STATISTICAL QUALITY CONTROL

Quality control is a powerful productivity technique for effective diagnosis of leg of quality (for conformity to settled standard) in any of the materials, process, machines or end products. It is essential that the end products possess the qualities that the consumer expects of them, for the progress of industry depends on the successful marketing of products. Quality control ensures this by insisting on quality specifications all along the line from the arrival of materials through each of their processing to the final delivery of goods.

Quality control, therefore, covers are the factors and processes of production which may be broadly classified as follows:

1. **Quality of materials.** Materials of good quality will result in smooth processing thereby reducing the waste and increasing the output. It will also give better finish to the end products.
2. **Quality of man-power**. Trained and qualified personnel will give increased efficiency due to the better quality production through the application of skill and also reduce production cost and waste.
3. **Quality of machines.** Better quality equipment will result in efficient work due to lack or scarcity of breakdowns and thus reduce the cost of defective.
4. **Quality of management**. A good management is imperative (अनिवार्य) for increase in efficiency, harmony in relations, and growth of business and markets.

BASIS OF STATISTICAL QUALITY CONTROL

The basis of statistical quality control is the degree of 'variability' in the size or the magnitude of a given characteristic of the product. Variation in the quality of manufactured product in the repetitive process in industry is inherent (जन्मजात) and inevitable (अटल). These variations are broadly classified as being due to two causes (i) chance causes, and (ii) assignable causes.

1. Chance causes. Some "stable pattern of variation" or "a constant cause system" is inherent in any particular scheme of production and inspection. This pattern results from many minor causes that behave in a random manner. The variation due to these causes is beyond the control of human hand and cannot be prevented or eliminated under any circumstances. One has got to allow for variation within this stable pattern, usually termed as allowable variation. The range of such variation is known as 'natural tolerance of the process'.

2. Assignable causes. The second type of variation attributed to any production process is due to non – random also called assignable causes and is termed as preventable variation. The assignable causes make creep in at any stage of the process, right from the arrival of the raw materials to the final delivery of goods. Some of the important factors of assignable causes of variation are sub – standard or defective raw materials, new techniques or operations, negligence of the operators, wrong or improper handling of machines, faulty equipment, unskilled or inexperienced technical staff, and so on. These causes can be identified and eliminated and are to be discovered in a production process before it goes wrong, i.e., before the production become defective.

S. No.	**Chance causes of variation**	**Assignable causes of variation**
1.	Consists of many individual causes.	Consist of just a few individual causes.
2.	Any one chance cause results in only a small amount of variation.	Any one assignable cause can result in a large amount of variation.
3.	Chance variation cannot economically be eliminated from a process.	The presence of assignable variation can be detected, and action to eliminate the cause is usually economically justified.
4.	Some typical chance causes of variations are: - Slight vibration of a machine. - Lack of human perfection in reading instruments and settling controls. - Voltage fluctuations and variations in temperature.	Some typical assignable causes of variations are: - Negligence of operators. - Defective raw material. - Faulty equipment. - In proper handling of machines.

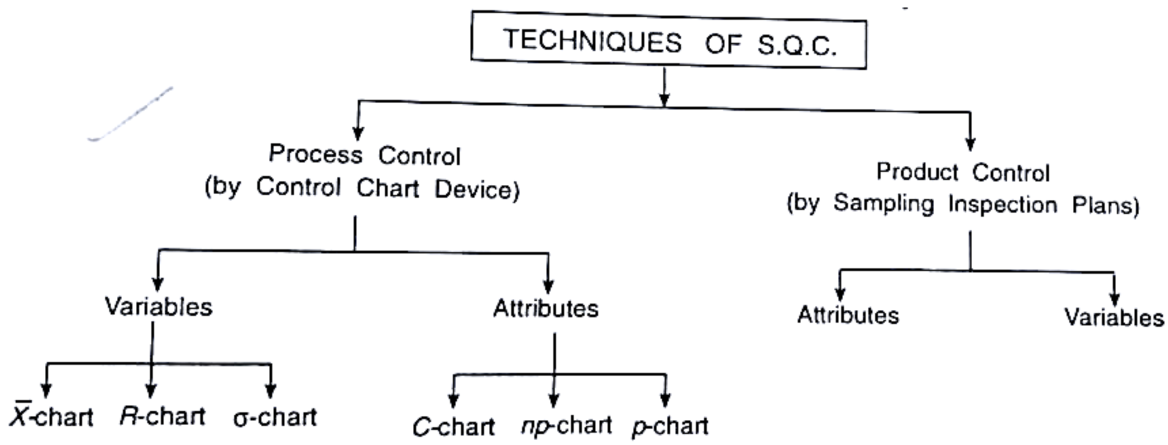
The main purpose of Statistical Quality Control (SQC) is to devise statistical techniques which would help us in separating the assignable causes from the chance causes, thus enabling us to take intermediate remedial (उपचारी) action whenever assignable causes are present. The elimination of assignable causes of erratic (अनियमित) fluctuations (उतार चढ़ाव) is described as bringing a process under control.

"A production process is said to be in a state of statistical control, if it is governed by chance causes alone, in the absence of assignable causes of variation."

Process control and Product control

As already stated the main objective in any production process is to control and maintain a satisfaction satisfactory quality level of the manufactured product so that it confirms to specified quality standard. In other words, we want to ensure that the proportion of defective items in the manufactured product is not too large. This is termed as 'process control' and is achieved through the technique of 'control charts'.

On the other hand, by product control we mean controlling the quality of the product by critical examination at strategic points and this is achieved through 'Sampling Inspection Plants'. Product control aims at guaranteeing a certain quality level to the consumer regardless of what quality level is being maintained by the producer. In other words, it attempts to ensure that the product marketed by sales department does not contain a large number of defective (unsatisfactory) items. Thus, product control is concerned with classification of raw materials, semi – finished goods or finished goods into acceptable or rejectable items.



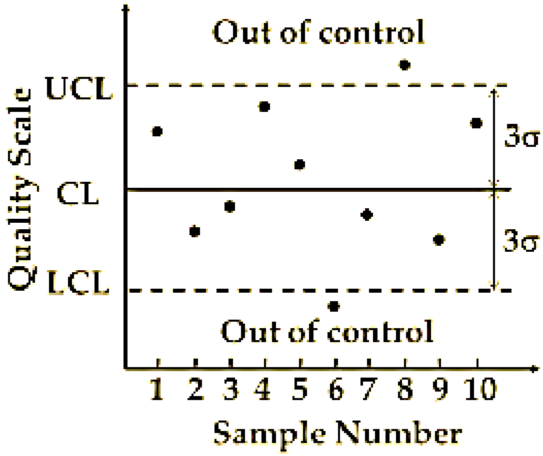
Control Limits, Specification Limits and Tolerance Limits

1. **Control Limits.** These are limits of sampling variation of a statistical measure (mean, range, or fraction-defective) such that if the production process is under control, the values of the measure calculated from different rational sub groups will lie within these limits. Points falling outside control limits indicate that the process is not operating under a system of chance cause, i.e., assignable causes of variation are present, which must be eliminated. Control limits are used in 'control charts'.
2. **Specification Limits.** When an article is proposed to be manufactured, the manufacturers have to decide upon the maximum and the minimum allowable dimensions of some quality characteristics so that the product can be gainfully utilised for which it is intended. If the dimensions are beyond these limits, the product is treated as defective and cannot be used. These maximum and minimum limits of variation of individual items, are known as specification limits.
3. **Tolerance Limits.** These are limits of variation of a quality measure of the product between which at least a specified proportion of the product is expected to lie (with a given probability), provided the process is in a state of statistical quality control. For example, we may claim with a probability of 0.99 that at least 90% of the products will have dimensions between some stated limits. These limits are also known as 'statistical tolerance limits.'

Major parts of a control chart. A control chart generally includes the following four major parts:

1. **Quality scale**. This is a vertical scale. The scale is marked according to the quality characteristics of each sample.
2. **Plotted samples**. The qualities of individual items of a sample are not shown on a control chart. Only the quality of the entire sample represented by a single value (a statistic) is plotted. The single value plotted on the chart is in the form of a dot.
3. **Sample (or Subgroup) Numbers**. The samples plotted on a control chart are numbered individually and consecutively on a horizontal line. The line is usually placed at the bottom of the chart. Generally, 25 sub – groups are used in constructing a control chart.
4. **The Horizontal Lines**. The central line represents the average quality of the samples plotted on the chart. The line above the central line shows the upper control limit (UCL) which is commonly obtained by adding 3σ

to the average (mean). The line below the central line is the lower control limit (LCL) which is obtained by subtracting 3σ from the average. The UCL and LCL are usually drawn as dotted lines and the central line is plotted as a bold line.



3σ Control Limits. 3σ control limits are proposed by Dr. Shewhart for his control charts from various considerations, the main being probabilistic considerations. Consider the statistic $t = t(x_1, x_2, .., x_n)$, a function of the sample observations $x_1, x_2, .., x_n$. Let

$$E(t) = \mu_t \quad and \quad Var(t) = {\sigma_t}^2$$

If the statistic t is normally distributed, then from the fundamental area property of the normal distribution, we have

$$P[\mu_t - 3\sigma_t < t < \mu_t + 3\sigma_t] = 0.9973$$

$$\Rightarrow \qquad P[\ |t - \mu_t| < 3\sigma_t\ ] = 0.9973$$

$$\Rightarrow \qquad P[\ |t - \mu_t| > 3\sigma_t\ ] = 0.0027$$

In other words, the probability that a random value of t goes outside the 3σ limits, is very small. Hence, if t is normally distributed, the limits of variation should between $\mu_t - 3\sigma_t$ and $\mu_t + 3\sigma_t$ which are termed as lower control limit and upper control limit respectively.

If the ith sample, the observed t_i lies between these two limits, there is nothing to worry as in such a case the process is in statistical control. It is only when ant observed t_i falls outside the control limits, it is considered to be a danger signal indicating that some assignable cause has crept in which must be identified and eliminated.

Control Charts for Variables

These charts may be applied to any quality characteristics that is measurable. In order to control a measurable characteristic, we have to exercise control on the measure of location as well as the measure of dispersion. Usually, $\bar{X}$ and R charts are employed to control the mean (location) and standard deviation (dispersion) respectively of the characteristic.

$\bar{X}$ and R Charts. No production process is perfect enough to produce all the items exactly alike. Some amount of variation, in the produced items, is inherent in any production scheme. This variation is the totality of numerous characteristics of the production process viz., Raw material, machine setting and handling, operators etc. As pointed out earlier this variation is the result of (i) chance causes, and (ii) assignable causes.

The control limits in the $\bar{X}$ and R charts are so placed that they reveal the presence or absence of assignable causes of variation in the

(a) Average – mostly related to the machine setting, and
(b) Range – mostly related to negligence on the part of the operator

Steps for $\bar{X}$ and R charts

1. Measurement. Actually the work of a control chart starts first with measurements. Any method of measurement has its own inherent variability. Errors in measurement can enter into the data by:
   (i) The use of faulty instruments,
   (ii) lack of clear-cut definitions of quality characteristics and the method of taking measurements, and
   (iii) lack of experience in the handling for use of the instrument etc.

2. Selection of Samples or Subgroups. In order to make the control chart analysis effective, it is essential to pay due regard to the rational selection of the samples or subgroups. The choice of the sample size n and the frequency of sampling, i.e., The time between the selection of two groups, depend upon the process and no hard and fast rules can be laid down for this purpose.

3. Calculation of $\bar{X}$ and R for each Subgroup. Let $X_{ij}, j = 1,2,..,n$ be the measurements on the ith sample $(i = 1,2,..,k)$. The mean $\bar{X}_i$, the range R_i and the standard deviation s_i for the ith sample are given by:

$$\bar{X}_i = \frac{1}{n}\sum_j X_{ij}\ , \qquad R_i = \max_j X_{ij} - \min_j X_{ij} \quad and$$

$$s_i^2 = \frac{1}{n}\sum_j \left(X_{ij} - \bar{X}_i\right)^2 \qquad (i = 1,2,..,n)$$

The next we find $\bar{\bar{X}}, \bar{R}\ and\ \bar{s}$, the averages of sample means, sample ranges and sample standard deviations, respectively, as follows:

$$\bar{\bar{X}} = \frac{1}{k}\sum_i \bar{X}_i \quad , \quad \bar{R} = \frac{1}{k}\sum_i R_i \ , \quad \bar{s} = \frac{1}{k}\sum_i s_i$$

4. Setting of control limits. It is well known that if σ is the process standard deviation, then the standard error of the sample mean is $\frac{\sigma}{\sqrt{n}}$, where n is the sample size, i.e., $S.E.(\bar{X}_i) = \frac{\sigma}{\sqrt{n}}$,
   Also from the sampling distribution of range, we know that $E(R) = d_2\sigma$ Where d_2 is a constant depending on the sample size. Thus an estimate of σ can be obtained from $\bar{R}$ by the relation:
   $$\bar{R} = d_2\sigma \qquad \Rightarrow \qquad \hat{\sigma} = \frac{\bar{R}}{d_2}$$
   Also $\bar{\bar{X}}$ gives an unbiased estimate of the population mean μ, since
   $$E(\bar{\bar{X}}) = \frac{1}{k}\sum_{i=1}^{k} E(\bar{X}_i) = \frac{1}{k}\sum_{i=1}^{k}\mu = \mu$$

Control limits for $\bar{X}$ chart:

Case I : when standards are given, i.e., both μ and σ are known. The 3σ control limits for $\bar{X}$ charts are given by:

$$E(\bar{X}) \pm 3\ S.E.(\bar{X})$$
$$= \mu \pm 3\frac{\sigma}{\sqrt{n}}$$
$$= \mu \pm A\,\sigma \qquad \left(A = \frac{3}{\sqrt{n}}\right)$$

A is a constant depending on n and its values are tabulated from different values of n.

Case II : If both μ and σ are unknown, then using their estimates $\bar{\bar{X}}$ and $\hat{\sigma}$, we get the 3σ control limits for $\bar{X}$ charts as:

$$\bar{\bar{X}} \pm 3\frac{\bar{R}}{d_2}.\frac{1}{\sqrt{n}} = \bar{\bar{X}} \pm \left(\frac{3}{d_2\sqrt{n}}\right)\bar{R}$$
$$= \bar{\bar{X}} \pm A_2\,\bar{R} \qquad \left(\frac{3}{d_2\sqrt{n}} = A_2\right)$$

A_2 is a constant depending on n and its values are tabulated from different values of n.

Control limits for $\boldsymbol{R}$ chart: R chart is constructed for controlling the variation in the dispersion (variability) of the product. The procedure of constructing $R-$ chart is similar to that for the $\bar{X}$ charts and involves the following steps:

1. Compute the range $R_i = \max_j X_{ij} - \min_j X_{ij}$, $(i = 1\,, 2, .\,., n)$ for each sample.
2. Compute the mean of the sample ranges:
$$\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i = \frac{1}{k}\ (R_1 + R_2 + \ldots + R_k).$$
3. Computation of control limits. The 3σ control limits for R charts are:
$$E(R) \pm 3\ \sigma_R.$$
$$= \bar{R}\ \pm 3\ d_3\ \hat{\sigma}$$
$$= \bar{R}\ \pm 3\ d_3\ \frac{\bar{R}}{d_2}$$
$$= \left(1\ \pm 3\frac{d_3}{d_2}\right)\bar{R}$$
Upper control limit
$$UCL_R = \left(1 + 3\frac{d_3}{d_2}\right)\bar{R} = \ D_4\bar{R}$$
Lower control limit
$$LCL_R = \left(1 - 3\frac{d_3}{d_2}\right)\bar{R} = \ D_3\bar{R}$$

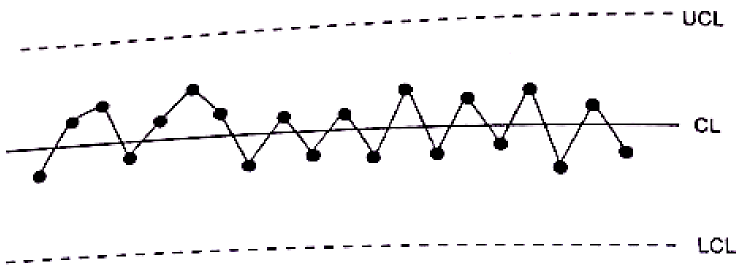
The values D_3 and D_4 depend only on n and tabulated for different values of n.

Criterion for Detecting Lack of Control in $\overline{\boldsymbol{X}}$ and $\boldsymbol{R}$ Chart. As pointed out earlier, the main object of the control chart is to indicate when a process is not in control. The criteria for detecting lake of control are therefore, of fundamental and crucial importance. The pattern of the sample points in a control chart is the key to proper interpretation of the working of the process. The following situation depict lack of control :

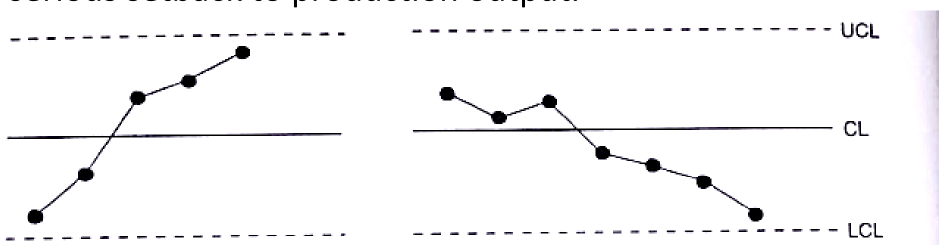
1. A point outside the control limits. A point going outside control limits is a clear indication of the presence of assignable causes of variation which must be searched and corrected. A point outside the control limits may result from an increased dispersion or change in level. Lack

of uniformity may be due to the variation in the quality of raw materials, deficiency in the skill of the operators, loss of alignment among machines coma change of working conditions, etc.

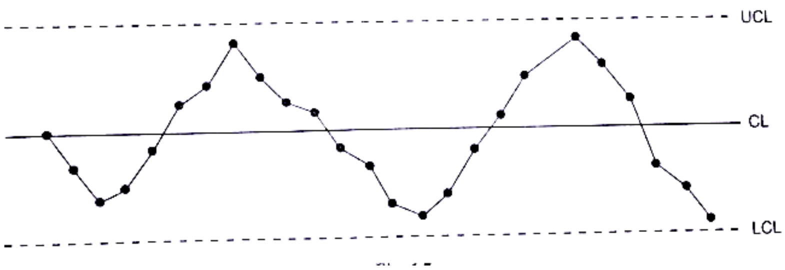
2. A run of seven or more points. Although all the sample points are within control limits, a run of 7 or more points above or below the central line in a in the control chart indicates shift in the process level. On R chat are run of points above the central line is indicative of increase in process spread and therefore represents and undesirable situation.
3. The sample points on $\bar{X}$ and R charts, too close to the central line, exhibit another form of assignable cause. This situation represents systematic differences within samples or subgroups and results from improper selection of samples and biases in measurements.



4. Presence of Trends. The trends exhibited by sample points on the control chart are also an indication of assignable cause. Trend pattern, a phenomenon usually observed in engineering industry, indicates the gradual shift in the process level. Trend may be upward or downward. Tools wear and the need for resetting machines often accounts for such a shift, and it is essential to determine when machine resetting becomes the desirable bearing in mind that too frequent adjustments are a serious setback to production output.



5. Presence of Cycles. In some cases the cyclic pattern of points in the control chart indicates the presence of assignable causes of variation. Such patterns are due to material or/and any mechanical reason.



Que 1. A machine is set to deliver the packets of a given weight. 10 samples of size 5 each were examined and the following results were obtained:

Sample no.	1	2	3	4	5	6	7	8	9	10
Mean	**43**	**49**	**37**	**44**	**45**	**37**	**51**	**46**	**43**	**47**
Range	**5**	**6**	**5**	**7**	**7**	**4**	**8**	**6**	**4**	**6**

Calculate the values for the central line and the control limits for the mean chart and range chart for stop comment on the state of control.

Solu: here $n = 5,\ k = 10$

$$\bar{\bar{X}} = \frac{1}{10}\ (43 + 49 + 37 + 44 + 45 + 37 + 51 + 46 + 43 + 47)$$

$$= \frac{442}{10} = 44.2$$

And

$$\bar{R} = \frac{1}{10}(5 + 6 + 5 + 7 + 7 + 4 + 8 + 6 + 4 + 6)$$

$$\frac{58}{10} = 5.8$$

For $\bar{X}$ chart

$CL_{\bar{X}} = \bar{\bar{X}} = 44.2$

$$UCL_{\bar{X}} = \bar{\bar{X}} + \frac{3\bar{R}}{d_2\sqrt{n}} = 44.2 + \frac{3(5.8)}{2.326\sqrt{5}} = 44.2 + 3.34 = 47.54$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - \frac{3\bar{R}}{d_2\sqrt{n}} = 44.2 - \frac{3(5.8)}{2.326\sqrt{5}} = 44.2 - 3.34 = 40.86$$

For R chart

$CL_R = \bar{R} = 5.8$

$UCL_R = D_4\,\bar{R} = (2.115)\ 5.8 = 12.26$

$LCL_R = D_3\,\bar{R} = (0)\,5.8 = 0$

Comments. Since the sample means corresponding to the sample numbers 2, 3, 6, and 7 are outside the control limits, the process average is out of control. This suggests the presence of assignable causes of variation which should be traced and corrected.

Since none of the sample ranges lies beyond the control limits of the R chart, the process variability is in control.

Since one of the control charts ($\bar{X}$ chart) shows lack of control, the process cannot be regarded in a state of statistical control.

Que 2. Construct a control chart for mean and the range for the following data on the basis of fuses, samples of 5 being taken every hour (each set of 5 has been arranged in ascending order of magnitude). Comment on whether the production seems to be under control, assuming that these are the first data.

42	**42**	**19**	**36**	**42**	**51**	**60**	**18**	**15**	**69**	**64**	**61**
65	**45**	**24**	**54**	**51**	**74**	**60**	**20**	**30**	**109**	**90**	**78**
75	**68**	**80**	**69**	**57**	**75**	**72**	**27**	**39**	**113**	**93**	**94**
78	**72**	**81**	**77**	**59**	**78**	**95**	**42**	**62**	**118**	**109**	**109**
87	**90**	**81**	**84**	**78**	**132**	**138**	**60**	**84**	**153**	**112**	**136**

Solu:

Sample No.	Sample Observation					Total	Sample mean	Sample Range
1	42	65	75	78	87	347	69.4	45
2	42	45	68	72	90	317	63.4	48
3	19	24	80	81	81	285	57.0	62
4	36	54	69	77	84	320	64.0	48
5	42	51	57	59	78	287	57.4	36
6	51	74	75	78	132	410	82.0	81
7	60	60	72	95	138	425	85.0	78
8	18	20	27	42	60	167	33.4	42
9	15	30	39	62	84	230	46.0	69
10	69	109	113	118	153	562	112.4	84
11	64	90	93	109	112	468	93.6	48
12	61	78	94	109	136	478	95.6	75
Total							859.2	716

From the above data we get

$$\bar{\bar{X}} = \frac{1}{12}\sum \bar{X}_i = \frac{1}{12}(859.2) = 71.6$$

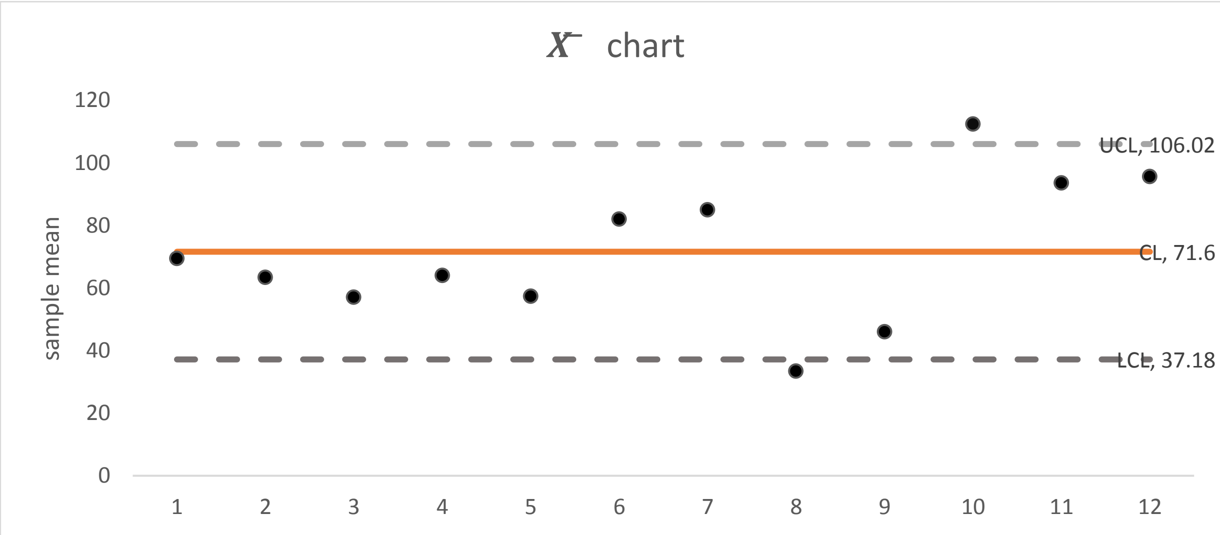
$$\bar{R} = \frac{1}{12}\sum R_i = \frac{1}{12}(716) = 59.66$$

For $\bar{X}$ chart

$$CL_{\bar{X}} = \bar{\bar{X}} = 71.6$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\,\bar{R} = 71.6 + 0.577\,(59.66) = 71.6 + 34.42 = 106.02$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\,\bar{R} = 44.2 - 0.577\,(59.66) = 71.6 - 34.42 = 37.18$$

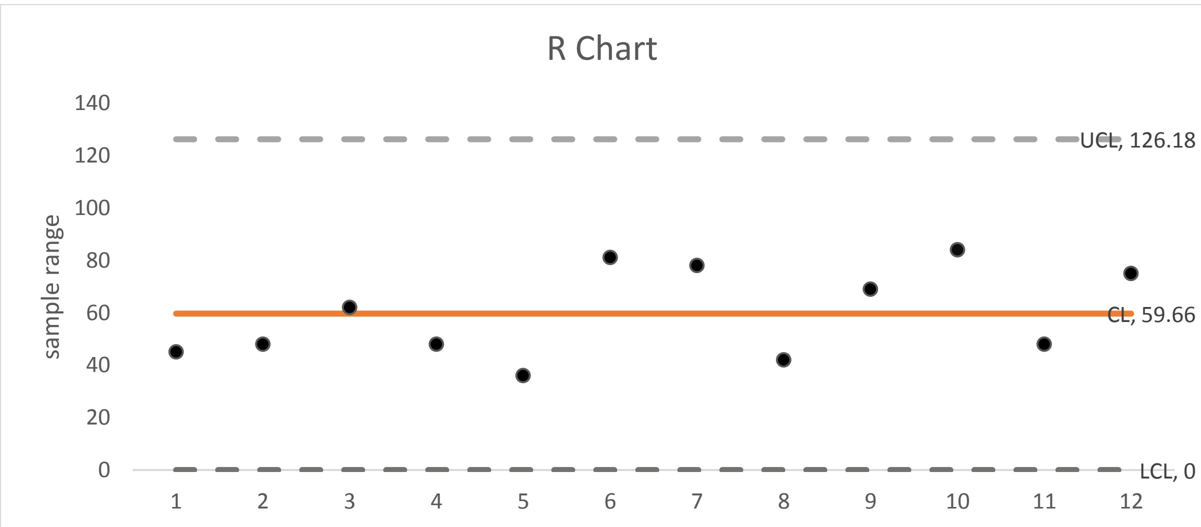


For R chart

$$CL_R = \bar{R} = 59.66$$

$$UCL_R = D_4\,\bar{R} = (2.115)\,59.66 = 126.18$$

$$LCL_R = D_3\,\bar{R} = (0)\,59.66 = 0$$



From the $\bar{X}$ chart, clearly the process average is out of control since the points corresponding to 8^{th} and 10^{th} samples lie outside the control limits.

In R chart, since all the sample points fall within control limits, this chart shows that process variability is in control.

The process cannot be regarded to be in statistical control since $\bar{X}$ chart shows lack of control.

CONTROL CHART FOR ATTRIBUTES

In spite of wide applications of $\bar{X}$ and R (or σ)charts as a powerful tool of diagnosis of sources of trouble in a production process, their use is restricted because of the following limitations:

1. They are charts for variables only, i.e., For quality characteristics which can be measured and expressed in numbers.
2. In certain situation they are impracticable and an – economical, e.g., If the number of measurable characteristics, each of which could be a possible candidate for $\bar{X}$ and R charts, is too large say 30000 or so then obviously there can't be 30,000 control charts.

As an alternative to $\bar{X}$ and R charts, we have the control chart for attributes which can be used for quality characteristics:

(i) which can be observed only as attributes by classifying an item as defective or non – defective i.e., confirming to specifications or not, and

(ii) which are actually observed as attributes even though they could be measured as variables.

There are two control charts for attributes:

(a) Control chart for fraction defective ($p -$ chart) or the number of defective (np or d chart).

(b) Control chart for the number of defects per unit (c chart).

Control Chart for Fraction Defective (p chart). While dealing with the attributes, a process will be adjudged in statistical control if all the samples or subgroups are ascertained (सुनिश्चित करना) to have the same population proportion P.

If $'d'$ is the number of defectives in a sample of size n, then the sample proportion defective is $p = d/n$ hence, d is a binomial variate with parameters n and P.

$$E(d) = nP \qquad and \qquad Var(d) = nPQ, \qquad (P + Q = 1)$$

Thus

$$E(p) = E\left(\frac{d}{n}\right) = \frac{1}{n}E(d) = \frac{nP}{n} = P \qquad and$$

$$Var(p) = Var\left(\frac{d}{n}\right) = \frac{1}{n^2}\,Var(d) = \frac{nPQ}{n^2} = = \frac{PQ}{n}$$

Thus, the $3 - \sigma$ control limits $p -$ chart are given by:

$$E(p) \pm 3\,S.E.(p) = P \pm 3\sqrt{\frac{PQ}{n}} = P \pm A\sqrt{PQ}$$

Where $A = 3/\sqrt{n}$ has been tabulated for different values of n.

Case I : standards specified. If P' is the given or known value of P, then

$$UCL_P = P' + A\sqrt{P'(1 - P')}$$

$$LCL_P = P' - A\sqrt{P'(1 - P')}$$

$$CL_P = P'$$

Case II : standards not specified. Let d_i be the number of defectives and p_i the fraction defective for the ith sample $(i = 1, 2, .., n)$ of size n_i. Then the population proportion P is estimated by the statistics $\bar{p}$ given by:

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{\sum n_i p_i}{\sum n_i}$$

Here $\bar{p}$ is an unbiased estimate of P, since

In this case

$$UCL_P = \bar{p} + A\sqrt{\bar{p}\,(1 - \bar{p})}$$

$$LCL_P = \bar{p} - A\sqrt{\bar{p}\,(1 - \bar{p})}$$

$$CL_P = \bar{p}$$

Control Chart for Number of Defectives (np chart). If instead of p, the sample proportion defective, we use d, the number of defectives in the sample, then the 3σ control limits for d – chart (or d – chart) are given by:

$$E(d)\ \pm 3\ S.E.(d)$$

$$= nP\ \pm 3\sqrt{nP\ (1-P)}$$

Case I : standards specified. If P' is the given or known value of P, then

$$UCL_{np} = nP' + A\sqrt{nP'(1-P')}$$

$$LCL_{nP} = nP' - A\sqrt{nP'(1-P')}$$

$$CL_P = nP'$$

Case II : standards not specified. using $\bar{p}$ as an estimate of P, we get

$$UCL_{nP} = n\bar{p}\ + A\sqrt{n\bar{p}\ (1-\bar{p}\ )}$$

$$LCL_P = n\bar{p}\ - A\sqrt{n\bar{p}\ (1-\bar{p}\ )}$$

$$CL_P = n\bar{p}$$

Since p cannot be negative, if LCL comes out to be negative, then its value is taken to be zero.

Que 3. The following are the figures of defectives in 22 lots each containing 2000 rubber belts:

425	**430**	**216**	**341**	**225**	**322**	**280**	**306**	**337**	**305**	**356**
402	**216**	**264**	**126**	**409**	**193**	**326**	**280**	**389**	**451**	**420**

Draw control chart for fraction defective and comment on the state of control of the process.

Solution: Here we have a fixed sample size $n = 2000$ for each lot. If d_i and p_i are respectively the number of defectives and the sample fraction defective for the ith lot, then

$$p_i = \frac{d_i}{2000}, \qquad (i = 1,2,..,22)$$

S. No.	d	$p = \frac{d}{2000}$	S. No.	d	$p = \frac{d}{2000}$

1.	425	0.2125	12.	402	0.2010
2.	430	0.2150	13.	216	0.1080
3.	216	0.1080	14.	264	0.1320
4.	341	0.1705	15.	126	0.0630
5.	225	0.1125	16.	409	0.2045
6.	322	0.1610	17.	193	0.0965
7.	280	0.1400	18.	326	0.1630
8.	306	0.1530	19.	280	0.1400
9.	337	0.1685	20.	389	0.1945
10.	305	0.1525	21.	451	0.2255
11.	356	0.1780	22.	420	0.2100
Total				**7019**	**3.5095**

$$\bar{p} = \frac{\sum p_i}{k} = \frac{3.5095}{22} = 0.1595$$

$$\bar{q} = 1 - \bar{p} = 1 - 0.1595 = 0.8405$$

3σ control limits for $p -$ chart are given by

$$UCL_P = \bar{p} \ \ + \frac{3}{\sqrt{n}} \sqrt{\bar{p} \ (1 - \bar{p} \ )}$$

$$UCL_P = 0.1595 \ \ + \frac{3}{\sqrt{2000}} \sqrt{0.1595 \, (0.8405)}$$

$$UCL_P = 0.1595 \ \ + (0.0670) \sqrt{0.1340}$$

$$UCL_P = 0.1595 \ \ + (0.0670) \, (0.3661)$$

$$UCL_P = 0.1595 \ \ + 0.0245$$

$$UCL_P = 0.1840$$

$$LCL_P = \bar{p} \ \ - \frac{3}{\sqrt{n}} \sqrt{\bar{p} \ (1 - \bar{p} \ )}$$

$$LCL_P = 0.1595 \ \ - \frac{3}{\sqrt{2000}} \sqrt{0.1595 \ (0.8405)}$$

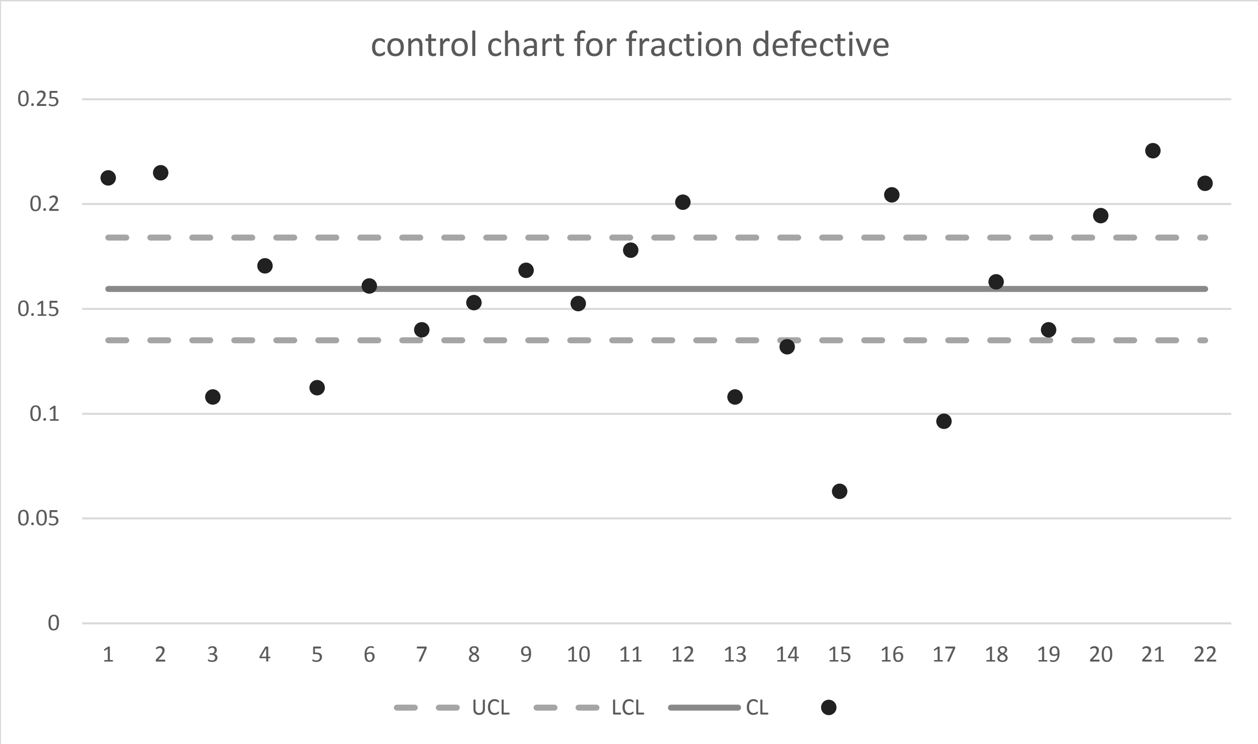
$$LCL_P = 0.1595 \ - (0.0670) \sqrt{0.1340}$$

$$LCL_P = 0.1595 \ - (0.0670) \, (0.3661)$$

$$LCL_P = 0.1595\ - 0.0245$$

$$LCL_P = 0.1350$$

$$CL_P = 0.1595$$



From the p chart, we find that multiple sample points Fall outside the control limits. Hence, the process cannot be regarded in statistical control.

Que 4. From the following inspection results, construct $3\ \sigma$ control limits for p chart:

Date:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
No of defectives	**22**	**40**	**36**	**32**	**42**	**40**	**30**	**44**	**42**	**38**	**70**	**80**	**44**	**22**	**32**
Date:	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
No of defectives	**42**	**20**	**46**	**28**	**36**	**66**	**50**	**46**	**32**	**42**	**46**	**30**	**38**	**40**	**24**

The sub groups, from which the defectives were taken out, were of the same size, i.e., 1000 items each. Without constructing the control chart, comment on the state of control of the process. If the process is out of control, then suggest the revised control limits for future use.

Solution:

$$p_i = \frac{d_i}{1000}$$

Date	No. of defectives	Fraction defective	Date	No. of defectives	Fraction defective
1	22	0.022	16	42	0.042
2	40	0.040	17	20	0.020
3	36	0.036	18	46	0.046
4	32	0.032	19	28	0.028
5	42	0.042	20	36	0.036
6	40	0.040	21	66	0.066
7	30	0.030	22	50	0.050
8	44	0.044	23	46	0.046
9	42	0.042	24	32	0.032
10	38	0.038	25	42	0.042
11	70	0.070	26	46	0.046
12	80	0.080	27	30	0.030
13	44	0.044	28	38	0.038
14	22	0.022	29	40	0.020
15	32	0.032	30	24	0.024
Total	**614**	**0.614**		**586**	**0.556**

$$\bar{p} = \frac{\sum p_i}{k} = \frac{0.614 + 0.586}{30} = \frac{1.200}{30} = 0.040$$

$$\bar{q} = 1 - \bar{p} = 1 - 0.1595 = 0.8405$$

3σ control limits for $p-$chart are given by

$$UCL_P = \bar{p} \ + \frac{3}{\sqrt{n}} \sqrt{\bar{p} \ (1 - \bar{p} \ )}$$

$$UCL_P = 0.04 \ + \frac{3}{\sqrt{1000}} \sqrt{0.04\ (0.96)}$$

$$UCL_P = 0.04 \ + (0.0948) \sqrt{0.0384}$$

$$UCL_P = 0.04 \ + (0.0948)\ (0.1959)$$

$$UCL_P = 0.04 \ + 0.0185$$

$$UCL_P = 0.0585$$

$$LCL_P = \bar{p} - \frac{3}{\sqrt{n}} \sqrt{\bar{p}\ (1 - \bar{p}\ )}$$

$$LCL_P = 0.04 - \frac{3}{\sqrt{1000}} \sqrt{0.04\,(0.96)}$$

$$LCL_P = 0.04 - (0.0948)\sqrt{0.0384}$$

$$LCL_P = 0.04 - (0.0948)\,(0.1959)$$

$$LCL_P = 0.04 - 0.0185$$

$$LCL_P = 0.0215$$

$$CL_P = 0.04$$

We observe that the sample points on 11th, 12th, 17th and 21st September were outside the control limits. Hence the process is not in state of statistical control.

Defect: Any instant of articles lack of conformity to specifications is a defect.

Defective: An article which does not confirm to one or more of the specifications, is termed as defective.

every defective contains one or more of the defects.

Control Chart for Number of Defects per Unit (c – chart). Unlike d or np chart which applies to the number of defectives in a sample, c – chart applies to the number of defects per unit. Sample size for c – chart may be a single unit like a radio or group of units or it may be a unit of fixed time, length, area, etc.

Control Limits for c – chart. In many manufacturing or inspection situations, the sample size n i.e., the area of opportunity is very large (since the opportunities for defects to occur are numerous) and the probability p of the occurrence of a defect in any one spot is very small such that np is finite. In such situations from statistical theory we know that the pattern of variations in data can be represented by Poisson distribution, and consequently 3σ control limits based on Poisson distribution are used. Since for a Poisson distribution, mean and variance are equal, we assume that c is a Poisson variate with parameter λ we get

$$E(c) = \lambda \quad and \quad Var(c) = \lambda$$

Thus $3-\sigma$ control limits for c – chart are given by:

$$UCL_c = E(c) + 3\sqrt{Var(c)} = \lambda + 3\sqrt{\lambda}$$

$$LCL_c = E(c) - 3\sqrt{Var(c)} = \lambda - 3\sqrt{\lambda}$$

$$CL_c = E(c) = \lambda$$

Case (i): When Standards are given: if λ' is the specified value of λ, then

$$UCL_c = \lambda' + 3\sqrt{\lambda'}$$

$$LCL_c = \lambda' - 3\sqrt{\lambda'}$$

$$CL_c = \lambda'$$

Case (i): When Standards are not given: If the value of λ is not known, it is estimated by the mean number of defects per unit. Thus, if c_i is the number of defects observed on the i^{th} $(i = 1, 2, \ldots., k)$ inspected unit, then an estimate of λ is given by:

$$\hat{\lambda} = \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i$$

it can be easily seen that $\bar{c}$ is an unbiased estimate of λ. The control limits, in this case, are given by:

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

$$CL_c = \bar{c}$$

Since c can not be negative, if LCL given by above formulae comes out to be negative, it is regarded as zero.

Application of c – chart

The universal nature of Poisson distribution makes the c – chart technique quite useful. In spite of the limited field of application of c – chart (as compared to $\bar{X}$, and R and p – charts), there do exist situations in industry where c – chart is definitely needed. Some of the representative types of defects to which c - chart can be applied with advantage are:

1. c is the number of imperfections (अपूर्णता) observed in a bale (गट्ठर) of cloth.

2. c is the number of surface defects observed in (i) role of coated paper or a sheet of photographic film, and (ii) a galvanized sheet or a painted, plated or enamelled () surface of given area.
3. c is the number of defects of all types observed in Aircraft sub – assemblies of final assembly.
4. c is the number of breakdowns at weak spots in insulation in a given length of insulated wire subject to a specified test voltage.
5. c is the number of defects observed in stains or blemishes (धब्बा) on a surface.
6. c is the number of soiled packages in a given consignment.
7. c – chart has been applied to sampling acceptance of procedures based on a number defects per unit, for example, in case of inspection of fairly complex assembled units such as T.V. sets, aircraft engines, tanks, machine guns, etc., in which there are very many opportunities for the occurrence of defects of various types and the total number of defects of all types found by inspection is recorded for each unit.
8. c- chart technique can be used with advantage in various fields other than industrial quality control, for example, it has been applied (i) to accident statistics (both of industrial accidents and highway accidents), (ii) in chemical laboratories and in epidemiology.

Que 5. In welding of seams, defect included pinholes, cracks, cold taps, etc. A record was made of the number of defects found in one seam each hour and is given below.

1.12.2005		**2.12.2005**		**3.12.2005**	
8 A.M.	**2**	**8 A.M.**	**5**	**8 A.M.**	**6**
9 A.M.	**4**	**9 A.M.**	**3**	**9 A.M.**	**4**
10 A.M.	**7**	**10 A.M.**	**7**	**10 A.M.**	**3**
11 A.M.	**3**	**11 A.M.**	**11**	**11 A.M.**	**9**
12 P.M.	**1**	**12 P.M.**	**6**	**12 P.M.**	**7**
1 P.M.	**4**	**1 P.M.**	**4**	**1 P.M.**	**4**
2 P.M.	**8**	**2 P.M.**	**9**	**2 P.M.**	**7**
3 P.M.	**9**	**3 P.M.**	**9**	**8 A.M.**	**12**

Draw the control chart for the number of defects and give your comments.

Solution: average number of defects per sample is

$$\bar{c} = \frac{1}{k}\sum c = \frac{1}{24} \times 144 = 6$$

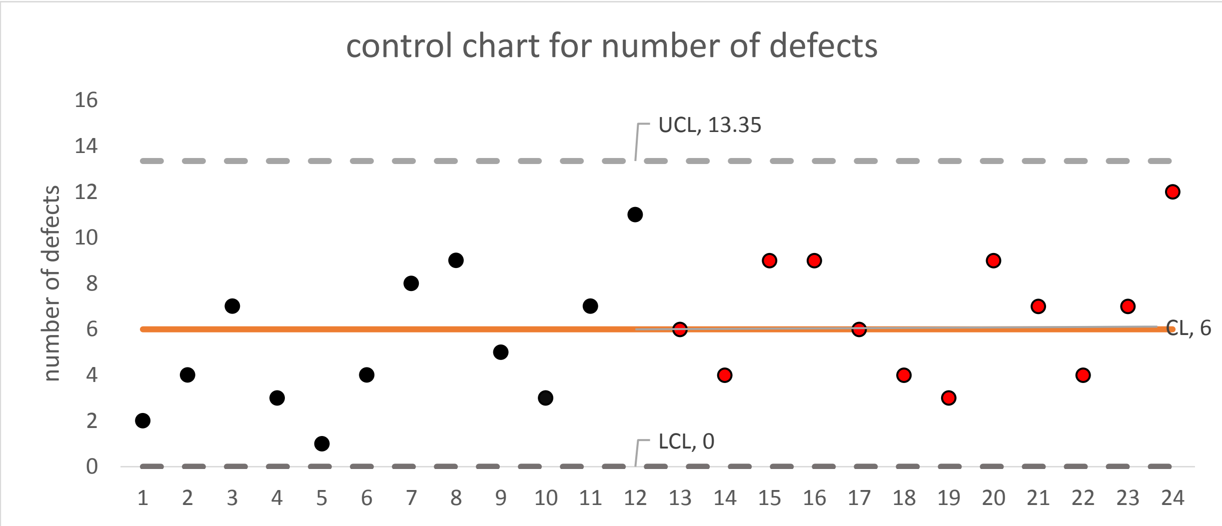
The control limits are given by

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 6 + 3\sqrt{6} = 13.34$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}} = 6 - 3\sqrt{6} = -1.34$$

$$CL_c = 6$$

Since the number of defects cannot be negative, so we consider the lower control limit is zero.



Since no sample point falls outside the control limits, process average may be regarded in state of statistical control.

Que 6. The number of defects in 19 pieces of cloth each of 100 meters length is given below:

1,3,3,1,6,4,3,7,10,2,2,6,4,3,2,1,5,6,4

draw the appropriate chart and say whether the process can be considered to be in control.