FERTILITY

Stationary population. *A population is said to be stationary if it is of constant size, and constant age and sex composition over time.*

Such a population may be conceived (कल्पना) of under the following conditions.

1. If every year, the number of births is exactly l_0 (say) and is equal to the number of deaths and these are distributed uniformly throughout the year, and
2. If the population is not affected by emigration (प्रवास).

Under the above conditions, in the long run, the population will be of the same size from year to year and will have the same age distribution so that the number of persons between ages x and $(x + 1)$ denoted by L_x will always be same. Thus the columns L_x and T_x of the life table may be interpreted as giving respectively the age distribution and the number of persons with age x or more in a stationary population.

Stable population. Concept of a stable population is due to A. J. Lotka and is very much akin (समान) to that of stationary population.

A population is said to be stable

1. If it has a fixed age and sex distribution,
2. If constant mortality and fertility rates are experienced at each age, and
3. If the population is close to immigration.

In other words, for a stable population the overall rates of births and deaths remain constant and consequently such a population increase at a constant rate, thus supporting the Malthus law 3 (compound interest law) of population growth.

In a stable population, mortality and fertility rates are constant but need not to be equal. In particular, in a stable population, if the constant over her birth and death rates are equal then the population size remains fixed and in this case stable population becomes a stationary.

Lotka and Dublin's model of for stable population

Assumptions. Lotka and s Dublin's stable population analysis is based on the following assumptions:

(i) The fertility (birth) rates are independent of time (t)
(ii) The mortality (death) rates are independent of time (t)
(iii) The age distribution between the ages x to $(x+\delta x)$ is independent of t.
(iv) The population is close to migration.
(v) The analysis is done with respect to female population (cohorts) only.

Notations:

$P(t) =$ Size of the population at any time t

$C(x,t)\,\delta x =$ The proportion of population in age interval $(x, x+\delta x)$ at time t.

$B(t) =$ Total number of births at time t

$P(x) =$ Probability that a female child (born alive) will survive upto age x under the given mortality conditions.

$i(x)\,\delta x =$ Probability that a women aged x will give birth to a female child in the age interval $(x, x+\delta x)$ under the given fertility conditions, (independent of time).

Then,

$P(t)C(x,t)\,\delta x =$ Population in the age group $x\ to\ (x+\delta x)$ at time t …(1)

$B(t-x)\,p(x)\,\delta x =$ A group of persons born $(t-x)$ years ago will survive upto age x or age interval $(x, x+\delta x)$ at time t.

= Number of persons or population in the age group $(x, x+\delta x)$ at time t. ………….(2)

From equation (1) and (2) we get the identity

$$P(t)C(x,t)\,\delta x = B(t-x)\,p(x)\,\delta x$$

Multiply by $i(x)$ both sides and integrating with respect to x over the interval $(0,\infty)$, we get

$$\int_0^\infty P(t)C(x,t)\,i(x)\,dx = \int_0^\infty B(t-x)\,p(x)\,i(x)\,dx \quad \ldots..(3)$$

Taking R.H.S. of equation (3)

$$R.H.S. = \int_0^\infty B(t-x)\,p(x)\,i(x)\,dx$$

$$= \int_0^\infty \begin{bmatrix} A\ group\ of\ women\ born\ (t-x)\ years\ ago\ will\ survive\ upto\ age\ x \\ and\ give\ birth\ to\ a\ female\ child\ in\ the\ age\ interval\ (x, x+\delta x) \end{bmatrix}$$

$$= A\ group\ of\ women\ born\ (t-x)\ years\ ago\ would\ replace\ themselves$$
$$by\ future\ mothers\ after\ a\ period\ of\ x\ years\ i.e.\ at\ time\ t.$$

$= Number\ of\ births\ at\ time\ t$

$= B(t)$

$$\Rightarrow \qquad B(t) = \int_0^\infty B(t-x)\, p(x)\, i(x)\, dx \qquad \dots .(4)$$

This is an integral equation which cannot be solved easily, Lotka and Dublin suggested a trial solution of the form

$$B(t) = \sum_{n=0}^{\infty} Q_n\, e^{r_n t}$$

Where $Q_0, Q_1, Q_2, \dots \dots$ Are the sizes of the population at the beginning of each year under consideration (treated here as constants) and $r_0, r, r_2, \dots \dots$ are the corresponding growth rates of population over time.

Substituting this in equation (4)

$$\sum_{n=0}^{\infty} Q_n\, e^{r_n t} = \int_0^\infty \left[\sum_{n=0}^{\infty} Q_n\, e^{r_n (t-x)}\right] p(x)\, i(x)\, dx$$

$$\Rightarrow \qquad \sum_{n=0}^{\infty} Q_n\, e^{r_n t} = \sum_{n=0}^{\infty} \left[Q_n e^{r_n t} \int_0^\infty Q_n e^{-r_n x}\ \ p(x)\, i(x)\, dx\right]$$

Which will be true if and only if

$$\int_0^\infty e^{-r_n x}\ \ p(x)\, i(x)\, dx = 1\ ; \qquad n = 0,1,2, \dots$$

$\Rightarrow$ $r_0, r, r_2, \dots \dots$ are the roots of the equation $\int_0^\infty e^{-r\,x}\ \ p(x)\, i(x)\, dx = 1$ .....(5)

This is known as Lotka's integral equation.

Lotka's integral equation can be rewritten as

$$\int_0^\infty e^{-r\,x}\ \ \emptyset\,(x)\,dx = 1$$

Where, $\emptyset\,(x) = \ p(x)\,i(x) =$ Net maternity function.

Lotka proved that: "Of all the infinite number of roots of the integral equation (5), only one root is real and the remaining all other roots are complex. Also, the only real root is the most dominant root of Lotka's integral equation and is supposed to ultimately reflect the changes in the population, i.e., it can be regarded as a measure of the growth rate in the population."

Central mortality rate. The central mortality or death rate is the probability that a person whose exact age is not known but lies in between x and $(x+1)$ will die within one year following the attainment of that age. It is denoted by m_x and is given by the expression:

$$m_x = \frac{Number\ of\ deaths\ within\ age\ interval\ x\ to\ (x+1)}{Average\ l_x\ of\ the\ cohort\ in\ that\ interval}$$

$$m_x = \frac{d_x}{L_x}$$

$$m_x = \frac{d_x}{l_x - \frac{1}{2}d_x} \qquad \left\{L_x = l_x - \frac{1}{2}d_x\right\}$$

Multiply numerator and denominator by $2/l_x$

$$m_x = \frac{\frac{2d_x}{l_x}}{2 - \frac{d_x}{l_x}}$$

$$m_x = \frac{2q_x}{2 - q_x} \qquad \left\{q_x = \frac{d_x}{l_x}\right\}$$

Force of mortality. So far, we have can find ourselves to the values of l_x for integral values of x. But since deaths occur at all ages and at every fraction of time of the year, $l_x$is continuous function of x. At any age x, the rate of decrease in l_x is given by the expression:

$$\lim_{t \to 0} \frac{l_x - l_{x+t}}{t} = -\lim_{t \to 0} \frac{l_{x+t} - l_x}{t} = -\frac{dl_x}{dx}$$

Where $\frac{dl_x}{dx}$ is the differential coefficient of l_x with respect to x.

The force of mortality age at age x is defined as the ratio of instantaneous rate of decrease in l_x to the value of l_x. It is denoted by μ_x and is given by the expression:

$$\mu_x = -\frac{1}{l_x} \cdot \frac{dl_x}{dx} = -\frac{d}{dx} (\log l_x)$$

it gives 'nominal annual rate of mortality', i.e., the probability of a person of age x exactly dying within the year if the risk of dying is same at every moment of the year as it is during the moment following the attainment of age x.

Fertility (उर्वरता)

In demography, the word fertility is used in relation to the actual production of children or 'occurrence of births, especially like births'. Fertility must be distinguished from fecundity (प्रजनन क्षमता) which refers to the capacity to bear children. In fact, fecundity provides an upper bound for fertility. As a measure of the rate of growth of population various fertility rates are computed.

In the following section we shall discuss briefly some of the important rates which are usually computed to have an idea about the fertility in the relevant section of the population.

Crude birth rate (C.B.R.). This is the simplest of all the measures of fertility and consists in relating the number of live births to the total population. This provides an index of the relative speed at which additions are being made through of child birth. The fertility pattern of the above mentioned measure is given by crude birth rate (C.B.R.) defined as follows:

$$C.B.R. = \frac{B^t}{P^t} \times k$$

Where,

$B^t =$ Total number of live births in the given region or locality during a given period, say t.

$P^t =$ Total population of the given region during the period t.

$k =$ A constant, usually 1000.

Merits

1. It is simple, easy to calculate and readily comprehensible.
2. It is based only on the number of births (B^t) and the total size of population (P^t) and does not necessitate the knowledge of these figures for different sections of community or the population.

Demerits

1. The crude birth rate, through simple, is only a crude measure of fertility and is unreliable since it completely ignores the age and sex distribution of the population.
2. C.B.R. is not a probability ratio, since the whole population (P^t) cannot be regarded as exposed to the risk of producing children. In fact, only the females and only those between the child bearing age group (usually 15 to 49 years) are exposed to the risk.
3. As a consequence of variation of climatic condition in various countries, the child bearing age groups are not identical in all these countries. In tropical countries, the period starts at an apparent at earlier date than in countries with cold weather. Accordingly, crude birth rate does not enable us to compare the fertility situations in different countries.
4. Crude birth rate assumes that women in all the ages have the same fertility, an assumption which is not true since younger women have, in general higher fertility than elderly women. C.B.R. thus gives us an estimate of a heterogeneous figure and is unsuitable for comprehensive studies.
5. The level of crude birth rate is determined by a number of factors such as age and sex distribution of the population, fertility of the population commerce sex ratio, marriage rate, migration, family planning measures and so on. Thus, a relatively high crude birth rate may be observed in a population with favourable age and sex structure even though fertility is low, i.e., a population with a large proportion of the individuals in the age group 15 to 49 years will have a high crude birth rate, other things remaining same.

General Fertility Rate (G.F.R.). This consists in relating the total number of live births to the number of females in the reproductive or child bearing ages and is given by the formula

$$G.F.R. = \frac{B^t}{\sum_{\lambda_1}^{\lambda_2} {}^f P_x} \times k$$

Where,

$B^t =$ Number of live births occurring among the population of a given geographic area during a given period t.

$\sum_{\lambda_1}^{\lambda_2} {}^f P_x =$ Female population in the reproductive age, in the given geographical region during the same time t.

$\lambda_1, \lambda_2 =$ Lower and upper limits of the female child bearing age,

and $k =$ A constant usually 1,000.

Thus, general fertility rate may be defined as the number of babies per k women in the reproductive age group.

Merits

1. General fertility rate is a probability rate since the denominator consists of the entire female population which is exposed to the risk of producing children.
2. G.F.R. reflects the extent to which the female population in the reproductive ages increases the existing population through live births. Obviously, G.F.R. takes into account the sex distribution of the population and also the age structure to a certain extent.

Demerits

1. G.F.R. Gives a heterogeneous figure since it overlooks the age composition of the female population in the childbearing age. Hence it suffers from the drawback of non-comparability in respect of time and country.

Specific Fertility Rate (S.F.R.). The concept of specific fertility rate originated from the fact that fertility is affected by a number of factors such as age, marriage, migration, state or region, etc. The fertility rate computed with respect to any specific factor is called specific fertility rate (S.F.R.) and is defined as:

$$S.F.R. = \frac{\begin{array}{c} Number\ of\ births\ to\ the\ female\ population \\ of\ the\ specified\ section\ in\ a\ given\ period \end{array}}{\begin{array}{c} Total\ number\ of\ female\ population\ in\ the \\ specified\ section \end{array}} \times k$$

Where $k = 1000$, usually.

Age specific fertility rate. In order to overcome the drawback of G.F.R. and get a better idea of the fertility situation prevailing in a community or locality it is necessary to compute the fertility rates for different age groups of reproductive age separately. The fertility rate so computed on the basis of specification with respect to age is called the age specific fertility rate. For its computation, the reproductive span is split into different subgroups and S.F.R. is worked out for each subgroup.

Symbolically, the age specific fertility rate for the age group x to $x + n$, denoted. by ${}_n l_x$, is given by the formula:

$${}_n l_x = \frac{{}_n B_x}{{}_n P_x} \times k$$

${}_n B_x =$ Number of births to the females in the age group $(x, x + n)$

i.e. age $\geq x$ but less than $x + n$ in the given geographical region

during a period t.

${}_n P_x =$ Average female population (i.e., average number of females) of

ages $(x\ to\ x + 1)$ in the given area during the period t.

and $k = 1000$, usually.

In particular, if we take $n = 1$ in the formula we get the so-called annual age-specific fertility rate given by:

$$i_x = \frac{B_n}{P_x} \times k$$

Merits.

1. Age-specific fertility rate is a probability rate. It removes the drawbacks of G.F.R. by taking into account the age composition of the women in the childbearing age group and is the suitable for comprehensive studies.

Demerits.

1. The use of age specific fertility rate for comparing the fertility situations of two regions (for of the same reason for two different periods) is not an easy job.
2. Generally, age-specific fertility rate will be higher for certain age groups and lower for the remaining age groups in one region than the other. it is difficult to say that if the fertility is higher or low in one region as compared to other.

Total Fertility Rate (T.F.R). As already pointed out age specific fertility rate is not of much practical utility. In order to arrive at more practical measure of the population growth, the age specific fertility rates for different groups have to be combined to get together to give a single quantity. A simple technique is to obtain a standardized fertility rate. This leads to total fertility rate (T.F.R.) which is obtained on adding the annual age-specific fertility rates. Thus, symbolically

$$T.F.R. = \sum_{\lambda_1}^{\lambda_2} i_x$$

$$= \sum_{\lambda_1}^{\lambda_2} \frac{B_n}{P_x} \times k$$

Where i_x is annual fertility rate and λ_1 and λ_2 are the lower and upper limits of the female reproductive period.

Thus, T.F.R. gives the number of children born per k $(= 1000$, usually) females in the child bearing age divided into different age groups. Thus T.F.R. for a particular region during a given period may be regarded as an index of the overall fertility conditions operating in that region during the same period.

Usually, $\lambda_1 = 15$ and $\lambda_2 = 49$. Thus, in order to compute T.F.R. we shall have to calculate 34 age specific fertility rates. The arithmetic may be reduced to a great extent by working with age groups, say x to $x + n$, where in general n, the width of the interval may very from one group to the other. In such a case, the T.F.R. is approximately given by the formula:

$$\text{T.F.R.} = \sum_x n\,({}_n i_x)$$

Where summation is taken over different age groups in the reproduction period. In particular, if we deal with queen quinquennial (पंचवार्षिक) age group $(n = 5)$ for each class then

$$\text{T. F. R.} = \sum_x 5\,(\,{}_5i_x) = 5\sum_x {}_5i_x$$

The calculation of T. F. R. are based on quinquennial age group requires only one fifth of the arithmetic as compared to T.F.R. based on a single age groups and from practical point of view it is almost as accurate.

Merits.

1. It allows us to compare the fertility pattern for two different regions or a same region for two different periods.
2. It is also a probability rate.

Demerits.

1. T. F. R. is a hypothetical figure giving the number of children born to a cohort of 1000 females (all born at same time) assuming that
   (i) None of them dies before reaching the age of child bearing age, i.e., all of them live till at least the age of 50 years. and
   (ii) at each age group (in the childbearing ages) they are subject to the fertility condition given by the observed age specific fertility rate.

Measurement of population growth

Having obtained the measure of mortality and fertility our next objective is to find out if the given population has a tendency to increase or decrease or remain stable. fertility rates are inadequate to give us any idea about the rate of population growth since they ignore the sex of the newly born children and their mortality. The population increases through female births. Thus, if a majority of births are those of girls, the population is bound to increase while it will have a downward trend if the majority of births are boys.

Similarly, if we ignore the mortality of the newly born children, we cannot for a correct idea of the rate of growth of the population, since it is possible that a number of female children may die before reaching the reproductive age. In the following sections we shall study some measures of the growth of population under assumption that in future also it is subjected to the current fertility and mortality rates.

Crude Rate of Natural Increase and Pearle's Vital Index.

The simplest measure of the population growth is known as crude rate of natural increase is defined as the difference between the crude birth rate (per thousand) and the crude death rate (per thousand) and is given by:

$$crude\ rate\ of\ natural\ increase = C.B.R. - C.D.R.$$

Since CBR (CDR) gives the proportion by which population increases (decreases) through births (deaths), the formula gives the net increase (or decrease) in population through births and deaths taken together.

Another indicator of population growth based on birth and death taken together is provided by R. Pearle's Vital Index, defined as follows:

$$Pearle's\ Vital\ Index = \frac{Number\ of\ births\ in\ the\ given\ period\ t}{Number\ of\ deaths\ in\ the\ given\ period\ t} \times 100$$

$$= \frac{B^t}{D^t} \times 100$$

Dividing numerator and denominator in formula by the population in given that in the given period t we get

$$Pearle's\ Vital\ Index = \frac{C.B.R.}{C.B.R.} \times 100$$

Both these measures suffer the drawbacks of C.B.R. and C.D.R. and as such are not suitable for comparative studies.

Gross Reproduction Rate (G.R.R.). In order to have a better idea about the rate of population growth, in addition to the age and sex composition of the population we must take into account the sex of the newly born children since it is ultimately the female births who are the potential future mothers and result in an increase in the population. The gross reproduction rate (G.R.R.) is a step in this direction and is defined as the sum of age specific fertility rates calculated from female births for each year of reproductive period.

Symbolically, if ${}^{f}B_x$ is the number of female births to the women of age x during the given period in the given region, then in the usual notations, we have

$$G.R.R. = \sum_{\lambda_1}^{\lambda_2} \frac{{}^{f}B_x}{{}^{f}P_x} \times k$$

$$= \sum_{\lambda_1}^{\lambda_2} {}^{f}i_x$$

Where ${}^{f}i_x = \frac{{}^{f}B_x}{{}^{f}P_x} \times k$

is termed as the female age specific fertility rate and $k = 1000$ usually. More precisely formula gives female gross reproduction rate.

Gross reproduction rate is thus a modified form of total fertility rate and gives the number of females expected to be born to k a newly born daughters if

(i) none of them is subjected to risk of mortality till attaining the age λ_2 the upper limit of the reproductive period, and
(ii) all of them experience, throughout the reproductive period, the current level of fertility as represented by ${}^{f}i_x$.

In other words G.R.R. exhibits the rate at which mothers would be replaced by daughters and the old generation by the new, under the above two assumptions.

Suppose now that instead of annual data, we are given the figures for different age groups of reproductive period. Let ${}_{n}^{f}B_x$ be the number of female babies born to the women, in the age group x to $x + n$ then in the usual notations, we get

$$G.R.R. = \sum_{\lambda_1}^{\lambda_2} n \left(\frac{{}_{n}^{f}B_x}{{}_{n}^{f}P_x}\right) \times k$$

$$= \sum_{\lambda_1}^{\lambda_2} n \left({}_{n}^{f}i_x\right)$$

Where $\left(\frac{{}_{n}^{f}B_x}{{}_{n}^{f}P_x}\right) \times k = {}_{n}^{f}i_x$

is the age specific fertility rate for the age group x to $x + n$ based on female births. In particular for the quinquennial data,

$$G.R.R. = 5 \sum_{\lambda_1}^{\lambda_2} \left({}_{5}^{f}i_x\right)$$

Net Reproduction Rate (N.R.R.). As already pointed out the principal limitation of G.R.R. is that it completely ignores the current mortality and takes into account only the current fertility.

Net Reproduction Rate (N.R.R.) is nothing but growth but gross reproduction rate adjusted for the effects of mortality. According to Benjamin, "N.R.R. measures the extent to which mothers produce female infants who survive to replace them. It measures the extent to which a generation of girl babies survive to reproduce themselves as they pass through the childbearing age group".

Let us now take into consideration the factor of mortality of mothers also in measuring the growth of growth of population. To formulate our ideas mathematically, to start with we construct a life table for females on the basis of age specific death rates for females, ${}^{f}m_x$. The values in the L_x column of the table, denoted by ${}_{n}^{f}L_x$ give the total number of years lived by the cohort of ${}^{f}l_0$ females in the age interval x to $x + n$. In the usual notations let ${}_{n}^{f}B_x$ be the number of female births to the women in the age group x to $x + n$ at any period t (say). Then

$$\frac{{}_{n}^{f}L_x}{{}^{f}l_0} \times {}_{n}^{f}B_x$$

gives the average number of female children that would be survive to the age group x to $x + n$. The quantity

$${}_{n}^{f}\pi_x = \frac{{}_{n}^{f}L_x}{{}^{f}l_0}$$

gives the average life table probability of survival of a female to the age interval x to $x + n$ and is called the survival rate. This implies that out of k newly born female babies $k \times \left({}_{n}^{f}\pi_x\right)$ will enter into the childbearing age interval x to $x + n$

$k \times \left({}_{n}^{f}\pi_{x+n}\right)$ into age group $x + n$ to $x + 2n$ and so on.

Hence instead of multiplying $\left[{}_{n}^{f}B_x \div {}_{n}^{f}P_x\right]$ by k alone as in G.R.R., we multiply it by the factor $k \times \left({}_{n}^{f}\pi_x\right)$ for each age interval x to $x + n$. Finally, a new measure of population growth, known as (female) Net Reproduction Rate (N.R.R.) is given by:

$$N.R.R. = k \sum_{x} n \left[\frac{{}_{n}^{f}B_x}{{}_{n}^{f}P_x} \times {}_{n}^{f}\pi_x\right]$$

From practical point of view, formula can be written as:

$$N.R.R. = k\sum_x\left[n\left({}_n^f i_x\right) \times {}_n^f\pi_x\right]$$

$$= k\sum_x[n(female\ Age\ S.F.R.) \times (Survival\ Factor)]$$

summation being taken over all the age groups of reproductive span.

Que 1. Compute (i) G.F.R., (ii) S.F.R., (iii) T.F.R., and (iv) G.R.R, from the following data. Assume that the proportion of female births is 46.2 percent.

Child bearing age group of females	**Number of women ('000)**	**Number of Births**
15 – 19	**16.0**	**260**
20 – 24	**16.4**	**2244**
25 – 29	**15.8**	**1894**
30 – 34	**15.2**	**1320**
35 – 39	**14.8**	**916**
40 – 44	**15.0**	**280**
44 – 49	**14.5**	**145**

Solution:

Child bearing age group of females	Number of women ('000) ${}_n^f P_x$	Number of Births ${}_nB_x$	Age S.F.R. ${}_n^f i_x = \frac{{}_nB_x}{{}_n^f P_x} \times 1000$
15 – 19	16.0	260	16.25
20 – 24	16.4	2244	136.83
25 – 29	15.8	1894	119.87
30 – 34	15.2	1320	86.84
35 – 39	14.8	916	61.89
40 – 44	15.0	280	18.67
44 – 49	14.5	145	10.00
Total	**107.7**	**7059**	**450.35**

(i) G.F.R.

$$G.F.R. = \frac{\sum_{\lambda_1}^{\lambda_2} {}_nB_x}{\sum_{\lambda_1}^{\lambda_2} {}_n^f P_x} \times 1000$$

$$= \frac{7059}{107700} \times 1000$$
$$= 65.54$$

(ii) Age S.F.R. are in column (iv) of above table for different ages.

(iii) T.F.R.

$$\text{T.F.R.} = 5 \sum_{x} {}_5 i_x$$
$$= 5\,(450.35)$$
$$= 2251.75$$

(iv) G.R.R.

$$G.R.R. = \sum_{\lambda_1}^{\lambda_2} n \left(\frac{{}_n^f B_x}{{}_n^f P_x} \right) \times k$$

Since we are not given ${}_n^f B_x$, number of female births out of total births, but we have the ratio of female births of total births. That is 46.2 %

$$G.R.R. = \frac{number\ of\ female\ births}{total\ number\ of\ births} \times\ T.F.R.$$
$$= \frac{46.2}{100} \times\ 2251.75$$
$$= 1040.3$$

Que 2. Calculate the general fertility rate, total fertility rate and the gross reproduction rate from the following data, assuming that for every 100 girls 106 boys are born.

Age of women	**15 – 19**	**20 – 24**	**25 – 29**	**30 – 34**	**35 – 39**	**40 – 44**	**45 – 49**
Number of women	**212619**	**198732**	**162800**	**145362**	**128109**	**106211**	**86753**
Age SFR (/1000)	**98.0**	**169.6**	**158.2**	**139.7**	**98.6**	**42.8**	**16.9**

Solution: since age specific fertility rate for any age x is given by

$${}_n^f i_x = \frac{{}_n B_x}{{}_n^f P_x} \times 1000$$

$$ {}_nB_x = \frac{{}_n^f i_x \times {}_n^f P_x}{1000} $$

Age group	Number of women ${}_n^f P_x$	Age SFR ${}_n^f i_x$	Number of births ${}_nB_x = \frac{{}_n^f i_x \times {}_n^f P_x}{1000}$
15 – 19	212619	98.0	20837
20 – 24	198732	169.6	33705
25 – 29	162800	158.2	25755
30 – 34	145362	139.7	20307
35 – 39	128109	98.6	12632
40 – 44	106211	42.8	4546
45 – 49	86753	16.9	1466
Total	**1040586**	**723.8**	**119248**

(i) G.F.R.

$$ G.F.R. = \frac{\sum_{\lambda_1}^{\lambda_2} {}_nB_x}{\sum_{\lambda_1}^{\lambda_2} {}_n^f P_x} \times 1000 $$

$$ G.F.R. = \frac{119248}{1040586} \times 1000 $$

$$ G.F.R. = 114.59 $$

(ii) T.F.R.

$$ \text{T.F.R.} = 5 \sum_x {}_5i_x $$

$$ \text{T.F.R.} = 5\ (723.8) $$

$$ \text{T.F.R.} = 3619 $$

(iii) G.R.R.

Since 100 girls are born to 106 boys

$$ G.R.R. = \frac{number\ of\ female\ births}{total\ number\ of\ births} \times T.F.R. $$

$$ G.R.R. = \frac{100}{100 + 106} \times 3619 $$

$$G.R.R. = 1756.79$$

Que 3. From the data given below calculate the gross re – production rate and the net reproduction rate.

Age Group	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40	41 – 45	46 – 50
Number of births/1000 women	150	1500	2000	800	500	200	100
Mortality rate /1000	120	180	150	200	220	230	250

Answer: G.R.R. = 2.52, N.R.R. = 2.07

Que 4. The number of births occurring in a country in a particular year is shown here classified according to the age of mother, together with the female population in each age group of the reproductive period.

Age – Group	Female population	Number of births to mothers
15 – 19	84796	2349
20 – 24	70018	14547
25 – 29	72660	16746
30 – 34	75924	10229
35 – 39	75109	5257
40 – 44	71625	1432
45 – 49	66660	93

The total population of the country during the year was 2285800. with the above data, determine

- **(i) The crude birth rate,**
- **(ii) The general fertility rate,**
- **(iii) The age – specific fertility rate, and**
- **(iv) The total fertility rate.**

Answers: (i) CBR = 22.164 (ii) GFR = 980.33 (iii) Age SFR = 3460.84

(iv) TFR = 1691.53