Date: / / Page no:

Practical- 27

Topic :- Two way Classification

Question :- Three varieties of coal were analysed by four chemists and the ash content in the varieties was found to be as under:

Varieties	Chemists 1	2	3	4
A	8	5	5	7
B	7	6	4	4
C	3	6	5	4

Do the varieties differ significantly in their ash- content?

formula used :-

1) R.S.S. $= \sum_i \sum_j y_{ij}^2$

$h=4, k=3$

2) Correction factor C.F. $= G^2/N$ where $= N = h \times k = 4 \times 3 = 12$

3) Total sum of square T.S.S = R.S.S - C.F.

4) Row sum of square $= \sum_i \frac{T_{i\cdot}^2}{h_i} - C.F.$

5) Column sum of square $= \sum_j \frac{T_{\cdot j}^2}{k_i} - C.F.$

6) Error sum of square = T.S.S. − Row S. S. − Column S.S.

Degrees of freedom

7) Row sum of square $= k - 1$

8) Column sum of square $= h - 1$

9) Error sum of square $= N - 1$

Scanned with CamScanner

Here the two factors of variation are say
A: Variates, represented along the rows of the Table
B: Chemists, represented along the columns of the Table.
Null Hypothesis :- H_{0A} : $\mu_{1.} = \mu_{2.} = \mu_{3.}$ i.e. there is no significant difference between the variates. H_{0B}: $\mu_{.1} = \mu_{.2} = \mu_{.3} = \mu_{.4}$ there is no significant difference b/t the method of variates.
Alternative Hypothesis :- H_{1A}: At least two type of $\mu_{1.}, \mu_{2.}, \mu_{3.}$ are different
H_{1B}: At least two of $\mu_{.1}, \mu_{.2}, \mu_{.3}, \mu_{.4}$ are different. y_{ij} = Response of the ith variate and jth chemist ($i = 1, 2, 3; j = 1, 2, 3, 4$) and $k = 3$, $h = 4$, $N = h \times k = 4 \times 3 = 12$

CALCULATION FOR VARIATES

variates / Chemists $i \downarrow$ / $j \rightarrow$	1	2	3	4	$T_{i.} = \sum y_{ij}$	$T_{i.}^2$
A	8	5	5	7	25	625
B	7	6	4	4	21	441
C	3	6	5	4	18	324
$T_{.j} = \sum y_{ij}$	18	17	14	15	64	1390
$T_{.j}^2$	324	289	196	225	1034	

i) $R.S.S = \sum_i \sum_j y_{ij}^2 = 366$

ii) $C.F. = \frac{G^2}{N} = \frac{64^2}{12} = 341.33$

iii) $T.S.S = R.S.S. - C.F = 366 - 341.33 = 24.66$

iv) Row sum of square $S.S.A = \sum T_{i.}^2/h - C.F. = 1390/4 - 341.33 = 6.17$

v) Column sum of square $S.S.B. = \sum T_{.j}^2/k - C.F. = 1034/3 - 341.33 = 3.33$

vi) Error sum of square $S.S.E = 24.66 - 6.17 - 3.33 = 15.16$

Source of variation (1)	d.f. (2)	S.S. (3)	Mean S.S. (4) = (3)/(2)	Variance Ratio (F)
factor A	$k-1 = 3$	6.17	2.05	$F_A = \frac{2.05}{2.61} = 0.785$
factor B	$h-1 = 2$	3.33	1.665	$F_B = \frac{1.665}{2.61} = 0.637$
Error	$3 \times 2 = 6$	15.66	2.61	
Total	$N-1 = 11$	25.16		

Conclusion- here Tabulated value of $F_{0.05}(3, \;) = 9.55$
Calculated value of $F_{0.05}(3, \;) =$

Date: / / Page no:

Practical- 28

Topic :- Two way classification

Question :- Four experimenters determine the moisture content of samples of a powder, each man taking a sample from each of six consignment. The assessments are:

observer	consignment 1	2	3	4	5	6
1	9	10	9	10	11	11
2	12	11	9	11	10	10
3	11	10	10	12	11	10
4	12	13	11	14	12	10

Carry out the ANOVA and discus whether there is any significant difference between consignments or between observer.

formula used:-

1) $R.S.S. = \sum_i \sum_j y_{ij}^2$ 2) $C.F. = G^2/N$

3) Total sum of square $T.S.S. = R.S.S - C.F.$

4) Row sum of square for A $= \frac{\sum_i T_{i.}^2}{h} - C.F.$

5) Column sum of square for B $= \frac{\sum_j T_{.j}^2}{k} - C.F.$

6) Error sum of square $= T.S.S. - Row\ S.S. - column\ S.S.$

Degree of freedom

7) Row S.S. $= k-1$ 8) Column SS. $= h-1$ 9) Error SS. $= h \times k$

Here the factor of variations are A and B
Null Hypothesis :- $H_{0A} = \mu_1 . \mu_2 . \mu_3 . \mu_4$ & $H_{0B} = \mu_{.1}, \mu_{.2}, \mu_{.3} - - , \mu_{.6}$
there is no significance difference b/t observer & consignment $\mu_1 . \mu_2 . \mu_3 . \mu_4$
Alternative hypothesis :- H_{1A} = At least two type of $\mu_{1.}, \mu_{2.}, \mu_{3.}, \mu_{4.}$ are different. H_{1B} = at least two of $\mu_{.1}, \mu_{.2}, - - - , \mu_{.6}$ are differ
y_{ij} = response of i^{th} observer & j^{th} consignment (i = 1, 2, 3, 4 and j = 1, 2, 3, 4, 5, 6) and $k = 4$, $h = 6$, $N = 24$

CALCULATION TABLE for VARIATES

observer i / j	Consignment 1	2	3	4	5	6	$T_{i.} = \sum y_{ij}$	$T_{i.}^2$
1	9	10	9	10	11	11	60	3600
2	12	11	9	11	10	10	63	3969
3	11	10	10	12	11	10	64	4096
4	12	13	11	14	12	10	72	5184
$T_{.j} = \sum y_{ij}$	44	44	39	47	44	41	259	16849
$T_{.j}^2$	1936	1936	1521	2209	1936	1681	11219	

i) $R.S.S = \sum_i \sum_j y_{ij}^2 = 2831$

ii) $C.F. = G^2/N = (259)^2/24 = 2795.04$

iii) $T.S.S = R.S.S. - C.F = 2831 - 2795.04 = 35.96$

iv) Row S.S. (A) $= \sum T_{i.}^2/h - C.F = 16849/6 - 2795.04 = 13.126$

v) Column S.S (B) $= \sum T_{.j}^2/k - C.F = 11219/4 - 2795.04 = 9.71$

vi) Error S.S. = T.S.S. − Row S.S − Column S.S. = 35.96 − 13.12 − 9.71 = 13.13

Degrees of freedom

vii) Row S.S = $k-1 = 4-1 = 3$ viii) Column S.S = $h-1 = 6-1 = 5$

ix) Error S.S. = $N-1 = 24-1 = 23$

ANOVA TABLE :-

Source of Variation (1)	d.f. (2)	Sum of square (3)	Mean S.S. (4)=(3)/(2)	Variance Ratio (F)
factor A	3	13.12	4.37	$\frac{4.37}{0.875} = 4.99$
factor B	5	9.71	1.942	$\frac{1.942}{0.875} = 2.219$
Error	5×3=15	13.13	0.875	
Total	N−1= 23			

Date: / / Page no:

Practical- 29

Topic:- Two way classification with M-observation

Question:- The data in Table show the birth-weights of babies born, classified according to the age of mother and order of gravida there being three observations per cell.

TABLE :- BIRTH-WEIGHTS (in lb.) OF BABIES BORN

order of gravida	Age group of mother. 15-20	20-25	25-30	30-35	35 & over.
1	5.1, 5.0, 4.8	5.0, 5.1, 5.3	5.1, 5.1, 4.9	4.9, 4.9, 5.0	5.0, 5.0, 5.0
2	5.2, 5.2, 5.4	5.3, 5.3, 5.5	5.3, 5.2, 5.2	5.2, 5.0, 5.5	5.1, 5.3, 5.9
3	5.8, 5.7, 5.9	6.0, 5.9, 6.2	5.8, 5.9, 5.9	5.8, 5.5, 5.5	5.9, 5.4, 5.5
4	6.0, 6.0, 5.9	6.2, 6.5, 6.0	6.0, 6.1, 6.0	6.0, 5.8, 5.5	5.8, 5.6, 5.5
5 & over	6.0, 6.0, 6.0	6.0, 6.1, 6.3	5.9, 6.0, 5.8	5.9, 6.0, 5.5	5.5, 6.0, 6.2

Test whether the age of mother and order of gravida significantly affect the birth-weight.

formula used :-

i) R.S.S. $= \sum_i \sum_j \sum_k y_{ijk}^2$

ii) Total S.S. = R.S.S. - C.F.

iii) C.F. $= G^2/N$

iv) S.S. due to order of gravida $= \dfrac{\sum_{i=1}^{p} T_{i..}^2}{mq} - C.F.$

v) S.S. due to mother's age $= \dfrac{\sum_{j=1}^{q} T_{.j.}^2}{mp} - C.F.$

vi) S.S. due to interaction $= \left(\sum_i \sum_j \dfrac{T_{ij.}^2}{m} - C.F.\right)$ - S.S. due to mother's age - S.S. due to order of gravida

Date : .... Experiment No. 29 Page No. ....

H_0 = (a) The age of mother and (b) The order of gravida do not significantly affect the height

TABLE : Calculation for various S.S.

order of gravida	Age group of mother 15-20	20-25	25-30	30-35	35 & over	Row totals ($T_{i..}$)	$T_{i..}^2$
1	14.9	15.4	15.1	14.8	15.0	75.2	5655.04
2	15.8	16.1	15.7	15.7	15.4	78.7	6193.69
3	17.4	18.1	17.6	16.8	16.8	86.7	7516.89
4	17.9	18.7	18.1	17.3	16.9	88.9	7903.21
5	18.0	18.4	17.7	17.4	17.7	89.2	7956.64
Column totals ($T_{.j.}$)	84.0	86.7	84.2	82.0	81.8	418.7	35225.47
$T_{.j.}^2$	7056	7516.89	7089.64	6724	6691.24	35077.77	

In the usual notation, given $m=3$, $p=5$, $q=5$, $N=75$

1) $R.S.S = \sum\sum\sum y_{ijk}^2 = (5.1)^2+(5.0)^2+\cdots+(6.0)^2+(6.2)^2 = 2351.19$

2) $C.F. = G^2/N = (418.7)^2/75 = 2337.46$

3) Total S.S. = R.S.S − C.F. = 2351.19 − 2337.46 = 13.73

4) S.S. due to order of gravida $= \sum_i T_{i..}^2/mq - C.F. = 35225.47/15 - 2337.46 = 10.90$

5) S.S. due to mothers age $= \sum_j T_{.j.}^2/mp - C.F. = 35077.77/15 - 2337.46 = 1.06$

6) $\sum_i\sum_j T_{ij.}^2 = (14.9)^2+(15.4)^2+\cdots+(17.4)^2+(17.7)^2 = 7049.33$

7) S.S. due to interaction $= \left(\sum_i\sum_j T_{ij.}^2/m - C.F.\right)$ − S.S. due to mothers − S.S. due to o.f.g.
$= (7049.33/3 - 2337.46) - 10.90 - 1.06 = 0.36$

8) S.S. due to error = 13.73 − 10.90 − 1.06 − 0.36 = 1.41

ANOVA TABLE

Source of variation (1)	d.f. (2)	Sum of Squares (3)	M.S.S. (4)=(3)/(2)	Variance Ratio (F)	F at 5% level
order of gravida	4	10.90	2.725	$\frac{2.725}{0.0282} = 96.63$	$F_{0.05}(4,50) = 2.57$
Mothers age	4	1.06	0.265	$\frac{0.265}{0.0282} = 9.39$	$F_{0.05}(4,50) = 2.57$
Interaction	16	0.36	0.023	$\frac{0.023}{0.0282} = <1$	$F_{0.05}(16,50) = 2.13$
Residual error	50	1.41	0.0282		
Total	74	13.73			

Date: / / Page no:

Practical- 30

Topic:-

Question:- Three different methods of analysis M_1, M_2, M_3 are used to determine in parts per million the amount of a certain constituent in the sample. Each method is used by five analysts, and the results are given in Table.

Analyst	Method		
	M_1	M_2	M_3
1	7.5	7.0	7.1
2	7.4	7.2	6.7
3	7.3	7.0	6.9
4	7.6	7.2	6.8
5	7.4	7.1	6.9.

a) Do these results indicate a significant variation either between the methods or between the analysts?

formula used :-

1) Raw S.S. (R.S.S.) = $\sum_i \sum_j y_{ij}^2$

2) G = Grand Total = $\sum_i \sum_j y_{ij}$

3) Correction factor = C.F. = G^2/N

4) Total S.S. (T.S.S.) = R.S.S. − C.F.

5) S.S.A. = S.S. due to factor A = $\frac{1}{n} \sum_i T_{i.}^2 - C.F.$

6) S.S.B. = S.S. due to factor B $= \frac{1}{k} \sum_{j} T_{\cdot j}^2 - C.F.$

7) S.S. due to Error (S.S.E.) $=$ T.S.S. $-$ S.S.A $-$ S.S.B

Date : .... Experiment No. ..30.......... Page No. ....

Sol^n

Here the two factors of variation are say A and B

Null Hypotheses: H_{0A} : $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$; i.e., there is no significant different between the analysis. H_{0B}: $\mu_{.1} = \mu_{.2} = \mu_{.3}$, i.e., there is no significant difference between the method of analysts.

Alternative Hypotheses: H_{1A}: At least two of $\mu_1, \mu_2, \ldots, \mu_5$ are different

H_{1B}: At least two of $\mu_{.1}, \mu_{.2}, \mu_{.3}$ are different.

y_{ij} = Response of the i^{th} analyst and the j^{th} method ($i = 1, 2, \ldots 5$; $j = 1, 2, 3$).

In the usual notations, we have $k = 5$, $h = 3$, and $N = h \times k = 3 \times 5 = 15$

TABLE: CALCULATIONS FOR VARIOUS

analyst	Method M_1	M_2	M_3	$T_{i.} = \sum_j y_{ij}$	$T_{i.}^2$
1	7.5	7.0	7.1	21.6	466.56
2	7.4	7.2	6.7	21.3	453.69
3	7.3	7.0	6.9	21.2	449.44
4	7.6	7.2	6.8	21.6	466.56
5	7.4	7.1	6.9	21.4	457.96
Column Total $T_{.j}$	37.2	35.5	34.4	$G = 107.1$	$\sum T_{i.}^2 = 2294.21$
$T_{.j}^2$	1383.84	1260.25	1183.36	$\sum T_{.j}^2 = 3827.45$	

$R.S.S. = \sum_i \sum_j y_{ij}^2 = (7.5)^2 + (7.0)^2 + \ldots (7.1)^2 + (6.9)^2 = 155.66 + 151.49 + 149.90 + 155.84 + 152.78 = 765.67$

G = Grand total $= \sum_i \sum_j y_{ij} = 107.1$

$C.F. = G^2/N = (107.1)^2/15 = 11470.41/15 = 764.694$

$T.S.S. = R.S.S. - C.F. = 765.67 - 764.694 = 0.976$

$S.S.A. = \frac{1}{h} \sum_i T_{i.}^2 - C.F. = \frac{2294.21}{3} - 764.694 = 764.737 - 764.694 = 0.043$

$S.S.B. = \frac{1}{k} \sum_j T_{.j}^2 - C.F. = \frac{3827.45}{5} - 764.694 = 765.49 - 764.694 = 0.796$

$S.S.E = T.S.S - S.S.A. - S.S.B = 0.976 - 0.043 - 0.796 = 0.137$

Source of variation (1)	d.f. (2)	S.S. (3)	Mean S.S. (4)=(3)/(2)	Variance Ratio (F)
factor A	$k-1 = 5-1 = 4$	0.043	0.0108	$F_A = \frac{0.0108}{0.0171} < 1$
factor B	$h-1 = 3-1 = 2$	0.796	0.3980	$F_B = \frac{0.3980}{0.0171} = 23.27$
Error	$4 \times 2 = 8$ or $14-(4+2) = 8$	0.137	0.0171	
Total	$N-1 = 15-1 = 14$	0.976		

Tabulated $F_{0.05}(2, 8) = 19.40$

Scanned with CamScanner

Practical - 31

Topic :- Two way classification

Question :- The following Table gives quality of then service stations by five professional raters:

RATER	SERVICE STATION									
	1	2	3	4	5	6	7	8	9	10
A	99	70	90	99	65	85	75	70	85	92
B	96	65	80	95	70	88	70	51	84	91
C	95	60	48	87	48	75	71	93	80	93
D	98	65	70	95	67	82	73	94	86	80
F	97	65	62	99	60	80	76	92	90	89

Analyse the data and discuss whether there is any significant difference between raters or between service stations.

formula used :-

1) $R.S.S = \sum_i \sum_j y_{ij}^2$

2) $C.F. = G^2/N$

3) Total S.S. = R.S.S. - C.F.

4) Row sum of squares $= \frac{\sum_i T_{i.}^2}{h} - C.F.$

5) Column sum of squares $= \frac{\sum_j T_{.j}^2}{k} - C.F.$

6) Error sum of squares = T.S.S. - Row S.S. - column S.S.