\ BDOCB}{Area\ of\ \Delta BOC}$$

But $Area\ of\ \Delta BOC = \frac{1}{2} \times BC \times OC = \frac{1}{2} \qquad \{BC = OC = 1\}$

$$\rho = 1 - 2 \times Area\ BDOCB$$

$$\rho = 1 - 2\int_0^1 q\ dp$$

$$\rho = 1 - 2\int_0^1 \left[1 - (1-p)^{\frac{1}{\delta}}\right] dp$$

$$\rho = 1 - 2\left|p + \frac{(1-p)^{\frac{1}{\delta}+1}}{\frac{1}{\delta}+1}\right|_0^1$$

$$\rho = 1 - 2\left(1 - \frac{1}{\frac{1}{\delta}+1}\right)$$

$$\rho = 1 - 2\left(1 - \frac{\delta}{1+\delta}\right)$$

But, $\delta = \frac{v}{v-1}$, on simplification

$$\rho = \frac{1}{2v-1}$$

As δ varies from 1 to ∞, ρ varies from 0 to 1. In particular, assuming Pareto's value of $v = 1.5$, we get

$$\rho = \frac{1}{2} = 0.5$$

Log Normal Distribution: a non – negative random variable X is said to have a logarithmic normal distribution, if its logarithm has a normal distribution. The $p.d.f.$ of the log normal distribution is given by

$$f(x;\ \mu, \sigma) = \frac{1}{x\,\sigma\,\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\,(\log x\ -\ \mu)^2\right],\ \ \mu > 0$$

It arises in problems of economics, biology, geology and reliability theory. In particular, it arises in the study of dimensions of particles under pulverisation (विनष्ट करना).

Moments of log normal distribution: the r[th] moment about origin is given by:

$$\mu_r{}' = E(X^r) = \int_{-\infty}^{\infty} x^r\ f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^r\ \frac{1}{x\,\sigma\,\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\,(\log x\ -\ \mu)^2\right] dx$$

$$\mu_r{}' = \frac{1}{\sigma\,\sqrt{2\pi}} \int_{-\infty}^{\infty} x^{r-1}\ \exp\left[-\frac{1}{2\sigma^2}\,(\log x\ -\ \mu)^2\right] dx$$

Let $\frac{(\log x - \mu)}{\sigma} = u \qquad \Rightarrow \log x\ =\ \mu +\ \sigma\,u$

$$\Rightarrow \quad x\ = e^{\,\mu + \,\sigma\,u}$$

$$dx = \sigma\,e^{\,\mu + \,\sigma\,u}.\ du$$

$$\mu_r{}' = \frac{1}{\sigma\,\sqrt{2\pi}} \int_{-\infty}^{\infty} (e^{\,\mu + \,\sigma\,u})^{r-1}\ e^{-\frac{u^2}{2}}\,.\ \sigma\,e^{\,\mu + \,\sigma\,u}.\ du$$

$$\mu_r{}' = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (e^{\,\mu + \,\sigma\,u})^r\ e^{-\frac{u^2}{2}}\,.du$$

$$\mu_r{}' = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\,\mu r + \,\sigma\,ur}.\ e^{-\frac{u^2}{2}}\,.du$$

$$\mu_r{}' = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\,\mu r + \,\sigma\,ur - \frac{1}{2}u^2}\,.du$$

$$\mu_r{}' = \frac{1}{\sqrt{2\pi}} e^{\,\mu r} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\sigma^2 r^2 + \frac{1}{2}\sigma^2 r^2 + \sigma ur - \frac{1}{2}u^2} . du$$

$$\mu_r{}' = \frac{1}{\sqrt{2\pi}} e^{\,\mu r + \frac{1}{2}\sigma^2 r^2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\sigma^2 r^2 + \sigma ur - \frac{1}{2}u^2} . du$$

$$\mu_r{}' = \frac{1}{\sqrt{2\pi}} e^{\,\mu r + \frac{1}{2}\sigma^2 r^2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(\sigma^2 r^2 - 2\sigma ur + u^2)} . du$$

$$\mu_r{}' = \frac{1}{\sqrt{2\pi}} e^{\,\mu r + \frac{1}{2}\sigma^2 r^2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(u - \sigma r)^2} . du$$

Again let $u - \sigma r = t \quad \Rightarrow \quad du = dt$

$$\mu_r{}' = \frac{1}{\sqrt{2\pi}} e^{\,\mu r + \frac{1}{2}\sigma^2 r^2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}t^2} . dt$$

$$\mu_r{}' = \frac{1}{\sqrt{2\pi}} e^{\,\mu r + \frac{1}{2}\sigma^2 r^2} . \sqrt{2\pi}$$

$$\boldsymbol{\mu_r{}' = e^{\,\mu r + \frac{1}{2}\sigma^2 r^2}}$$

Now for mean $r = 1$

$$\mu_1{}' = e^{\,\mu.1 + \frac{1}{2}\sigma^2 1^2} = e^{\,\mu + \frac{1}{2}\sigma^2}$$

Now if $r = 2$

$$\mu_2{}' = e^{\,2\mu + \frac{1}{2}\sigma^2 2^2} = e^{\,2\mu + 2\sigma^2}$$

Now variance

$$\mu_2 = \mu_2{}' - \mu_1{}'^2$$

$$\mu_2 = e^{\,2\mu + 2\sigma^2} - \left(e^{\,\mu + \frac{1}{2}\sigma^2}\right)^2$$

$$\mu_2 = e^{\,2\mu} . e^{2\sigma^2} - e^{2\mu} . e^{\sigma^2}$$

$$\mu_2 = e^{\,2\mu}\left(e^{2\sigma^2} - e^{\sigma^2}\right)$$

$$\mu_2 = e^{\,2\mu} . e^{\sigma^2} \left(e^{\sigma^2} - 1\right)$$

$$\mu_2 = e^{\,2\mu + \sigma^2} \left(e^{\sigma^2} - 1\right)$$