ACCEPTANCE SAMPLING INSPECTION PLANS

"Acceptance sampling plans refer to the use of Sampling inspection by a purchaser to decide whether to accept or to reject a lot of given product. In statistical quality control terminology, it is also known as product control. In case of acceptance sampling by attributes the decision for accepting or rejecting a given lot is taken on the basis whether the sample items possess a particular attribute or not. In other words, for acceptance sampling by attributes is an attribute-based inspection that merely grades the product as defective or non-defective if the product is found defective, it is rejected and if it is found non defective, it is accepted.

Acceptable Quality Level (AQL). This is the quality level of a good lot. It is the percent defective that can be considered satisfactory as a process average, and represents a level of quality which the producer wants accepted with a high probability of acceptance. In other words, if α is the producer risk, then the level of quality which result in $100\ (\ 1\ -\ \alpha\ )\ \%$ acceptance of the good Lots submitted for inspection is called the acceptable quality level.

A lot with relatively small fraction defective (i.e., sufficiently good quality) say, p_1 that we do not wish to reject more often than a small proportion of time is sometimes referred to as a good Lot. Usually,

$$P\ (rejecting\ a\ lot\ of\ quality\ p_1\ )\ =\ 0.05$$

$$P(accepting\ a\ lot\ of\ quality\ p_1)\ =\ 0.95$$

p_1 is known as the 'Acceptance Quality Level' and a lot of this quality is considered as satisfactory by the consumer.

Lot Tolerance Proportion or Percentage Defective (LTPD). The lot tolerance proportion defective, usually denoted by p_t, is the lot quality which is considered to be bad by the consumer. The consumer is not willing to accept Lots having proportioned effective p_t or greater. $100\ p_t$ is called Lot Tolerance Percentage Defective. In other words, this is the quality level which the consumer regards as rejectable and is usually abbreviated as R.Q.L. (Rejecting Quality Level). A lot of quality p_t stands to be accepted some arbitrary and small fraction of time, use only 10%.

Consumer's Risk. Any sampling scheme would involve certain risk on the part of the consumer, in the sense that he has to accept certain percentage of undesirably bad lots, that is lots of quality p_t or greater fraction defective. More precisely, the probability of accepting a lot with fraction defective p_t is termed as consumers risk and is written, as P_c. Usually it is denoted by β. This is taken by Dodge and Romig as 10% or 0.10.

$$consumer's\, risk = P_c = P[Accepting\ a\ lot\ of\ quality\ p_t] = \beta$$

Producer's Risk: the producer has also to face the situation that some good lots will be rejected. He might demand adequate protection against such contingencies (आकस्मिक व्यय) happening too frequently just as the consumer can claim reasonable protection against accepting too many bad lots.

"The probability of rejecting a lot with $100\,\bar{p}$ as the process average percentage defective is called the producer's risk P_p and is usually denoted by α." Thus

$$producer'risk = P_p = P(rejecting\ a\ lot\ of\ quality\ \bar{p}) = \alpha$$

Average Outgoing Quality Limit (**AOQL**): let the producer's fraction defective, i.e., lot quality before inspection be $'p'$. This is known as 'incoming quality'. The fraction defective of the lot after inspection is known as 'outgoing quality' of the lot. The expected fraction defective remaining in the lot after the application of sampling inspection plans is termed as average outgoing quality (AOQ). Obviously, it is a function of the incoming quality p.

Average Sampling Number (ASN) and Average Amount of Total Inspection (ATI). The average sample number (ASN) is the expected value of the sample size required for coming to decision about the acceptance or rejection of the lot in an acceptance – rejection sampling plan. Obviously, it is a function of the incoming lot quality p.

On the other hand, the expected number of items inspected per lot to arrive at a decision in an acceptance – rectification (शोधन) sampling plan calling for 100% inspection of the rejected lots is called average amount of total inspection (ATI). Obviously ATI is also a function of the lot quality p.

We observe that

$$ATI = ASN + (Average\ size\ of\ inspection\ of\ the\ remainder\ in\ the\ rejected\ lots)$$

Thus, if the lot is accepted on the basis of the sampling inspection plan then

$$ATI = ASN$$

Otherwise, $ATI > ASN$

In other words, ASN gives the average number of units inspected per accepted lot.

For example, if a single sampling acceptance – rejection plan is used, the number of items inspected from each lot will be the corresponding sample size $n, i.e.$,

$$ASN = n$$

And this will be true, independently of the quality of the submitted lots.

OC Curve: Operating Characteristic (OC) curve of a sampling inspection plan is a graphic representation of the relationship between the probability of acceptance $P_\alpha(p)$ or generally denoted by $L(p)$, for variation in the incoming lot quality 'p' (fraction defective in the lot).

SAMPLING INSPECTION PLANS FOR ATTRIBUTES

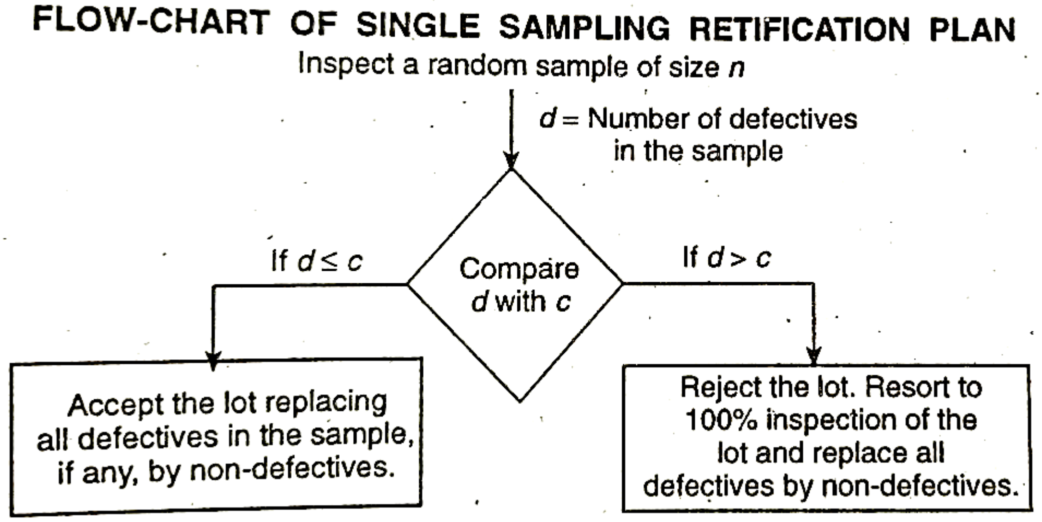
The commonly used sampling inspection plans for attributes and count of defects are:

(i) Single sampling plan,
(ii) Doubles sampling plan, and
(iii) Sequential sampling plan.

1. **Single Sampling Plan**. If the decision about accepting or rejecting a lot is taken on the basis of one sample only, the acceptance plan is described as single sampling plan. It is completely specified by three numbers N, n and c, where:

   $N =$ is the lot size,
   $n =$ is the sample size, and
   $c =$ is the acceptance number, i.e., maximum allowable number of defectives in the sample.

The following flow chart elegantly displays the single sampling plan.



The single sampling plan may be described as follows:

1. Select a random sample of size n from a lot size N.
2. Inspect all the articles included in the sample. Let d be the number of defectives in the sample.
3. If $d \leq c$, accept the lot, replacing defectives pieces found in the sample by non-defective (standard) ones.
4. If $d > c$, reject the lot. In this case we inspect the entire lot and replace all the defective pieces by standard ones.

2. **Double Sampling Plan**. In this method, a second sample is permitted if the first sample fails i.e., if the data from the first sample is not conclusive on either side (about accepting or rejecting the lot), then a definite decision is taken on the basis of the second sample. Such a rectifying double sampling inspection plan for attributes is briefly described below:

   N = lot size from which samples are taken

   n_1 = size of sample 1

   n_1 = size of sample 2

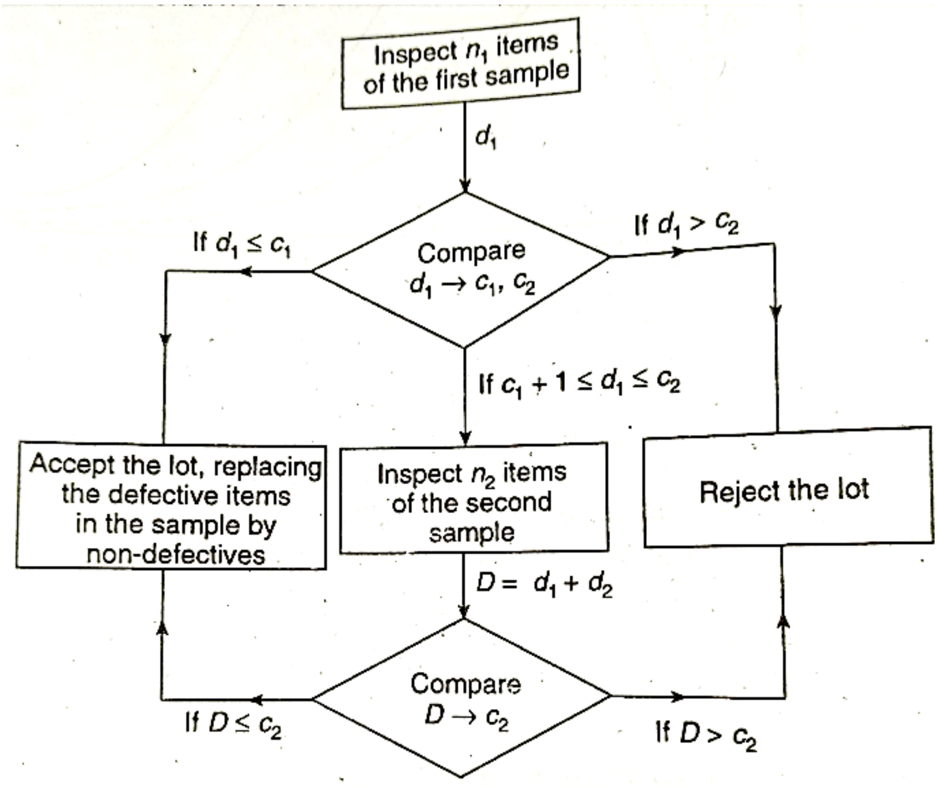
   c_1 = Acceptance number for first sample, i.e., maximum permissible number of defectives in first sample if lot is to be accepted without taking another sample.

   c_2 = Acceptance number for samples 1 and 2 combined, i.e., maximum permissible number of defectives in combined samples if lot is to be accepted.

$d_1 =$ Number of defectives in sample 1.

$d_2 =$ Number of defectives in sample 2.

The sampling procedure is displayed in the following flow chart



Procedure:

1. Take a sample of size n_1 from the lot of size N.
2. If $d_1 \leq c_1$, accept the lot replacing the defectives found in the sample by non – defectives.
3. If $d_1 > c_2$, reject the lot. Detail the lot 100%, replacing all bad items by good ones.
4. If $c_1 + 1 \leq d_1 \leq c_2$, take a second sample of size n_2 from the remaining lot.
5. If $d_1 + d_2 \leq c_2$, accept the lot, replacing defective items by standard ones.
6. If $d_1 + d_2 > c_2$, reject the whole lot. Inspect the rejected lot 100%, replacing all the defective items by good one.

Single Sampling vs Double Sampling Plans

1. Single sampling plans are simple, easy to design and administer, and since each sample can be plotted on a control chart, maximum information concerning the lot can be obtained.

2. It seems unfair to reject a lot on the basis of one sample alone and appears more convincing to say that the lot was rejected after inspecting two samples.
3. The double sampling schemes being more complicated and the necessity of inspecting second sample being unpredictable, the unit cost of inspection for a double sampling procedure may be higher than foe single sampling procedure.

Sequential Sampling Plan. In sequential sampling, items are examined one at a time and after each item inspected one of the three decisions, viz., to accept the lot, to reject the lot or to continue sampling is taken. This scheme provides for a minimum amount of inspection.

Sequential Probability Ratio Test: A sampling plan satisfying the condition that the probability of rejecting the lot does not exceed α whenever $p \le p_0$ and the probability of accepting lots does not exceed β whenever $p \ge p_0$ is given by the sequential probability ratio test (SPRT), pioneered by Dr. Abraham Wald, for testing the hypothesis $H_0: p = p_0$ against the hypothesis $H_1: p = p_1$.

Here if we take AQL = p_0 ; LTPD $= 100\, p_1$ or lot tolerance fraction defective p_1: $\alpha =$ probability of type I error and $\beta =$ probability of type II error then α and β are the maximum producer's and consumer's risk respectively. SPRT is defined as follows:

Let the result of the inspection of the i[th] unit be denoted by a Bernoulli variate X_i, i.e.,

$$X_i = 1, \qquad if\ ith\ item\ inspected\ is\ found\ to\ be\ defective$$
$$= 0, \quad otherwise.$$

For the incoming lot quality 'p', if $f(x, p)$ represents the probability function of X then

$$f(1, p) = p \qquad and \qquad f(0, p) = 1 - p$$

Let p_{1m} and p_{0m} be the probability of getting d_m defectives in the sample $(X_1, X_2, \dots, X_m)$ of size m under H_1 and H_0 respectively. Then the likelihood ratio test λ_m is given by:

$$\lambda_m = \frac{p_{1m}}{p_{0m}} = \frac{\prod_{i=1}^{m} f(x_i, p_1)}{\prod_{i=1}^{m} f(x_i, p_0)}$$

$$= \prod_{i=1}^{m} \frac{f(x_i, p_1)}{f(x_i, p_0)}$$

$$= \frac{{p_1}^{dm}(1 - p_1)^{m-dm}}{{p_0}^{dm}(1 - p_0)^{m-dm}}$$

SPRT is carried out as follows: At each stage of the experiment, at the inspection of the mth unit for each possible integral value of m, we compute λ_m and

(i) If $\lambda_m \geq A$, we terminate the process with rejection of the lot.
(ii) If $\lambda_m \leq B$, we terminate the process with acceptance of the lot.
(iii) If $B < \lambda_m < A$, we continue the sampling by taking an additional observation,

Where A and B are constants determined in terms of $\alpha \ and \ \beta$ are given by

$$A = \frac{(1-\beta)}{\alpha} \quad and \quad B = \frac{\beta}{(1-\alpha)}$$

For computational point of view, it would be much easier to deal with $\log \lambda_m$ rather than with λ_m. Thus SPRT can be restated as:

(i) If $\log \lambda_m \geq \log A$, reject of the lot.
(ii) If $\log \lambda_m \leq \log B$, accept of the lot. And
(iii) If $\log B < \log \lambda_m < \log A$, continue sampling by taking one more observation.