Cryptocurrency Volatility-Scaling: does it work?*

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Abstract

Volatility scaling is a technique extensively studied in the stock market but not in the cryptocurrency market. In this study, we investigated whether volatility scaling increases the Sharpe ratio of eight common risk factors in the cryptocurrency market by scaling these factors by their last week's standard deviation. Similar to the stock market, it increases the Sharpe ratio of both the market factor and momentum-related factors. Our results are robust to illiquidity, and our scaled factors generate positive and significant alphas when accounting for the cryptocurrency 3-factor model. However, investors cannot access the improvement in momentum factors due to short-selling constraints.

* This preliminary manuscript provides a brief overview of my work on volatility scaling. It requires further revision and editing.

Introduction

Several studies have shown that volatility scaling enhances the performance of stocks' pricing factors. Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) showed that volatility-managed momentum strategies virtually eliminate momentum crashes and nearly double the Sharpe ratio of the original momentum strategy. Moreira and Muir (2017) extended the analysis to nine equity factors and found that volatility-scaled factors increase the Sharpe ratio and also produce significantly positive alphas relative to their unscaled counterparts. However, Cederburg et al. (2020) considered 103 factors and asserted that, while most scaled factors produce positive alphas in spanning regressions on their unscaled counterparts, these alphas cannot simply translate into higher Sharpe ratios. Therefore, we should rely on Sharpe ratio comparisons to evaluate volatility scaling. Pedro Barroso and Andrew Detzel (2021) claim that after transaction costs, volatility management of asset-pricing factors, besides market return, generally produces zero abnormal returns and significantly reduces Sharpe ratios. In contrast, abnormal returns of the volatility-managed market portfolio are robust to transaction costs. Wang and Yan (2021) showed that strategies scaled by downside volatility exhibit significantly better performance than strategies scaled by total volatility.

Volatility scaling has not been studied in the crypto market. As an initial check of volatility-scaling potential, we focus on the risk-return trade-off in the time-series of cryptocurrency market returns. Specifically, we categorized all weeks into three terciles based on their previous week's volatility. Figure 1 shows the average volatility, average return, and average E(R)/Var(R) for each tercile. Panel A clearly shows a positive relationship between the volatility in two consecutive weeks. However, as shown in panel B, there isn't a clear positive relationship between a week's return and the previous week's volatility. Consequently, the price of risk is high following a low volatility week, and low following a high volatility week. In other words, volatility changes do not necessarily result in proportionate changes in returns. Therefore, employing a strategy that takes greater risk when recent volatility is low and takes less risk when recent volatility is high can lead to a higher return per unit of risk, thereby enhancing the Sharpe ratio. One notable example of such a strategy is volatility-scaling. This strategy, when applied to a portfolio, in each period, scales the risk exposure to that portfolio by the volatility of its returns in the previous period.

For a mean-variance investor, the optimal risk exposure is proportional to the return-to-variance ratio. Panel C illustrates this ratio. While a perfect downtrend would be optimal for benefiting from volatility scaling, the market doesn't exhibit such a trend. Therefore, it appears that volatility scaling would enhance performance by raising exposure during calm periods. However, it would miss out on some positive performance during volatile times.

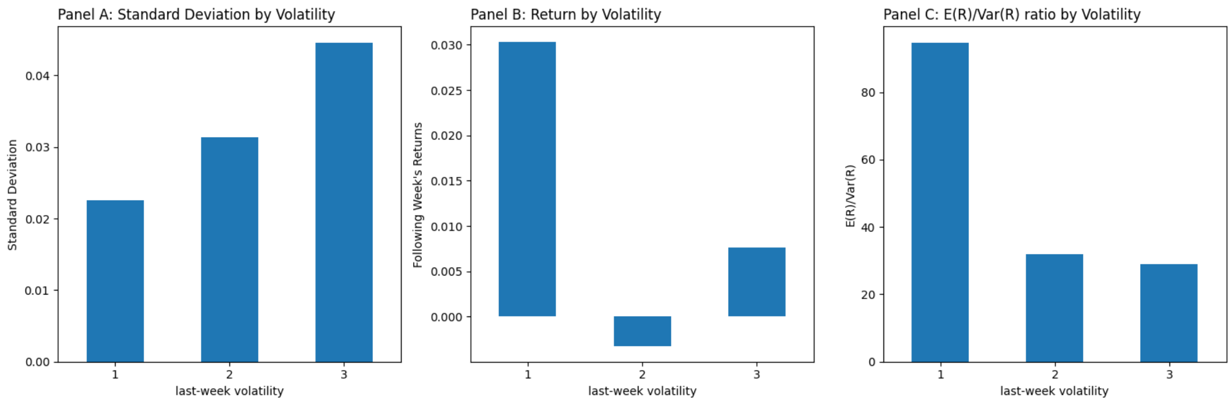

Figure 1

This study investigates whether volatility scaling can increase the Sharpe ratio of cryptocurrency market risk factors, similar to its success in the stock market. In the asset pricing literature, extensive research focused on stock returns, resulting in numerous risk factors. However, the cryptocurrency market is relatively new and has seen limited development in this regard. In an extensive study, Liu et al. (2019) examined 25 cryptocurrency factors and discovered statistically significant returns for 9 factors, including market capitalization, price, maximum price, 1-week momentum, 2-week momentum, 3-week momentum, 4-week momentum, dollar volume, and standard deviation of dollar volume. They also introduced a three-factor model comprising the cryptocurrency market factor (CMKT), cryptocurrency size factor (CSMB), and cryptocurrency momentum factor (CMOM), effectively accounting for excess returns. Our study replicates their 9+3 factor construction from 2016 to 2022 and investigates volatility scaling effects on 8 factors. We exclude 4-week momentum and dollar volume factors because their returns were not significantly different from zero in our sample period. Additionally, we omit the size (CSMB) and momentum (CMOM) factors because they closely resemble MCAP and 3-week momentum, respectively.

The scaled versions of the four factors have higher Sharpe ratios and also yield abnormal returns not attributable to the cryptocurrency 3-factor model. Next, we test the robustness of our results to illiquidity, as the crypto market includes many small and illiquid coins. Then, we check if the positive effect of volatility scaling exists in all years. Finally, we investigate whether these improvements are available for real investors in the cryptocurrency market. Specifically, we consider the effect of volatility scaling on the long portfolio of factors, as the short portfolio involves short selling and is not widely available for many coins.

In conclusion, this study investigates the applicability of volatility scaling in the cryptocurrency market, aiming to enhance risk-adjusted returns. By replicating and validating the 9+3 factor model proposed by Liu et al. (2019) and assessing the effectiveness of volatility scaling, our research contributes to bridging the gap in volatility-management literature specific to cryptocurrencies. The empirical evidence presented supports the thesis that implementing volatility scaling can improve the Sharpe ratio of selected cryptocurrency market risk factors.

The structure of this paper is as follows: In Section I, we present our data description and methodology. In Section II, we outline our empirical findings and engage in a discussion about them. Section III offers robustness checks. Finally, Section IV concludes the paper.

I. Data and Methodology

1) Data:

We obtain historical daily data from Coinmarketcap.com, a reliable source of crypto data, as confirmed by David Vidal-Tomás (2021). We consider all coins with available data on closing prices, market capitalizations, and trading volumes from 2016 to 2022 which are 3061 coins. We use the daily close price of coins to calculate their daily and weekly returns.

2) Factor Building:

We construct both weekly and daily factors in a manner consistent with Liu et al. (2019). Table 1 defines all 3+9 factors.

category	Factor	Definition
3 Factor model	CMKT	Cryptocurrency Market Factor
	CSMB	Cryptocurrency Size Factor
	CMOM	Cryptocurrency Momentum Factor
Size	MCAP	Log last day market capitalization in the portfolio formation week
Size	PRC	Log last day price in the portfolio formation week
Size	MAXDPRC	The maximum price of the portfolio formation week
Momentum	r_1_0	One-week momentum
Momentum	r_2_0	Two-week momentum
Momentum	r_3_0	Three-week momentum
Momentum	r_4_0	Four-week momentum
Volume	PRCVOL	Log average daily volume times price in the portfolio formation week
Volatility	STDPRCVOL	Log standard deviation of dollar volume in the portfolio formation week

Table 1

We consider all coins to calculate the CMKT factor. However, for other factors we exclude coins with a market cap below $1 million at the end of the portfolio formation week. In addition, we exclude coins with zero trading volume. Then for each factor, we sort the returns of individual coins into quintile portfolios based on the value of that factor. We track the return of each portfolio in the following week and calculate the average excess return over the risk-free rate for each portfolio. We then form the long-short strategy based on the difference between the cap-weighted fifth and first quintiles.

We capped coins' market capitalization by the 99^{th} percentile each week before building a cap-weighted portfolio. This eliminates the allocation of extreme weights to a few coins.

Liu et al. (2019) reported significant positive returns for all of these factors during 2014 to 2018. We present the mean weekly returns of these factors from 2016 to 2022 in table 2. This confirms that all of these factors yield significant positive returns except the 4-week momentum factor and the PRCVOL factor. Thus, we excluded them from our study.

	Mean Excess Returns	p_value	Sharpe Ratio
CMKT	0.018996	0.003420	1.112186
CSMB	0.024682	0.004287	1.076869
CMOM	0.018721	0.015915	0.905360
MCAP	0.041653	0.006259	1.033384
PRC	0.094567	0.025423	0.845408
MAXDPRC	0.093206	0.026369	0.839979
r_1_0	0.028463	0.004318	1.077077
r_2_0	0.034083	0.000417	1.337315
r_3_0	0.024696	0.008188	0.996151
r_4_0	0.011255	0.154322	0.530117
PRCVOL	0.056887	0.102969	0.615038
STDPRCVOL	0.062133	0.072363	0.678176

Table 2

3) Volatility scaling:

We construct our scaled portfolios by scaling each factor's return by the inverse of its conditional standard deviation. This approach dynamically adjusts the risk exposure to the portfolio each week, either increasing or decreasing it, based on the realized standard deviation in the previous week. Let f_t be the buy-and-hold factor excess return in week t. We construct its scaled version as follows:

$$f_t^{\sigma} = \frac{C^*}{\sigma_{t-1}} f_t$$

Where c* is a constant and σ_{t-1} is the realized standard deviation of daily returns in the previous week. The scaling parameter, c*, is determined such that the scaled factor has the same unconditional volatility as the original factor over the entire sample period.

II. Empirical results:

1) Sharpe-Ratio Comparison

We present the results of applying volatility scaling to 8 factors in Table 3. As observed, volatility scaling enhances the performance of 4 factors, while its impact on other factors is negative or not statistically significant. The scaled factors maintain the same volatility level as their original versions by design. Consequently, the scaled cryptocurrency market factor (CMKT) achieves 28.9% higher annualized returns on average while taking the same level of risk. In line with the stock market, volatility scaling exhibits the best effect on MOM-related factors.

	R(Scaled) - R(Original)	SR(Original)	SR(Scaled)	SR2-SR1	p-value
CMKT	0.254568	1.112186	1.398816	0.286630	0.010230
MCAP	0.654796	1.037941	1.350347	0.312406	0.045780
PRC	-0.827055	0.847050	0.704864	-0.142186	0.727522
MAXDPRC	-0.414921	0.841634	0.769725	-0.071910	0.829527
r_1_0	0.633466	1.084027	1.545011	0.460984	0.008648
r_2_0	0.586243	1.344522	1.786871	0.442349	0.016927
r_3_0	0.528141	1.003560	1.413235	0.409675	0.257640
STDPRCVOL	0.011036	0.680181	0.682497	0.002316	0.725439

Table 3*

* all p-values are from the Jobson and Korkie (1981) test of the null that $SR(f\sigma) - SR(f) = 0$.

2) Cumulative Return

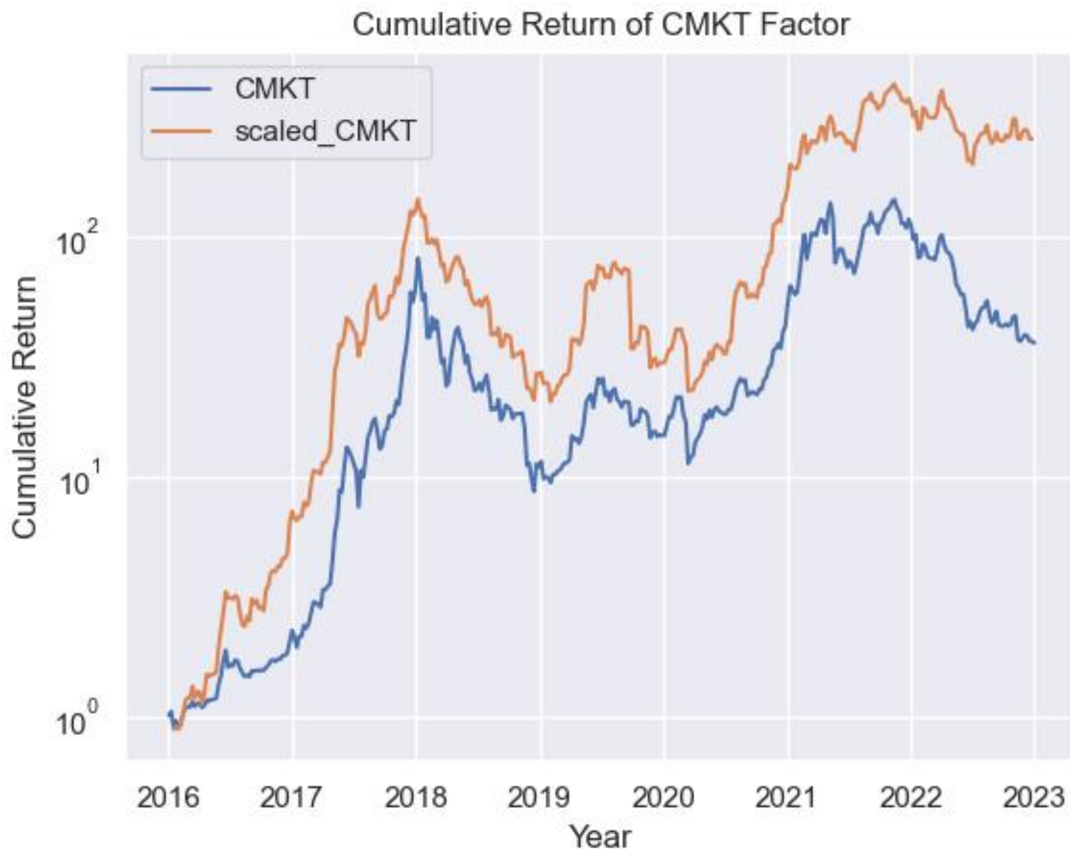

Figure 2

Figure 2 displays the cumulative return of both the CMKT factor and its scaled version. As is evident, if you invested $1 in each at the beginning of 2016, by the end of 2022, you would have approximately $40 and $250, respectively.

3) Control for 3-factor Model:

In this step we examine if our positive findings can be explained by the cryptocurrency 3-factor model. For this purpose, we regress the time-series of each weekly factor return on the 3 factors as independent variables.

$$f_t^{\sigma} \approx \alpha + \beta_1(R_{\mathrm{market},t}) + \beta_2(R_{\mathrm{size},t}) + \beta_3(R_{\mathrm{mom},t})$$

Table 4 demonstrates that all scaled factors yield significant abnormal returns. Therefore, volatility scaling generates benefits not available using the 3-factor model.

	alpha	t_stat
CMKT	0.009345	2.286666
MCAP	0.020872	2.129489
r_1_0	0.033214	3.913703
r_2_0	0.035445	4.752073

Table 4

III. Robustness Checks

1) applying liquidity filter

However, we only consider coins with a market capitalization above $1 million to construct our factors' portfolios; the long leg of the MCAP factor mostly consists of relatively small coins. This raises the concern that the observed positive performance of volatility scaling on the MCAP may stem from illiquid coins. In such a scenario, this positive performance could quickly diminish with even a slight increase in trading volume, preventing investors from realizing those returns. Additionally, the CMKT factor includes numerous small coins and may encounter a similar issue. To ensure the robustness of the beneficial effect of volatility scaling to illiquidity, we employ the Amihud illiquidity measure to exclude data for each coin during its illiquid times. We calculate the daily Amihud measure for each coin using the formula:

$$\mathrm{Amihud}_d = \frac{|\mathrm{ret}_d|}{\mathrm{volume}_d}$$

We also calculate the weekly Amihud measure by averaging its value each week. Subsequently, for each period, we filter out all coins with an Amihud illiquidity measure above 0.0001.* This is equivalent to excluding coins when they yield more than a 1% return for each $100 trading volume.

* Determining an exact threshold to identify illiquid coins is beyond the scope of this paper. Nevertheless, our results remain consistent across a wide range of thresholds.

Next, we compute the returns for both CMKT and MCAP factors, as well as their scaled versions using the filtered dataset. Table 5 illustrates that the primary positive impact of volatility scaling on $MCAP_{long}$ diminishes when our illiquidity filter is applied. This suggests that the risk-return trade-off in the time-series of returns for small coins is weak due to limited trading volume, and it would strengthen as more volume flows into them.

	R(Scaled) - R(Original)	SR(Original)	SR(Scaled)	SR2-SR1	p-value
CMKT	0.282326	1.048836	1.408170	0.359334	0.001303
MCAP	-0.057288	0.966983	0.898975	-0.068008	0.514841
r_1_0	0.590102	1.321566	1.797947	0.476380	0.001215
r_2_0	0.522306	1.632840	2.059866	0.427026	0.031320

Table 5

2) performance in subperiods

In this step, we test whether the positive impact of volatility scaling is pervasive among all years of our sample period.

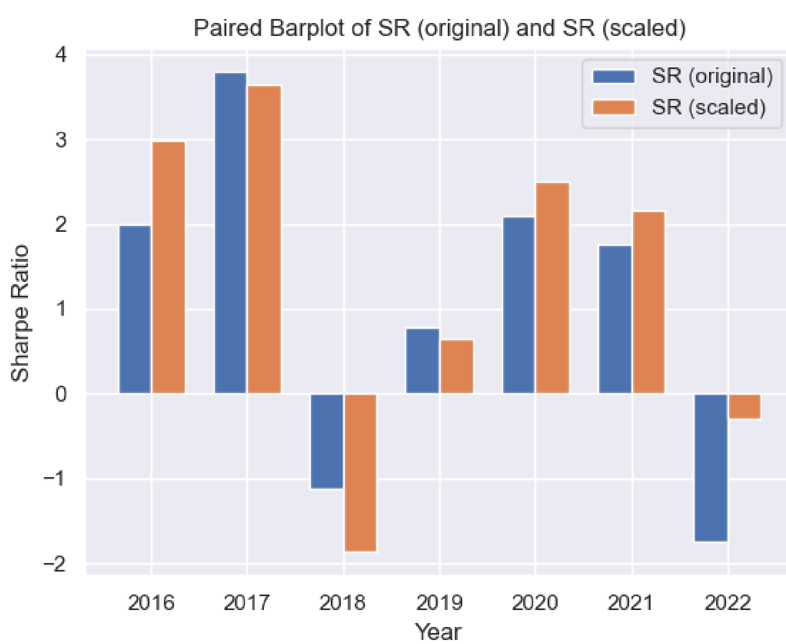

Figure 3

3) long strategies

Up to this point, we have documented the positive impact of volatility scaling on certain factors. Nevertheless, the enhancements observed in the MCAP, r_1_0, and r_2_0 factors may not be accessible to investors. This is because constructing portfolios for these factors involves a short leg, and short sales are not widely available for all coins in the cryptocurrency market. Consequently, we analyze the impact of volatility scaling on long strategies corresponding to these factors and present the results in Table 6. As evident from the results, volatility scaling does not improve the long portfolio's performance for the 1-week and 2-week momentum factors. This implies that volatility scaling benefits on these factors are not achievable. Nevertheless, the long position of the MCAP factor exhibits positive performance when volatility scaling is applied, consistent with its long-short version.

	R(Scaled) - R(Original)	SR2-SR1	p-value
MCAP	0.011602	0.263415	0.028324
r_1_0	-0.005157	-0.187787	0.849646
r_2_0	-0.010940	-0.386354	0.294655

Table 6

IV. Conclusion

In conclusion, this paper investigates the application of volatility scaling in the cryptocurrency market and its impact on the risk-adjusted returns of selected market risk factors. Drawing inspiration from the extensive literature on volatility scaling in the stock market, the study replicates and validates the 9+3 factor model proposed by Liu et al. (2019) for the cryptocurrency market and assesses the effectiveness of volatility scaling on eight factors. The empirical evidence presented in this study contributes to bridging the gap in volatility management literature specific to cryptocurrencies.

The key findings of the research indicate that volatility scaling has the potential to enhance the Sharpe ratio of certain cryptocurrency market risk factors. Similar to its success in the stock market, volatility scaling exhibits positive effects on the market and momentum factors. The scaled versions of these factors demonstrate higher Sharpe ratios and generate abnormal returns not explained by the cryptocurrency 3-factor model. However, it is noteworthy that short-selling constraints limit investors from fully realizing the improvements in momentum factors.

Furthermore, the study addresses concerns related to illiquidity in the cryptocurrency market by applying robustness checks. The positive impact of volatility scaling on the MCAP factor diminishes when accounting for illiquidity. This suggests that the risk-return trade-off for small coins may be weak due to limited trading volume.

The research also explores volatility scaling performance across different years of the sample period. It demonstrates that the positive impact is not uniform across all years. Additionally, the study evaluates the practicality of volatility scaling for real investors in the cryptocurrency market. It emphasizes the limitations imposed by short-selling constraints on the availability of improved performance of momentum factors.

In summary, this paper provides valuable insights into the potential benefits and limitations of volatility scaling in the cryptocurrency market. The findings contribute to the growing body of literature on risk management strategies in the emerging field of cryptocurrencies. Future research may further explore volatility scaling dynamics, considering evolving market conditions and additional risk factors. This will enhance our understanding of its applicability to the rapidly evolving cryptocurrency landscape.

References

Amihud, Y. (2002). "Illiquidity and stock returns: cross-section and time-series effects." *Journal of Financial Markets, 5*(1), 31-56.

Barroso, P., & Detzel, A. (2021). "Do limits to arbitrage explain the benefits of volatility-managed portfolios?" *Journal of Financial Economics, 140*(3), 744-767.

Barroso, P., & Santa-Clara, P. (2015). "Momentum has its moments." *Journal of Financial Economics, 116*(1), 111-120.

Cederburg, S., O'Doherty, M. S., Wang, F., & Yan, X. S. (2020). "On the performance of volatility-managed portfolios." *Journal of Financial Economics, 138*(1), 95-117.

Daniel, K., & Moskowitz, T. J. (2016). "Momentum crashes." *Journal of Financial Economics, 122*(2), 221-247.

Harvey, C. R., Hoyle, E., Korgaonkar, R., Rattray, S., Sargaison, M., & Van Hemert, O. (2018). "The impact of volatility targeting." *The Journal of Portfolio Management, 45*(1), 14-33.

Jobson, J.D. and Korkie, B.M., 1981. Performance hypothesis testing with the Sharpe and Treynor measures. *Journal of Finance*, pp.889-908.

Liu, Y., Tsyvinski, A., & Wu, X. (2022). "Common risk factors in cryptocurrency." *The Journal of Finance, 77*(2), 1133-1177.

Moreira, A., & Muir, T. (2017). "Volatility-managed portfolios." *The Journal of Finance, 72*(4), 1611-1644.

Vidal-Tomás, D. (2022). "Which cryptocurrency data sources should scholars use?" *International Review of Financial Analysis, 81*, 102061.

Wang, F., & Yan, X. S. (2021). "Downside risk and the performance of volatility-managed portfolios." *Journal of Banking & Finance, 131*, 106198.